Heating up QGP: how far to charm quark chemical equilibration?

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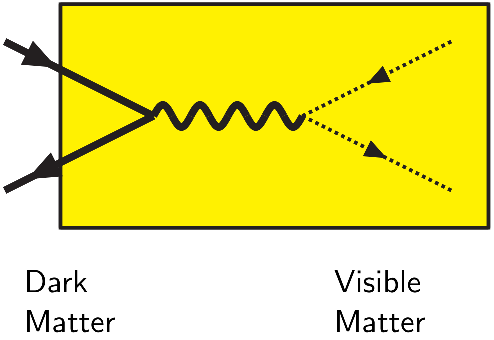
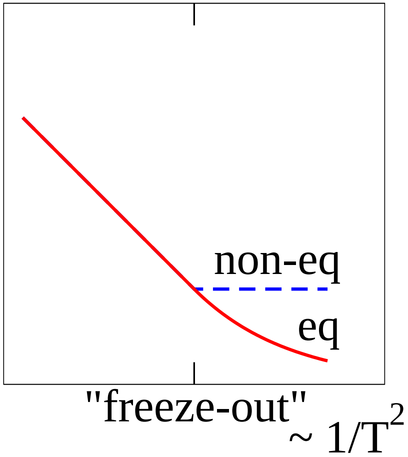
For inspiration: cosmological analogue

Origin of Weakly Interacting Massive Dark Matter

The system thermalizes after inflation, but then chemically decouples when pair annihilation is not fast enough to track the equilibrium distribution, which is $\sim (\frac{MT}{2\pi})^{3/2} e^{-M/T}$ at $T \ll M$.





"WIMP miracle": for typical parameter values the outcome is indeed of the astronomically observed order of magnitude.

Back of the envelope estimate

Equate Hubble rate (H) with annihilation rate ($\Gamma \sim n\sigma v$):

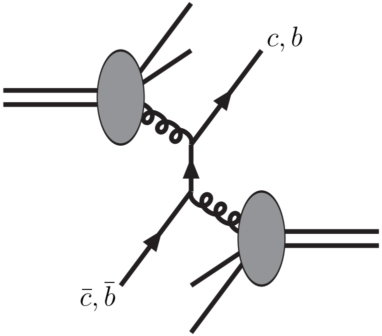
$$\begin{aligned} H &\sim n\sigma v \\ \Leftrightarrow \frac{T^2}{m_{\mathsf{Pl}}} &\sim \left(\frac{MT}{2\pi}\right)^{3/2} e^{-M/T} \, \frac{\alpha_w^2}{m_W^2} \left(\frac{T}{M}\right)^{1/2} \\ \Rightarrow \frac{M}{T} &\sim \ln\left[\frac{\alpha_w^2 \, m_{\mathsf{Pl}} \, M}{m_W^2 (2\pi)^{3/2}}\right] \sim 30 \, . \end{aligned}$$

A more precise estimate gives $M/T \sim 25$. Then the exponential suppression is $e^{-25} \sim 10^{-11}$. If the same were true in HIC, even bottom quarks would be comfortably in chemical equilibrium!

Heavy ion collisions: a sketch

(i) Initial production

Initial state is out-of-equilibrium, with a non-thermal abundance of heavy quarks with hard momenta:[1]



If nothing happens afterwards, quarks and antiquarks need to be modelled with separate ("non-relativistic") chemical potentials.[2]

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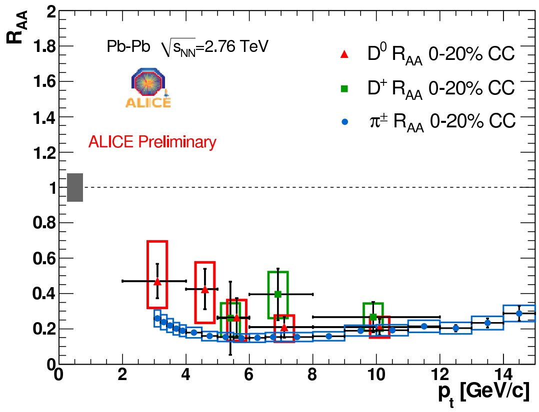
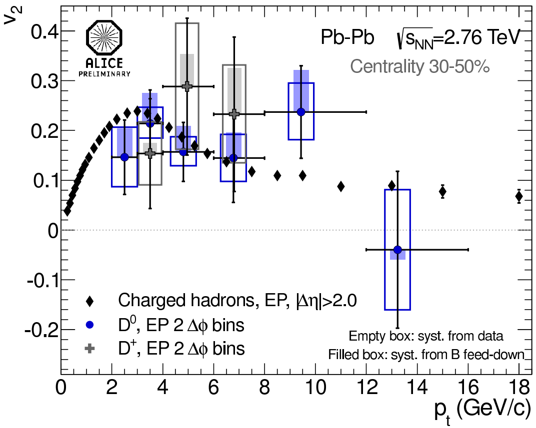
[1] e.g. M. Cacciari *et al*, Phys. Rev. Lett. 95 (2005) 122001 [hep-ph/0502203].

