

# USING ENERGY-PEAKS FOR MEASURING (OLD AND NEW) PARTICLE MASSES

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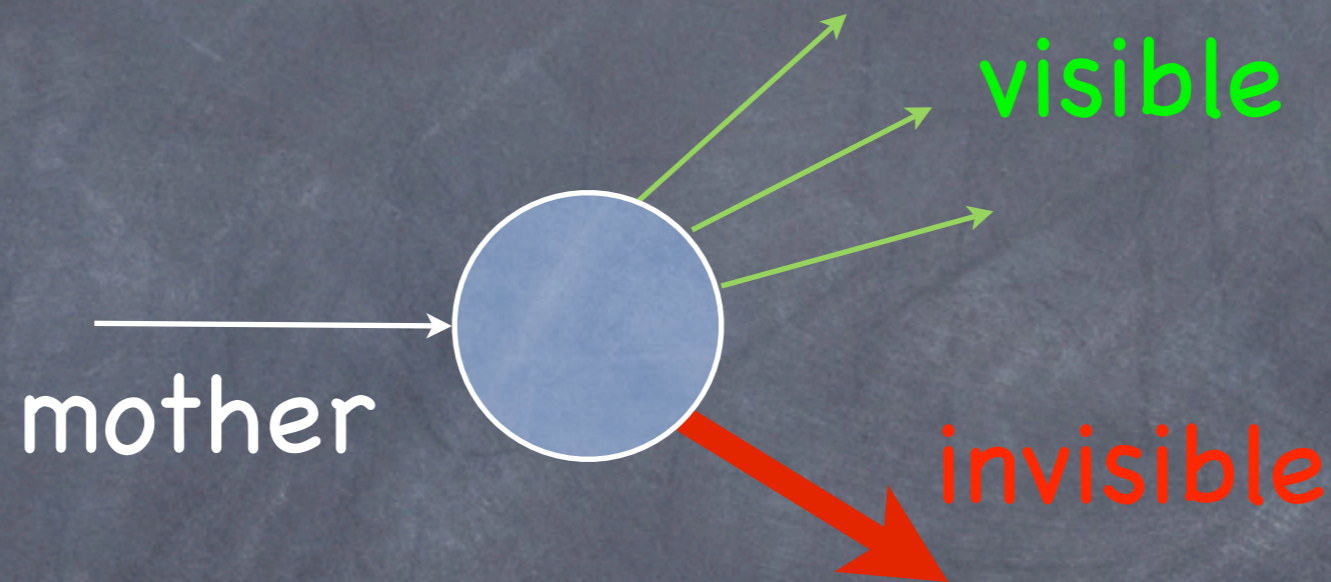
Kaustubh Agashe (University of Maryland)



(with Roberto Franceschini, Sungwoo Hong, Doojin Kim, Kyle Wardlow: 1209.0772; 1212.5230;  
1309.4776 and to appear)

# Basic goal (simple!)

- determine **mass** of mother by **measuring** energy/momentum of (visible) decay products

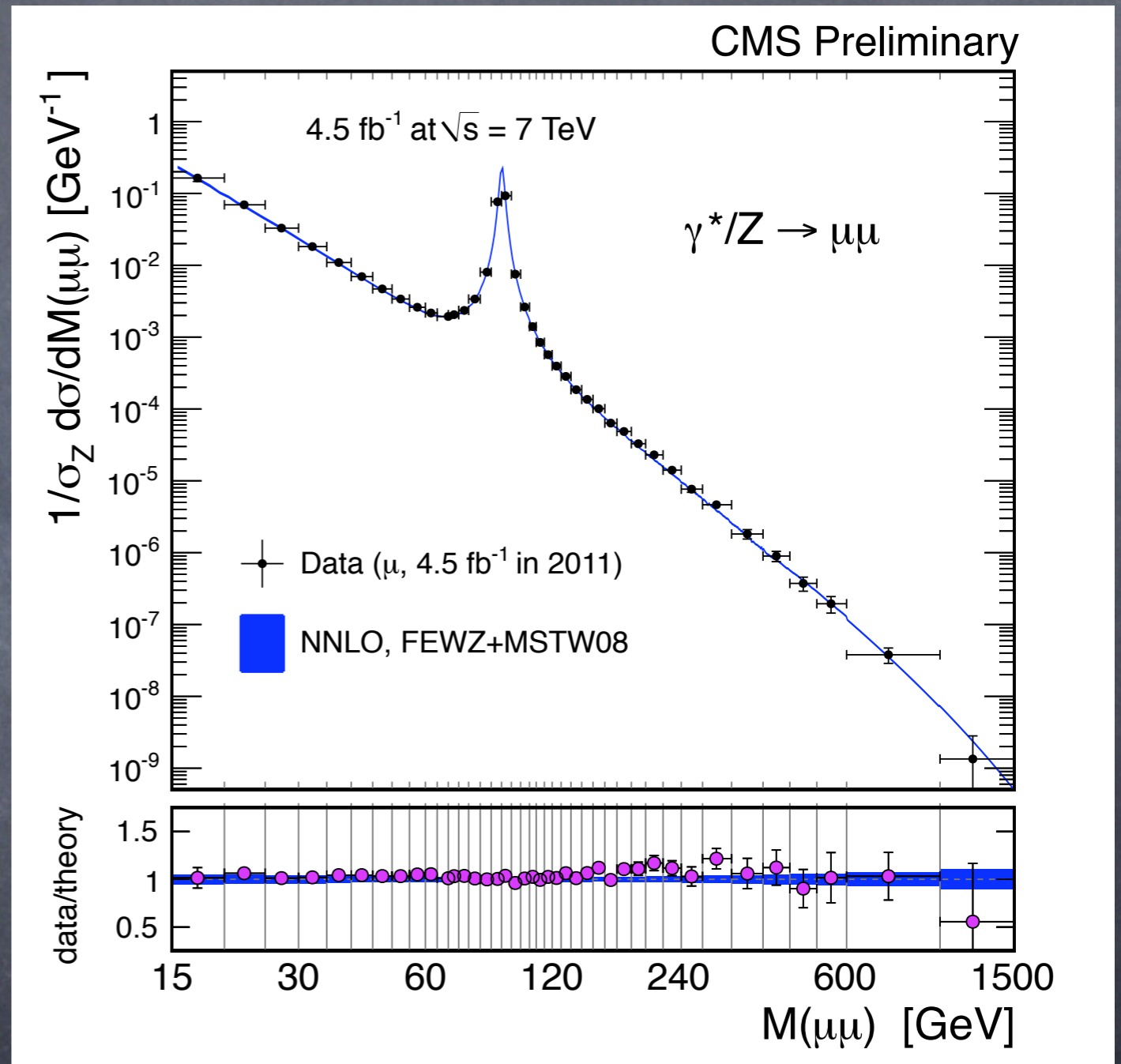
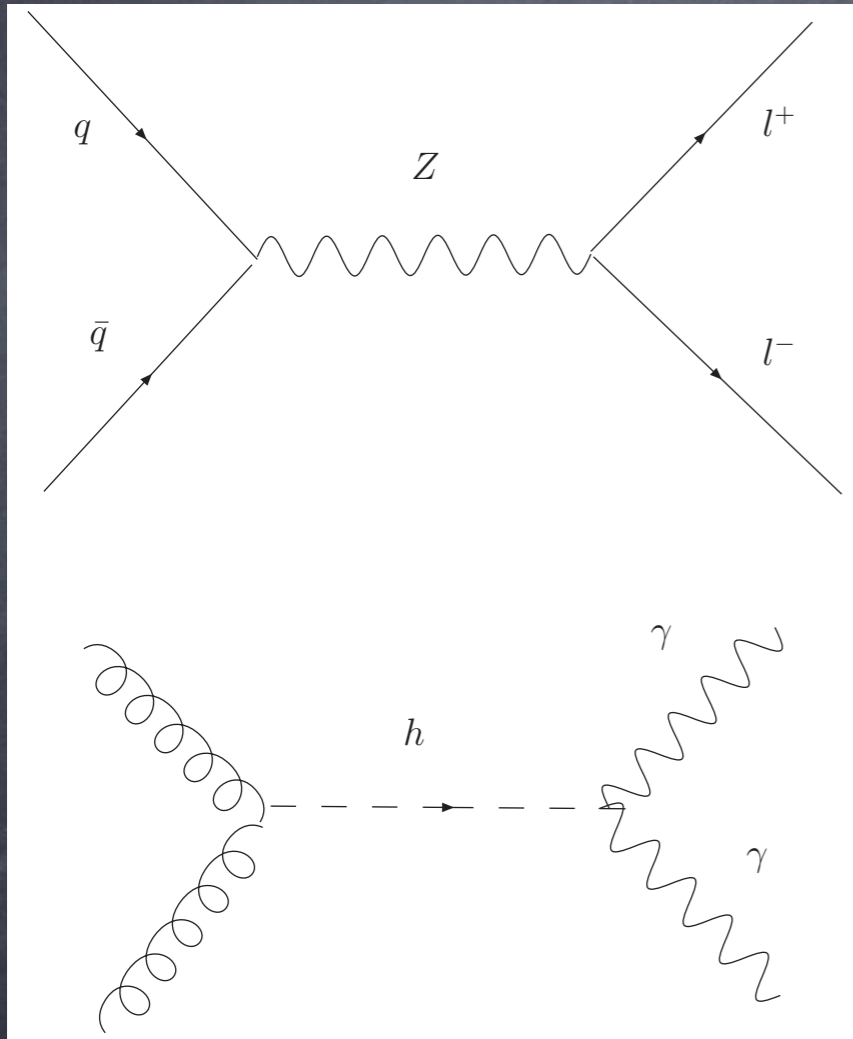


- **kinematics**-based (**i**ndependent of **p**roduction mechanism)

SEVERAL TECHNIQUES  
SO FAR  
(MANY CASES)

# Fully visible I (“golden”)

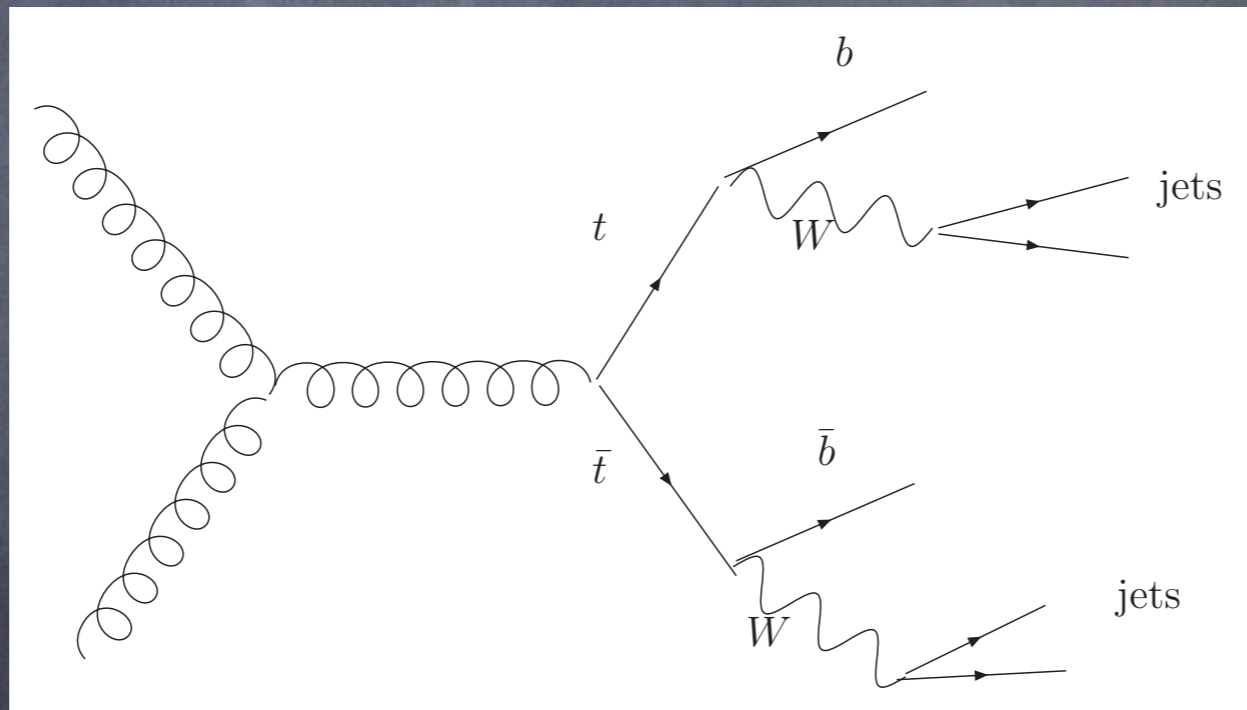
- invariant mass of decay products has **Breit-Wigner** peak



- have to be “lucky”!

# Fully visible II (**not** so easy)

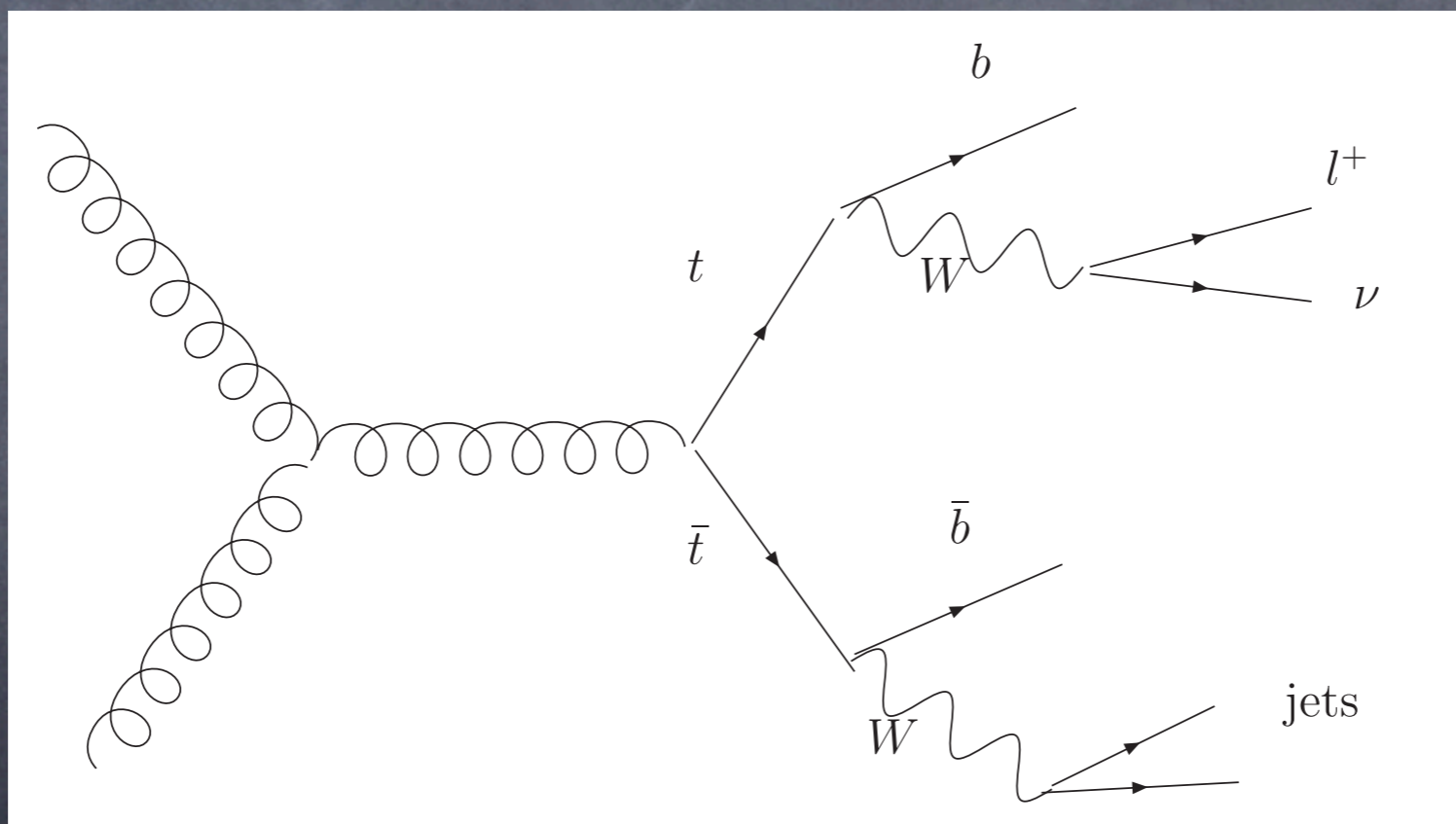
- **fully** hadronic top decay



- problem: **all** jetty + **combinatorics** (**compounded** by jets from initial/final state radiation)

# “Partially” visible I (can be reconstructed)

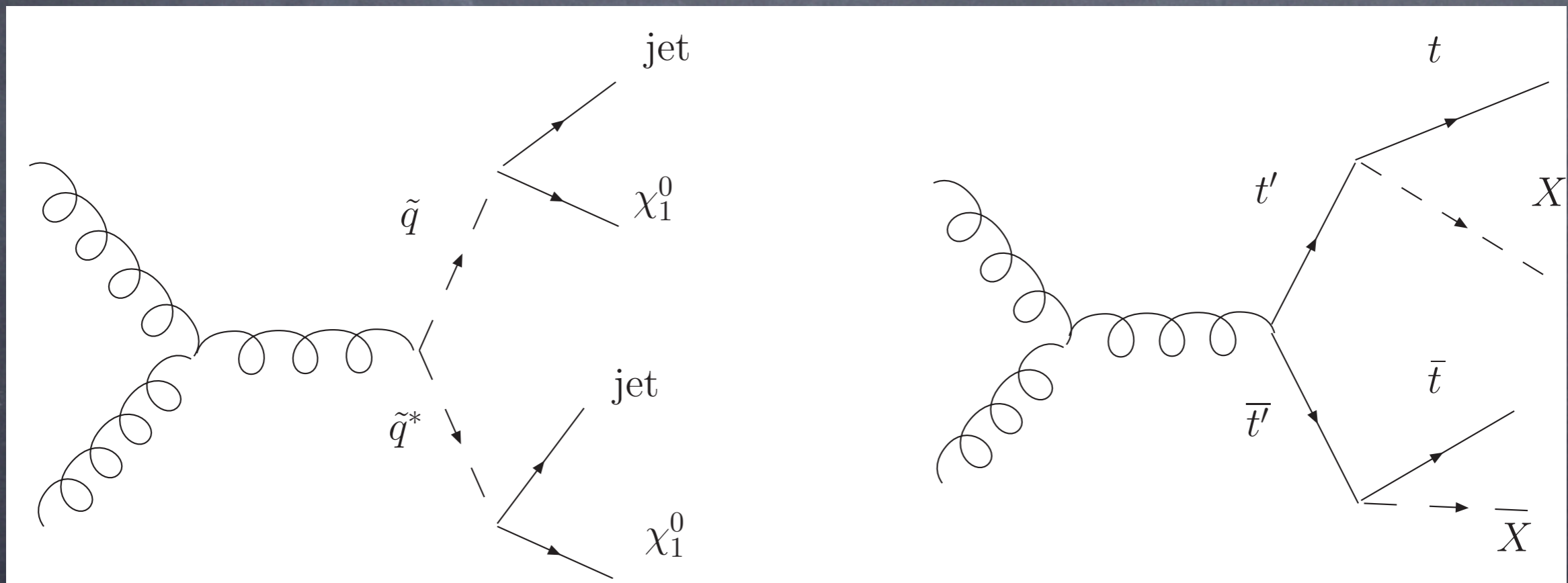
- 1 daughter fully visible, other partially
- semileptonic top decay (“less” jetty)



- still “issues”: discrete ambiguity in reconstructing  $W$ ; uses MET; still combinatorics (which  $W$  with which  $b$ )...

