Numerical relativity and holographic metal

Toward theory of Strongly interacting many-body system

Sang-Jin Sin (Hanyang U.)

2014.12.09@CERN

Based on arXiv:1409.8346 with

Keun-Young Kim*, Kyung-Kyu Kim*, Yunseok Seo#

*GwangJu Institute of Science and technology, #Hanyang univ.

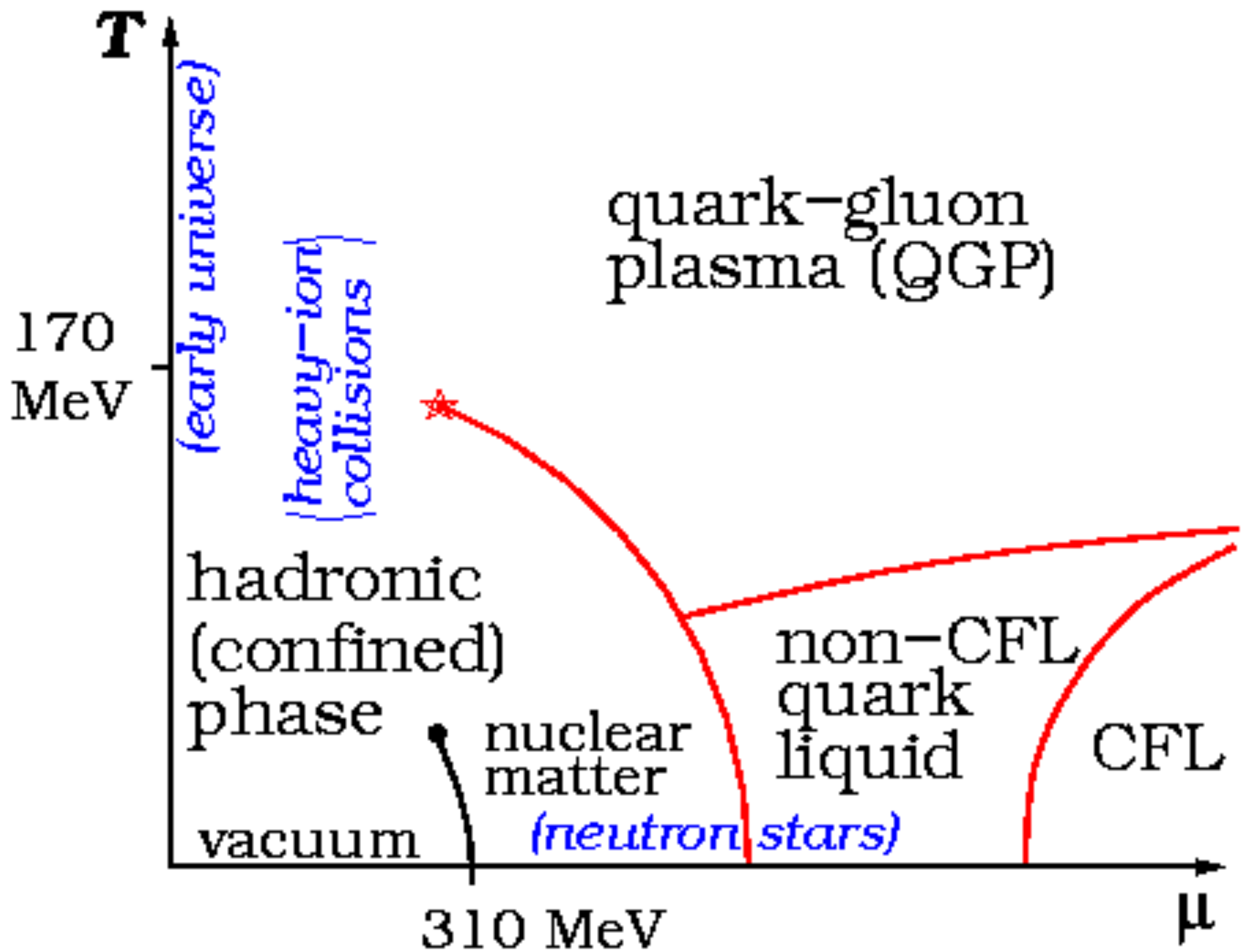
References

T. Andrade and B. Withers, *A simple holographic model of momentum relaxation*, *JHEP* **1405** (2014) 101, [arXiv:1311.5157].

Introduction

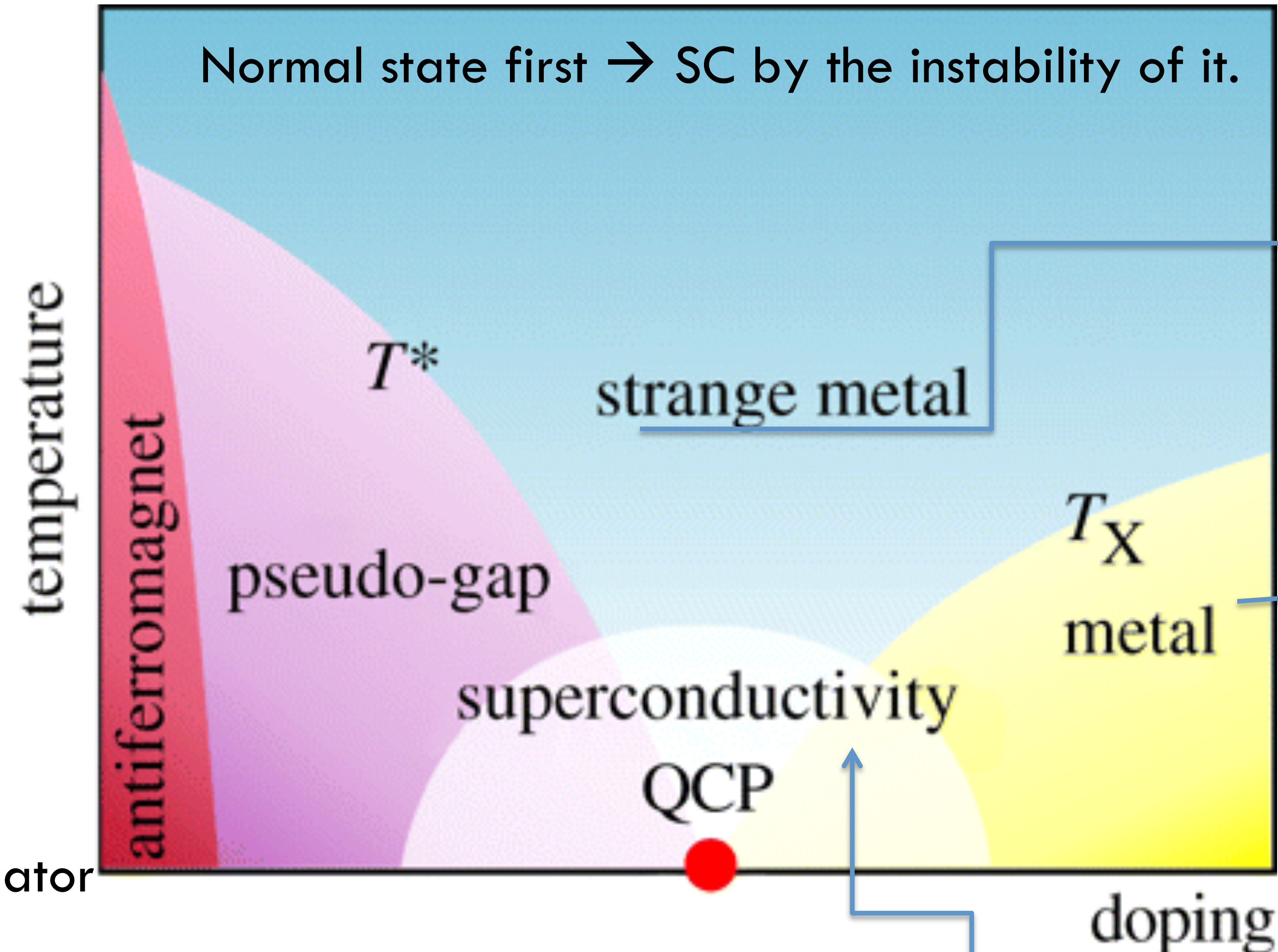
One Challenging problem of 21C physics is to understand dynamics of strongly interacting many body system.

QCD phase diagram



Physics Goal

understand the Cuprate phase diagram Quantitatively



Non-Fermi Liquid
Linear Registivity

Fermi Liquid
T^2 registivity
Wiederman-Frant Law

Mott Insulator

d-wave condensation

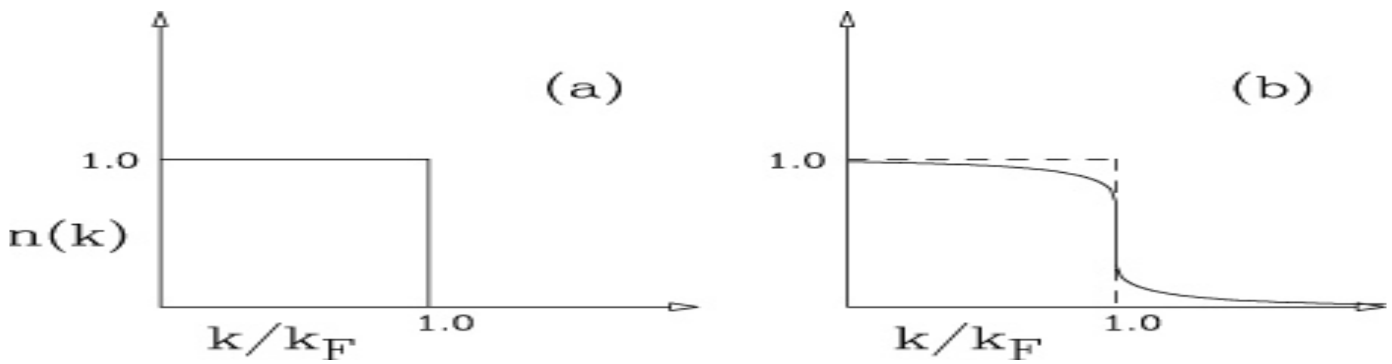
Based on Royal Society publishing, D. Galanakis et.al

Same Origin of the difficulty.

- No calculational tool.
- Even numerical work is not feasible due to the fermion sign problem. (chem. Pot is an axis of the diagram.)

(Non) Fermi liquid of Landau

1. Free fermions and Fermi Sea



2. Interaction weak→ dressed particles (q-p)
   →Fermi liquid (stable: irrelevance of most perturbation
   & SC is an instability channel.
3. Strong interaction→ fermi surface disappear→NFL
4. System property is not reduced to that of individual particles. So Band picture lose meaning. Pauli principle ?

If no quasi-particle, what to measure?

what is correct degree of freedom?

How to even formulate the problem?

