HOLOGRAPHIC QUANTUM REVIVALS

Javier Mas
Universidad de Santiago de Compostela

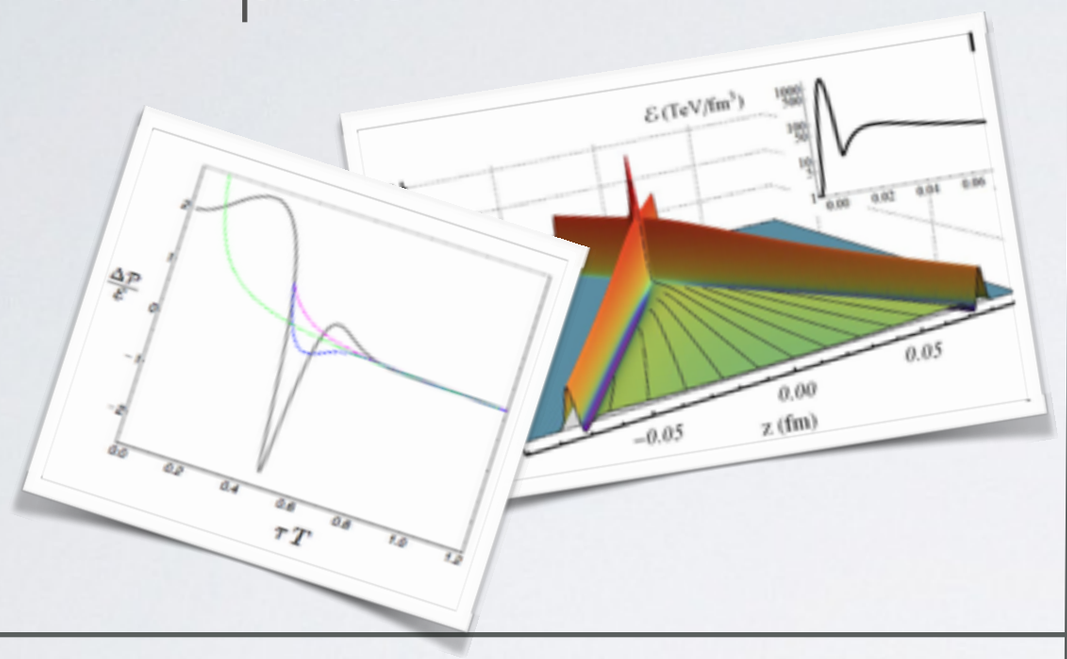
CERN-CKC TH Institute on
Numerical Holography
Dec. 2014

Javier Abajo-Arrastia, Emilia da Silva, Esperanza López, J.M. & Alexandre Serantes.
arXiv:1403.2632 and 1501.xxxx

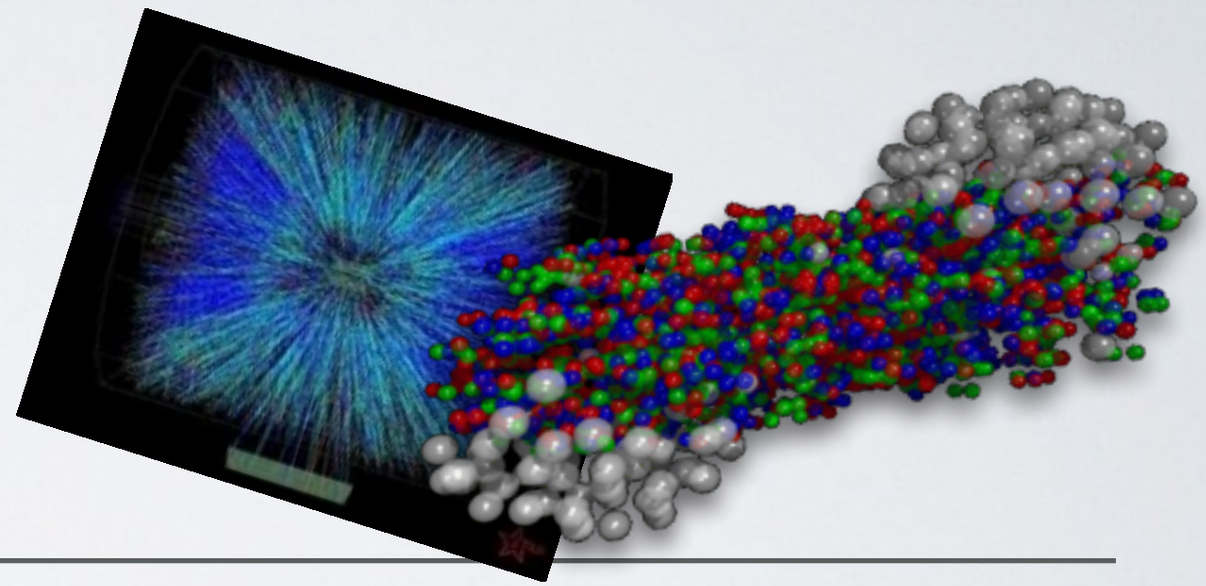
MOTIVATION

time dependent AdS/CFT

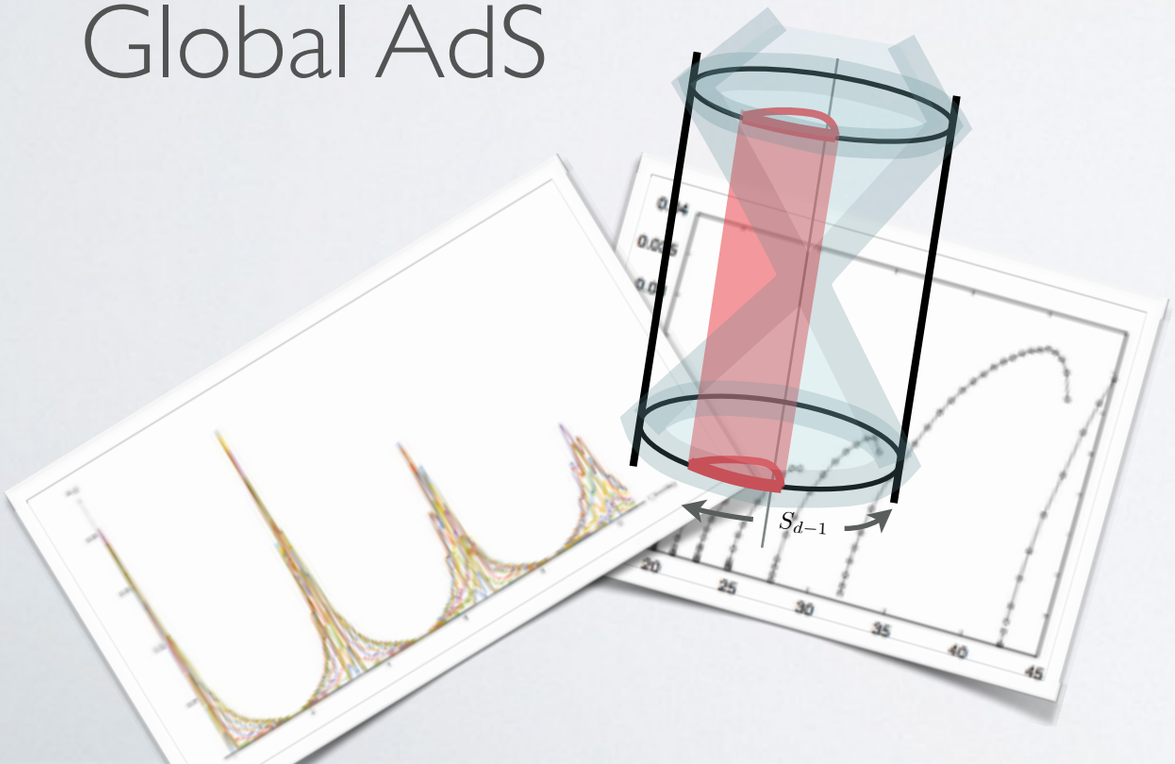
Poincaré patch



Heavy Ion Collisions



Global AdS



PLAN

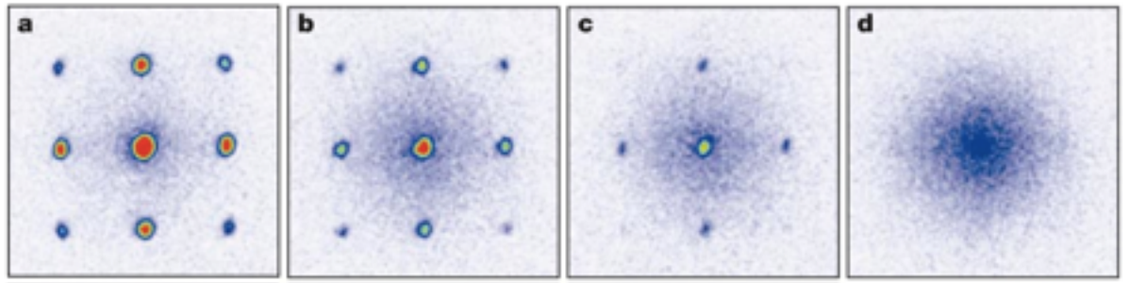
- Revivals
- Entanglement Entropy
- Holographic Revivals

Revivals

Collapse and revival of the matter wave field of a Bose–Einstein condensate

Markus Greiner, Olaf Mandel, Theodor W. Hänsch & Immanuel Bloch

2002



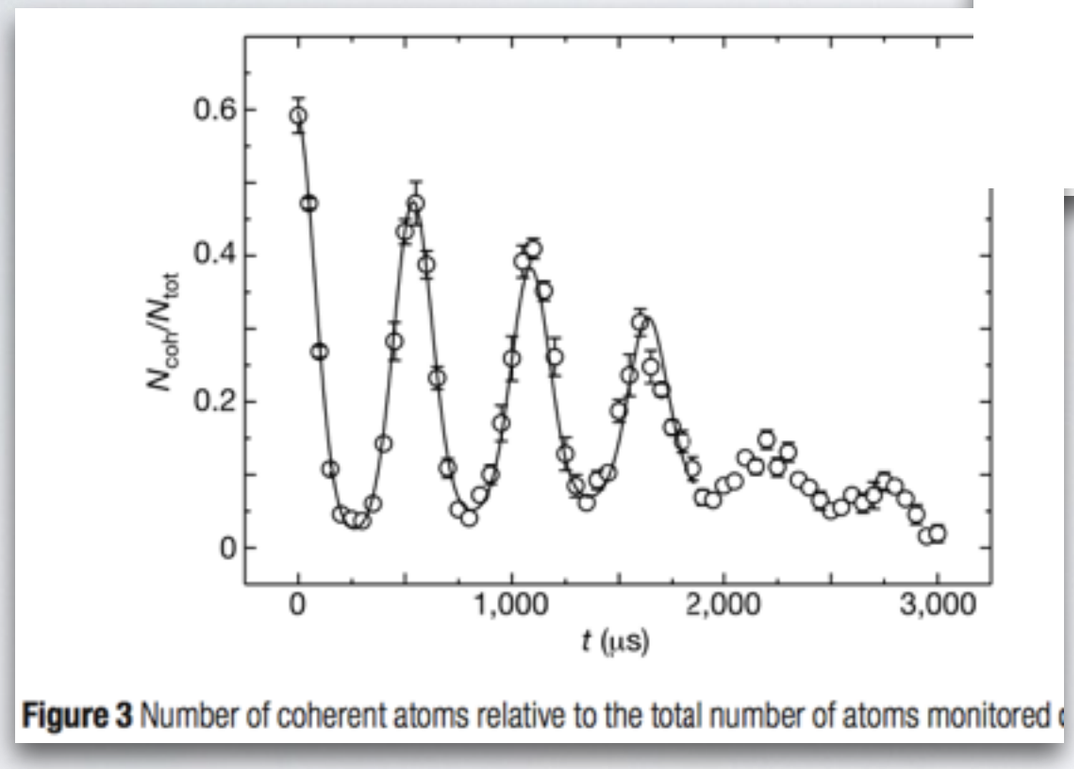
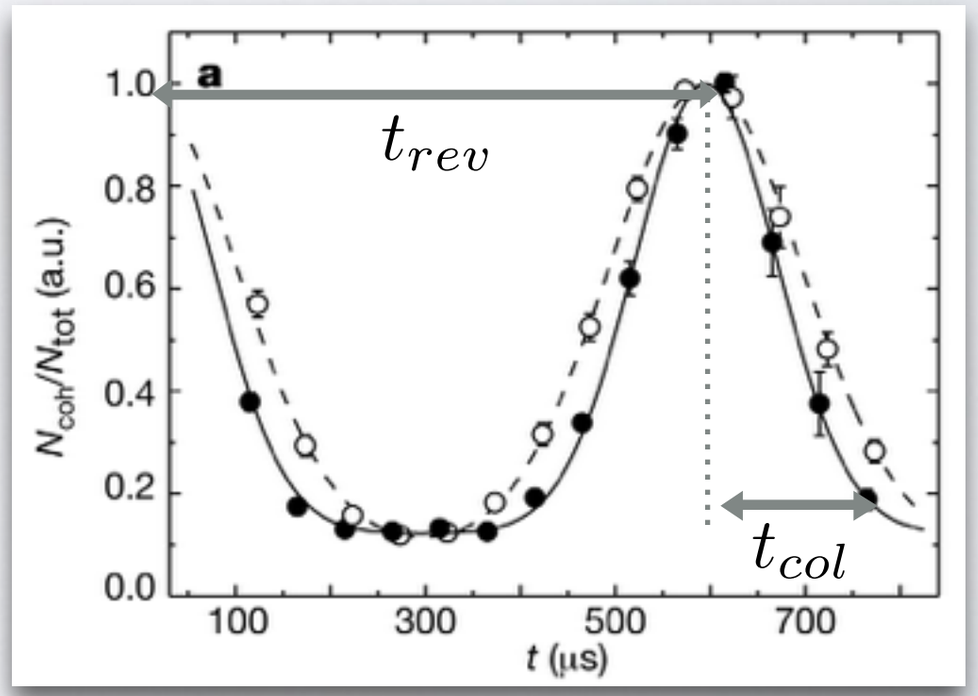


Figure 3 Number of coherent atoms relative to the total number of atoms monitored



$$\hat{H} = \frac{U}{2}\hat{n}(\hat{n}-1)$$

$$\psi(t) = \sqrt{\bar{n}}\, e^{-\bar{n}(1-\cos Ut)} e^{i\bar{n}\sin Ut}$$

$$t_{rev} = \frac{2\pi}{U} \qquad t_{col} = \frac{1}{\bar{n}U}$$

Revivals

The Finite Group Velocity of Quantum Spin Systems

Elliott H. Lieb*
Dept. of Mathematics, Massachusetts Institute of Technology
Cambridge, Massachusetts, USA

Derek W. Robinson**
Dept. of Physics, Univ. Aix-Marseille II, Marseille-Luminy, France

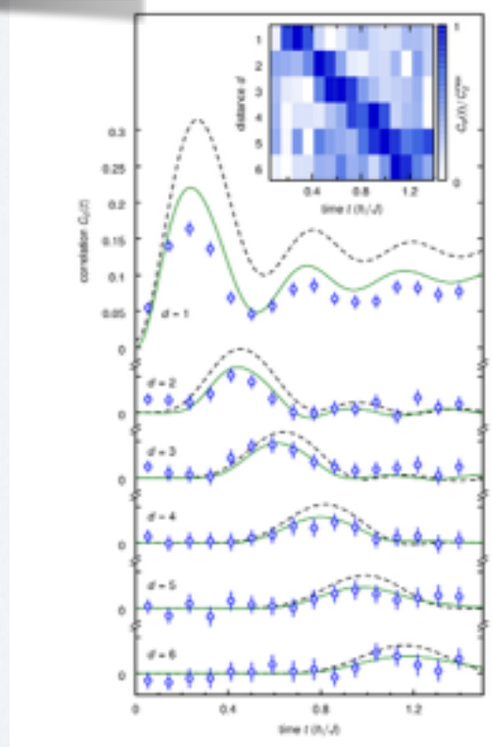
1972

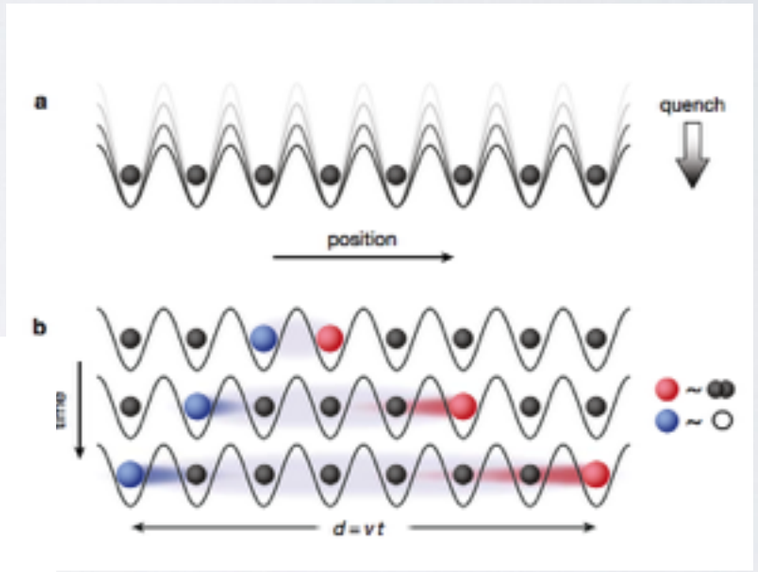
Quantum systems described by a sum of local Hamiltonians → emergent maximum speed v_{LR}

Light-cone-like spreading of correlations in a quantum many-body system

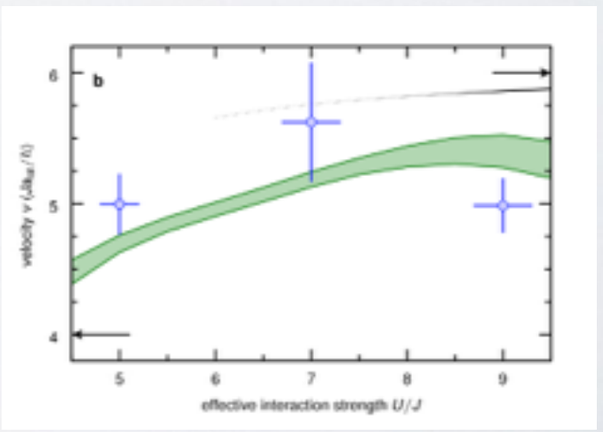
Marc Cheneau, Peter Barmettler, Dario Poletti, Manuel Endres, Peter Schauß, Takeshi Fukuhara, Christian Gross, Immanuel Bloch, Corinna Kollath & Stefan Kuhr

2012





what is the speed v_{LR}



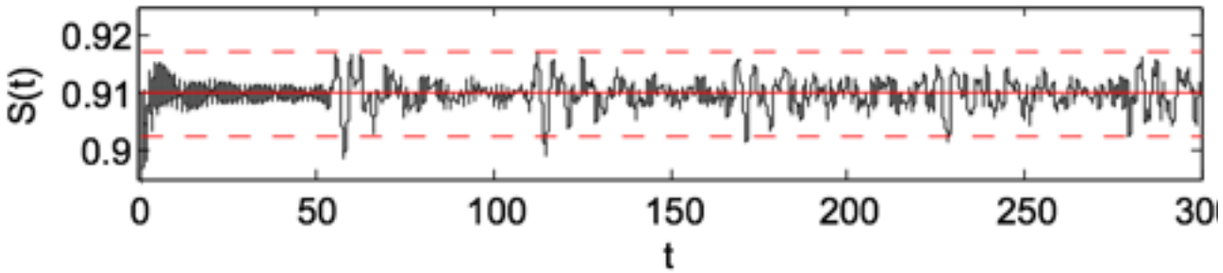
Revivals

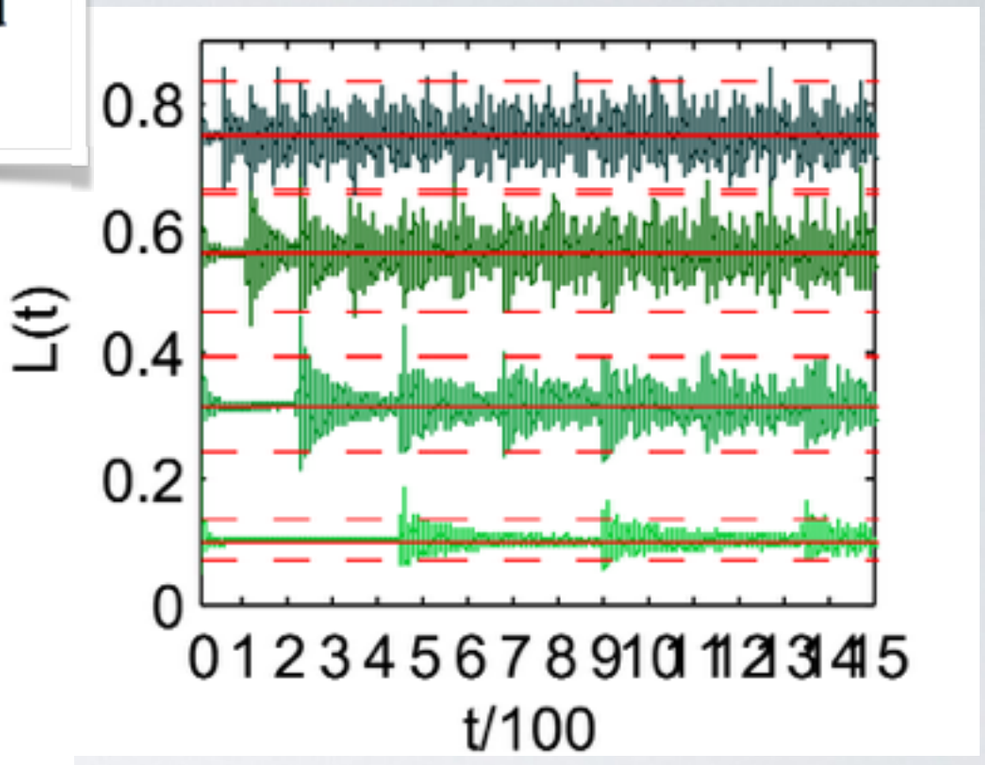
Revivals of a closed quantum system and Lieb-Robinson speed

Juho Häppölä[4],[1,2] Gábor B. Halász,[3,4] and Alioscia Hamma[4]

2010

$$t_{rev} = p\frac{L}{v_{LR}}$$

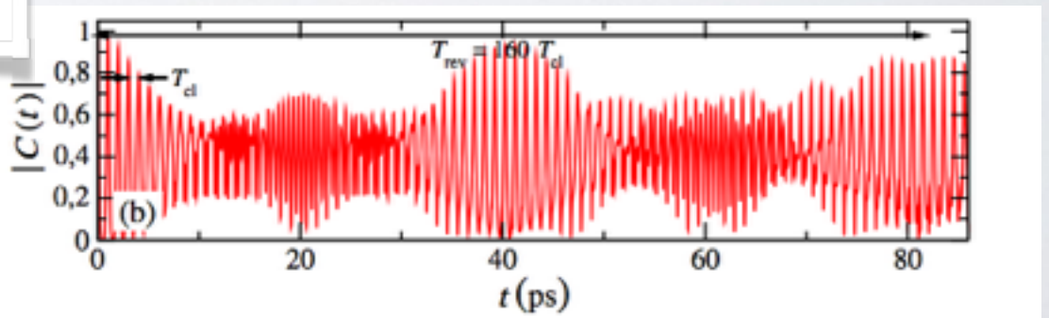




Revivals of quantum wave packets in graphene

Viktor Krueckl[1,3] and Tobias Kramer[1,2,3]

2009



R.W. Robinett, "*Quantum wave packet revivals*", Physics Reports 392 (2004) 1-119

Quantum Field Theory

- are there revivals?

T. Takayanagi & T. Ugajin 2010
CFT = 1+1 Free Fermion

John Cardy 2014
*R*CFT =minimal models

- at strong coupling ?

