Generalized Unitarity Techniques for Higgs Physics

Teaching Old Loops New Tricks

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- Outline

  - Introduction
  - Historical review of spinor helicity

    - Gluons, gluons, and gluons
    - Tree level properties and MHV
    - Massless and massive spinors
    - Modern notation

  - Our process ($0 \longrightarrow \bar{q}qQ\bar{Q}\Phi$)
  - Loops... Old and New

    - Traditional approach
    - Generalized unitarity
    - New method and stability
    - Automation

  - Comparison of methods
  - Outlook and work left to do

• Introduction

- ▷ Theory calculations have not entirely kept pace with projected experimental precision
- ▷ Many important processes which need attention (Les Houches 2007)
  - ⟶ Tree level is well under control (Born and real radiation)
  - ⟶ Dipole subtraction is virtually automatic at NLO
  - ⟶ Splitting functions are known to three-loops
  - ⟹ The virtual (loop) corrections are the sticking point, particularly at high multiplicity and multiple scales
- ▷ Traditional diagrammatic techniques are robust, well understood, but not particularly efficient (large expressions until end)
- ▷ Spinor helicity exploits gauge invariance
- ▷ This reduces complexity of intermediate (final) expressions
- ▷ When combined with cutting rules (generalized unitarity), we can get at virtual corrections in a highly efficient manner

On-shell unitarity is built on the spinor helicity formalism

- Spinor helicity... brief background (massless)

  ▷ 1966: Bjorken and Chen, Phys Rev **154** 1335
  ▷ 1988: Berends and Giele (mostly gluons)
  ▷ The Ubiquitous Photon [OUP 1990, Gastmans and Wu]
  Multiparton amplitudes in gauge theories [Phys Rept **200** 301 (1991)]

$$u_\pm(p_i) = v_\mp(p_i) \equiv |p_i^\pm\rangle \equiv |i^\pm\rangle \qquad \langle i^\pm| \equiv \langle p_i^\pm| \equiv \overline{u_\pm(p_i)} = \overline{v_\mp(p_i)}$$

$$\langle ij\rangle \equiv \langle i^-|j^+\rangle = \overline{u_-(p_i)}u_+(p_j) \qquad [ij] \equiv [i^+|j^-] = \overline{u_+(p_i)}u_-(p_j)$$

$$\langle ij\rangle = -\langle ji\rangle, \quad [ij] = -[ji] \qquad \langle i\gamma^\mu j]\langle k\gamma_\mu l] = 2\langle ik\rangle[lj]$$

$$\langle ij\rangle\langle kl\rangle = \langle ik\rangle\langle jl\rangle + \langle il\rangle\langle kj\rangle \qquad \langle ij\rangle[ji] = 2p_i\cdot p_j$$

$$\langle ii\rangle = [ii] = \langle ij] = [ij\rangle = 0 \qquad \langle ij\rangle^\star = \mathrm{sign}(i\cdot j)[ji] \quad \text{(outgoing)}$$

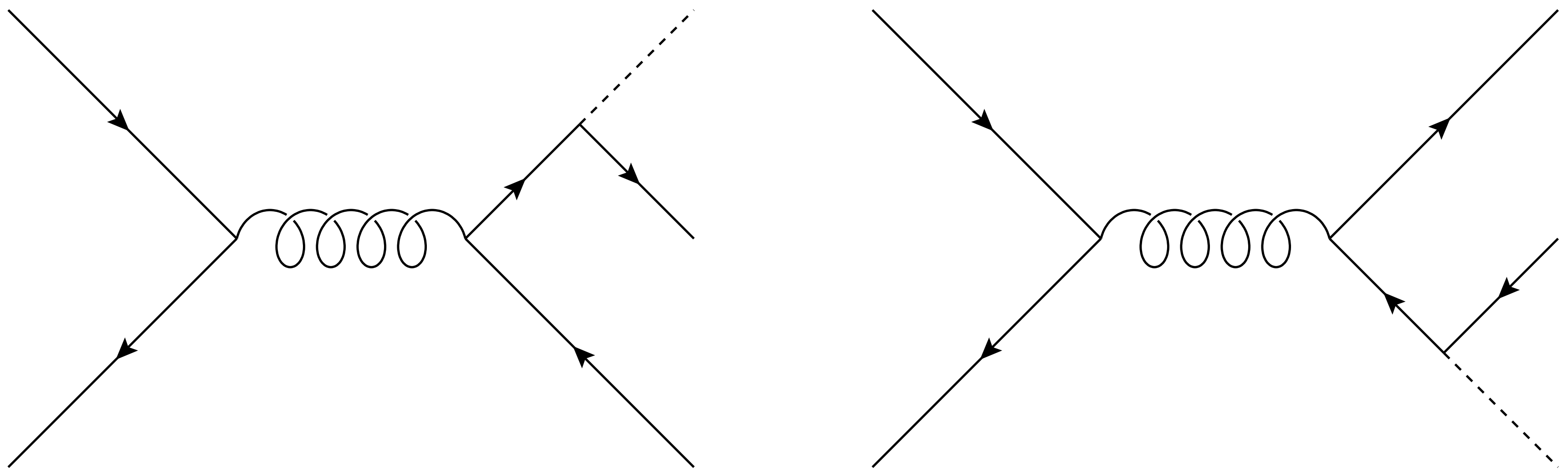
$$\epsilon_\mu^\pm(p,q) = \pm\frac{\langle q^\mp|\gamma_\mu|p^\mp\rangle}{\sqrt{2}\langle q^\mp|p^\pm\rangle}$$

▷ The subject seemed to be closed... until 1995 with an observation about gluon amplitudes in $\mathcal{N}=1$, and $\mathcal{N}=4$ SUSY by Bern, Dixon, Dunbar, and Kosower