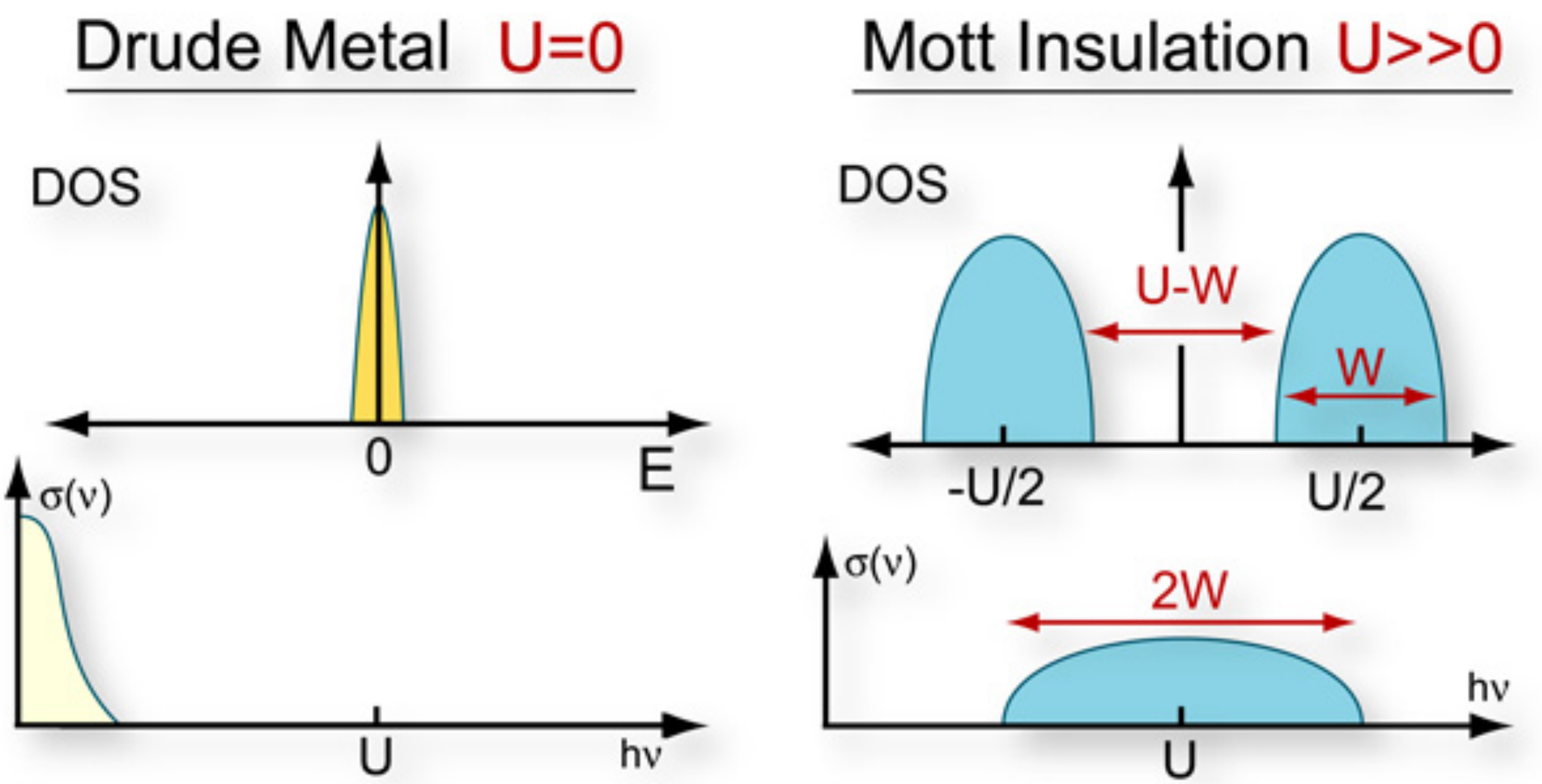
…………..

What experimentalists are doing?

Method of St.Int.Manybody

Example: Meta-insulator transition

Spectral function / Optical conductivity



<JJ> Green function
→ DC/AC conductivity

1. Metal/insulator, Good/Bad metal
2. Normal/Strange
3. Gap and pseudo gap
4. Coherent and incoherent metal

5. 1,410,000 image for optical conductivity in google image. standard probe of materials in experiment. Photo electric effect.
6. Comes from Two point function. Precisely defined.

Remark: what about the ARPES?

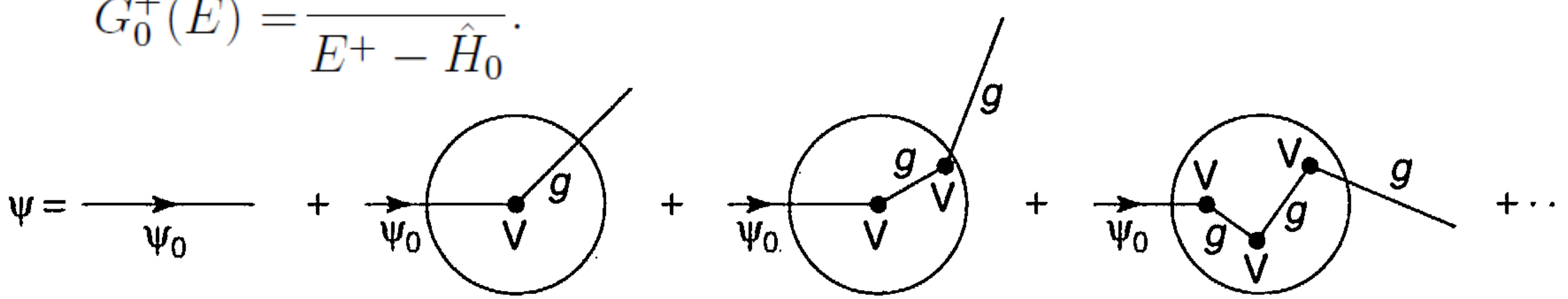
Q: Strong interaction in e-e int.?

1. For QCD: yes g^2 ~ 1 for low energy.
2. But in condensed matter, Isn't it

   e^2 ~ 1/137<<1
3. How ee interaction can be strong …?
4. Lippman-Schiwinger eq.

$$|\psi^+(E)\rangle = G_0^+(E)\hat{V}|\psi^+(E)\rangle + |\phi(E)\rangle = \sum_{n=0}^{\infty}\left(\hat{G}_0^+\hat{V}\right)^n|\phi\rangle$$

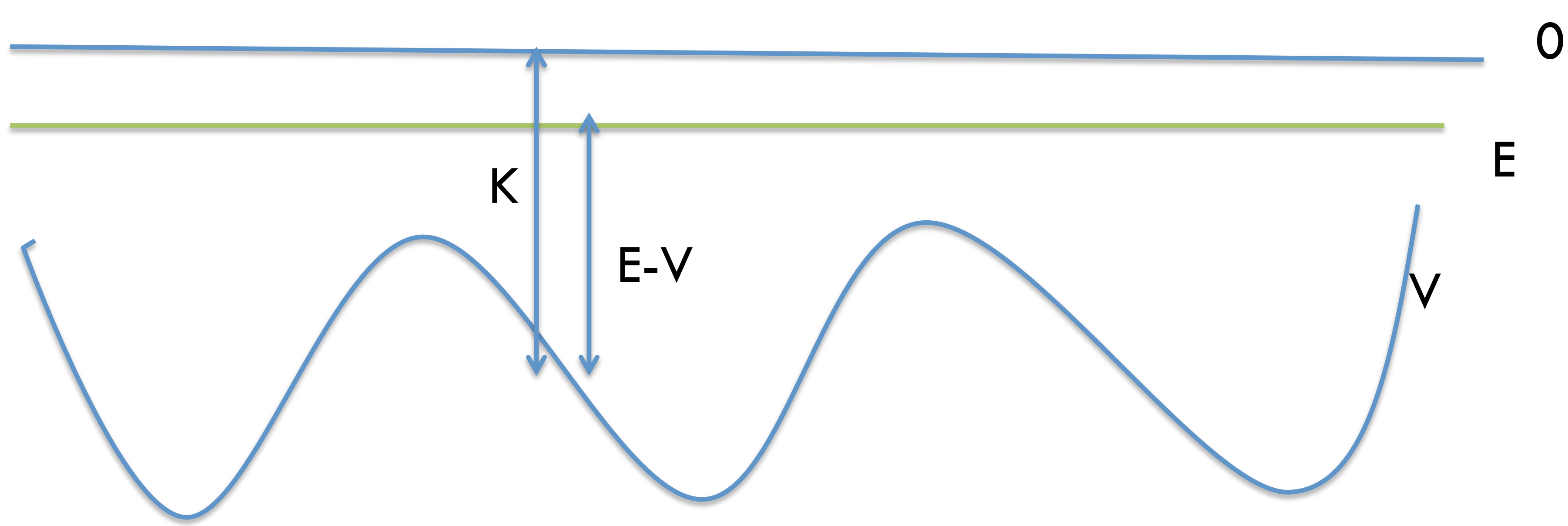
$$\hat{G}_0^+(E) = \frac{1}{E^+ - \hat{H}_0}.$$



$$|E^+ - K| \leq K$$

$$|\hat{G}_0^+ V| \geq |V/K|$$ Slow electrons interact strongly.

