Black rings in global AdS

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Outline

1. Introduction
2. Numerical construction of AdS black rings
3. Physics of AdS black rings
4. Summary and conclusions

1. Introduction

Introduction

Four dimensional stationary asymptotically flat vacuum black holes are well-understood in classical general relativity:

- They are topologically spherical [Hawking]
- They rotate rigidly with respect to asymptotic observers [Hawking]
- They are uniquely specified by their mass M and angular momentum J and they are given, explicitly, by the Kerr family of solutions [Carter; Robinson; Bunting; Mazur]
- The methods to explicitly construct all 4d black holes are also known [Kerr; Belinskii and Zakharov]
- Likely to be stable

Introduction

In recent years there has been a growing interest in studying GR in dimensions > 4 and/or asymptotically anti-de Sitter spaces because:

Gravity (GR) in such settings can exhibit fundamentally new phenomena:

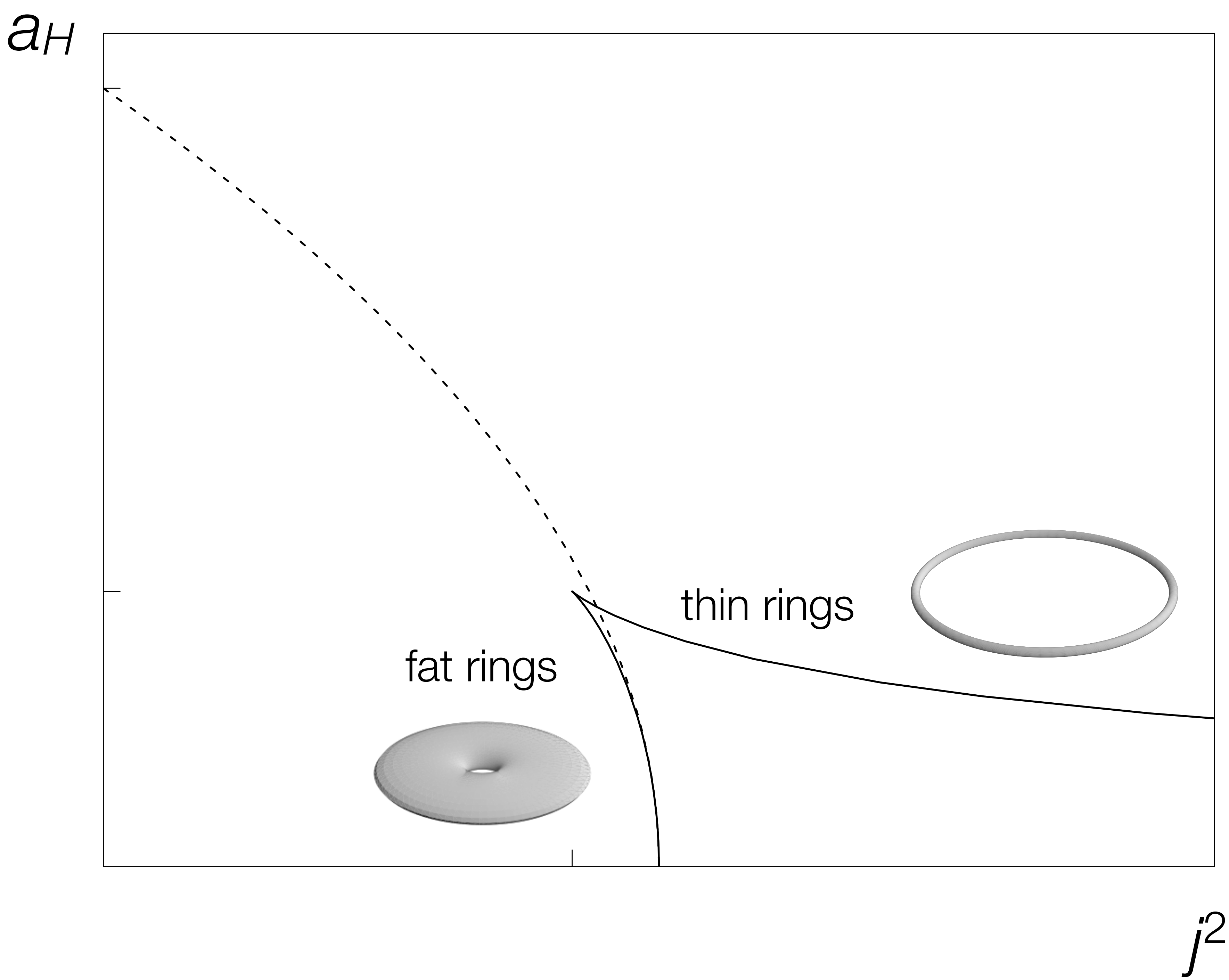
- Stationary black holes need not be spherical: black rings [Emparan and Reall], black saturns [Elvang and PF], … are explicitly known but more general solutions are expected to exist

- Non-uniqueness

- Dynamical instabilities can be generic but the endpoints are not known

Introduction

$M = 1$



Introduction

But many of the classical results on black holes do *NOT* hold in AdS:

- Horizon topology: even in 4d, in AdS we can have planar and/or hyperbolic horizons
- Black holes can have non-Killing horizons [Wiseman and PF; Fischetti, Marolf and Santos]
- Uniqueness?
- Fewer symmetries?
- Instabilities?

⇒ black holes in AdS are much richer!

- The methods to explicitly construct all 4d asymptotically flat black holes [Kerr; Belinskii and Zakharov] do not seem to work in AdS

⇒ we need new methods!

Introduction

AdS/CFT

Stationary black holes in global AdS $=$ Equilibrium finite temperature states of $\mathcal{N}$=4 super Yang-Mills on S^3 at strong coupling

Introduction

AdS/CFT

Stationary black holes in global AdS	=	Equilibrium finite temperature states of $\mathcal{N}$=4 super Yang-Mills on S^3 at strong coupling

Some questions arise:

- Do non-spherical black holes with compact horizons exist in global AdS?
- What is the dominant state at a given temperature T_H and angular velocity Ω_H?
- How can the field theory tell about the horizon topology of the dual black hole?

2. Numerical construction of AdS black rings

AdS black rings: setup

We want to solve the Einstein equations in *D* dimensions with a negative cosmological constant to find a *stationary* space-time (*M*,*g*):

$$R_{ab} + \frac{(D-1)}{\ell^2} g_{ab} = 0$$

- Dynamical equations
- Constraints

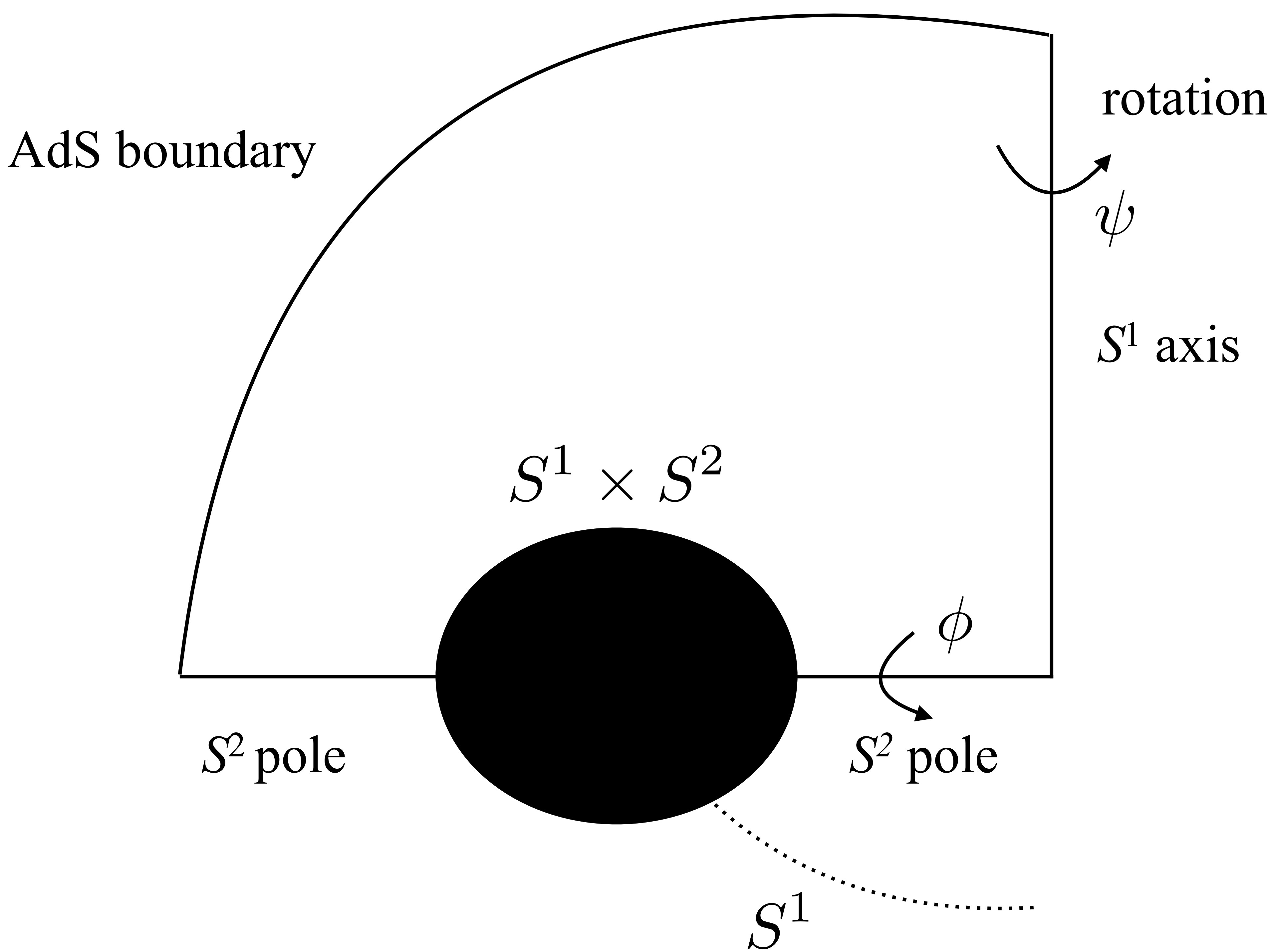
$\Rightarrow$ No definite character

Modern approach: covariant gauge fixing [Choquet-Bruhat; Garfinkle; Pretorius; Headrick, Kitchen and Wiseman]

