Coupling hydrodynamics to nonequilibrium degrees of freedom in strongly interacting quark-gluon plasma

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Introduction

Insights gained from holographic studies of $\mathcal{N} = 4$ SYM plasma are not directly applicable in QCD, but

- some results may be universal
- may lead to phenomenological approaches directly in 4d

Main inspiration for this talk:

- studies of the hydrodynamic expansion at high orders
- numerical simulations of Bjorken flow

Bjorken flow: in proper time-rapidity coordinates

$$t = \tau \cosh y \ , \quad z = \tau \sinh y \ ,$$

observables depend only on τ and not on y or $x_\perp$.

$$\langle T^\mu_\nu \rangle = \mathrm{diag}(-\mathcal{E}, p_{||}, p_\perp, p_\perp) \ ,$$

where (imposing conservation and tracelessness)

$$p_{||} = -\mathcal{E} - \tau \dot{\mathcal{E}} \ , \quad p_\perp = \mathcal{E} + \frac{1}{2}\tau \dot{\mathcal{E}}$$

Everything is expressed in terms of the **energy density** $\mathcal{E}(\tau)$.

The energy density $\mathcal{E}(\tau)$ has been calculated using AdS/CFT in two ways:

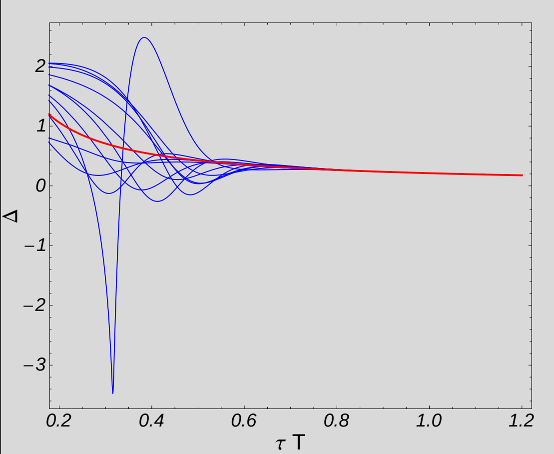
- in the hydrodynamic gradient expansion (analytically up to 3rd order and numerically up to 240th)
- exactly via numerical evolution starting from some initial geometries

Comparing the results leads to a number of concludsions.

Lessons from holographic simulations of Bjorken flow:

- The **gradient expansion** matches numerical results early, while the pressure anisotropy is still significant
- 2nd and 3rd order terms are usually less important than the **viscosity**
- The pressure anisotropy **oscillates** as the hydrodynamic regime is reached

(Chesler & Yaffe 0906.4426 & 1011.3562, Heller et al. 1103.3452, Jankowski et al. 1411.1969).



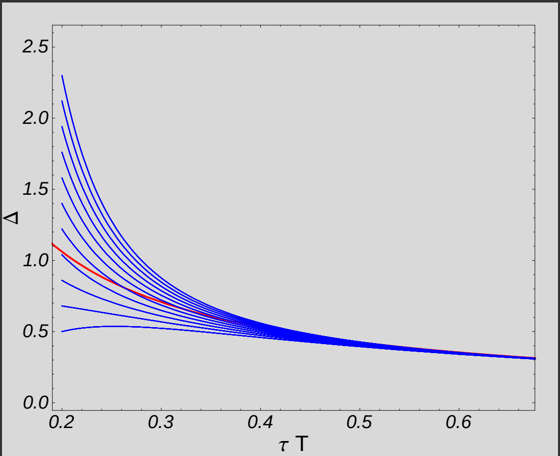
Pressure anisotropy $\Delta \equiv (p_\perp - p_{||})/\mathcal{E}$ for a sample of numerical solutions.

The large and oscillating pressure anisotropy suggests that

- **applicability of hydrodynamics** is not limited by the size of gradient corrections to the perfect fluid stress tensor, but rather by the presence of degrees of freedom not described by hydrodynamics

- phenomenological attempts to capture features of **pre-equilibrium dynamics** in heavy ion collisions need to incorporate effects of these degrees of freedom

Similar (and more precise) conclusions follow from anaylsis of high orders of the hydrodynamic expansion (as described by Przemek Witaszczyk on Friday).



Pressure anisotropy $\Delta \equiv (p_\perp - p_{||})/\mathcal{E}$ for a sample of solutions of the Müller-Israel-Stewart equations.