[2] e.g. A. Andronic *et al*, Nucl. Phys. A 789 (2007) 334 [nucl-th/0611023].

(ii) Kinetic equilibration

Charm (and even bottom) do equilibrate kinetically: jets get quenched,[3] quarks adjust their velocities to hydrodynamic flow.[4]





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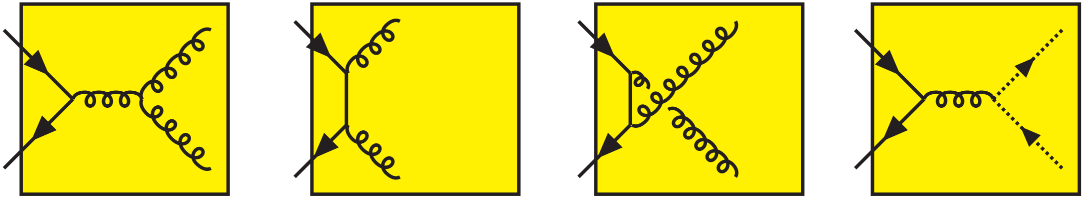
[3] e.g. A. Dainese [ALICE Collaboration], 1106.4042.

[4] e.g. G. Ortona [ALICE Collaboration], 1207.7239.

(iii) Chemical equilibration: how fast does pair creation or annihilation take place?



The computation is in principle the same as for strangeness,[5] and near equilibrium the answer can be expressed as:

$$\Gamma_{\text{chem}} = \frac{g^4 C_F}{8\pi M^2} \left(N_{\text{f}} + 2C_F - \frac{N_{\text{c}}}{2} \right) \left(\frac{TM}{2\pi} \right)^{\frac{3}{2}} e^{-M/T} \, .$$

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[5] T.S. Biró and J. Zimányi, Phys. Lett. B 113 (1982) 6; J. Rafelski and B. Müller, Phys. Rev. Lett. 48 (1982) 1066 [Erratum-ibid. 56 (1986) 2334]; T. Matsui, B. Svetitsky and L.D. McLerran, Phys. Rev. D 34 (1986) 783 [Erratum-ibid. D 37 (1988) 844].

Numerical estimates:[6]

$$\Gamma_{\text{chem}} \simeq \frac{2\pi\alpha_s^2 T^3}{9M^2}\left(\frac{7}{6}+N_{\rm f}\right)\frac{\chi_f}{\chi_0}\,,$$

where χ_f, χ_0 are massive and massless quark number susceptibilities. For $N_{\rm f} = 3$, $\alpha_s \sim 0.3$, $M \sim 1.5$ GeV, and χ_f/χ_0 from the lattice,[7] yields:

$$\Gamma^{-1}_{\text{chem}} \gtrsim 60\,\text{fm/c}\,, \quad \text{for } T \sim 400 \text{ MeV}\,,$$

$$\Gamma^{-1}_{\text{chem}} \sim 10\,\text{fm/c}\,, \quad \text{for } T \sim 600 \text{ MeV}\,.$$

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[6] D. Bödeker and ML, JHEP 07 (2012) 130 [1205.4987].

[7] H.-T. Ding *et al*, PoS LATTICE 2010 (2010) 180 [1011.0695]; S. Borsanyi *et al*, PoS LATTICE 2011 (2011) 201 [1204.0995].

Open questions

- Validity of the weak-coupling expansion?
- Validity of the non-relativistic expansion?
- Which is the "correct" mass to use for order-or-magnitude estimates? [PDG: $m_c(\bar{\mu}_{\text{ref}}) = 1.275(25)$ GeV with $\bar{\mu}_{\text{ref}} = 2$ GeV; "pole mass" $M \simeq 1.5$ GeV; $m_{D^0} = 1.86$ GeV.]
- Non-equilibrium effects? (Since Γ_{chem} is proportional to the density of the antiquarks, the annihilation rate is faster if heavy antiquarks appear in overabundance.)