- what observables are nice to monitor

i) local: $\langle \hat{A} \rangle(t) = \langle \psi(t) | \hat{A} | \psi(t) \rangle$

ii) non-local:
- Wilson line $W_C = \mathrm{Tr}\left(\mathcal{P} \exp \frac{i}{\hbar} \int_C A \right)$

- **entanglement entropy**

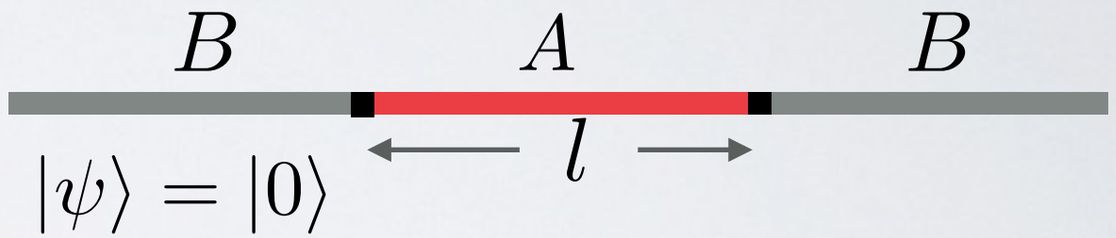
Quantum Field Theory

$$S_A = -\mathrm{Tr}\,\hat{\rho}_A \log \hat{\rho}_A \qquad \hat{\rho}_A = \mathrm{Tr}_B \hat{\rho}$$

Exact results for CFT in 1+1 dimensions

P. Calabrese & J. Cardy 2003

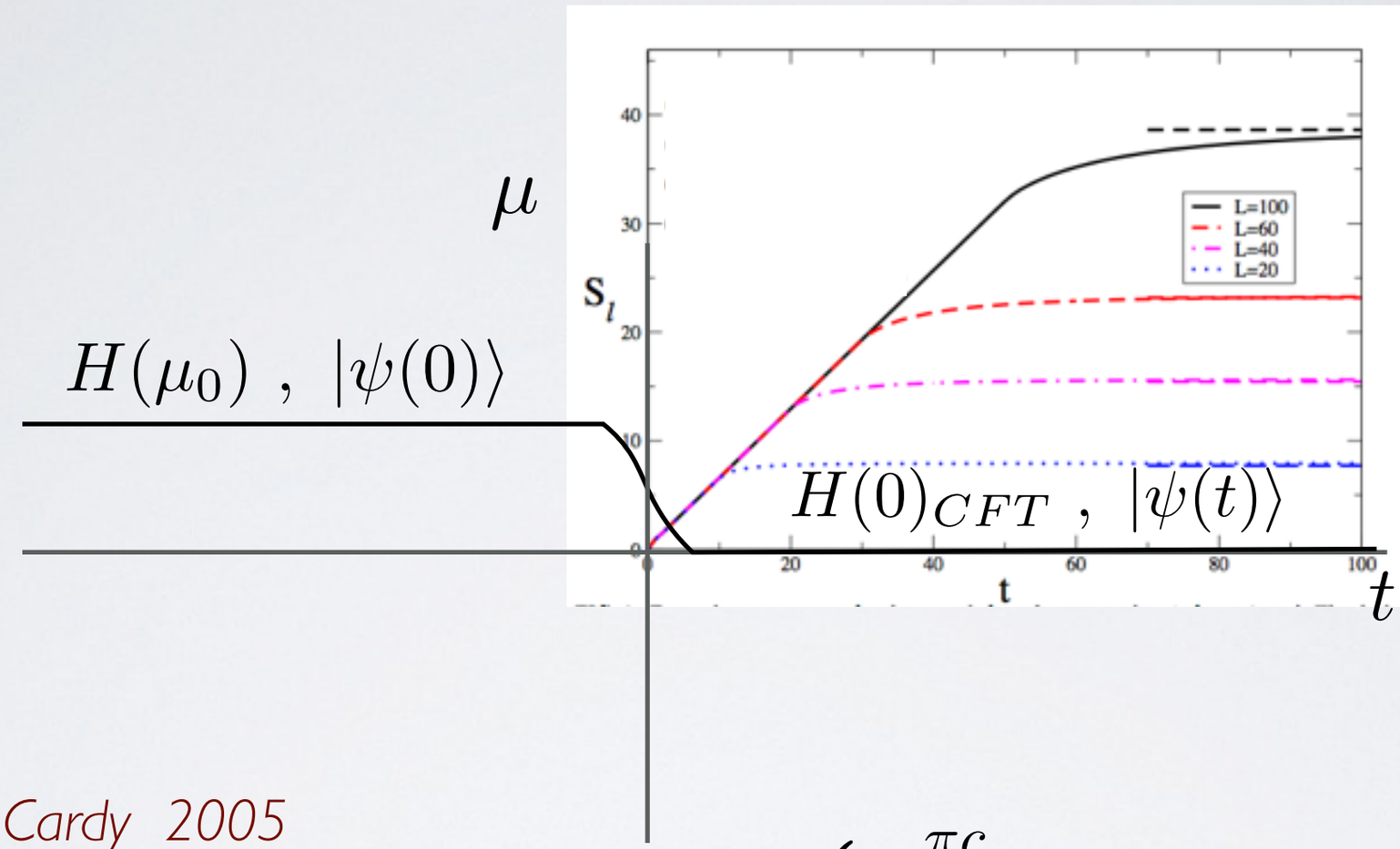
$$T = 0 \qquad S_A = \frac{c}{3} \log\left(\frac{l}{\epsilon}\right)$$



$$T = \frac{1}{\beta} \neq 0 \qquad S_A^\beta = \frac{c}{3} \log\left(\frac{\beta}{\pi\epsilon} \sinh \frac{\pi l}{\beta}\right) \longrightarrow \begin{cases} \xrightarrow{l \ll \beta} & \frac{c}{3} \log \frac{l}{\epsilon} \\ \xrightarrow{l \gg \beta} & \frac{c}{3} \log\left(\frac{\beta}{2\pi\epsilon}\right) + \frac{\pi c}{3\beta} l + \ldots \end{cases}$$

ENTANGLEMENT ENTROPY AFTER A QUENCH

At t = 0 the Hamiltonian changes abruptly, leaving an excited state. Watch it evolve.



P. Calabrese & J. Cardy 2005

for $l, t \gg \tau_0 \sim 1/m$

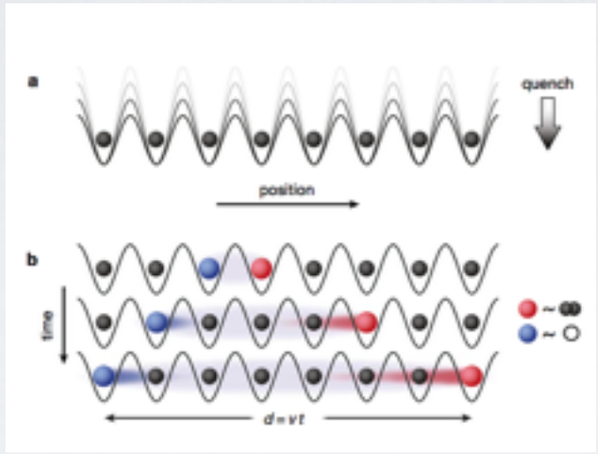
$$S_l(t) \sim \begin{cases} \dfrac{\pi c}{6\tau_0}\, t & t < l/2, \\ \dfrac{\pi c}{12\tau_0}\, l & l/2 < t \end{cases}$$

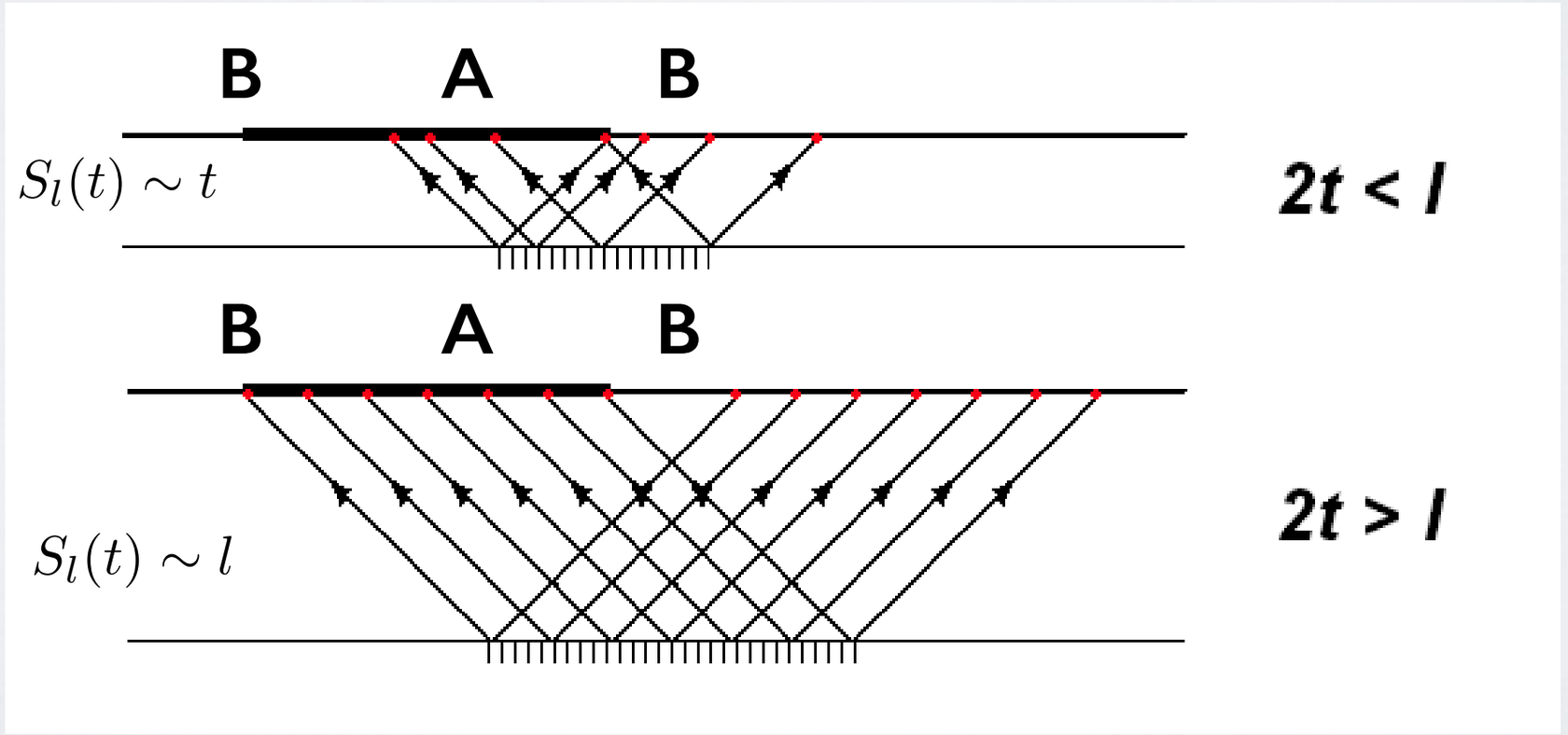
reaches an extensive EE with a $T \sim 1/4\tau_0$

ENTANGLEMENT ENTROPY AFTER A QUENCH

kinematical argument: $t = 0$ quench leaves an excited sea of quasiparticle pairs

$0 \leq t \leq l/2$ entangled pairs fly apart at the speed of light

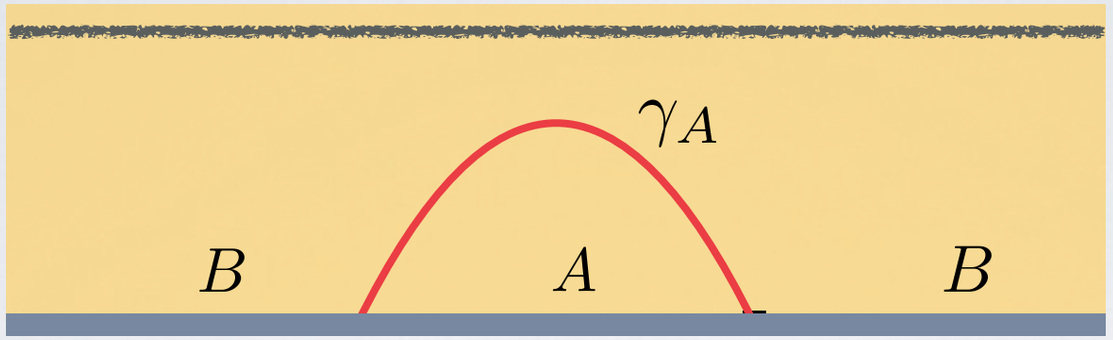




HOLOGRAPHIC ENTANGLEMENT ENTROPY

entanglemente entropy ⟷ minimal surface homologous to A

$$S(\theta) = \frac{\text{Area}(\gamma_A)}{4G_{d+1}}$$



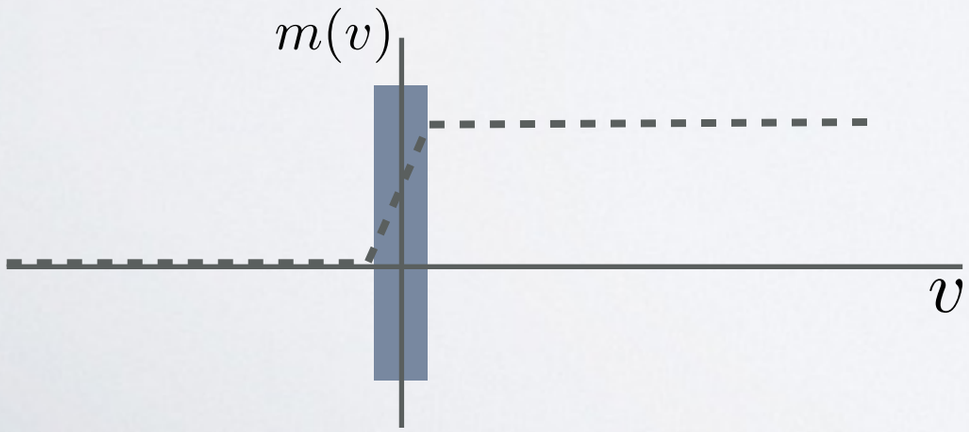
S. Ryu & T. Takayanagi 2006

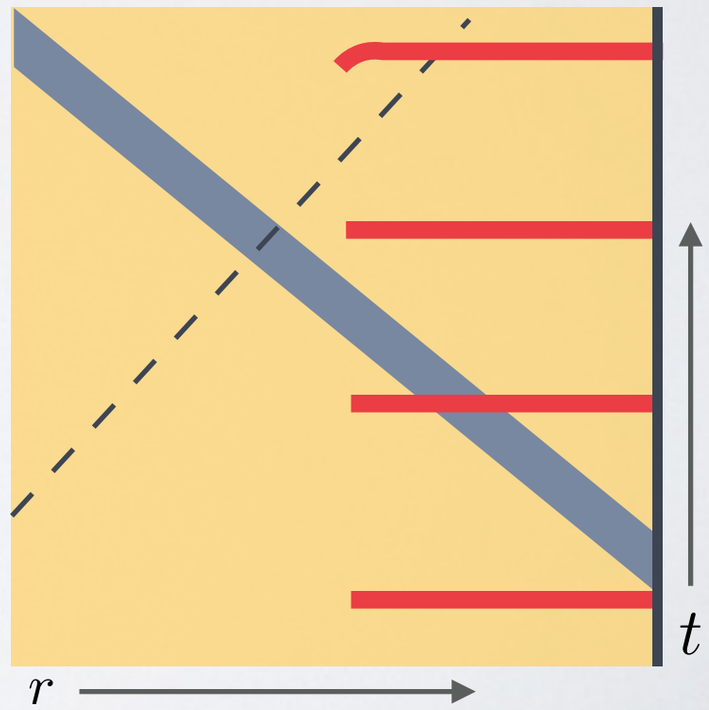
quench in CFT_d ⟷ shell collapse in AdS_{d+1}

analytic case: AdS_{d+2} -Vaidya: collapse of radiation shell

$$ds^2 = -\left(r^2 - \frac{m(v)}{r^{d-1}}\right) dv^2 + 2drdv + r^2 \sum_{i=1}^{d} dx_i^2$$

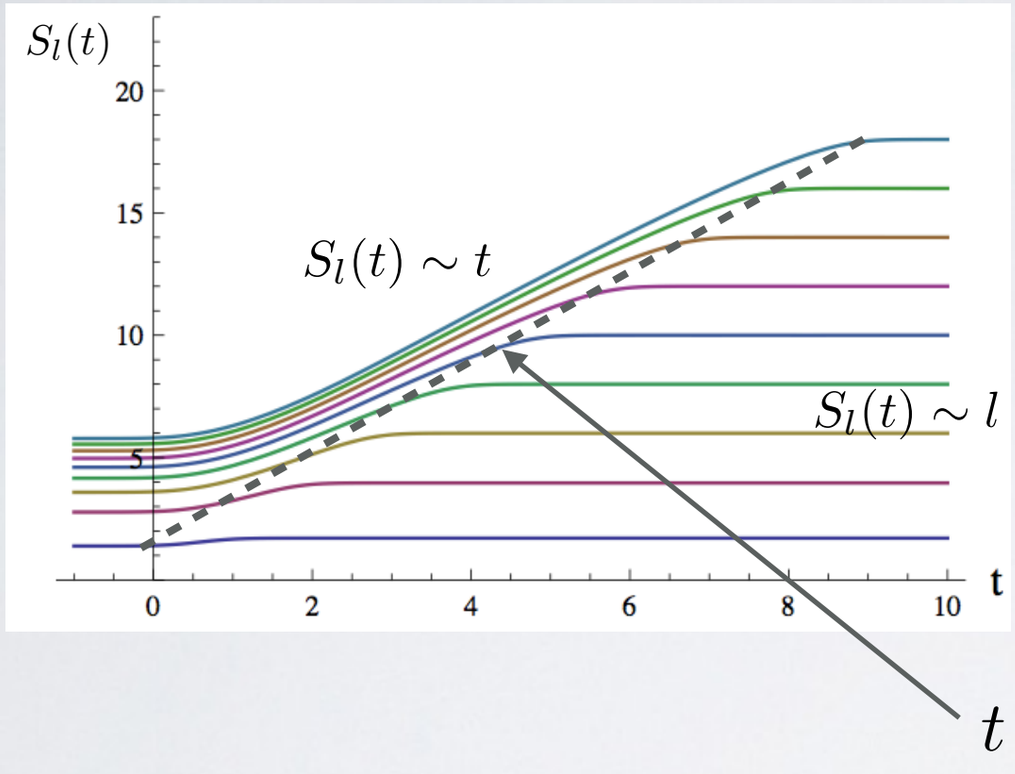
$$T_{vv} = d\frac{m'(v)}{2r^d}$$

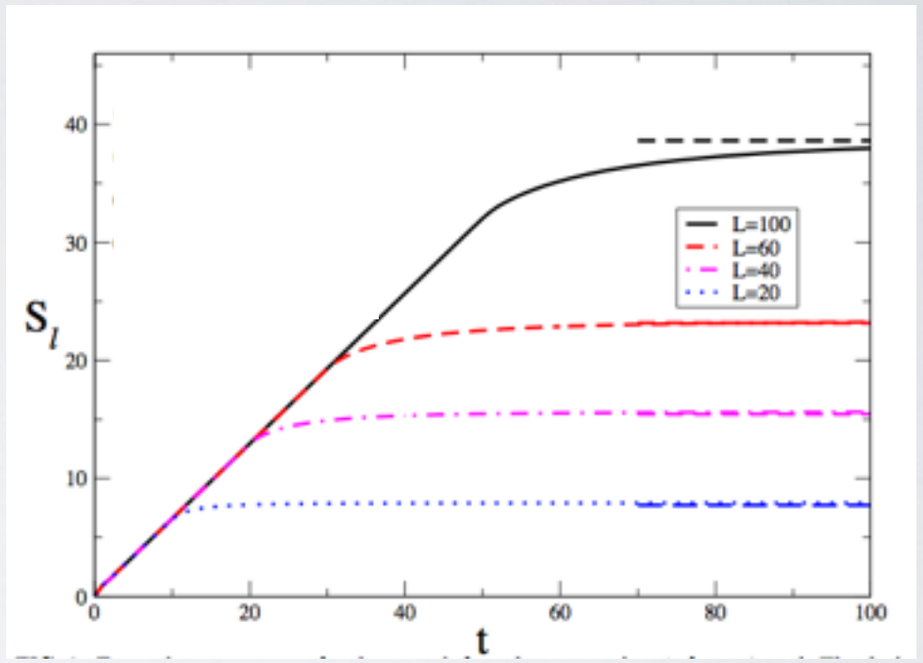




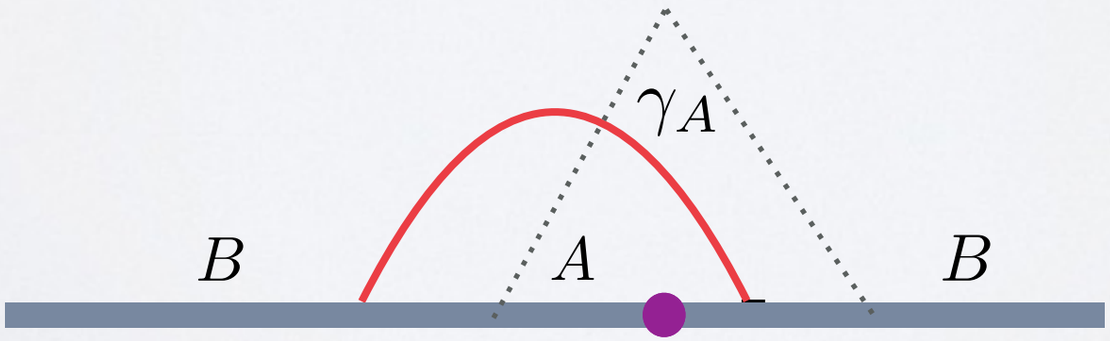
HOLOGRAPHIC ENTANGLEMENT ENTROPY

J. Abajo-Arrastia, J. Aparicio & E. López 2010
T. Albash & C.V. Johnson 2011



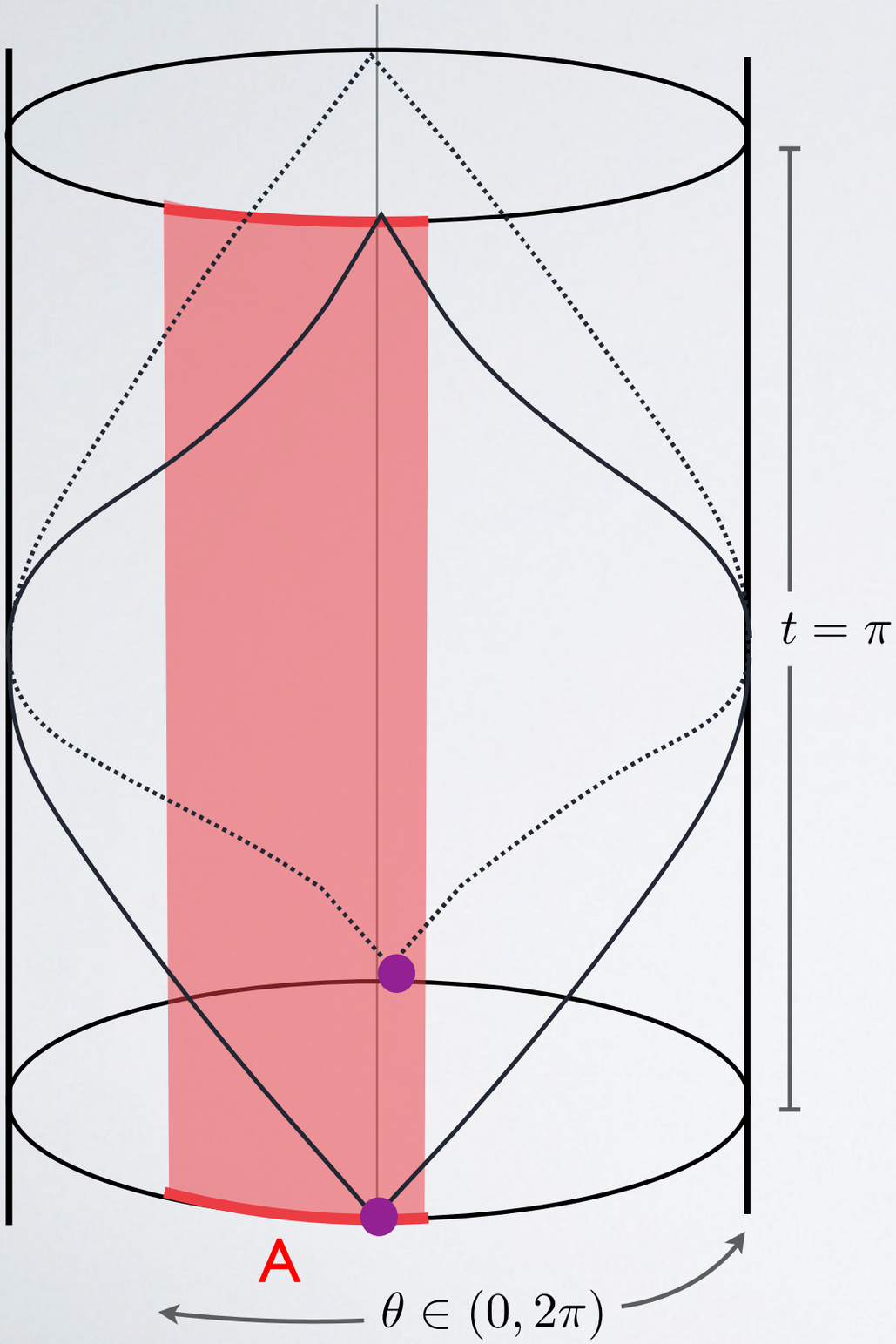


- are we seeing a light-cone-like effect for entangled quasiparticles?