# “Partially” visible II (cannot be reconstructed)

- 1 daughter fully visible, other fully invisible (maybe DM)
- R-parity conserving SUSY, top-partner in T-parity little Higgs models...



- (generalized) transverse mass ( $M_{T2}$ ): uses MET
- razor:  $M_R$  based on (plausible) assumptions about boosts

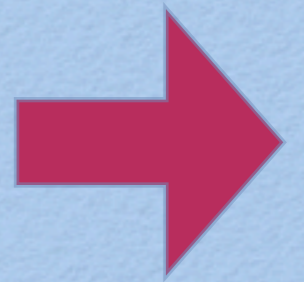
# Bottomline: (in my opinion)

## no slam dunk!

- useful to have **more** techniques, especially simpler; **complementary** (**different** systematics, e.g., **avoid** MET or combinatorics or assumptions about boosts)

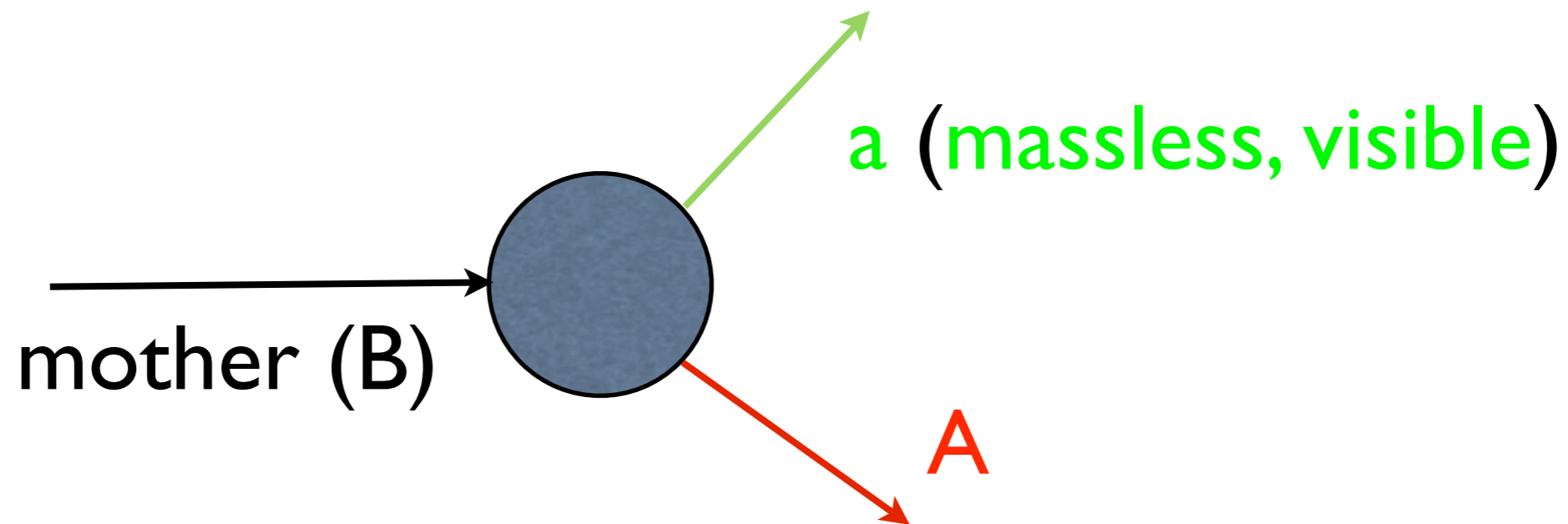


NEW OBSERVATION  
TECHNIQUE



# Basic assumptions

- 2-body decay: one daughter (fully) **visible, massless**:



- ...other (A) **don't** care (almost)!
- more assumptions later
- extensions/generalizations later

# Energy (**not** invariant) of daughter

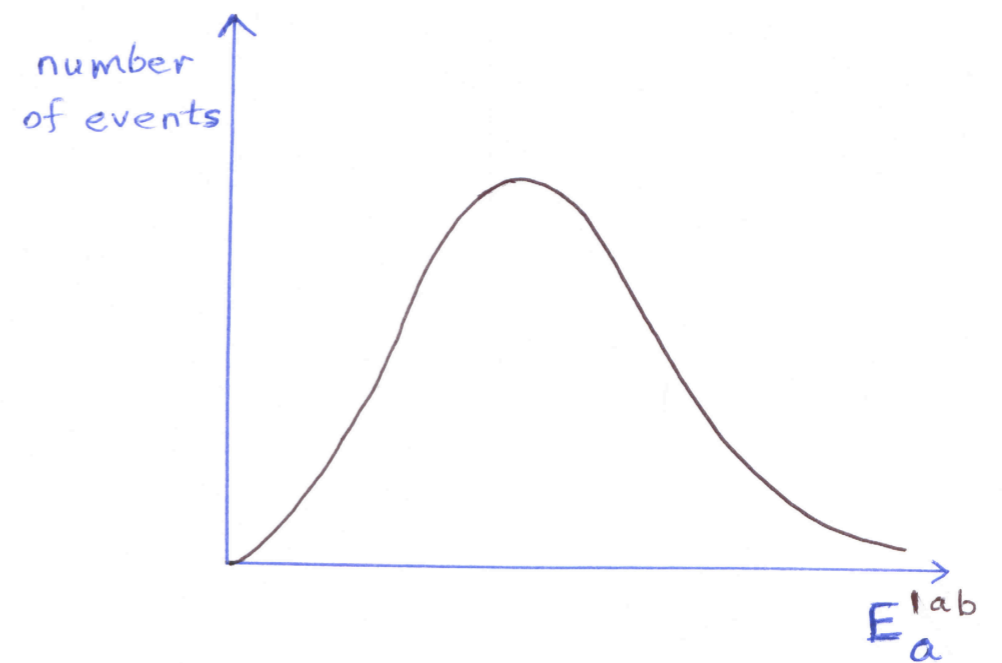
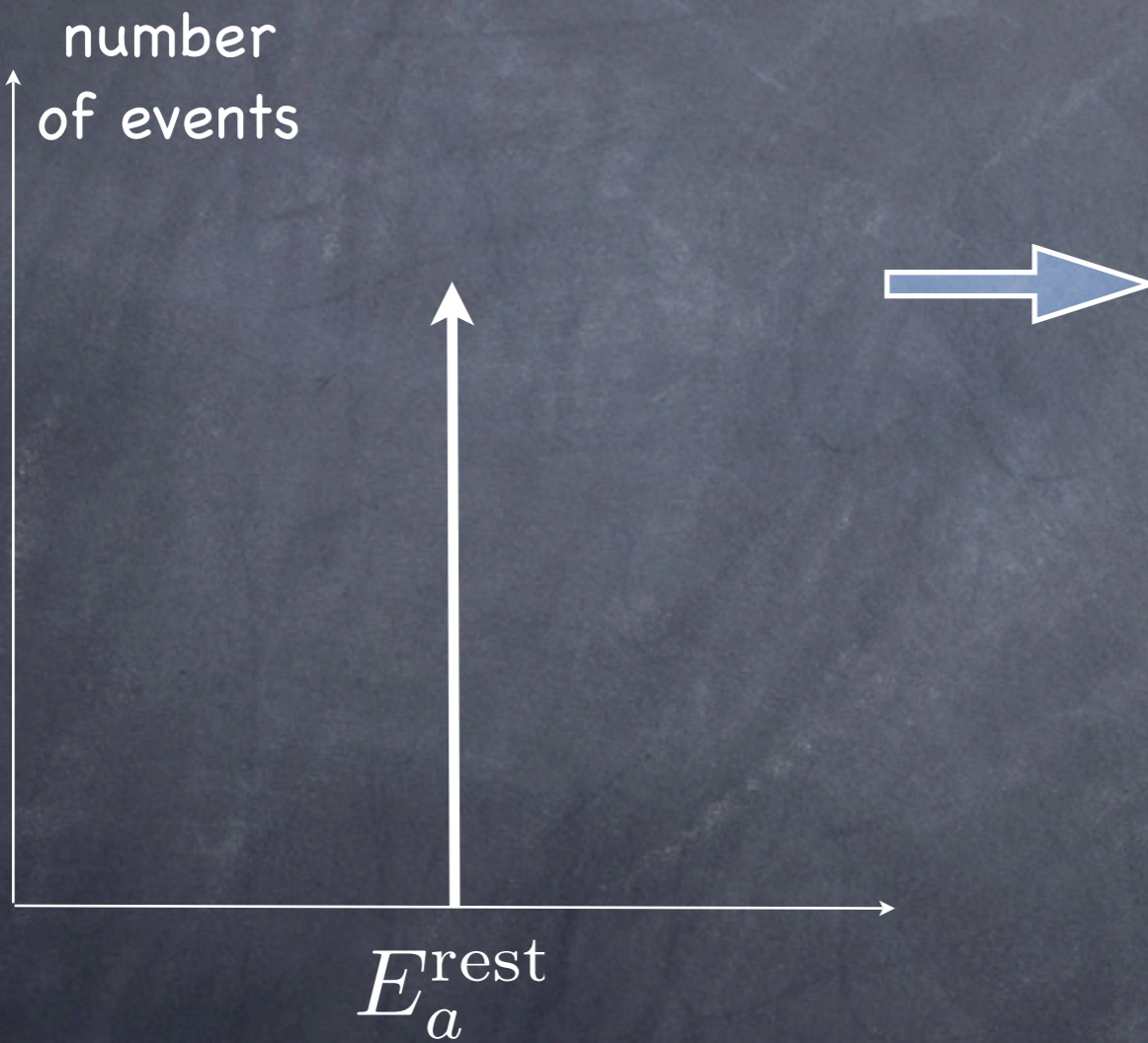
- **mono-chromatic** and **simple** function of masses in **rest** frame of mother:

$$E_a^{\text{rest}} = \frac{M_B^2 - M_A^2}{2M_B}$$

- **determine**  $M_B$  if  $M_A$  known and  $E_a^{\text{rest}}$  measured

...**too** simple to be practical/useful?!

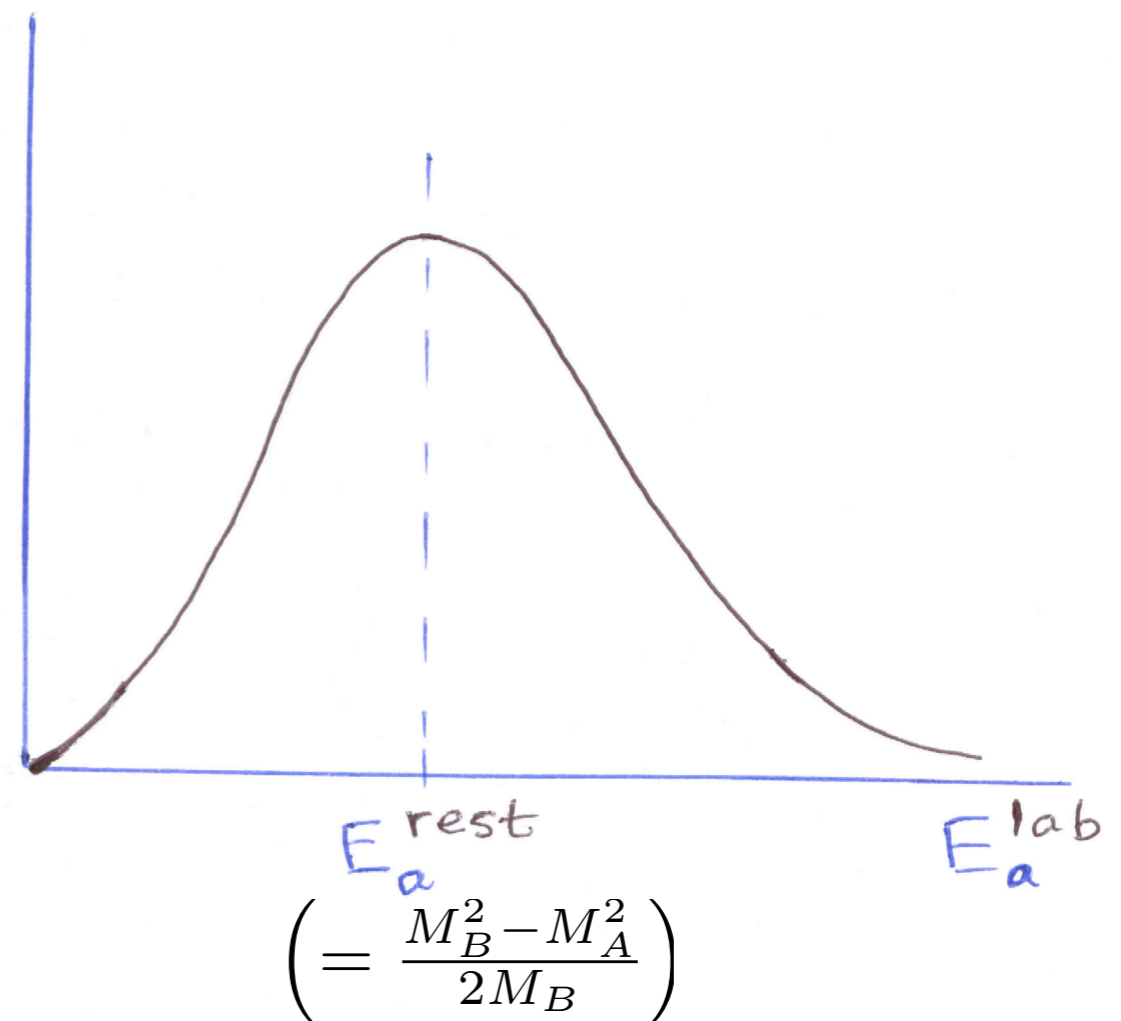
- hadron collider: mother has **unknown boost**;  
varies event to event  $\rightarrow$  **distribution** in  $E_a^{\text{lab}}$



- lose** rest-frame information?!

# Outline

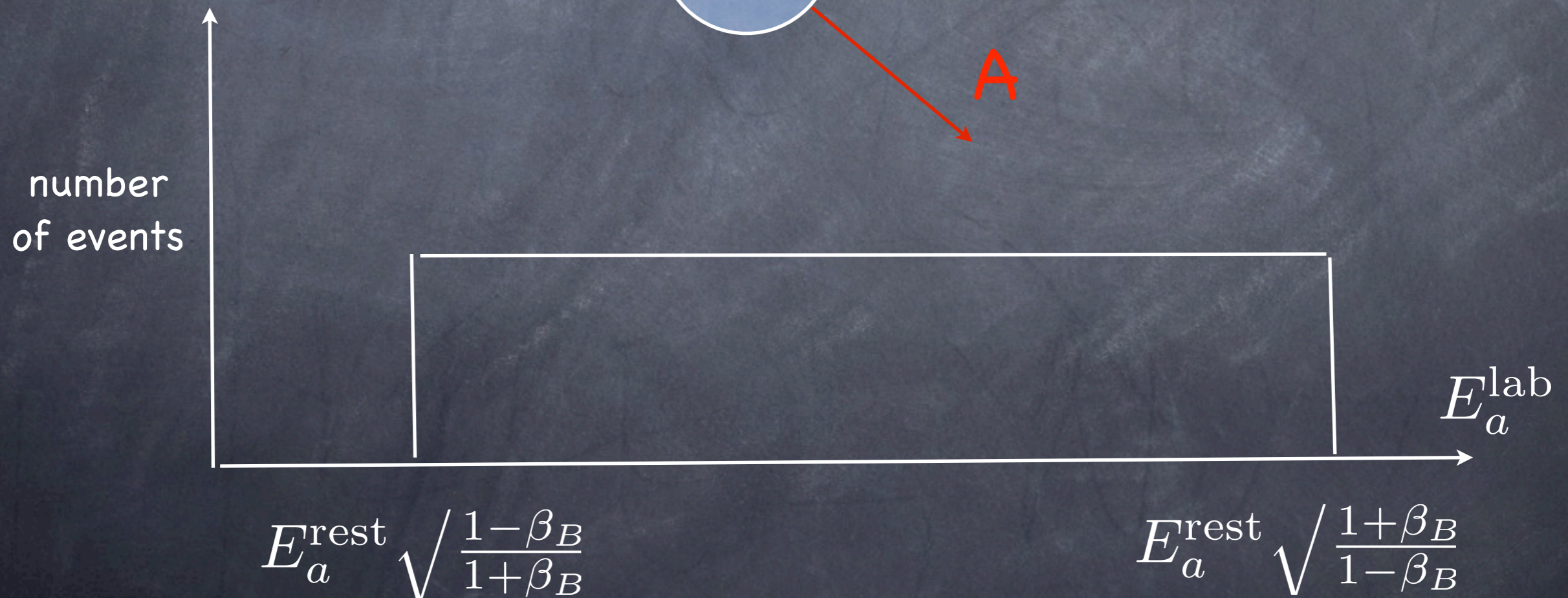
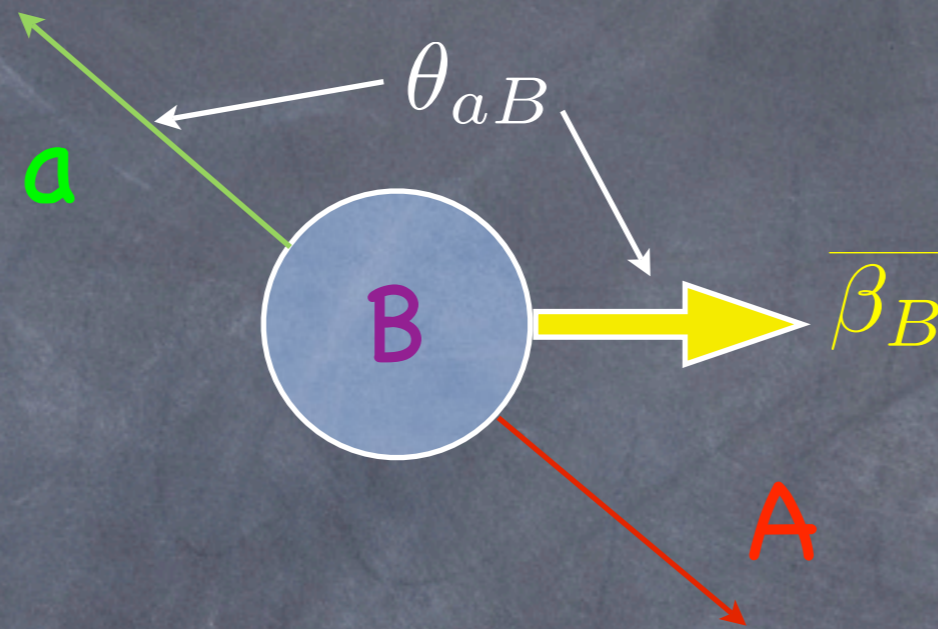
- **Peak** (of lab. distribution) still **retains** this information...as **simply, precisely, robustly!**
- “Test” **application** (**top** mass):  
obtain approximation to theory curve  
Fit it to (simulated) data for extracting peak
- **New physics** (**Cascade** decay): *number of events*  
**general** idea  
**SUSY** example
- **Three**-body decay
- **Conclusions**