Beyond weak coupling
(but staying non-relativistic)

Reminder: heavy quark kinetic equilibration near rest

For kinetic equilibration, the leading-order expression gives a rate suppressed by $\alpha_s^2 T^2/M$ and too small to play a role in practice.

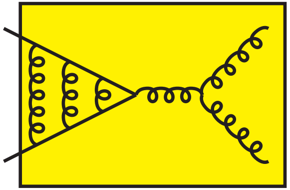
However there is a large positive NLO correction ($\sim$ factor 5). Model studies and AdS/CFT computations also support rapid kinetic equilibration, as do preliminary lattice investigations. All this goes in the direction of the empirical observations.

Q: Could there be similar problems with the weak-coupling expansion for the chemical equilibration rate?

A: This is not excluded: even the leading-order expression is not quite correct for $T \ll M$!

Sommerfeld effect (i)

Pair-annihilating particles have strong "initial state" interactions; pair-created particles have strong "final state" interactions.



The methods have been elucidated in cosmology, where the "Sommerfeld effect" may also play an important role.[8]

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[8] J. Hisano, S. Matsumoto, M. Nagai, O. Saito and M. Senami, *Non-perturbative effect on thermal relic abundance of dark matter,* Phys. Lett. B 646 (2007) 34 [hep-ph/0610249]; J.L. Feng, M. Kaplinghat and H.-B. Yu, *Sommerfeld Enhancements for Thermal Relic Dark Matter,* Phys. Rev. D 82 (2010) 083525 [1005.4678]; A. Hryczuk, R. Iengo and P. Ullio, *Relic densities including Sommerfeld enhancements in the MSSM,* JHEP 03 (2011) 069 [1010.2172]; A. Strumia, *Sommerfeld corrections to type-II and III leptogenesis,* Nucl. Phys. B 809 (2009) 308 [0806.1630].

Sommerfeld effect (ii)

Consider two heavy particles of mass M, interacting through an attractive Coulomb-like potential

$$V(r) = -\frac{g^2 C_{\rm F}}{4\pi r} \;,$$

where $r = |\mathbf{r}_1 - \mathbf{r}_2|$ is the relative distance. Recalling that the reduced mass is $M/2$, and denoting by v the velocity with respect to the center-of-mass frame ($v = v_{\rm rel}/2$), the stationary Schrödinger equation takes the form

$$\left(-\frac{\nabla^2}{M} + V(r)\right)\psi = M v^2 \psi \;.$$

The probability that the two particles meet, allowing them to co-annihilate, is proportional to $|\psi|^2(0)$.

Sommerfeld effect (iii)

Now, we could first solve the problem with free particles, obtaining a plane-wave solution, and an r-independent $|\psi|^2_{(g^0)}$.

However, because of the attractive force, there is an increased probability for the particles to meet.

This increase constitutes the **Sommerfeld effect**, and is characterized by the coefficient

$$S_1 \equiv \frac{|\psi|^2_{(g^2)}(0)}{|\psi|^2_{(g^0)}(0)} \, .$$

[This can be defined separately for s-wave, p-wave, ...]

Sommerfeld effect (iv)

Remarkably, the value of S_1 can be determined in closed form for the s-wave case:[9]

$$S_1 = \frac{X_1}{1 - e^{-X_1}} \, , \quad X_1 = \frac{g^2 C_\mathrm{F}}{4v} \, .$$

If we then consider a thermal environment, the factor needs to be averaged over the thermal ensemble:

$$\bar{S}_1 \equiv \frac{4}{\sqrt{\pi}} \left(\frac{M}{T} \right)^{3/2} \int_0^\infty \mathrm{d}v \, v^2 e^{-Mv^2/T} \, S_1 \, .$$

If $T \lesssim g^4 M$, so that $v \lesssim g^2$, then $\bar{S}_1 \gtrsim 1$.