Our goal: extend the range of applicability of hydro by accounting for the least damped nonhydrodynamic degrees of freedom visible in numerical holography simulations of the approach to equilibrium in $\mathcal{N} = 4$ SYM .

- We formulate **new hydrodynamic evolution equations** valid for general flows.
- We compare their solutions in the case of $\mathcal{N} = 4$ SYM plasma with numerical simulations of **Bjorken flow** based on holography.

(Work with Heller, Janik and Witaszczyk arXiv:1409.5087).

Hydrodynamics

EOM: conservation of energy momentum

$$\langle T^{\mu\nu} \rangle = \mathcal{E}\, u^\mu u^\nu + \mathcal{P}(\mathcal{E})(\eta^{\mu\nu} + u^\mu u^\nu) + \Pi^{\mu\nu}$$

where

- $u_\nu u^\nu = -1$
- $u_\mu \Pi^{\mu\nu} = 0$

In CFTs:

$$\mathcal{P}(\mathcal{E}) = \frac{1}{3}\mathcal{E}\,, \quad \Pi^\mu_\mu = 0$$

Relativistic Navier-Stokes theory assumes

$$\Pi^{\mu\nu} = -\eta(\mathcal{E})\sigma^{\mu\nu}$$

where

$$\sigma^{\mu\nu} = \partial^{\mu} u^{\nu} + \dots$$

This leads to a framework which

- accounts for the key physical effect of dissipation
- admits an entropy current with nonnegative divergence

However, the linearization of NS equations around equilibrium reveals modes which **violate causality**.

Müller-Israel-Stewart

MIS hydro is a **UV completion** of Navier Stokes relativistic hydrodynamics which

- introduces a spurious, exponentially decaying mode with relaxation time τ_Π/T (a **non-hydrodynamic DOF**)
- is designed to reproduce the general form of gradient corrections up to 2nd order in the gradient expansion

The shear tensor gets its own evolution equation:

$$\left(\tau_\Pi \frac{1}{T}\mathcal{D} + 1\right) \Pi^{\mu\nu}_{MIS} = -\eta\sigma^{\mu\nu} + \ldots$$

This is written in terms of

$$\mathcal{D} \equiv u^\mu \mathcal{D}_\mu$$

where the $\mathcal{D}_\mu$ is the **Weyl-covariant derivative** constructed using the "Weyl connection"

$$\mathcal{A}_\mu = u^\lambda \nabla_\lambda u_\mu - \frac{1}{3} \nabla_\lambda u^\lambda u_\mu .$$

Explicitly

$$\mathcal{D}\Pi^{\mu\nu} = u^\lambda \left(\nabla_\lambda + 4\mathcal{A}_\lambda \right) \Pi^{\mu\nu} - 2\mathcal{A}_\lambda u^{(\mu} \Pi^{\nu)\lambda}$$

(Loganayagam 0801.3701)

By iteration one finds

$$\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu} + \tau_\Pi \frac{1}{T}\mathcal{D}(\eta\sigma^{\mu\nu}) + \dots$$

If one introduces the 3 elided terms in the MIS evolution equation, one gets the most general form of the shear stress tensor allowed by conformal symmetry, with 4 transport coefficients: τ_Π, $\lambda_{1,2,3}$ (Baier et al. 0712.2451).

Note that in this scheme the **"relaxation time"** τ_Π is identified with a 2nd order transport coefficient.

The MIS evolution equations

- are hyperbolic, causal and stable **if** $\tau_\Pi \geq 2\eta/s$
- admit an entropy current with nonnegative divergence

This comes at the cost of introducing a **nonhydrodynamic mode** whose interpretation is unclear.

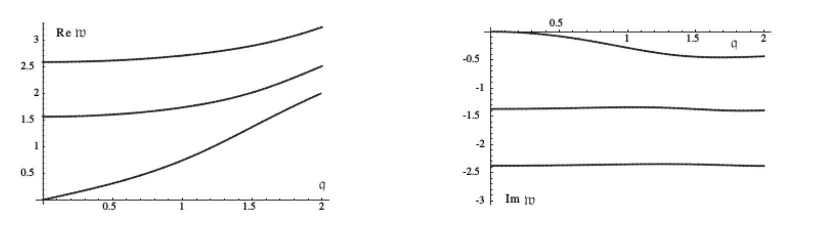
Can we instead achieve hyperbolicity, causality and stability by including nonhydro modes whose interpratation we understand?