“INVARIANCE” OF TWO-  
BODY DECAY KINEMATICS

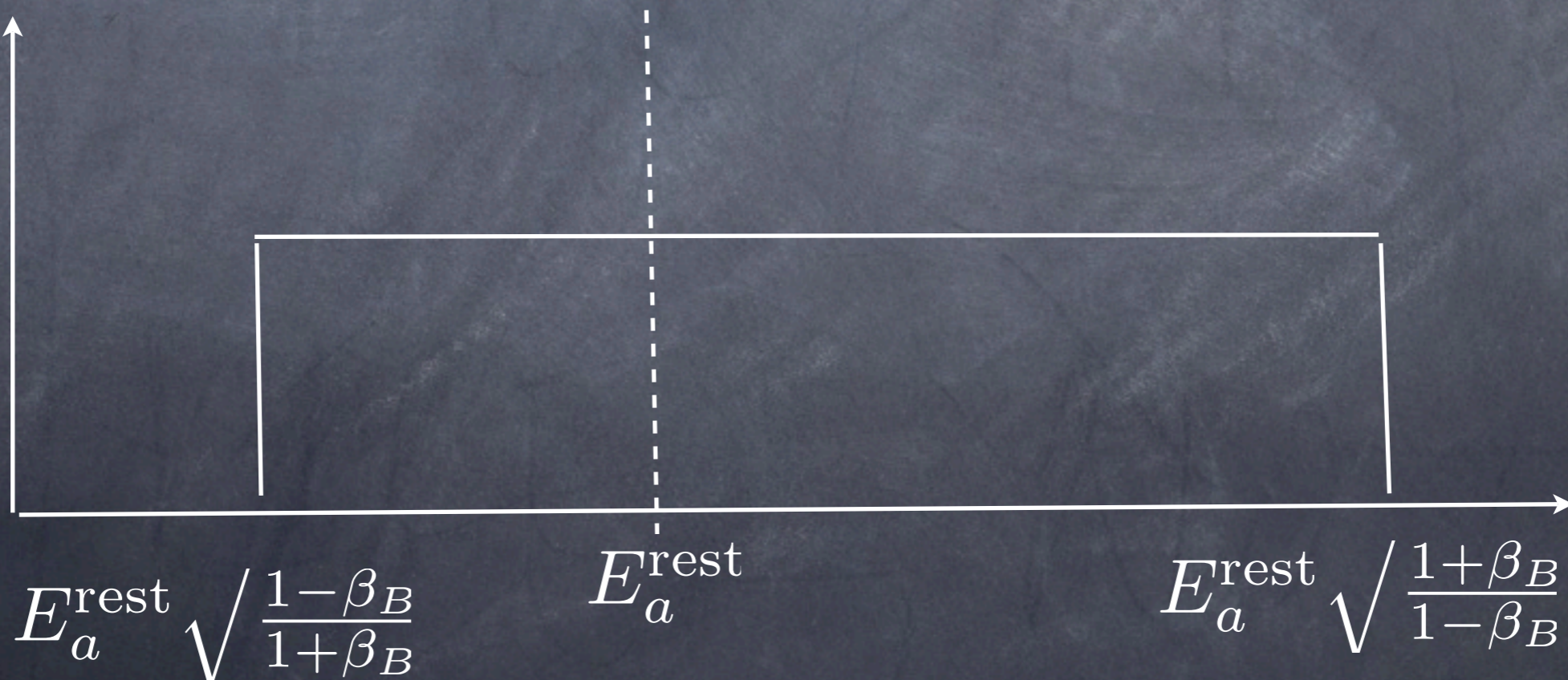
# Rectangle for **fixed**, but **arbitrary** boost

- In general:  $E_a^{\text{lab}} = E_a^{\text{rest}} \gamma_B (1 + \beta_B \cos \theta_{aB})$
- Assume unpolarized mother:  $\cos \theta_{aB}$  is flat



# Rectangle vs. rest energy

- contains  $E_a^{\text{rest}}$  (for **any** boost)
- no other**  $E_a^{\text{lab}}$  gets **larger** contribution from given boost than does  $E_a^{\text{rest}}$
- no other**  $E_a^{\text{lab}}$  is contained in **every** rectangle (e.g.,  $\beta_B \rightarrow 0$ )
- asymmetric** on linear (symmetric on **log**...)



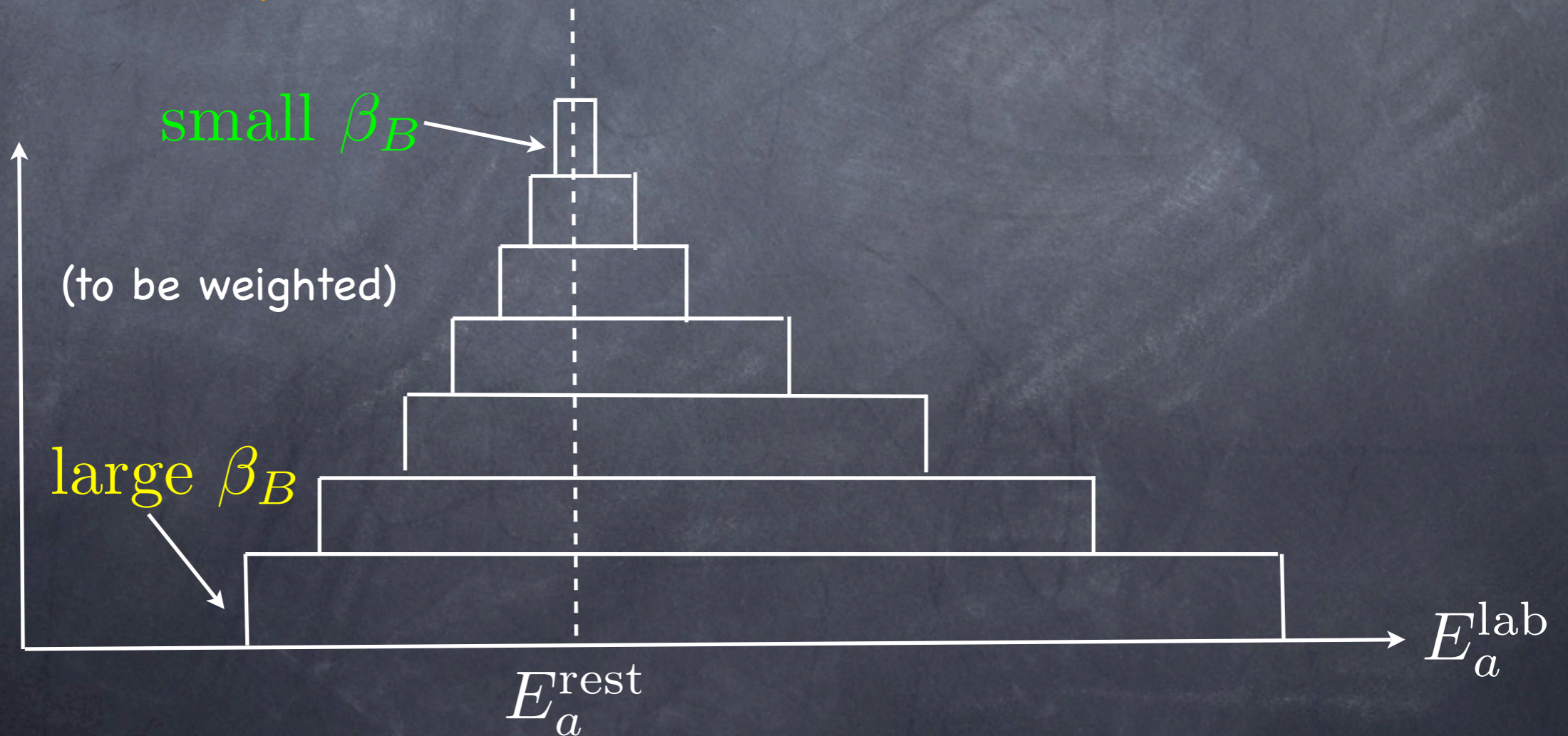


# (Generic) Boost distribution: "stacking" up rectangles

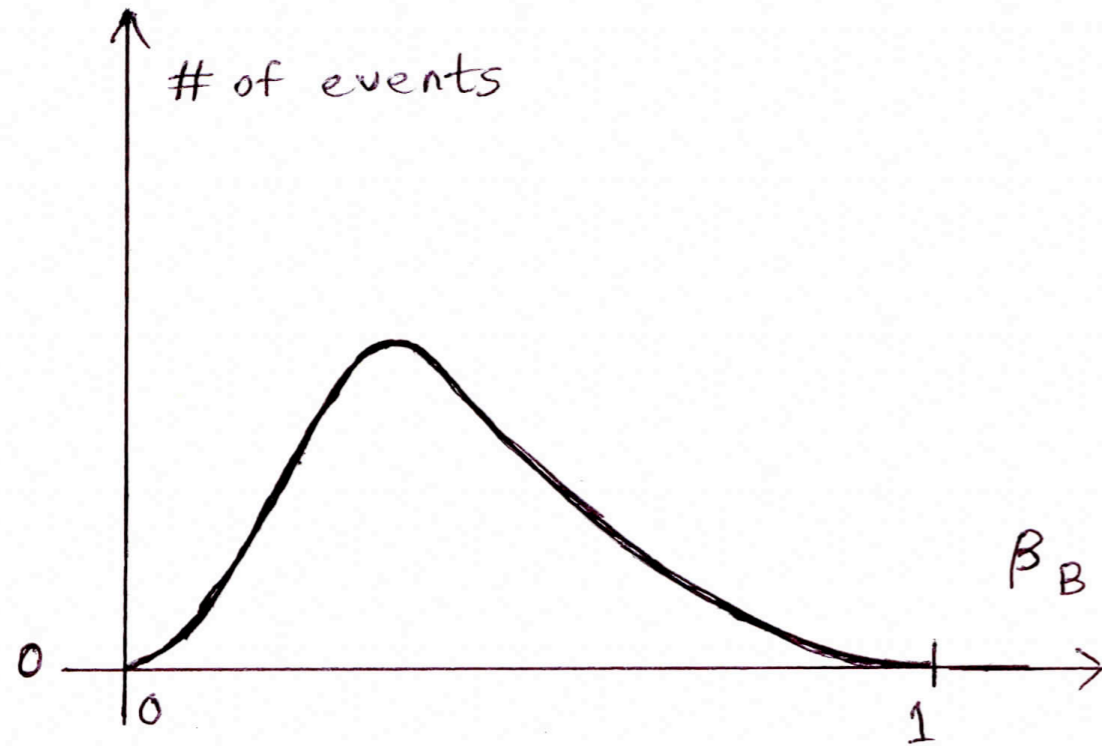
(KA, Franceschini, Kim: 1209.0772)

(see also Stecker: "Cosmic gamma rays")

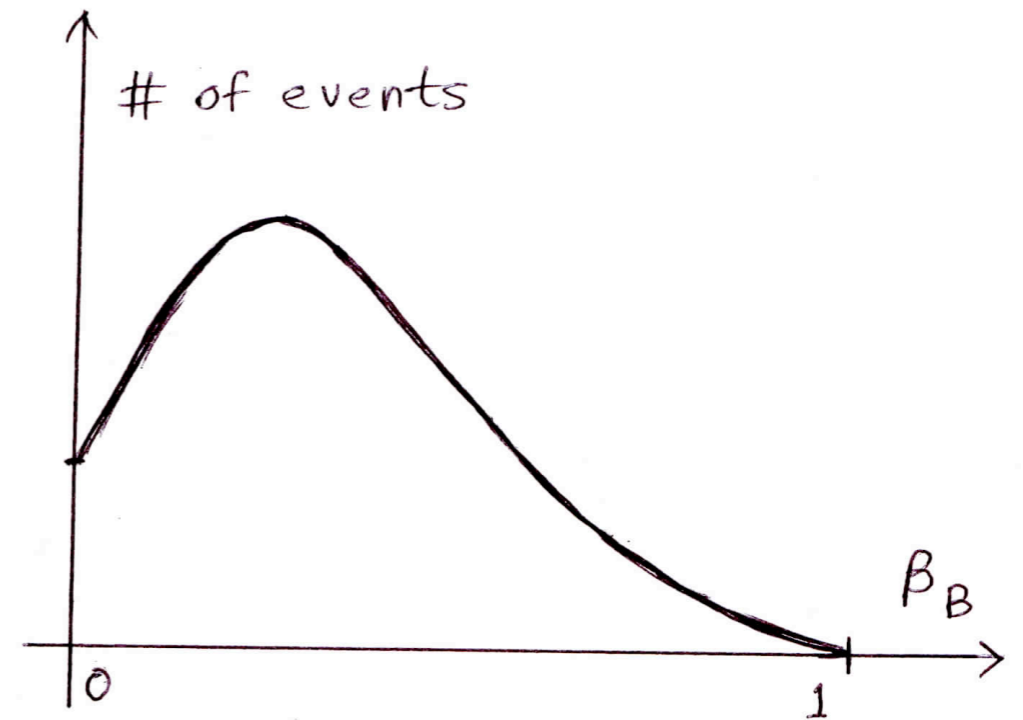
- distribution of  $E_a^{\text{lab}}$  has **peak** at  $E_a^{\text{rest}}$
- ...**no matter** what is the **boost distribution!**
- boost distribution depends on **production mechanism, mother mass, PDF's...**



# Boost distributions: I & II



boost distribution for  $2 \rightarrow 2$  (previous)

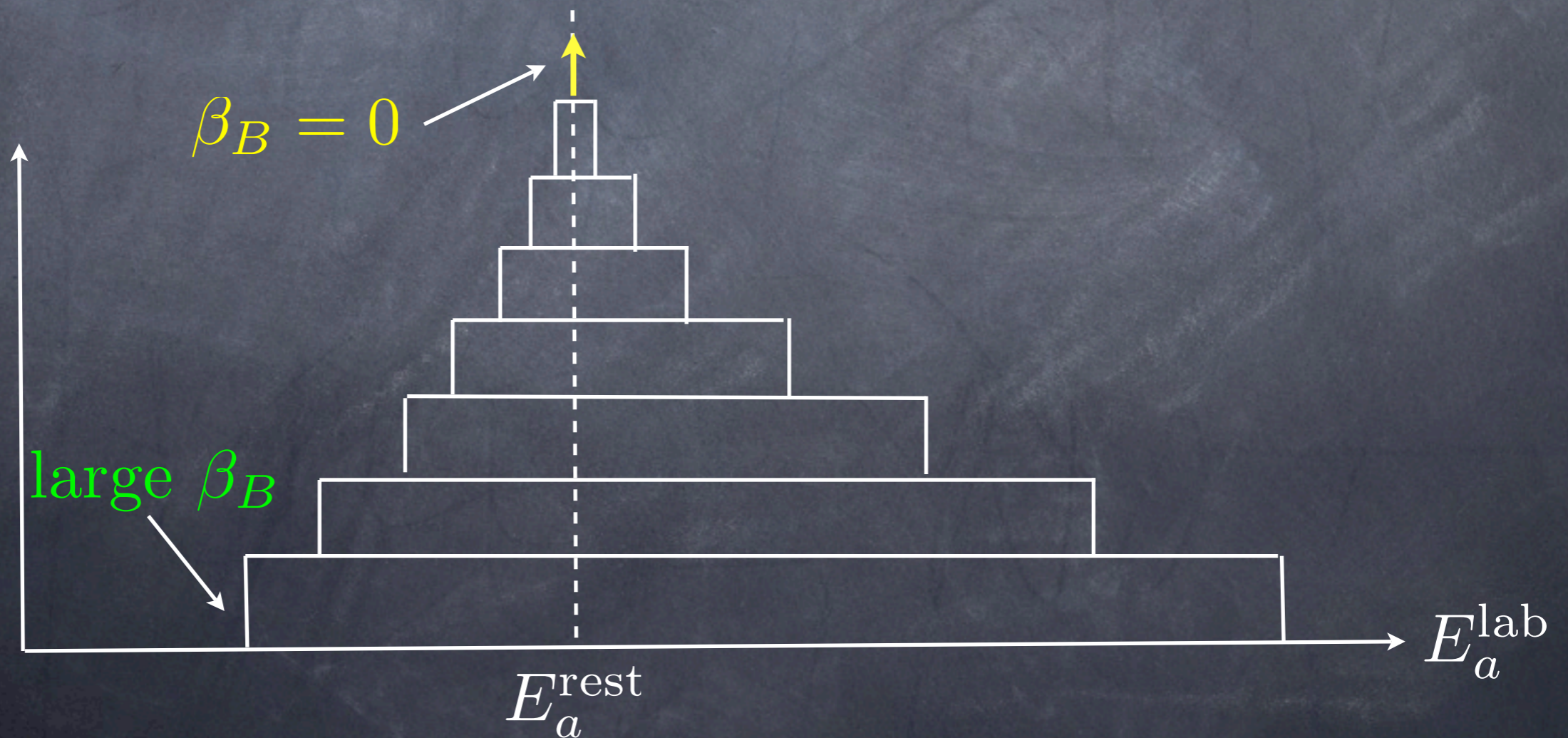


boost distribution for  $2 \rightarrow 1$  (next)