- Spinor helicity... continued

  - ▷ Color ordering drastically simplifies calculations
    - ⟶ Strip color factors, use distinct cyclic orderings of external legs
    - ⟶ Particularly useful in gluon calculations (unphysical)
    - ⟶ Spurious singularities much easier to find (not true traditionally)
    - ⟶ Manifest gauge cancellations makes for compact expressions
    - ⟶ Color structure and gauge invariance are separate issues
  - ▷ Twistor inspiration:
    - ⟶ if $p_i \in \mathbb{R}$, then $u_\pm(p_i)$ are related
    - ⟶ if $p_i \in \mathbb{C}$, this is not true generally! Think ψ and χ
    - ⟶ This gives us new non-zero three-vertices
  - ▷ Recursion relations at tree level
    - ⟶ $\mathcal{A}_n = \sum_{r,h} \mathcal{A}^h_{r+1} \frac{1}{P_r^2} \mathcal{A}^{-h}_{n-r+1}$
    - ⟶ $\langle j, l]$ shift $\quad k_j^\mu \to k_j^\mu - \frac{z}{2}\langle j\gamma^\mu l] \qquad k_l^\mu \to k_l^\mu + \frac{z}{2}\langle j\gamma^\mu l]$
    - ⟶ $\mathcal{A}_n(z) \to 0$ as $z \to \infty$ (meromorphic)
  - ▷ Yields *very* simple expressions for certain helicity combinations (MHV)
  - ▷ Sew diagrams together to get *all* tree level expressions

- Sewing example $0 \to \bar{q}qQ\bar{Q}\Phi$



Two tree (Born) level diagrams

The process we have in mind is $b\bar{b}A^{(0)}$ production (not in SM)

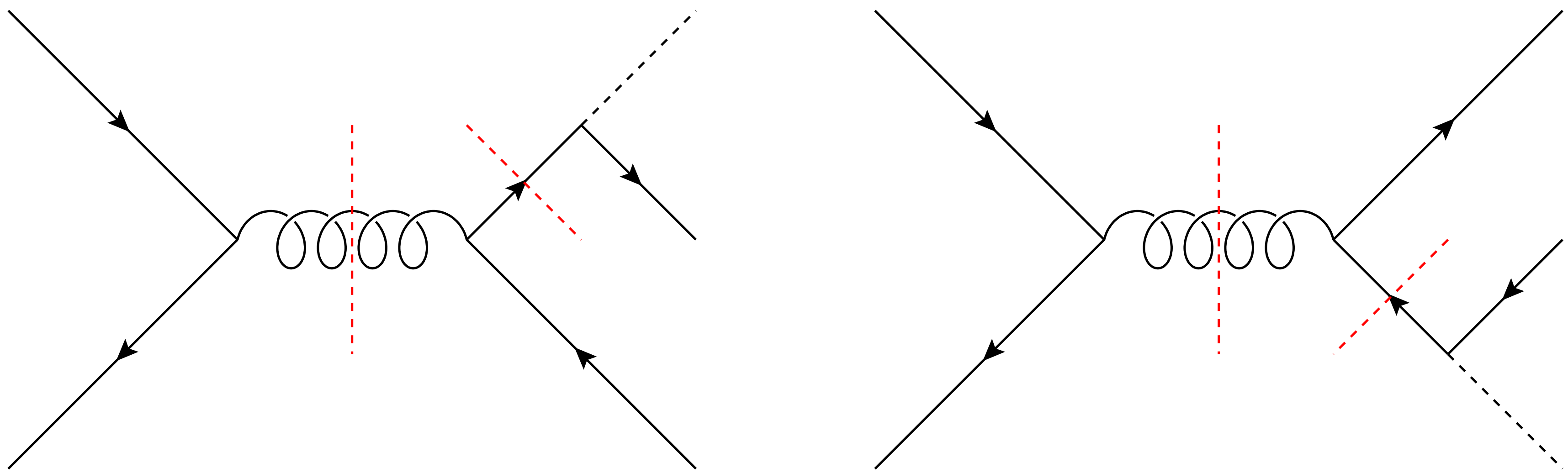
$\longrightarrow$ Discovery channel, lots of neat phenomenology

We can check the method with $t\bar{t}h/b\bar{b}h$ calculations that are already available

[Reina, Dawson, and Wackeroth PRD **65** 053017 (2002)],

[Dawson, Jackson, Reina, and Wackeroth Mod Phys Lett **A21** 89 (2006)]

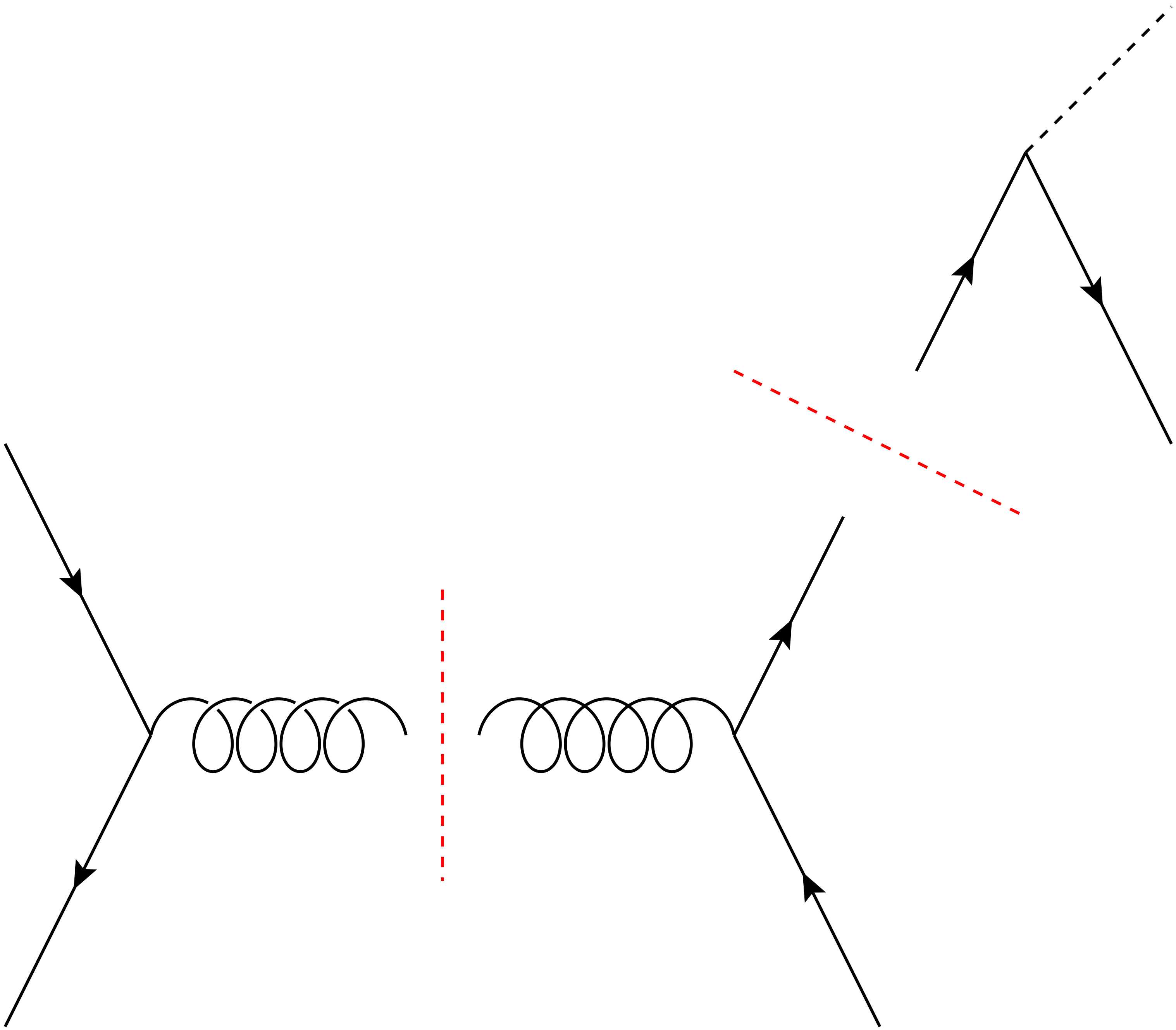
- Sewing example $0 \to \bar{q}qQ\bar{Q}\Phi$



