

Extension and Consolidation of Mathematical Packages for LHC Beam Optics and Geometry Calculations

A. M. DeMaio

BE-ABP-HSS

Steps...

- 1. Database**
- 2. Web availability**
- 3. Automation**

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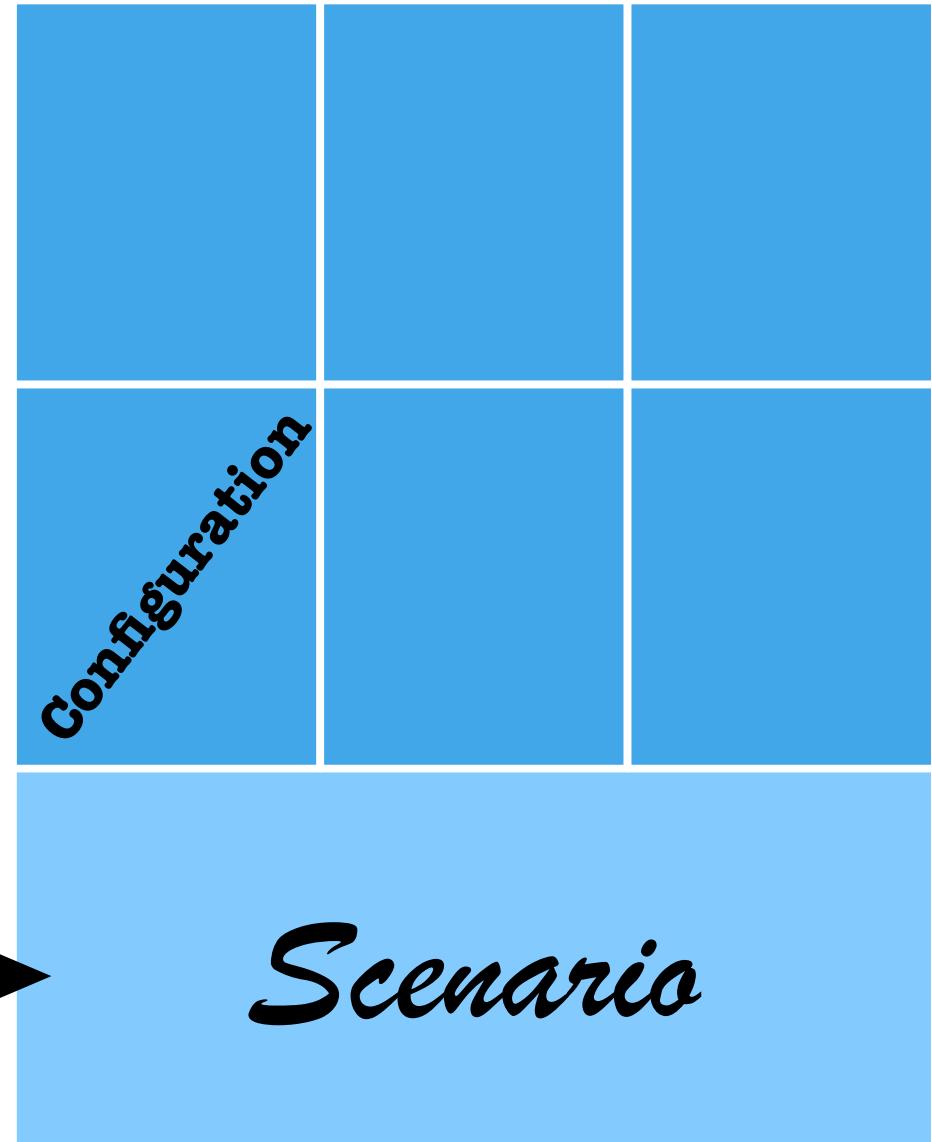
1. Database
2. Web availability
3. Automation



Scenario

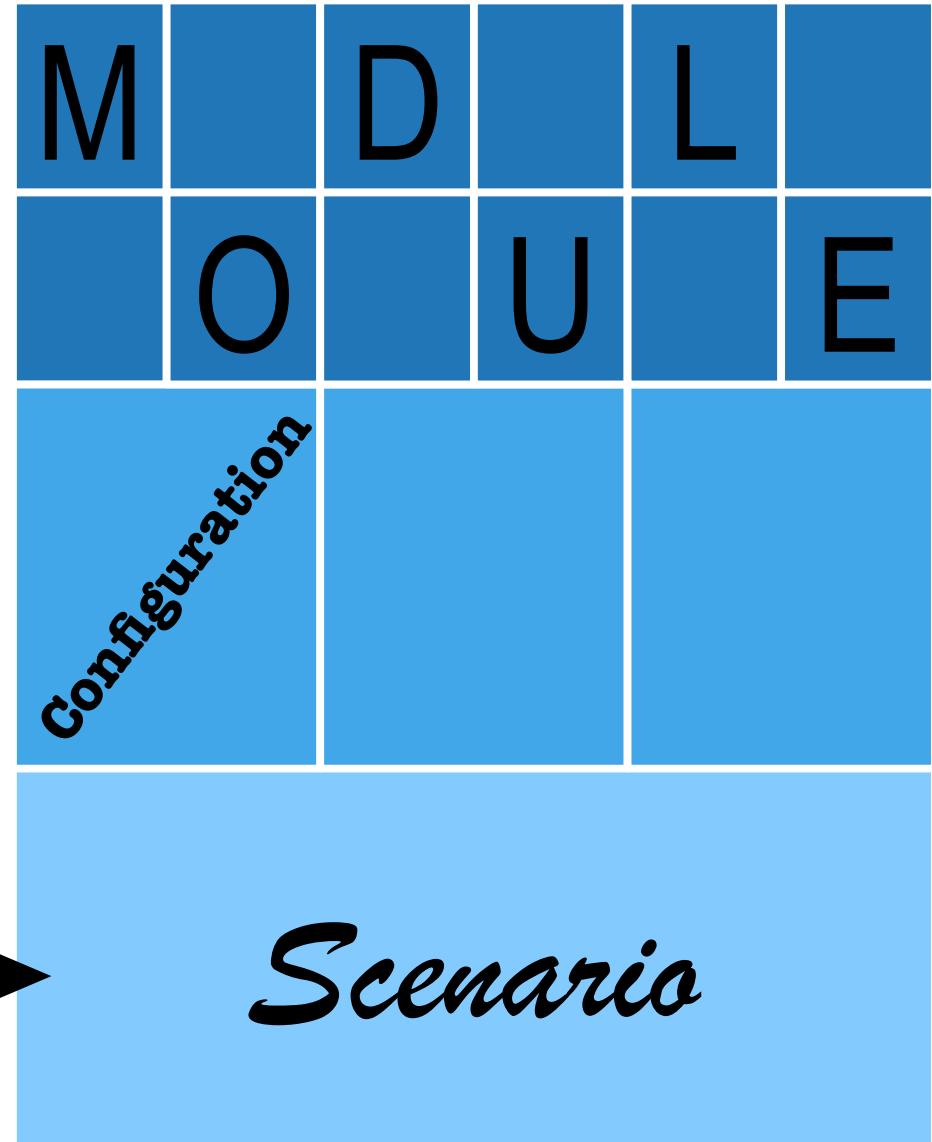
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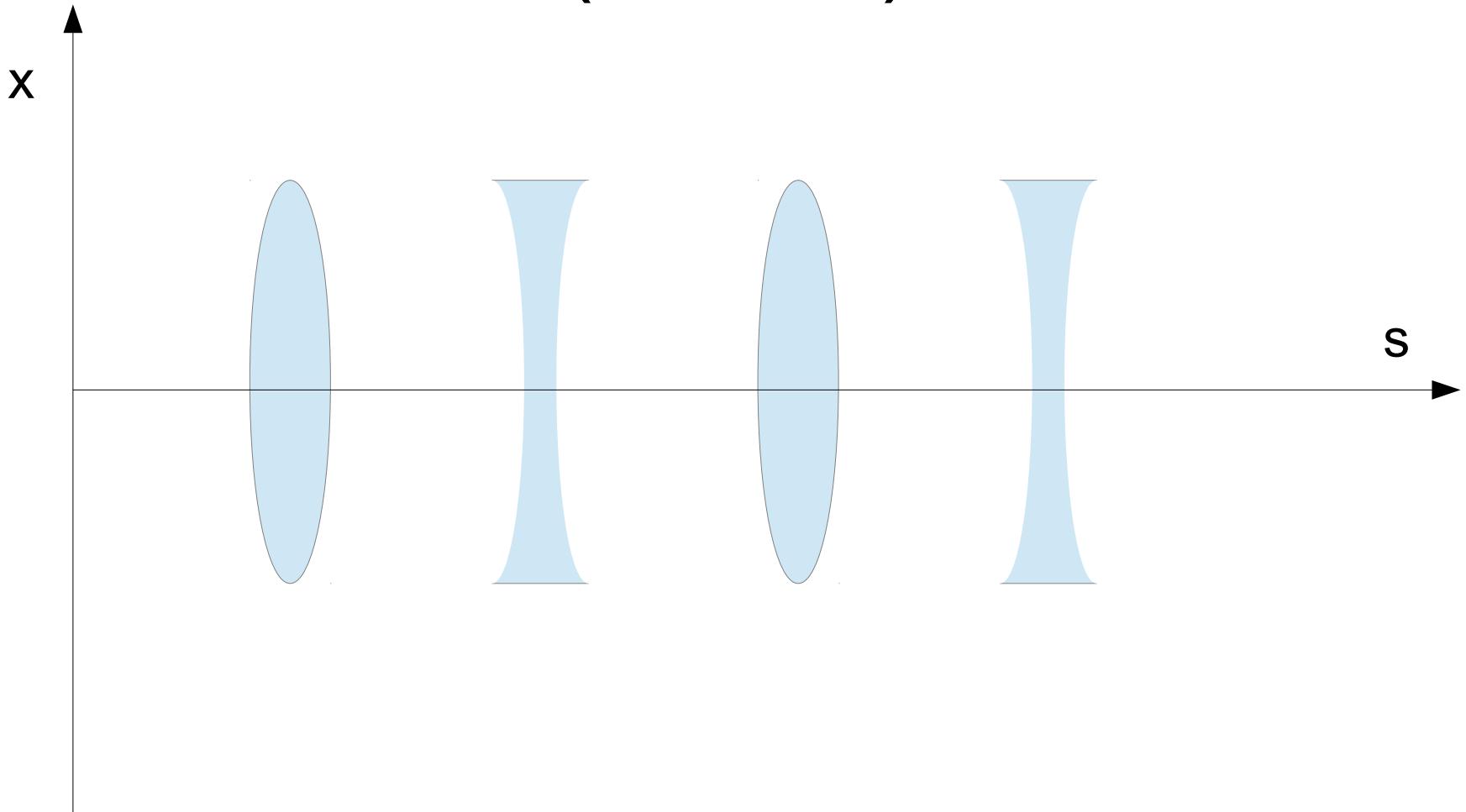