- is the radial position of the shell dual to the entangled pair separation?

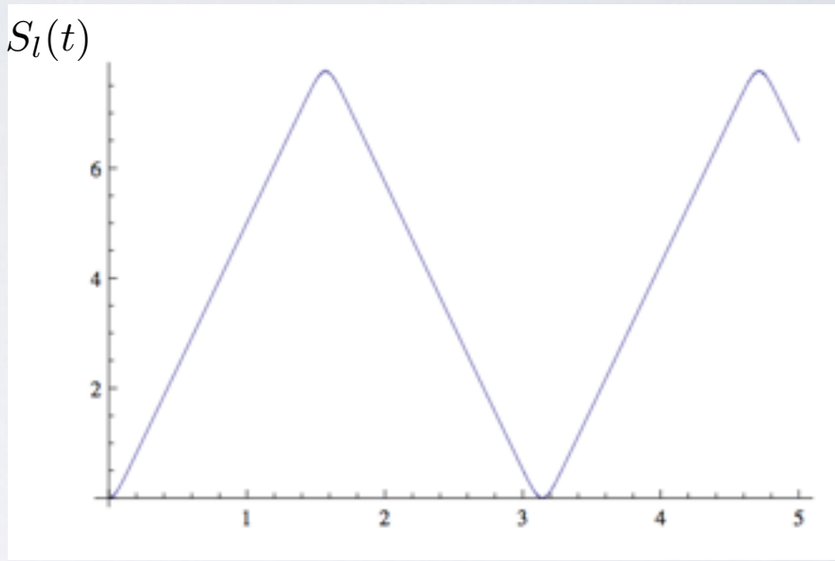
ENTANGLEMENT EVOLUTION IN COMPACT SPACE



Confirmed for:

CFT = 1+1 Free Fermion

T. Takayanagi & T. Ugajin 2010

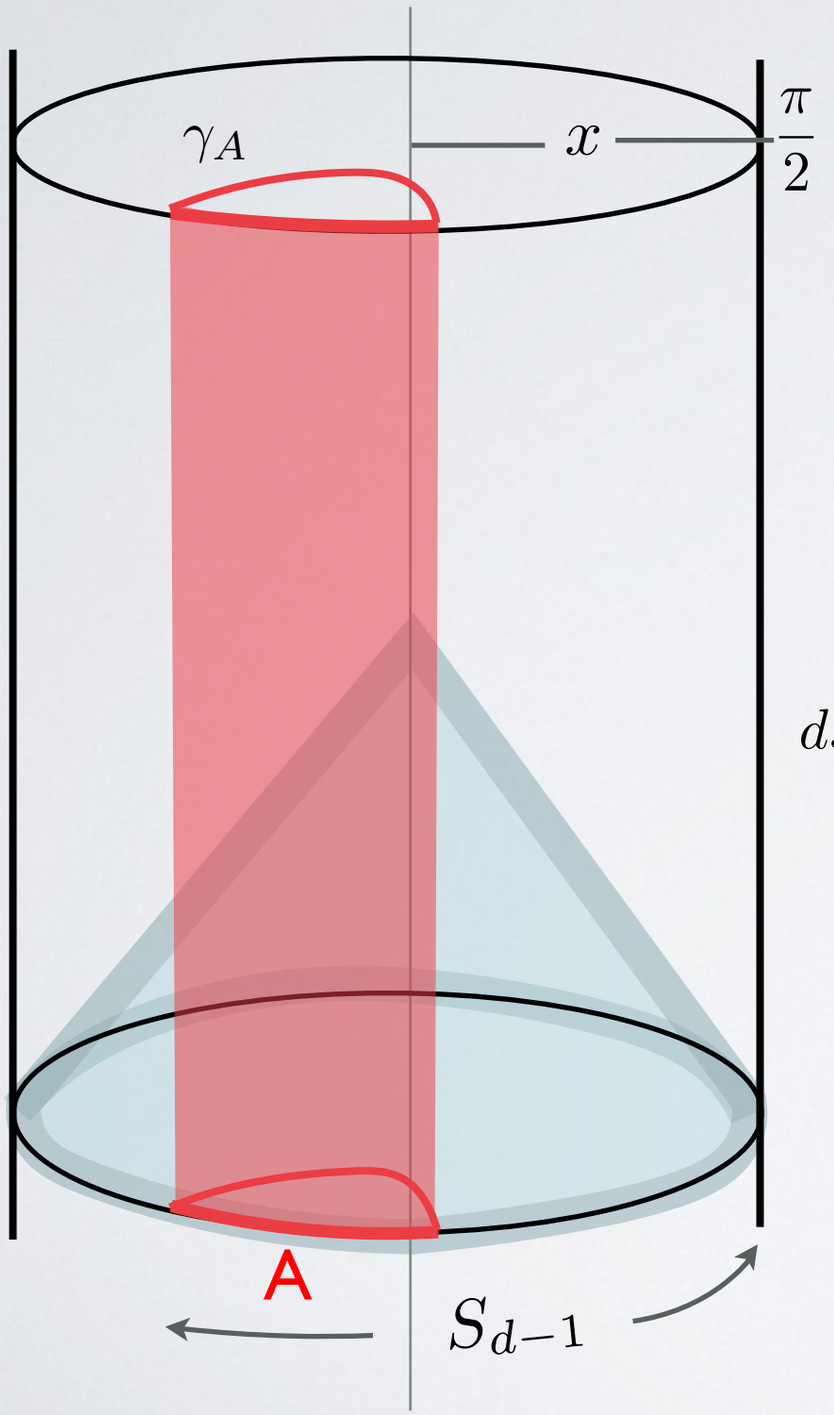


*R*CFT =minimal models

John Cardy 2014

ENTANGLEMENT EVOLUTION IN GLOBAL AdS

AdS-Vaydia leads to a direct black hole formation



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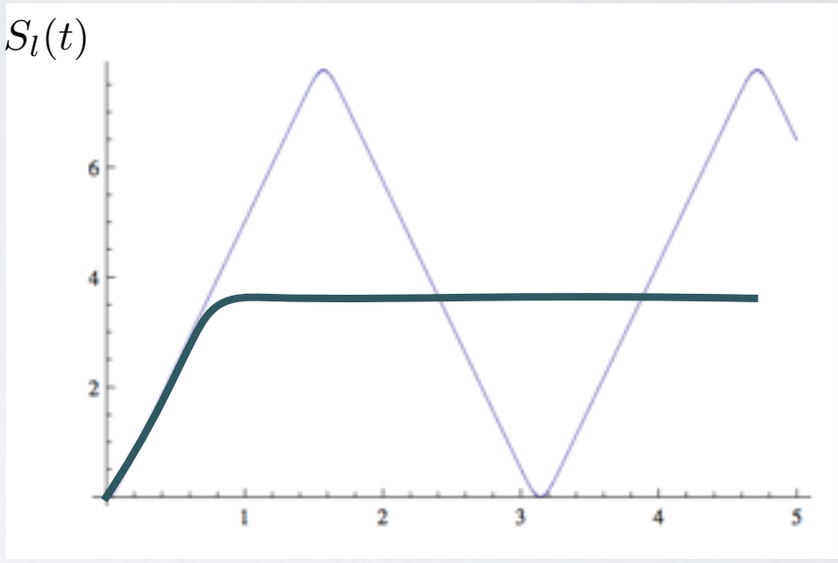
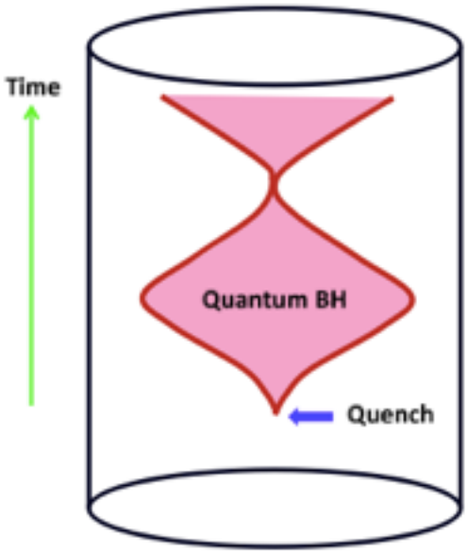


$$ds^2 = \frac{1}{\cos^2 x}\left(-dt^2 + dx^2 + \sin^2 x\, d\Omega_{d-1}^2\right)$$



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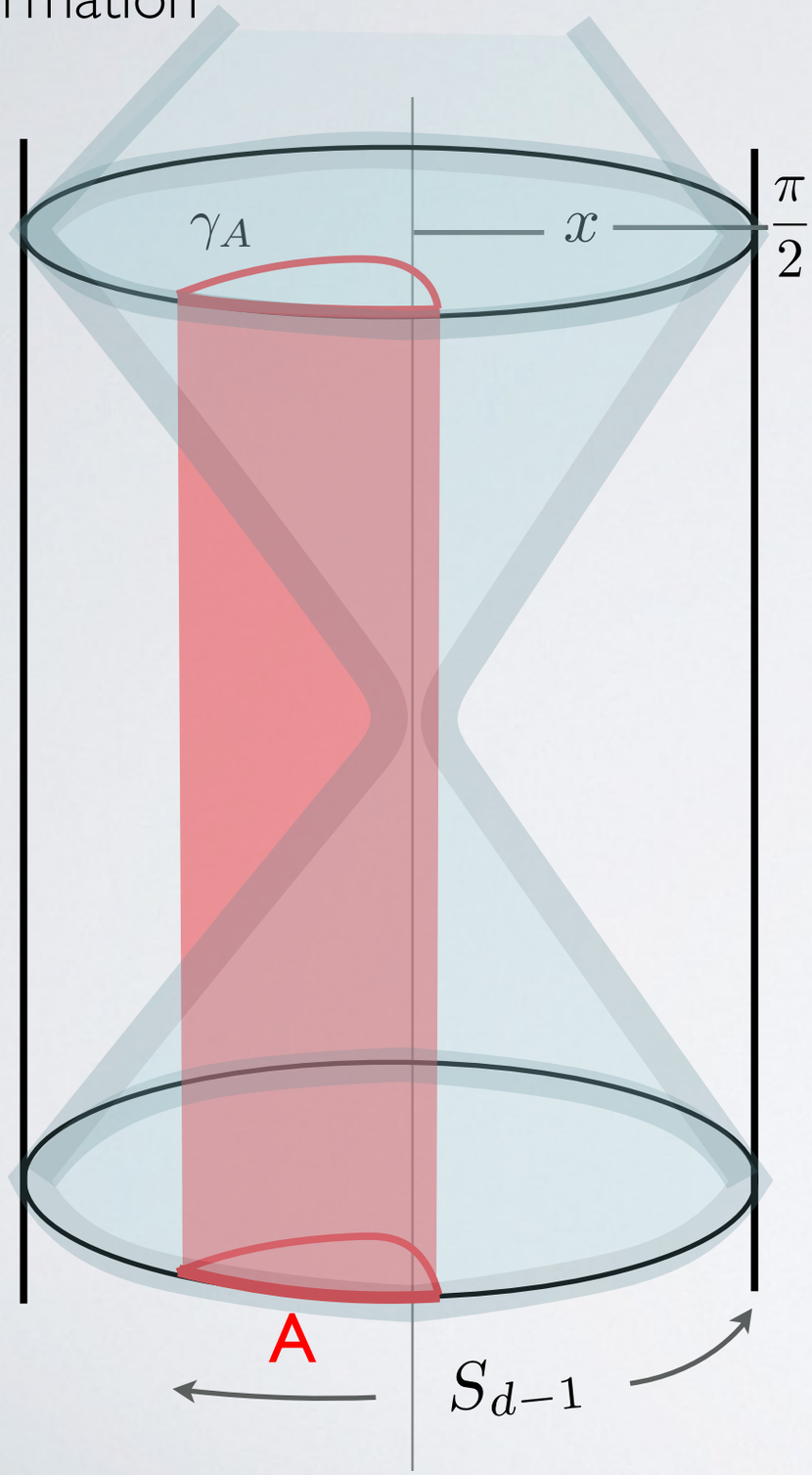


Collapse of a massless scalar field in AdS_{d+1}

$$S = \int d^{d+1}x\sqrt{g}\left(\frac{1}{2\kappa^2}R + \frac{d(d-1)}{l^2} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi\right)$$

homogeneous ansatz (Bizon & Rostworowski 2011)

$$ds^2 = \frac{1}{\cos^2 x}\left(-A(x,t)e^{-2\delta(x,t)}dt^2 + \frac{dx^2}{A(x,t)} + \sin^2 x\, d\Omega_d^2\right)$$

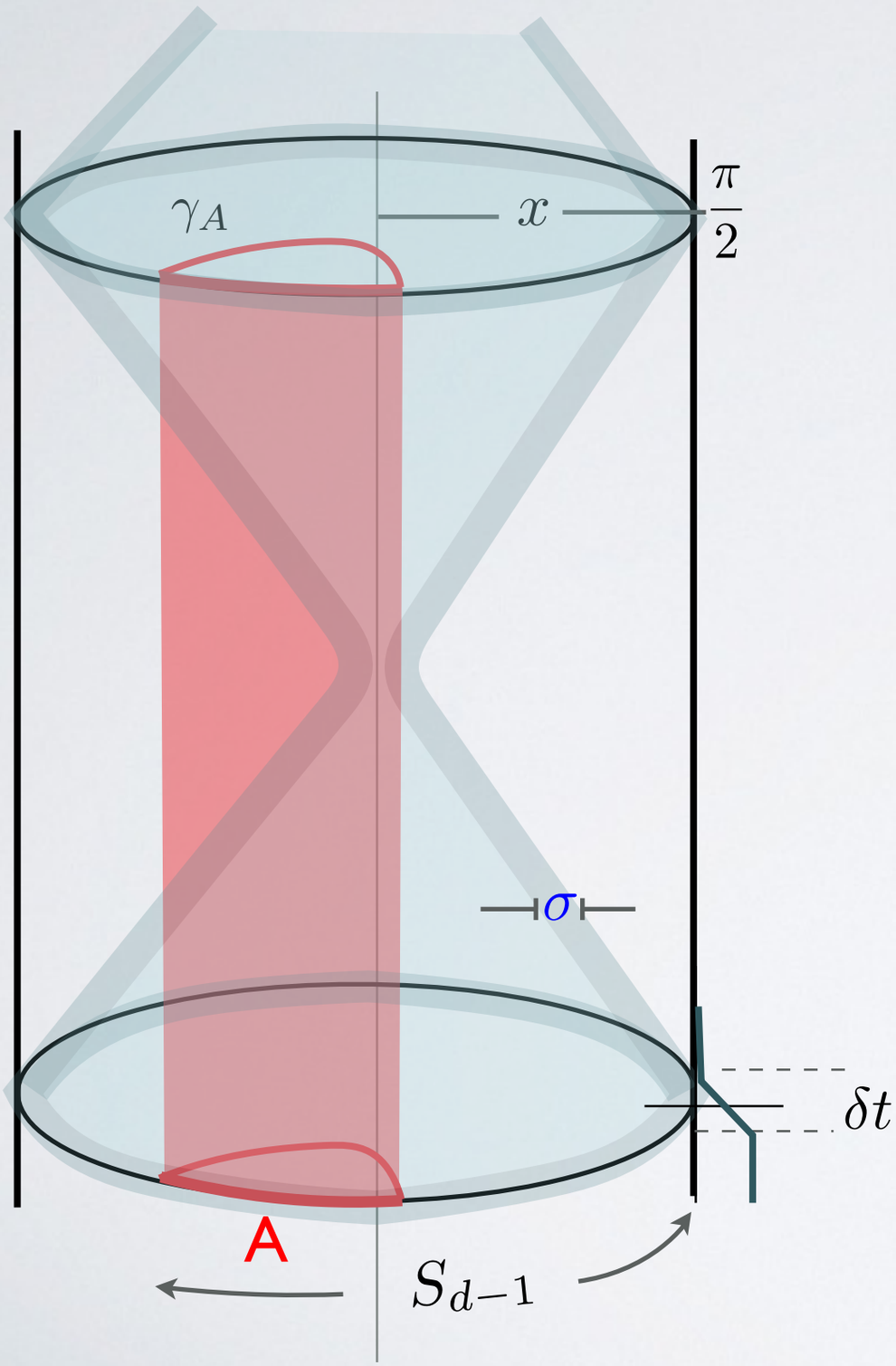
$$0 \leq x \leq \pi/2$$

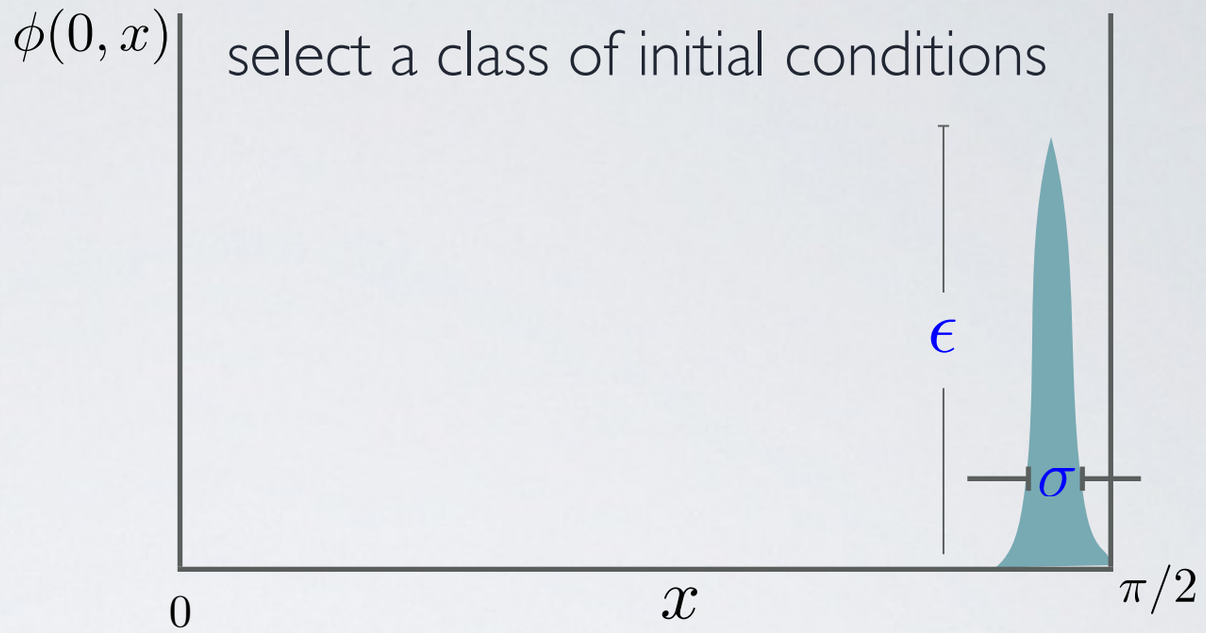
Equations of motion + boundary conditions

$$\begin{aligned}
\phi(x,t) &\sim (\tfrac{\pi}{2} - x)^d\phi_\infty + \ldots \\
A(x,t) &\sim 1 - (\tfrac{\pi}{2} - x)^d M + \ldots \\
\delta(x,t) &\sim (\tfrac{\pi}{2} - x)^{2d}\phi_\infty + \ldots
\end{aligned}$$

apparent horizon forms whenever $A(x_h, t_h) = 0$

ENTANGLEMENT EVOLUTION IN GLOBAL AdS





$$\phi(0,x) = \epsilon\,\frac{12}{\pi}\exp\left(-\frac{4\tan^2(\frac{\pi}{2}-x)}{\sigma^2}\right)\cos^d x$$