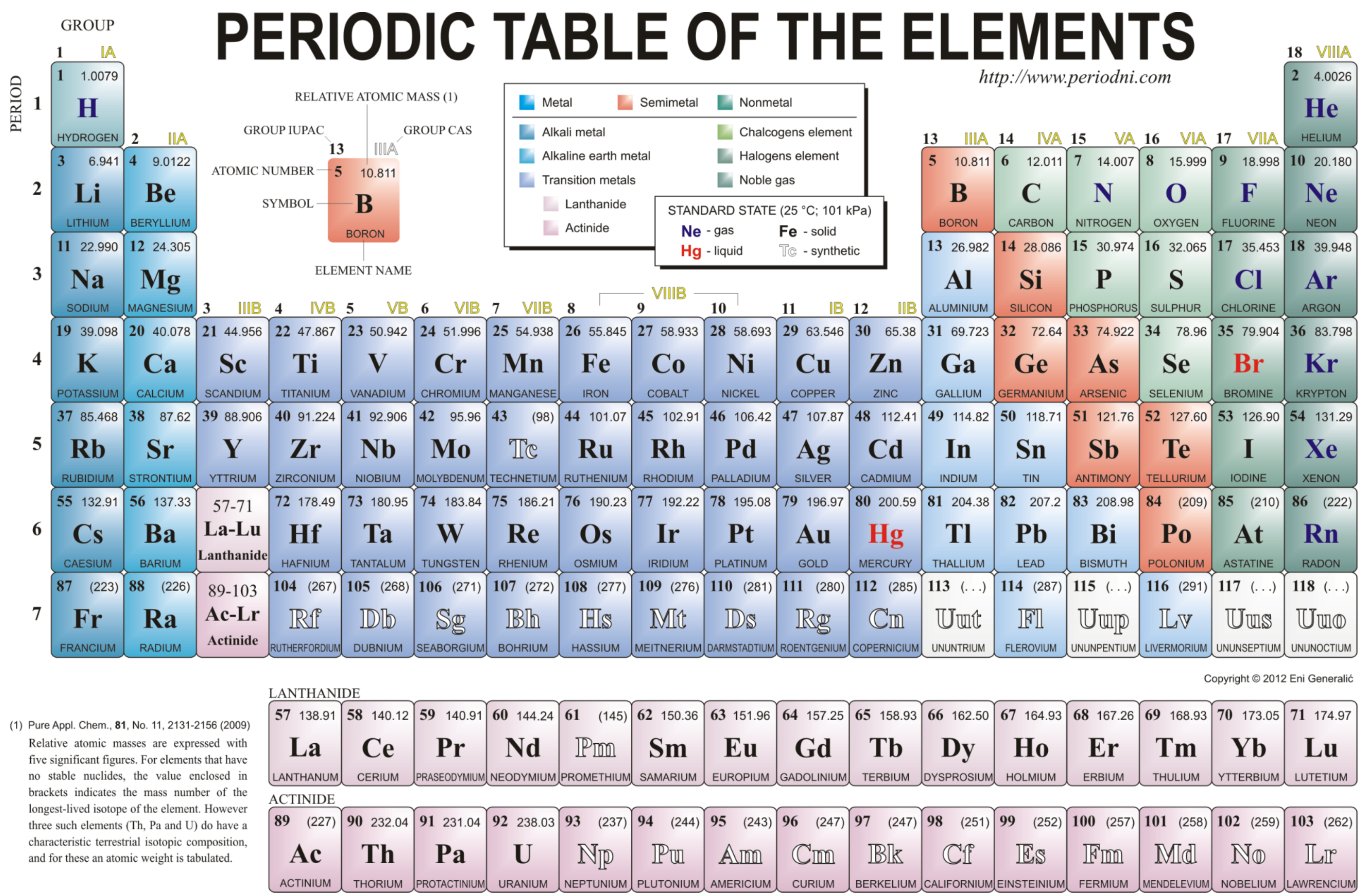
K-0>K-E



3d 4f electrons are interacting strongly.

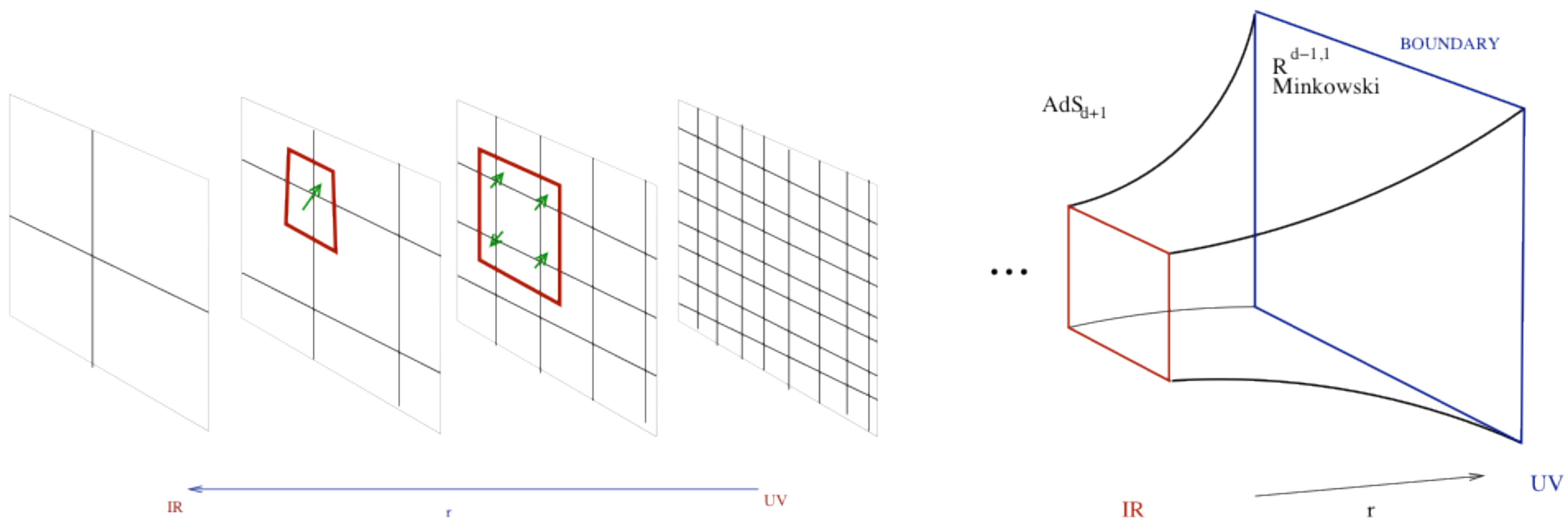
Electrons in the valence band progressively more localized in the sequence 5d → 4d → 3d → 5f → 4f

Such electrons are slower in this order.



Holography for calculability

- Collection of view of system along the RG scale,



McGreevy

- RG unning controlled by Einstein equation.
- Exact duality only for N=4SYM/AdS5. Why?
- Enough apology by previous speakers.

FAQ

- Why QM → Classical?

  Curvature radius = $(Ng)^{1/4}$

- How eV scale not string scale?

  near conformal point. /QCP is important.

1st Job. Have tv screen with Knob for gravity channels

1. Charged BH
2. Haired BH
3. Lifshitz geometry.
4. Hyper scaling violating solutions
5. Electron star
6. linear dilaton

   ....

Numerical holography

Need to handle multi fields coupled system.

2nd Job: Lattice at boundary

- Much later…….

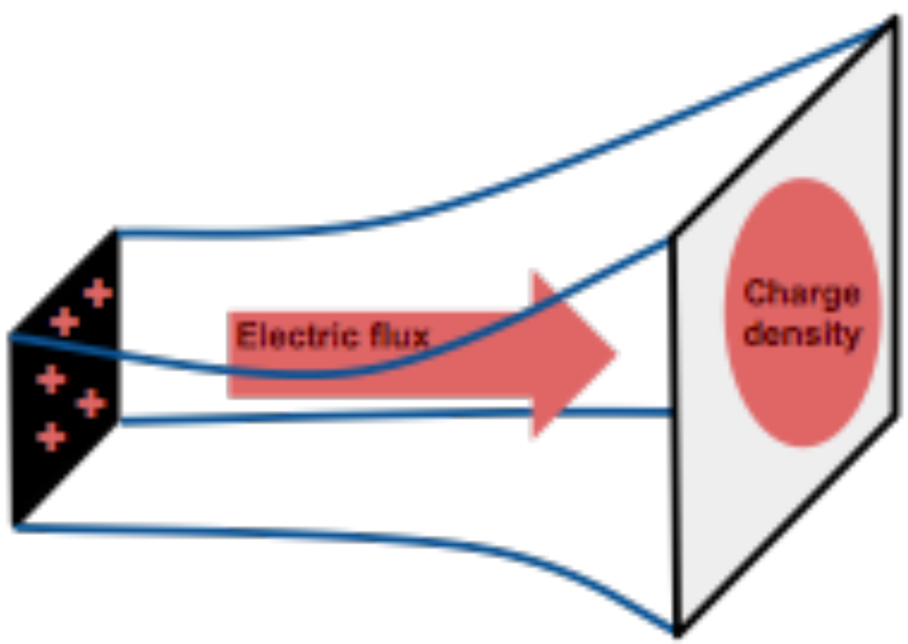
- So far, motivations

Review 1/4: charged AdS BH and conductivity

- Einstein-Maxwell system

$$S_{\rm EM} = \int_M \mathrm{d}^4x\sqrt{-g}\left[R - 2\Lambda - \frac{1}{4}F^2\right] - 2\int_{\partial M}\mathrm{d}^3x\sqrt{-\gamma}K$$

- Reissner-Nordstrom-AdS black hole
~ Boundary field theory at finite temperature and density



Hartnoll, 1106.4324

Eq. of Motion +
B.C @Horizon & boundary

boundary value pr.→ initial value pr.

Shooting ? Yes in I field.

If more than 1 coupled fields, shooting is not practical.

Review 2/4: charged AdS BH and conductivity

Electric, Thermoelectric, Thermal conductivity:

- Fluctuations

$$\delta g_{ti}(t,r) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega t} r^2 h_{ti}(\omega, r),$$
$$\delta A_i(t,r) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega t} a_i(\omega, r),$$

- EOMs

$$\boxed{\begin{aligned} -\frac{\mu a_x'}{r^4} - \frac{4h_{tx}'}{r} - h_{tx}'' &= 0 \\ \frac{\mu a_x}{r^4} + h_{tx}' &= 0 \\ \frac{f' a_x'}{f} + \frac{\mu h_{tx}'}{f} + \frac{\omega^2 a_x}{f^2} + a_x'' &= 0 \end{aligned}}$$

- Boundary action

$$S_{\text{ren}}^{(2)} = \lim_{r\to\infty} V_2 \frac{1}{2} \int d\omega \boxed{\left[-m_0\, h_{tx} h_{tx} - \mu\, a_{tx} h_{tx} - f(r) a_{tx} a_{tx}' + r^4 h_{tx} h_{tx}'\right]} \qquad m_0 = \left(1 + \frac{\mu^2}{4}\right)$$

$$\begin{pmatrix} G^R_{J_x J_x} & G^R_{J_x T_{tx}} \\ G^R_{T_{tx} J_x} & G^R_{T_{tx} T_{tx}} \end{pmatrix} = \begin{pmatrix} \frac{a_x^{(1)}}{a_x^{(0)}} & -\mu \\ -\mu & -m_0 \end{pmatrix}$$

$$=: \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix}$$

Linear response

$$\begin{pmatrix} \langle J_x \rangle \\ \langle T_{tx} \rangle \end{pmatrix} = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} \begin{pmatrix} \delta a_x^s \\ \delta h_{tx}^s \end{pmatrix}$$

$$\begin{pmatrix} \langle J_x \rangle \\ \langle Q_x \rangle \end{pmatrix} = \begin{pmatrix} \sigma & \alpha T \\ \bar{\alpha} T & \bar{\kappa} T \end{pmatrix} \begin{pmatrix} E_x \\ -(\nabla_x T)/T \end{pmatrix}$$

- Two issues for generalisation

1. more than one equation
2. identify the sources and currents

Hartnoll 0903.3246

$$\boxed{\begin{pmatrix} \sigma & \alpha T \\ \bar{\alpha} T & \bar{\kappa} T \end{pmatrix} = \begin{pmatrix} -\frac{iG_{11}}{\omega} & \frac{i(G_{11}\mu - G_{12})}{\omega} \\ \frac{i(G_{11}\mu - G_{21})}{\omega} & -\frac{i(G_{22} + \mu(-G_{12} - G_{21} + G_{11}\mu))}{\omega} \end{pmatrix}}$$