➡ Solve the "harmonic" (or "DeTurck") Einstein equations

$$R^H_{ab} = R_{ab} - \nabla_{(a}\xi_{b)} + \frac{(D-1)}{\ell^2} g_{ab} = 0 \qquad \xi^a = g^{bc}(\Gamma^a{}_{bc} - \bar{\Gamma}^a{}_{bc})$$

AdS black rings: setup



AdS black rings: construction

Divide (arbitrarily) the geometry into two regions:

- Far region: asymptotic region far from the black ring
  $\Rightarrow \partial_t + \Omega_H\, \partial_\psi$ does not become null anywhere
- Near region: neighbourhood of the black ring horizon
  $\Rightarrow \partial_t + \Omega_H\, \partial_\psi$ becomes null at the horizon

Choose coordinates adapted to each of the two regions:

- Far region: adapted coordinates to global AdS

- Near region: adapted coordinates to the near horizon region of a black ring

Transfer information between the two patches in the overlapping region:

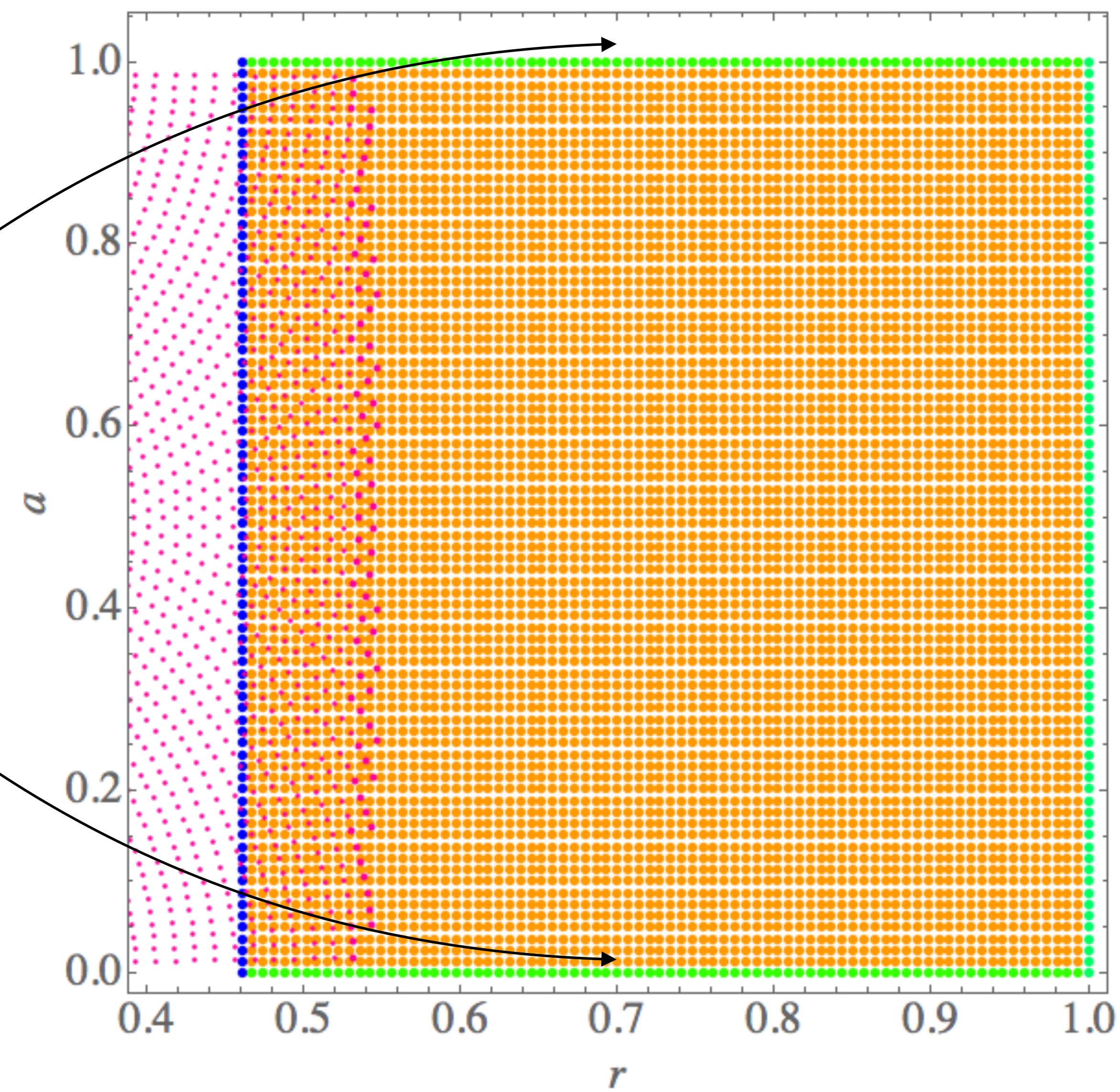
- The values of the functions in one patch are obtained by interpolation

AdS black rings: construction

Far region:
near AdS boundary

$$ds^2 = -\left(1+\frac{r^2}{\ell^2}\right) e^T\, dt^2 + \frac{e^A}{1+\frac{r^2}{\ell^2}}\,(dr - F\,da)^2$$
$$+\, r^2\left[\frac{\pi^2}{4}\, e^B\, da^2 + \cos^2(\tfrac{\pi\, a}{2})\, e^R\, d\phi^2 + \sin^2(\tfrac{\pi\, a}{2})\, e^S\,(d\psi - W_0\, W\, dt)^2\right]$$

pole of S^3

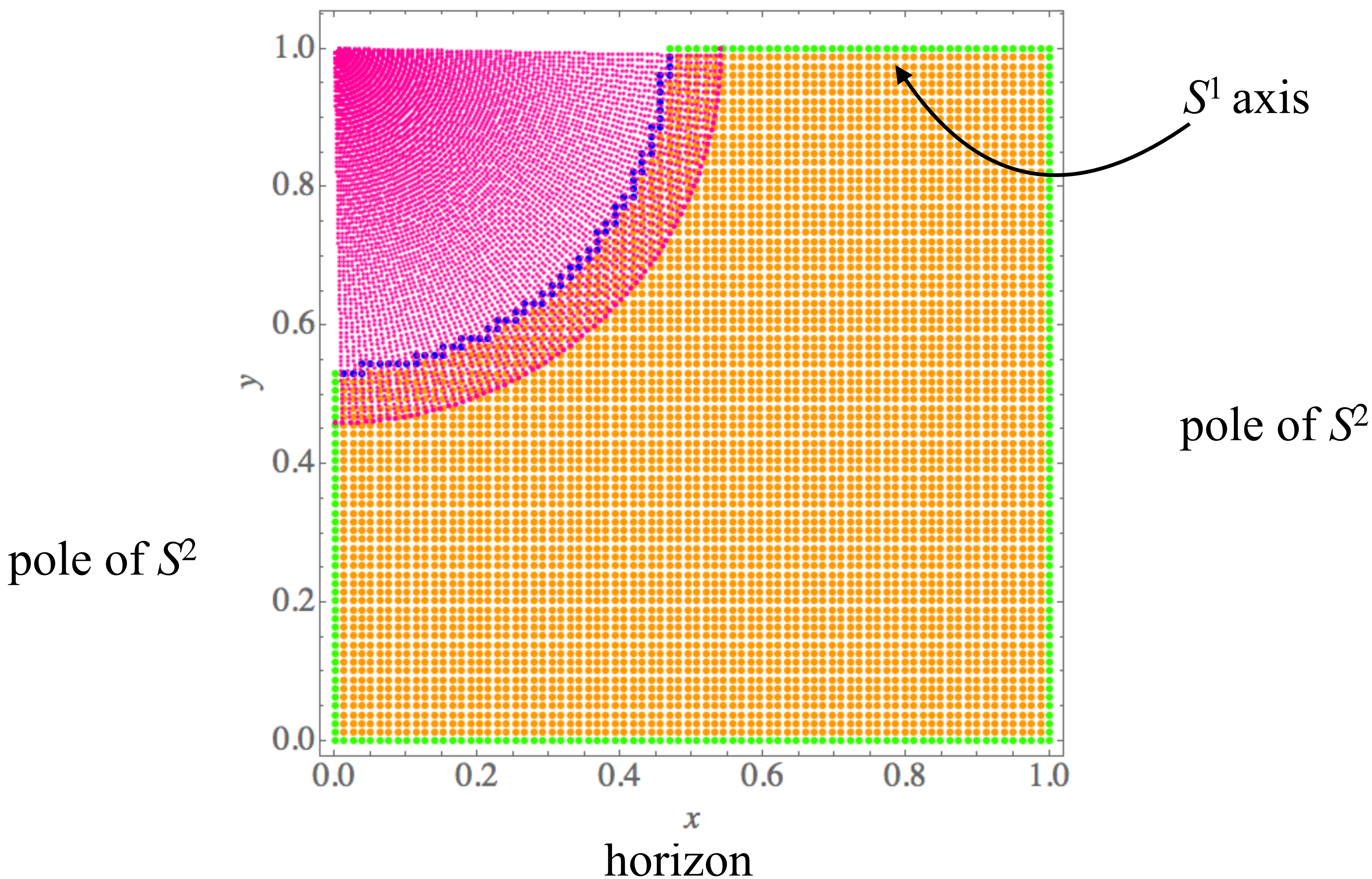


AdS boundary

$$r \to \frac{r}{1-r^2}$$

AdS black rings: construction

Near horizon region: $$ds^2 = -\,T_0\,e^{T'}\,dt^2 + S_0\,e^{S'}(d\psi - W_0\,W'\,dt)^2 + R_0\,e^{R'}\,d\phi^2 + A_0\,e^{A'}\,dx^2 + B_0\,e^{B'}(dy - F'\,dx)^2$$