# Single mother production

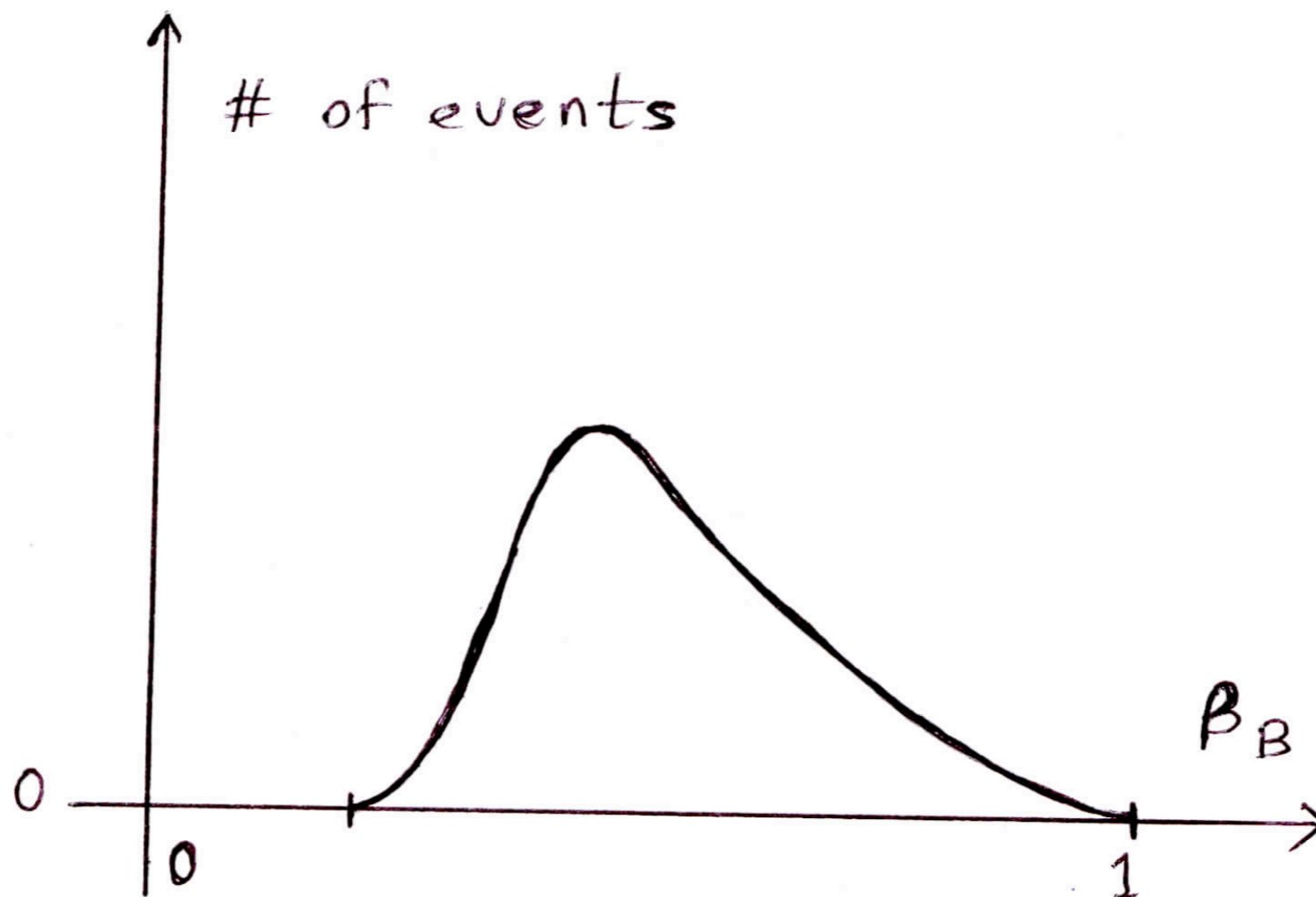
[e.g.,  $gg \rightarrow \text{Higgs}$ ; KK graviton (?): Chen, Davoudiasl, Kim]

- distribution of  $E_a^{\text{lab}}$  has "kink" at  $E_a^{\text{rest}}$



# Boost distributions: III

- Due to cuts (or highly boosted secondary mother), boost distribution vanishes close to  $\beta_B = 0$



...plateau is not "generic"

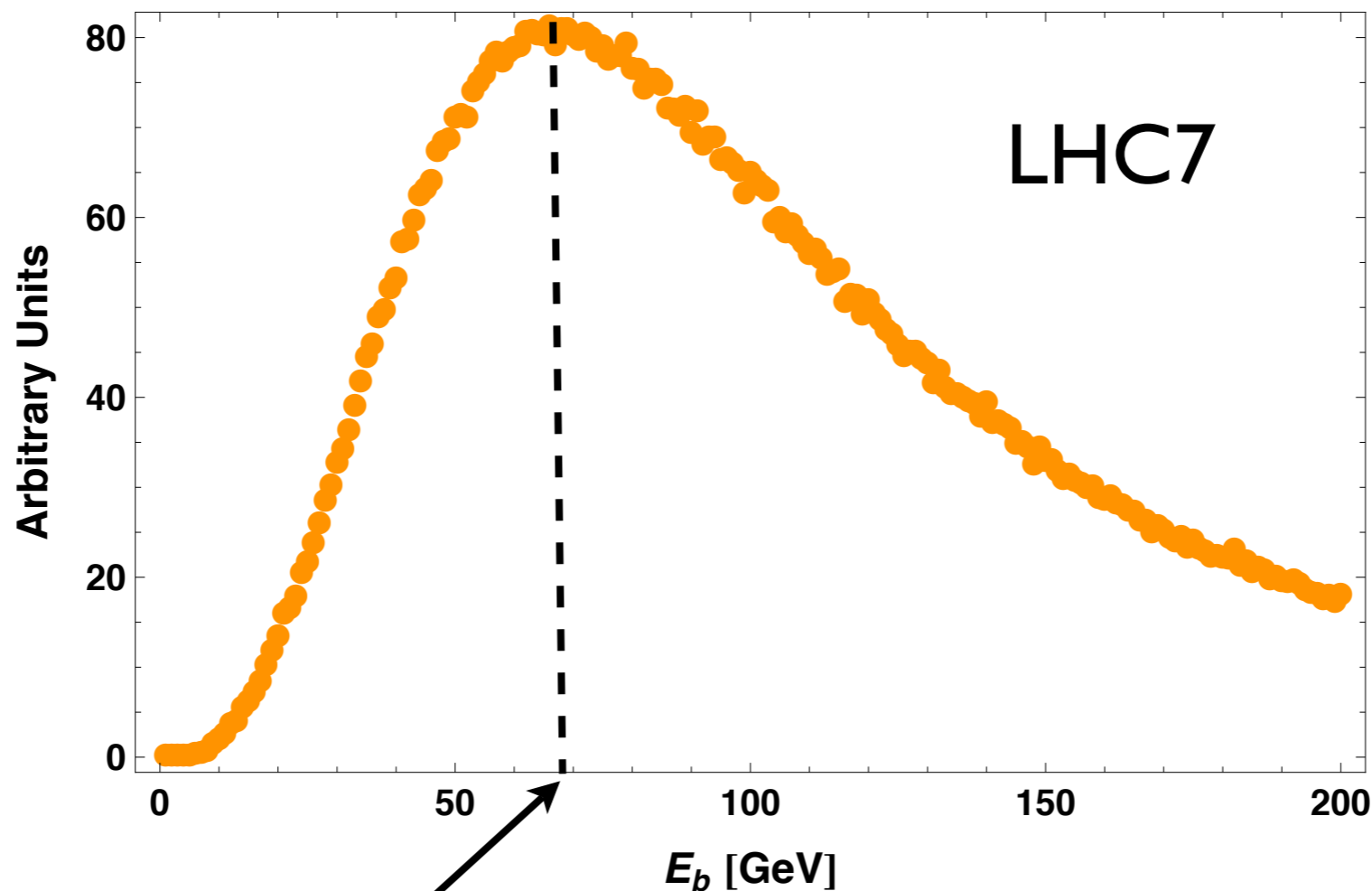


# No need really, but anyway, actual calculation...

- bottom from top quark decay as example:  
bottom mass negligible  $\Rightarrow$  peak is **not** expected to

shift from  $E_b^{\text{rest}} = \frac{M_t^2 - M_W^2 + m_b^2}{2M_t}$

modified

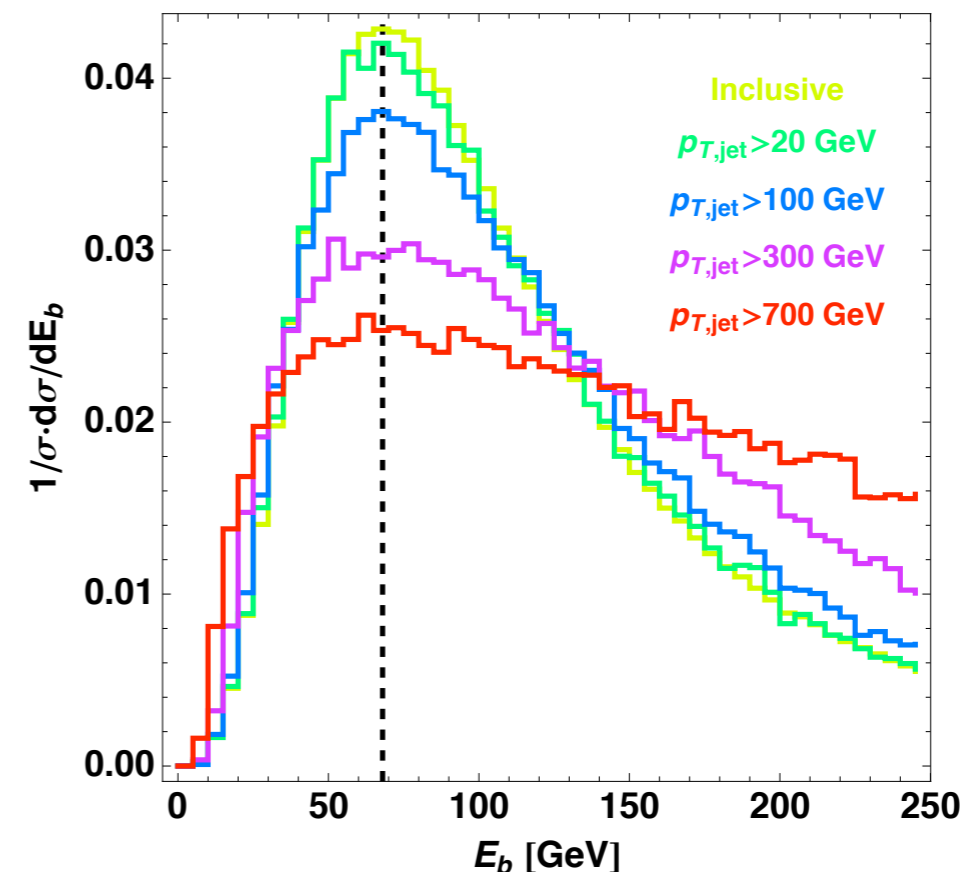
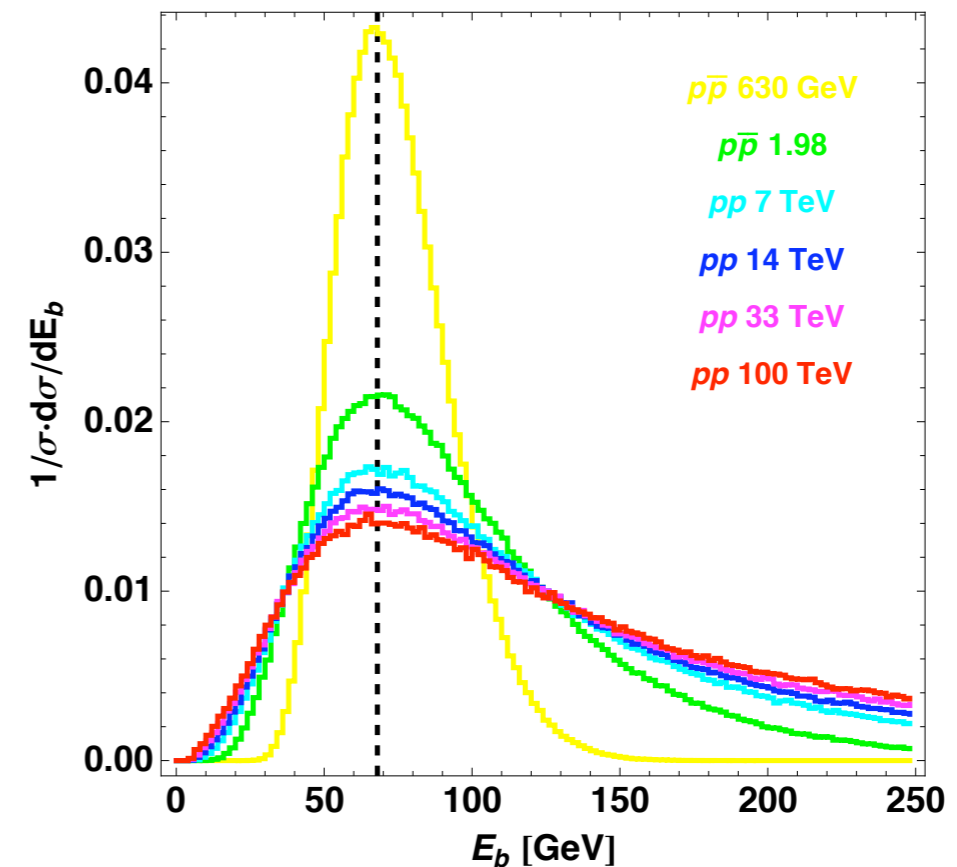


$68 \text{ GeV} = \frac{M_t^2 - M_W^2 + m_b^2}{2M_t} !$

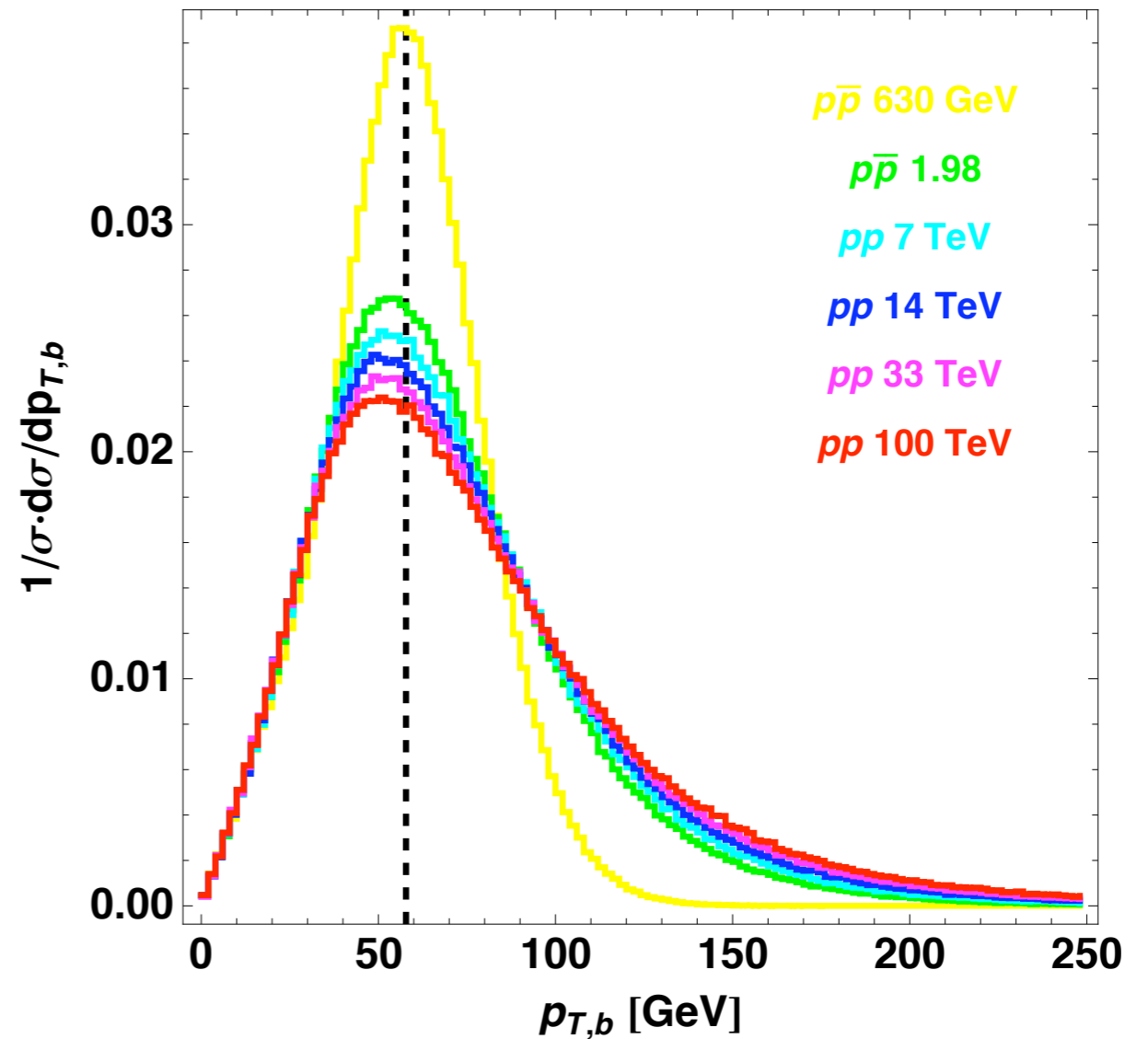
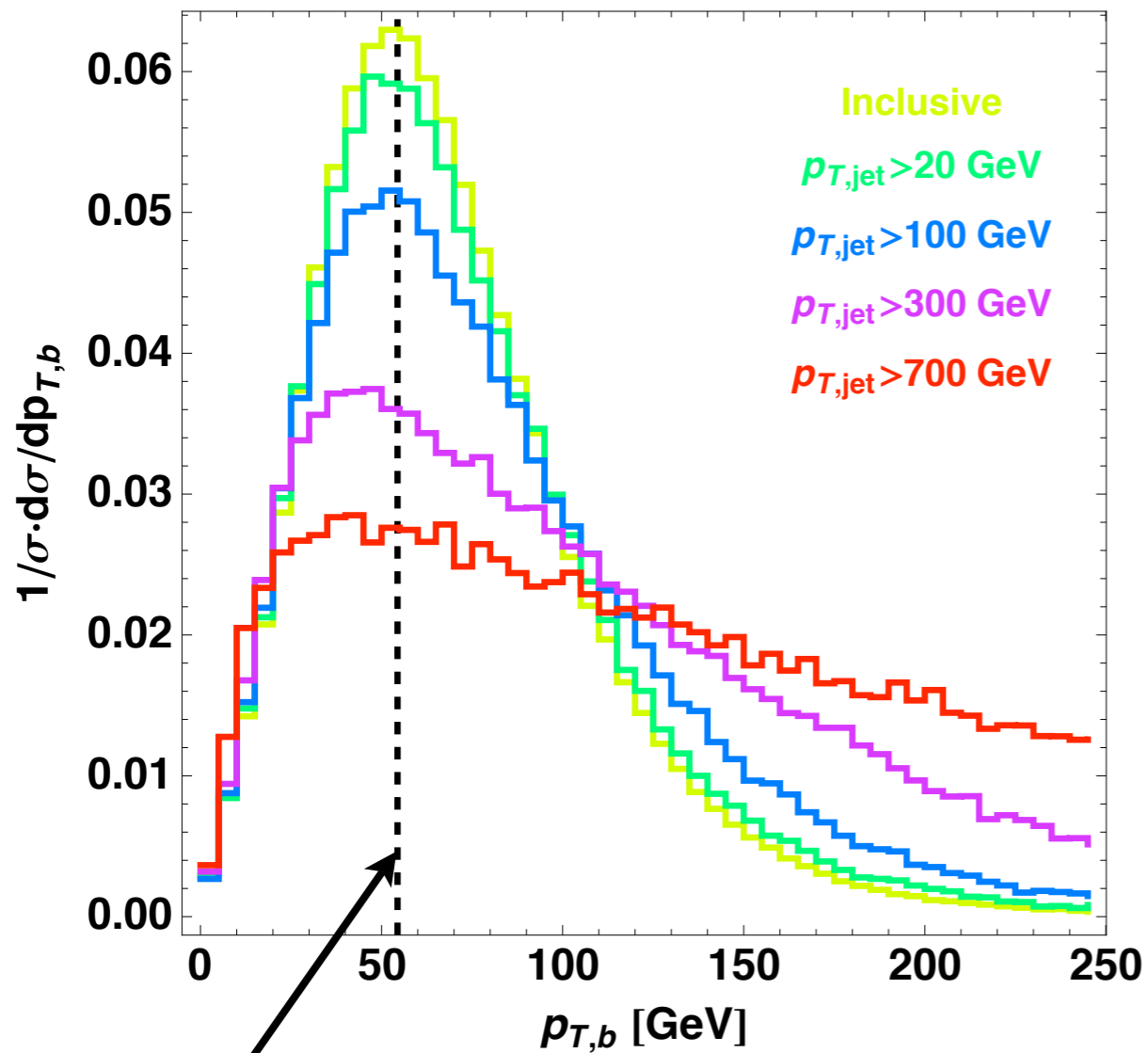
- ...maybe an “accident”?!