Beam optics: the study of particle paths

Lenses!

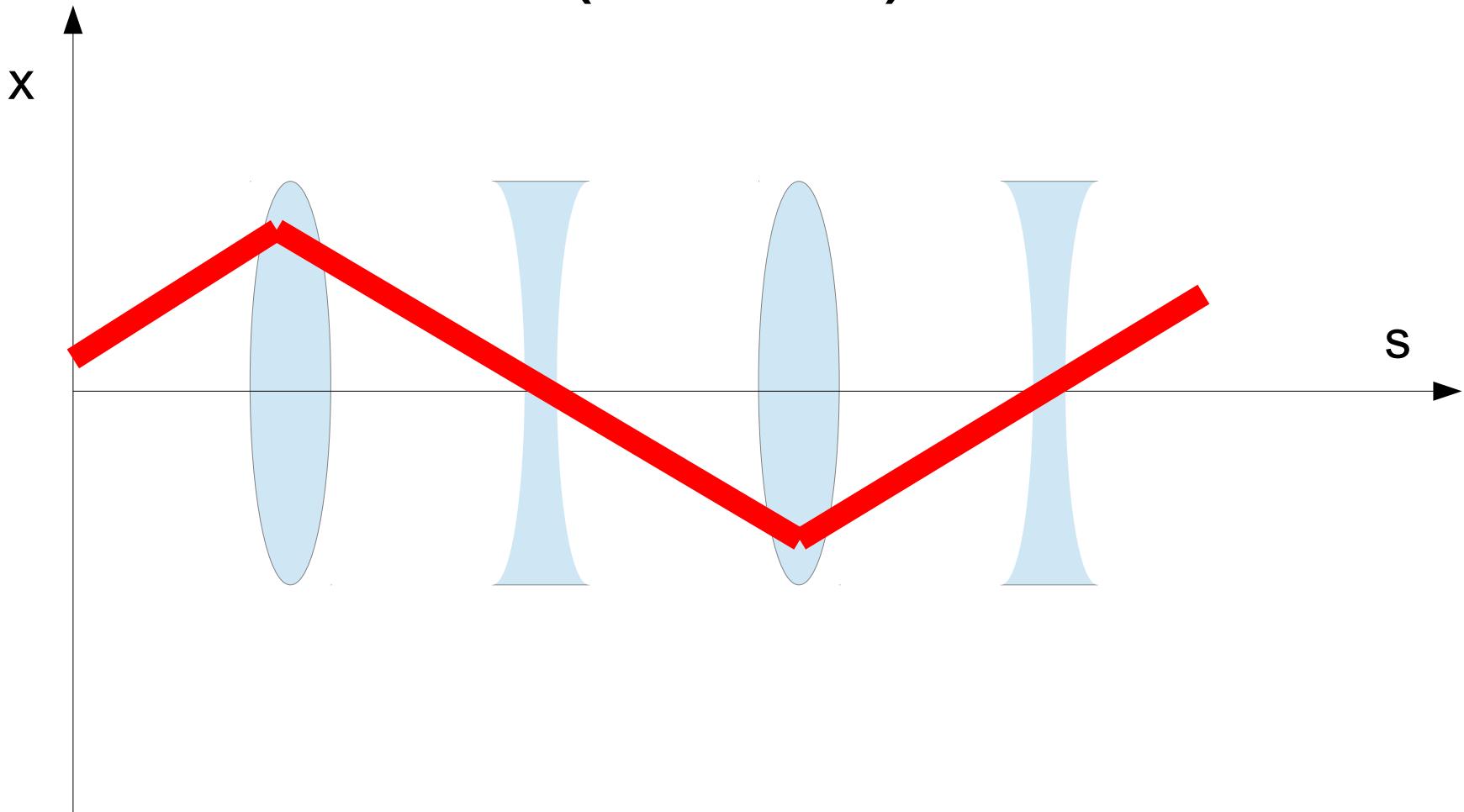
(quadrupole magnets)

Strong Focusing (FODO)



$$M = M_n \cdots M_2 M_1$$

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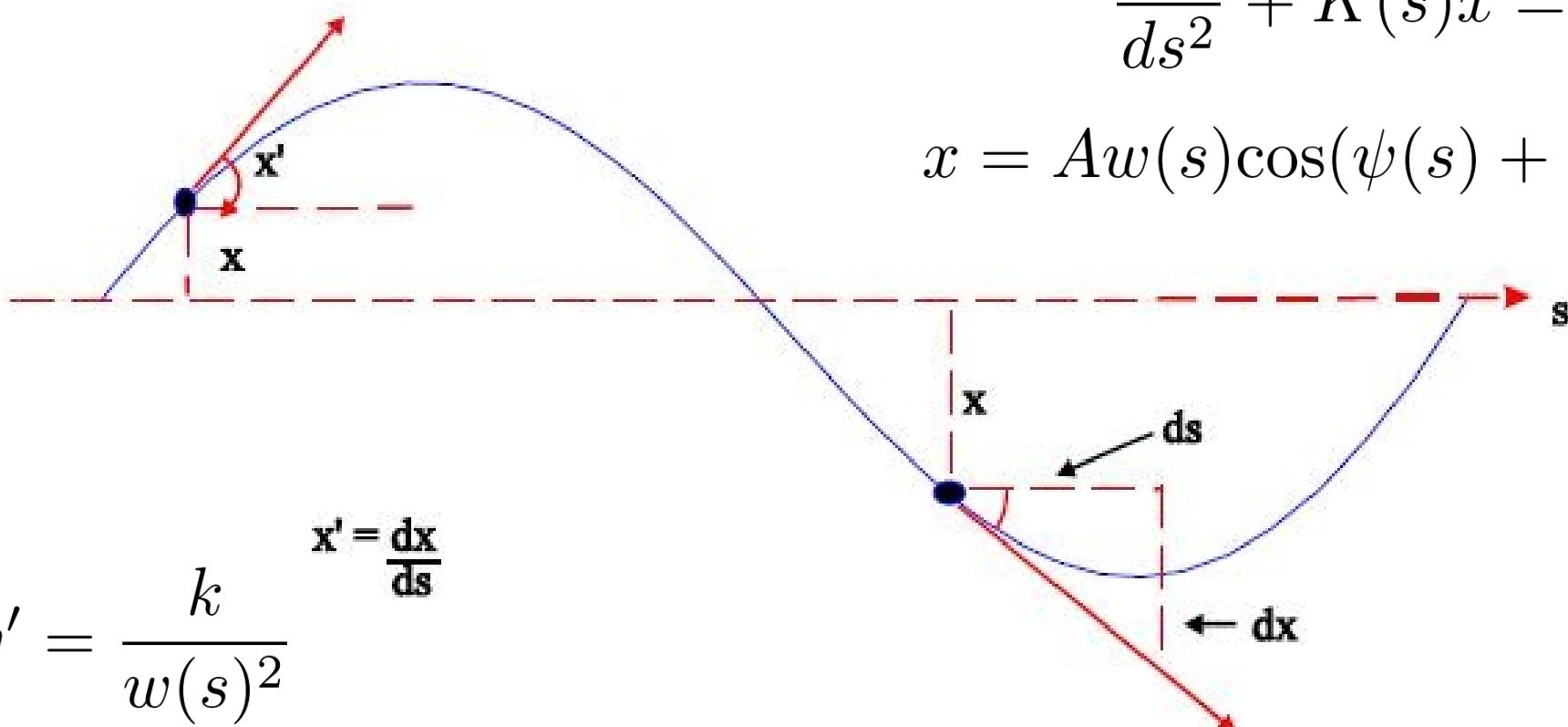
$$M = \begin{pmatrix} 1 - \frac{L}{f} - \left(\frac{L}{f}\right)^2 & 2L + \frac{L^2}{f} \\ -\frac{L}{f^2} & 1 + \frac{L}{f} \end{pmatrix} \quad \left| \frac{L}{2f} \right| \leq 1$$