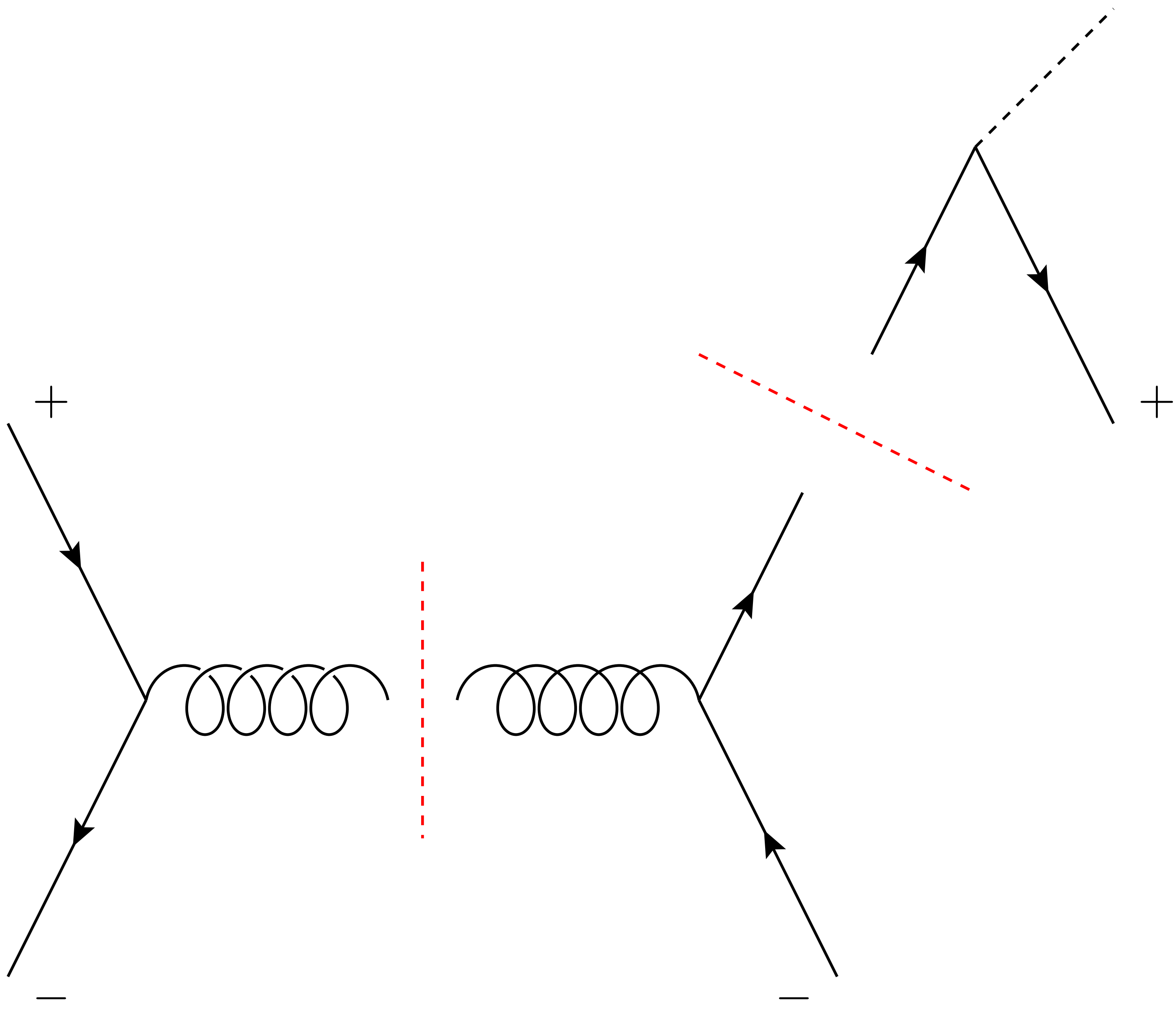
We can cut these into three new vertices...

