بسم الله الرحمن الرحيم

اللهم انفعنى بهذا العمل واجعله خالصا لوجهك الكريم

واغفر لكل من قام بمساعدتى فى اكماله واتمامه

اللهم آمين

Introduction to Quantum Field Theory (QFT)

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Outlines

Quantum Field Theory (QFT)

S-matrix

Feynman Amplitude

The decay width

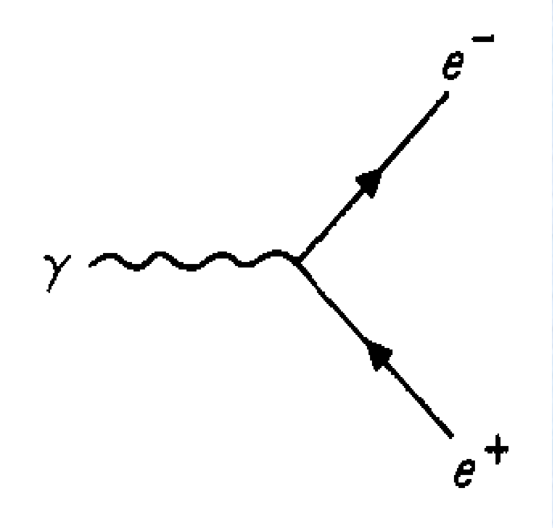
In 1900, Planck **postulated that the process of emission and absorption of radiation by atoms occurs discontinuously in quanta.**

In 1905 , Einstein **concluded that it was not merely the atomic mechanism of emission and absorption of radiation which is quantized, but also, the electromagnetic radiation.**

In 1927, Dirac **maid the foundations of a systematic quantum theory of fields in his famous paper on 'The Quantum Theory of the Emission and Absorption of Radiation'.**

From the quantization of the electromagnetic field one is naturally led to the quantization of any classical field, the quanta of the field being particles with well-defined properties.

The interactions between particles is brought about by other fields whose quanta are other particles. For example, we can think of the interaction between electrons and positrons by photon.



Since the spins and actions of most subatomic particles are comparable to the reduced Planck's constant $\hbar$. So we need *Quantum Mechanics*. Also, the sub-atomic particles often travel with velocities close to that of light c, and hence relativistic effects will also be important.

QM

Relativity

Quantum Field Theory (QFT)

So the Quantum Field Theory is the reconciling of Quantum Mechanics with the special relativity, which implies that particle number is not conserved.

For example if we have three fields interacting. The first field goes to its vacuum state, by annihilation, while the other two fields jump to one of their excitations, so we have two particles as output.

- **Transition amplitude square** is the probability that a certain in-state goes to a certain out-states.
- QFT allows us to calculate the different amplitudes of the different final states and thus their probability to occur.

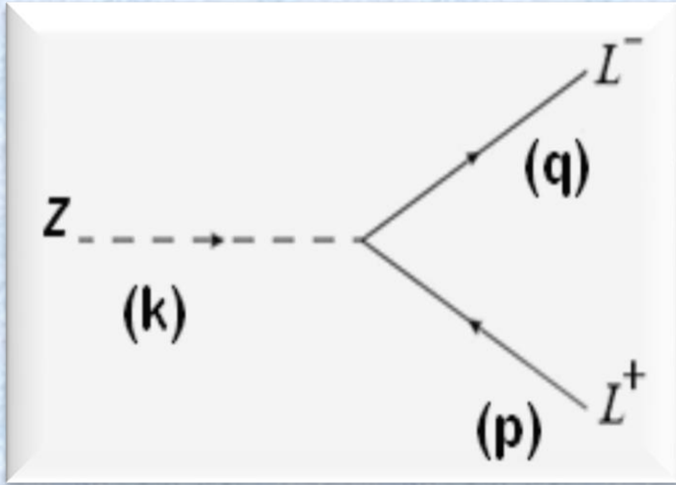
S-matrix relates the final state to the initial one through $|\Phi(\infty)\rangle = S|i\rangle$

- The S-matrix expansion is given by

$$= \sum_{n=0}^{\infty} S^{(n)} = \sum_{n=0}^{\infty} \frac{-i^{n}}{n!} \int d^4x_1 d^4x_2 \ldots d^4x_n T\{\mathcal{H}_I(x_1)\mathcal{H}_I(x_2)\ldots\mathcal{H}_I(x_n)\}$$