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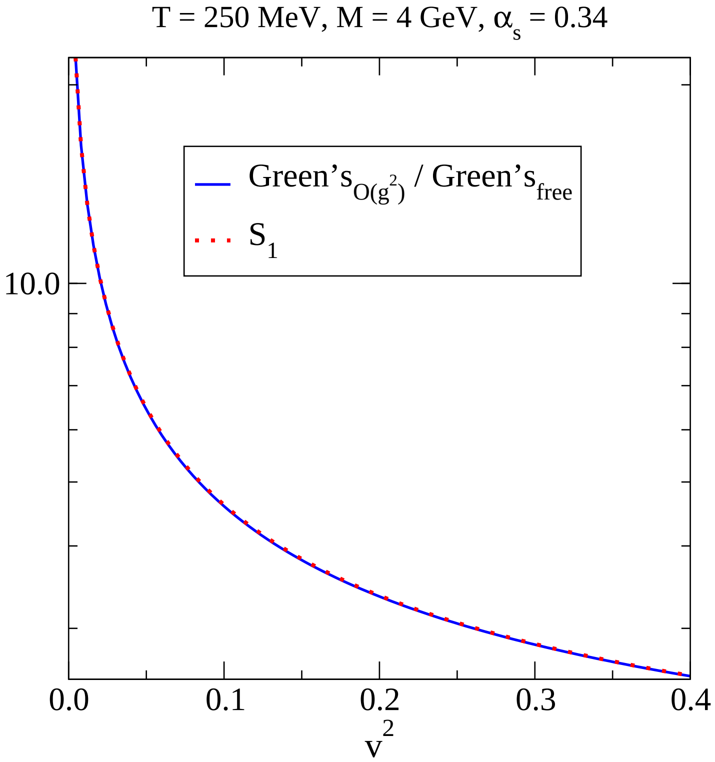
[9] L.D. Landau and E.M. Lifshitz, *Quantum Mechanics, Non-Relativistic Theory*, Third Edition, §136; V. Fadin, V. Khoze and T. Sjöstrand, *On the threshold behavior of heavy top production*, Z. Phys. C 48 (1990) 613.

Sommerfeld effect (v)

Typical values, obtained for QCD-like parameters (here b!):



Sommerfeld effect (vi)

As it happens, in QCD the process splits up into two parts, the “colour-singlet” discussed here as well as a “colour-octet” one, in which case the interaction is repulsive, and $\bar{S}_8 < 1$.[10]

$$
\begin{aligned}
\Gamma_{\text{chem}} &= \frac{g^4 C_{\rm F}}{8\pi M^2} \left(\frac{MT}{2\pi} \right)^{3/2} e^{-M/T} \\
&\times \left[\frac{1}{N_{\rm c}} \bar{S}_1 + \left(\frac{N_{\rm c}^2 - 4}{2N_{\rm c}} + N_{\rm f} \right) \bar{S}_8 \right] .
\end{aligned}
$$

The colour-octet channel is weighted more than the colour-singlet channel (with $\bar{S}_1 \simeq 3.4$). So, accidentally, the numerical effect on heavy quark chemical equilibration in QCD is small.

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[10] D. Bödeker and ML, JHEP 01 (2013) 037 [1210.6153].

Beyond leading order in the non-relativistic regime?

In NRQCD pair creation / annihilation is represented by 4-quark operators with complex coefficients[11]

$$\begin{aligned}\delta\mathcal{L}_M &= \frac{f_1({}^1S_0)}{M^2}\,\psi^\dagger\chi\,\chi^\dagger\psi + \ldots\,, \\ \mathrm{Im}\, f_1({}^1S_0) &= \frac{C_\mathrm{F}}{2N_\mathrm{c}}\,\pi\alpha_s^2 + O(\alpha_s^3)\ .\end{aligned}$$

In principle could use these to compute Γ_chem at full NLO, or even measure it on the lattice.

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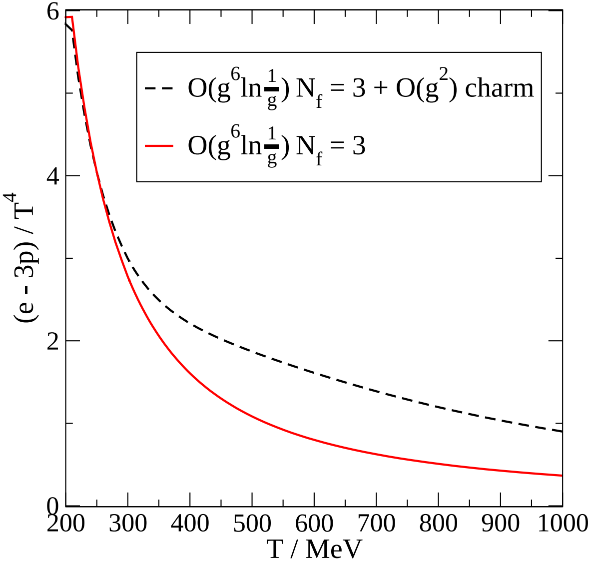
[11] G.T. Bodwin, E. Braaten and G.P. Lepage, *Rigorous QCD analysis of inclusive annihilation and production of heavy quarkonium*, Phys. Rev. D 51 (1995) 1125 [Erratum-ibid. D 55 (1997) 5853] [hep-ph/9407339].