Quasinormal modes

For small amplitude **perturbations of equilibrium plasma**

$$\tilde{\Pi}^{\mu\nu} \sim e^{-i\omega(k)t + i\vec{k}\cdot\vec{x}}$$

one finds modes with complex ω: the black brane QNM (Kovtun, Starinets hep-th/0506184).



At late times the **least damped mode** is dominant.

Ignoring the weak dependence of QNM frequencies on k (**ultralocality**) one finds a damped harmonic oscillator equation:

$$\left(\frac{1}{T}\partial_t\right)^2 \tilde{\Pi}^{\mu\nu} + 2\,\omega_I \left(\frac{1}{T}\partial_t\right) \tilde{\Pi}^{\mu\nu} + |\omega|^2 \tilde{\Pi}^{\mu\nu} = 0.$$

where

$$\frac{\omega}{T} = \omega_R(k/T) + i\,\omega_I(k/T).$$

and $|\omega|^2 \equiv \omega_I^2 + \omega_R^2$.

For $\mathcal{N} = 4$ supersymmetric Yang-Mills SYM

$$\omega_R \approx 9.800 \quad \text{and} \quad \omega_I \approx 8.629.$$

Hydro plus QNM

We wish to generalize the QNM equation to describe the least damped QNM on top of a hydrodynamic background

$$\Pi^{\mu\nu} = \Pi^{\mu\nu}_{\text{hydro}} + \tilde{\Pi}^{\mu\nu}$$

where

$$\Pi^{\mu\nu}_{\text{hydro}} = -\eta\sigma^{\mu\nu} + \dots$$

We will

- not try to reproduce second order terms
- ensure only that viscosity is correctly accounted for

Naively we might replace

$$\partial_t \to u^\mu \partial_\mu$$

in

$$\left(\frac{1}{T}\partial_t\right)^2 \tilde{\Pi}^{\mu\nu} + 2\,\omega_I \left(\frac{1}{T}\partial_t\right) \tilde{\Pi}^{\mu\nu} + |\omega|^2 \tilde{\Pi}^{\mu\nu} = 0$$

but this does not preserve

- the transversality of the shear tensor
- conformal symmetry

Consistency requires that we use the Weyl covariant derivative:

$$\partial_t \to u^\mu \mathcal{D}_\mu$$

which leads to

$$(\frac{1}{T}\mathcal{D})^2 \tilde{\Pi}_{\mu\nu} + 2\omega_I \frac{1}{T}\mathcal{D}\tilde{\Pi}_{\mu\nu} + |\omega|^2 \tilde{\Pi}_{\mu\nu} = 0$$

Two key features

- consistent with $\Pi^{\mu\nu}$ transforming homogeneously under Weyl transformations
- preserves transversality and tracelessness due to the fact that $\mathcal{D}\, u^\mu = 0$

The simplest way to incorporate QNM degrees of freedom into a hyperbolic, causal and stable description is to set

$$\Pi^{\mu\nu} = \Pi^{\mu\nu}_{MIS} + \tilde{\Pi}^{\mu\nu}$$

with $\Pi^{\mu\nu}_{MIS}$ satisfying the MIS equation

$$\left(\tau_\Pi \frac{1}{T}\mathcal{D} + 1\right) \Pi^{\mu\nu}_{MIS} = -\eta\sigma^{\mu\nu} + \ldots$$

and $\tilde{\Pi}^{\mu\nu}$ the QNM equation.

These traceless and transverse quantities are coupled by the conservation of energy-momentum.

The resulting theory

- is valid for general flows
- satisfies the same causality and stability properties as vanilla MIS hydro
- at the linearized level, in addition to the standard hydrodynamic modes it contains a nonhydro mode corresponding to the least damped QNM
- still contains the spurious decaying mode of MIS theory

In order to minimize the impact of the MIS mode

- as initial condition for this mode we set

$$\Pi^{\mu\nu}_{MIS} = -\eta\sigma^{\mu\nu}$$

- we set τ_Π to the smallest value allowed by causality in order to maximize the damping of this mode

Eliminating the MIS mode

One can in fact decouple the MIS mode completely

$$\left((\frac{1}{T}\mathcal{D})^2 + 2\omega_I \frac{1}{T}\mathcal{D} + |\omega|^2 \right) \Pi^{\mu\nu} = -\eta|\omega|^2\sigma^{\mu\nu} - c_\sigma \frac{1}{T}\mathcal{D}\left(\eta\sigma^{\mu\nu}\right) + \dots$$

Linearization around equilibrium leads to a system of PDEs which is

- hyperbolic for $c_\sigma \geq 0$
- causal for $c_\sigma \leq \frac{s}{4\eta}$

Stability depends on values of the QNM frequencies, the viscosity to entropy ratio and c_σ.