AdS black rings: construction

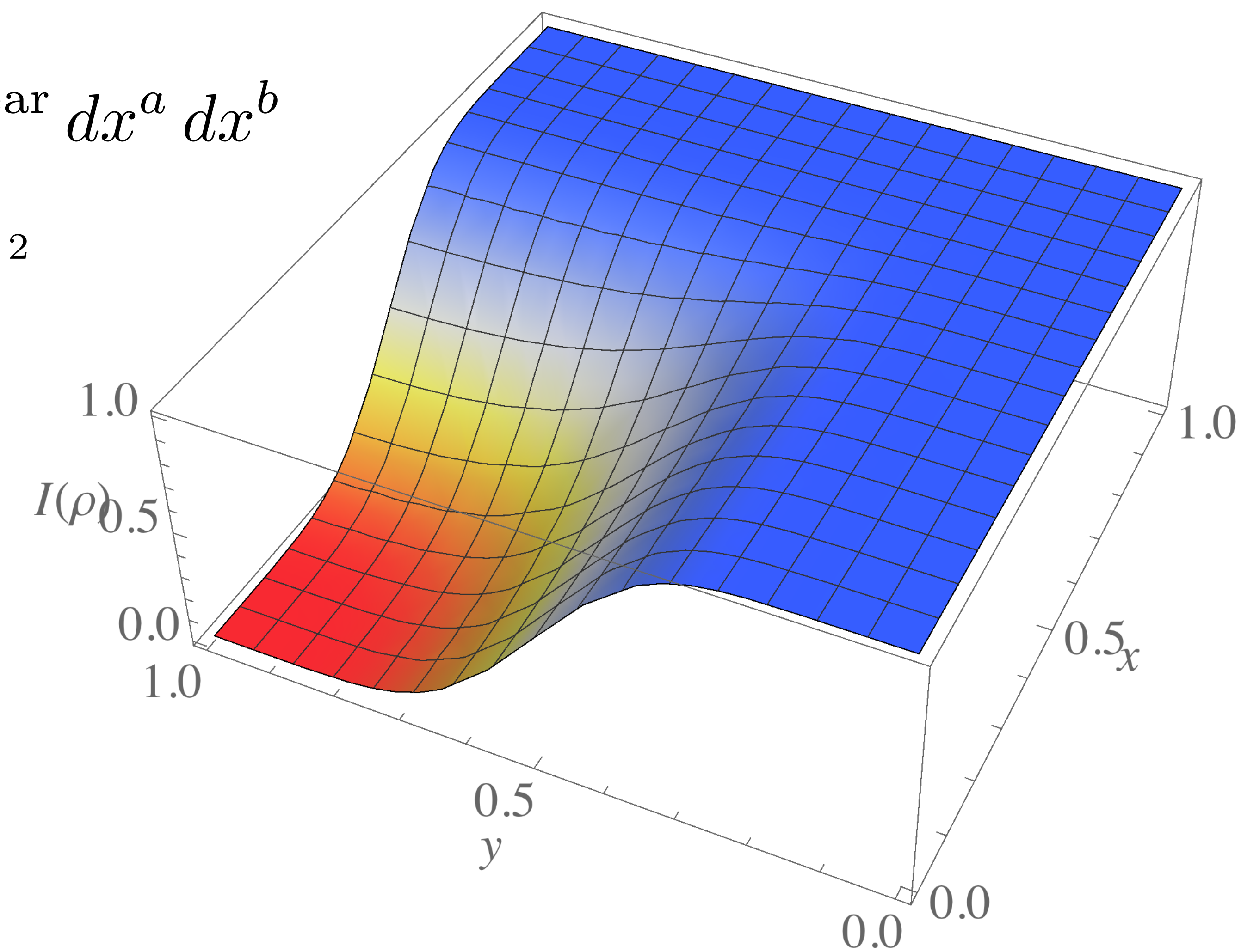
Coordinate transformation:

$$x = (1-r)\cos\left(\tfrac{\pi\, a}{2}\right)\ , \quad y = (1-r)\sin\left(\tfrac{\pi\, a}{2}\right)$$

Reference metric:

$$d\bar{s}^2 = (1 - I(\rho))\, g^{\text{Far}}_{ab}\, dx^a\, dx^b + I(\rho)\, g^{\text{Near}}_{ab}\, dx^a\, dx^b$$

$$I(\rho) = 1 - \sin^8(\tfrac{\pi\,\rho}{2})\left(6 - 8\,\sin^2(\tfrac{\pi\,\rho}{2}) + 3\,\sin^4(\tfrac{\pi\,\rho}{2})\right)^2$$



AdS black rings: construction

Parameters in our solutions: $T_H\,,\,\Omega_H\,,\,\ell$

Move along the branch of solutions by fixing $T_H\,\ell$ and vary $\Omega_H\,\ell$

Solve the equations numerically using 4th order or 6th order finite differences

3. Physics of AdS black rings

Geometry of AdS black rings

Geometry of AdS black rings

Induced metric on the horizon:

$$\underbrace{ds_H^2 = R_{||}(x)^2\, d\psi^2}_{S^1} + \underbrace{R_\perp(x)^2\, d\phi^2 + A_0(x,0)\, e^{A(x,0)}\, dx^2}_{S^2}$$

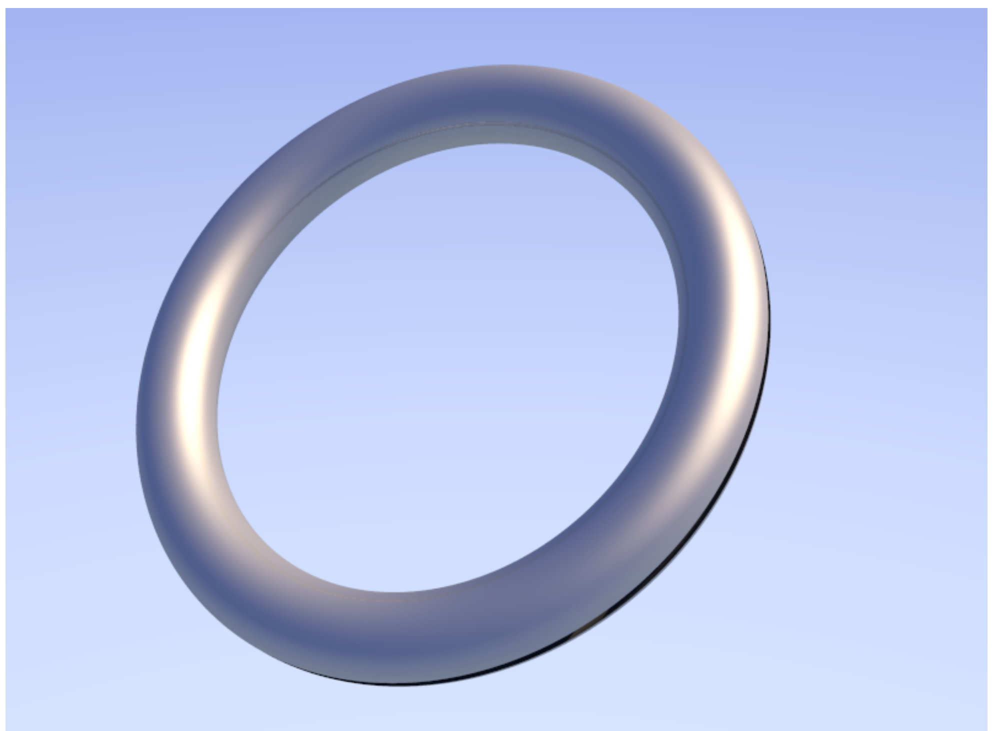
- Embeddings of the horizon S^2 into $\mathbb{E}_3$

$$ds_H^2 = A_0(x,0)\, e^{A(x,0)}\, dx^2 + R_\perp(x)^2\, d\phi^2 \quad \hookrightarrow \quad ds_{\mathbb{E}_3}^2 = du^2 + d\rho^2 + \rho^2\, d\phi^2$$

