Driven Holographic CFTs

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Numerical Holography Institute, December 2014

Based on:

M.R., Mukund Rangamani, Anson Wong, to appear shortly.
M.R., Mukund Rangamani, Mark van Raamsdonk, in progress.

Outline:

Introduction and Motivation

Interesting physics in driven dissipative many-body systems:

- Non-thermal steady states in open systems
- Non-equilibrium phase transitions (also in open systems).

  Dynamics and universality in noise driven dissipative systems.

  Emanuele G. Dalla Torre,Eugene Demler, Thierry Giamarchi, Ehud Altman. arXiv 1110.3678.
- New types of Universality, e.g.

  Universal energy fluctuations in thermally isolated driven systems.

  Guy Bunin, Luca D'Alessio, Yariv Kafri, Anatoli Polkovnikov. arXiv 1102.1735.
- New phenomena

  Many-body energy localization transition in periodically driven systems.

  Luca D'Alessio, Anatoli Polkovnikov. arXiv 1210.2791.
- New regimes obtained in cold atom experiments (also, relation to cosmology).

Even more interesting issues have to do with spatial inhomogeneities (e.g. topological defects, spatial disorder). This talk, however, is about the simplest example: homogeneous periodically driven system.

Introduction and Motivation

Note the difference from the well-studied holographic *quenches*. We look at universal features of the system at late times, while it is still being driven. This probes different aspects of the system from those relevant to quenches and return to static equilibrium. Any late time steady state in not equilibrium state.

Previous studies in the context of holography

- Perturbative study of similar systems, in the linear response regime.

  R. Auzzi, S. Elitzur, S. B. Gudnason and E. Rabinovici,
  On periodically driven AdS/CFT.
  JHEP 1311, 016 (2013), arXiv:1308.2132.
- W. J. Li, Y. Tian and H. b. Zhang,
  Periodically Driven Holographic Superconductor,
  JHEP **1307**, 030 (2013). arXiv:1305.1600.

Driving the Geometry I

We consider gravity coupled to a scalar field with $m^2 = -2$ in asymptotically AdS_4 space. The scalar field is used to drive the geometry: choose the non-normalizable mode to be of the form

$$\phi_0(t) = A\cos(\omega t)$$

Importantly, this is a relevant perturbation, with a time scale. Some of the observed phenomena are absent when driving the system by e.g. a massless scalar or a gauge field.

We start the system at equilibrium at initial temperature T_0. The dimensional scales in the problem are then T_0, A, ω. We usually measure all quantities, including the time t, in units of the period $P = \frac{2\pi}{\omega}$, but sometimes in units of the initial temperature T_0. At late times the initial temperature T_0 should drop out. Everything should then depend only the dimensionless strength of the external driving force.

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Driving the Geometry II

Another interesting option: drive the system via the metric, i.e consider a cosmological background. Look at boundary 3+1 dimensional FRW times a Scherk-Schwarz circle (to provide a length scale).

$$ds^2_{bdy} = -dt^2 + a^2(t)\, ds^2_{3,k} + L^2\, dw^2$$

$$ds^2_{3,k} = \frac{d\rho^2}{1 - k\,\rho^2} + \rho^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

The holographic dual is asymptotically AdS_6. To make connection to cosmology choose $a(t)$ to expand, e.g. between two asymptotic values of the scale factor.

The types of observables interesting in this context is also different (particle production, cosmological fluctuations). Most of the results below are for the first model, ask me privately about the second one.

Metric Ansatz

Back to the first model. We use the Bondi-Sachs form of the metric to utilize the characteristic formulation

$$ds^2 = -2Adt^2 + 2e^{\chi}dtdr + \Sigma^2\left(dx^2 + dy^2\right)$$

We gauge fix Σ, in a form which fixes difeomorphisrm invariance up to a radial shift parametrized by $\lambda(t)$. The gauge parameter $\lambda(t)$ is chosen to fix the coordinate location of the apparent horizon.