Lets look at the first diagram (we label these tree level $B_2^{(1)}$ and $B_2^{(2)}$)

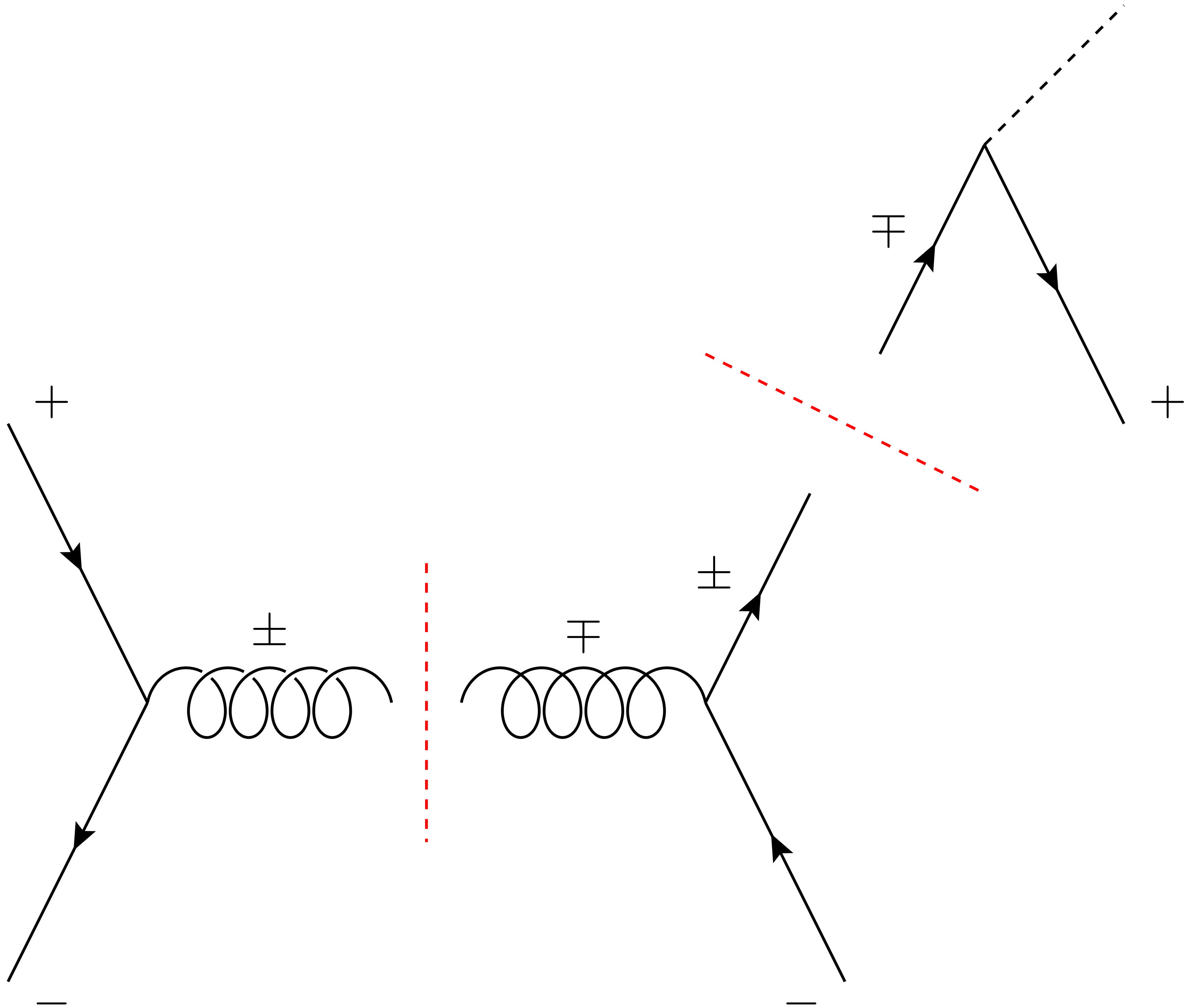
- Sewing example $0 \to \bar{q}qQ\bar{Q}\Phi$ for tree level $B_2^{(1)}$



- Sewing example $0 \to \bar{q}qQ\bar{Q}\Phi$ $(+-+-)$ for tree level $B_2^{(1)}$



- Sewing example $0 \to \bar{q}qQ\bar{Q}\Phi$ $(+-+-)$ for tree level $B_2^{(1)}$



- $0 \to \bar{q}q Q\bar{Q}\Phi$ tree level helicities (there are four)

	$B_2^{(1)}$			$B_2^{(2)}$		
	$\bar{q}qG(p_{12})$	$Q\bar{Q}G(p_{12})$	$Q\bar{Q}\Phi(p_5)$	$\bar{q}qG(p_{12})$	$Q\bar{Q}G(p_{12})$	$Q\bar{Q}\Phi(p_5)$
$\mathcal{M}_{(0)}^{+-+-}$	$-++$	$+--$	$-+$	$-++$	$++-$	$--$
	$-+-$	$+-+$	$-+$	$-+-$	$+-+$	$+-$
	$-+-$	$--+$	$++$	$-++$	$+--$	$+-$
$\mathcal{M}_{(0)}^{+--+}$	$-++$	$++-$	$--$	$-++$	$-+-$	$-+$
	$-+-$	$-++$	$+-$	$-+-$	$-++$	$-+$
	$-++$	$-+-$	$+-$	$-+-$	$--+$	$++$
$\mathcal{M}_{(0)}^{-++-}$	$+-+$	$+--$	$-+$	$+-+$	$++-$	$--$
	$+--$	$+-+$	$-+$	$+--$	$+-+$	$+-$
	$+--$	$--+$	$++$	$+-+$	$+--$	$+-$
$\mathcal{M}_{(0)}^{-+-+}$	$+-+$	$++-$	$--$	$+-+$	$-+-$	$-+$
	$+--$	$-++$	$+-$	$+--$	$-++$	$-+$
	$+-+$	$-+-$	$+-$	$+--$	$--+$	$++$

Now we just need expressions for these vertices, and to connect them with propagators. But what about these helicity violating terms? How do we account for masses? What do you mean by massive 'helicity'?

- Spinor helicity... with masses
  - $\longrightarrow$ Massive spinors can be built (light-cone projection)
  - $\longrightarrow$ Add a massless reference spinor q (take from our process)
  - $\longrightarrow$ [Kleiss and Stirling NPB **262** 235 (1985)], [Ozeren and Stirling Eur Phys J **C48** 159 (2006)] and [Schwinn and Weinzierl JHEP **0505** 006 (2007)]

$$k^2 = m^2, \qquad k^\mu \xrightarrow{\flat} k^{\flat,\mu} = k^\mu - \frac{m^2}{2k\cdot q} q^\mu, \qquad (k^\flat)^2 = q^2 = 0$$

$$\bar{u}_+(p_1) = \frac{\langle q|(\not{p}_1 + m)}{\langle q1\rangle} \qquad v_-(p_2) = \frac{(\not{p}_2 - m)|q]}{[2q]}$$

$$\bar{u}_-(p_1) = \frac{[q|(\not{p}_1 + m)}{[q1]} \qquad v_+(p_2) = \frac{(\not{p}_2 - m)|q\rangle}{\langle 2q\rangle}$$

▷ We now have the building blocks needed for generalized unitarity

▷ What do the vertices look like?