Equations of Motion

$$\frac{d^2x}{ds^2} + Kx = 0$$

$$\frac{d^2x}{ds^2} + K(s)x = 0$$

$$x = Aw(s)\cos(\psi(s) + \phi)$$



$$\beta(s) \equiv \frac{w^2(s)}{k} \rightarrow \Delta\psi_C = \int_{s_0}^{s_0+C} \frac{ds}{\beta(s)}$$

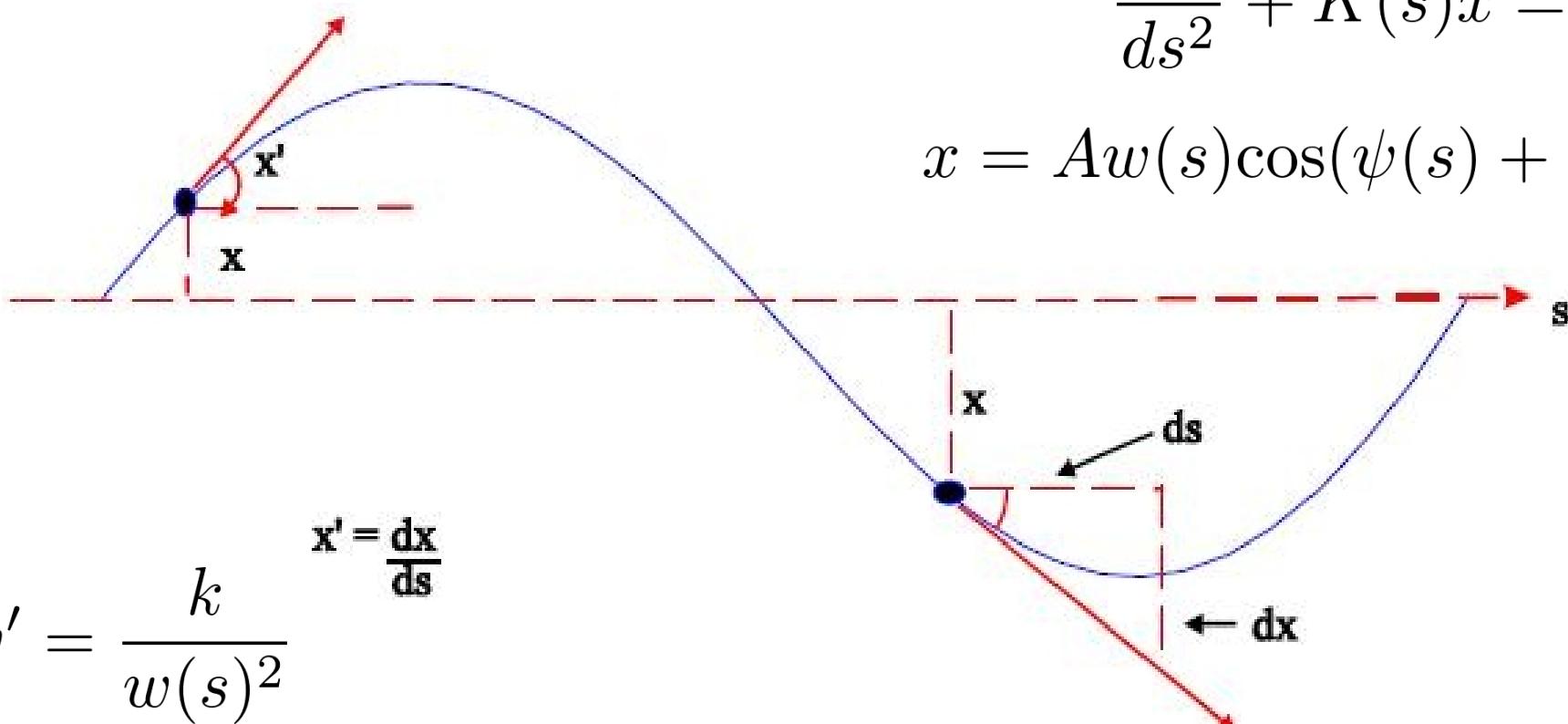
$$x = A\sqrt{\beta(s)}\cos(\psi(s) + \phi) \rightarrow x = \sqrt{\epsilon \cdot \beta(s)}\cos(\psi(s) + \phi)$$