We treat $\phi(t), \lambda(t)$ as dynamical variables, and solve the constraints for A, χ at each time step. Detailed procedure is a variation on:

P. M. Chesler and L. G. Yaffe, JHEP **1407**, 086 (2014) [arXiv:1309.1439 [hep-th]].
K. Balasubramanian and C. P. Herzog, Class. Quant. Grav. **31**, 125010 (2014) [arXiv:1312.4953 [hep-th]].

Details of the Numerics

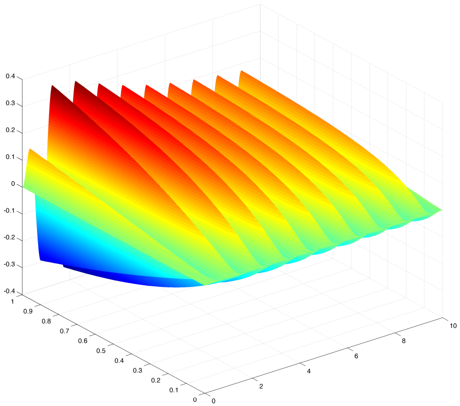
Some gratuitous technicalities:

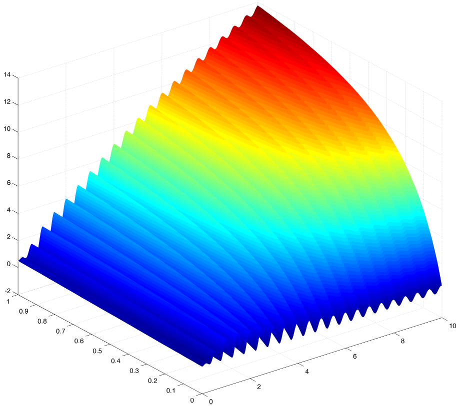
- Discretize the radial direction on Chebyshev grid. For most of the plots there are 61 grid points (likely an overkill).
- For time evolution use for the most part explicit or singly-diagonal implicit RK45 method: Runge-Kutta of order 4 with an adaptive step size.
- For the purpose of retaining precision near the boundary and horizon, use compensated summation when evaluating the equations.
- Source is ramped up gradually, getting to the advertised amplitude over two periods.

Most of the changes in the code are due to the difference between quench and return to equilibrium versus continuous drive. At late times the system is very far from the initial configuration.

Bulk Solutions

The solutions look like what you'd expect: on the left is the scalar field (which grows towards the horizon since it is relevant). On the right is g_{tt}.





Observables

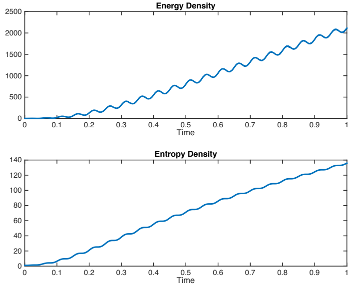
Types of observables we display to exhibit different aspects of the physics:

- **Phase Portrait** Scalar field response as function of source, good tool to display phase information. Alternatively, define conductivity as $\sigma(\omega) = \frac{\phi_1}{i\omega\phi_0}$, though we are not in the linear response regime.
- **Cycle Averages** of entropy and energy densities, as function of time. Scaling relations between them $s \sim \epsilon^\gamma$. At equilibrium $\gamma = \frac{2}{3}$, we'll see scaling relation (for late times) with $\gamma > \frac{2}{3}$ for us.
- **Fluctuations** (in each cycle) of scalar response, entropy and energy. Note the difference from ensemble fluctuations.
- **Entanglement Entropy** For discs and strips of various sizes.

Monotonicity of Entropy

- Since we measure entropy, in time-dependent settings, by the area of apparent horizon, it is not clear it has to be monotonic.
- Examples where the apparent horizon area if not monotonic are known.
- However, with the additional assumption of spatial homogeneity, the area of the apparent horizon is monotonic.
- This is a non-trivial check of the numerics, one needs to control the near-horizon behaviour of the fields.