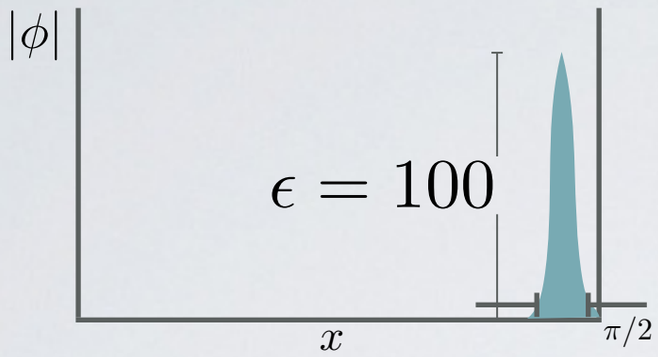
ϵ is related to the quench energy

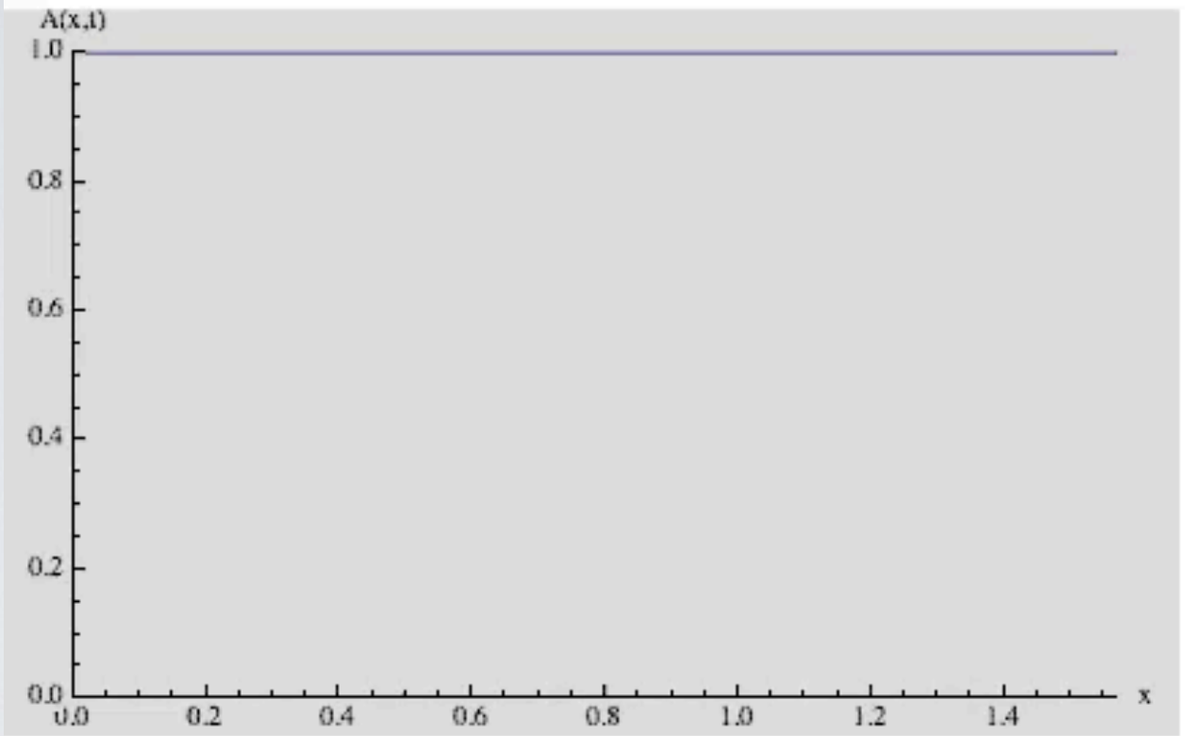
σ is related to the quenching time δt

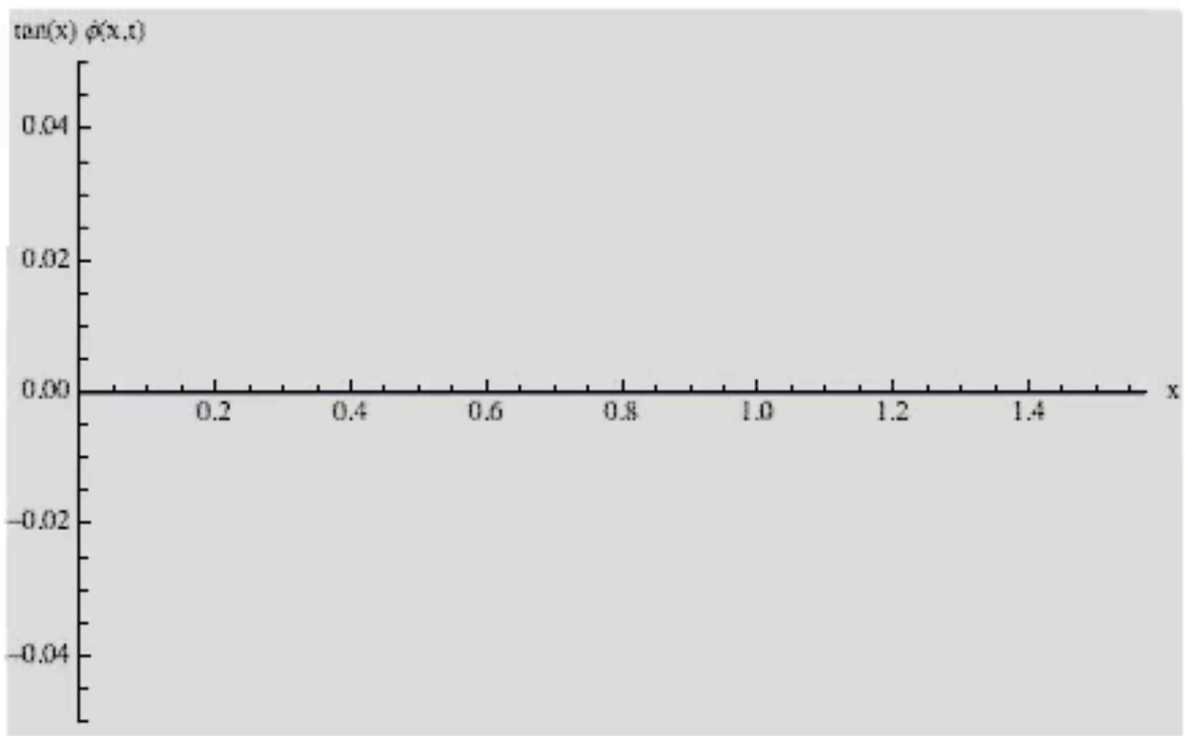
$$M(\epsilon,\sigma) = \epsilon^2 f(\sigma)$$

AdS_4 : COLLAPSES

$\sigma = 1/16$







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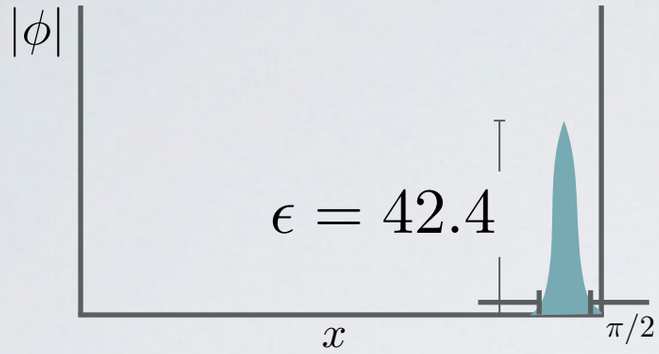
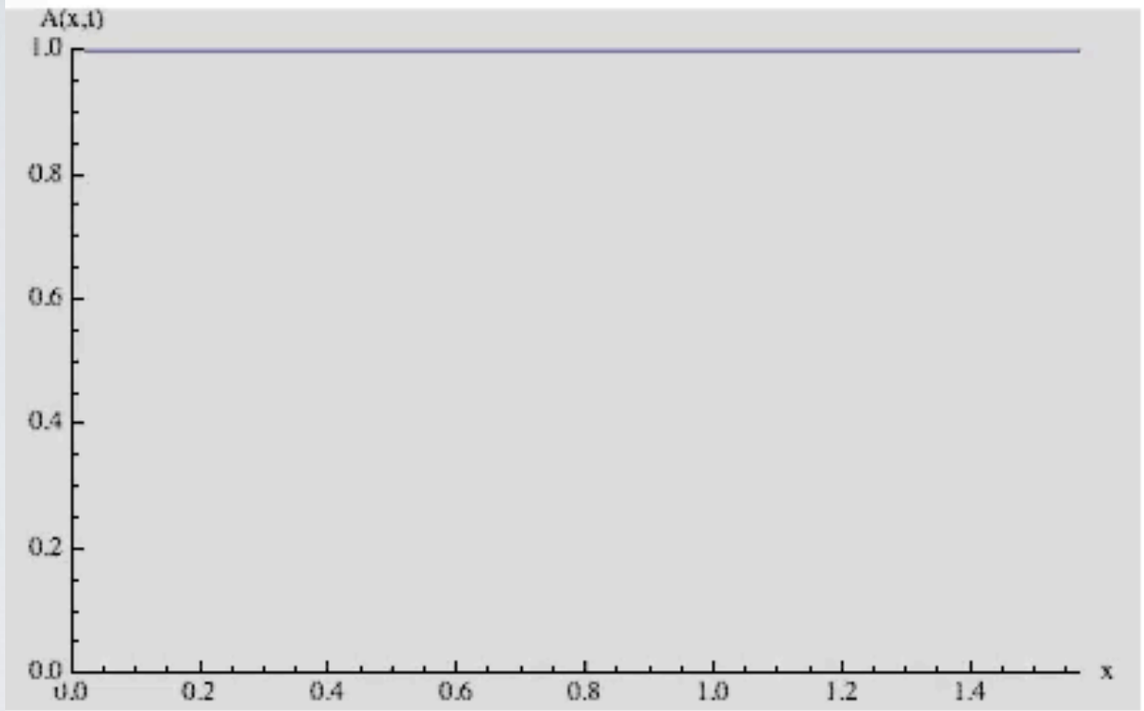
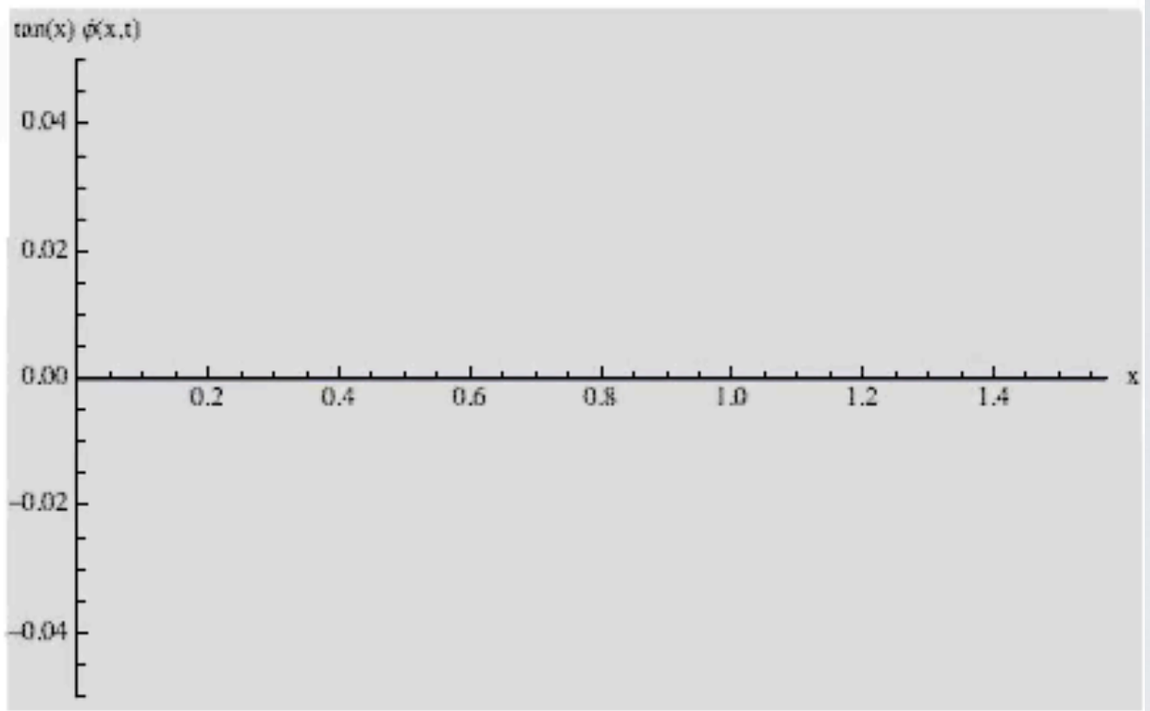
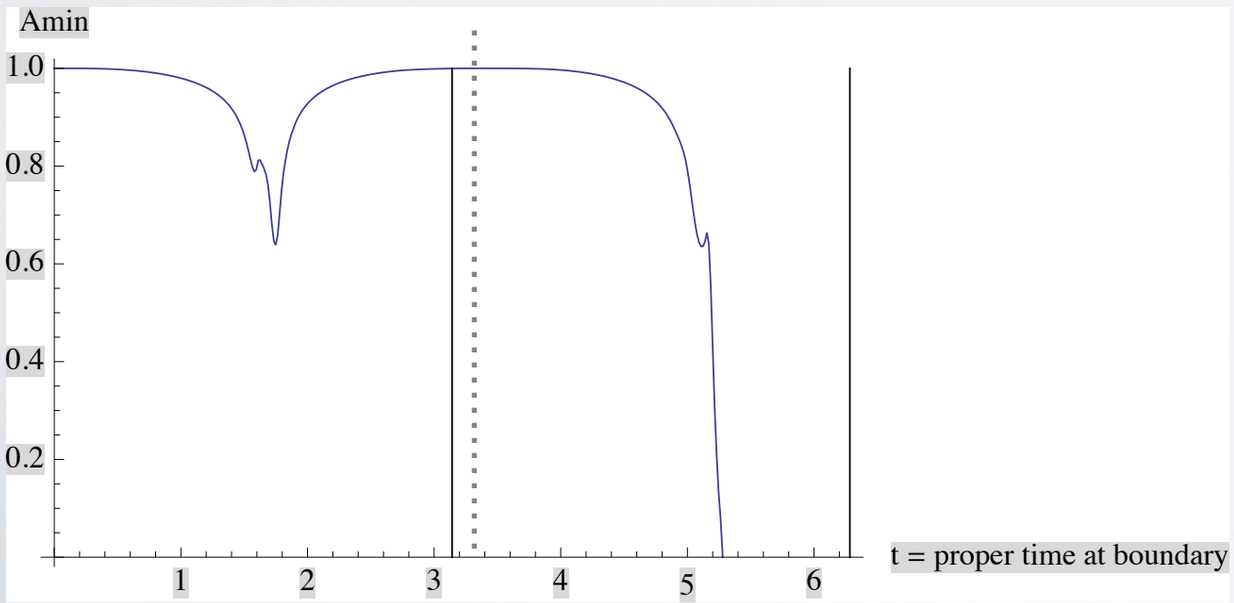






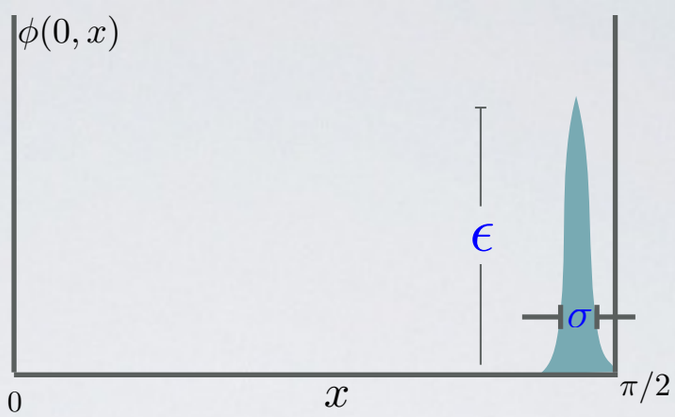


weak turbulence in action

the periodicity is $\geq \pi$!

AdS$_4$: COLLAPSES

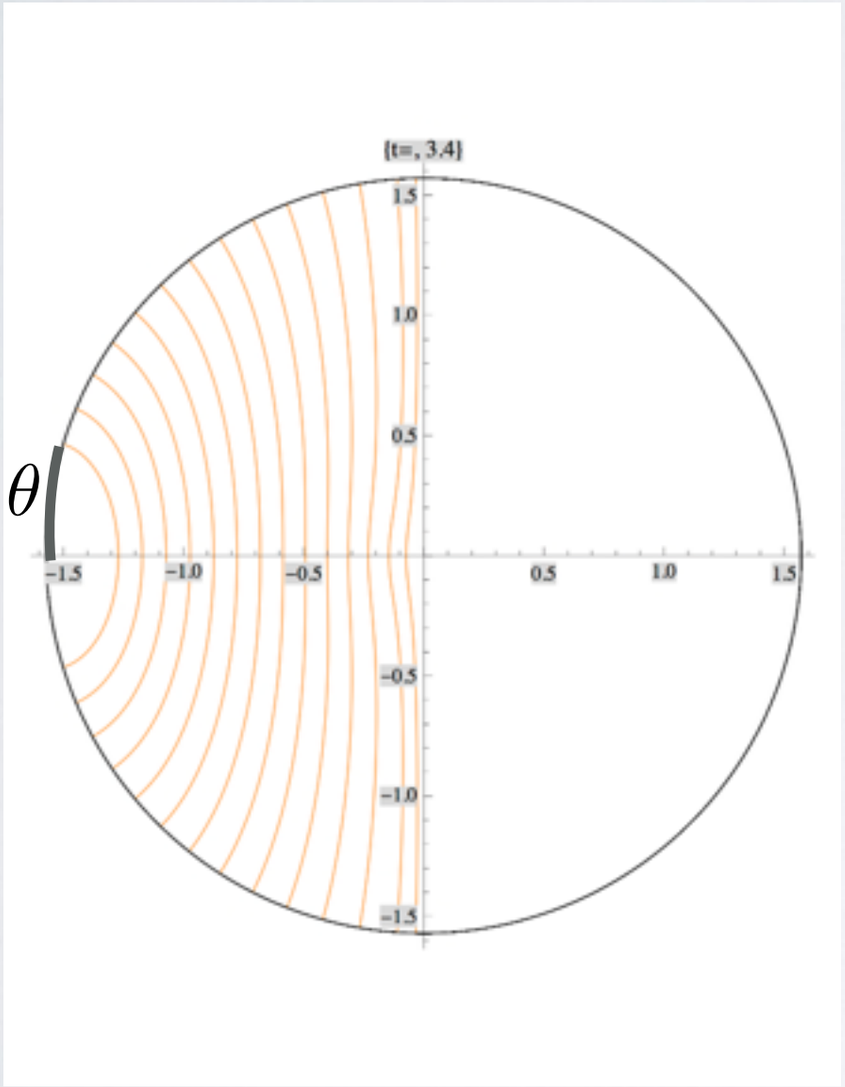
$$\sigma = \frac{1}{16}$$



AdS$_4$: ENTANGLEMENT ENTROPY

Pre-collapse

$$\theta = 0.3, 0.4, ..., 1.4$$



J. Abajo-Arrastia, E da Silva, E. López, J.M. & A. Serantes.

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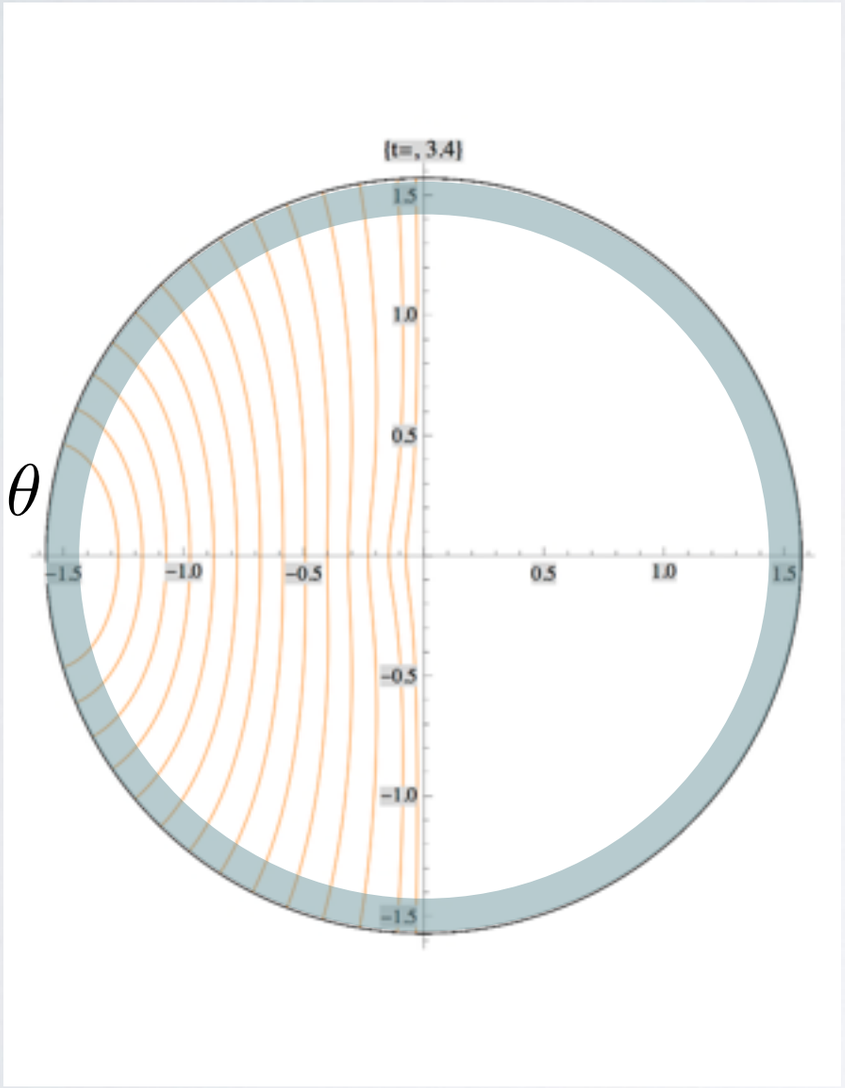

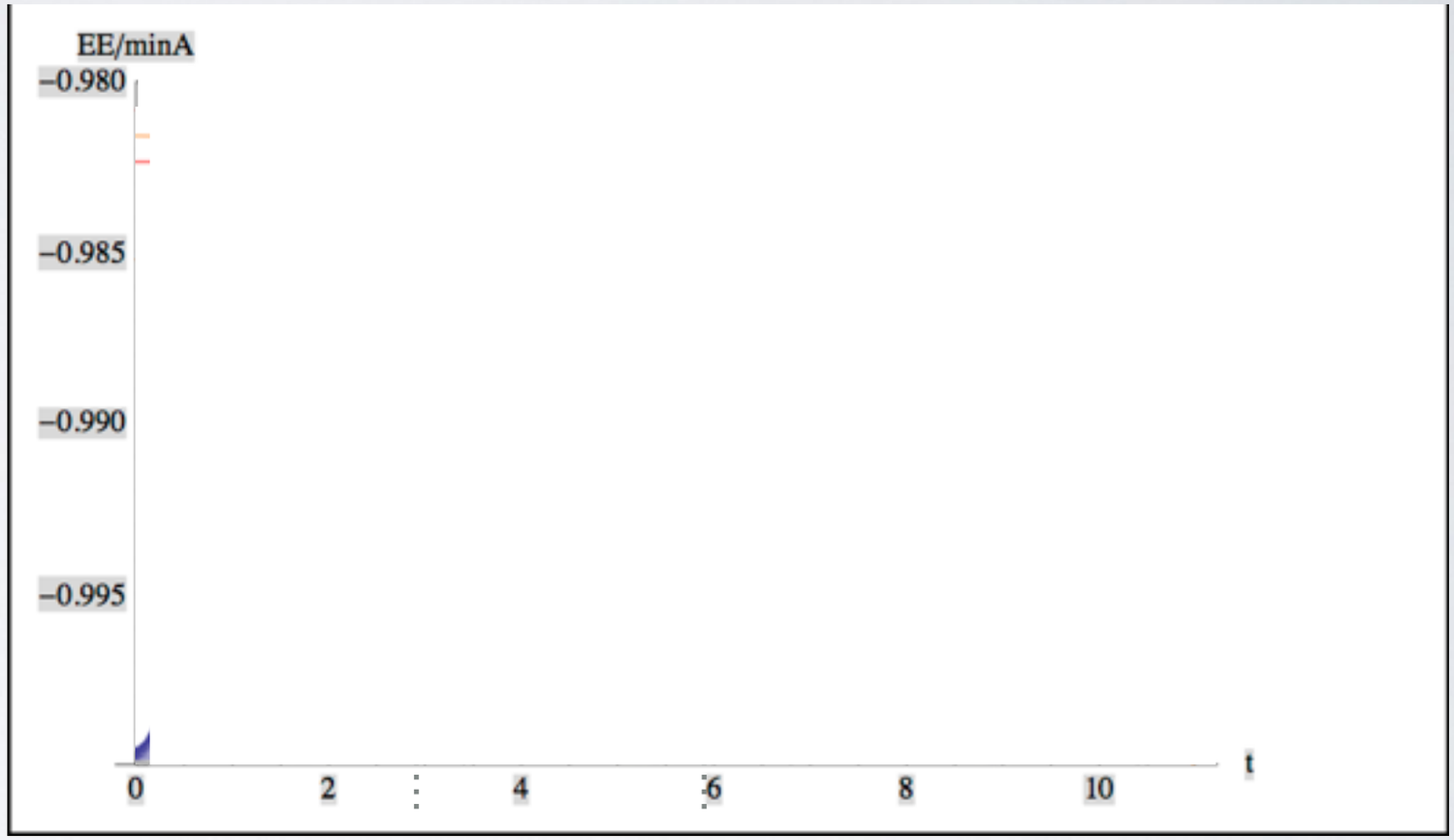

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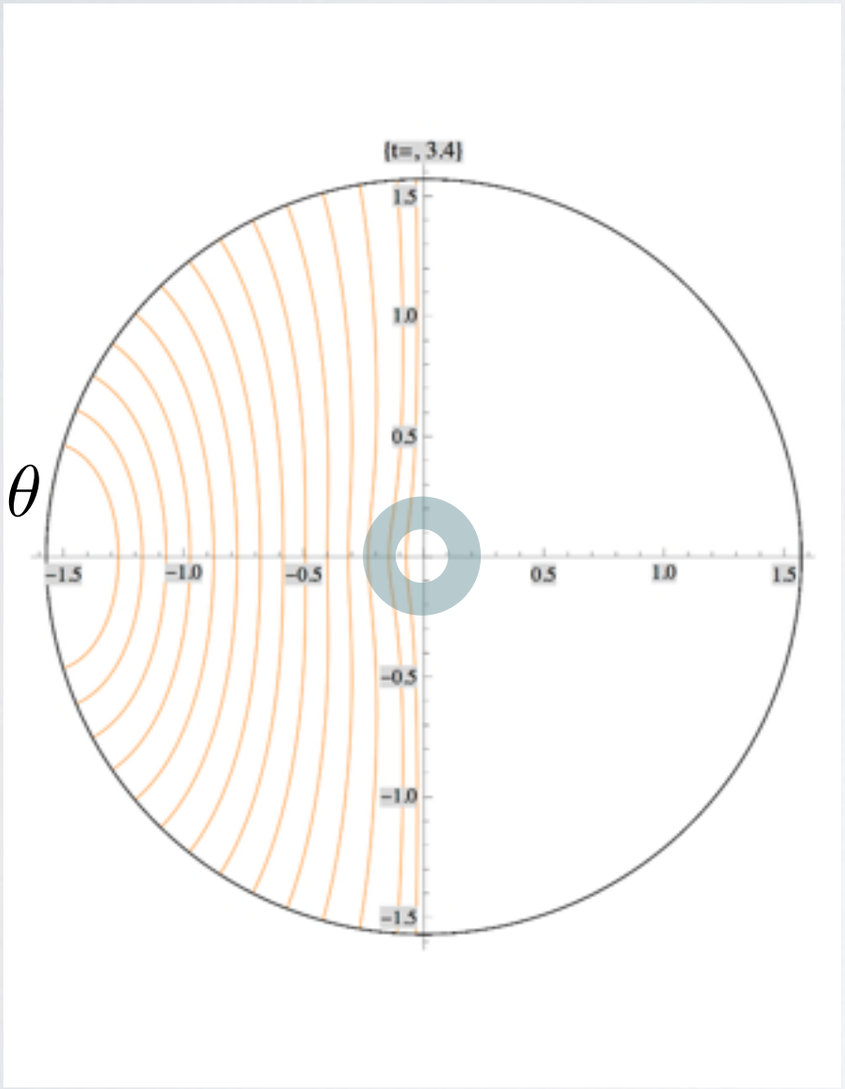