$Z \to L^{+} + L^{-}$

$$\mathcal{L}_{\bar{L}LZ} = \frac{g}{2\cos\theta_{\omega}} Z_{\alpha}\, \bar{L}\gamma^{\alpha}(g_{V} - g_{A}\gamma_{5})L$$



$$g_{V} = 2\sin^{2}\theta_{\omega} - ½ \,, \quad g_{A} = -½$$

$$|i\rangle = |Z(j,\mathbf{k}),0,0\rangle = a_{j}^{\dagger}(\boldsymbol{k})|0,0,0\rangle$$

$$|f\rangle = L(s,\boldsymbol{q}),\bar{L}(r,\boldsymbol{p})\rangle = d_{r}^{\dagger}(\boldsymbol{p})C_{s}^{\dagger}(\boldsymbol{q})|0,0,0\rangle$$

$$S_{fi} = \langle f|S|i\rangle$$

$$S_{fi} = \delta_{fi} + \frac{-g}{2\cos\theta_{\omega}}\langle 0,0,0|d_{r}(\boldsymbol{p})C_{s}(\boldsymbol{q})\{-i\int d^{4}x\, Z_{\alpha}{}^{+}\,\bar{L}^{-}\gamma^{\alpha}(g_{V} - g_{A}\gamma_{5})L^{-}\}a_{j}^{\dagger}(\boldsymbol{k})|0,0,0\rangle$$

$$Z_\alpha^{\;+} = \sum_{\grave{\mathbf{k}},j} \frac{1}{\sqrt{2V\omega_{\grave{\mathbf{k}}}}} \varepsilon_{j\alpha}(\grave{\mathbf{k}}) a_j(\grave{\mathbf{k}}) e^{-i\grave{k}x}$$

$$L^{\;-} = \sum_{\grave{\mathrm{r}},\grave{\mathbf{p}}} \sqrt{\frac{m_l}{VE_{\grave{\mathbf{p}}}}} d_{\grave{\mathrm{r}}}^{\dagger}(\grave{\mathbf{p}}) v_{\grave{\mathrm{r}}}(\grave{\mathbf{p}}) e^{i\grave{p}x}$$

$$\overline{L}^{\;-} = \sum_{\grave{\mathrm{s}},\grave{\mathbf{q}}} \sqrt{\frac{m_l}{VE_{\grave{\mathbf{q}}}}} C_{\grave{\mathrm{s}}}^{\dagger}(\grave{\mathbf{q}}) \overline{u}_{\grave{\mathrm{s}}}(\grave{\mathbf{q}}) e^{i\grave{q}x}$$

$$= \delta_{fi} + \frac{ig}{2\cos\theta_\omega} \sum_{\grave{\mathrm{r}},\grave{\mathbf{p}},\grave{\mathrm{s}},\grave{\mathbf{q}},\grave{\mathbf{k}},\grave{j}} \frac{1}{V} \sqrt{\frac{m_l\, m_l}{2VE_{\grave{\mathbf{q}}}E_{\grave{\mathbf{p}}}\omega_{\grave{\mathbf{k}}}}} \varepsilon_{\grave{j}\alpha}(\grave{\mathbf{k}}) \int d^4x e^{i(\grave{k}+\grave{q}-\grave{p})x} \overline{u}_{\grave{\mathrm{s}}}(\grave{\mathbf{q}}) \gamma^\alpha (g_V - g_A\gamma_5) v_{\grave{\mathrm{r}}}(\grave{\mathbf{p}}) \langle 0,0,0| C_r(\boldsymbol{p}) C_s(\boldsymbol{q}) C_{\grave{\mathrm{s}}}^{\dagger}(\grave{\mathbf{q}}) d_{\grave{\mathrm{r}}}^{\dagger}(\grave{\mathbf{p}}) a_{\boldsymbol{j}}(\grave{\mathbf{k}}) a_j^{\dagger}(\boldsymbol{k}) |0,0,0\rangle$$

$$\left[a_l(\mathbf{k}), a_{\boldsymbol{\grave{l}}}^{\dagger}(\grave{\mathbf{k}})\right] = \delta_{\boldsymbol{k},\grave{\mathbf{k}}} \delta_{\boldsymbol{l},\grave{\boldsymbol{l}}}$$