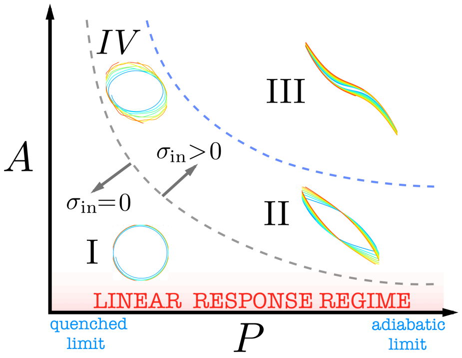
Example of non-monotonic energy increase, versus monotonic entropy increase.



Phase Diagram

Note that our scalar is linear, gravity is the source of both non-linearity and dissipation (later we will compare to non-linear scalar fields).

Qualitative behaviour as we change parameters. Explanation of main features of various phases to follow.



Dissipation Dominated Regime

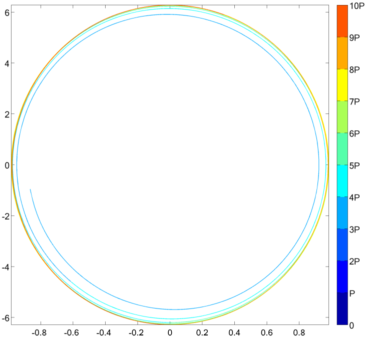
This includes the previously studied linear response regime and the high frequency regime. In gravity: all the supplied energy falls into the black hole without interacting, i.e. dissipated. No work in done on the scalar field. Note that evolution can be rapid ($\frac{1}{T_0}\frac{\dot{s}}{s} >> 1$).

Other manifestations of the simplicity in this regime:

– Simple phase portrait where the response always orthogonal to the source, i.e. the conductivity is purely real (dissipative).

– Steady state reached almost immediately, periods are closed and regular.

– Scaling relation $s \sim \epsilon^{\gamma}$ with $\gamma = \frac{2}{3}$ or slightly above. Small fluctuations in ϵ, s.



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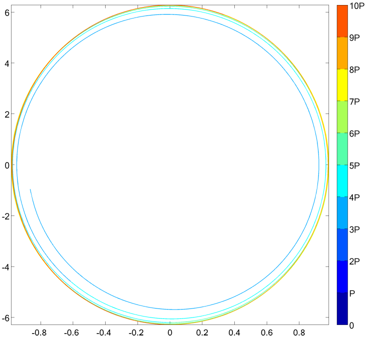
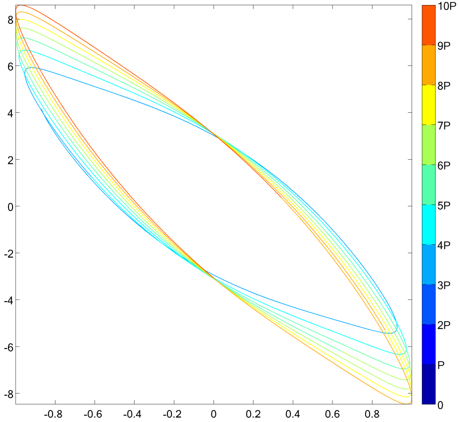


Dynamical Crossover

Increase the dimensionless coupling AP, get qualitative change in the scalar response:

–Most striking feature: the phase portrait is no longer closed, it precesses. The precession means that the discrete time translation invariance associated with the driving force is now broken.

– Tilt in the phase diagram means conductivity has non-trivial imaginary part. The external force is doing work on the scalar field.



Perturbative Interactions

Correspondingly, energy and entropy of black hole grow at a slower pace, since some of the injected energy goes towards doing work on the scalar field.

Interestingly, there is still a scaling relation between them at late times, $s \sim \epsilon^{\gamma}$ with $\gamma > \frac{2}{3}$ throughout the phase diagram. The entropy production is less affected by the work done on the system.

Fluctuations in thermodynamic quantities are still small.

Those features can be mimicked by considering non-linear scalar field with polynomial self-interactions, in the probe limit. Considering different potentials allows us to separate the two features of the phase portrait above.