- Vertices (helicity violating in red)

$q(p_1)\,\bar{q}(p_2)\,G(p_3)$	mod $ig\sqrt{2}$
$+-+$	$-\dfrac{[13]^2}{[12]}$
$-+-$	$+\dfrac{\langle 13\rangle^2}{\langle 12\rangle}$
$+--$	$-\dfrac{\langle 23\rangle^2}{\langle 12\rangle}$
$-++$	$+\dfrac{[23]^2}{[12]}$
$--+$	$+m\dfrac{\langle 12\rangle^2}{\langle 23\rangle\langle 13\rangle}$
$++-$	$+m\dfrac{[12]^2}{[23][13]}$

$q(p_1)\,\bar{q}(p_2)\,h^0(p_3)$	mod $i\,m/v$
$++$	$+[12]$
$--$	$+\langle 12\rangle$
$+-$	$+m\left(\dfrac{\langle q2\rangle}{\langle q1\rangle}-\dfrac{[1q]}{[2q]}\right)$
$-+$	$+m\left(\dfrac{[q2]}{[q1]}-\dfrac{\langle 1q\rangle}{\langle 2q\rangle}\right)$

$q(p_1)\,\bar{q}(p_2)\,A^0(p_3)$	mod $i\,m/v$
$++$	$-[12]$
$--$	$+\langle 12\rangle$
$+-$	$+m\left(\dfrac{\langle q2\rangle}{\langle q1\rangle}+\dfrac{[1q]}{[2q]}\right)$
$-+$	$-m\left(\dfrac{[q2]}{[q1]}+\dfrac{\langle 1q\rangle}{\langle 2q\rangle}\right)$

- Generalized (Perturbative) Unitarity

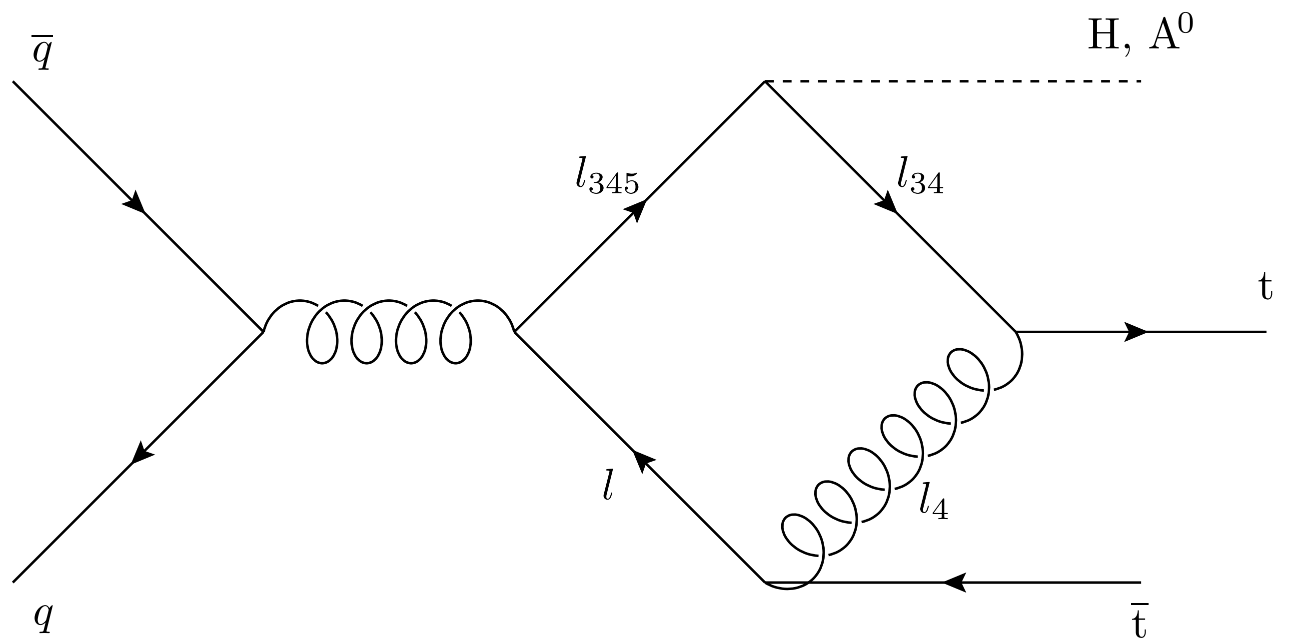
[Britto, Cachazo, Feng, Witten, Dixon, Forde, Kilgore, Kosower, Ossola, Papadopoulos, Pittau,...]

- ▷ Unitarity and Renormalizability *are* gauge theories
- ▷ Cutkosky rules (largest-time equation)
- ▷ Disc $\mathcal{A} = \mathcal{A}^\dagger \mathcal{A}$ (optical theorem, imaginary part across branch cut)
- ▷ Very limited applications
  - $\longrightarrow$ Optical theorem is about $2 \to 2$ forward scattering
  - $\longrightarrow$ No interesting resolved final states

$$\mathcal{M}_n^{(1)} = \sum_k C_4^k D_0^k + \sum_k C_3^k C_0^k + \sum_k C_2^k B_0^k + \sum_k C_1^k A_0^k + R_n$$

- ▷ If we cut a diagram twice (cut two propagators in two different ways), we find the discontinuity in two ways and can solve for the box scalar integral coefficient directly
- ▷ Box diagrams are the easiest (contrast to traditional methods)
  - $\longrightarrow$ Several boxes can share a given branch cut, which is resolved by the sewing procedure
- ▷ The other integral coefficients take some work, but can also be obtained in this manner

- Our process ($0 \to \bar{q}qQ\bar{Q}\Phi$), sample diagram (49 others)