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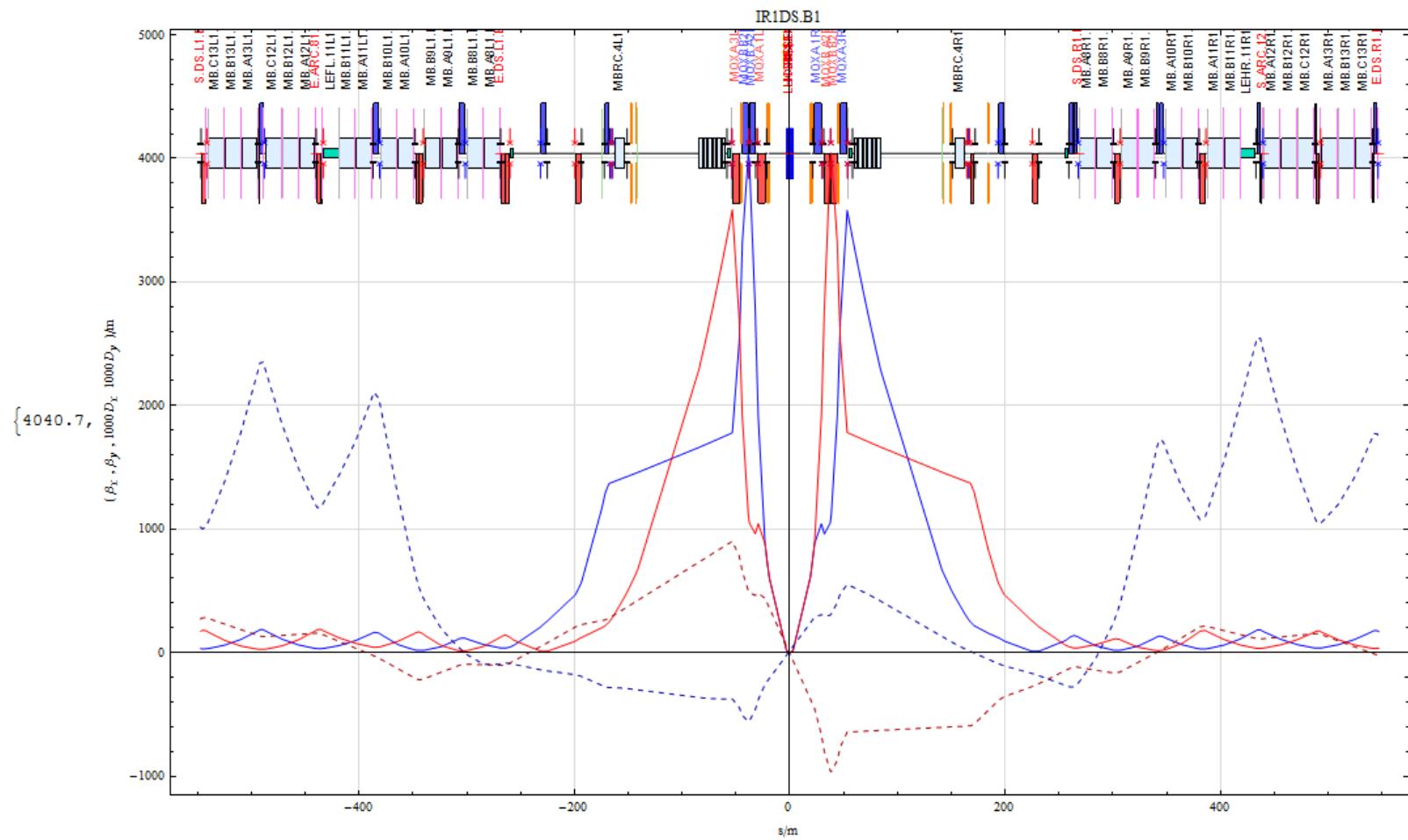
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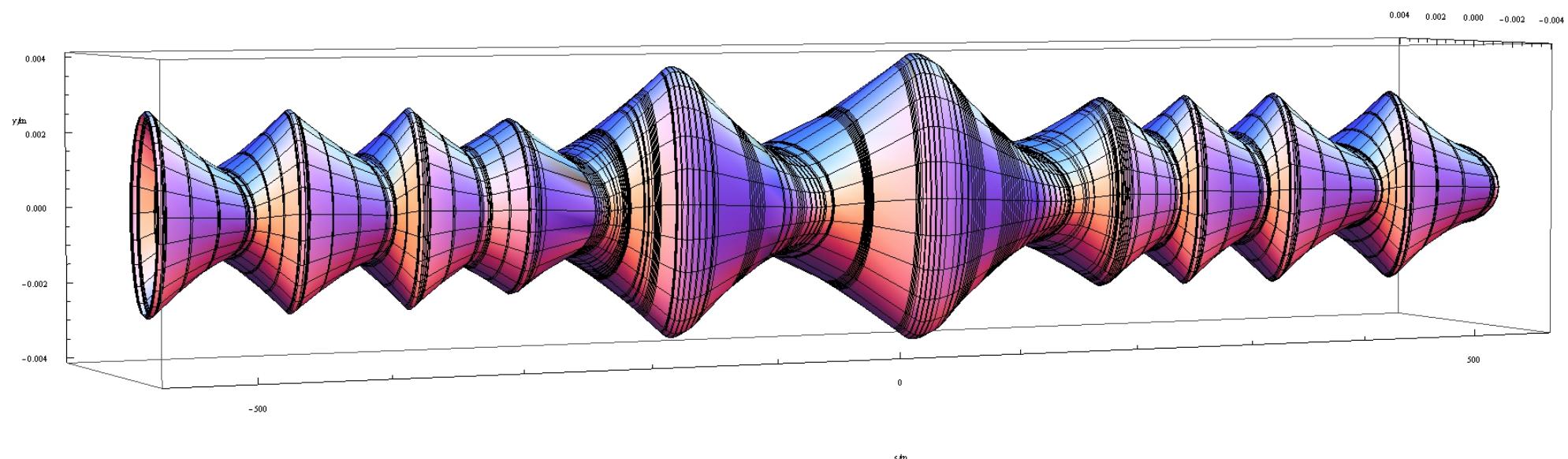
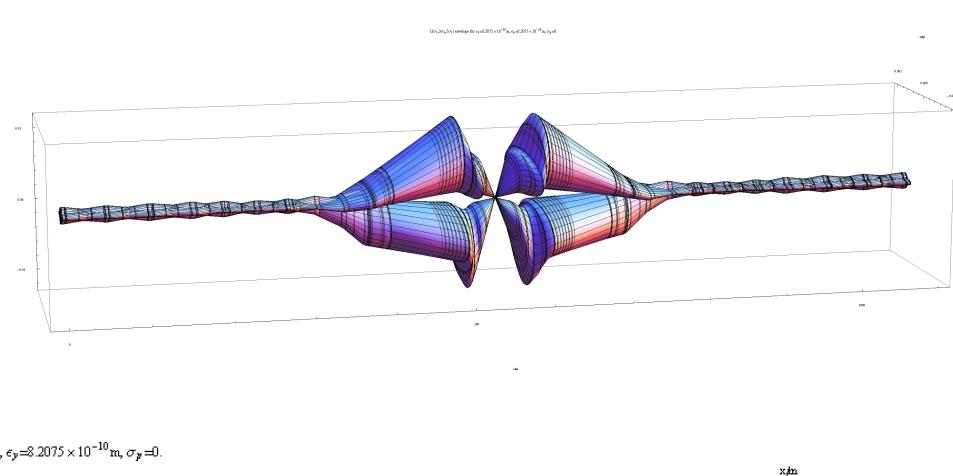
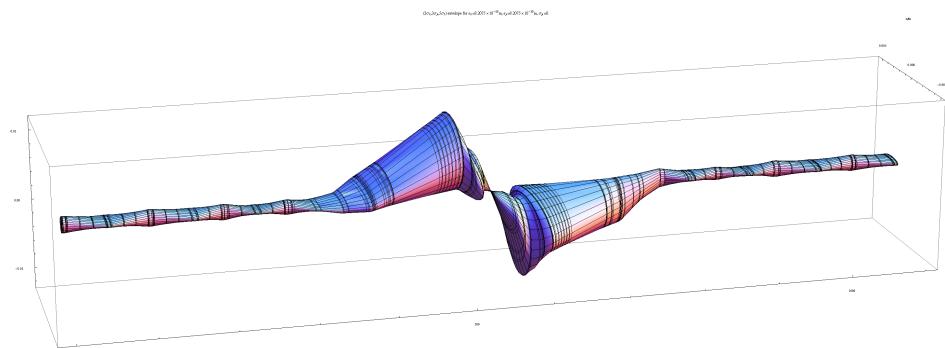
Momentum Dispersion

Chromatic aberration!

$$x = D(p, s) \frac{\Delta p}{p_0}$$

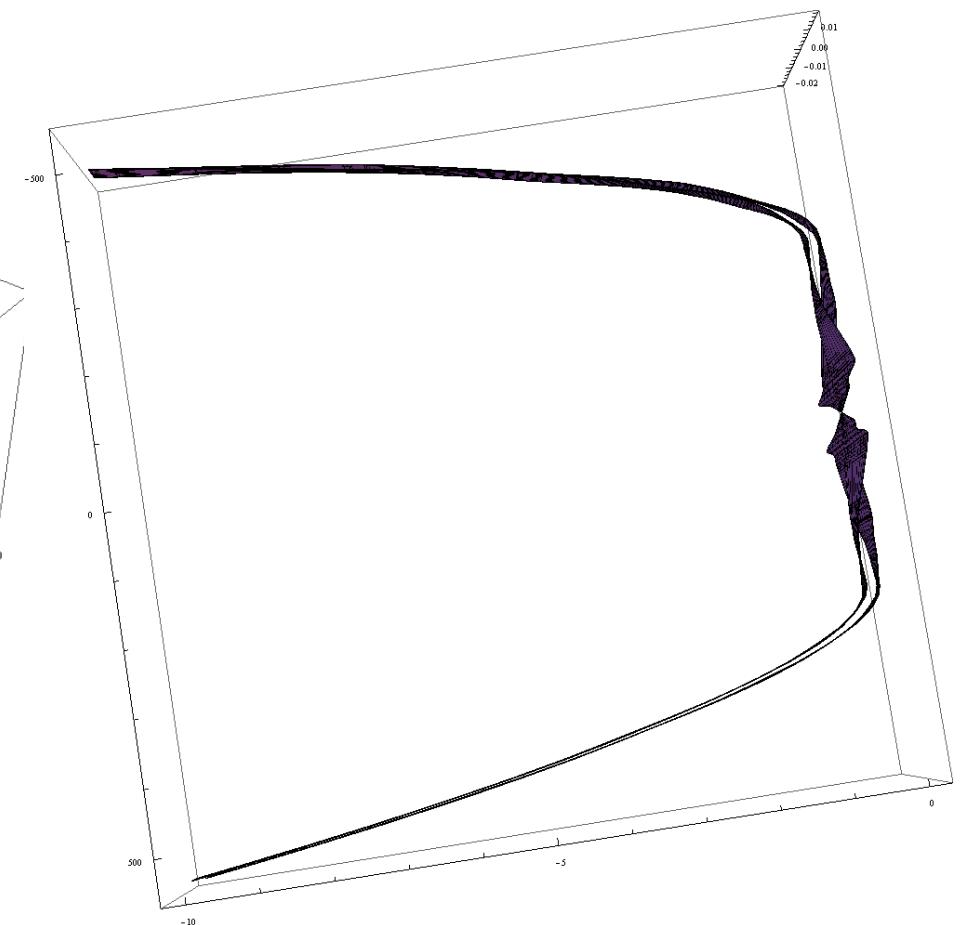
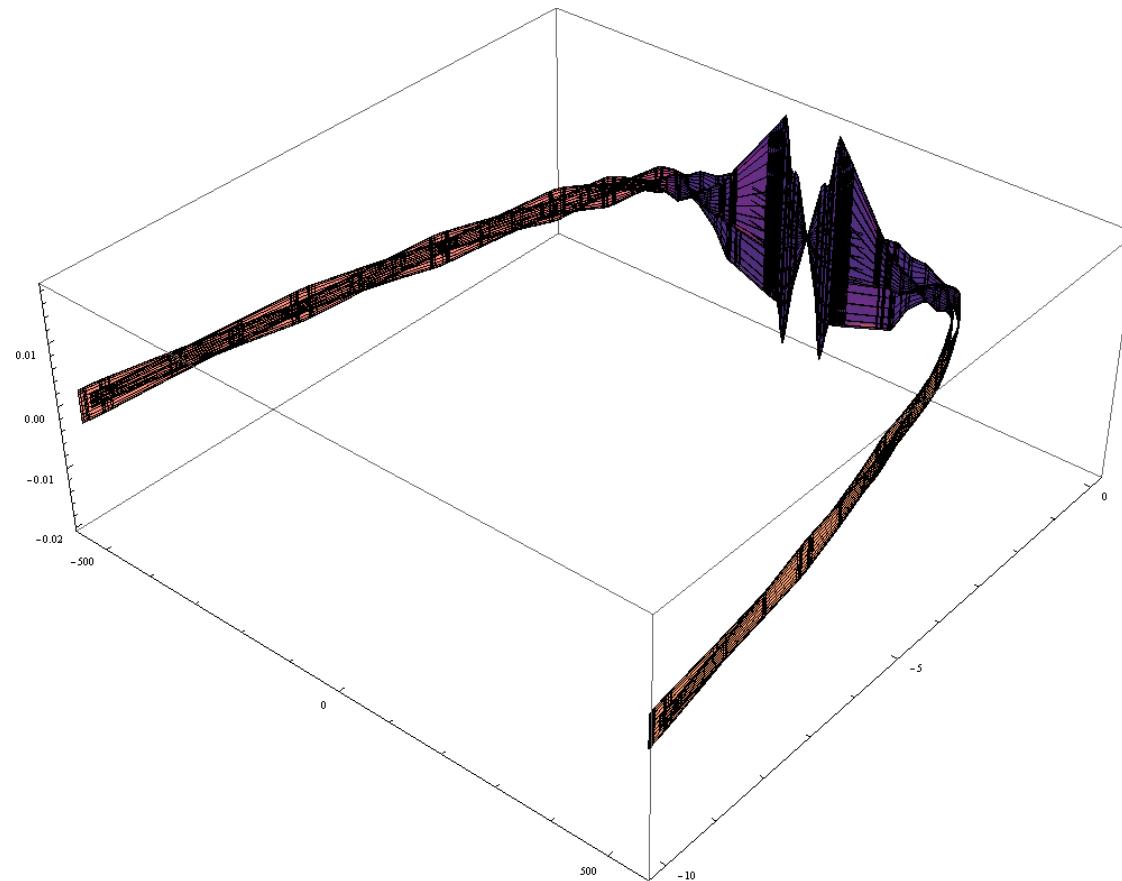
$$D'' + \frac{1}{\rho^2} \frac{2p_0 - p}{p} + \frac{B'}{B\rho} \frac{p_0}{p} D = \frac{1}{\rho} \frac{p_0}{p}$$