Beyond non-relativistic regime

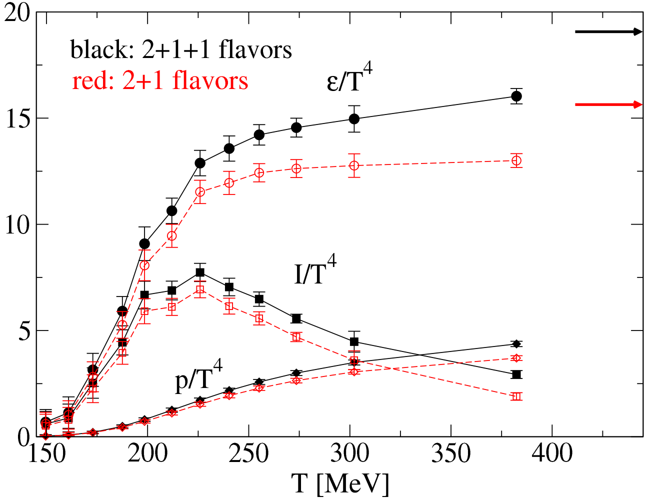
How severe is the exponential suppression?

Assuming chemical equilibration, when do charm quarks become visible in the equation of state?[12]

pQCD



lattice (w/o extrapolations)



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[12] Pert.theory: ML and Y. Schröder, Phys. Rev. D 73 (2006) 085009 [hep-ph/0603048]. Lattice: M. Cheng [RBC-Bielefeld Collaboration], PoS LAT2007 (2007) 173 [0710.4357]; C. DeTar *et al*, Phys. Rev. D 81 (2010) 114504 [1003.5682]; S. Borsanyi *et al*, PoS LATTICE 2011 (2011) 201 [1204.0995].

Holy grail: 2-point correlator of trace anomaly.

Trace of the energy-momentum tensor:

$$T^{\mu}{}_{\mu} = \underbrace{c_\theta\, g_{\rm B}^2 F^{a\mu\nu} F^{a}_{\mu\nu}}_{\equiv\,\theta} + \underbrace{\bar\psi M_{\rm B} \psi}_{\equiv\, S}\,, \quad c_\theta = -\frac{b_0}{2} - \frac{b_1 g^2}{4} + \ldots\ .$$

Bulk viscosity:

$$\zeta = \frac{1}{9} \lim_{\omega\to 0^+} \left\{ \frac{1}{\omega} \int_{\mathcal{X}} e^{i\omega t} \left\langle \frac{1}{2} [T^{\mu}{}_{\mu}(\mathcal{X}), T^{\mu}{}_{\mu}(0)] \right\rangle_T \right\} .$$

Heavy-quark contribution:[13]

$$\delta\zeta = \frac{1}{18T} \lim_{\omega\to 0^+} \left\{ \frac{2M^2 \chi_f \Gamma_{\rm chem}}{\omega^2 + \Gamma_{\rm chem}^2} \right\} = \frac{M^2 \chi_f}{9T\Gamma_{\rm chem}} .$$