Unlike in the case of MIS equations, using $\mathcal{N}=4$ SYM values and c_σ in the causal window there are still exponentially unstable modes with high k.

In general this renders these equations unsuitable for numerical evaluation **if** we wish to compare with the results of simulations of $\mathcal{N}=4$ SYM .

In the following tests we stick to the equations which include the MIS mode.

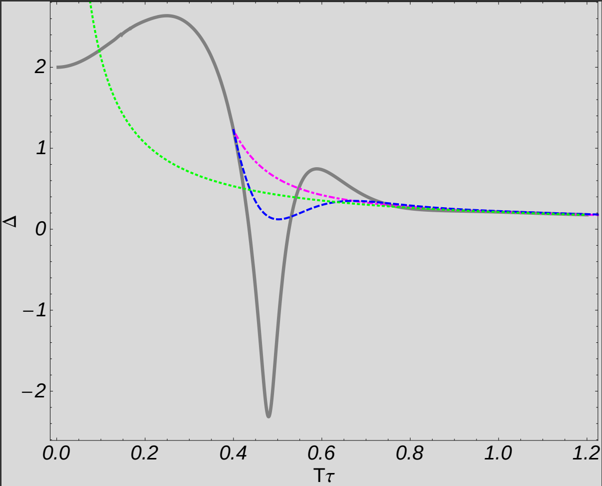
Tests

We tested the new equations against microscopic numerical computations of Bjorken flow of $\mathcal{N} = 4$ supersymmetric Yang-Mills SYM plasma based on the AdS/CFT correspondence.

This requires setting the parameters η and $\omega_{R,I}$ to values known from holographic calculations in $\mathcal{N} = 4$ supersymmetric Yang-Mills SYM .

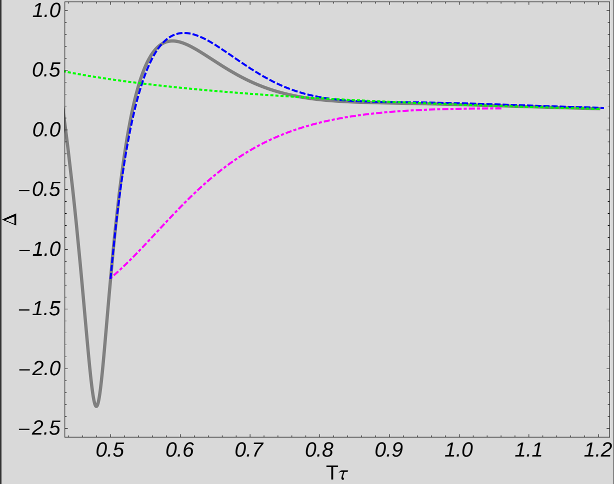
To maximally damp the MIS mode we also set $\tau_{\Pi} = 1/(2\pi)$, which is the smallest value allowed by causality.

Case 1: $\tau T = 0.4$



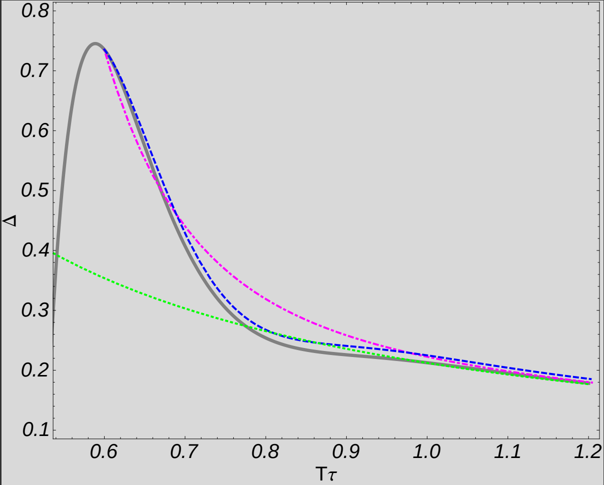
gray: AdS/CFT; green: hydro; magenta: MIS; blue: MIS+QNM