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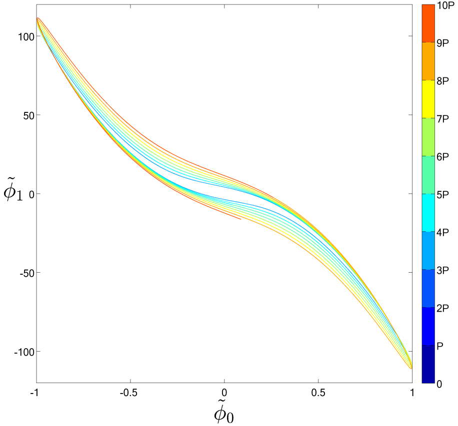
Non-perturbative Interactions

Finally, when we increase the drive strength AP further we enter another new regime:

– Time translation invariance is restored, periods are closed again.

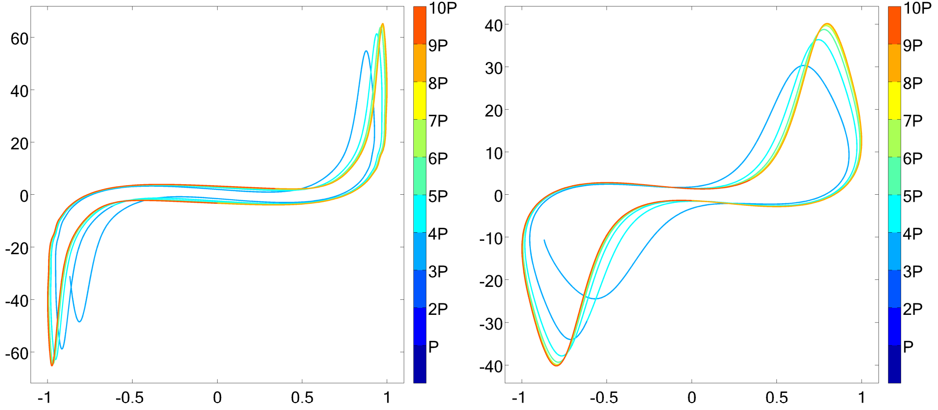
–Phase portrait is narrow, and response almost in phase with source: regime of large excitation of the scalar field.

– Non-linear resonant response: maximal response is very large and increases rapidly with small changes in amplitude.



Non-perturbative Interactions

These features can be reproduced by non-polynomial scalar (where $\phi \to \sinh\phi$, for example), but not by a polynomial potential. For example the phase portraits, for two different non-linear potentials.



Energy Fluctuations

As with the previous dynamical regime, average entropy and energy seem to still relate by some scaling with scaling exponent $\gamma > \frac{2}{3}$. However, *fluctuations* in energy are enhanced in this regime.

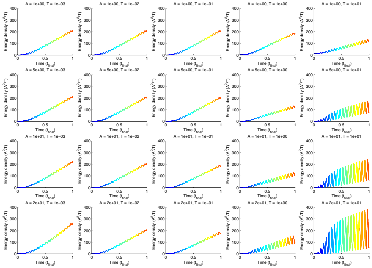
- *Ensemble* energy fluctuations are known to exhibit phase transition as function of parameters in some driven systems.

  Universal energy fluctuations in thermally isolated driven systems.

  Guy Bunin, Luca D'Alessio, Yariv Kafri, Anatoli

  Polkovnikov. arXiv 1102.1735.

- The relation to *cycle* fluctuations we see is not clear to us.



Future Directions

In this project:

- Finish calculating the entanglement entropy.
- Organize the above phenomenology in terms of a coherent narrative, especially for the strongly non-linear regime.

Some projects in progress, and a wish list:

- Holographic cosmology (MR, Mark van Raamsdonk, Mukund Rangamani, in progress).
- Spatially inhomogeneous drives and quenches (MR, Mukund Rangamani, Alex Vincart-Emard, in progress).
- Ensemble fluctuations, but: what boundary conditions?
- Noisy systems and non-equilibrium phase transitions.

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