# “Invariant” (under boost distributions) feature in non-invariant (energy) distribution: subtle!

- vary collider energy
- vary ISR
- ...but, peak stays put, even though shape changes (broadens for more boosted top)



...accidents don't happen: **no**  
such invariance for  $p_T$ !



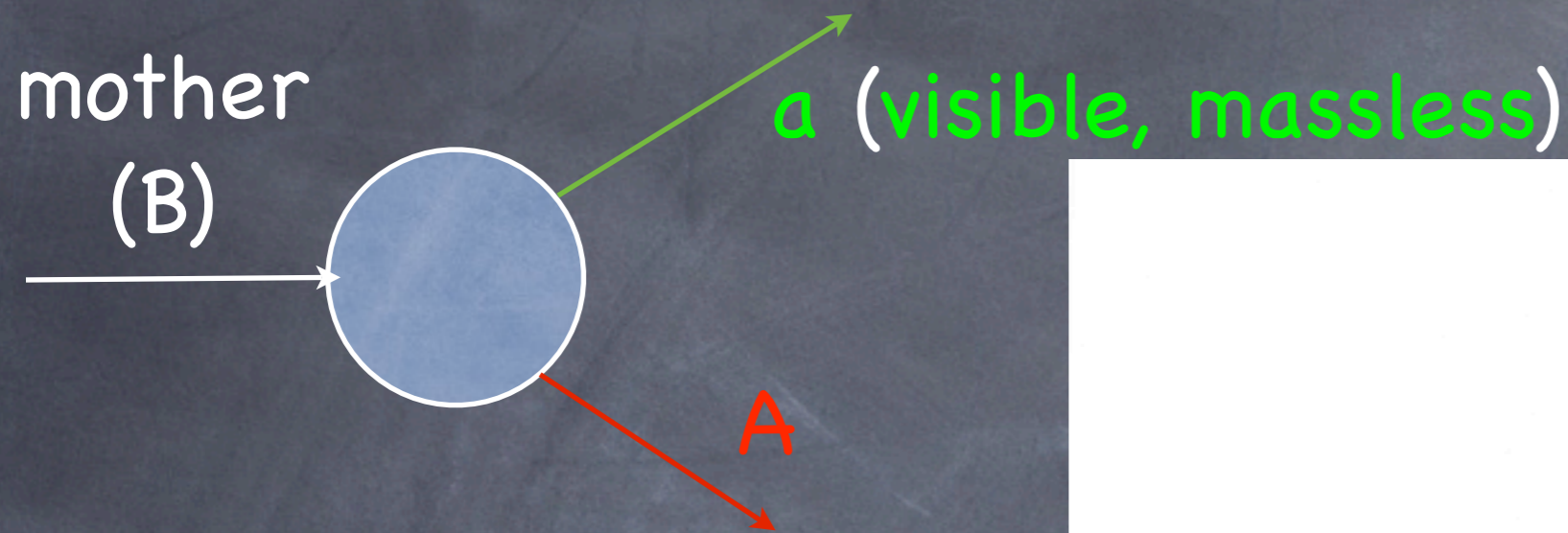
not 68 GeV

- peak (and shape) **change**...



(POSSIBLE) APPLICATIONS:  
"BETTER" ONES TO COME?!

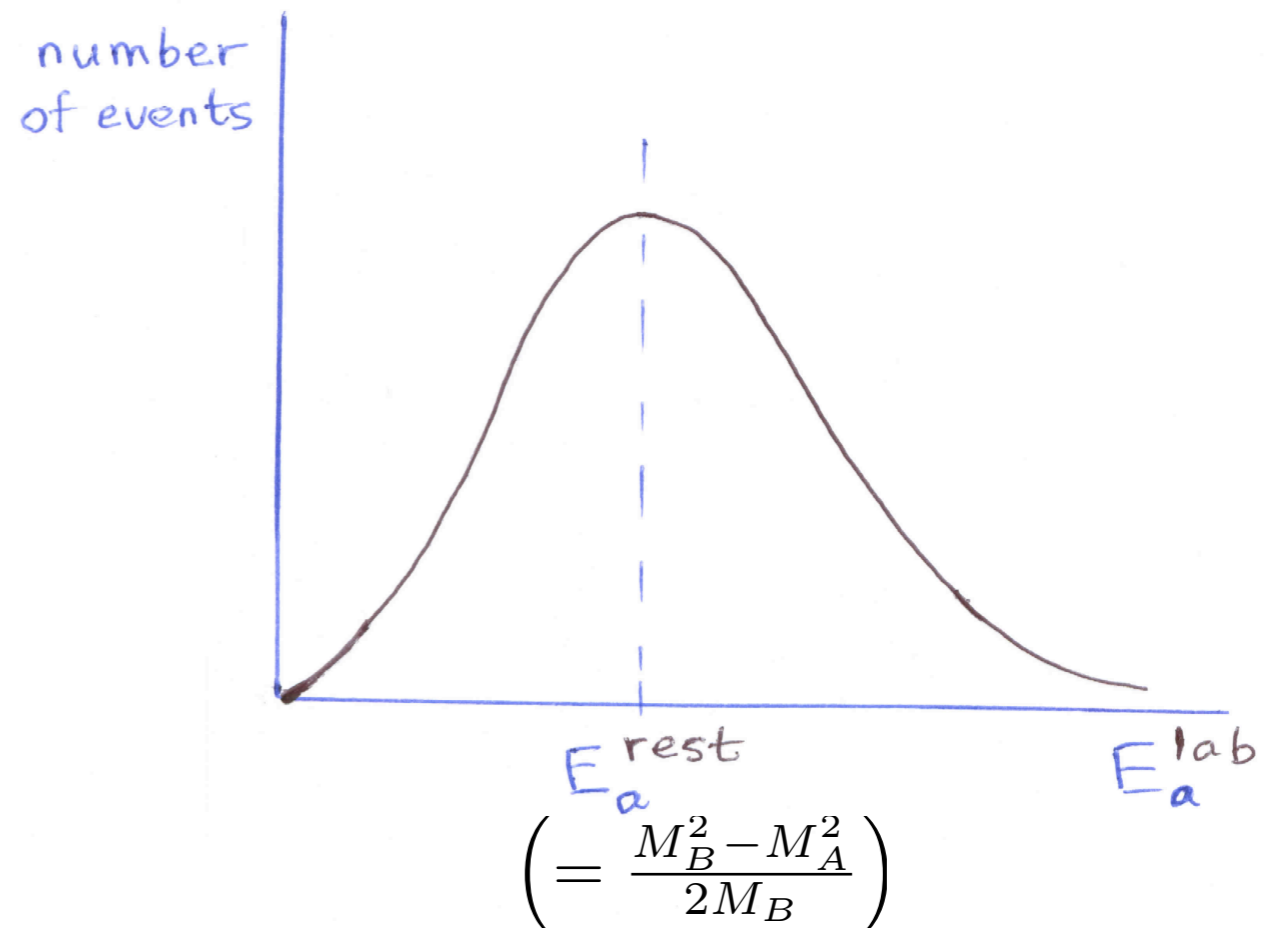
# General, simple Idea



- determine  $M_B$  (if  $M_A$  known) using  $E_a^{\text{rest}}$  (measured from peak in  $E_a^{\text{lab}}$ )

$$E_a^{\text{rest}} = \frac{M_B^2 - M_A^2}{2M_B}$$

(independent of production mechanism of unpolarized mother)



# Measuring the peak

- peak can be **wide** (**difficult** to read-off value "by eye")
- extract peak by fitting to "theory curve":  
a la **Breit-Wigner** [**simple** (2-parameter), analytic, model-**independent** function]
- ...but exact, **analytic** formula **difficult** to obtain here  
(depends on boost distribution, thus PDF's...)



APPROXIMATION TO  
THEORY CURVE

# Do know (analytically) properties of distribution, $f$

- value of  $f(x)$  remains the same under  $x \leftrightarrow \frac{1}{x}$  ( $x \equiv \frac{E_a^{\text{lab}}}{E_a^{\text{rest}}}$ )
- $f$  is maximized at  $x = 1$
- $f$  vanishes as  $x$  approaches 0 or  $\infty$
- $f$  becomes a  $\delta$ -function in some limit of its parameters

# Ansatz (based on properties)

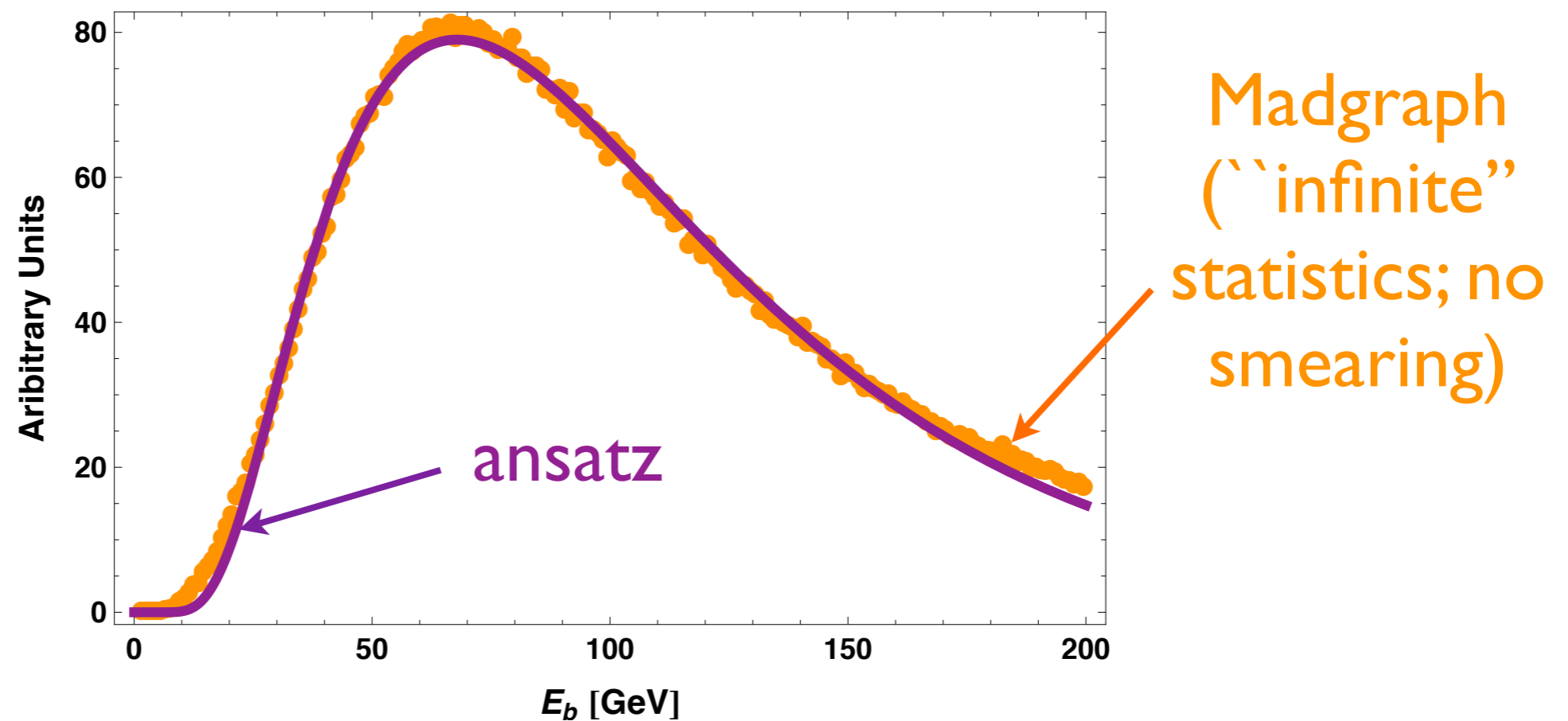
$$f(x) = K_1^{-1}(p) \exp \left[ -\frac{p}{2} \left( x + \frac{1}{x} \right) \right]$$

Bessel function (normalization)

width parameter

- simple, but not unique "peak finder"...

# “Test” on b-jet energy from top quark decay (production unpolarized...)



- bottom (almost) “massless”: peak does not shift, shape property negligibly violated
- good fit for heavier “top” quark as well: different PDF’s, boost distribution (width parameter encompasses this variation)

# "New" Breit-Wigner

- Based on theory fits, **assume**

$$f(x) = K_1^{-1}(p) \exp \left[ -\frac{p}{2} \left( x + \frac{1}{x} \right) \right]$$



FURTHER TEST: FIT TO  
(SIMULATED) DATA

# (Again) Top quark decay: basic idea

neglect  $m_b$  in  $E_b^{\text{rest}}$



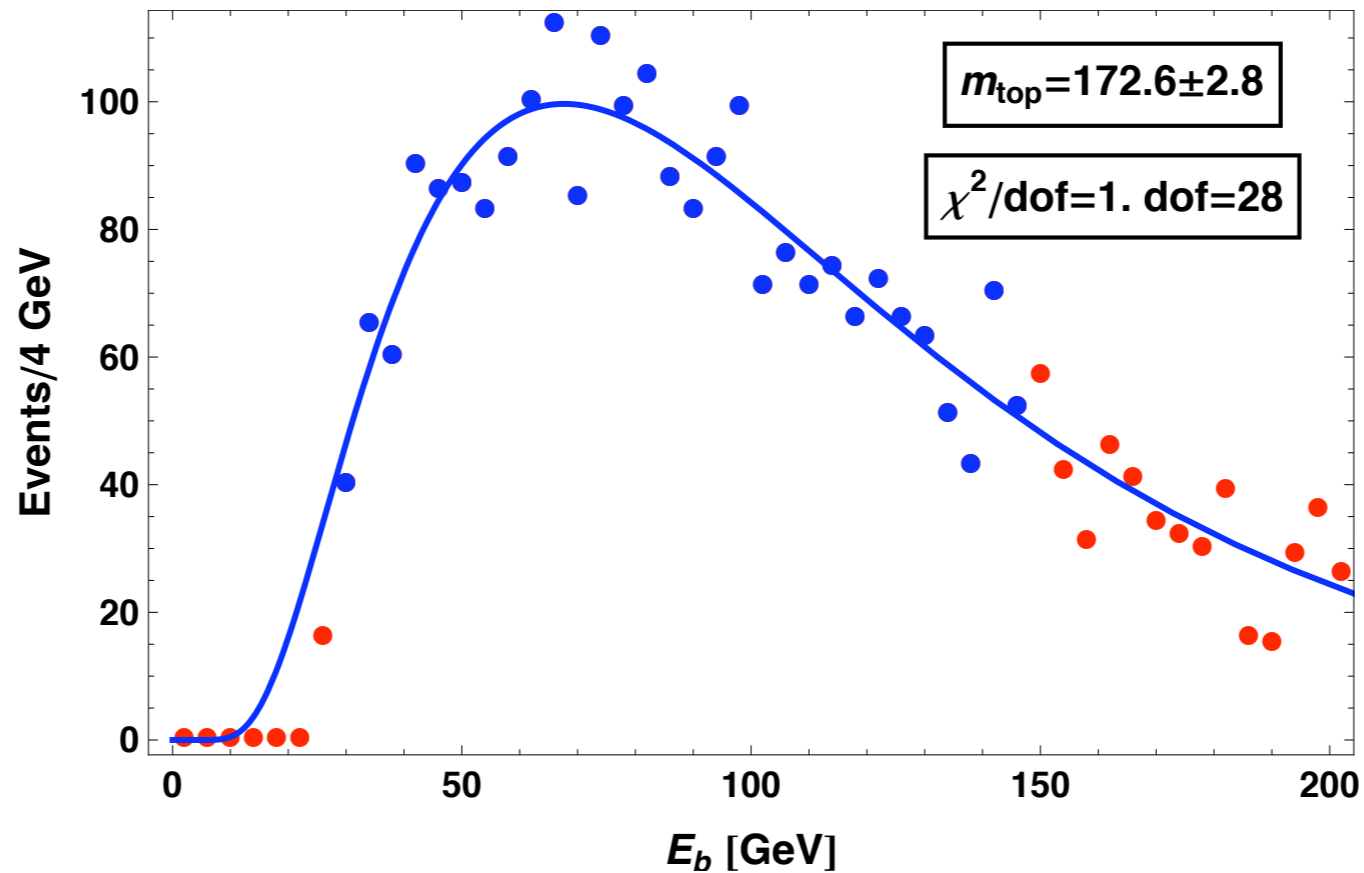
- Peak in measured b-jet energy distribution  $\approx \frac{M_t^2 - M_W^2}{2M_t}$
- Assuming  $M_W$  (but **no** need to **detect** it at all!), get  $M_t$

# Top mass measurement: details

- Fully **leptonic** (**opposite** flavor) and 2 b-tags, with 5/fb at LHC7: expect 4000 **S** vs. 200 **B**
- Madgraph  $\longrightarrow$  Pythia  $\longrightarrow$  Delphes/Fastjet
- 100 pseudo-experiments
- ATLAS/CMS choice of (**mild**) **cuts**:  
**1209.2393; ATLAS-CONF-2012-097**
- **neglected** background