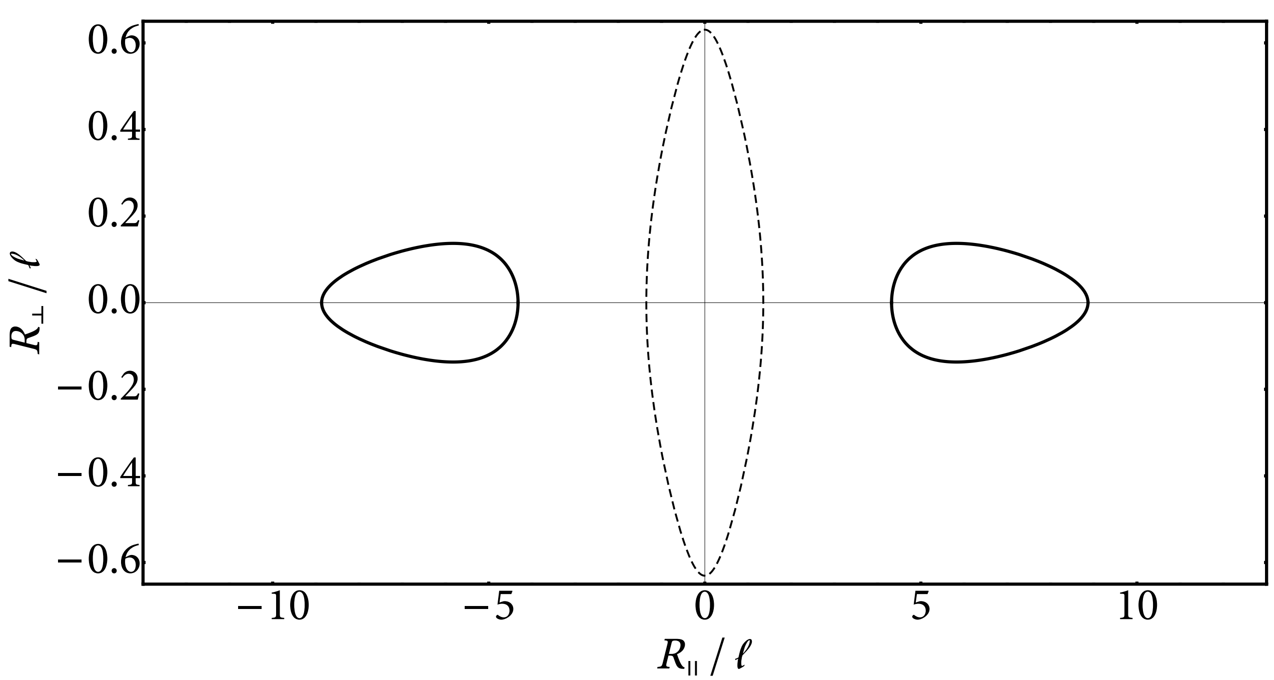
- Plots of the invariant radii: $R_\perp(x)$ vs. $R_{||}(x)$