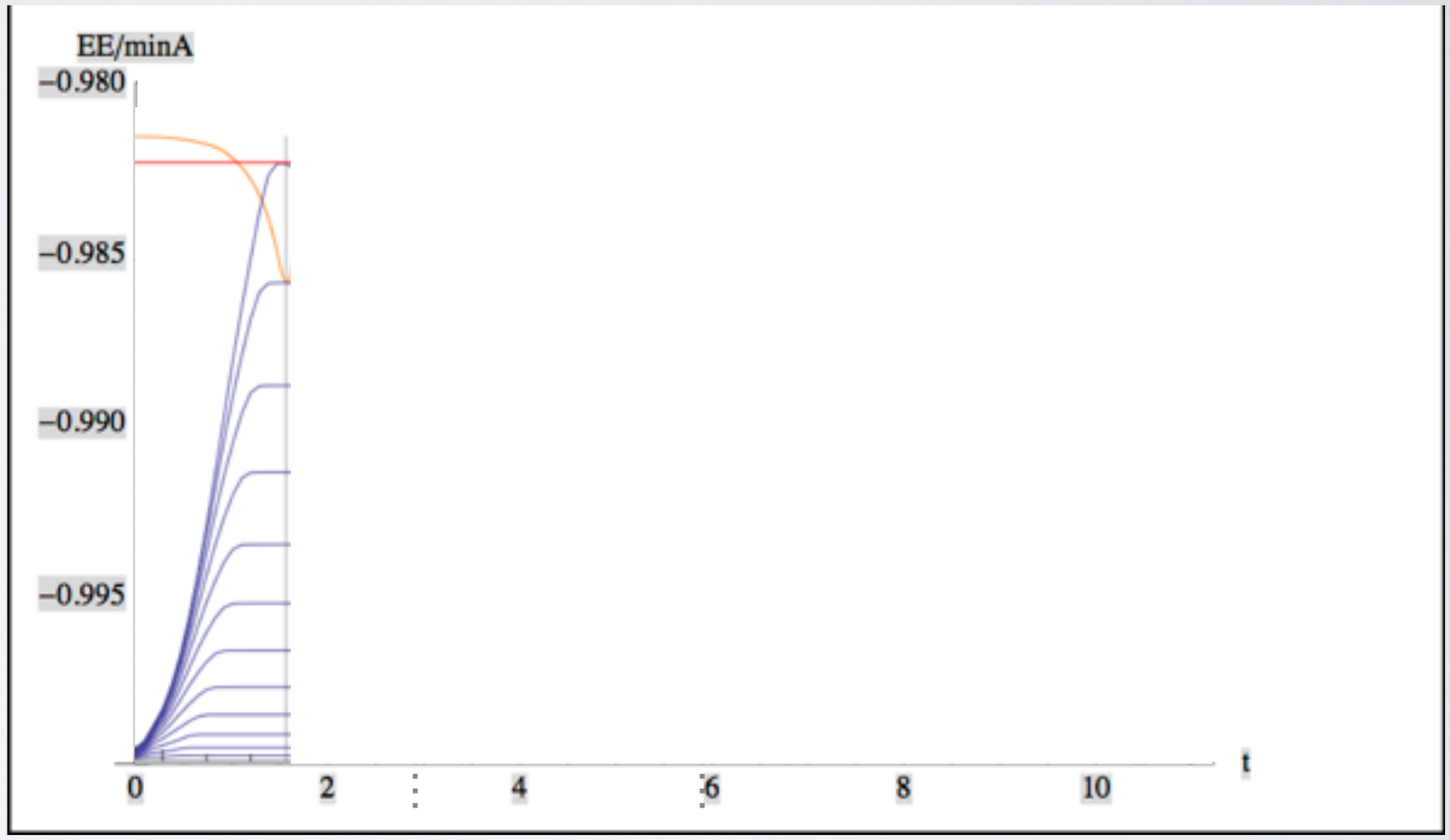

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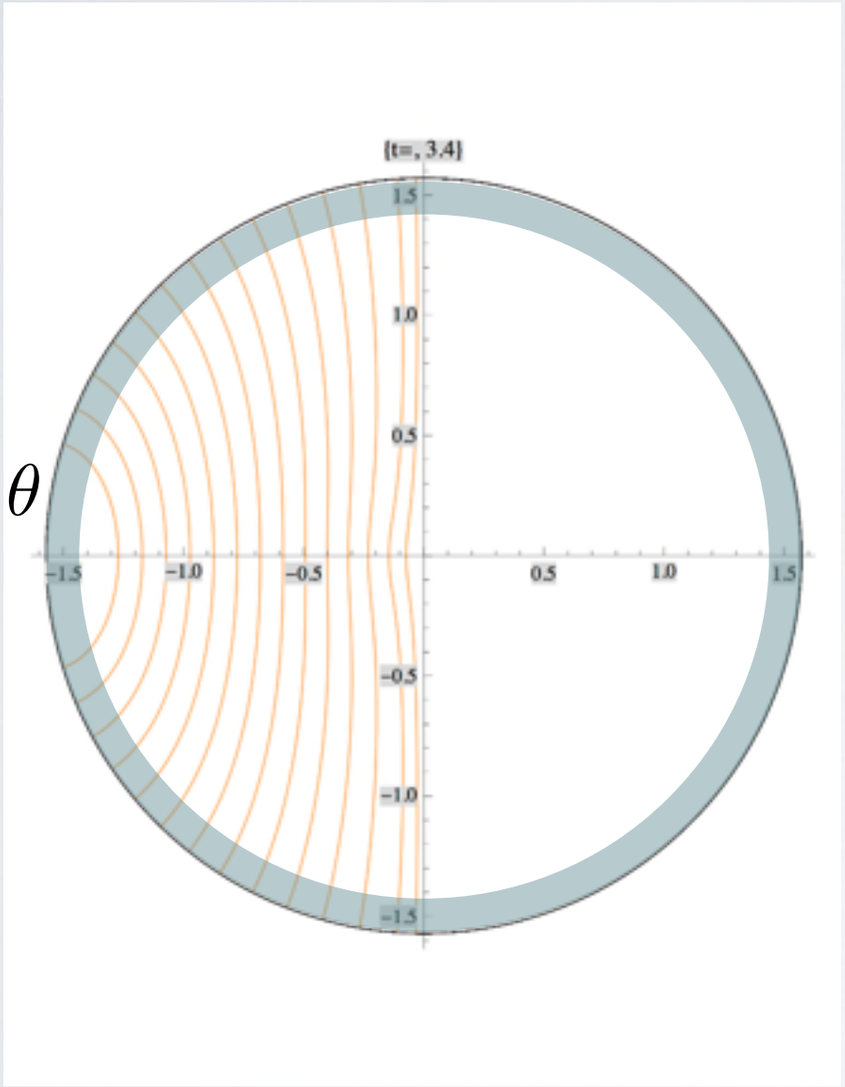
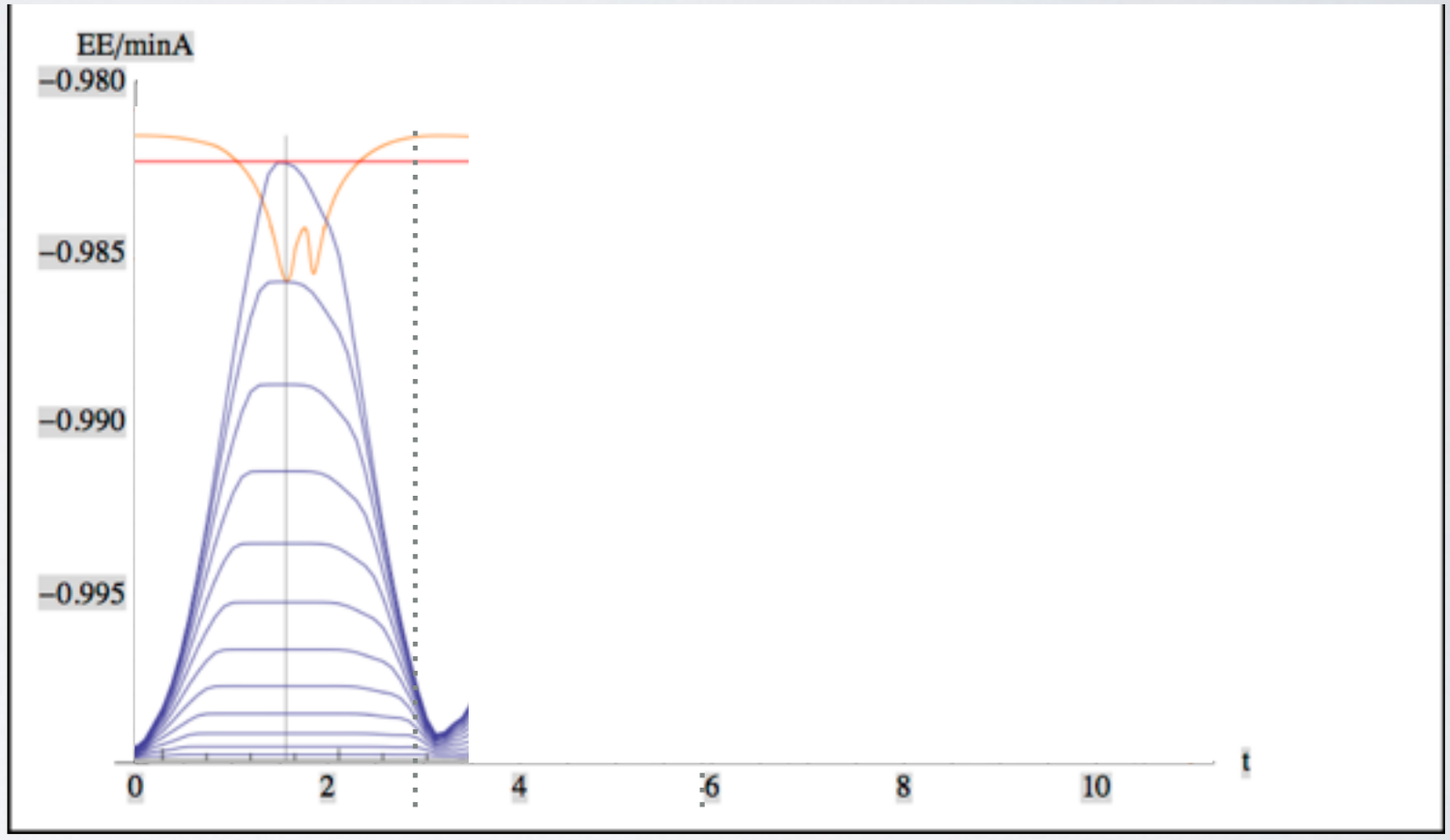
Pre-collapse

$$\theta = 0.3, 0.4, ..., 1.4$$



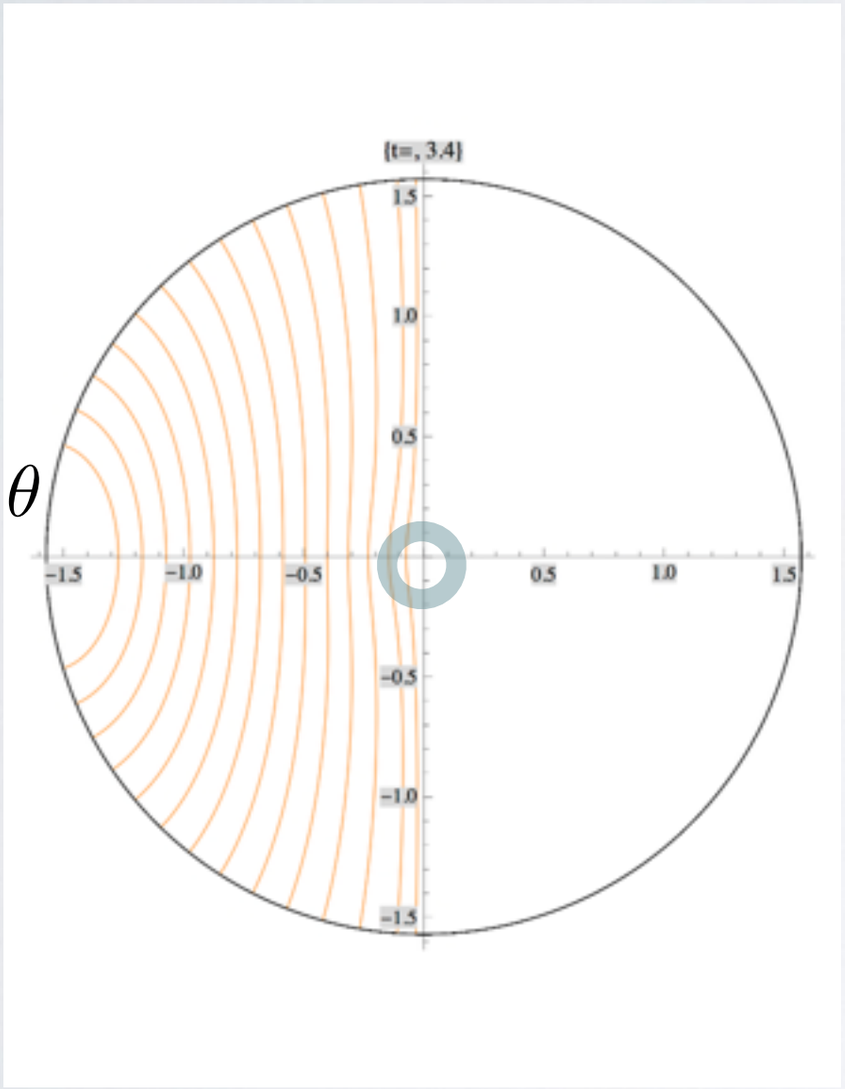


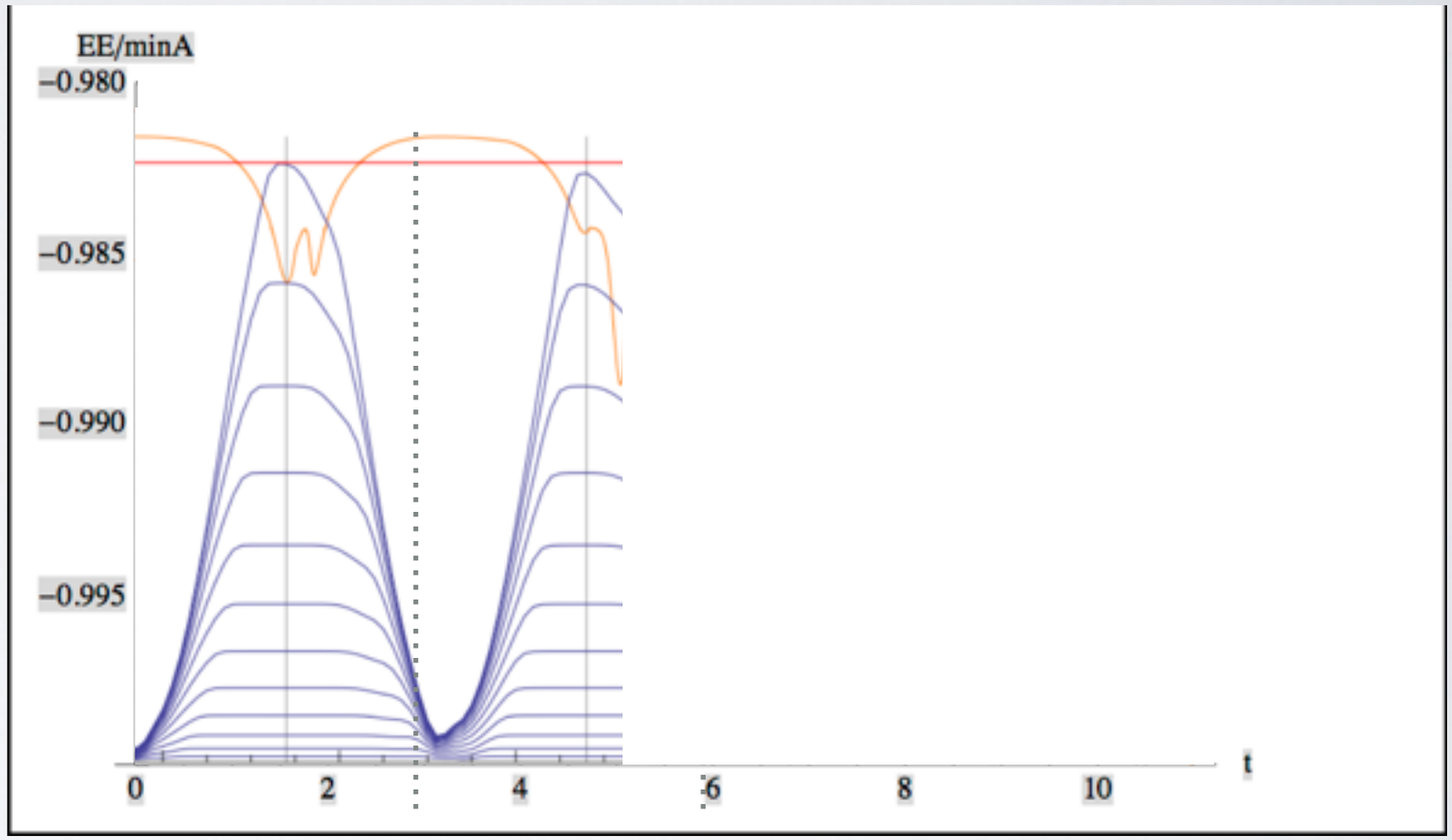
J. Abajo-Arrastia, E da Silva, E. López, J.M. & A. Serantes.

AdS_4 : ENTANGLEMENT ENTROPY

Pre-collapse

$$\theta = 0.3, 0.4, ..., 1.4$$



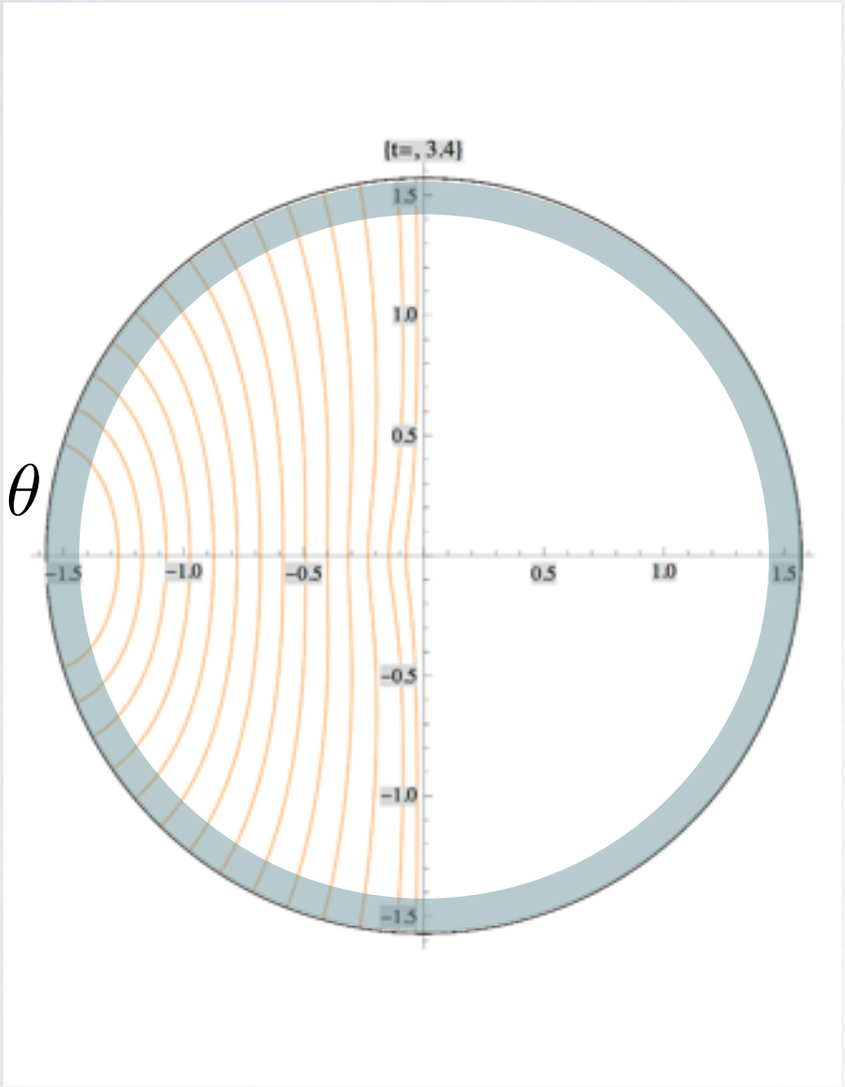


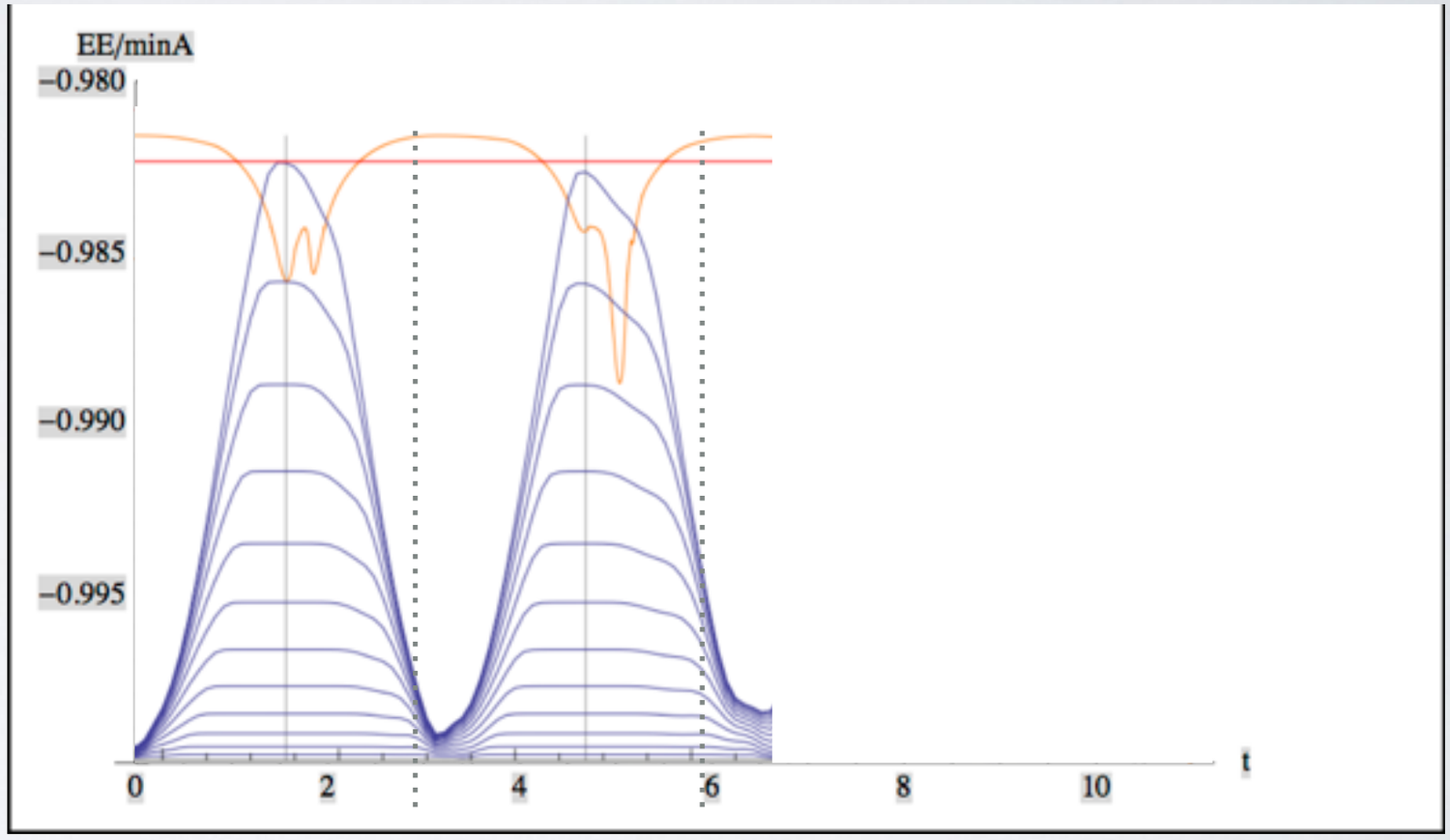
J. Abajo-Arrastia, E da Silva, E. López, J.M. & A. Serantes.