# Result

(1 pseudo-experiment shown)

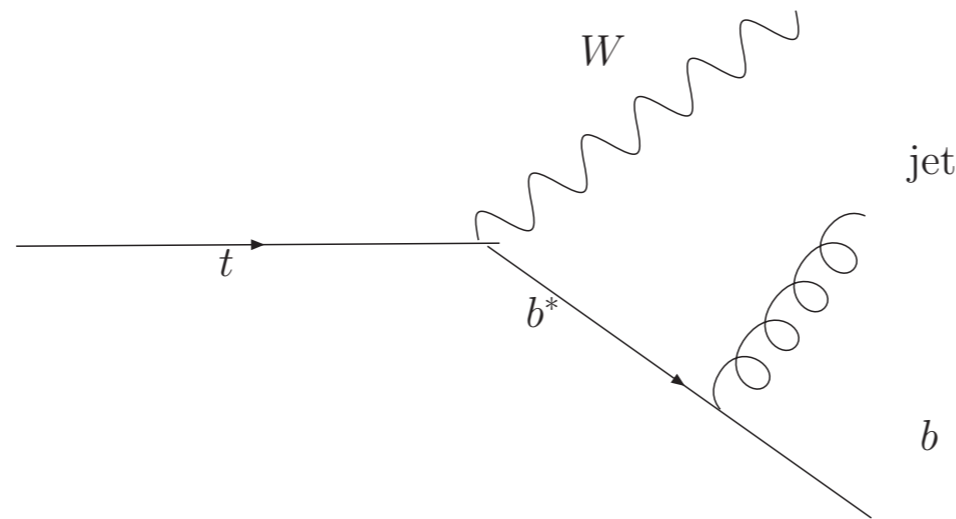


(use only  
blue dots)

- consistent with input value
- fitting not spoiled by cuts or detector effects


# Discussion

- **neglected hard** radiation from bottom (**3-body**):  
suppressed by  $\alpha_s/\pi$  + **jet-veto** (**calculable** in **QCD**)



- **safe** from **soft** radiation off of bottom
- **safe** from **initial** state radiation
- **no** combinatorics (include both  $b$ 's)
- **in**dependent of production mechanism (single or pair; uncertainty in PDF's; new physics or SM) as long as **unpolarized**

# Comparison (simplified!) of methods for top mass

- **complementary**/cross-check: **different** systematics, e.g., use of MET in some earlier methods vs. **not** here
- **theory** systematics, based on (**small**) parameters:  
 $\delta_{\text{prod}}$  (PDF's, new physics);  $\epsilon_{\text{FSR}}$  (NLO, jet-veto);  $f_{\text{pol.}}$  (new physics)
- error in top mass with
  - matrix element; full reconstruction (combinatorics):  $\delta_{\text{prod}}$
  - **b-jet energy-peak**:  $\epsilon_{\text{FSR}} \times \delta_{\text{prod.}} + f_{\text{pol.}}$   
  
 $\epsilon_{\text{FSR}}$  for QCD production is **calculable**
- “**test**” for applications to **new** physics

# What about real data?

• CMS email to us in July, 2013:

“...I guess you will be pleased to hear that we have now someone within CMS who is planning to try an mt extraction with the 8 TeV data following **your** Ansatz....

However, since that group is only starting now, we can't expect to see results **too soon...**”

...October, 2013: contact with M. Irfan Asghar from CMS about actual implementation!

(**Preliminary** results with **semileptonic** **and** dileptonic)

• Can **ATLAS** be far behind?!

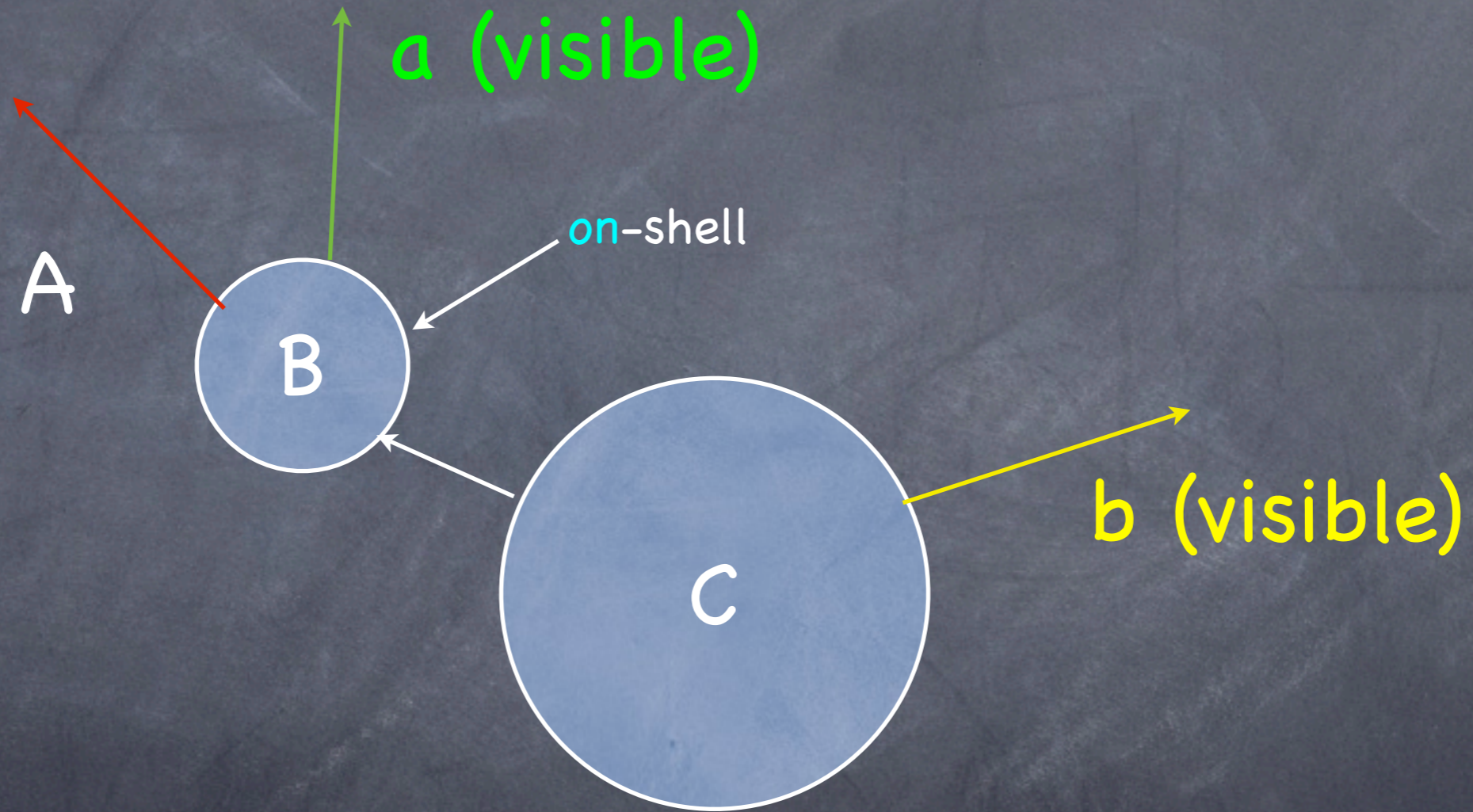
A **NEW** PHYSICS APPLICATION  
(METHOD "TESTED" ON TOP MASS):  
**CASCADE** DECAY

(KA, Franceschini, Kim: 1309.4776)



# In **General**: Topology

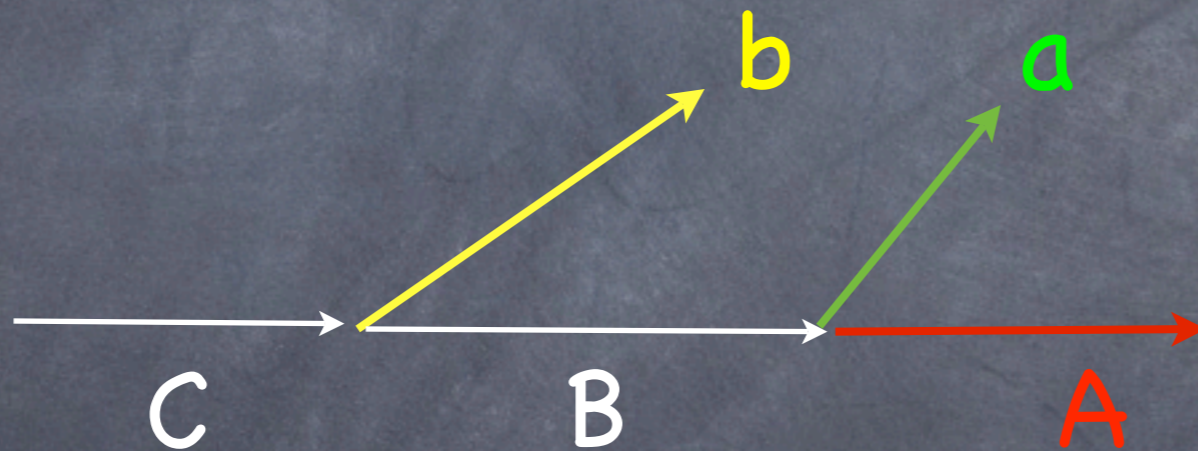
- **Two** 2-body decays: primary (C) and secondary (B) mothers)



# Two energy peaks

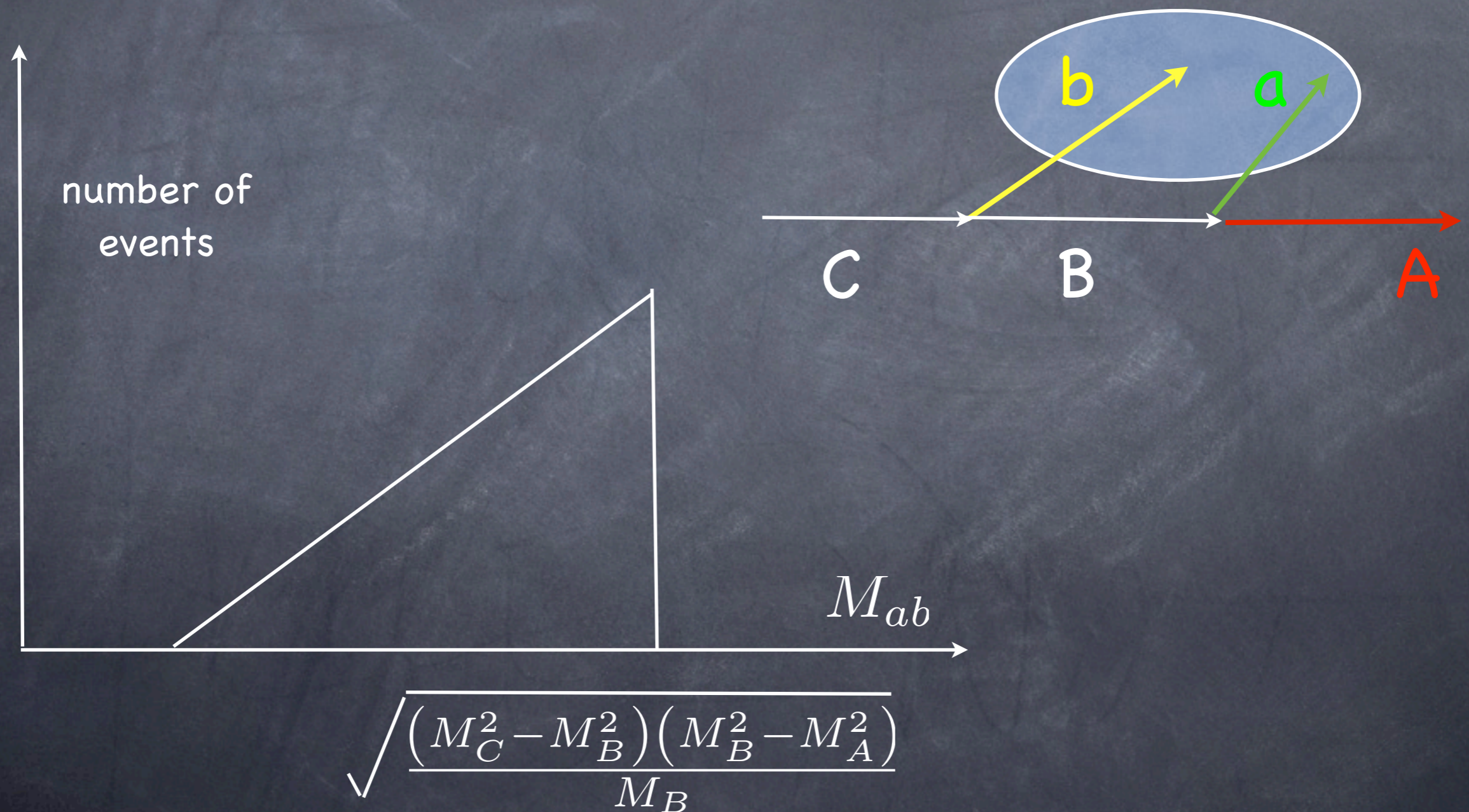
Based on **new** observation:

$$E_b^{\text{peak}} = \frac{M_C^2 - M_B^2}{2M_C} \quad \text{and} \quad E_a^{\text{peak}} = \frac{M_B^2 - M_A^2}{2M_B}$$



# Edge in invariant mass (old)

- On-shell intermediate particle  $\longrightarrow$  (sharp) edge

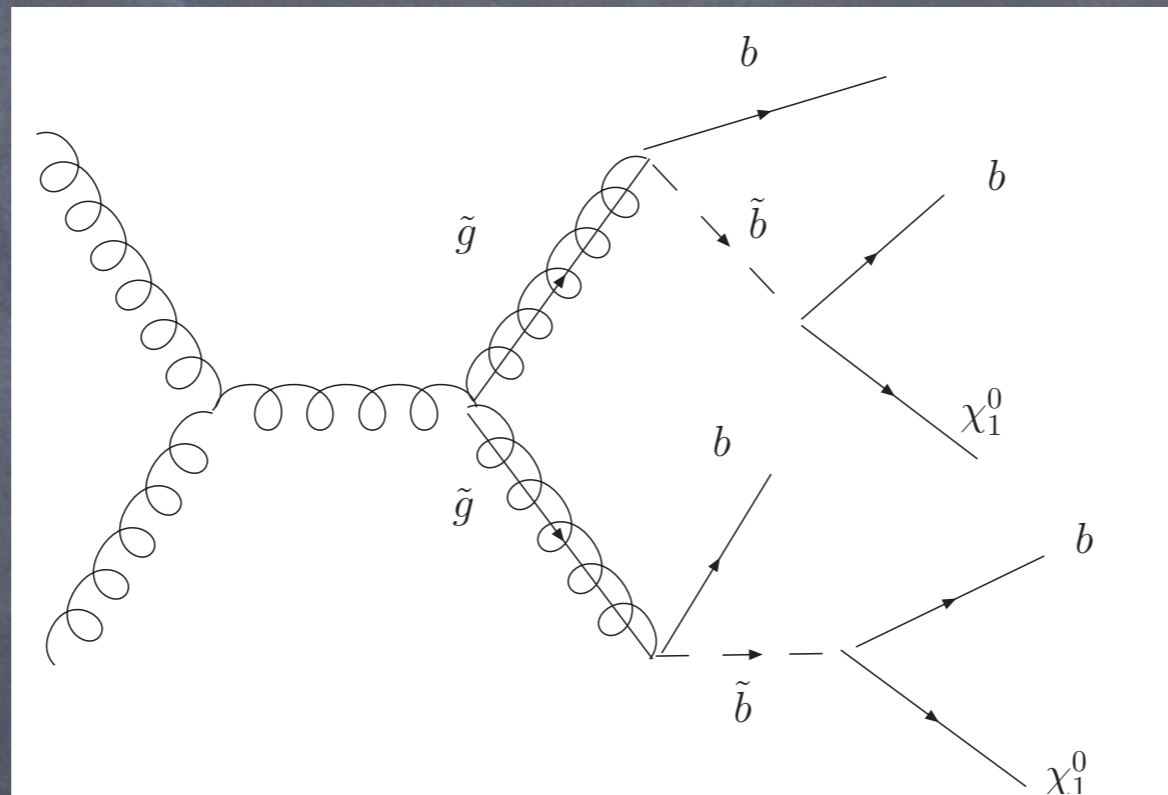


= 3 (independent)  
observables for  
determining 3 masses!

...(in principle) determine invisible particle  
mass without measuring MET!