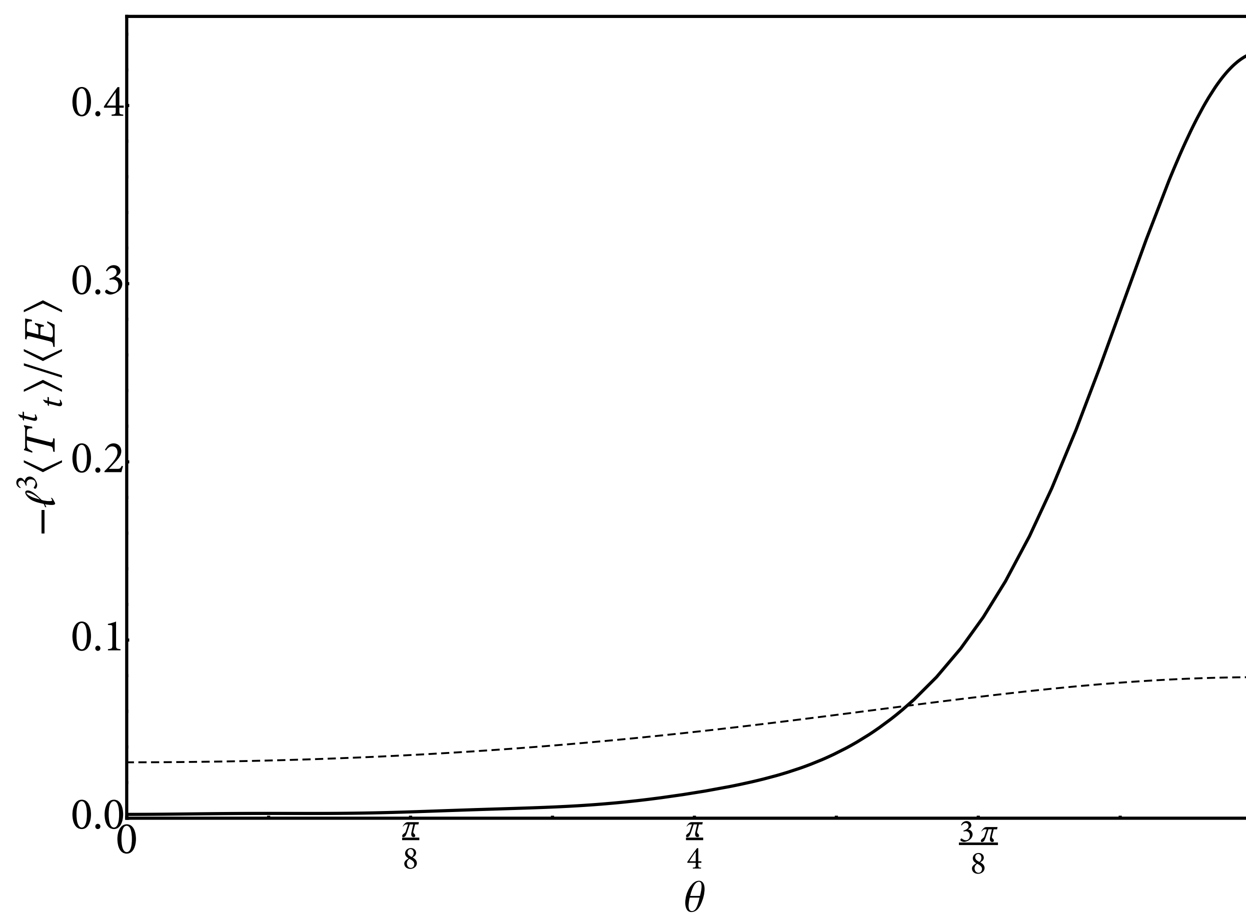
Geometry of AdS black rings

High temperature: thin rings



$$\kappa\,\ell = 2.0 \quad \Omega_H\,\ell = 1.07704$$

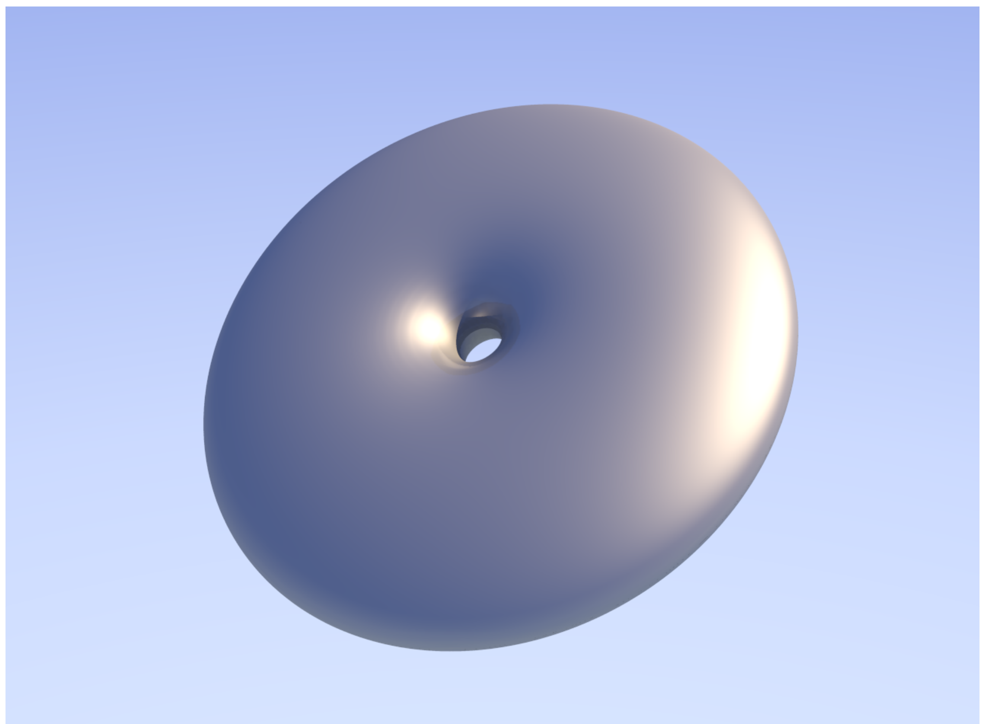


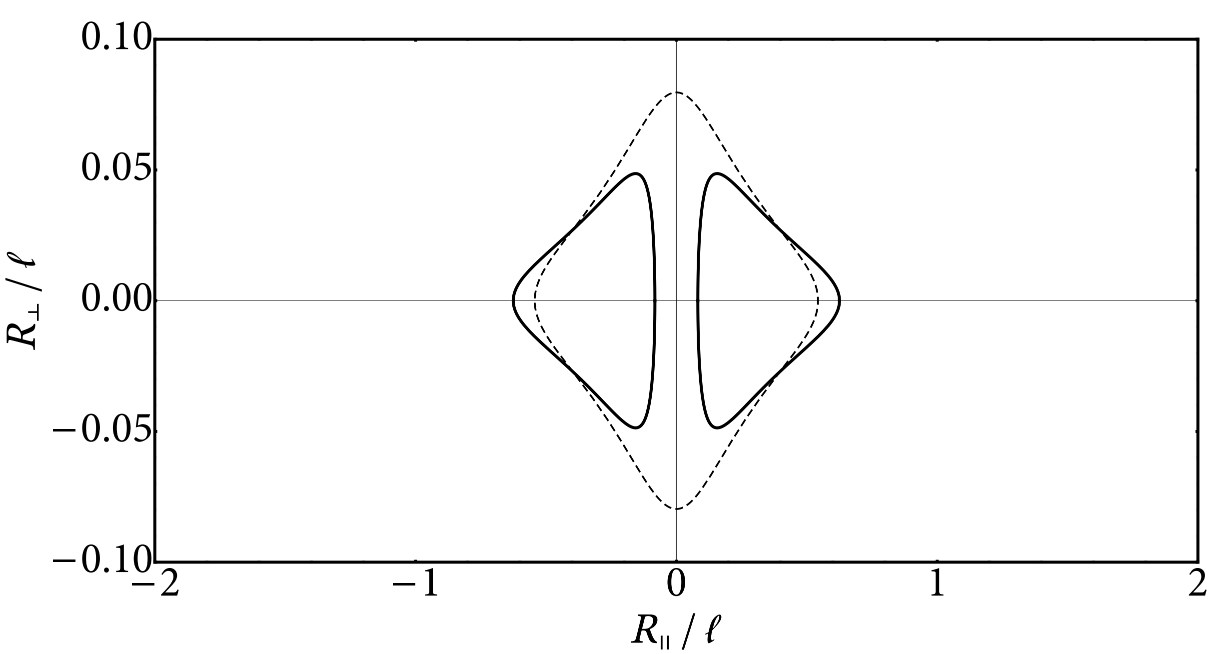


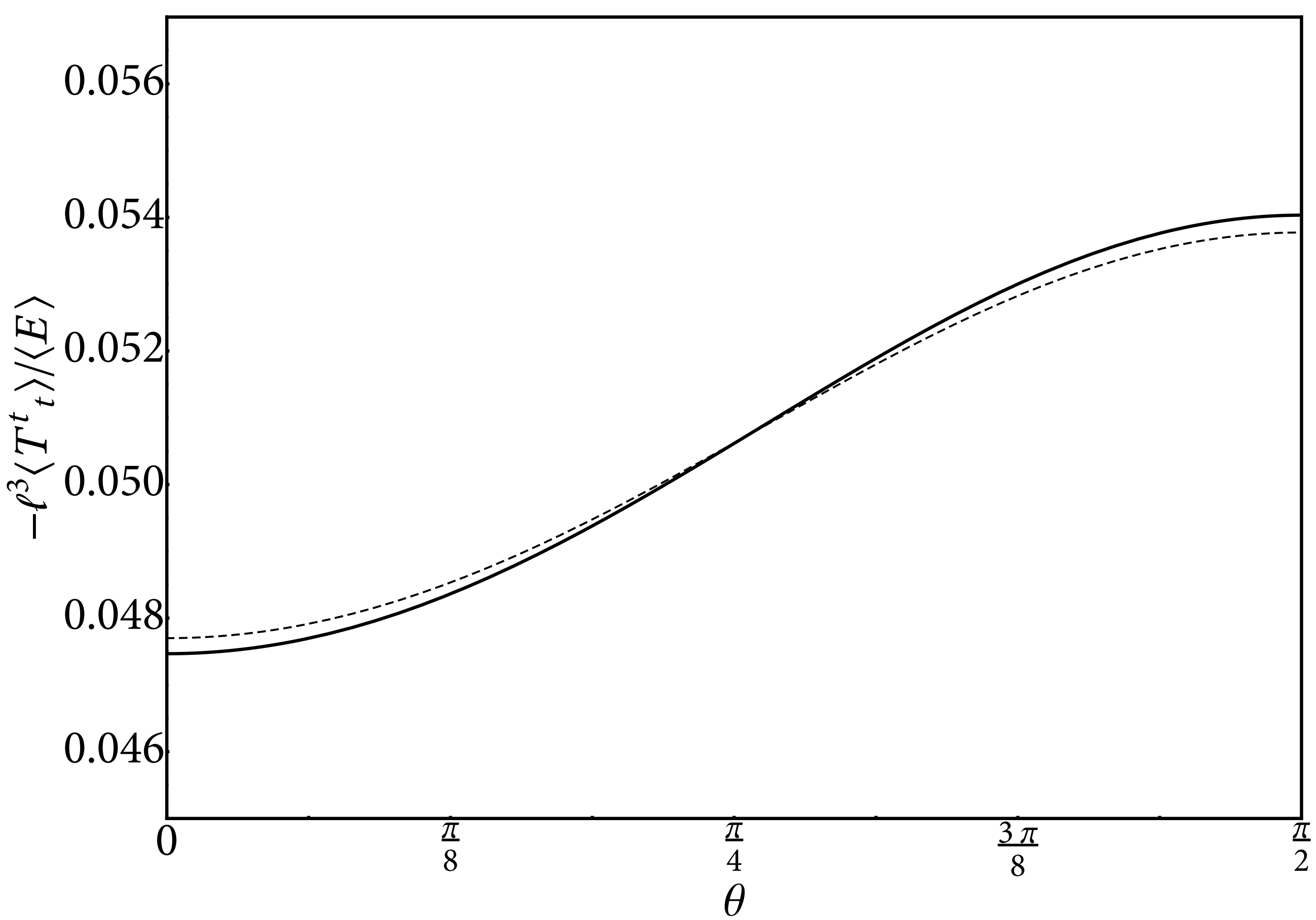
Geometry of AdS black rings

High or low temperature: fat rings

$$\kappa\,\ell = 2.0 \quad \Omega_H\,\ell = 4.53557$$