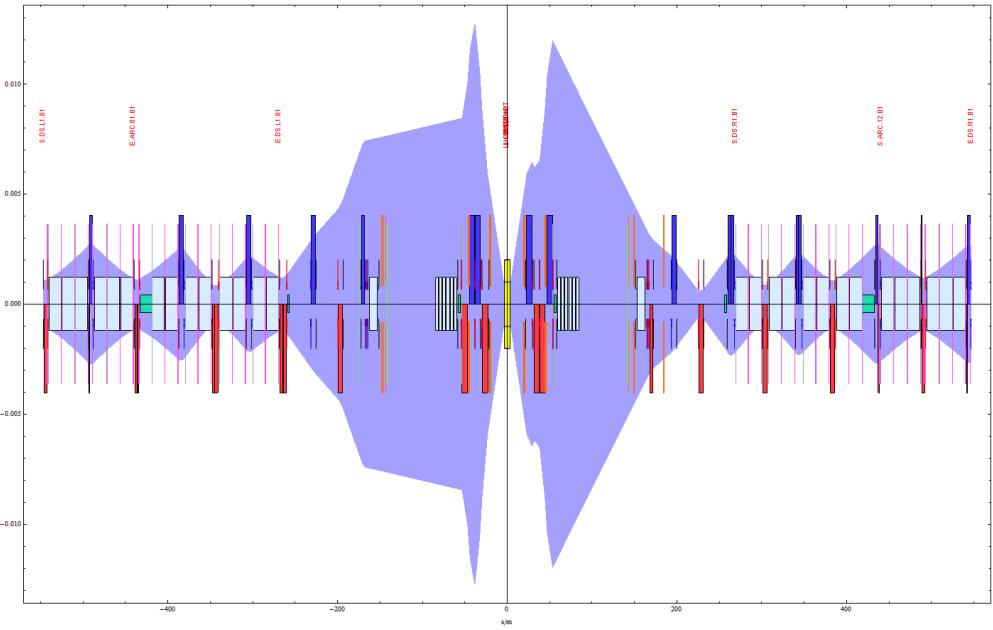


$$\sigma_x^2 = \frac{\beta_x \epsilon_x}{\pi} + D_x^2 \sigma_\epsilon^2$$

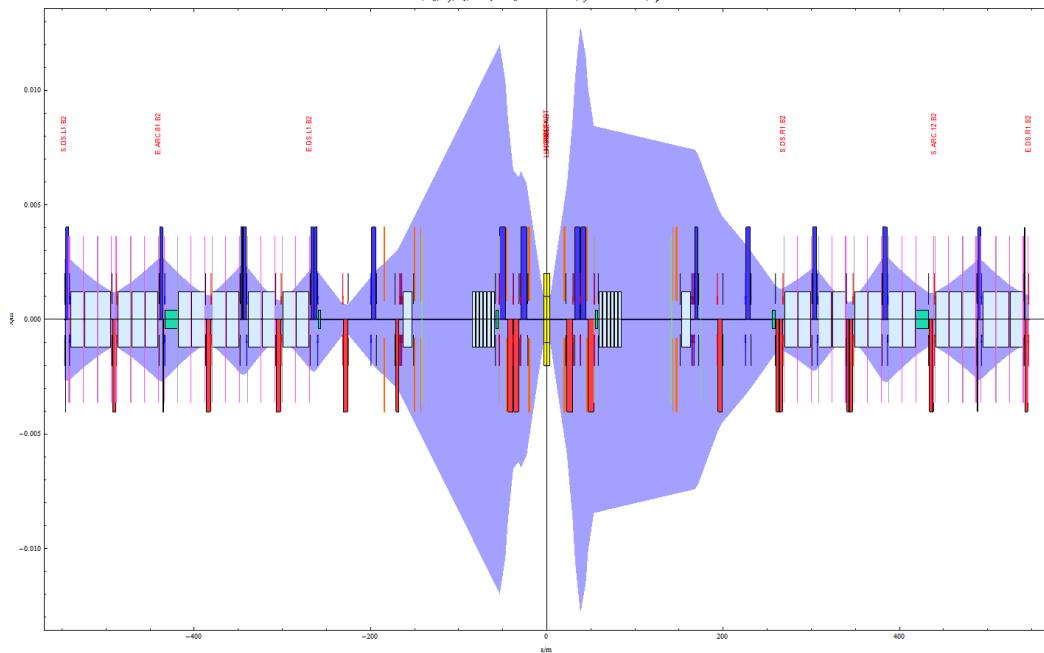
Global Cartesian Envelope Plots



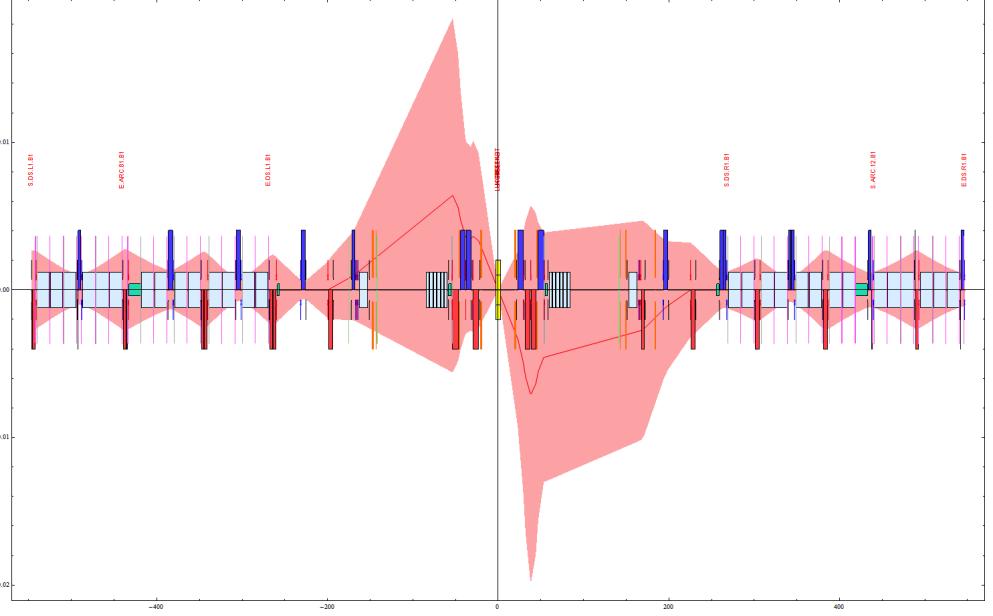
$(\sigma_x, \sigma_y, \sigma_z)$ envelope for $\epsilon_1=8.2075 \times 10^{-12} m$, $\epsilon_2=8.2075 \times 10^{-12} m$, $\sigma_p=0$.