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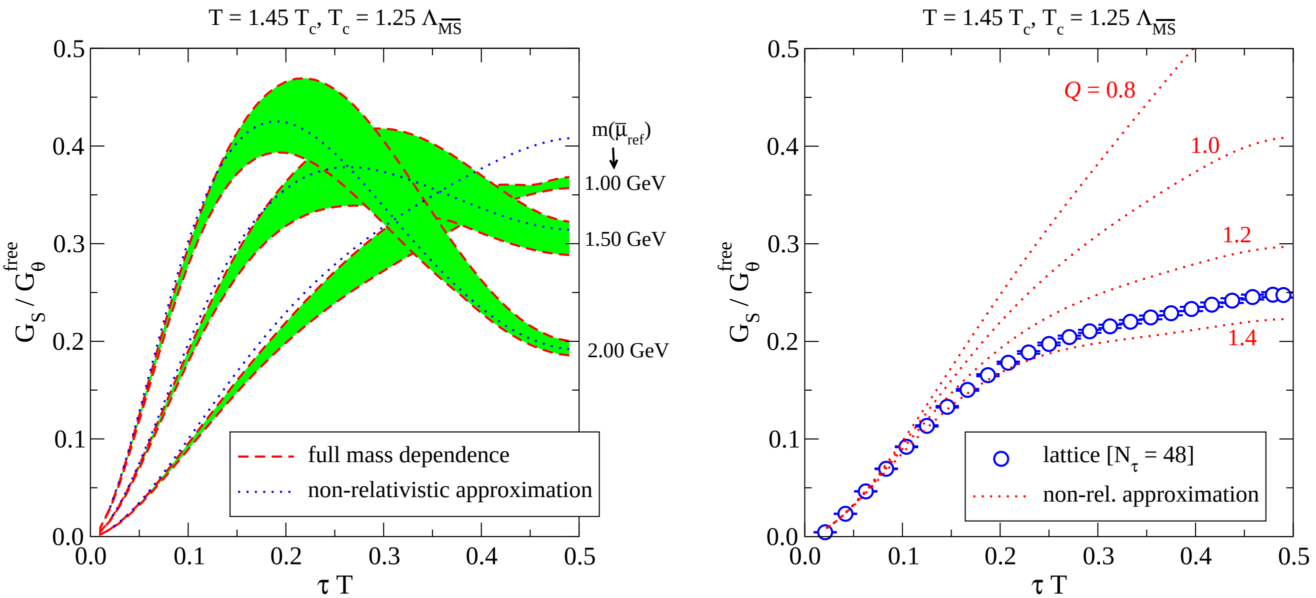
[13] Y. Burnier and ML, JHEP 11 (2013) 012 [1309.1573].

Imaginary-time computations and measurements do exist.[14]



⇒ Here charm has 25-30% influence even at $T \sim 300$ MeV. There is a strong mass dependence. Simulations appear feasible.

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[14] Y. Burnier and ML, 1310.6124; H.-T. Ding *et al*, Phys. Rev. D 86 (2012) 014509 [1204.4945]. In the simulations, $m_{\rm C}(\bar\mu_{\rm ref}) \approx 0.97$ GeV. In the plot, $Q \sim M/m_{\rm C}(\bar\mu_{\rm ref}) = 1 + 4g^2(\bar\mu_{\rm ref})C_{\rm F}/(4\pi)^2 + \mathcal{O}(g^4) \simeq 1.2$.

Summary: can one mimic WIMP freeze-out at HIC@FCC?

For two reasons (charm quarks may be “lighter” than naively expected; their interactions may be stronger), their chemical equilibration (pair creation/annihilation) could be rather fast.

There is a prospect for more precise (NLO) computations and lattice estimates of their chemical equilibration rate in the future.

The chemical equilibration time scale shortens rapidly as the temperature increases.

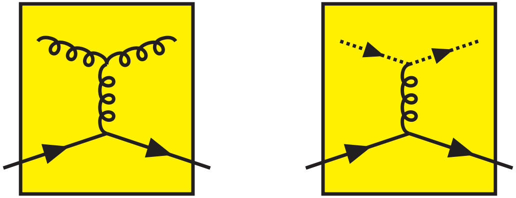
Conservative (?) numbers:

$$\begin{aligned} \Gamma^{-1}_{\text{chem}} &\gtrsim 60\,\text{fm}/\text{c}\;, \quad \text{for } T \sim 400\ \text{MeV}\;, \\ \Gamma^{-1}_{\text{chem}} &\sim 10\,\text{fm}/\text{c}\;, \quad \text{for } T \sim 600\ \text{MeV}\;. \end{aligned}$$

Backup slides

Reminder: heavy quark kinetic equilibration near rest (i)