$$\left[C_s(\boldsymbol{q}), C_{\grave{\mathrm{s}}}^{\dagger}(\grave{\mathbf{q}})\right]_+ = \delta_{\boldsymbol{q},\grave{\mathbf{q}}} \delta_{\boldsymbol{s},\grave{\mathrm{s}}}$$

$$\int d^4x e^{i(q+k-p)x} = (2\pi)^4 \delta^{(4)}(q+k-p)$$

$$\therefore S_{fi} = \delta_{fi} + \left[(2\pi)^4\delta^{(4)}(k+q-p)\sqrt{\frac{1}{2V\omega_k}}\sqrt{\frac{m_l}{VE_q}}\sqrt{\frac{m_l}{VE_p}}\right]\frac{ig}{2\cos\theta_\omega}\varepsilon_{j\alpha}(\mathbf{k})\bar{u}_s(\mathbf{q})\gamma^\alpha(g_V - g_A\gamma_5)v_r(\mathbf{p})$$

Comparing this expression with the general expression of the S-matrix element;

$$S_{fi} = \delta_{fi} + \left[(2\pi)^4\delta^{(4)}(p_f - p_i)\prod_{ext}\sqrt{\frac{1}{2V\omega}}\prod_{ext}\sqrt{\frac{m}{VE}}\right]\mathcal{M}$$

The Feynman amplitude will be:

$$\mathcal{M} = \frac{ig}{2\cos\theta_\omega}\varepsilon_{j\alpha}(\mathbf{k})\bar{u}_s(\mathbf{q})\gamma^\alpha(g_V - g_A\gamma_5)v_r(\mathbf{p})$$

$$d\Gamma = (2\pi)^4\delta^4\left(\sum \grave{p}_f - p\right)\frac{1}{2E}\left(\prod_l 2m_l\right)\left(\prod_f \frac{d^3\grave{p}_f}{(2\pi)^3 2\grave{E}_f}\right)|\mathcal{M}|^2$$

- The Feynman amplitude square value is :

$$|\mathcal{M}|^2 = \varepsilon_{j\alpha}(\mathbf{k})\bar{u}_s(\mathbf{q})\{\frac{ig\gamma^\alpha}{2\cos\theta_\omega}(g_V - g_A\gamma_5)\}v_r(\mathbf{p})\varepsilon_{j\beta}(\mathbf{k})v_r^\dagger(\mathbf{p}) * [\frac{-ig}{2\cos\theta_\omega}(g_V - g_A{\gamma_5}^\dagger)\gamma^{\beta\dagger}]\gamma^{0\dagger}u_s(\mathbf{q})$$

Averaging over initial spin states and summating over final spin states

$$\sum_{j=1}^{3}\sum_{r,s=1}^{2}|\mathcal{M}|^2 = \frac{g^2}{12\cos^2\theta_\omega}\sum_{j=1}^{3}\varepsilon_{j\alpha}(\mathbf{k})\varepsilon_{j\beta}(\mathbf{k})\sum_{s=1}^{2}(u_s(\mathbf{q}))_d(\bar{u}_s(\mathbf{q}))_a\{\gamma^\alpha(g_V - g_A\gamma_5)\}_{ab} * \sum_{r=1}^{2}(v_r(\mathbf{p}))_b(\bar{v}_r(\mathbf{p}))_c\{(g_V + g_A\gamma_5)\gamma^\beta\}_{cd}$$

$$\sum_{r=1}^{2} v_r(p)_a\bar{v}_r(p)_b = (\frac{\not{p} - m}{2m})_{ab}$$

$$\sum_{r=1}^{2} u_r(p)_a\bar{u}_r(p)_b = (\frac{\not{p} + m}{2m})_{ab}$$

$$\sum_{r=1}^{3}\varepsilon_{r\mu}(\mathbf{k})\varepsilon_{r\nu}(\mathbf{k}) = -g_{\mu\nu} + \frac{k_\nu k_\mu}{m_z^2}$$