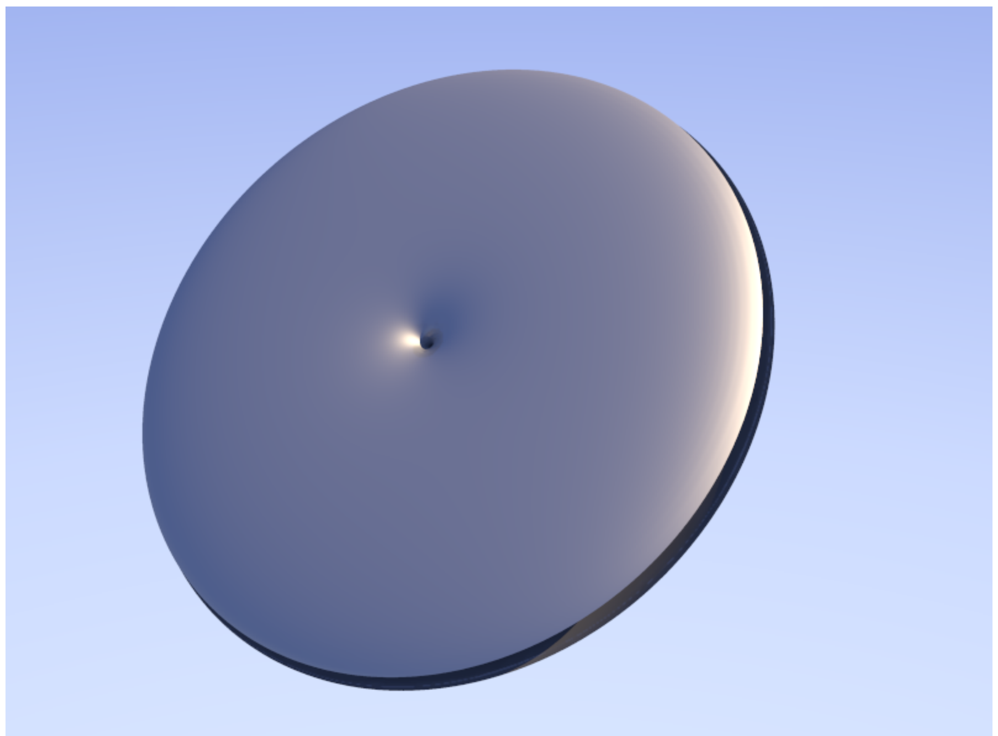




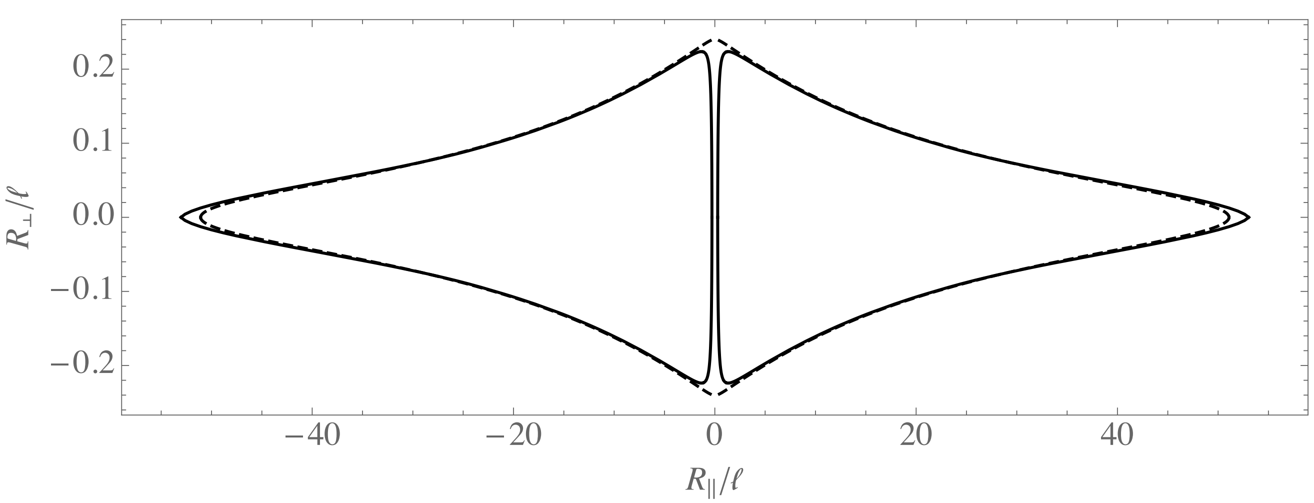


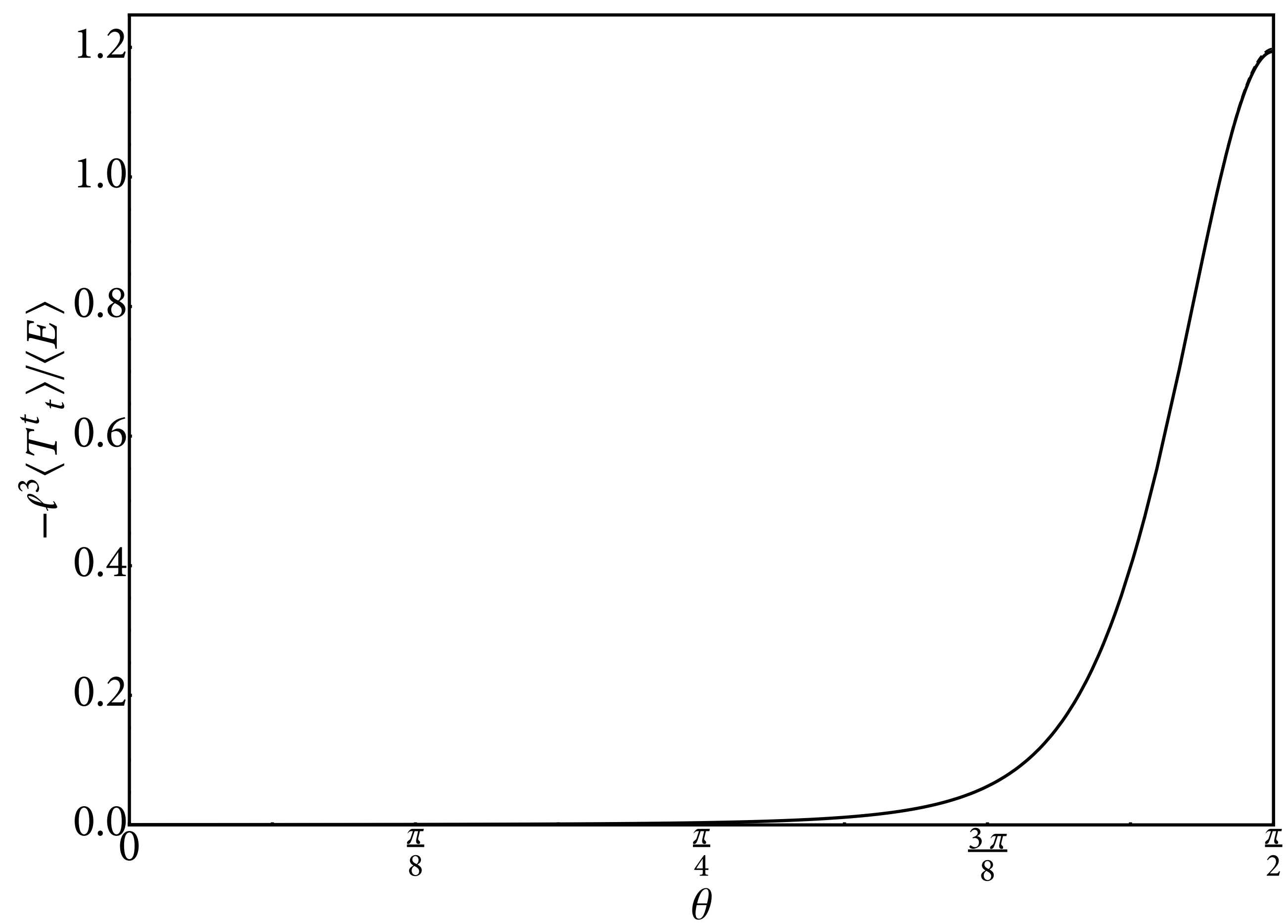
Geometry of AdS black rings

Low temperature: membrane rings



$$\kappa\,\ell = 0.5 \quad \Omega_H\,\ell = 1.03750$$





Thermodynamics: static case

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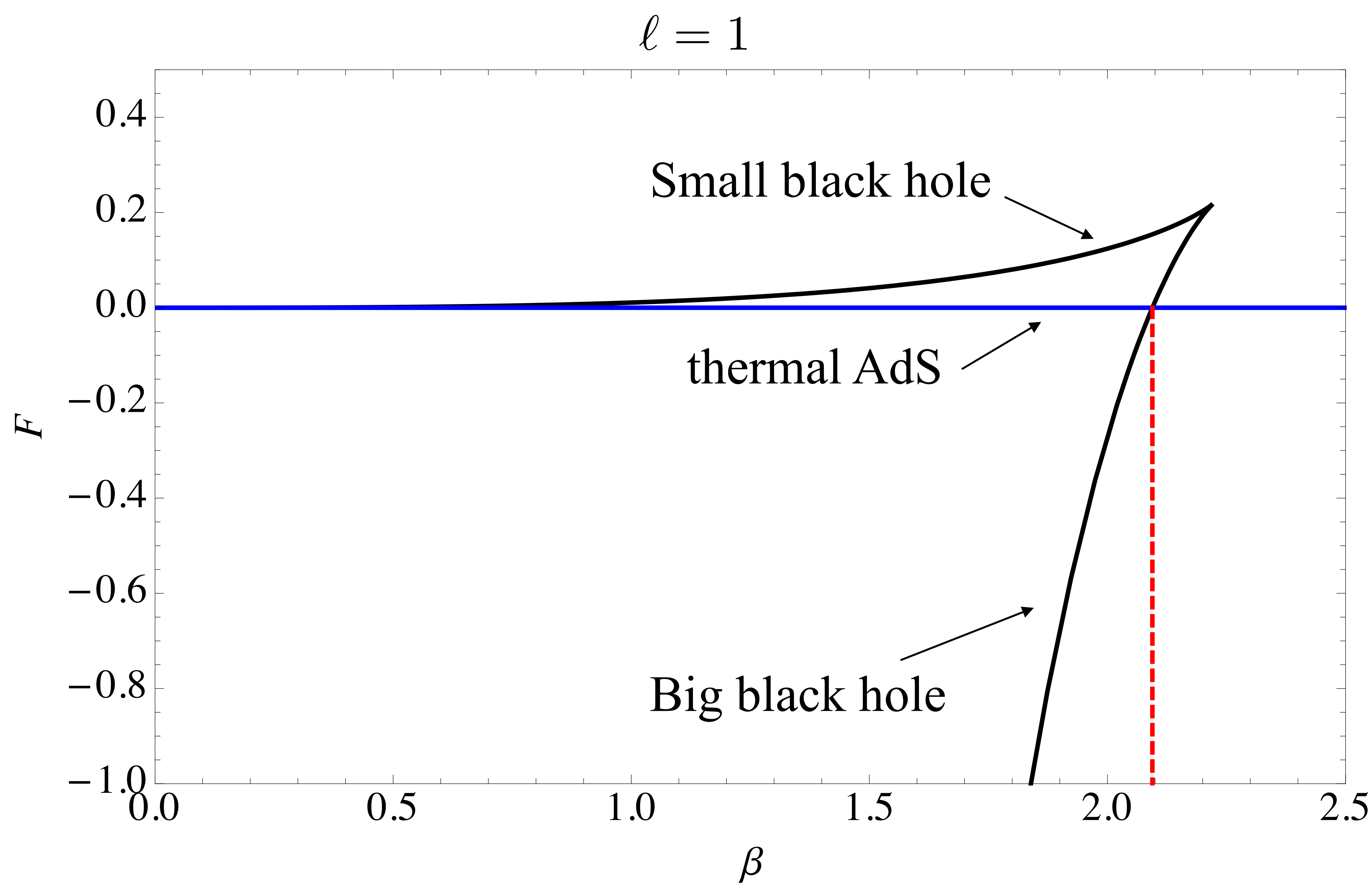