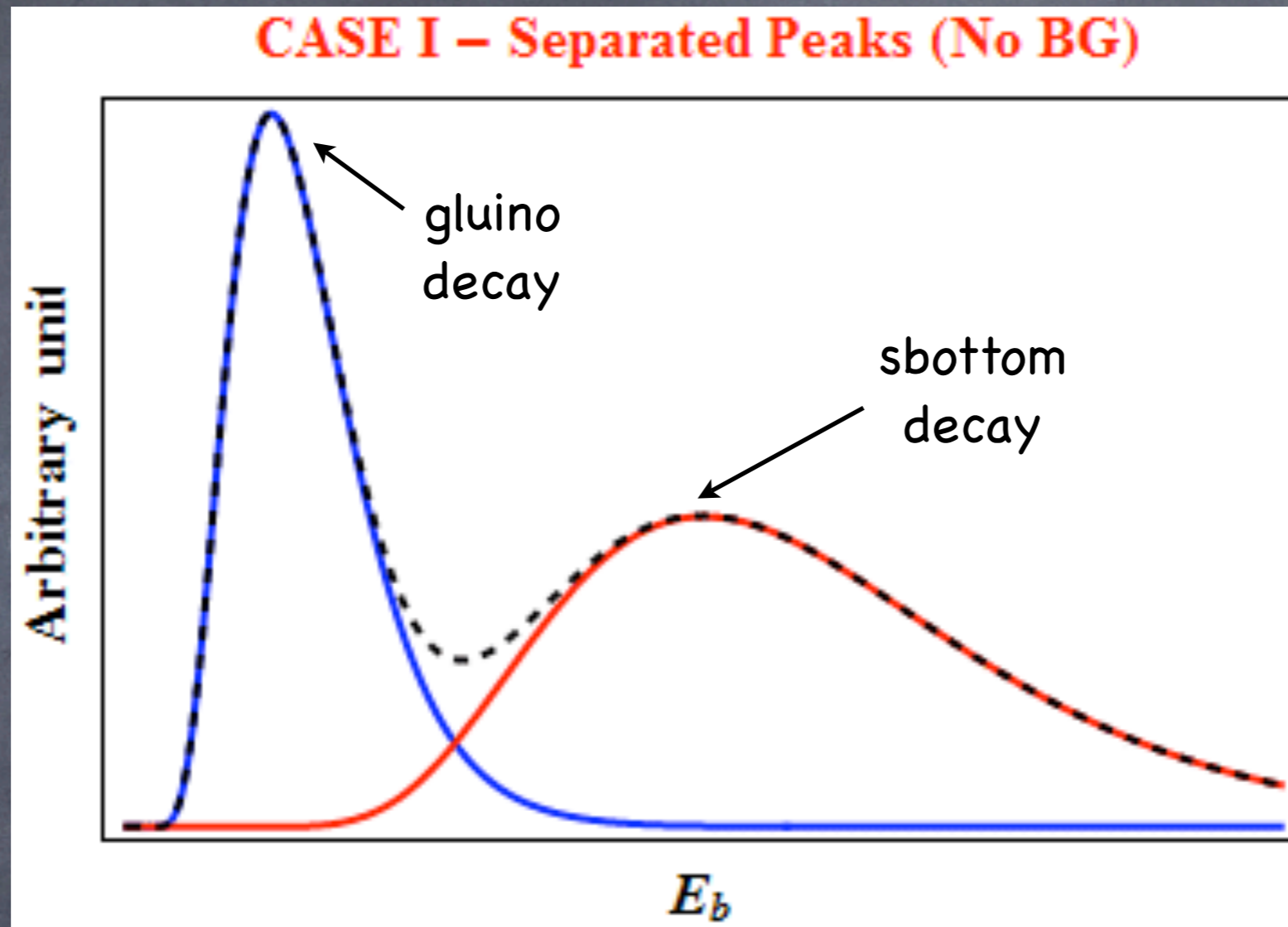
CASCADE DECAY IN SUSY  
(AN EXAMPLE)

# Glauino, sbottom, neutralino



- **natural SUSY**: 1st/2nd generation squarks heavy, stop/sbottom and gluino, Higgsino light

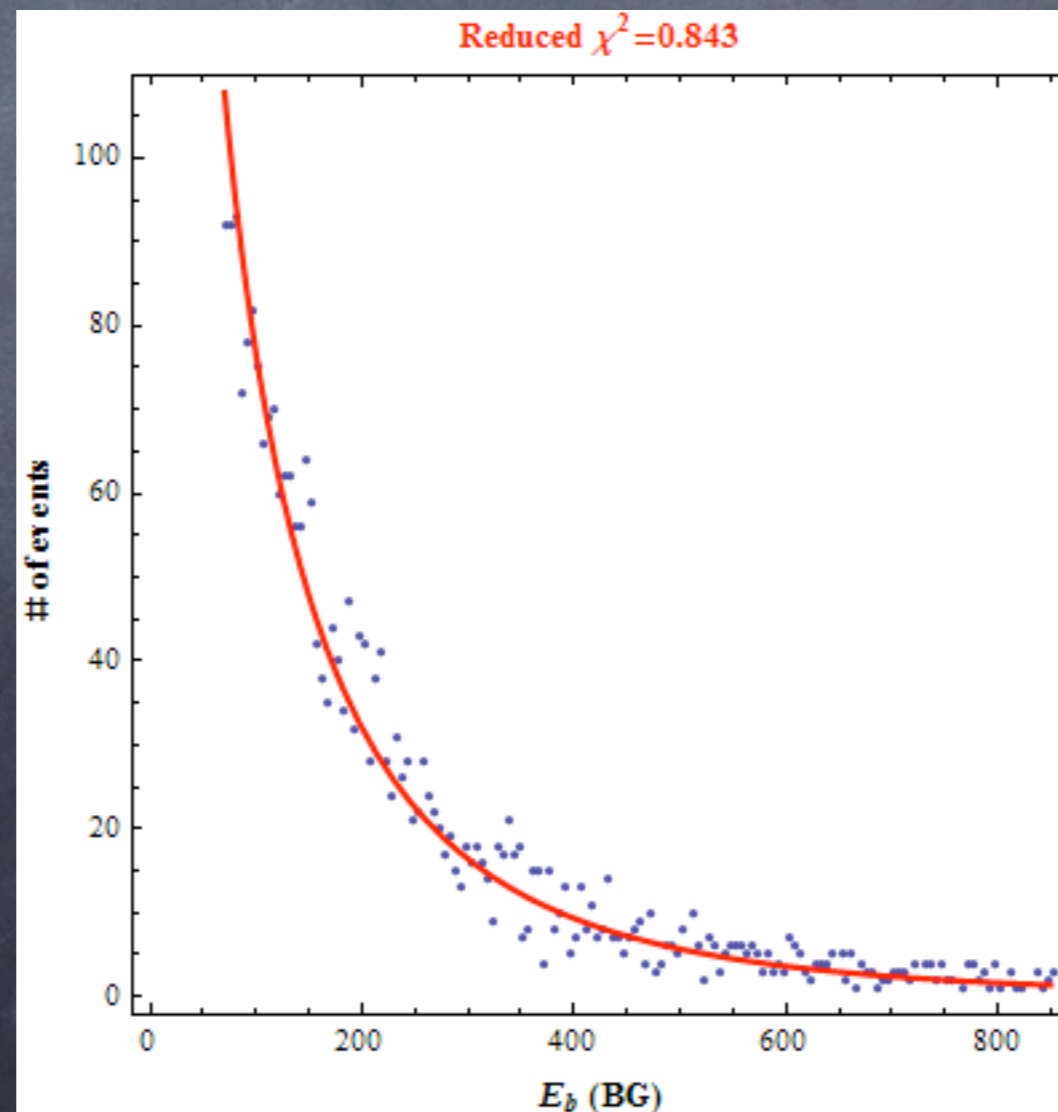
# Double (b-jet energy) peak



• mass hierarchy:  $M_{\tilde{g}} \approx M_{\tilde{b}} \gg M_{\chi_1^0}$   $\longrightarrow$  "soft" & hard b-jets

# Background

- $t\bar{t}b\bar{b}$  reducible and  $Z + 4b$  irreducible
- **template** for background:  $N_{p'} \exp(-p' \sqrt{E})$



$Z + 4b$

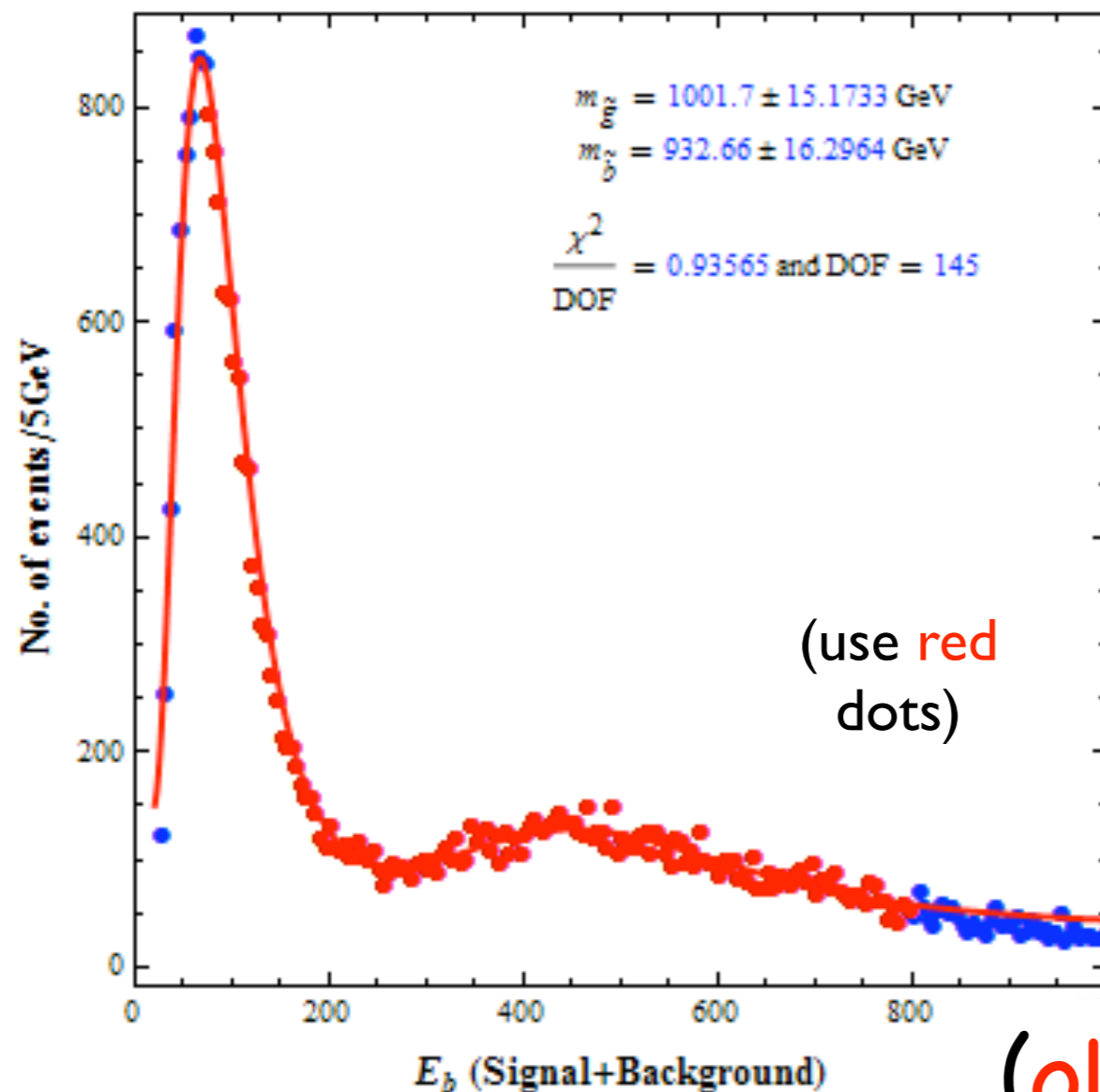
(old plot)



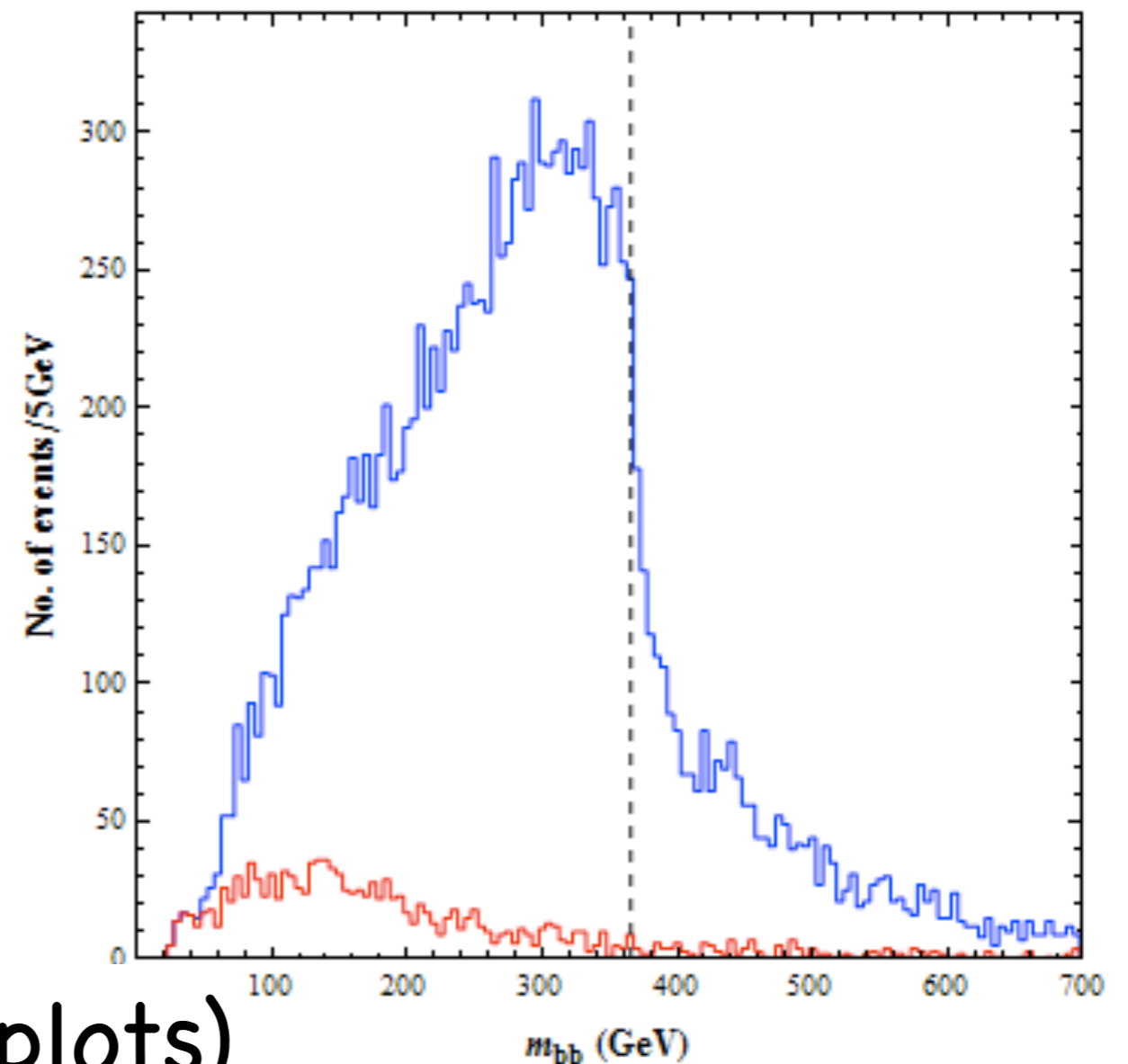
# Results

- $M_{\tilde{g}} = 1000$  GeV;  $M_{\tilde{b}} = 930$  GeV and  $M_{\chi_1^0} = 100$  GeV with 300 / fb at LHC14
- 3 (2 signal + 1 background) **template** fit (assume this model)
- **little** sensitivity to  $M_{\chi_1^0}$ :  $2\sqrt{E_b^{\text{peak } 1} E_b^{\text{peak } 2}} \approx M_{bb}^{\text{max}}$

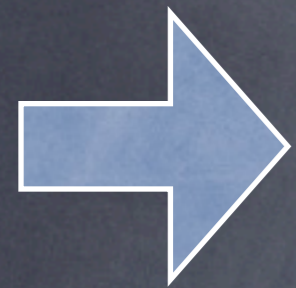
CASE I (S/B=10)



CASE I (S/B=10)



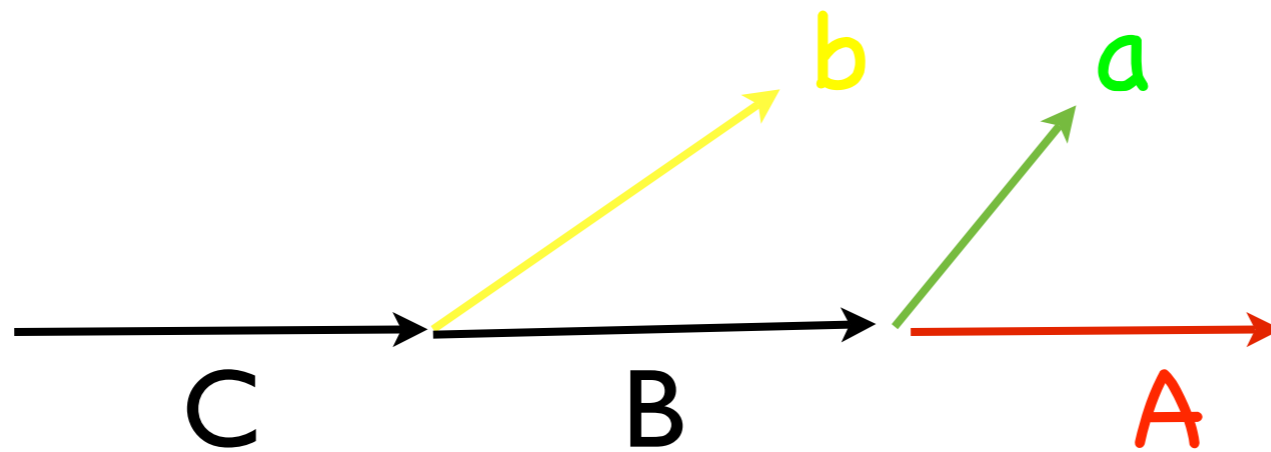
(old plots)



ansatz/fitting function  
works for (boost  
distribution of) a  
“secondary” mother as  
well!