Case 2: $\tau T = 0.5$



gray: AdS/CFT; green: hydro; magenta: MIS; blue: MIS+QNM

Case 3: $\tau T = 0.6$

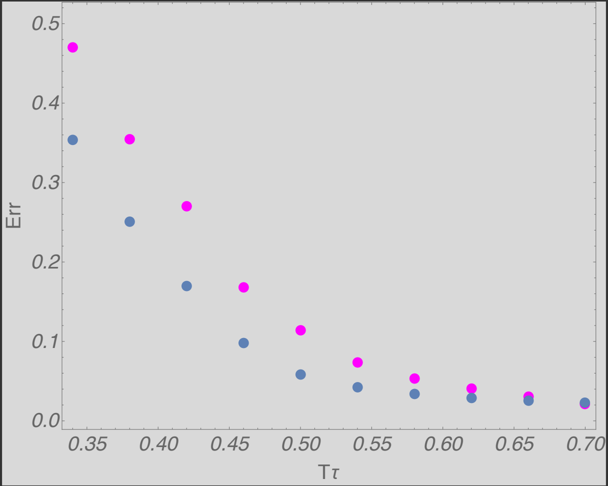


gray: AdS/CFT; green: hydro; magenta: MIS; blue: MIS+QNM

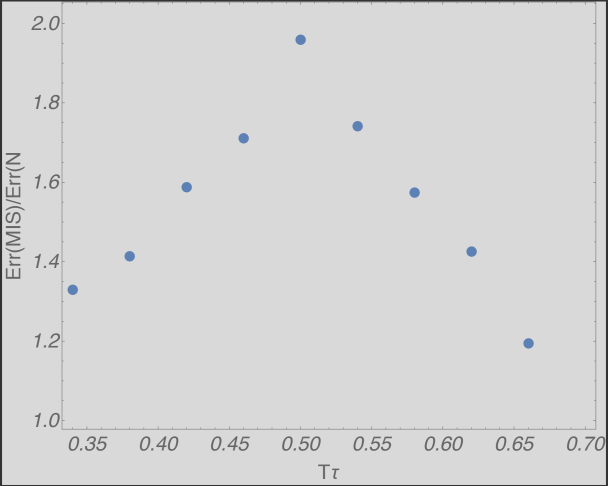
We see that:

- at late times everything converges to viscous hydrodynamics
- the final temerature is estimated reasonably accurately both by MIS and by MIS+QNM
- with earlier initialization MIS+QNM is better at estimating the pressure anisotropy

“Error” averaged over 99 runs



$$Err = \langle Max(Abs(\Delta_{MIS} - \Delta_{MIS+QNM}))\rangle$$



2.0
1.8
1.6
1.4
1.2
1.0
Err(MIS)/Err(N
0.35
0.40
0.45
0.50
0.55
0.60
0.65
0.70
Tτ

We also ran simulations of **two dimensional flows** (without comparing to AdS/CFT numerics however):

- the equations which include the MIS mode along with the QNM showed no instabilities
- the equations with the QNM only (i.e. no MIS mode) lead to instabilities where expected (i.e. sufficiently high k)

Caveats:

- we need initial conditions not just for the shear stress tensor, but also for its derivative
- for initial conditions involving many QNMs the agreement at early times should not be as good
- for initial conditions where no nohydrodynamic modes are excited at early times, effects of second and higher order (or possibly resummed hydrodynamics) may become important

Summary

What we did:

- assumed ultralocality to write a simple equation satisfied by QNM contributions to the shear stress tensor
- coupled the least damped QNM to MIS hydrodynamics
- eliminated the MIS mode completely
- compared with numerical simulations of Bjorken flow

Outlook

We proposed an exploratory 4d model incorporating some nonequilibrium physics of strongly coupled SYM (for which we normally need holography) :

- it describes oscillations of the pressure anisotropy seen in numerical holography
- initial conditions require the energy-momentum tensor and its time derivative
- practical applications to HIC require heuristics for the initial conditions (perhaps based on perturbative QCD)