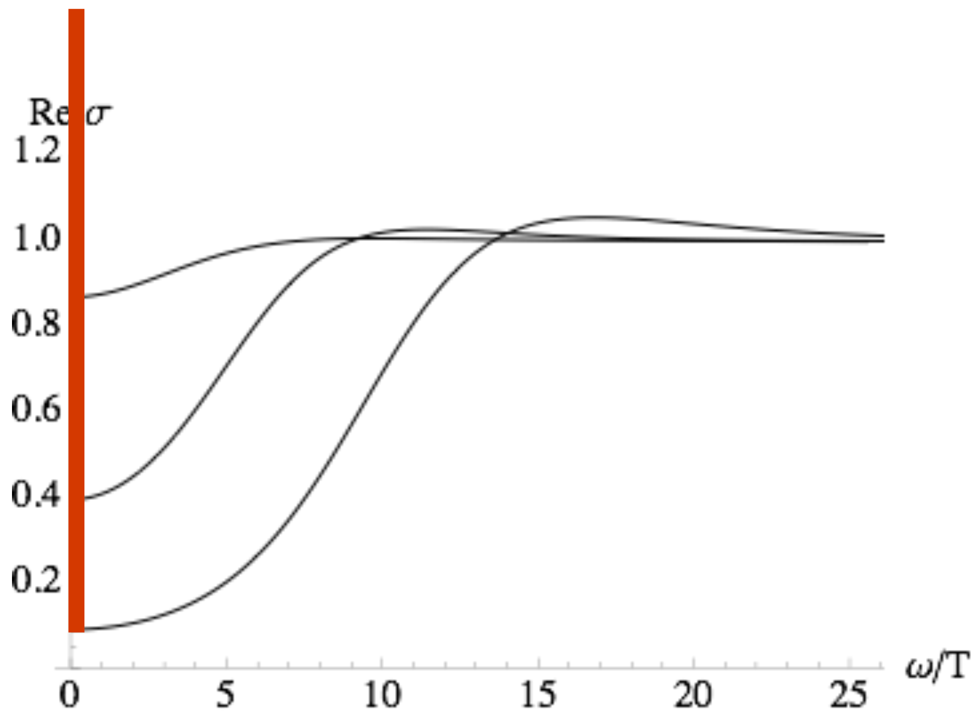
Review 3/4: charged AdS BH and conductivity

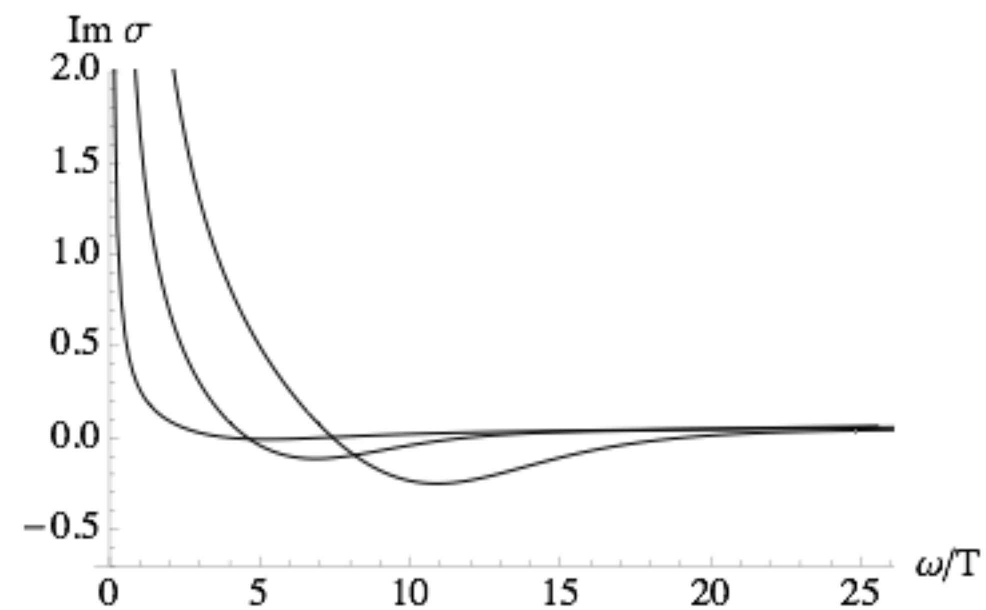
- Conductivity

2007

Herzog, Kovtun, Sachdev and Son; Hartnoll

$$A_x = \frac{E_x}{i\omega} e^{i\omega t} + \frac{J_x}{r} + \cdots \qquad J_x = \sigma E_x$$





- Kramers-Kronig relation

$$\mathrm{Im}\,\sigma \sim 1/\omega \quad \Leftrightarrow \quad \mathrm{Re}\;\sigma(\omega) \sim \delta(\omega)$$

Translation invariance + finite density

Problem is traced back to the translation invariance

Review 4/4: 3 class of Models for Momentum relaxation

1. Breaking Translation Symmetry in Holographic model

'Ionic' Lattice $A_t \sim 1 + A_0 \cos(k_0 x)$ Horowitz, Santos, Tong: 1209.1098

$\phi \sim A_0 \cos(k_0 x)$ Horowitz, Santos, Tong: 1204.0512

- Fluctuations:11 PDEs in two variables

$$\{\bar{h}_{tt}, \bar{h}_{tz}, \bar{h}_{tx}, \bar{h}_{zz}, \bar{h}_{zx}, \bar{h}_{xx}, \bar{h}_{yy}, \bar{b}_t, \bar{b}_z, \bar{b}_x, \bar{\eta}\}$$

Other methods

2. Massive gravity model:

Vegh(1301), Davison(1306)

Add graviton mass→ momentum cons broken

$$m(\mathrm{Tr}K^2 - (\mathrm{Tr}\,K)^2)$$

3. Use of solution that breaks translation invariance

$$\nabla^\nu \langle T_{\nu\mu}\rangle = F_{\mu\nu}\langle J^\nu\rangle + \partial_\mu \phi \langle \mathcal{O}\rangle \,,$$

Metal-Insulator transition in holography: Donos and Hartnoll(1212)
Q-lattice model: Donos and Gauntlett (1311)
....

Andrade and Withers 1311.5157→ we work here.
Simplest model with P dissipation and w. analytic background

- Momentum relaxation simplified (ODE): Find and Use a Translation inv. Breaking Exact solution

$$\psi_I = \beta_{Ii} x^i = \beta \delta_{Ii} x^i \,, \rightarrow \sigma_{DC} = 1 + \frac{\mu^2}{\beta^2}$$

But not AC

So we want to calculate
AC Seeback, AC thermal conductivity as well as AC electric conductivity

Bardoux, Caldarelli, Charmousis (2012), Andrade, Withers(2013)

- Actions

$$S_{\rm EM} = \int_M \mathrm{d}^4x\sqrt{-g}\left[R - 2\Lambda - \frac{1}{4}F^2\right] - 2\int_{\partial M}\mathrm{d}^3x\sqrt{-\gamma}K\,,$$

$$S_\psi = \int_M \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{2}\sum_{I=1}^{2}(\partial\psi_I)^2\right]$$

$$S_{\rm c} = \int_{\partial M}\mathrm{d}x^3\sqrt{-\gamma}\left(-4 - R[\gamma] + \frac{1}{2}\sum_{I=1}^{2}\gamma^{\mu\nu}\partial_\mu\psi_I\partial_\nu\psi_I\right)$$

- EOMs

$$R_{MN} = \frac{1}{2}g_{MN}\left(R - 2\Lambda - \frac{1}{4}F^2 - \frac{1}{2}\sum_{I=1}^{2}(\partial\psi_I)^2\right) + \frac{1}{2}\sum_I \partial_M\psi_I\partial_N\psi_I + \frac{1}{2}F_M{}^PF_{NP}$$

$$\nabla_M F^{MN} = 0\,, \qquad \nabla^2\psi_I = 0\,.$$

- RN-AdS solution + two scalars

$$\mathrm{d}s^2 = G_{MN}\mathrm{d}x^M\mathrm{d}x^N = -f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j\,,$$

$$m_0 = r_0^3\left(1 + \frac{\mu^2}{4r_0^2} \boxed{- \frac{\beta^2}{2r_0^2}}\right)$$

$$f(r) = r^2 - \frac{\beta^2}{2} - \frac{m_0}{r} + \frac{\mu^2}{4}\frac{r_0^2}{r^2}\,, \qquad m_0 = r_0^3\left(1 + \frac{\mu^2}{4r_0^2} - \frac{\beta^2}{2r_0^2}\right)$$

$$T = \frac{f'(r_0)}{4\pi} = \frac{1}{4\pi}\left(3r_0 - \frac{\mu^2 + \boxed{2\beta^2}}{4r_0}\right)$$

$$A = \mu\left(1 - \frac{r_0}{r}\right)\mathrm{d}t\,,$$

$$\psi_I = \beta_{Ii}x^i = \beta\delta_{Ii}x^i\,,$$

$$r_0 = \frac{2\pi}{3}\left(T + \sqrt{T^2 + 3(\mu/4\pi)^2 + 6(\beta/4\pi)^2}\right)$$

- Fluctuations

$$\delta g_{tx}(t,r) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} e^{-i\omega t} \frac{r^2}{r_0^2} h_{tx}(\omega,r),$$
$$\delta\psi_1(t,r) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} e^{-i\omega t} \chi(\omega,r)\,,$$
$$\delta A_x(t,r) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} e^{-i\omega t} a_x(\omega,r)$$

- EOMs

$$\frac{\beta^2 h_{tx}}{r^2 f} + \frac{i r_0^2 \beta\omega\chi}{r^2 f} - \frac{\mu r_0^3 a_x'}{r^4} - \frac{4h_{tx}'}{r} - h_{tx}'' = 0\,,$$
$$\frac{i\beta r_0^2 f\chi'}{r^2\omega} + \frac{\mu r_0^3 a_x}{r^4} + h_{tx}' = 0\,,$$
$$\frac{f' a_x'}{f} + \frac{\mu h_{tx}'}{r_0 f} + \frac{\omega^2 a_x}{f^2} + a_x'' = 0\,,$$
$$\frac{f'\chi'}{f} - \frac{i\beta\omega h_{tx}}{r_0^2 f^2} + \frac{\omega^2\chi}{f^2} + \frac{2\chi'}{r} + \chi'' = 0\,.$$

- Boundary action

$$S_{\mathrm{ren}}^{(2)} = \lim_{r\to\infty} \frac{V_2}{2} \int \mathrm{d}\omega \left[-m_0\, h_{tx}h_{tx} - \mu\, a_x h_{tx} - f(r) a_x a_x' + r^4 h_{tx} h_{tx}' - r^2 f(r)\chi\chi'\right]$$

Numerical methods 1

$S_{ren}^{(2)} = \frac{1}{2}\int dxdy \int d\omega\, (-3\, h_0\, h_3 - m_0\, h_0^2 - \mu\, h_0\, a_0 + a_0\, a_1 + 3\, \chi_0\, \chi_3)$

Near the boundary ($r \to \infty$) the asymptotic solutions read

$$
\begin{aligned}
h_{tx} &= h_{tx}^{(0)} + \frac{1}{r^2} h_{tx}^{(2)} + \frac{1}{r^3} h_{tx}^{(3)} + \ldots, \\
a_x &= a_x^{(0)} + \frac{1}{r} a_x^{(1)} + \cdots, \\
\chi &= \chi^{(0)} + \frac{1}{r^2}\chi^{(2)} + \frac{1}{r^3}\chi^{(3)} + \cdots
\end{aligned}
$$

$$\vec{J}(\omega) = (a_0, h_0, \chi_0), \quad \Pi(\omega) = (a_1, h_3, \chi_3),$$

$$\Pi^a = C_{ab} J^b \ \text{ by Eq. of M + boundary condition}$$

$$S_{ren}^{(2)} = \frac{V}{2}\int d\omega J^a(-\omega) G_{ab}^R J^b(\omega)$$

Gauge invariance: all the gauge non-invariant quantity does not enter into Renormalized on shell action.