# Other/cleaner possibilities

- $a \neq b$ : peaks in **different** distributions (no “pollution” between peaks)



- **lepton** instead of jet

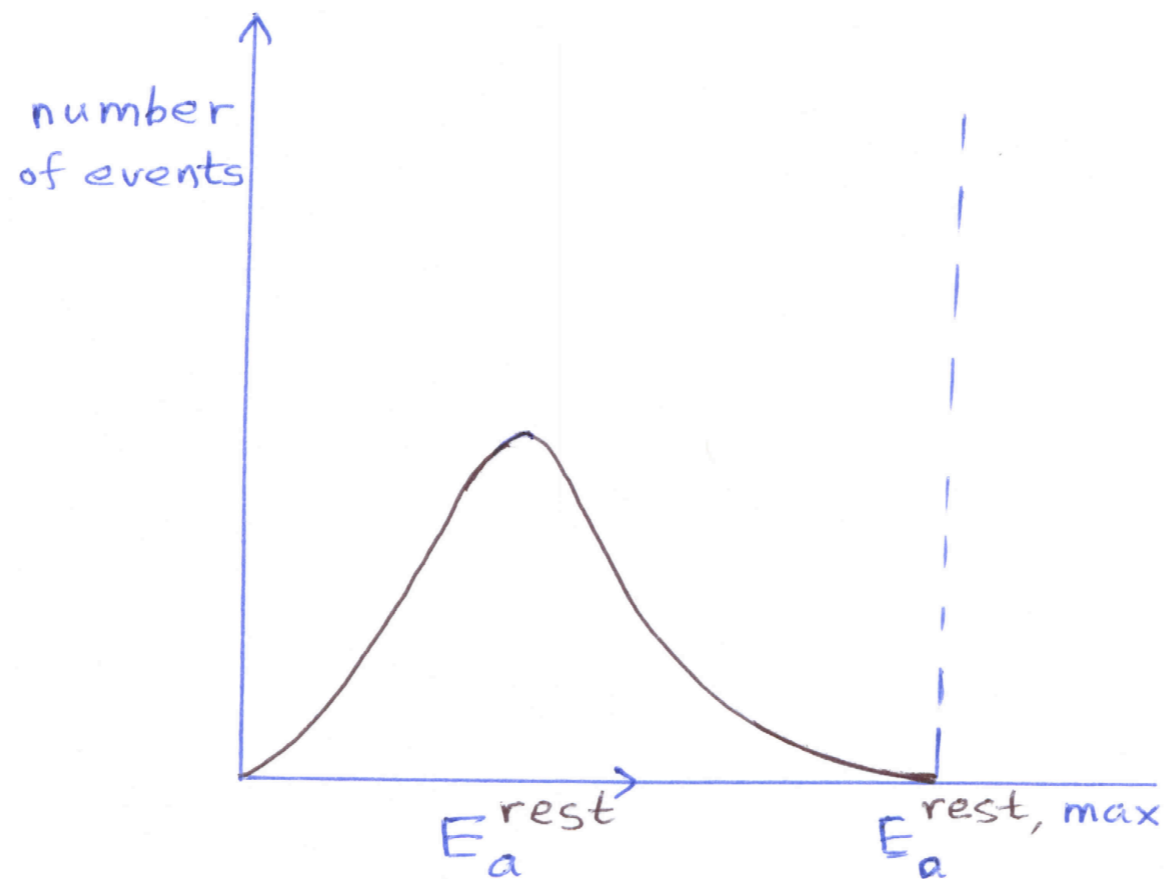
# Generalizations

- **Massive** daughter
- Three-body decay with **2** visible (e.g., **off-shell s** bottom): for **fixed** invariant mass of 2 visible, apply 2-body result

THREE-BODY DECAY: ONE  
VISIBLE

# Endpoint of distribution in **rest** frame

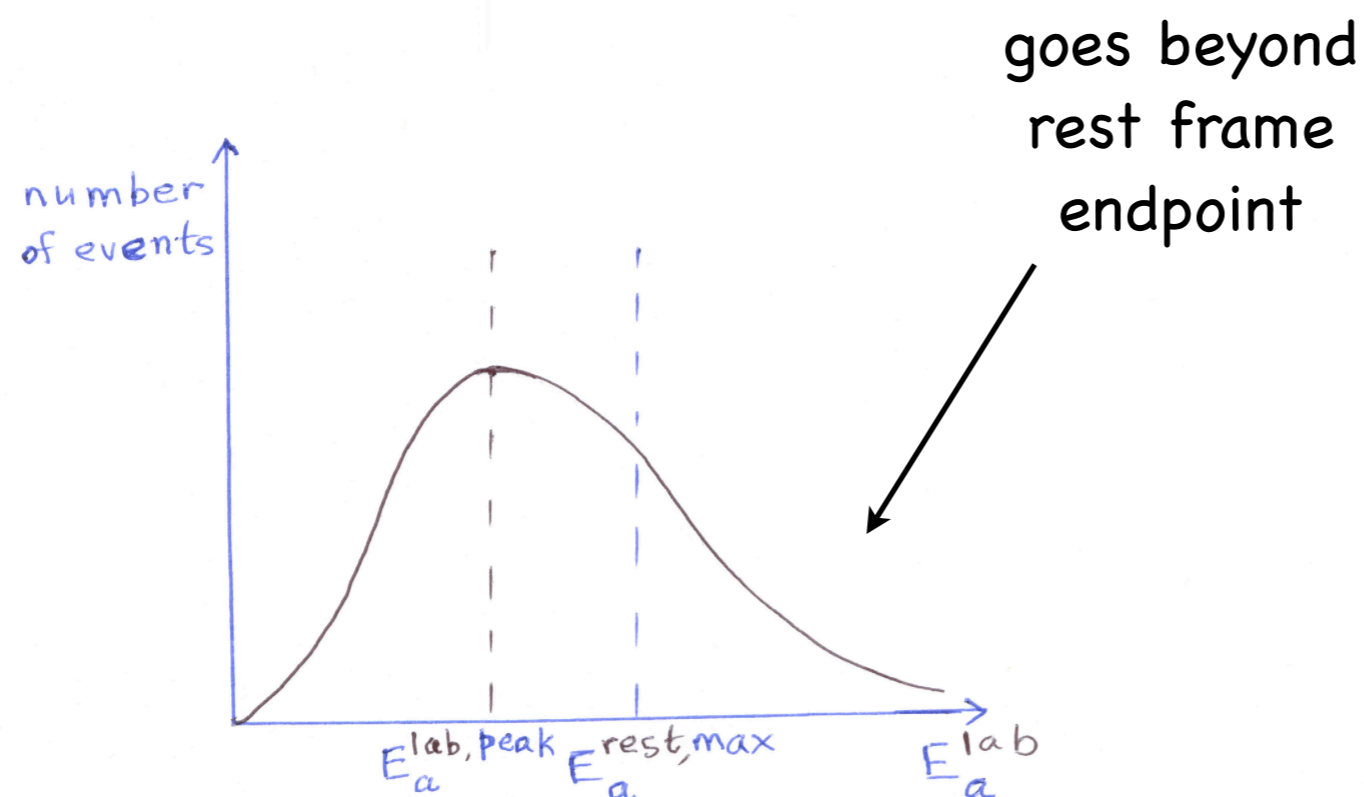
- Endpoint related **simply** to masses



# Peak of distribution in lab frame

$$E_a^{\text{lab,peak}} < E_a^{\text{rest,max}}$$

- Obtain **inequality** for masses
- distinguishing  $Z_3$  vs.  $Z_2$ -stabilized dark matter (DM):  
decay into 1 visible + **2** vs. **1** DM ("same" final state!)



(KA, Franceschini, Kim, Wardlow:  
1212.5230)

# Conclusions

- **Two** body decay of **unpolarized** mother at hadron colliders:  
**peak in energy distribution of massless daughter same as rest frame energy** (simple function of masses)
- Obtain **approximation** to **theory curve** (for fitting to data to extract peak)
- **Application(s):**  
**top** quark mass (test)  
**new particles decaying semi-invisibly**: extract **all** masses from cascade decay (e.g., gluino to sbottom...)



BACK-UP

# Formal proof

- Single Rectangle ( $x = \frac{E_a^{\text{lab}}}{E_a^{\text{rest}}}$ ):

$$\frac{1}{\Gamma} \frac{d\Gamma}{dx} \Big|_{\text{fixed } \gamma_B} = \frac{\Theta\left(x - \gamma_B + \sqrt{\gamma_B^2 - 1}\right) \Theta\left(-x + \gamma_B + \sqrt{\gamma_B^2 - 1}\right)}{2\sqrt{\gamma_B^2 - 1}}$$

- Stacking up rectangles:

$$f(x) \equiv \frac{1}{\Gamma} \frac{d\Gamma}{dx} = \int_{\frac{1}{2}\left(x + \frac{1}{x}\right)}^{\infty} d\gamma_B \frac{g(\gamma_B)}{2\sqrt{\gamma_B^2 - 1}}$$

- Slope:

$$f'(x) = \frac{\text{sgn}(1-x)}{2x} g\left(\frac{1}{2}\left(x + \frac{1}{x}\right)\right)$$

- Behavior at  $x = 1$ :

$f'(x = 1) \propto g(1) = 0 \Rightarrow$  extremum or

$f'(x)$  flips its sign at  $x = 1 \Rightarrow$  a cusp

$f(x)$  is positive and vanishes for both  $x \rightarrow 0$  and  $x \rightarrow \infty$

$\Rightarrow$  peak at  $E_a^{\text{rest}}$

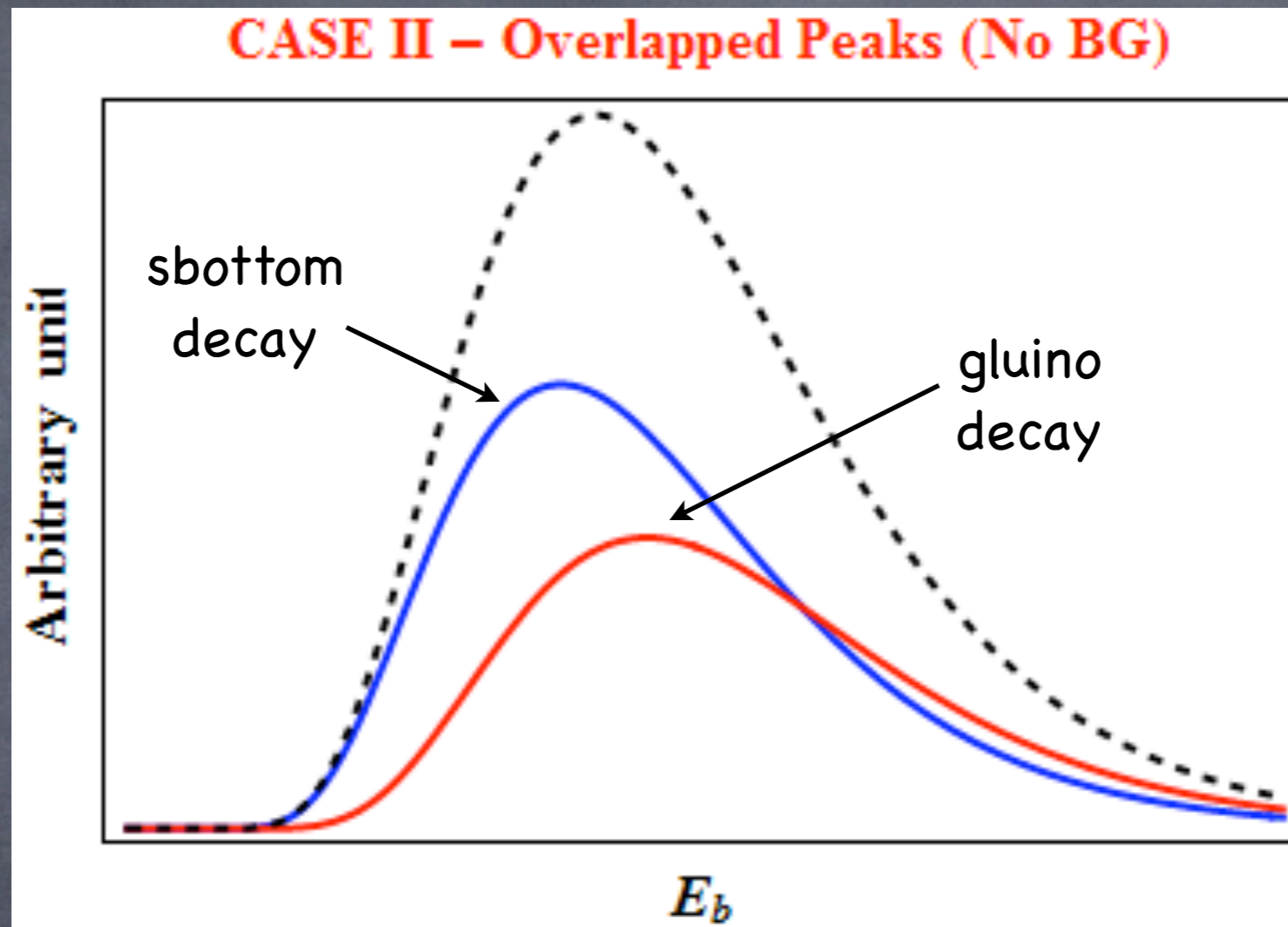
# ``Massive'' daughter

- argument goes thru' (rectangle contains  $E_a^{\text{rest}}$  ...) even for **massive** daughter if **boost distribution restricted** to  $\gamma_B < \left[ 2 (\gamma_a^{\text{rest}})^2 - 1 \right]$
- This **critical** boost is typically large value for massive, but ``light'' daughter: e.g., for **bottom** from top quark decay ( $\gamma_b^{\text{rest}} \approx 15 \Rightarrow \gamma_{\text{top}} \lesssim 500$  suffices)

# Another SUSY spectrum: sensitivity to **neutralino** mass

• **mass hierarchy**:  $M_{\tilde{g}} \gg M_{\tilde{b}} \gtrsim M_{\chi_1^0}$   **both b-jets hard**

# Overlapping peaks



- Ansatz can extract 2 peaks separately (assuming this model)

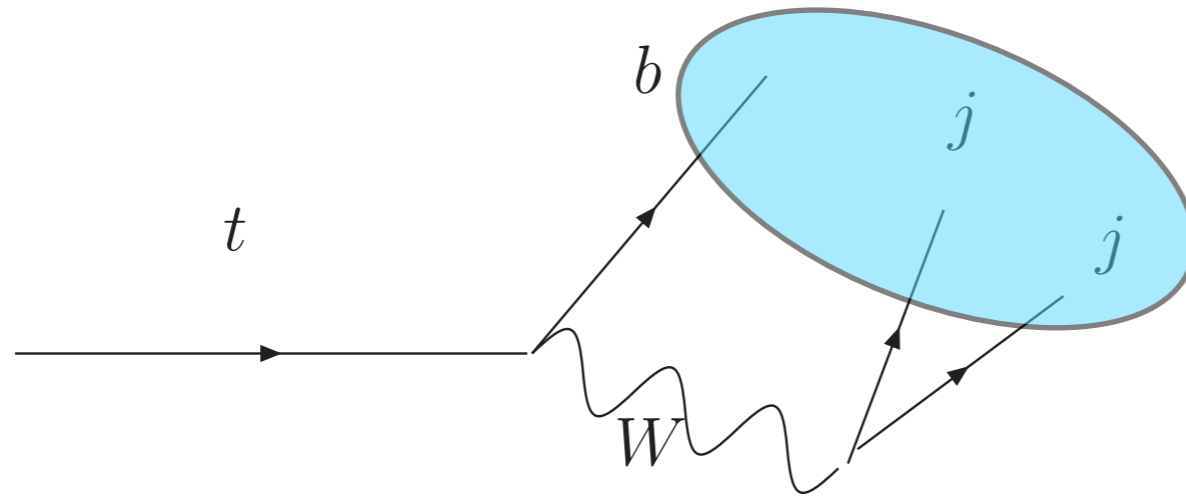
# Other (possibly) related work

- razor (pair-produced polarized/unpolarized mothers):  
 $M_R = 2E_a^{\text{rest}}$ , but *assuming* (a) mother at rest in COM of two mothers *and* (b) no transverse boost of this COM in lab frame
- 1107.4460, 1305.6150: use **entire** energy distribution, **no** (explicit) mention of location of **peak** (local feature)
- “Jacobian” peak at  $M_W$  in  $2 \times p_T^{\text{lepton}}$  and  $M_T$  (only) for **single** W production (+ events at this peak **different** than at energy-peak)

(Motivation for top mass: fundamental parameter of SM; enters calculation of other observables)

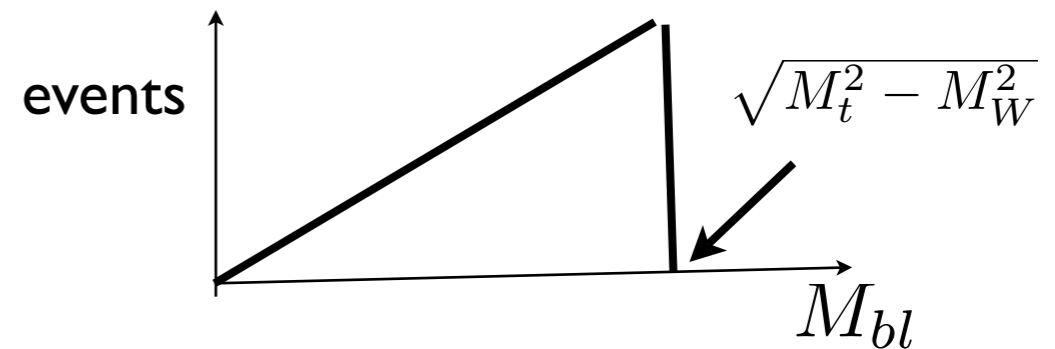
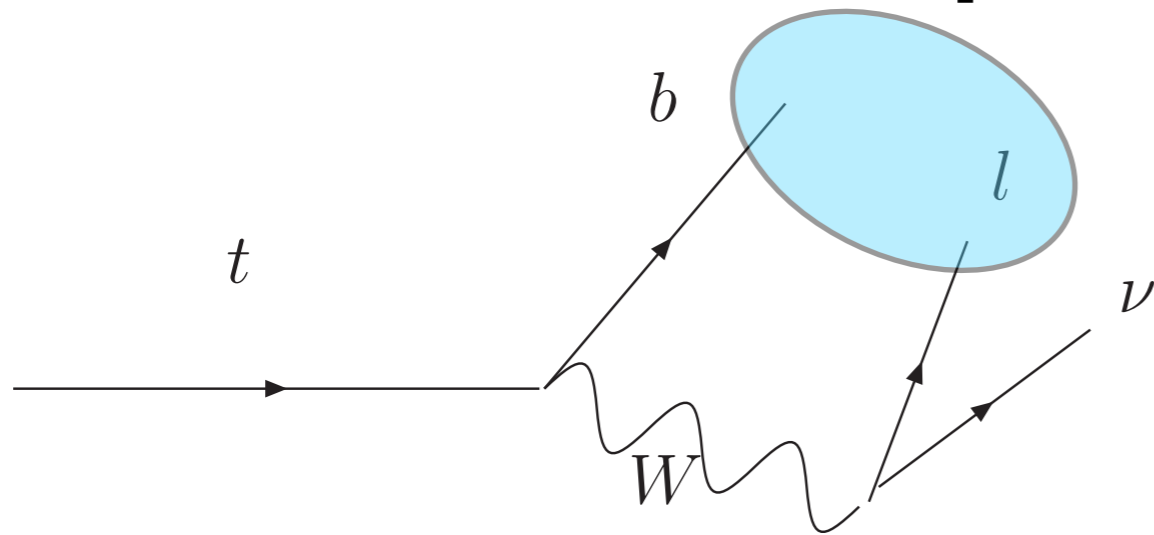
# Conventional methods

- Basic idea: reconstruct (full) decay of top



- can achieve  $O(0.6 \text{ GeV})$  uncertainty at LHC14, with 300/fb
- further gain may be possible with 3000/fb by using a more extended approach to constraining uncertainties using data
- Simulation (using SM matrix element in production) is used to handle combinatorics

# Latest: endpoint of $M_{bl}$



- more cleanly interpreted as measurements of the pole quark mass
- combinatorics resolved *without* assuming SM matrix element in production  
→  
resulting top quark mass immune to possible contaminations from New Physics in production of top quarks
- can provide precision competitive with more conventional methods, especially using 3000/fb at LHC14



# Comparison between $M_{bl}$ and energy-peak

- energy-peak has larger statistics
- $M_{bl}$  more robust (against hard radiation from bottom; polarized top quarks)

# Using energy-peak for searches

- if background is flat or peaks elsewhere from signal
- Stops (Low: 1304.0491):

for  $\tilde{t} \rightarrow b\tilde{\chi}_1^+$ , peak in  $E_b^{\text{lab}}$  at  $(M_{\tilde{t}}^2 - M_{\tilde{\chi}_1^+}^2) / (2M_{\tilde{t}}) \dots$

can be  $\gg (M_t^2 - M_W^2) / (2M_t)$  from  $t\bar{t}$  background (from SM or from  $\tilde{t} \rightarrow t\tilde{\chi}_1^0$ )