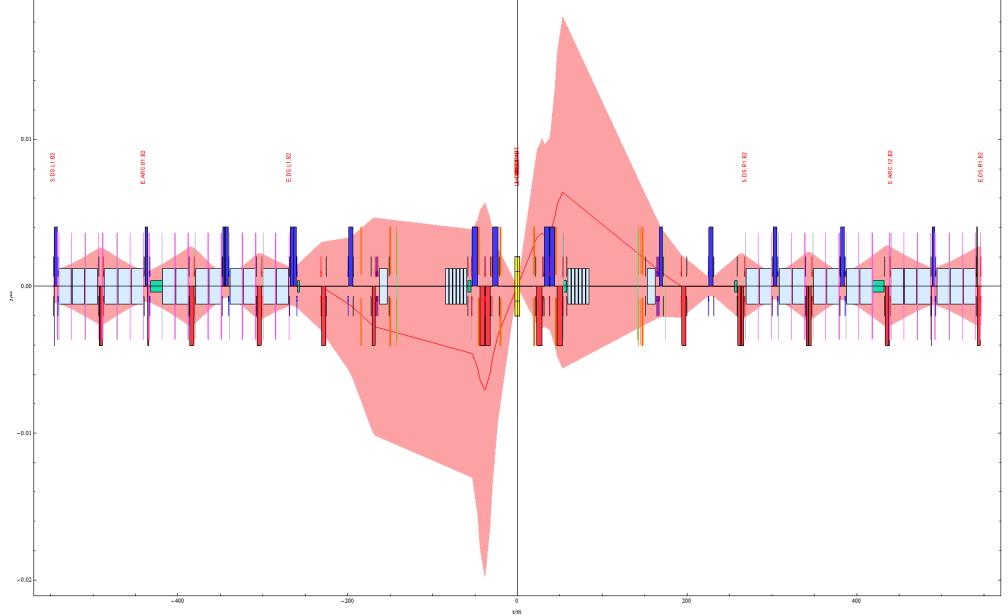
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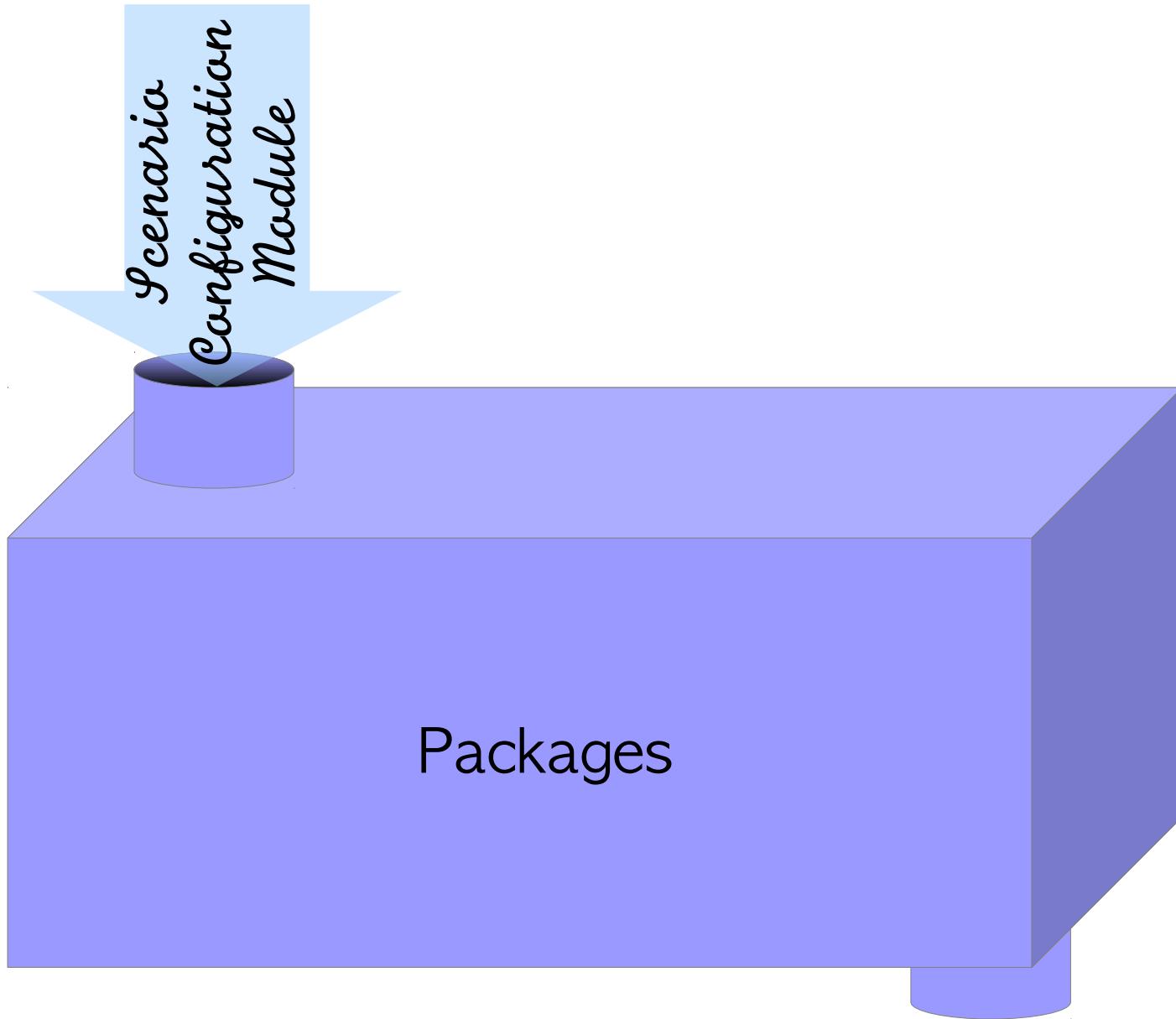


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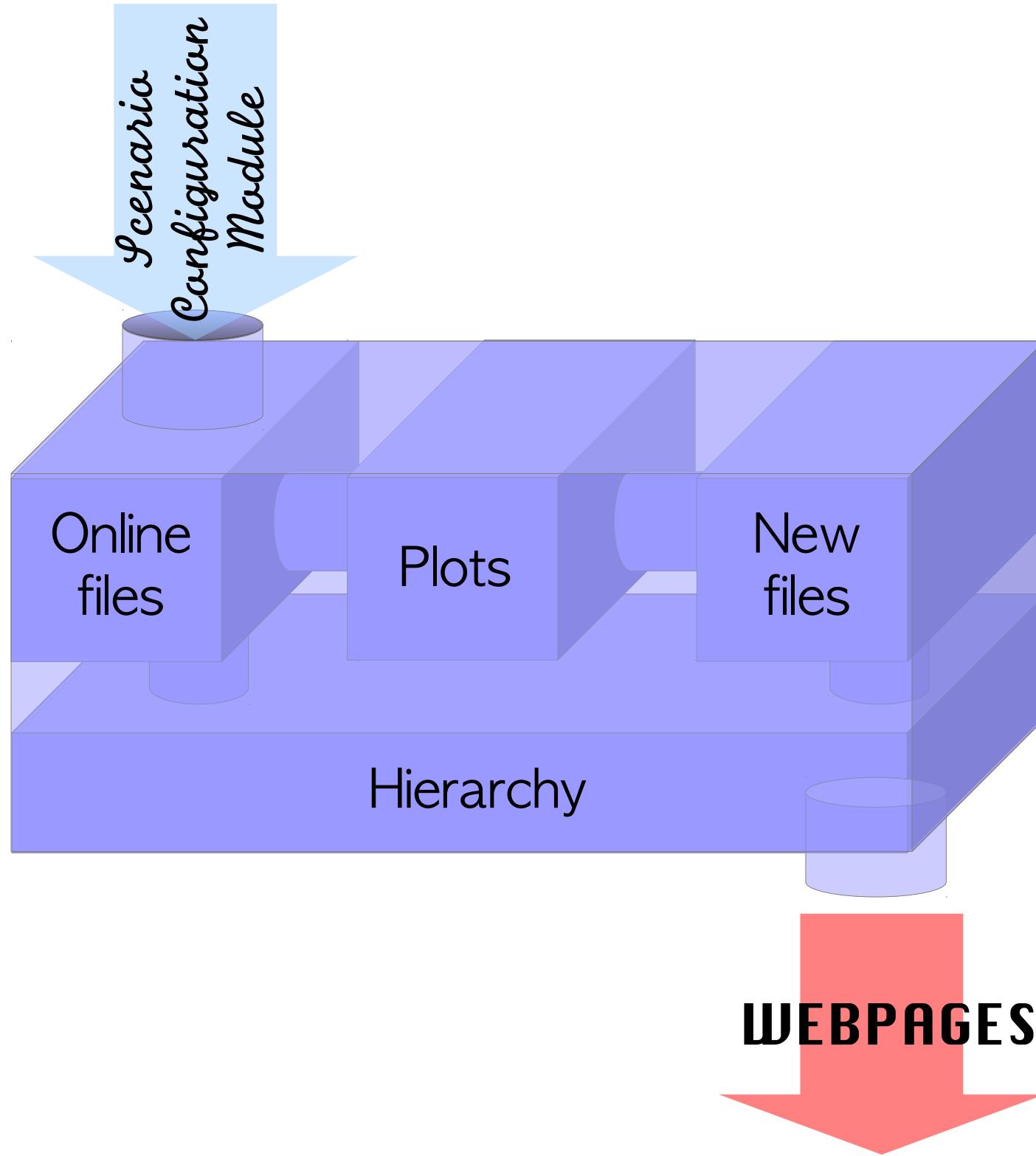


$(\sigma_x, \sigma_y, \sigma_z)$ envelope for $\epsilon_1=8.2075 \times 10^{-12} m$, $\epsilon_2=8.2075 \times 10^{-12} m$, $\sigma_p=0$.