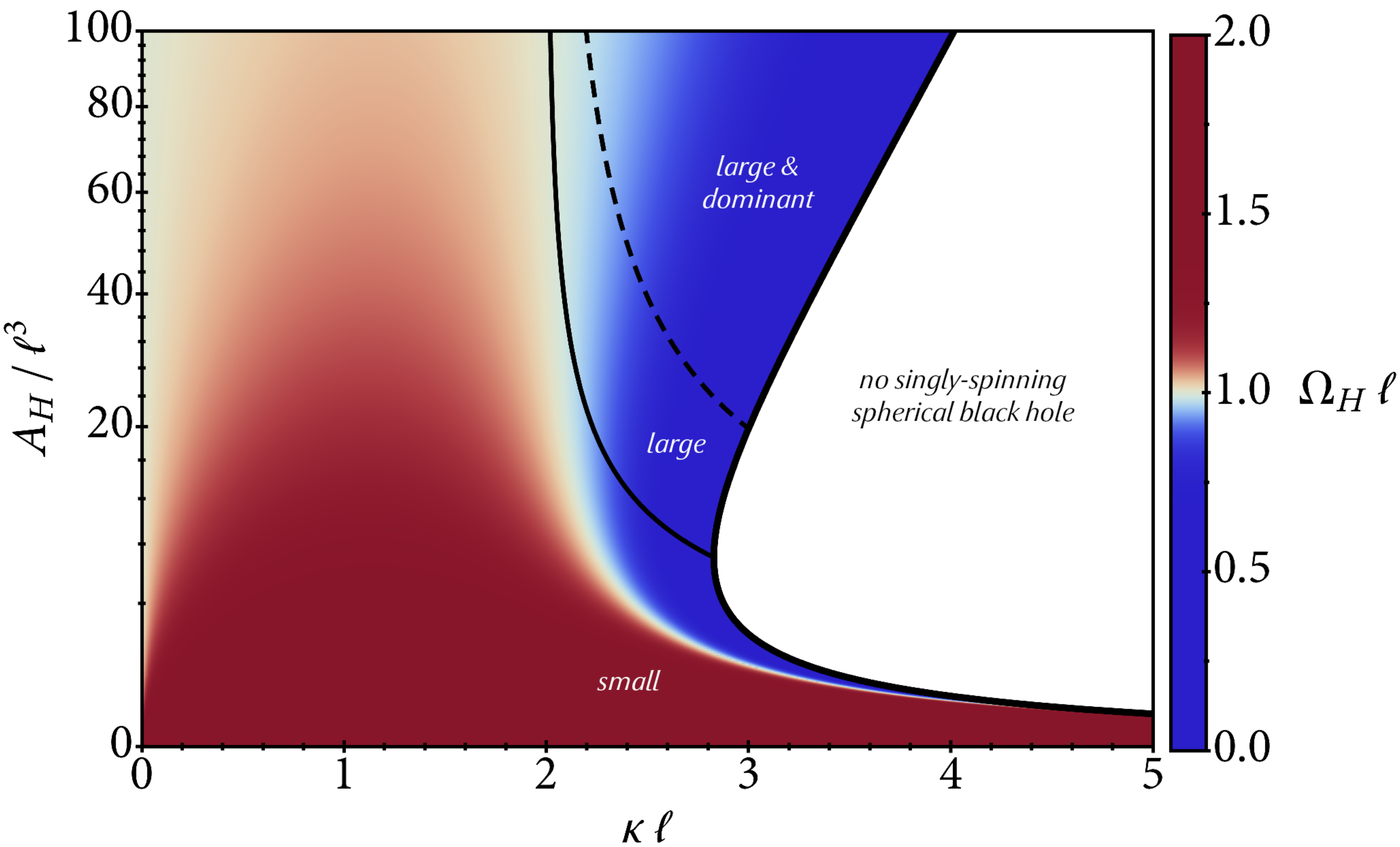
⇒ Phase transition when β ~ ℓ : Hawking-Page

⇒ Confinement/deconfinement: $F \sim O(1)$ for small T and $F \sim c_{\text{eff}} \sim O(N_c^2)$ for high T

[Witten]

Thermodynamics: rotating case

Grand canonical potential: $\Phi = E - T_H\, S - \Omega_H\, J$



Thermodynamics: where do rings sit?

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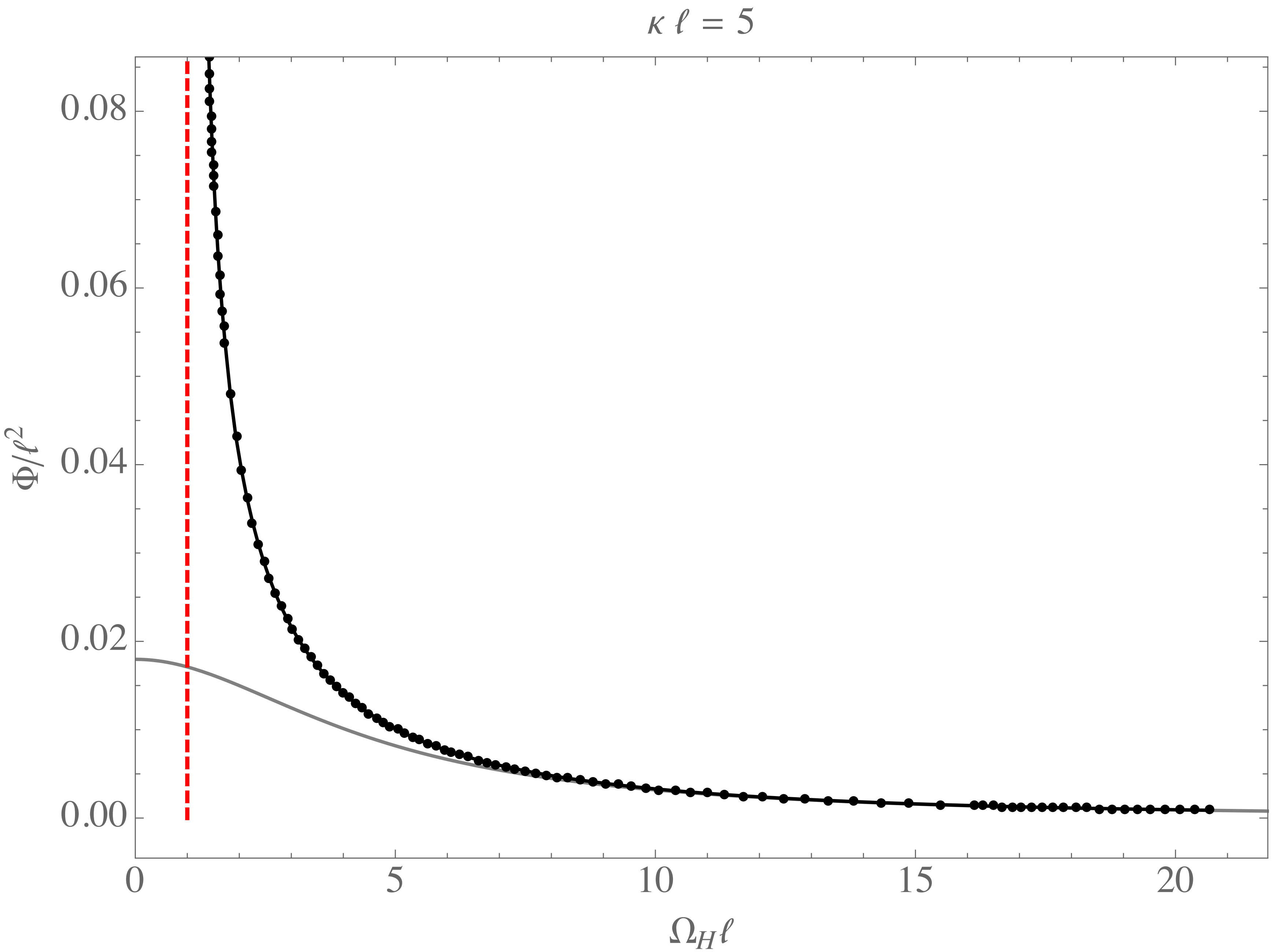


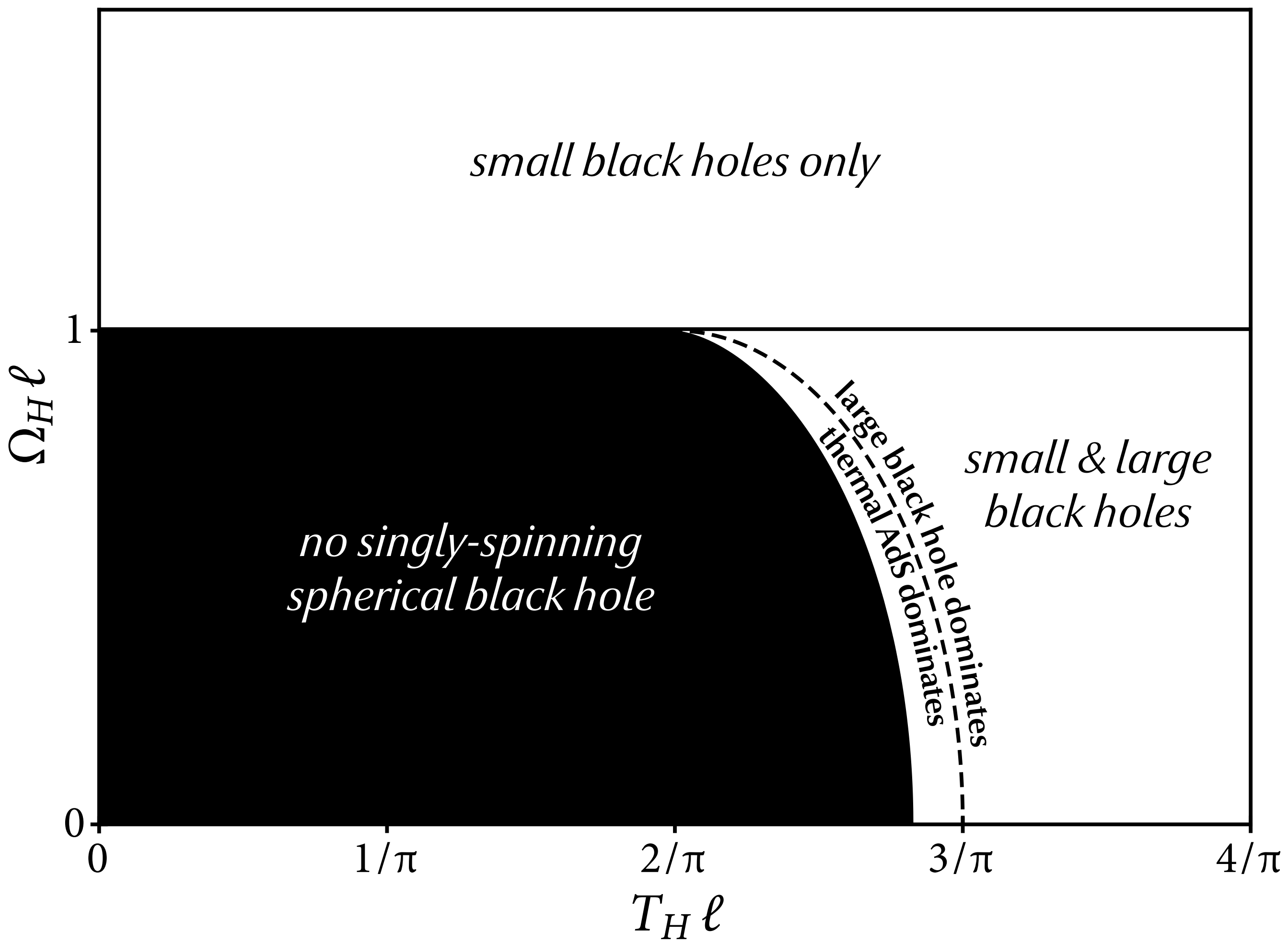
Thermodynamics: where do rings sit?