Numerical methods 2: express Pi by J

$$s^{(2)}_{\rm ren} \equiv \frac{S^{(2)}_{\rm ren}}{V_2} = \lim_{r\to\infty} \int \frac{\mathrm{d}\omega}{2\pi} \left[\Phi^a_{-\omega}(r)\mathbb{A}_{ab}(r,\omega)\Phi^b_\omega(r) + \Phi^a_{-\omega}(r)\mathbb{B}_{ab}(r,\omega)\partial_r\Phi^b_\omega(r)\right],$$

with

$$\Phi^a = \begin{pmatrix} a_x \\ h_{tx} \\ \chi \end{pmatrix}, \quad \mathbb{A} = \begin{pmatrix} 0 & -\mu/2 & 0 \\ -\mu/2 & -m_0 & 0 \\ 0 & & 0 \end{pmatrix}, \quad \mathbb{B} = \begin{pmatrix} -f(r) & 0 & 0 \\ 0 & r^4 & 0 \\ 0 & 0 & -r^2 f(r) \end{pmatrix}$$

Near horizon:

$$\Phi^a(r) = (r-1)^{\nu_{a\pm}} \left(\varphi^a + \tilde{\varphi}^a(r-1) + \cdots\right)$$

$$\nu_\pm = \pm i 12\omega/(-12 + 2\beta^2 + \mu^2)$$

$$h_{tx} = (r-1)^{\nu_\pm+1}(h^{(I)}_{tx} + h^{(II)}_{tx}(r-1) + \cdots),$$
$$a_x = (r-1)^{\nu_\pm}(a^{(I)}_x + a^{(II)}_x(r-1) + \cdots),$$
$$\chi = (r-1)^{\nu_\pm}(\chi^{(I)} + \chi^{(II)}(r-1) + \cdots)$$

In falling BC: choose + only→ keep half of d.o.f

If we give h^(I), etc then second term and all higher order terms determined recursively.

Solve three times, with independent initial conditions,

$$\begin{pmatrix} \varphi^a_1 & \varphi^a_2 & \varphi^a_3 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$$

Then, we have 3 solutions

$$\Phi_i^a(r) \to \mathbb{S}_i^a + \frac{\mathbb{O}_i^a}{r^{\delta_a}} + \cdots \qquad \text{(near boundary)}\,,$$

The general solution is a linear combination of them: let

$$\begin{aligned}\Phi^a(r) &= \Phi_i^a(r) c^i \\ &\to J^a + \frac{\Pi^a}{r^{\delta_a}} + \cdots\end{aligned}$$

$$J^a = \mathbb{S}_i^a c^i \quad \longrightarrow \quad c^i = (\mathbb{S}^{-1})_a^i J^a\,.$$

$$\Pi^a = \mathbb{O}_i^a c^i = (\mathbb{O} \cdot \mathbb{S}^{-1})_b^a J^b$$

So, we achieved our goal, if every thing is OK.

Numerical solution 4: Constraint and Symmetry

However, NOT every thing is OK

$$h_{tx} = (r-1)^{\nu_\pm+1}(h_{tx}^{(I)} + h_{tx}^{(II)}(r-1) + \cdots),$$
$$a_x = (r-1)^{\nu_\pm}(a_x^{(I)} + a_x^{(II)}(r-1) + \cdots),$$
$$\chi = (r-1)^{\nu_\pm}(\chi^{(I)} + \chi^{(II)}(r-1) + \cdots)$$

$\longrightarrow$ Eq. of M

→ Last compnent is determined by the first two.
So only two indendent solution can be generated !

This is rooted to the diffeomorphism invariance of gravity system:
Some of the diffeo. Leaves the Linearized Eq. inv.
even after gauge fixing : g_{rx}=0

Problem maker solves the Problem

$$\text{denote Eq. of . M by } \mathbb{M}\cdot\vec{\Phi}(r)=0$$

Under Diffeo. generated by ξ^μ whose non-vanishing component is $\xi^x = \epsilon e^{-i\omega t}$,

$$\delta\mathbb{M}=0, \longrightarrow \mathbb{M}\cdot\delta_\xi\Phi=0$$

Therefore $\delta_\xi\Phi=(0,-i\omega,\beta)\xi^x$ Is a solution, r-independent

$$\Longrightarrow \vec{S}_0=(a_x^0,h_{tx}^0,\chi^0)^T=(0,1,i\beta/\omega)^T.$$ Hartnoll

Can be added as the third vector to invert

$$\vec{S}_1c^1+\vec{S}_2c^2+\vec{S}_0c^0=\vec{J}$$

Only 2 degree of freedom. S0 is on shell but gauge degree of freedom.

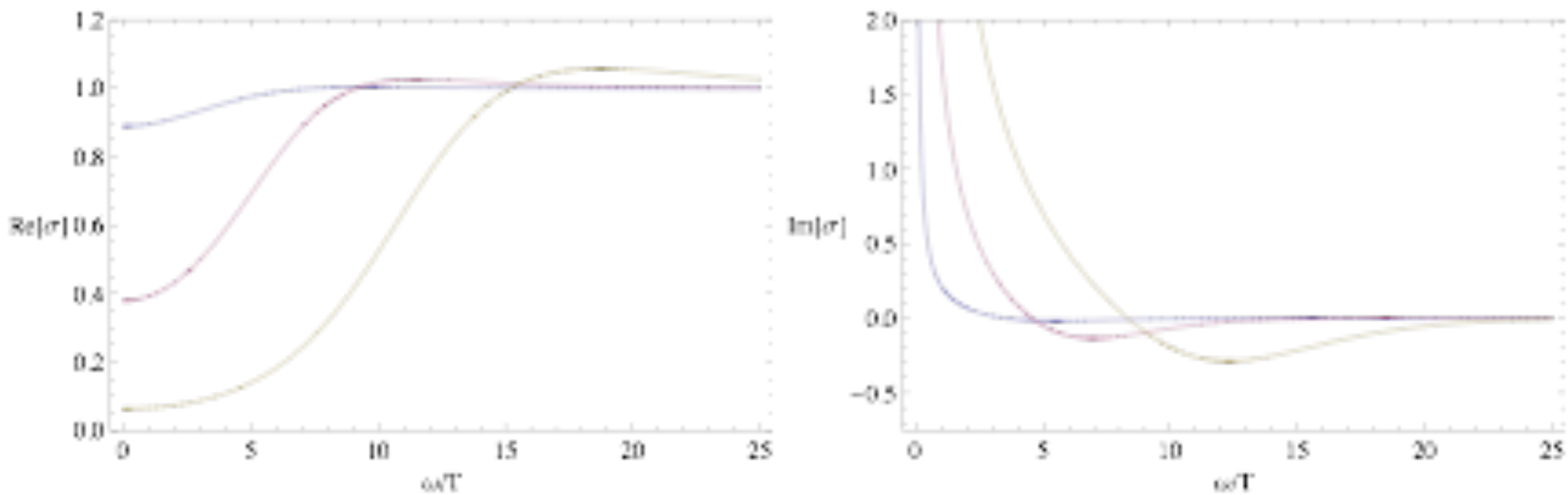
comments: Basis independence

r-evolution is a linear map by left multiplication
While basis change is Right multiplication.
Change in basis can be undone by change in c^I
In more detail,

$$\vec{\varphi}_j \text{ by } \phi_i^a \to \phi_j^a R_i^j \text{ which results in } S_j^a R_i^j$$

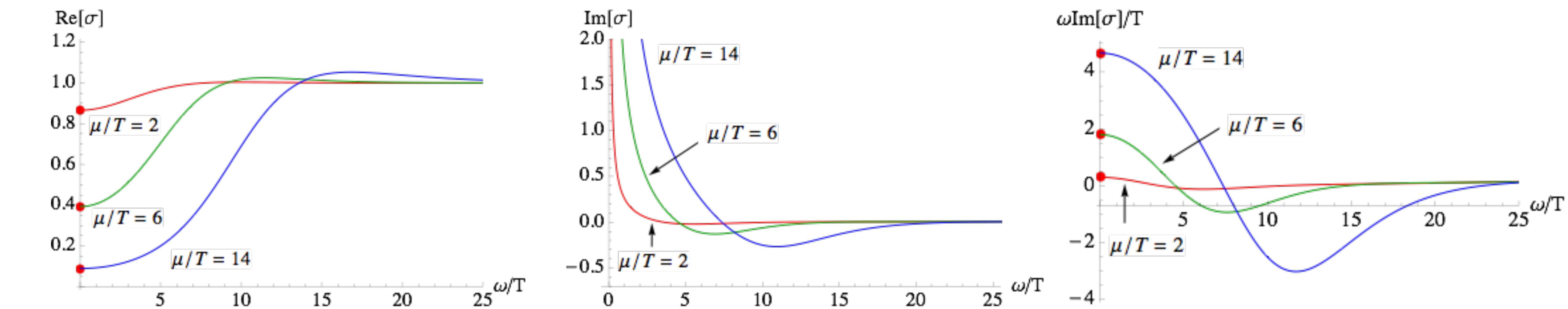
$$\text{undone by changing the } c^i \to c^j (R^{-1})_j^i,$$

Checking code with known results



Hartnoll 0903.3234

Our results



$$\sigma = \sigma_Q + i\frac{K}{\omega}$$

$$\sigma_Q = \left(\frac{3-\frac{\mu^2}{4r_0^2}}{3+\frac{3\mu^2}{4r_0^2}}\right)^2 \qquad K = r_0\frac{\frac{\mu^2}{r_0^2}}{3+\frac{3\mu^2}{4r_0^2}}$$