WEBPAGES



Webpages

pprun1, coll_4tev, IR1DS

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[Download B2 data as .csv](#)

[Beamlne Plots](#)

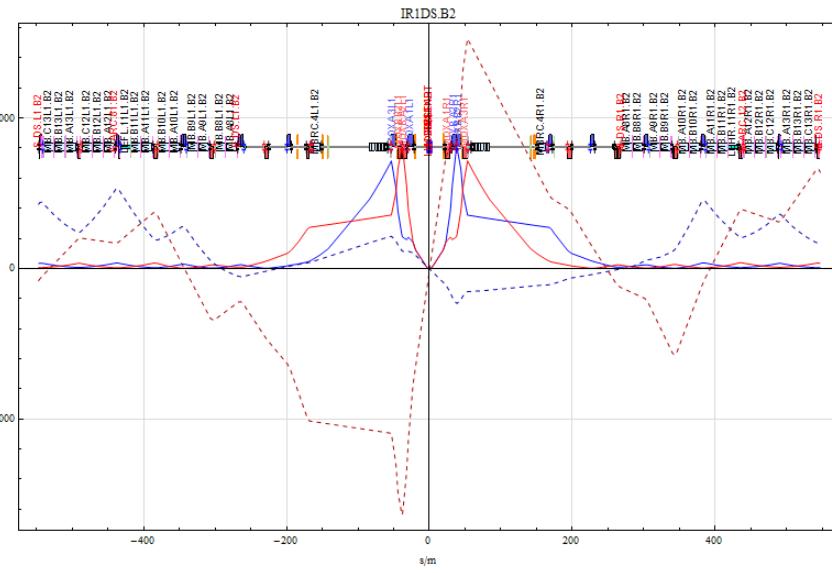
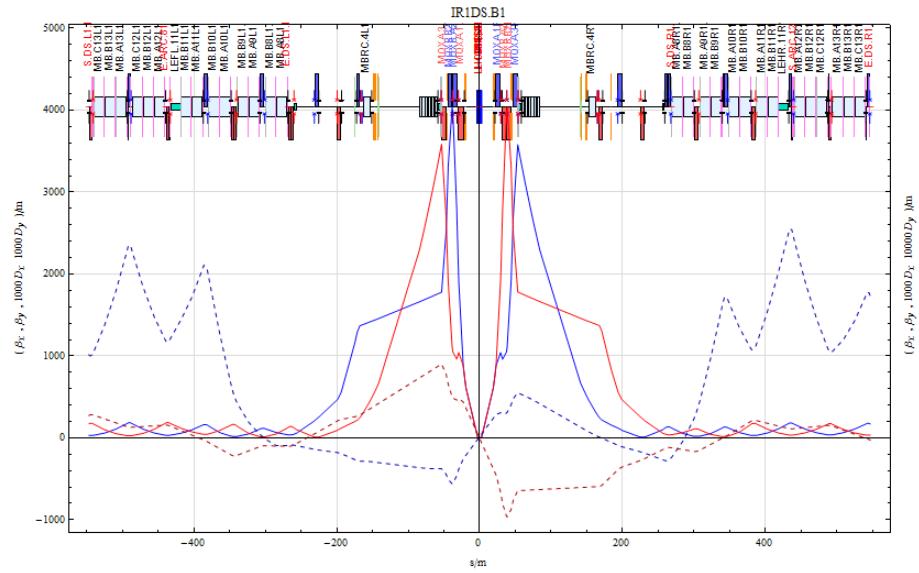
[2D Envelope Plots](#)

[3D Envelope Plots \(CS Coordinates\)](#)

[3D Global Envelope Plots](#)

Beamline Plots

β_x shown in red solid line, β_y shown in blue solid line, DispersionX shown in red dotted line, DispersionY shown in blue dotted line

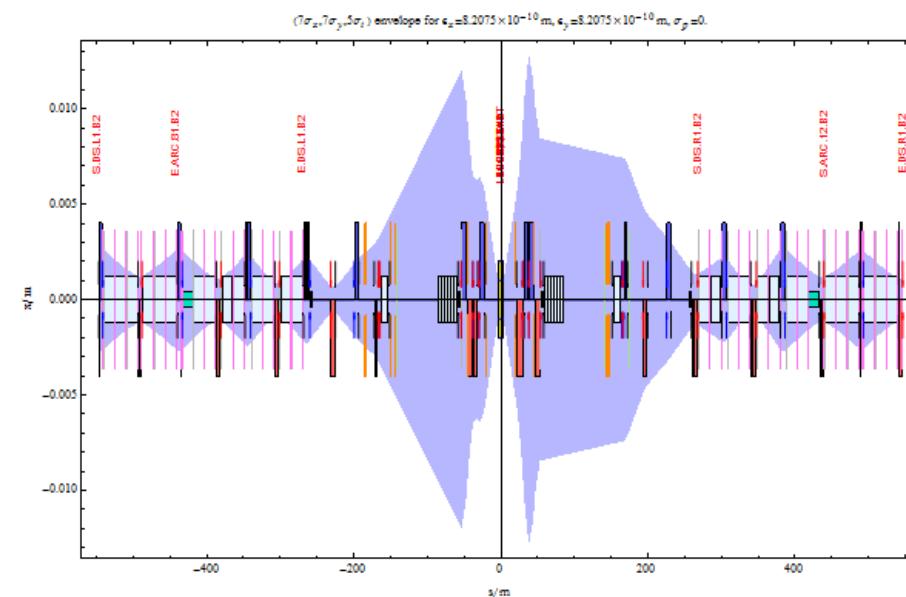
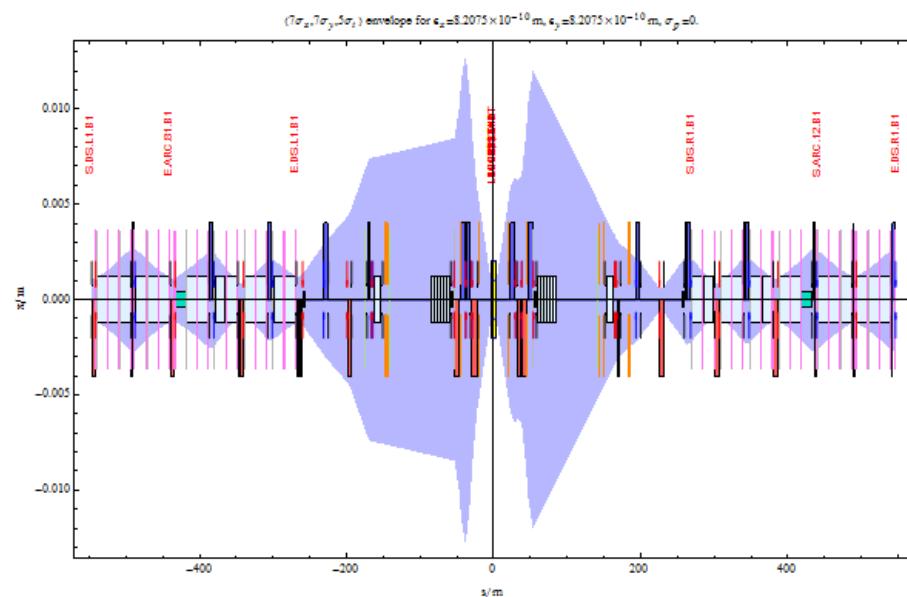