AdS$_4$: ENTANGLEMENT ENTROPY

Pre-collapse

$$\theta = 0.3, 0.4, ..., 1.4$$



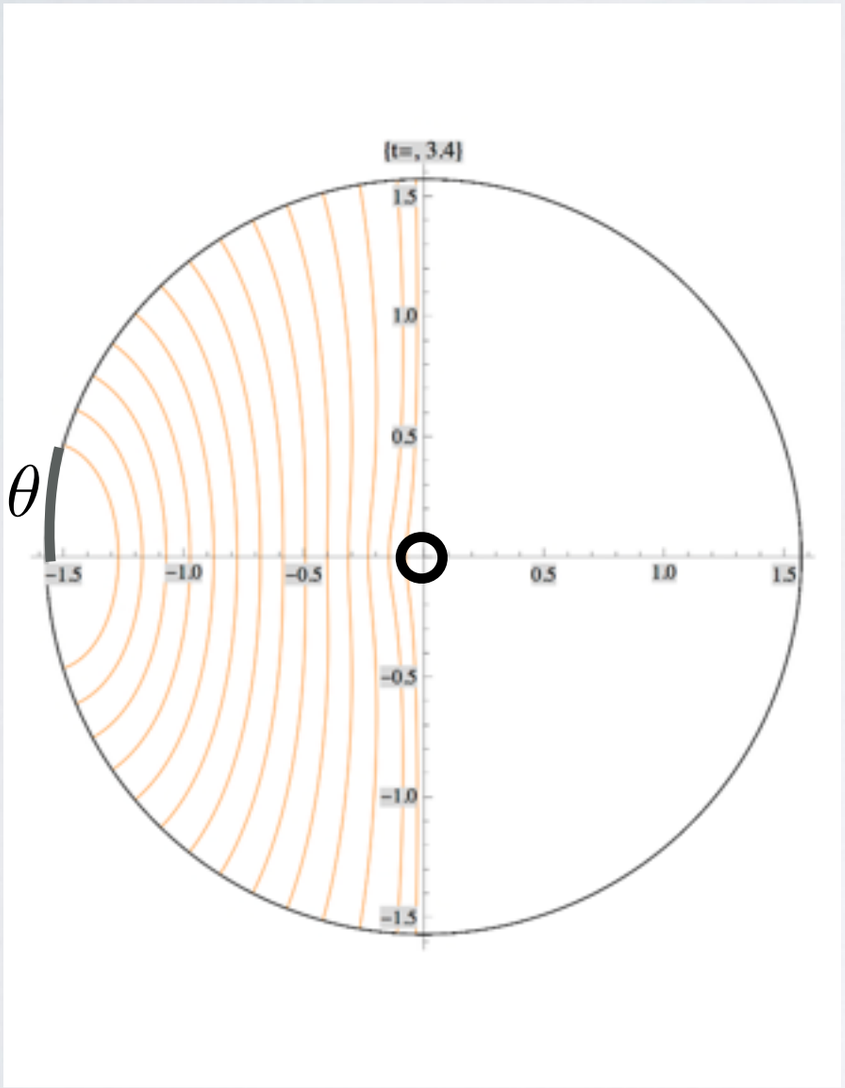


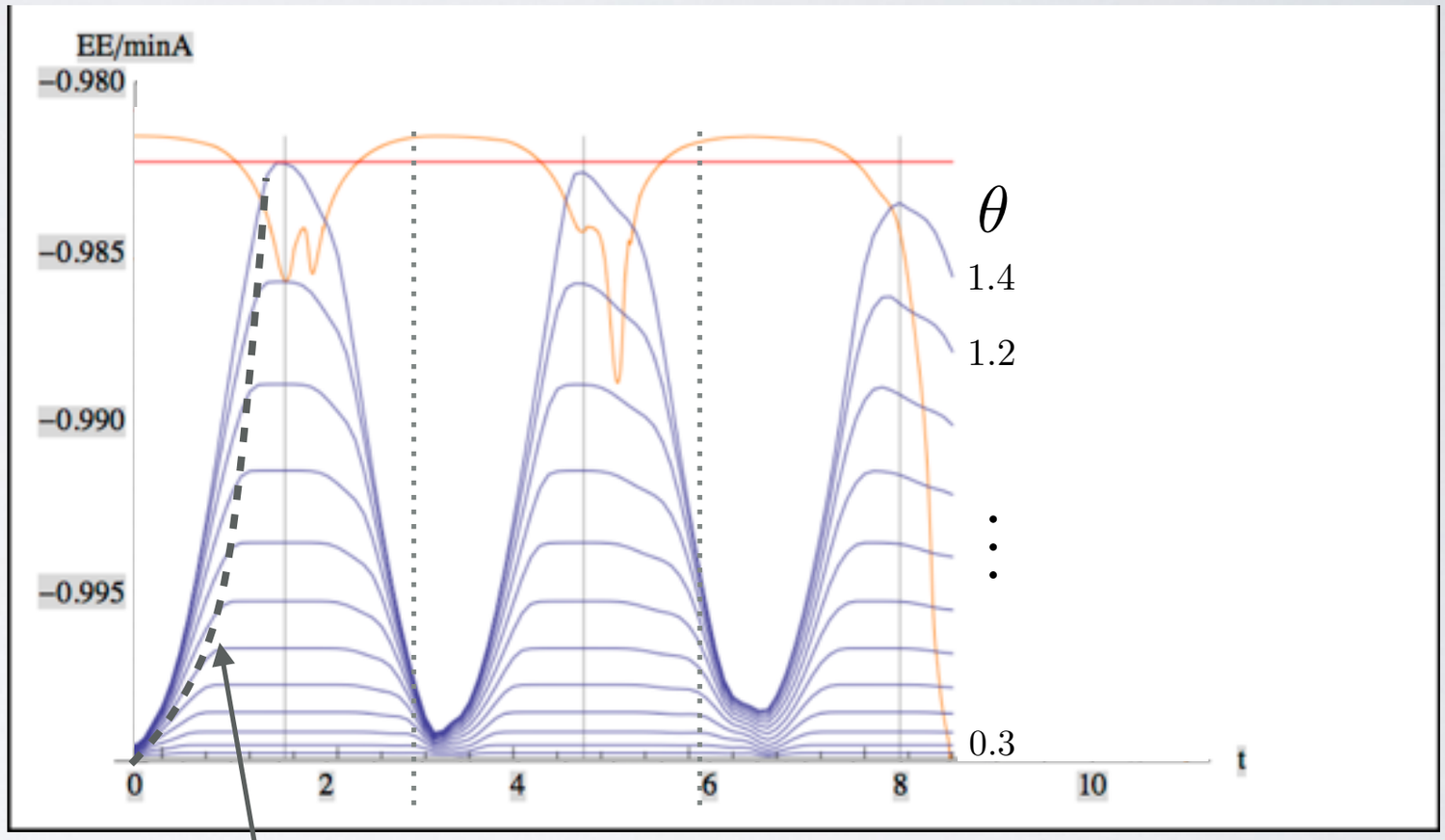
J. Abajo-Arrastia, E da Silva, E. López, J.M. & A. Serantes.

AdS_4 : ENTANGLEMENT ENTROPY

Pre-collapse

$$\theta = 0.3, 0.4, ..., 1.4$$



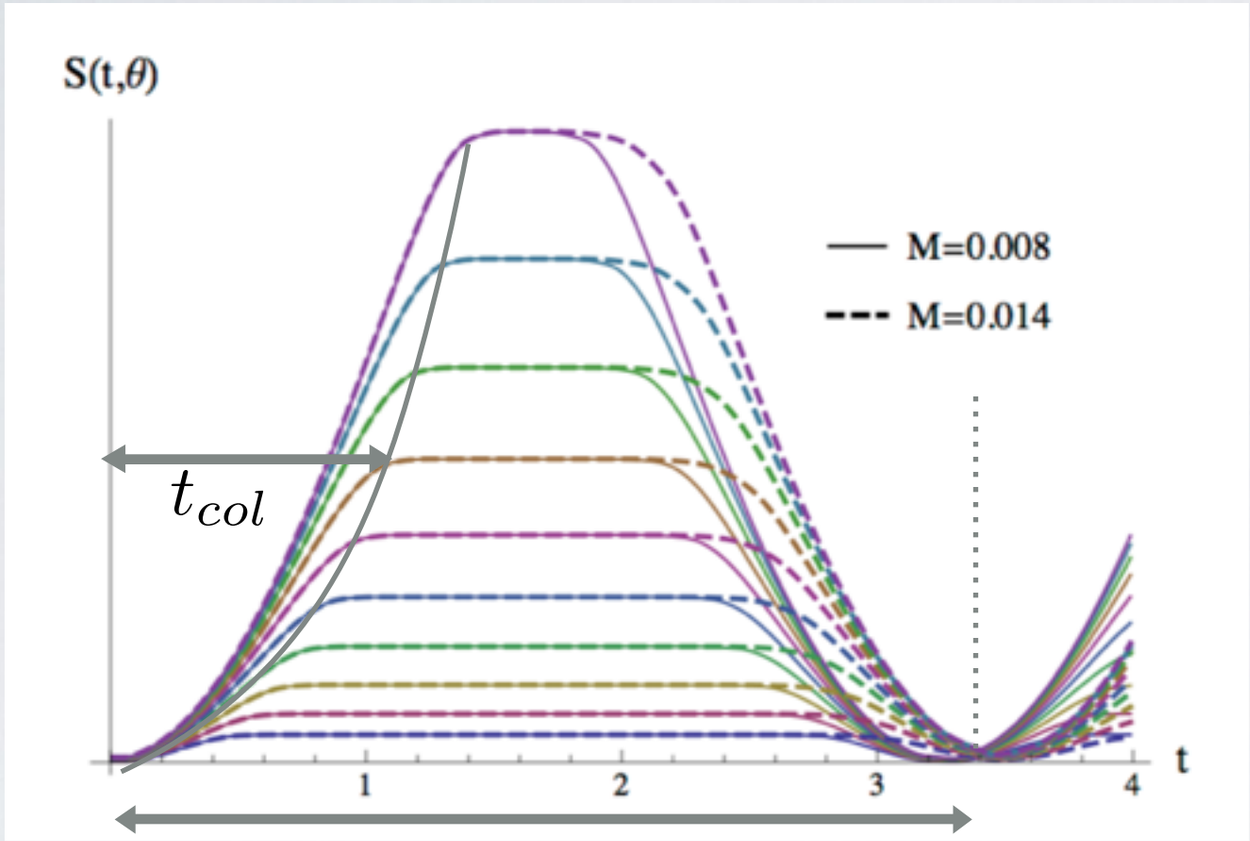


$t = \theta$

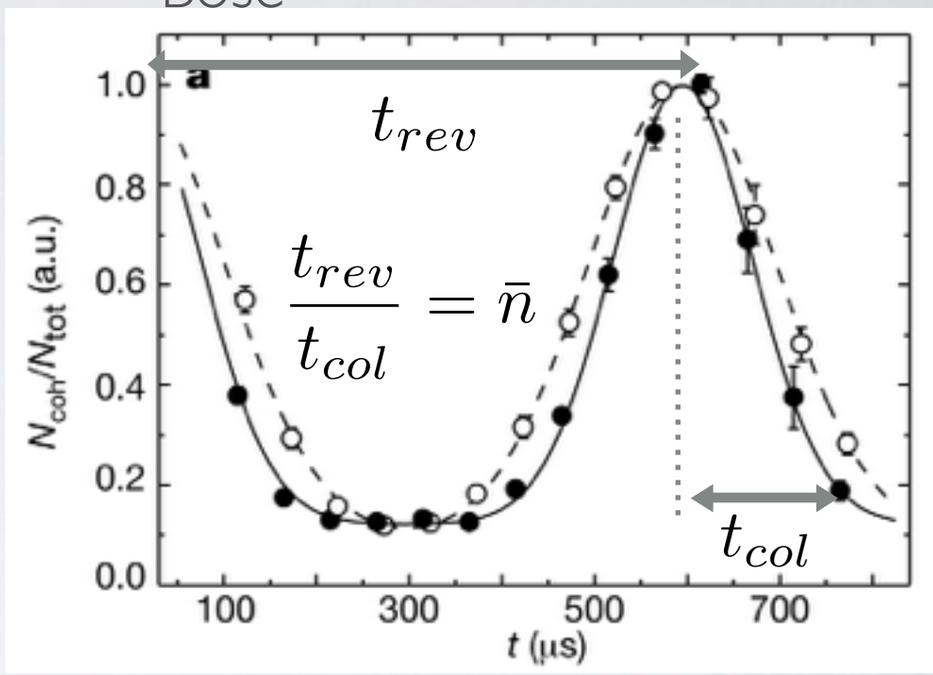
Javier Abajo-Arrastia, Emilia da Silva, Esperanza López, J.M. & Alexandre Serantes. arXiv:1403.2632

AdS_4 : ENTANGLEMENT ENTROPY

Bouncing phase: period changes with mass: two time scales



Bose



collapse time:

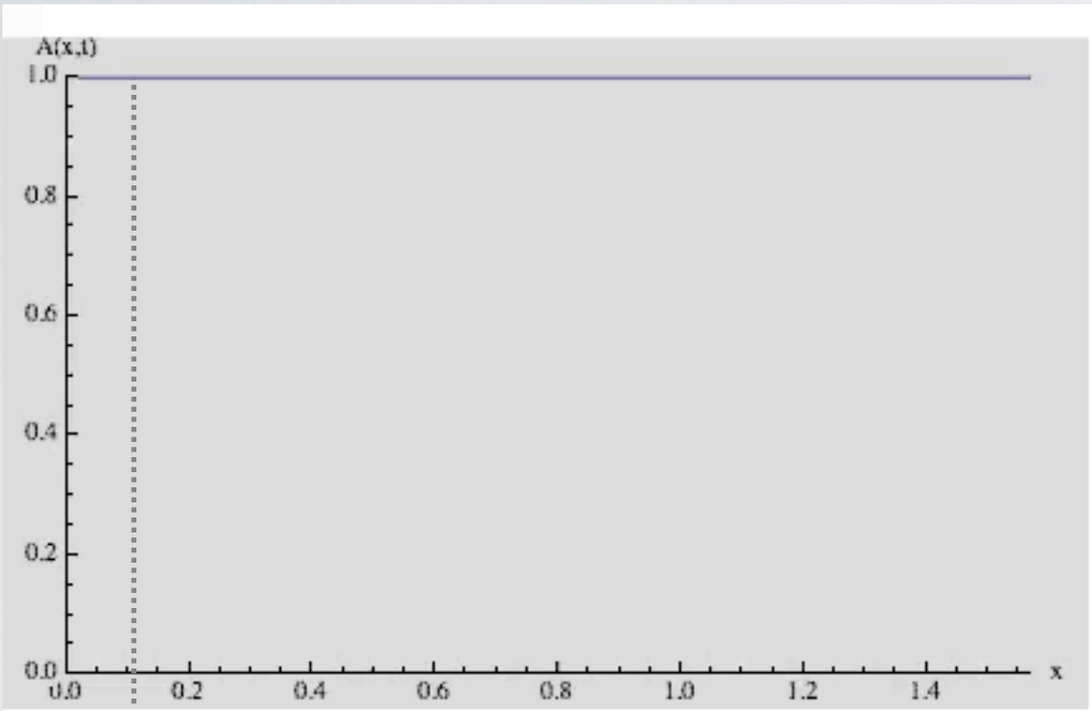
$$t_{col} = \theta$$

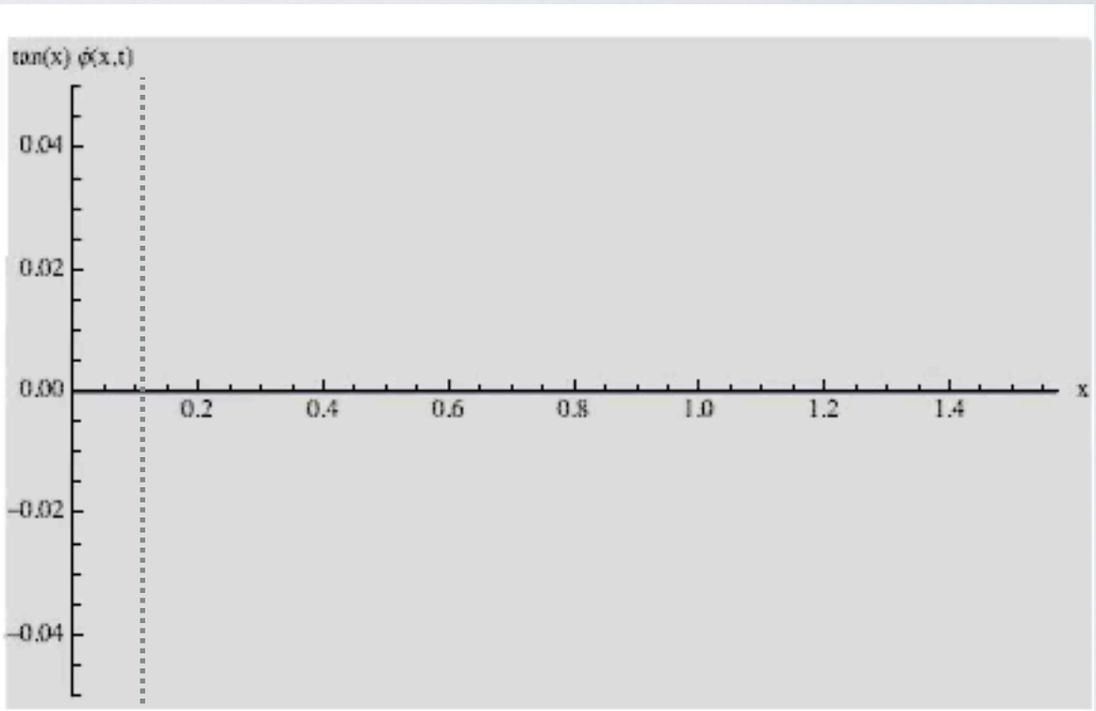
revival time:

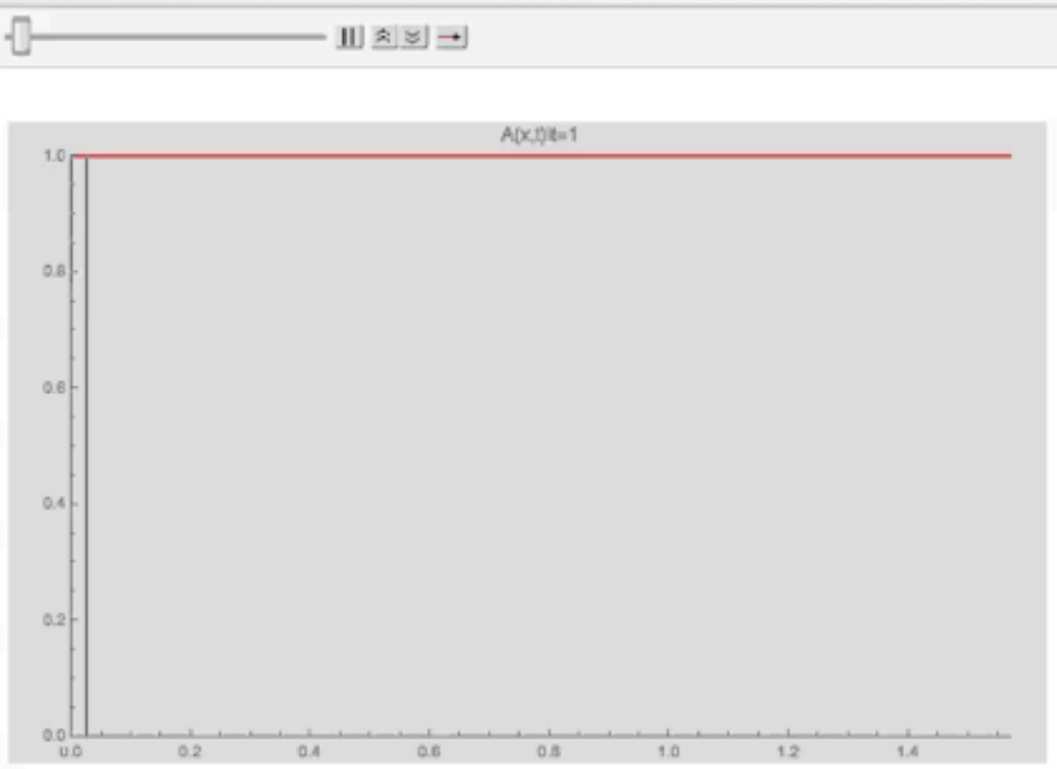
$$t_{rev} = f(M)$$

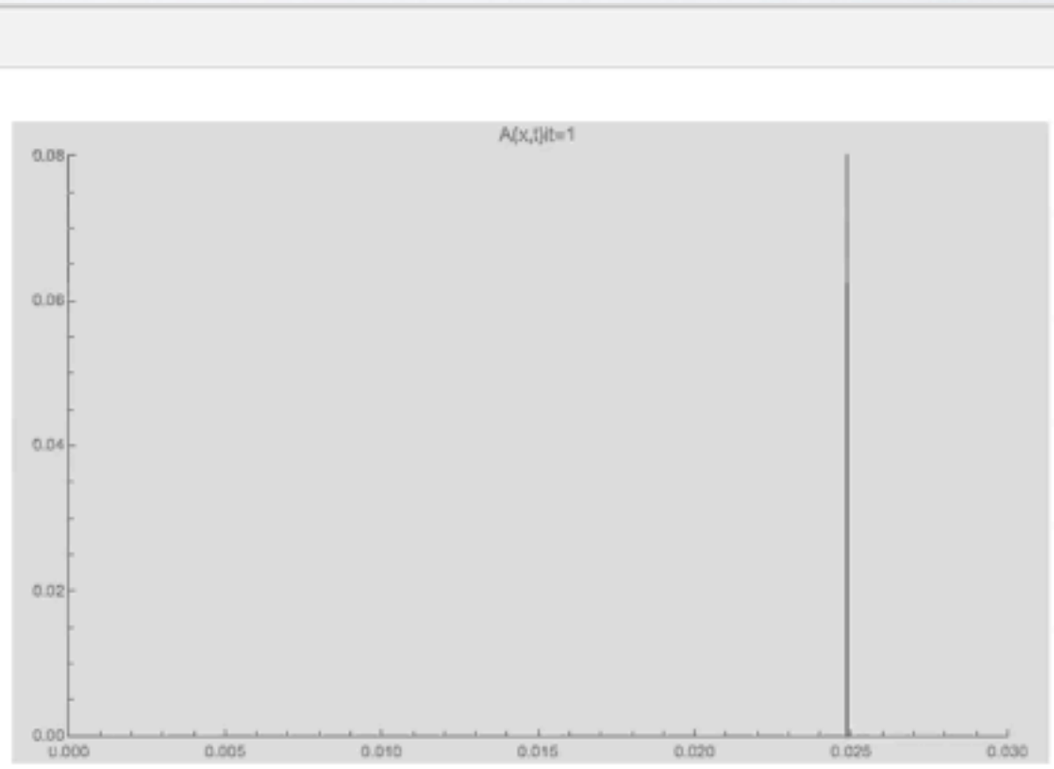
AdS_4 : ENTANGLEMENT ENTROPY

Absortive phase

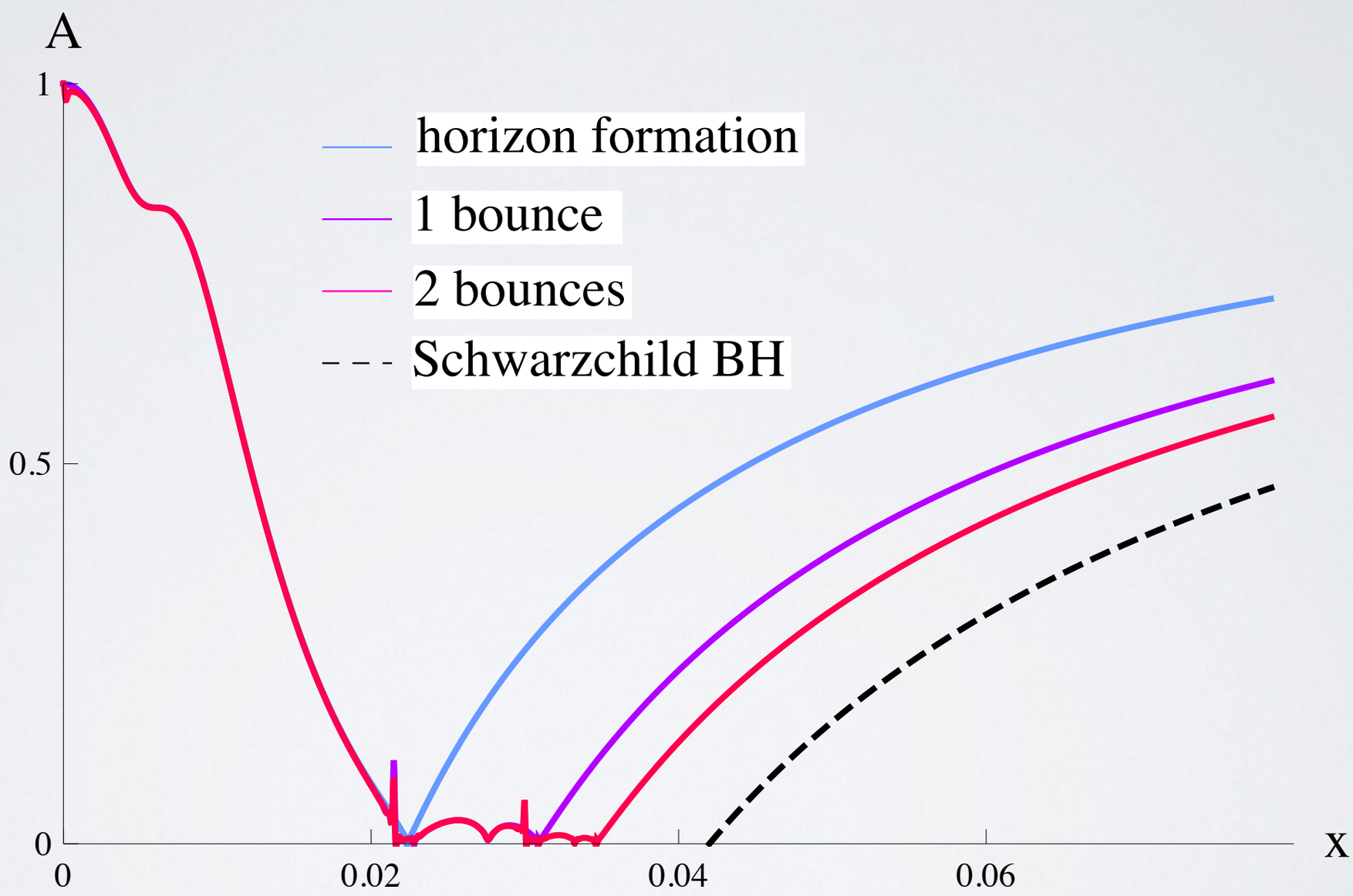






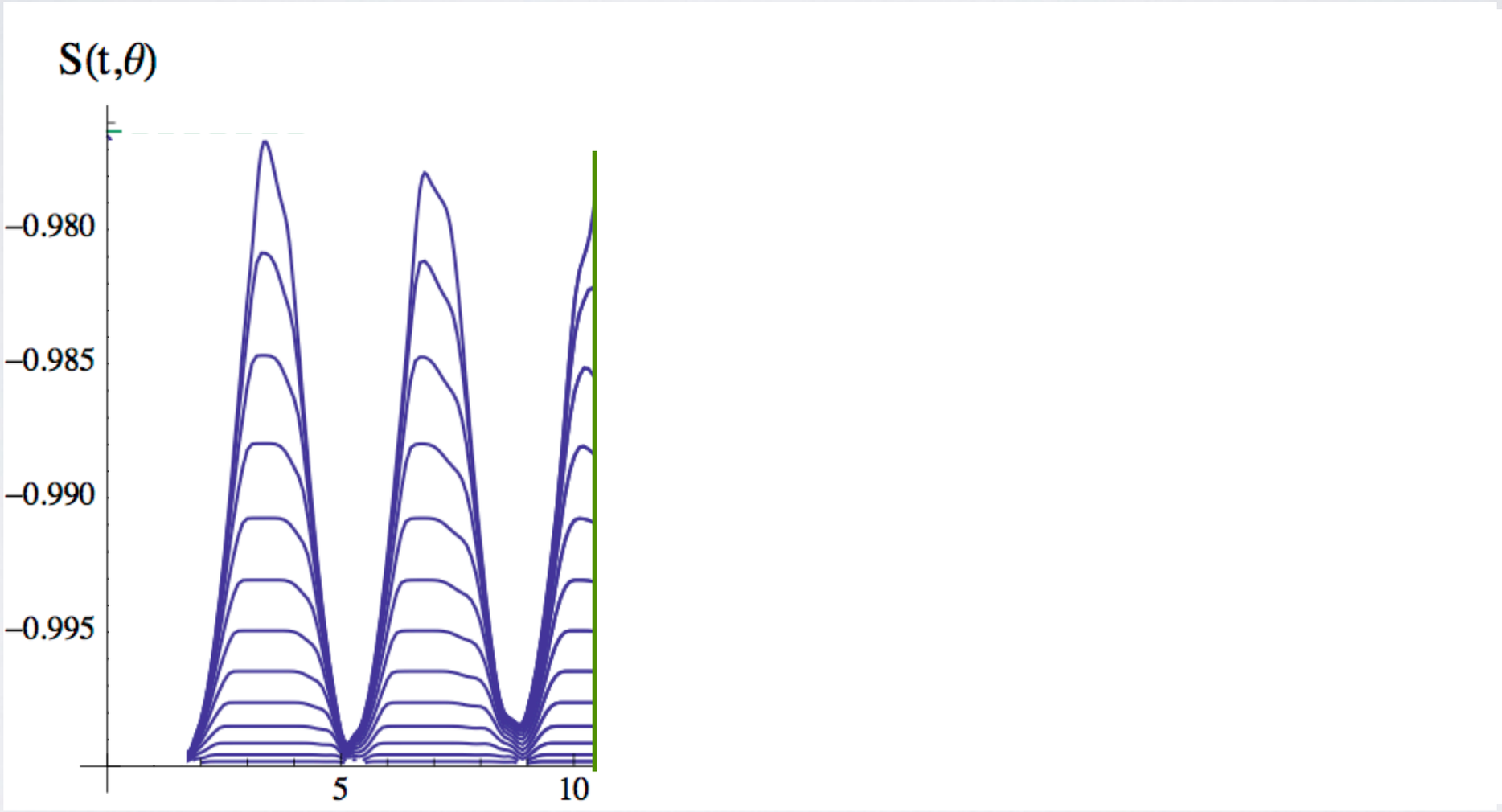


AdS_4 : ENTANGLEMENT ENTROPY



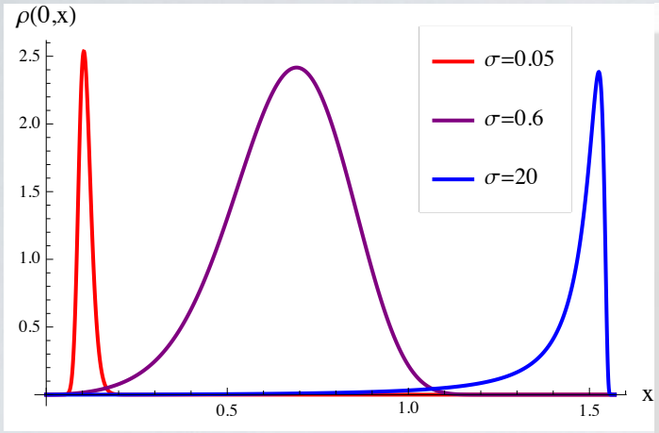
AdS_4 : ENTANGLEMENT ENTROPY

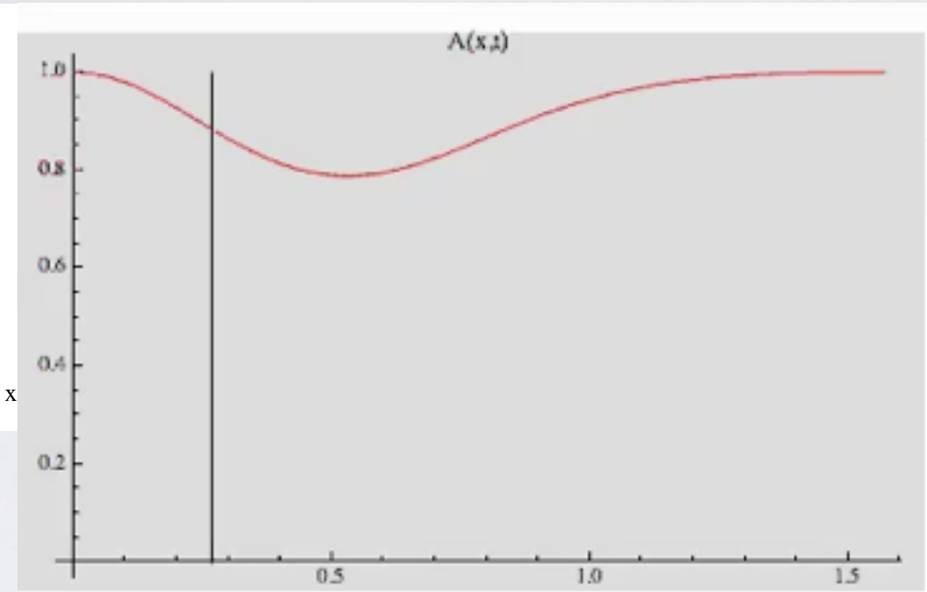
Post-collapse: two regimes, elastic and absorptive

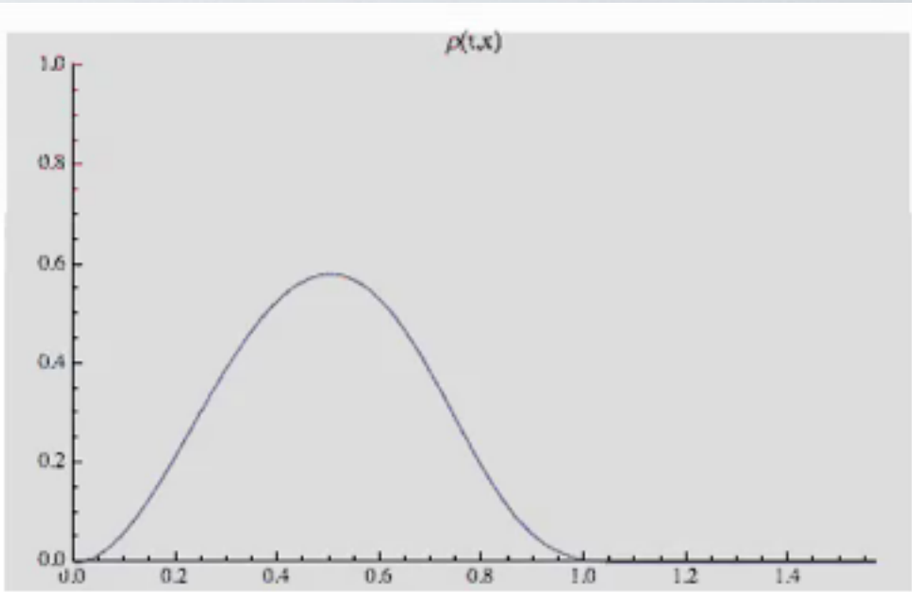


AdS_4 : BROAD PULSES

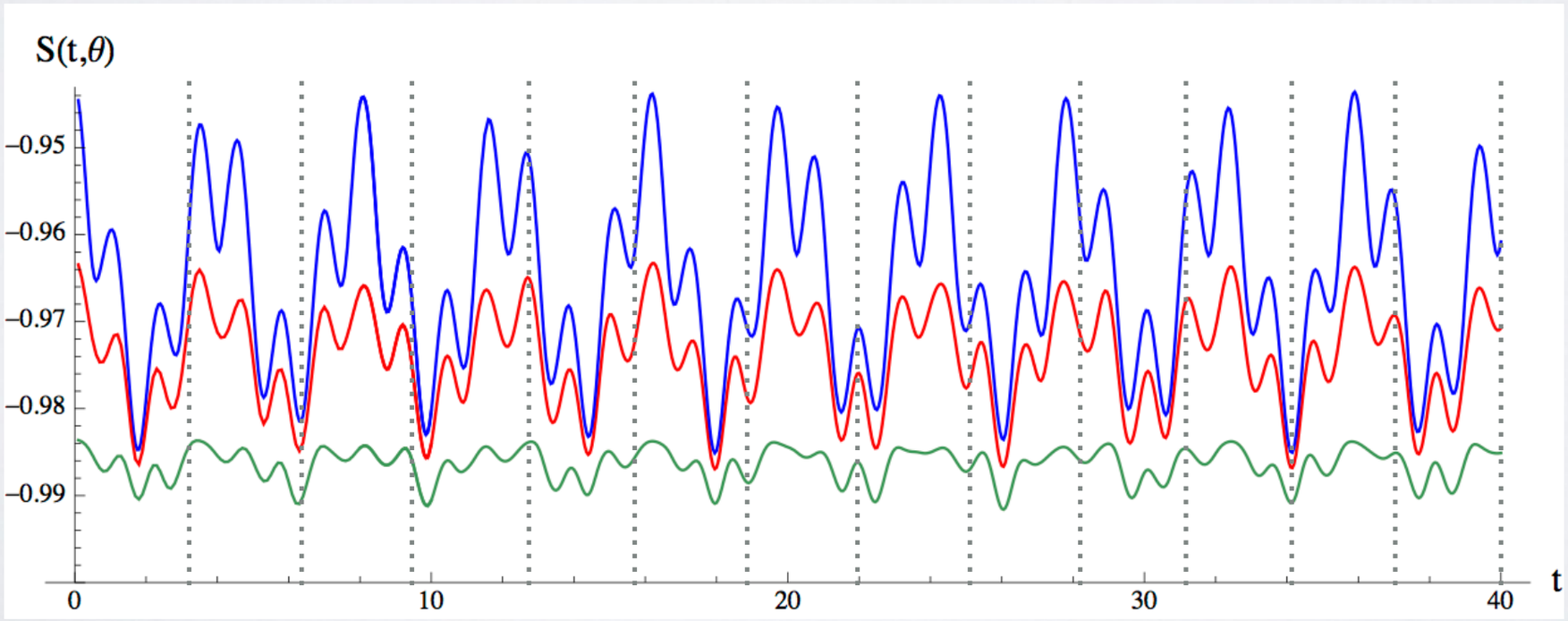
- **no collapse** occurs for $0.3 \leq \sigma \leq 16$ Buchel, Liebling & Lehner 1304.4166







- new $\pi/3$ period appears



AdS_3

$$\sigma = \frac{1}{4}$$

ϵ

Collapse

$M = 1$

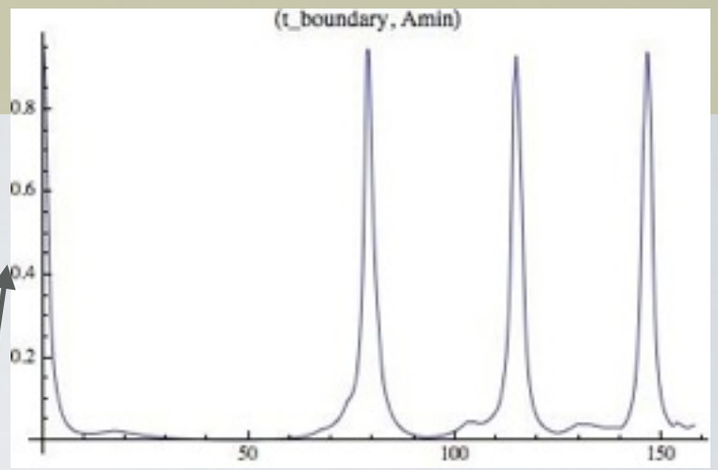
Bounce

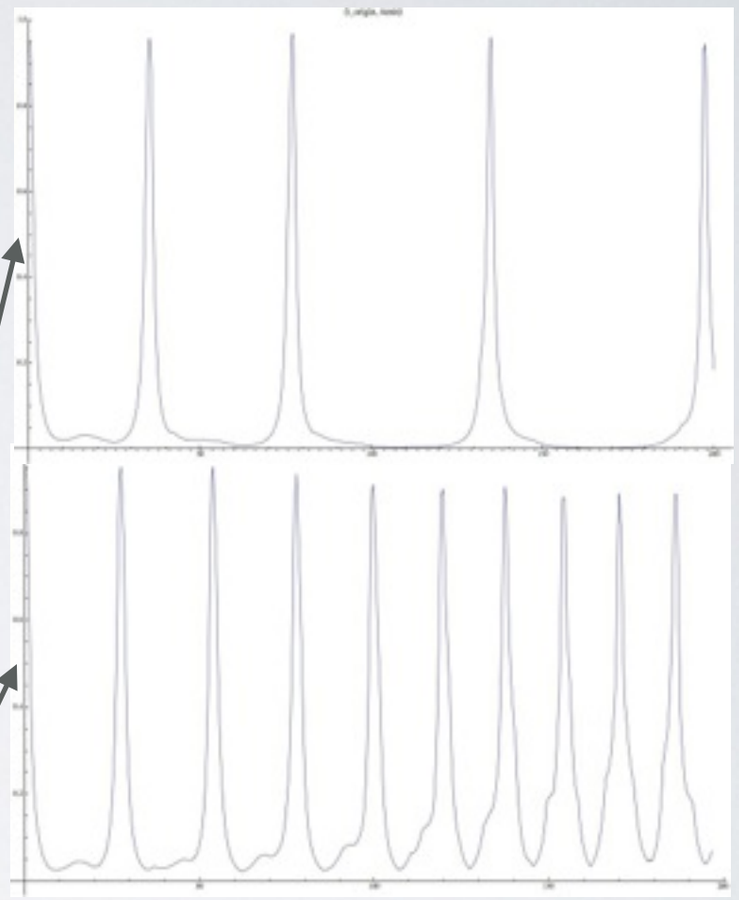
Unstable (Caotic)

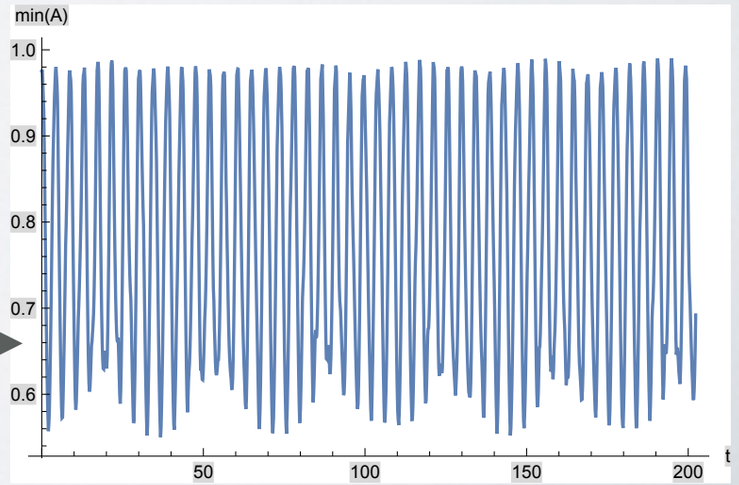
Stable (Integrable)