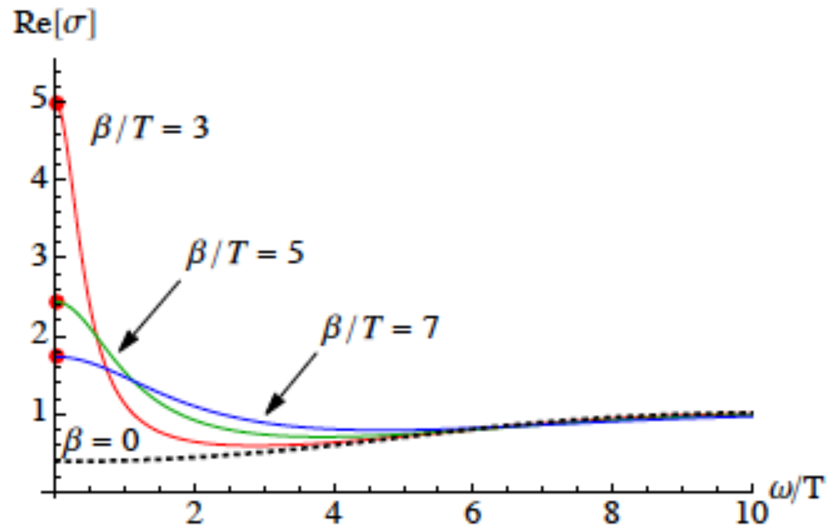
$$r_0 = \frac{2\pi}{3}\left(T+\sqrt{T^2+3(\mu/4\pi)^2}\right)$$

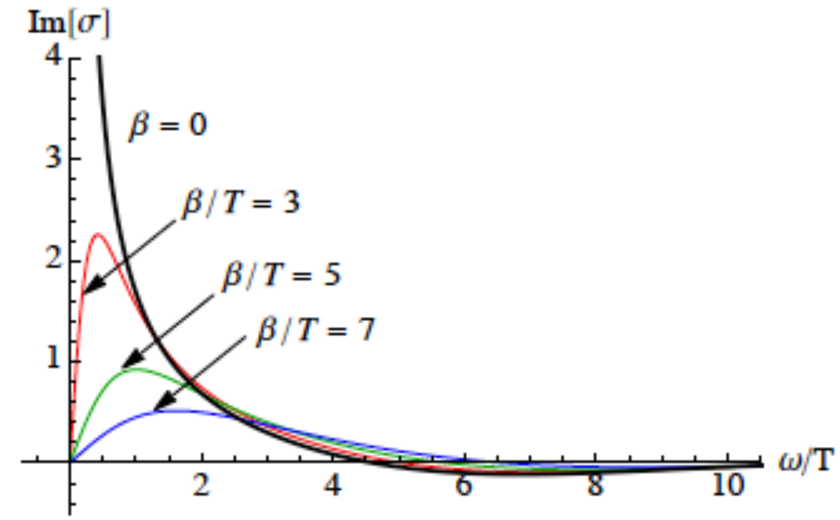
Ge, Jo, and Sin, 1012.2515

$$\begin{pmatrix} G^R_{J_xJ_x} & G^R_{J_xT_{tx}} \\ G^R_{T_{tx}J_x} & G^R_{T_{tx}T_{tx}} \end{pmatrix} = \begin{pmatrix} \frac{a_x^{(1)}}{a_x^{(0)}} & -\mu \\ -\mu & -m_0 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{a_x^{(1)}}{a_x^{(0)}} & -\frac{\mu}{2}-\frac{h_{tx}^{(1)}}{a_x^{(0)}} \\ -\frac{\mu}{2}-\frac{h_{tx}^{(1)}}{a_x^{(0)}} & -m_0 \end{pmatrix}$$

Main Result : AC electric conductivity



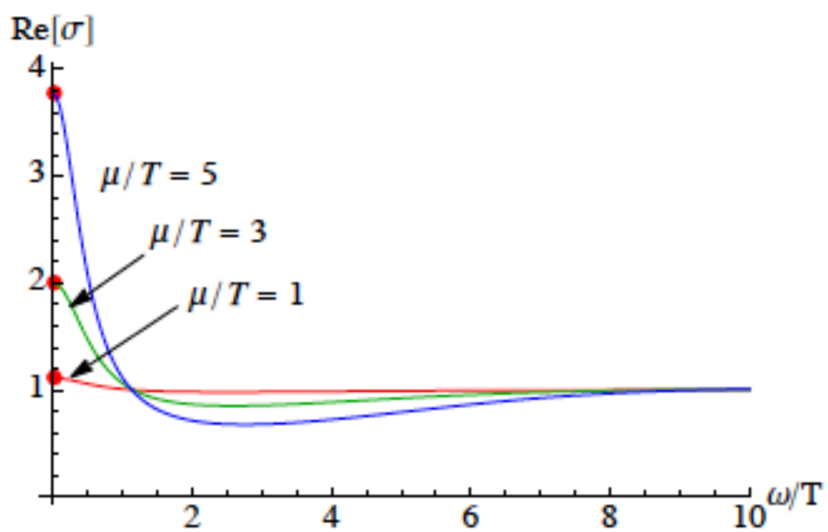


$\mu/T = 6$

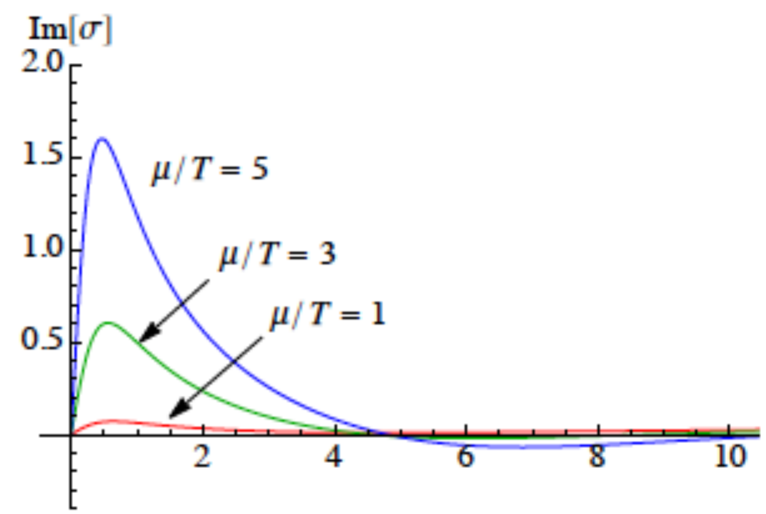
DC limit

$$\sigma = 1 + \frac{\mu^2}{\beta^2}$$

Andrade and Withers
1311.5157



(a) Re σ

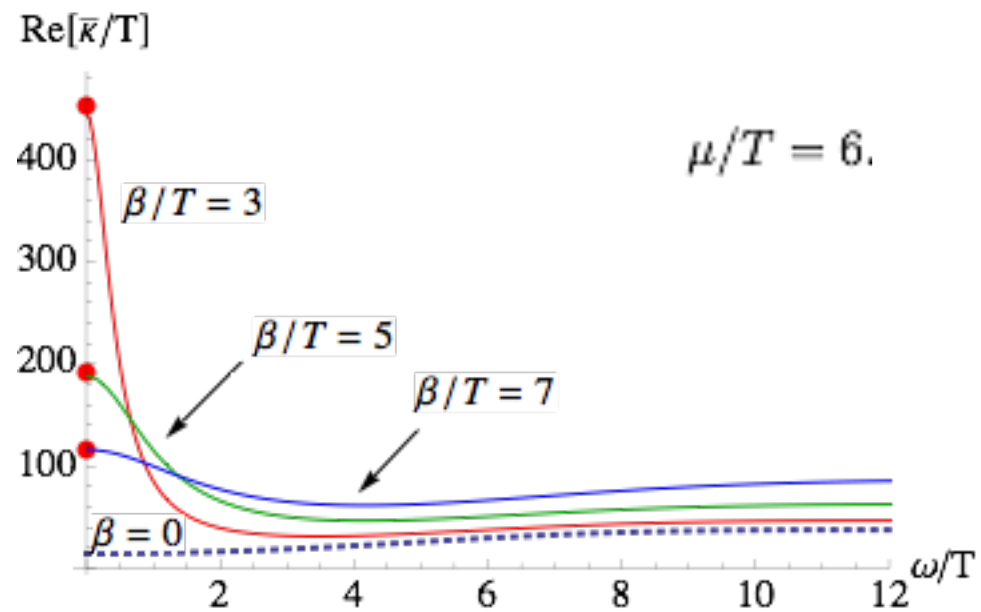


(b) Im σ

$\beta/T = 3$

Thermal and thermoelectric conductivity

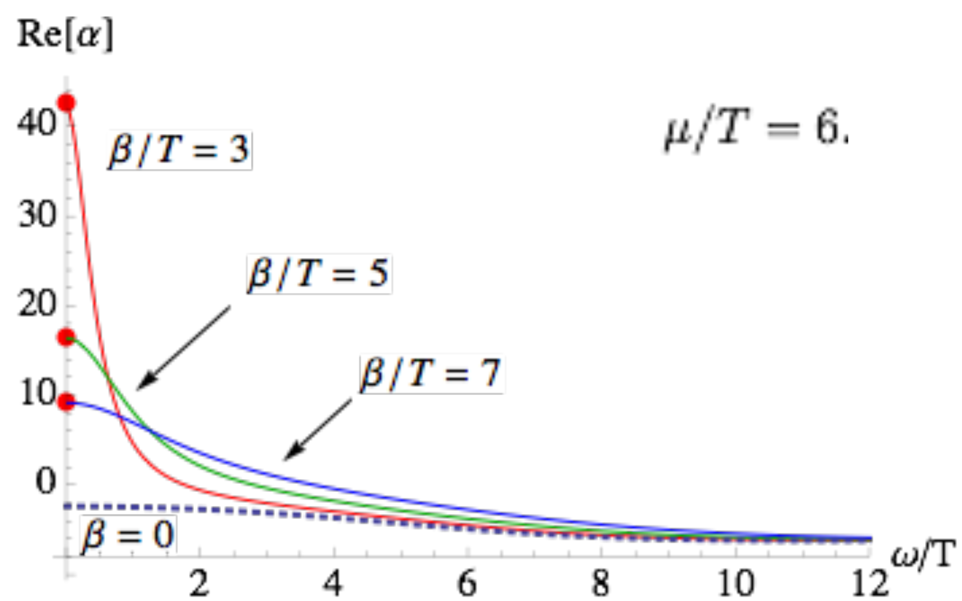
$$\frac{\bar{\kappa}}{T} = \frac{(4\pi)^2}{\beta^2} r_0^2$$



$$\frac{\bar{\kappa}}{T} \to \frac{\mu^2+\beta^2}{T^2}$$

DC results:
Donos and Gauntlett
1406.4742

$$\alpha = \frac{4\pi\mu}{\beta^2} r_0$$



$$\alpha \to -\frac{\mu}{T}$$

Drude-like? Intermediate scaling?