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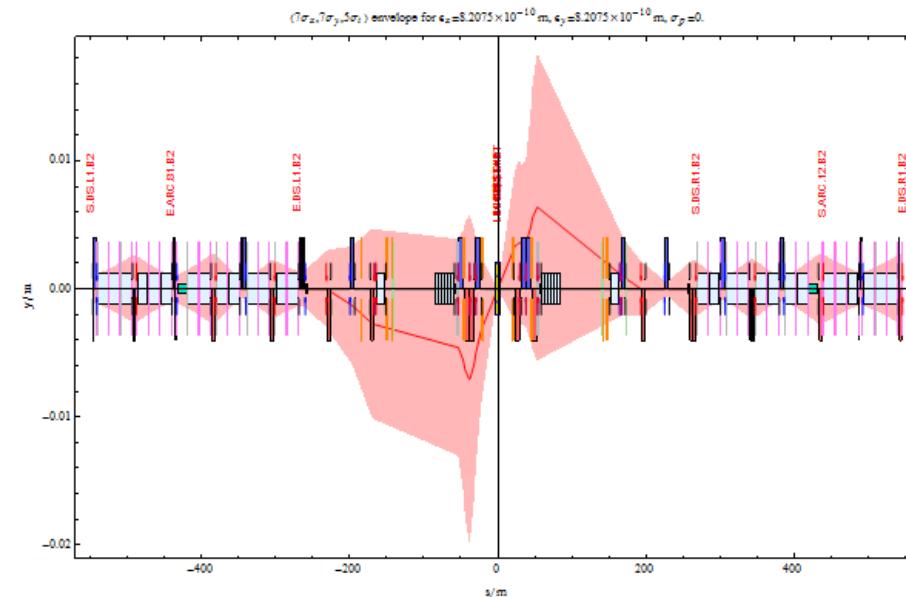
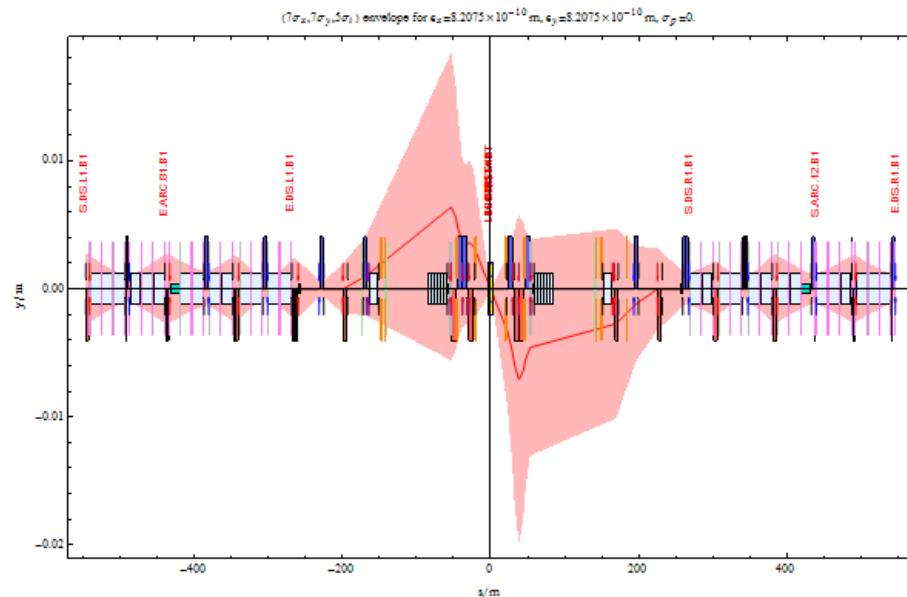
2D Envelope Plots

X shown in blue, Y shown in red



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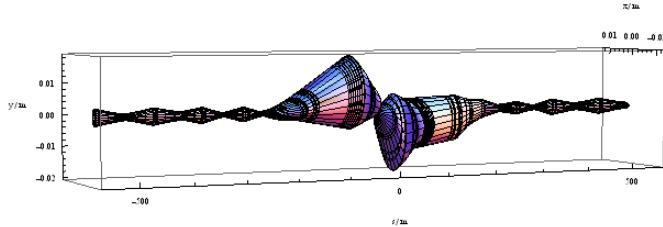


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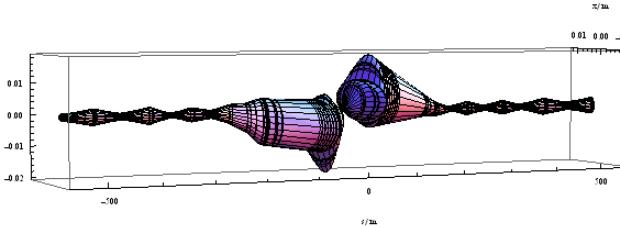
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3D Envelope Plots (CS Coordinates)

(7 σ_1 , 7 σ_2 , 5 σ_3) envelope for $a = 5.2075 \times 10^{-10}$ m, $c = 5.2075 \times 10^{-10}$ m, $\sigma_f = 0$.



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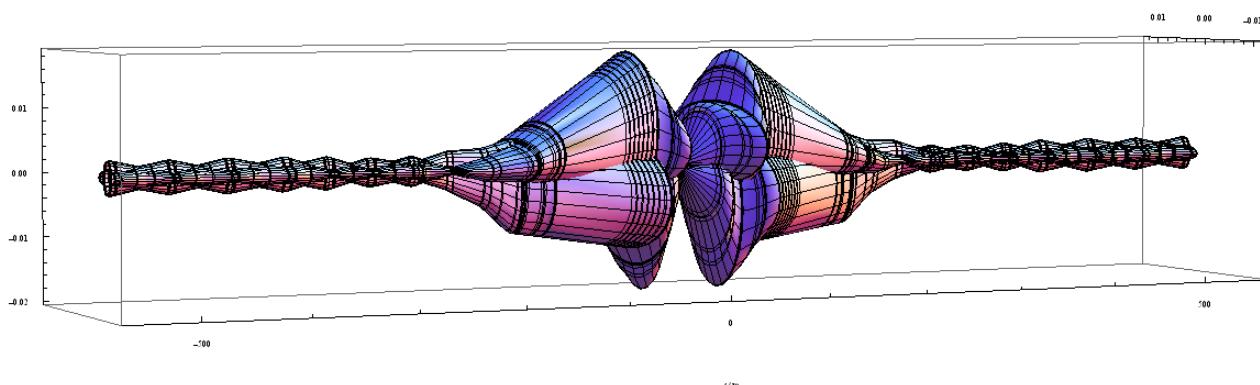


\$Plot5\$

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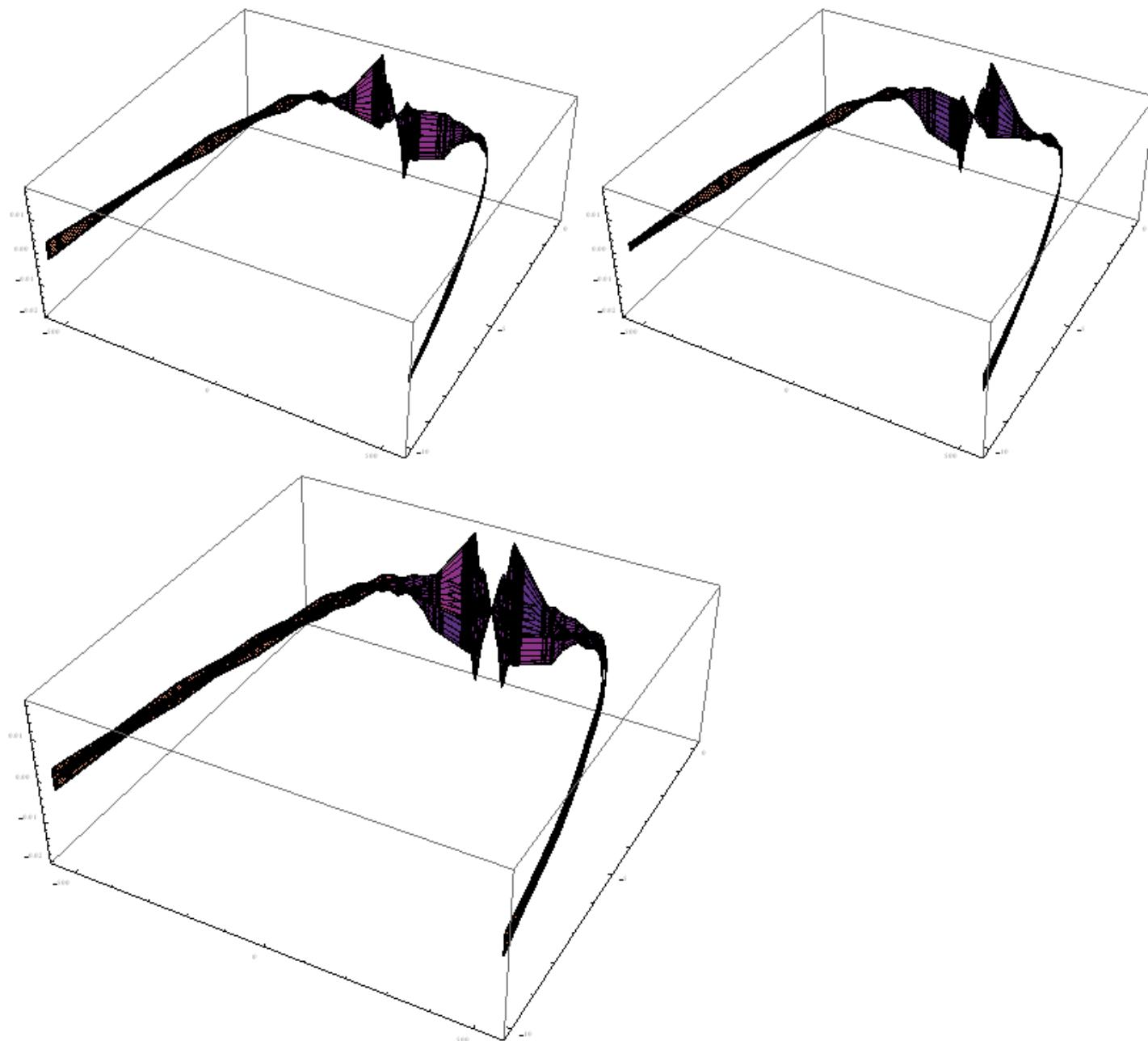
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[Download superposition plot as .m](#)

3D Global Envelope Plots



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Future Work

Additional plots/simulations
(squeeze, aperture, etc.)

LHC upgrades = new scenarios

Thank you

Dr. John Jowett

University of Michigan

National Science Foundation

References

1. Edwards D A and Syphers M J 1993 An Introduction to the Physics of High Energy Accelerators (New York: Wiley)
2. [Web image] Retrieved from <http://www.lhc-closer.es/1/4/1/1>

Questions?