$$\therefore \sum_{j=1}^{3}\sum_{r,s=1}^{2}|\mathcal{M}|^2 = \frac{g^2}{12\cos^2\theta_\omega}\left[-g_{\alpha\beta}+\frac{k_\alpha k_\beta}{m_z^2}\right]\left(\frac{\not{q}+m_l}{2m_l}\right)_{da}\{\gamma^\alpha(g_V-g_A\gamma_5)\}_{ab}\left(\frac{\not{p}-m_l}{2m_l}\right)_{bc} * \{(g_V+g_A\gamma_5)\gamma^\beta\}_{cd}$$

$$\therefore \sum_{j=1}^{3}\sum_{r,s=1}^{2}|\mathcal{M}|^2 = \frac{g^2}{48m_l^2\cos^2\theta_\omega}\left[-g_{\alpha\beta}+\frac{k_\alpha k_\beta}{m_z^2}\right]Tr[(\not{q}+m_l)\{\gamma^\alpha(g_V-g_A\gamma_5)\}(\not{p}-m_l)\{(g_V+g_A\gamma_5)\gamma^\beta\}$$

$$Tr[\gamma^\alpha\gamma^5] = Tr[\gamma^\alpha\gamma^\beta\gamma^5] = Tr[\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^5] = 0$$

$$Tr[\not{p}\,\not{q}] = 4pq$$

$$\begin{aligned} Tr[\gamma^{\alpha_1}\gamma^{\alpha_2}\gamma^{\alpha_3}\gamma^{\alpha_4}\dots\dots\gamma^{\alpha_{2n-1}}\gamma^{\alpha_{2n}}] &= g^{\alpha_1\alpha_2}Tr[\gamma^{\alpha_3}\dots\dots\gamma^{\alpha_{2n}}] \\ &- g^{\alpha_1\alpha_3}Tr[\gamma^{\alpha_2}\gamma^{\alpha_4}\dots\dots\gamma^{\alpha_{2n}}] \\ &+ \cdots g^{\alpha_1\alpha_{2n}}Tr[\gamma^{\alpha_2}\gamma^{\alpha_3}\gamma^{\alpha_4}\dots\dots\gamma^{\alpha_{2n-1}}] \end{aligned}$$

$$= \frac{g^2}{12m_l^2\cos^2\theta_\omega}\{(g_V^2+g_A^2)[2pq]+4m_l^2(g_V^2-g_A^2)+\frac{1}{m_z^2}\{(g_V^2+g_A^2)[2(kp)(kq)-(pq)k^2]-m_l^2(g_V^2-g_A^2)k^2\}$$

$$\therefore d\Gamma = \frac{g^2}{24 m_z \cos^2\theta_\omega}\frac{\delta^4(p+q-k)}{(2\pi)^2}\Big\{(g_V^2+g_A^2)[2pq]+4m_l^2(g_V^2-g_A^2)+\frac{1}{m_z^2}\{(g_V^2+g_A^2)[2(kp)(kq)-(pq)k^2]-m_l^2(g_V^2-g_A^2)k^2\}\Big\}\frac{d^3p\,d^3q}{E_pE_q}$$

$$\Gamma = \frac{g^2}{24 m_z (2\pi)^2\cos^2\theta_\omega}\int d^3\mathbf{p}\,d^3\mathbf{q}\,\frac{\delta^4(p+q-k)}{E_pE_q}\{(g_V^2+g_A^2)[2pq]+4m_l^2(g_V^2-g_A^2)+\frac{1}{m_z^2}\{(g_V^2+g_A^2)[2(kp)(kq)-(pq)k^2]-m_l^2(g_V^2-g_A^2)k^2\} \tag{3.153}$$

$$\therefore \Gamma(Z \longrightarrow L^+ + L^-) = \frac{g^2}{48\pi\, m_z\cos^2\theta_\omega}\sqrt{1-4\frac{m_l^2}{m_z^2}}\{(4\sin^4\theta_\omega - 2\sin^2\theta_\omega + ½)(m_z^2 - m_l^2) + 3m_l^2(4\sin^4\theta_\omega - 2\sin^2\theta_\omega)\}$$

$\Gamma(Z \to e^+ + e^-)$	$\Gamma(Z \to \mu^+ + \mu^-)$
0.083486917 GeV	0.083486249 GeV

Thank You

سبحانك اللهم وبحمدك أشهد أن لا إله الا أنت أستغفرك وأتوب اليك