Drude peak

- Drude model

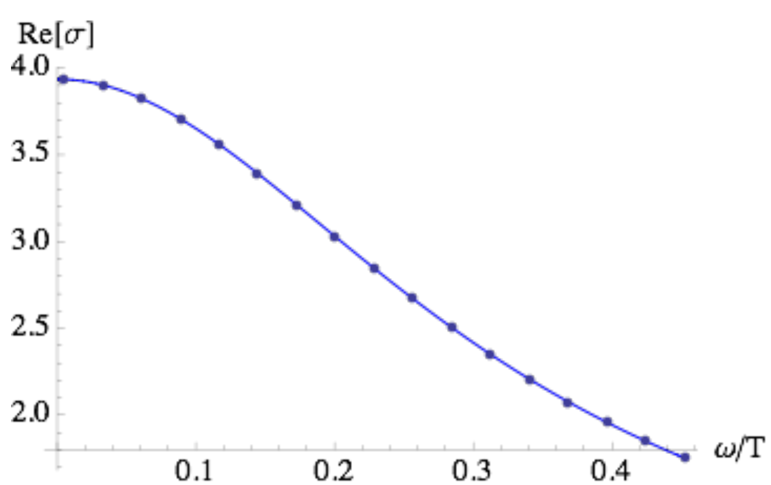
$$\frac{dp}{dt} = -\frac{1}{\tau}p + qE\,. \qquad\qquad \sigma(\omega) = \frac{K\tau}{1 - i\omega\tau}$$

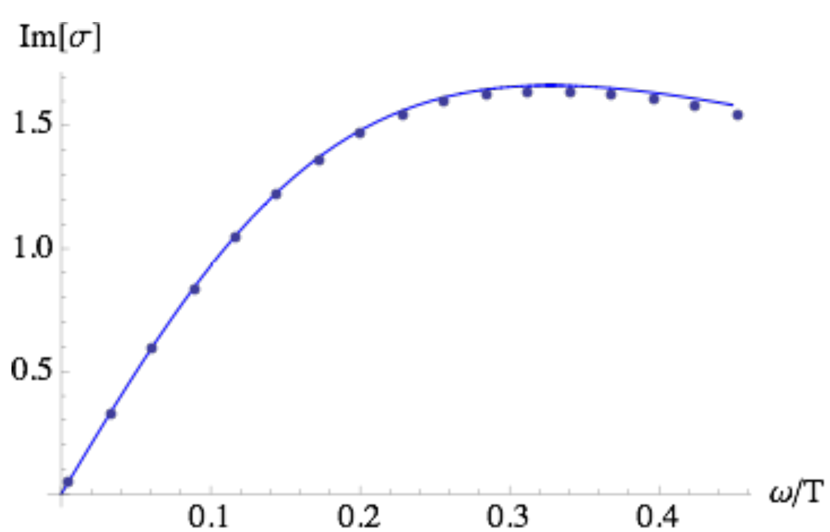
- Ward identity

$$\nabla^{\nu}\langle T_{\nu\mu}\rangle = F_{\mu\nu}\langle J^{\nu}\rangle + \partial_{\mu}\phi\langle \mathcal{O}\rangle\,,$$

$$\partial_t\langle \delta p_x\rangle = \beta\langle \delta\mathcal{O}\rangle + \langle J^t\rangle \delta E_x$$

- Fitting $\quad \sigma(\omega) = \dfrac{B\tau}{1 - i\omega\tau} + A$





Analytic expressions of Drude peak parameters

$$\sigma_Q = \left(\frac{3 - \frac{\mu^2}{4r_0^2}}{3 + \frac{3\mu^2}{4r_0^2}} \right)^2 , \quad K = r_0 \frac{\frac{\mu^2}{r_0^2}}{3 + \frac{3\mu^2}{4r_0^2}} ,$$

$$r_0 = \frac{2\pi}{3} \left(T + \sqrt{T^2 + 3(\mu/4\pi)^2 + 6(\beta/4\pi)^2} \right)$$

$$\sigma \to \sigma_Q + i\frac{K}{\omega}$$

$$\tau \to \infty \ (\beta \to 0)$$

$$\boxed{\sigma(\omega) = \frac{K\tau}{1 - i\omega\tau} + \sigma_Q}$$

$$\omega \to 0$$

$$\sigma \to K\tau + \sigma_Q = 1 + \frac{\mu^2}{\beta^2}$$

$$\begin{aligned} \tau &= \frac{1 + \frac{\mu^2}{\beta^2} - \sigma_Q}{K} \\ &= \frac{1}{4\pi T} \cdot \frac{45\tilde{\beta}^4 + 36\tilde{\mu}^4 + 2(1+\Delta) + 6\tilde{\beta}^2(4 + 12\tilde{\mu}^2 + 3\Delta) + 3\tilde{\mu}^2(5 + 4\Delta)}{\tilde{\beta}^2(1+\Delta)(1 + 3\tilde{\beta}^2 + 6\tilde{\mu}^2 + \Delta)} \end{aligned}$$

$$\Delta \equiv \sqrt{1 + 3\tilde{\mu}^2 + 6\tilde{\beta}^2} , \quad \tilde{\mu} \equiv \frac{\mu}{4\pi T} , \quad \tilde{\beta} \equiv \frac{\beta}{4\pi T}$$

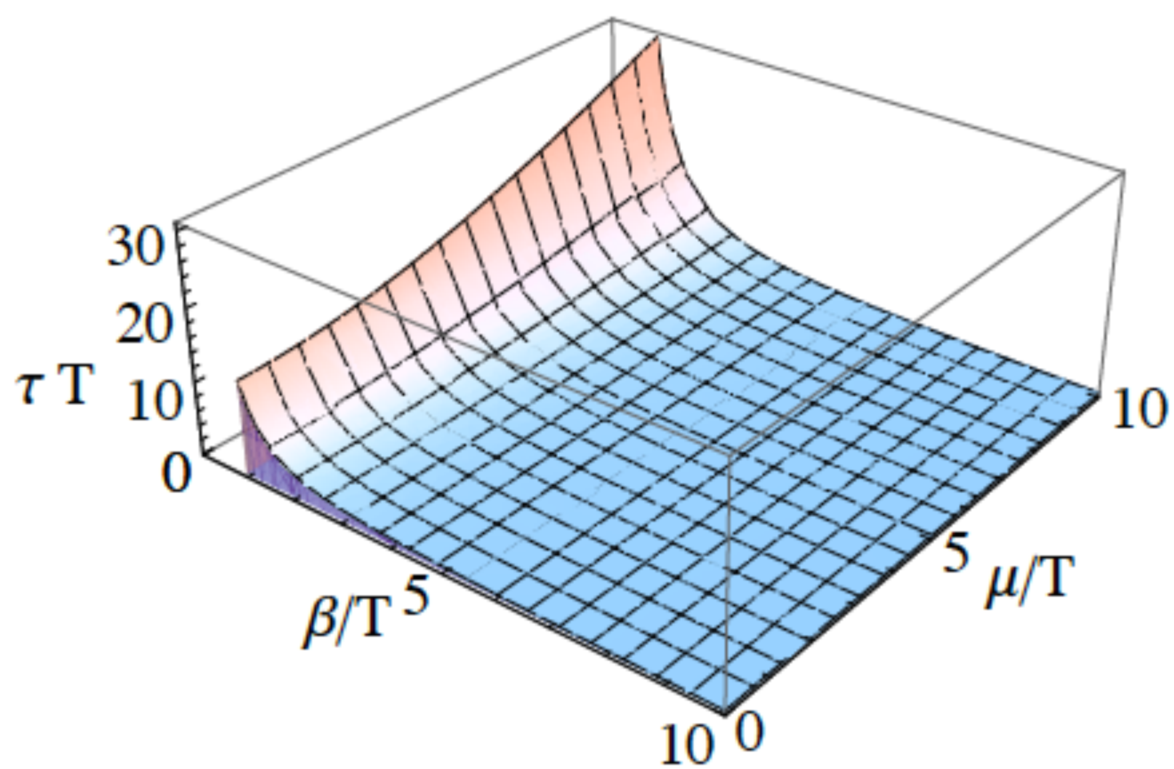
Relaxation time

$$\tau = \frac{1 + \frac{\mu^2}{\beta^2} - \sigma_Q}{K}$$

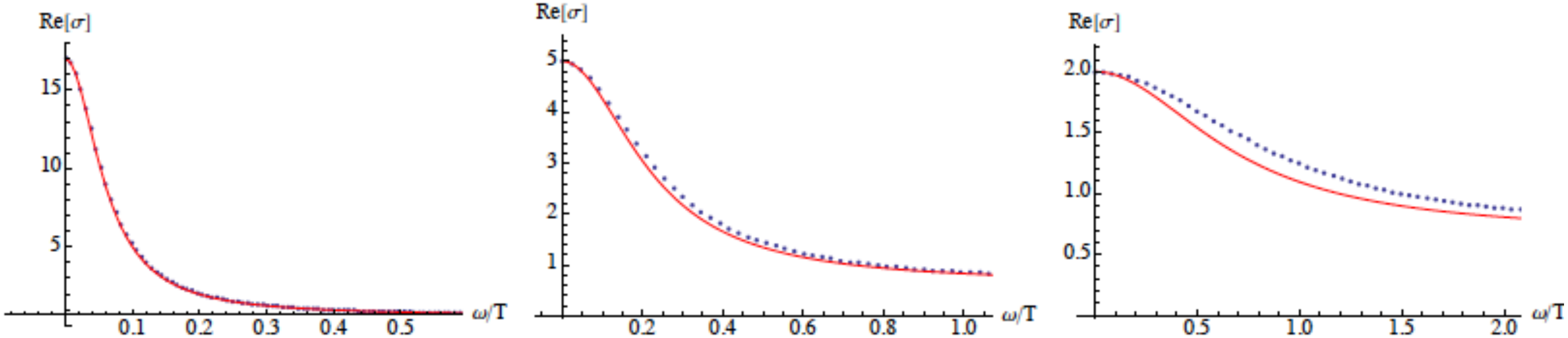
$$= \frac{1}{4\pi T} \cdot \frac{45\tilde{\beta}^4 + 36\tilde{\mu}^4 + 2(1+\Delta) + 6\tilde{\beta}^2(4 + 12\tilde{\mu}^2 + 3\Delta) + 3\tilde{\mu}^2(5 + 4\Delta)}{\tilde{\beta}^2(1+\Delta)(1 + 3\tilde{\beta}^2 + 6\tilde{\mu}^2 + \Delta)}$$

$$\Delta \equiv \sqrt{1 + 3\tilde{\mu}^2 + 6\tilde{\beta}^2}, \quad \tilde{\mu} \equiv \frac{\mu}{4\pi T}, \quad \tilde{\beta} \equiv \frac{\beta}{4\pi T}$$