Thermodynamics: where do rings sit?

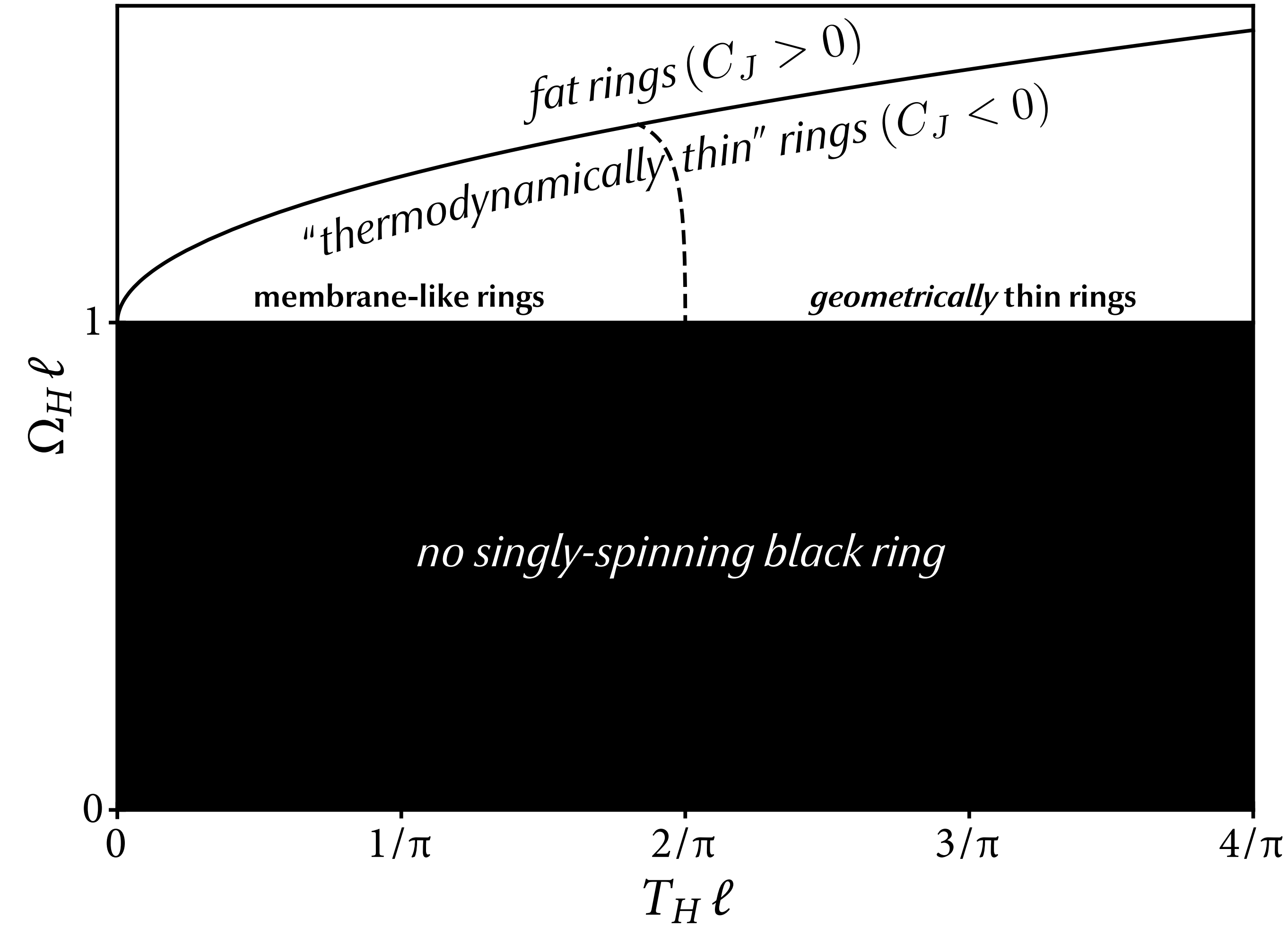
AdS black rings never dominate
the grand canonical ensemble!

Singly spinning black hole phases in AdS

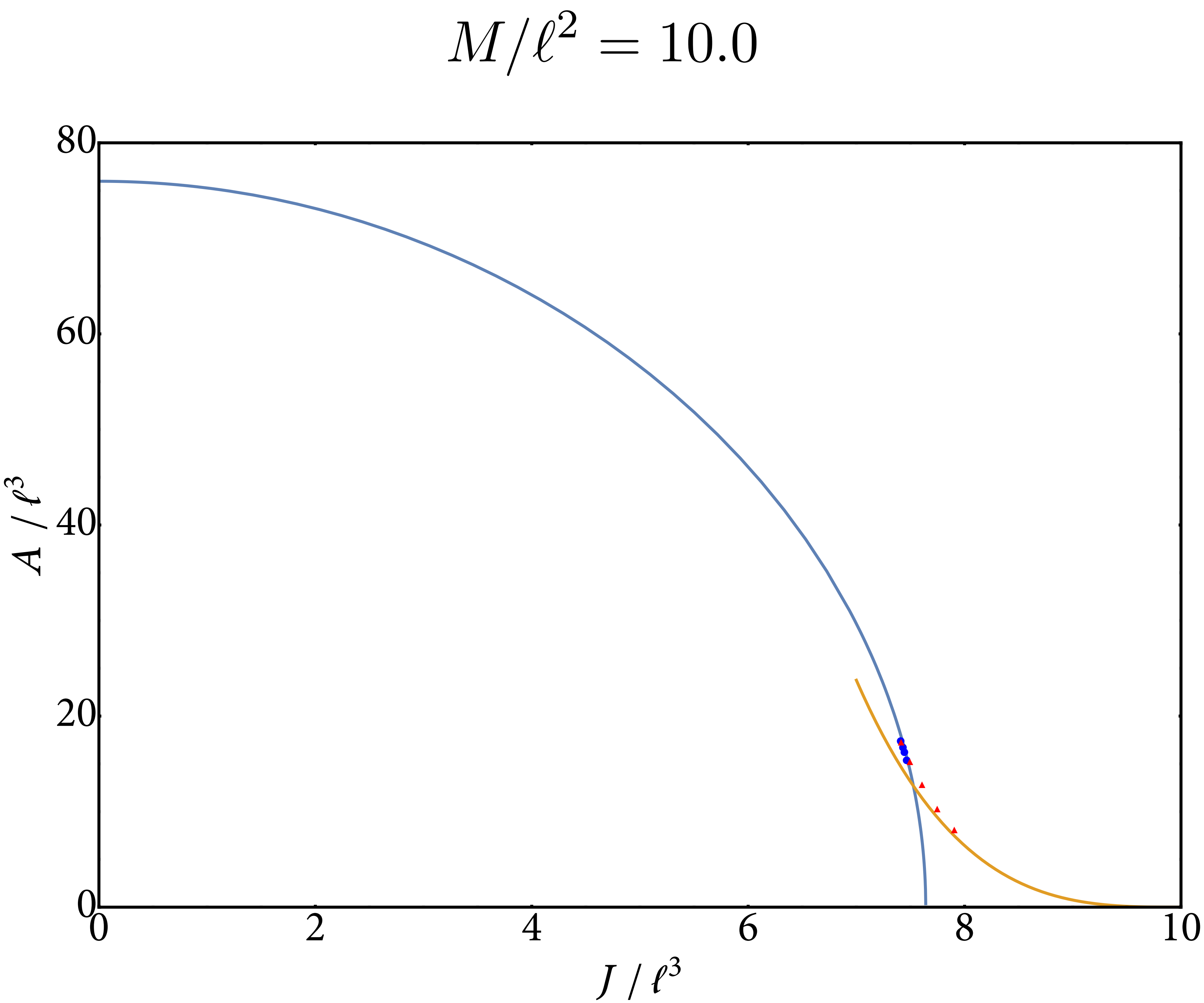
rotating black hole



black ring



AdS black ring phases



3. Summary and conclusions

Summary and conclusions

1. We have constructed black rings in global AdS
2. Black rings in AdS never dominate the grand canonical ensemble
3. At high temperatures, there can be thin and fat rings but at sufficiently low temperatures only fat and membrane rings exist
4. There are no large thin rings
5. Black rings are not describable in hydrodynamics

Summary and conclusions

1. We have constructed black rings in global AdS
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4. There are no large thin rings
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Are AdS black rings dynamically stable? No because $\Omega_H \, \ell > 1$

Thank you for your attention!!!