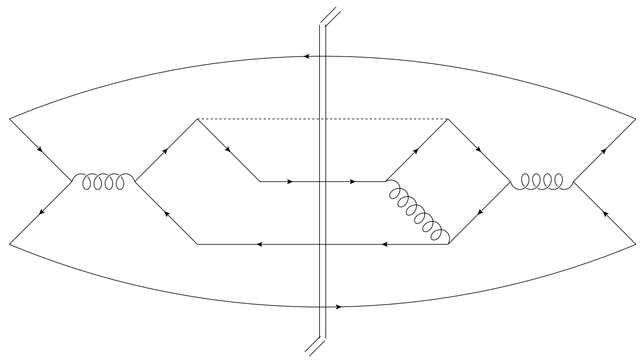
$$i\mathcal{M} = g^4\,(T^aT^bT^aT^b)\frac{1}{s_{12}}\frac{m}{v}\times$$

$$\bar{v}_2\gamma^\nu u_1\,\bar{u}_3\left\{\int\frac{d^4l}{(2\pi)^4}\;\gamma^\rho\frac{(\not{l}_{34}+m_Q)}{l_{34}^2-m_Q^2}(\gamma^5)\frac{(\not{l}_{345}+m_Q)}{l_{345}^2-m_Q^2}\gamma_\nu\frac{(\not{l}+m_Q)}{l^2-m_Q^2}\gamma_\rho\frac{1}{l_4^2}\right\}v_4$$

Amplitude is first step, need to interfere for physics (and reduction)...

- Loops... Old (Part of NLO calculation)

$$\left(\mathcal{M}_{(0)}\mathcal{M}^{\dagger}_{(1)}\right) = T_{11} + T_{12} + T_{21} + T_{22}, \qquad \text{just } B_2^{(1,2)}$$



$$T_{11} = [\gamma^{\mu} u_1 \bar{u}_1 \gamma^{\nu} v_2 \bar{v}_2]\, [\gamma^5 (\not{p}_{35} + m) \gamma_{\mu} v_4 \bar{v}_4 \gamma^{\rho} (\not{l} + m) \gamma_{\nu} (\not{l}_{345} + m) \gamma_5 (\not{l}_{34} + m) \gamma_{\rho} u_3 \bar{u}_3]$$

$$T_{12} = [\gamma^{\mu} u_1 \bar{u}_1 \gamma^{\nu} v_2 \bar{v}_2]\, [\gamma^5 (\not{p}_{35} + m) \gamma_{\mu} v_4 \bar{v}_4 \gamma^{\rho} (\not{l}_{34} + m) \gamma_5 (\not{l}_{345} + m) \gamma_{\nu} (\not{l} + m) \gamma_{\rho} u_3 \bar{u}_3]$$

$$T_{21} = [\gamma^{\mu} u_1 \bar{u}_1 \gamma^{\nu} v_2 \bar{v}_2]\, [\gamma_{\mu} (\not{p}_{45} + m) \gamma^5 v_4 \bar{v}_4 \gamma^{\rho} (\not{l} + m) \gamma_{\nu} (\not{l}_{345} + m) \gamma_5 (\not{l}_{34} + m) \gamma_{\rho} u_3 \bar{u}_3]$$

$$T_{22} = [\gamma^{\mu} u_1 \bar{u}_1 \gamma^{\nu} v_2 \bar{v}_2]\, [\gamma_{\mu} (\not{p}_{45} + m) \gamma^5 v_4 \bar{v}_4 \gamma^{\rho} (\not{l}_5 + m) \gamma_5 (\not{l} + m) \gamma_{\nu} (\not{l}_{345} + m) \gamma_{\rho} u_3 \bar{u}_3]$$

Tensor integrals need to be reduced to scalar integrals...

- Loops... Old (4D Passarino-Veltman reduction)

$$A_0(m_1) \equiv (2\pi\mu)^{4-n}\int \frac{d^n l}{i\pi^2}\frac{1}{D_1}$$

$$B_{0,\mu,\mu\nu}(p_1,m_1,m_2) \equiv (2\pi\mu)^{4-n}\int \frac{d^n l}{i\pi^2}\frac{\{1,l_\mu,l_{\mu\nu}\}}{D_1\,D_2}$$

$$C_{0,\mu,\mu\nu,\mu\nu\rho}(p_1,p_2,m_1,m_2,m_3) \equiv (2\pi\mu)^{4-n}\int \frac{d^n l}{i\pi^2}\frac{\{1,l_\mu,l_{\mu\nu},l_{\mu\nu\rho}\}}{D_1\,D_2\,D_3}$$

$$D_{0,\mu,\mu\nu,\mu\nu\rho,\mu\nu\rho\sigma}(p_1,p_2,p_3,m_1,m_2,m_3,m_4) \equiv (2\pi\mu)^{4-n}\int \frac{d^n l}{i\pi^2}\frac{\{1,l_\mu,l_{\mu\nu},l_{\mu\nu\rho},l_{\mu\nu\rho\sigma}\}}{D_1\,D_2\,D_3\,D_4}$$

$$D_1 = l^2 - m_1^2 + i\epsilon$$

$$D_2 = (l+p_1)^2 - m_2^2 + i\epsilon$$

$$D_3 = (l+p_1+p_2)^2 - m_3^2 + i\epsilon$$

$$D_4 = (l+p_1+p_2+p_3)^2 - m_4^2 + i\epsilon$$

- Loops... Get harder and harder (especially boxes)

