HERAPDF2.0(prel.) fitted to new FINAL combined inclusive cross sections from HERA NCe+p, NCe-p, CCe+p, CCe-p proton beam energies, 920, 820, 575, 460 GeV

HERAPDF2.0 NLO and NNLO fits performed for both $Q^2 > 3.5 \text{ GeV}^2$ and $Q^2 > 10 \text{ GeV}^2$

Compared to HERAPDF1.0 NLO and HERAPDF1.5NNLO, some noteworthy differences.
DIS is the best tool to probe proton structure

**Neutral current:**

\[
\frac{d^2 \sigma^{NC}}{dx dQ^2} = \frac{2\alpha_s^2}{x Q^4} (Y_+ F_2 - Y_- x F_3 - y^2 F_L)
\]

\[F_2 \propto \sum_i c_i (x q_i + x q_i^*) \]

\[x F_3 \propto \sum_i (x q_i - x q_i^*)\]

\[F_L \propto \alpha_s \times g\]

Gluon distributions, quark from scaling violation

**Charged current:**

\[
\frac{d^2 \sigma^{CC}}{dx dQ^2} = \frac{G_F^2}{2\pi} \frac{M_W^2}{M_W^2 + Q^2} (u + c + (1 - y^2)(\bar{d} + s))
\]

\[
\frac{d^2 \sigma^{\bar{C}C}}{dx dQ^2} = \frac{G_F^2}{2\pi} \frac{M_W^2}{M_W^2 + Q^2} (\bar{u} + \bar{c} + (1 - y^2)(d + s))
\]

Gluon from the scaling violations: DGLAP

\[
\frac{dq(x, Q^2)}{d\ln Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[ P_{qq} \left( \frac{x}{y} \right) q(y, Q^2) + P_{qg} \left( \frac{x}{y} \right) g(y, Q^2) \right]
\]

\[
\frac{dg(x, Q^2)}{d\ln Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[ P_{qg} \left( \frac{x}{y} \right) q(y, Q^2) + P_{gg} \left( \frac{x}{y} \right) g(y, Q^2) \right]
\]
HERAPDF approach uses only HERA data

Some of the debates about the best way of estimating PDF uncertainties concern the use of many different data sets with varying levels of consistency.

The combination of the HERA data yields a very accurate and consistent data set for 4 different processes: e+p and e-p Neutral and Charged Current reactions.

The use of the single consistent data set allows the usage of the conventional $\chi^2$ tolerance $\Delta\chi^2 = 1$ when setting 68%CL experimental errors

NOTE the use of a pure proton target means d-valence is extracted without need for heavy target/deuterium corrections or strong iso-spin assumptions these are the only PDFs for which this is true

Furthermore, the kinematic coverage at low-x ensures that these are the most crucial data when extrapolating predictions from W, Z and Higgs cross-sections to the LHC

HERAPDF evaluates model uncertainties and parametrisation uncertainties in addition to experimental uncertainties
PDFs are parametrised at the starting scale $Q_0^2=1.9 \text{ GeV}^2$ as follows:

\[
\begin{align*}
    xg(x) &= A_g x^B_g (1-x)^C_g - A'_g x'^B_g (1-x)^C'_g, \\
xu_v(x) &= A_{u_v} x^{B_{u_v}} (1-x)^{C_{u_v}} \left(1 + D_{u_v} x + E_{u_v} x^2\right), \\
dx_v(x) &= A_{d_v} x^{B_{d_v}} (1-x)^{C_{d_v}}, \\
x\bar{U}(x) &= A_{\bar{U}} x^{B_{\bar{U}}} (1-x)^{C_{\bar{U}}} \left(1 + D_{\bar{U}} x\right), \\
x\bar{D}(x) &= A_{\bar{D}} x^{B_{\bar{D}}} (1-x)^{C_{\bar{D}}}. \\
\end{align*}
\]

QCD Sum rules constrain Normalisation parameters: $A_g, A_{u_v}, A_{d_v}$

And the condition that:

\[xu \to xd\text{ as } x \to 0.\]

relate $A_{\bar{U}}$ to $A_{\bar{D}}$, and with $xs = f_s x\bar{D}$

- Due to increased precision of data, more flexibility in functional form is allowed $\Rightarrow$ 15 free parameters
- PDFs are evolved via evolution equations (DGLAP) to NLO and NNLO (alphas(MZ)=0.118)[QCDNUM]
- Thorne-Roberts GM-VFNS for heavy quark coefficient functions – as used in MSTW
- Chi2 definition used in the minimisation [MINUIT] accounts for correlated uncertainties:

\[
\chi^2 = \sum_i \frac{\left[\mu_i - m_i \left(1 - \sum_j \gamma_i^j b_j\right)\right]^2}{\delta_{i,\text{unc}}^2 m_i^2 + \delta_{i,\text{stat}}^2 \mu_i m_i \left(1 - \sum_j \gamma_i^j b_j\right)} + \sum_j b_j^2 + \sum_i \ln \left(\frac{\delta_{i,\text{unc}}^2 m_i^2 + \delta_{i,\text{stat}}^2 \mu_i m_i}{\delta_{i,\text{unc}}^2 \mu_i^2 + \delta_{i,\text{stat}}^2 \mu_i^2}\right)
\]