J. Jalmuzna & P. Bizón 2013

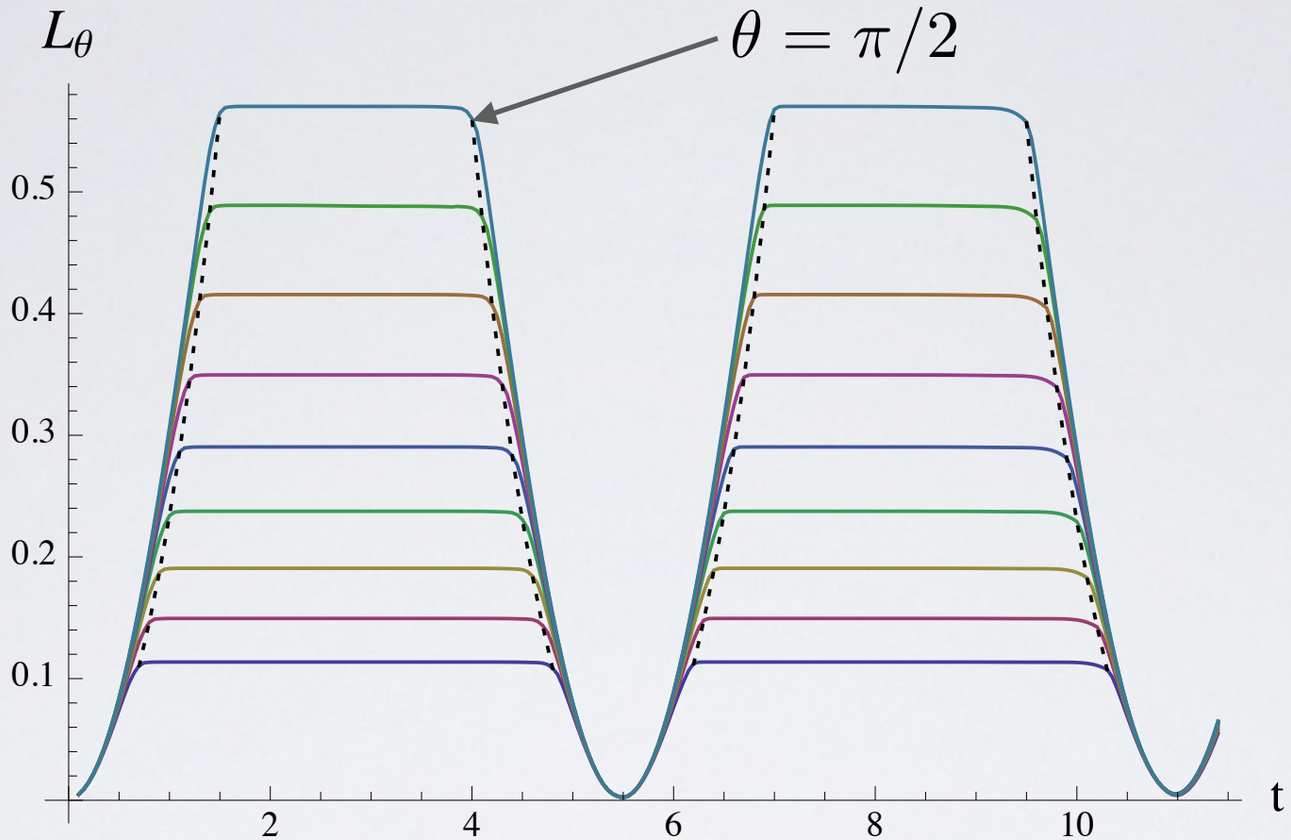
x_H

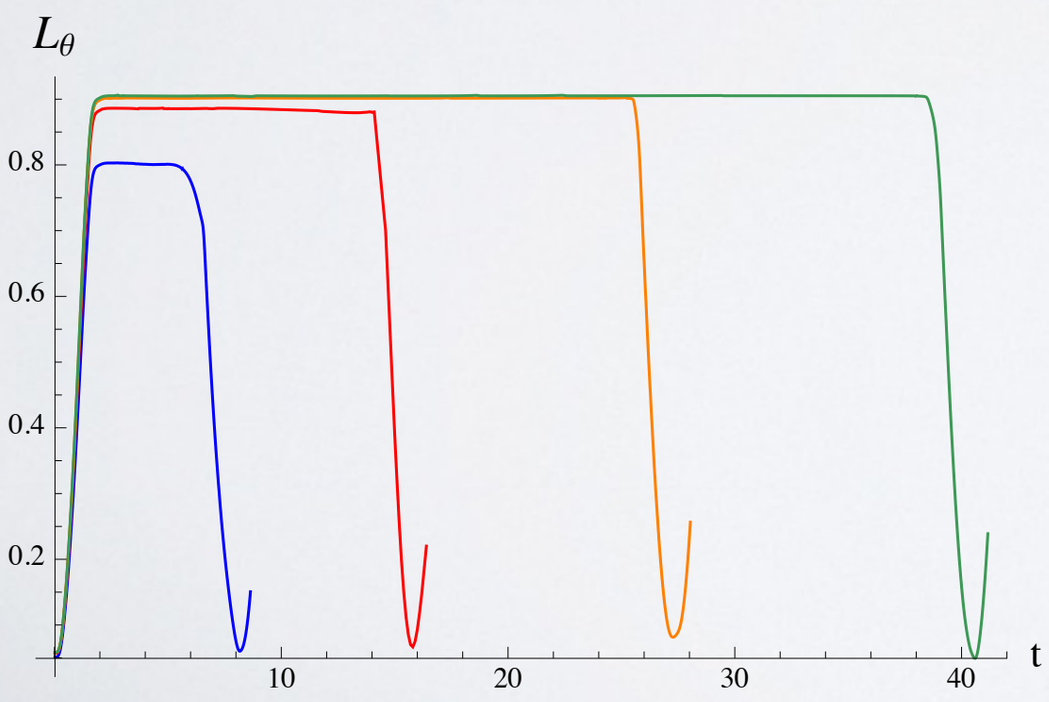


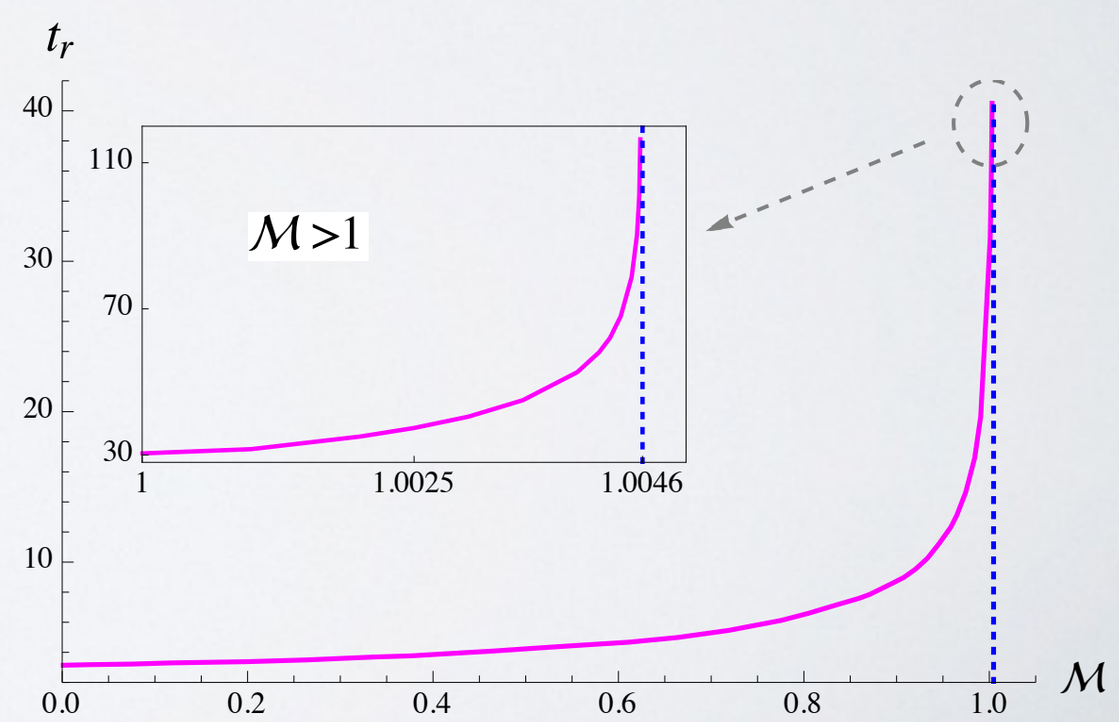




AdS_3


L_θ
$\theta = \pi/2$
0.5
0.4
0.3
0.2
0.1
2
4
6
8
10
t

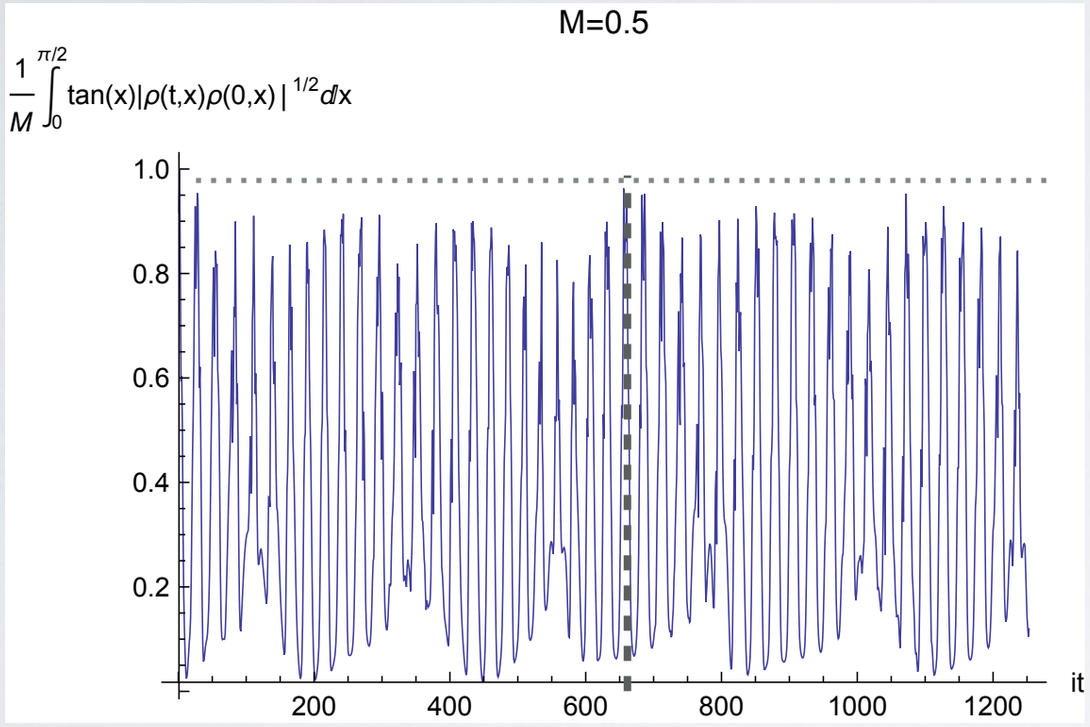

L_θ
0.8
0.6
0.4
0.2
10
20
30
40
t

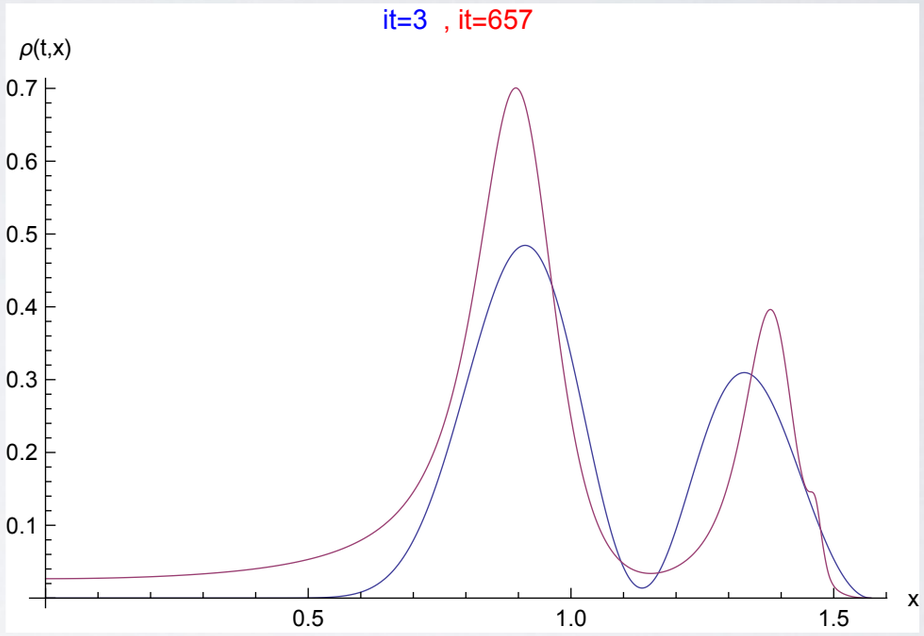

t_r
40
30
20
10
110
70
30
$\mathcal{M}>1$
1
1.0025
1.0046
0.0
0.2
0.4
0.6
0.8
1.0
$\mathcal{M}$

AdS_3

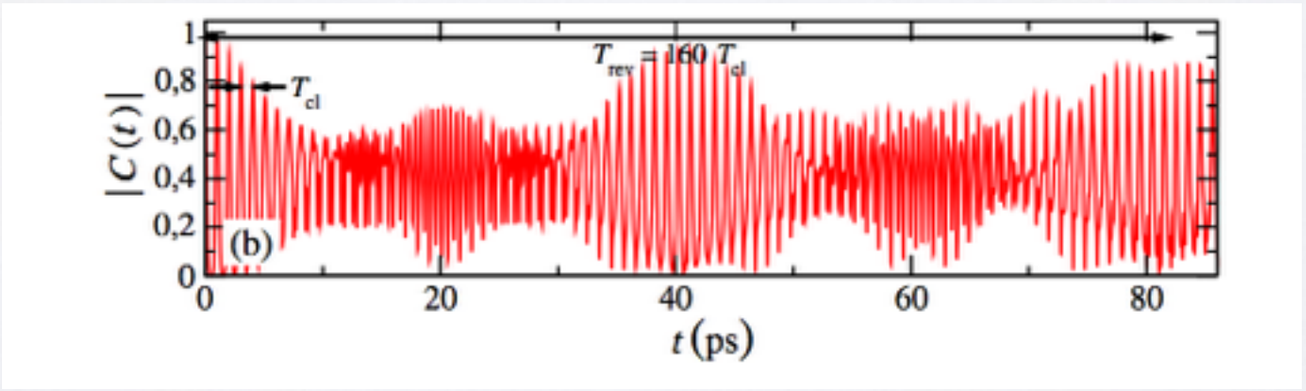
Autocorrelation superperiod

$$C(t) = \frac{1}{M}\int_0^{\pi/2} \tan(x)|\rho(t,x)\rho(0,x)|^{1/2}dx$$





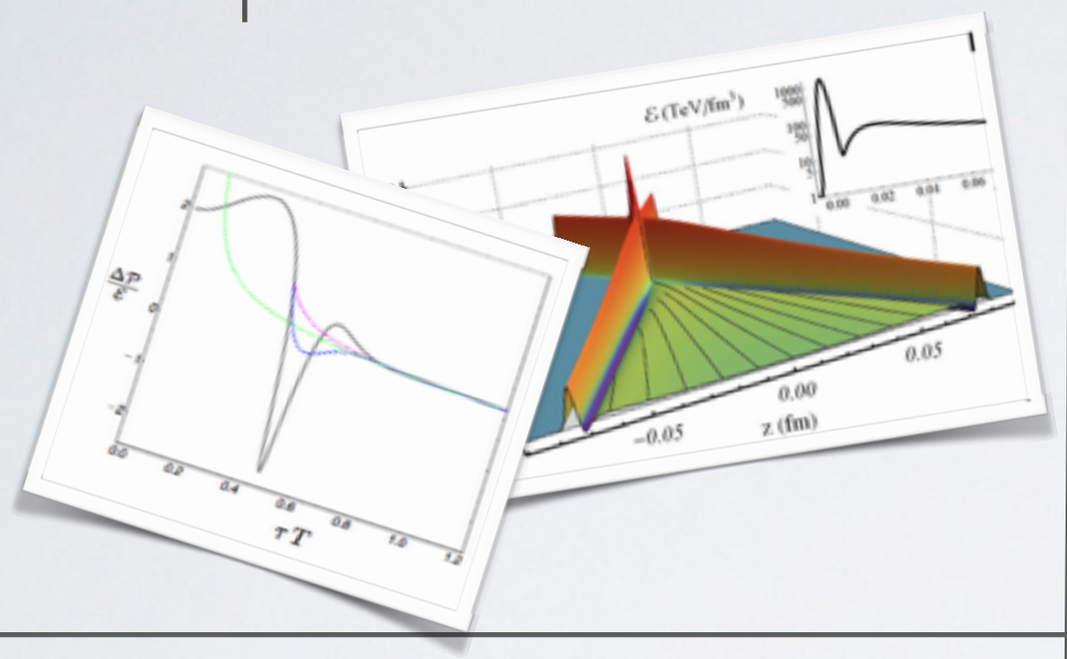
in graphene



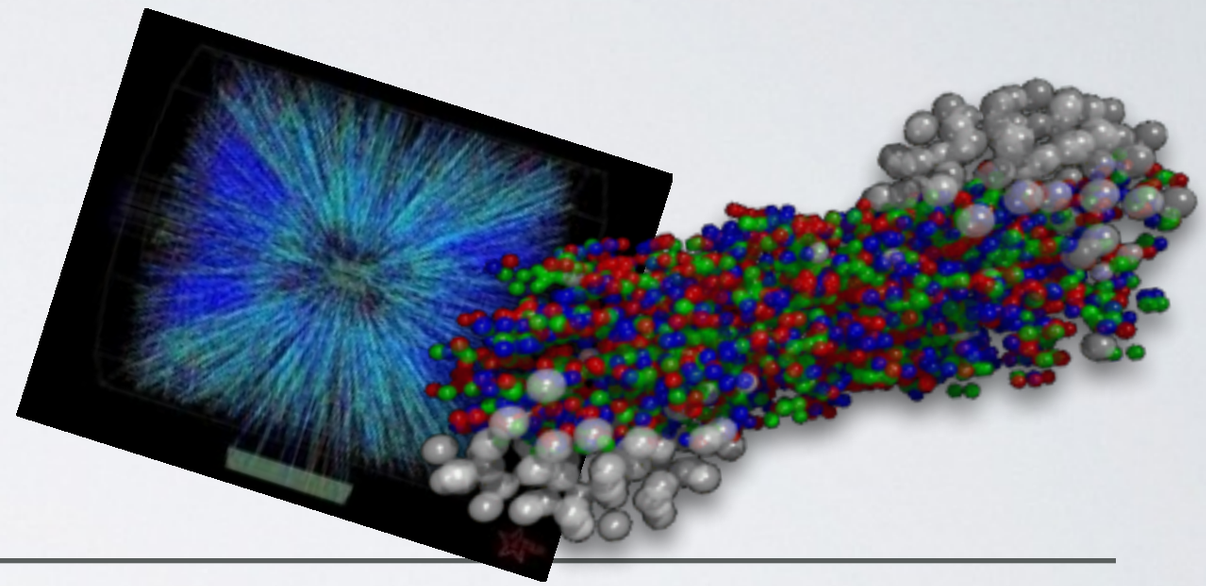
CONCLUSION

AdS/CFT

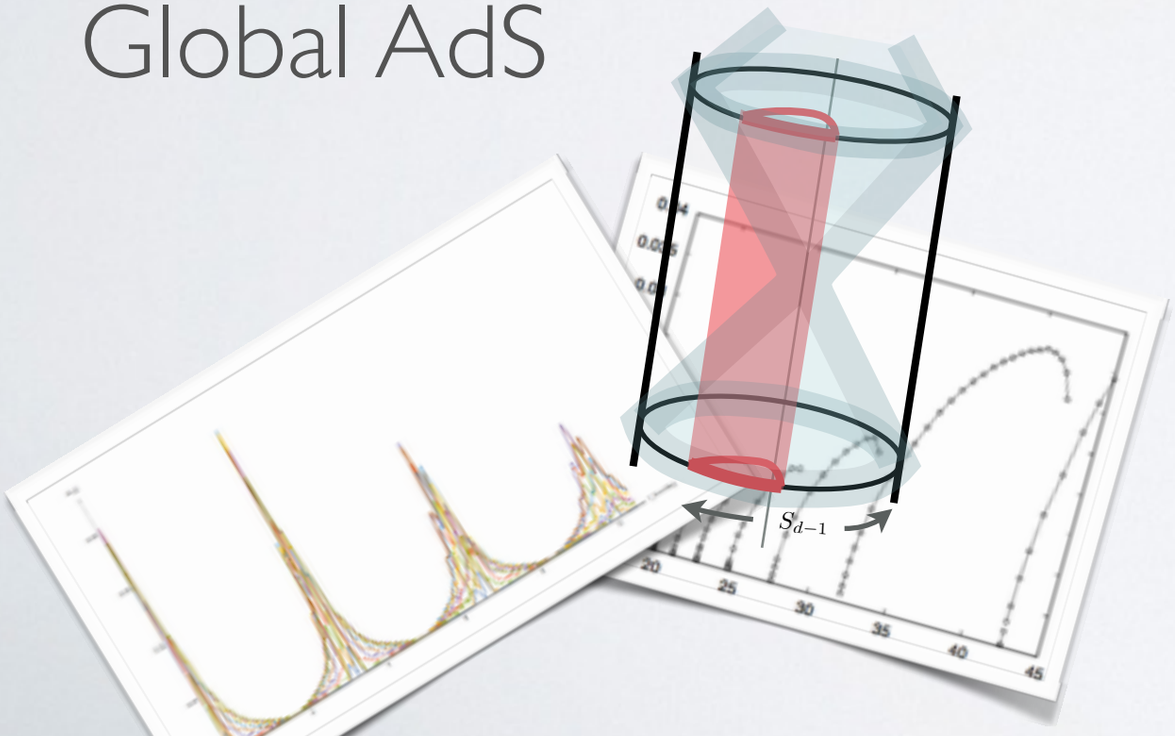
Poincaré patch



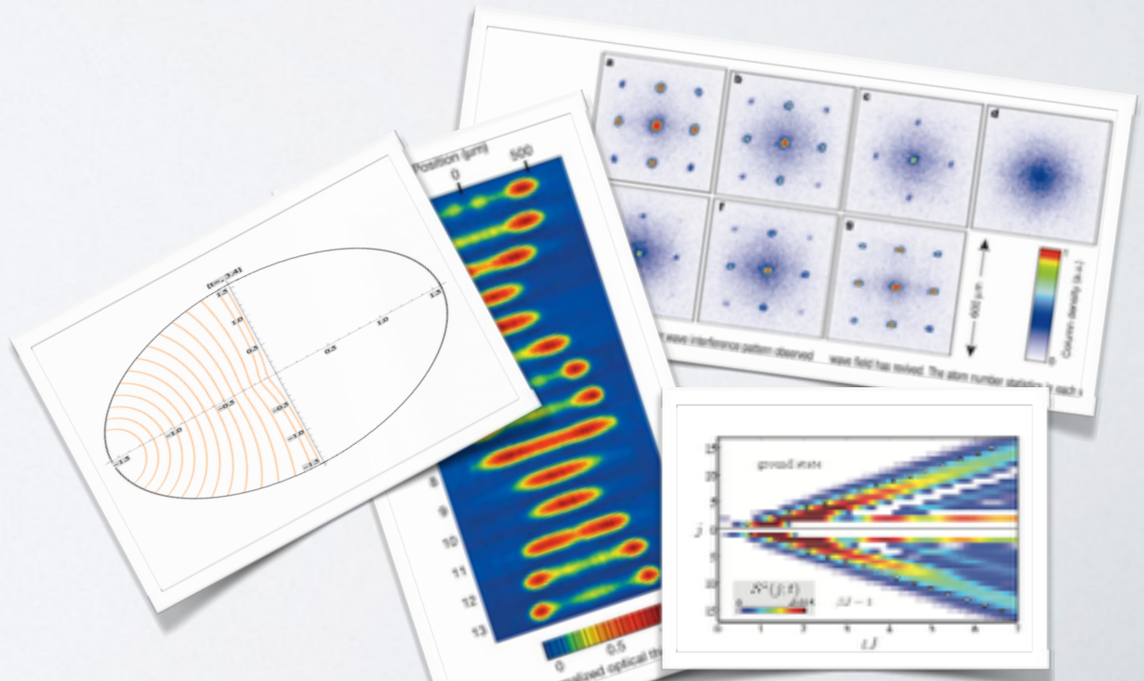
Heavy Ion Collisions



Global AdS



Revivals



thank you

&

nice weekend