Decompose into coefficients

$$
\begin{aligned}
D_\mu &= p_{1\mu} D_{11} + p_{2\mu} D_{12} + p_{3\mu} D_{13} \\
D_{\mu\nu} &= p_{1\mu} p_{1\nu} D_{21} + p_{2\mu} p_{2\nu} D_{22} + p_{3\mu} p_{3\nu} D_{23} \\
&\quad + p_{1\mu} p_{2\nu} D_{24} + p_{1\mu} p_{3\nu} D_{25} + p_{2\mu} p_{3\nu} D_{26} + g_{\mu\nu} D_{27} \\
D_{\mu\nu\rho} &= p_{1\mu} p_{1\nu} p_{1\rho} D_{31} + p_{2\mu} p_{2\nu} p_{2\rho} D_{32} + p_{3\mu} p_{3\nu} p_{3\rho} D_{33} \\
&\quad + \{p_1 p_1 p_2\}_{\mu\nu\rho} D_{34} + \{p_1 p_1 p_3\}_{\mu\nu\rho} D_{35} + \{p_1 p_2 p_2\}_{\mu\nu\rho} D_{36} \\
&\quad + \{p_1 p_3 p_3\}_{\mu\nu\rho} D_{37} + \{p_2 p_2 p_3\}_{\mu\nu\rho} D_{38} + \{p_2 p_3 p_3\}_{\mu\nu\rho} D_{39} \\
&\quad + \{p_1 p_2 p_3\}_{\mu\nu\rho} D_{310} + \{p_1 g\}_{\mu\nu\rho} D_{311} + \{p_2 g\}_{\mu\nu\rho} D_{312} + \{p_3 g\}_{\mu\nu\rho} D_{313} \\
D_{\mu\nu\rho\sigma} &= \cdots
\end{aligned}
$$

$$
\{p_i p_j p_k\}_{\mu\nu\rho} \equiv \sum_{\sigma(i,j,k)} p_{\sigma(i)\mu} p_{\sigma(j)\nu} p_{\sigma(k)\rho} \qquad \{p_i g\}_{\mu\nu\rho} \equiv p_{i\mu} g_{\nu\rho} + p_{i\nu} g_{\mu\rho} + p_{i\rho} g_{\mu\nu}
$$

- Loops... and they have stability issues

$$X = \begin{pmatrix} p_1^2 & p_1 \cdot p_2 & p_1 \cdot p_3 \\ p_1 \cdot p_2 & p_2^2 & p_2 \cdot p_3 \\ p_1 \cdot p_3 & p_2 \cdot p_3 & p_3^2 \end{pmatrix} \text{ Gram determinant } (\det X \neq 0)$$

$$\begin{pmatrix} p_1^\mu \\ p_2^\mu \\ p_3^\mu \end{pmatrix} D_\mu = X \begin{pmatrix} D_{11} \\ D_{12} \\ D_{13} \end{pmatrix} = \begin{pmatrix} R_{20} \\ R_{21} \\ R_{22} \end{pmatrix}$$

$$R_{20} = \frac{1}{2}[C_0(1,3,4) - C_0(2,3,4) + (m_2^2 - m_1^2 - Q_2^2)D_0]$$
$$R_{21} = \frac{1}{2}[C_0(1,2,4) - C_0(1,3,4) + (m_3^2 - m_2^2 - Q_3^2 + Q_2^2)D_0]$$
$$R_{22} = \frac{1}{2}[C_0(1,2,3) - C_0(1,2,4) + (m_4^2 - m_3^2 - Q_4^2 + Q_3^2)D_0]$$

$$Q_1 = 0, \qquad Q_i = \sum_{j=1}^{i-1} p_j$$

Gram determinant (can) introduce non-physical singular points, high complexity, error prone, diagrams talk to each other

- Loops... New

$$\mathcal{M}_n^{(1)} = \sum_k C_4^k D_0^k + \sum_k C_3^k C_0^k + \sum_k C_2^k B_0^k + \sum_k C_1^k A_0^k + R_n$$

▷ Reconstruct coefficients from unitarity cuts (with present propagators)

$$\frac{i}{p^2 - m^2} \to (2\pi)\delta^{(+)}(p^2 - m^2)$$

$$\int \frac{d^n l}{(2\pi)^n} \left[\frac{f(l)}{[l^2 - m_1^2 + i\epsilon][l_1^2 - m_2^2 + i\epsilon][l_{12}^2 - m_3^2 + i\epsilon][l_{123}^2 - m_4^2 + i\epsilon]} \right] \Bigg|_{l^n \to l^4}$$

$$\to \int \frac{d^n l}{(2\pi)^n} [\, f(l)\delta^{(+)}(l^2 - m_1^2)\, \delta^{(+)}(l_1^2 - m_2^2)\, \delta^{(+)}(l_{12}^2 - m_3^2)\, \delta^{(+)}(l_{123}^2 - m_4^2)\,]$$

$$= \frac{1}{2} \sum_{l^\pm} f(l), \qquad \text{completely determines the loop momentum}$$

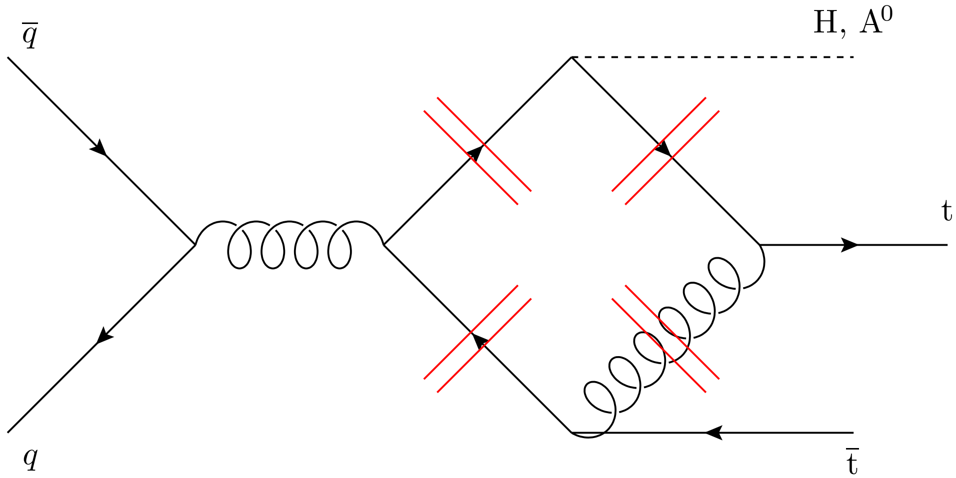
▷ We find following equation to solve for our box integral coefficient