Perturbation theory gives a nice and simple answer:[15]



$$\Gamma_{\rm kin} = \frac{g^2 C_{\rm F} m_{\rm D}^2}{12\pi M}\left(\ln\frac{2T}{m_{\rm D}} + \frac{1}{2} - \gamma_{\rm E} + \frac{\zeta'(2)}{\zeta(2)} + \frac{N_{\rm f}\ln 2}{2N_{\rm c}+N_{\rm f}}\right)$$

$$m_{\rm D}^2 = g^2T^2\left(\frac{N_{\rm c}}{3}+\frac{N_{\rm f}}{6}\right) .$$

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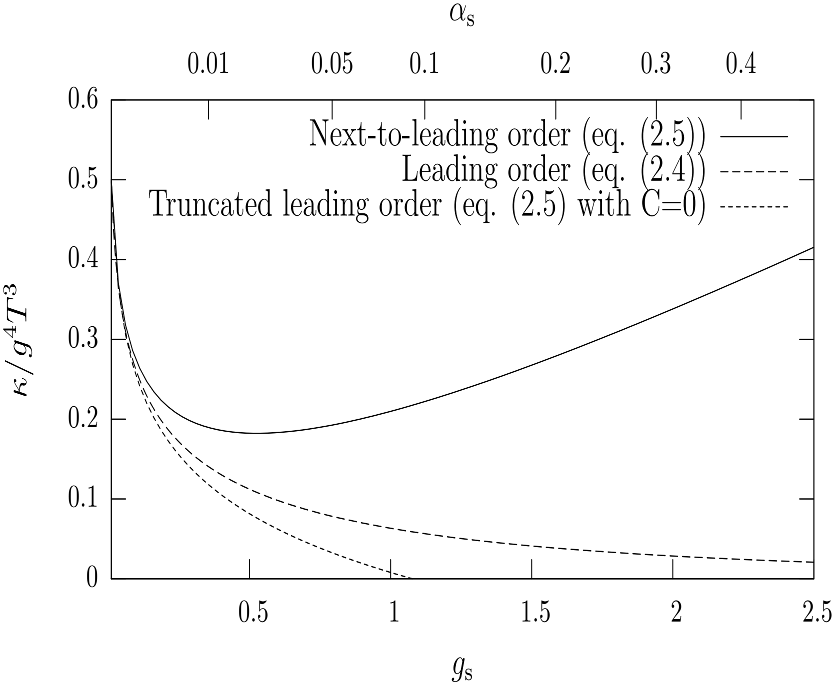
[15] B. Svetitsky, Phys. Rev. D 37 (1988) 2484; E. Braaten and M.H. Thoma, Phys. Rev. D 44 (1991) 2625; G.D. Moore and D. Teaney, Phys. Rev. C 71 (2005) 064904 [hep-ph/0412346].

Reminder: heavy quark kinetic equilibration near rest (ii)

However that are large positive NLO corrections.[16]



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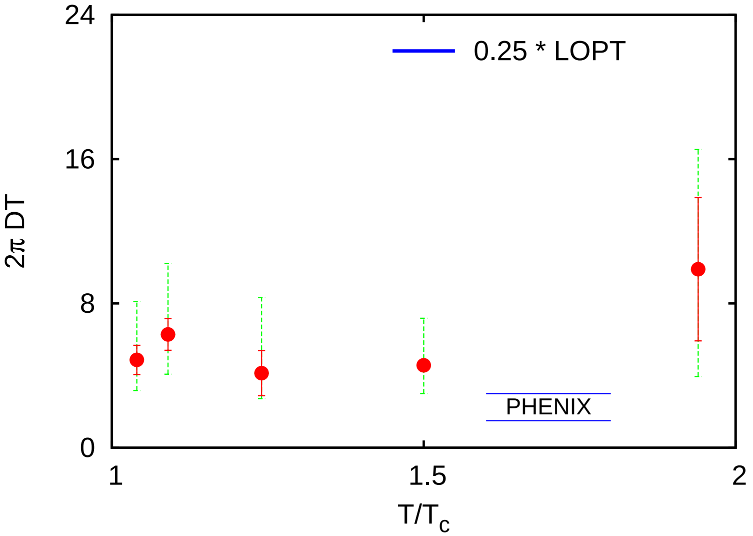
[16] S. Caron-Huot and G.D. Moore, JHEP 02 (2008) 081 [0801.2173].

Reminder: heavy quark kinetic equilibration near rest (iii)

Preliminary lattice studies, still lacking a continuum limit, appear to confirm a large increase of $\Gamma_{\rm kin}$ (here $D = T/(\Gamma_{\rm kin} M)$): [17]



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[17] A. Francis *et al*, PoS LATTICE 2011 (2011) 202 [1109.3941]; D. Banerjee *et al*, Phys. Rev. D 85 (2012) 014510 [1109.5738].