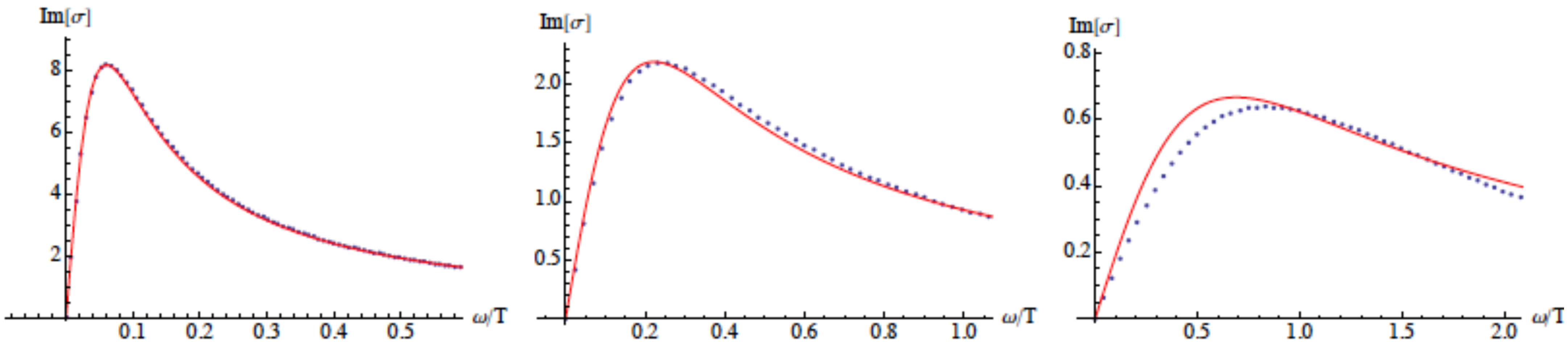
$$\tau \approx 2\sqrt{3}\frac{\mu}{\beta^2}$$



Coherent to incoherent transition



(a) Re σ at $\beta/\mu = 1/4$ (b) Re σ at $\beta/\mu = 1/2$ (c) Re σ at $\beta/\mu = 1$



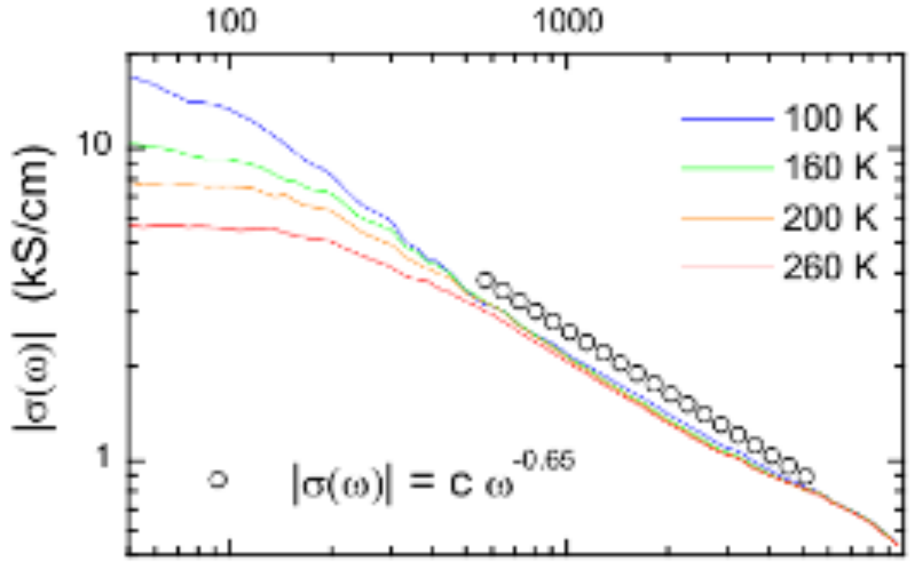
(d) Im σ at $\beta/\mu = 1/4$ (e) Im σ at $\beta/\mu = 1/2$ (f) Im σ at $\beta/\mu = 1$

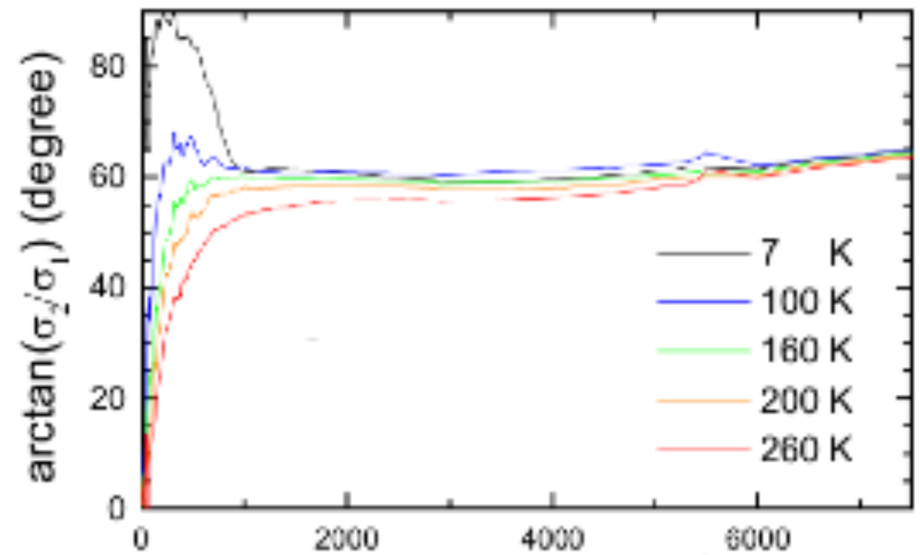
Low $T < \beta, \mu$ $\qquad \sigma(\omega) = \dfrac{K\tau}{1 - i\omega\tau} + \sigma_Q$

$\beta/\mu \leq 1/2$	'Clean' region	Drude	Coherent metal	$\tau \approx \dfrac{\mu}{\beta^2}$
Beyond:	'Dirty' region	~~Drude~~	Incoherent metal	

Scaling law: exp v.s theory a la HST

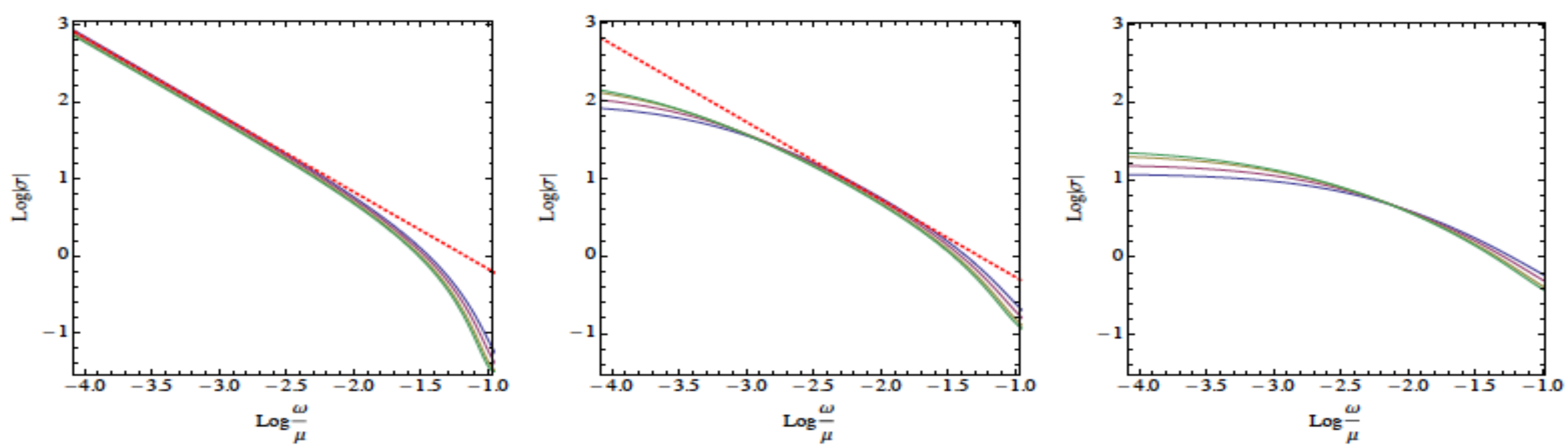
- AC conductivity $\sigma(\omega) \to (i/\omega)^{\nu} \quad \nu \approx 0.65$





•In our study: No. → breaking TI in boundary and Bulk gives different result. → Importatant data.

•



(a) $\beta/r_0 = 0.1$ (b) $\beta/r_0 = 1$ (c) $\beta/r_0 = 1.5$

Physics:

Drue peak without quasi partilce

$$\tau \simeq \frac{\mu}{\beta^2}$$

Coherence is time scale: If beta is small enough, momentum relaxation is slow and there is a drude peak.

1/mu ~ inter-particle distance.

1/beta~ inter-impurity distance.

Wiedermann-Franz Law

$$\bar{L} \equiv \frac{\bar{\kappa}}{\sigma T} = \frac{1}{\tilde{\mu}^2 + \tilde{\beta}^2}\frac{r_0^2}{T^2} = \frac{4\pi^2\left(1+\sqrt{1+3(2\tilde{\beta}^2+\tilde{\mu}^2)}\right)^2}{9(\tilde{\beta}^2+\tilde{\mu}^2)} \approx \frac{4\pi^2}{3}\frac{2\beta^2+\mu^2}{\beta^2+\mu^2}$$

$$\bar{L} = \begin{cases} \frac{4\pi^2}{3} \ (\mu \gg \beta) \\ \frac{8\pi^2}{3} \ (\beta \gg \mu) \end{cases} .$$

numerical values are different from the Fermi-liquid case

Conclusion

- In RN black hole with translation symmetry broken by

$$\psi_I = \beta_{Ii} x^i = \beta \delta_{Ii} x^i$$

$$\sigma(\omega) = \frac{K\tau}{1 - i\omega\tau} + \sigma_Q \qquad \tau \approx \frac{\mu}{\beta^2}$$

- AC electric conductivity → Coherent /Iincoherent transition can be discussed by the impurity parameter beta.

- Thermoelectric conductivity has the same relaxation time

- Systematic numerical recipe established.

- No intermediate scaling in our case as well as massive gravity. → breaking TI in boundary and Bulk gives different result.

-

Strongly interacting many body system.

Hard to calculate in strongly interaction regime in field theory.
→mimic Holographic calculation of N=4 Gauge theory

However, out of Three pillars of proving duality
(Susy, large N, conformality) at least two should be broken
Analogy: 1st law of thermodynamics
or Schroedinger eq. without/with potential

$$\frac{-\hbar^2}{2m}\nabla^2\Psi = E\Psi \rightarrow [\frac{-\hbar^2}{2m}\nabla^2 + V(x)]\Psi = E\Psi$$

Only Experiment can tell us the validity!
Meanwhile we practice and develop new intuitition about model and phenomena.