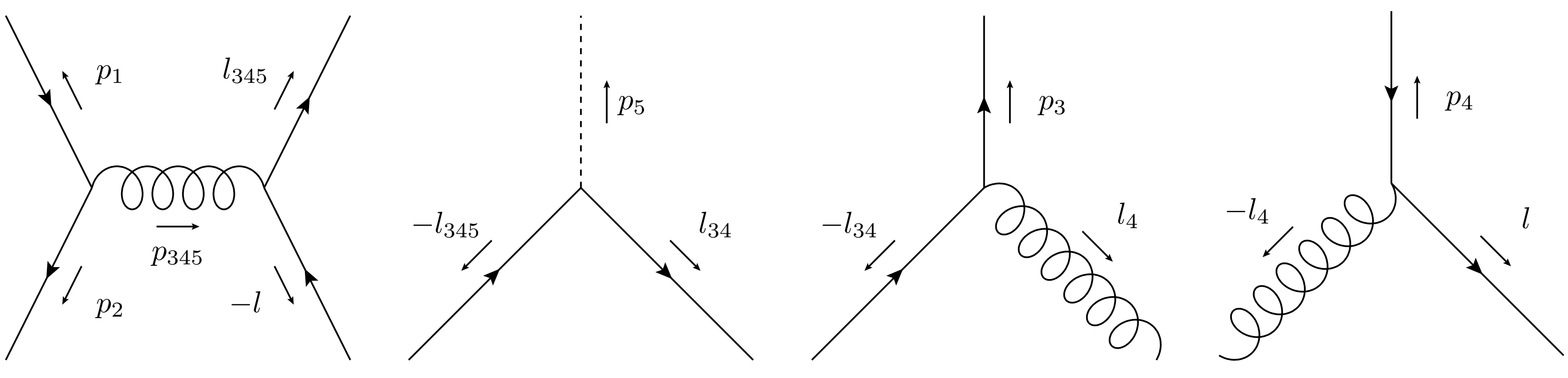
$$\{ l \,|\, l^2 = m_1^2, \quad l_1^2 = m_2^2, \quad l_{12}^2 = m_3^2, \quad l_{123}^2 = m_4^2 \},$$

$$l^\pm = \alpha p_1 + \beta p_2 + \sigma p_3 \pm \rho P, \qquad P^\mu = i\epsilon^{\mu\nu\rho\sigma}\, p_{1\nu}\, p_{2\rho}\, p_{3\sigma}, \qquad \text{find } \alpha, \beta, \sigma, \rho$$

$$\Longrightarrow \qquad C_4^k = \frac{1}{2} \sum_{l^\pm} A_1\, A_2\, A_3\, A_4$$

- Our loop process ($0 \to \bar{q}qQ\bar{Q}\Phi$), the orphan (for check)





- $0 \to \bar{q}qQ\bar{Q}\Phi \; B_2^{(1)}$ loop helicities (independent)

$\mathcal{M}^{+-+-}$			
$\bar{q}q\, G^{\lambda}\, Q\bar{Q}$	$Q\bar{Q}\Phi$	$Q\bar{Q}G$	$Q\bar{Q}G$
$+ - \pm - +$	$++$	$- - +$	$+ - -$
$+ - \pm + -$	$--$	$+ - +$	$+ + -$
$+ - \pm + +$	$--$	$- - +$	$+ + -$
$+ - \pm - +$	$-+$	$- - +$	$+ + -$
$+ - \pm + +$	$+-$	$- - +$	$+ - -$
$+ - \pm + -$	$+-$	$+ - -$	$+ - +$
$+ - \pm + -$	$+-$	$+ - +$	$+ - -$
$+ - \mp - +$	$++$	$- - +$	$+ - -$
$+ - \mp - -$	$++$	$+ - -$	$+ - +$
$+ - \mp - -$	$++$	$+ - +$	$+ - -$
$+ - \mp + -$	$--$	$+ - +$	$+ + -$
$+ - \mp - -$	$-+$	$+ - +$	$+ + -$
$+ - \mp - +$	$-+$	$- - +$	$+ + -$
$+ - \mp + -$	$+-$	$+ - +$	$+ - -$
$+ - \mp + -$	$+-$	$+ - -$	$+ - +$

$\mathcal{M}^{-+-+}$			
$\bar{q}q\, G^{\lambda}\, Q\bar{Q}$	$Q\bar{Q}\Phi$	$Q\bar{Q}G$	$Q\bar{Q}G$
$- + \mp + -$	$--$	$+ + -$	$- + +$
$- + \mp - +$	$++$	$- + -$	$- - +$
$- + \mp - -$	$++$	$+ + -$	$- - +$
$- + \mp + -$	$+-$	$+ + -$	$- - +$
$- + \mp - -$	$-+$	$+ + -$	$- + +$
$- + \mp - +$	$-+$	$- + +$	$- + -$
$- + \mp - +$	$-+$	$- + -$	$- + +$
$- + \pm + -$	$--$	$+ + -$	$- + +$
$- + \pm + +$	$--$	$- + +$	$- + -$
$- + \pm + +$	$--$	$- + -$	$- + +$
$- + \pm - +$	$++$	$- + -$	$- - +$
$- + \pm + +$	$+-$	$- + -$	$- - +$
$- + \pm + -$	$+-$	$+ + -$	$- - +$
$- + \pm - +$	$-+$	$- + -$	$- + +$
$- + \pm - +$	$-+$	$- + +$	$- + -$

• Comparison of methods

▷ Passarino-Veltman

⟶ Well understood and implemented for years

⟶ Largely automated

⟶ Complex, unwieldy, unstable (as is, can be improved)

▷ Unitarity

⟶ Drastic drop in complexity of expressions

⟶ Utility in massive calculations? Massless is much easier...

⟶ Easy (straightforward) to automate [BlackHat, Rocket, ...]

⟶ Singularities are under better control numerically

⟶ Diverge like $\frac{1}{\langle ij \rangle} \sim \frac{1}{\sqrt{s_{ij}}}$

▷ Timing and interest in theory community will be deciding factor

▷ Results are paramount at this point

- Outlook

▷ Still have triangles, bubbles, tadpoles, and rational terms

$\longrightarrow$ Challenging, but understood (not presented here)

▷ Numeric checks between different methods are still underway

▷ Proof of concept calculation for massive process

▷ Still some concern about speed of implementation at tree level

$\longrightarrow$ Adding in color slows calculations

$\longrightarrow$ Could be solved with representation of spinors?

$\longrightarrow$ Clearly, still needs to be optimized for MC work

▷ Strength is in wide applicability to NLO processes

▷ Automation will lead to low "time to curves"

▷ Worst case, good for new signal processes

Generalized Unitarity $\Leftrightarrow$ New Tool for Phenomenology