$m_i$ is the theoretical prediction

$\mu_i$ is the measured cross section

$\delta_{i,\text{stat}}$, $\delta_{i,\text{unc}}$ statistical and uncorrelated systematic uncertainty

$\gamma_i^j$ correlated systematic uncertainties

$b_j$ shifts
Sources of HERAPDF uncertainties

**Experimental:**
- Hessian method is used to evaluate experimental uncertainties
- Consistent data sets → use $\Delta \chi^2 = 1$

**Model:**
- Following variations have been considered

<table>
<thead>
<tr>
<th>Variation</th>
<th>Standard Value</th>
<th>Lower Limit</th>
<th>Upper Limit</th>
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<tbody>
<tr>
<td>$f_3$</td>
<td>0.4</td>
<td>0.3</td>
<td>0.5</td>
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<tr>
<td>$M_c^{\text{opt}}$ (NLO) [GeV]</td>
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<td>7.5</td>
<td>12.5</td>
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<td>$Q_2^{\text{min}}$ [GeV^2]</td>
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<td>2.5</td>
<td>5.0</td>
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<tr>
<td>$Q_0^2$ [GeV^2]</td>
<td>1.9</td>
<td>1.6</td>
<td>2.2</td>
</tr>
</tbody>
</table>

**Parametrisation:**
- An envelope is formed from PDF fits using variants of parametrisation form
  - Scanning of 16 parameter space with D or E as extra parameters of $(1 + D x + E x^2)$
  - $Q_0^2$ variation → dominant parametrisation uncertainty

Values of $M_c$ and its uncertainties from scanning $\chi^2$ for fits including HERA charm combination data
Value of $f_3$ from considering ATLAS result AND $\nu$ di-muon results
Reconsider the Q$^2$ cut on the data
Traditionally Q$^2$>3.5 GeV$^2$

For Q$^2_{\text{min}}= 3.5$ GeV$^2$
Chi2/dof (NLO) = 1386/1130
Chi2/dof(NNLO) = 1414/1130

For Q$^2_{\text{min}}= 10$ GeV$^2$
Chi2/dof (NLO) = 1156/1001
Chi2/dof(NNLO) = 1150/1001

Fits for two Q$^2$ cuts will be presented: Q$^2 > 3.5$ and Q$^2 > 10$ GeV$^2$

Note that HERA kinematics is such that cutting out low Q$^2$ also cuts the lowest x values
These are the comparisons of the fit to the NCe+p data at low $Q^2$
The fit with $Q^2>10$ misses the lower $Q^2$ data in a systematic matter – worse at low-$x$ and low $Q^2$--- (not just at high-$y$)
Going to higher orders does not improve the fit at low-$Q^2$, low-$x$
So let’s take a look at the fits starting with NLO Q2>3.5
Comparison to data

**H1 and ZEUS**

Comparison to data

**H1 and ZEUS preliminary**

Improvement since HERAPDF1.0
Data vs fit comparison NLO $Q^2 > 3.5$

NLO $Q^2 > 3.5 \text{ GeV}^2$
NLO
$Q^2 > 3.5 \text{ GeV}^2$
H1 and ZEUS preliminary

\( Q^2 \gtrsim 3.5 \text{ GeV}^2 \)

\[ \sigma_{T,NC}(x,Q^2) \times 2 \]

\[ \sqrt{s} = 318 \text{ GeV} \]

- HERA NC e^+p (prel.) 0.5 fb^{-1}
- HERAPDF2.0 (prel.)
- NLO, \( Q_{\text{min}}^2 = 3.5 \text{ GeV}^2 \)

\( x = 0.00005, i=21 \)
\( x = 0.00008, i=20 \)
\( x = 0.00013, i=19 \)
\( x = 0.00020, i=18 \)
\( x = 0.00032, i=17 \)
\( x = 0.0005, i=16 \)
\( x = 0.0008, i=15 \)
\( x = 0.0013, i=14 \)
\( x = 0.0020, i=13 \)
\( x = 0.0032, i=12 \)
\( x = 0.005, i=11 \)
\( x = 0.008, i=10 \)
\( x = 0.013, i=9 \)
\( x = 0.02, i=8 \)
\( x = 0.032, i=7 \)
\( x = 0.05, i=6 \)
\( x = 0.08, i=5 \)
\( x = 0.13, i=4 \)
\( x = 0.18, i=3 \)
\( x = 0.25, i=2 \)
\( x = 0.40, i=1 \)
\( x = 0.65, i=0 \)
NLO
$Q^2 > 3.5 \text{ GeV}^2$

**H1 and ZEUS**

Improvement since HERAPDF1.0
H1 and ZEUS preliminary

\[ \sigma_{\text{NC}}^{+}(x, Q^2) \]

\[ Q^2 = 2 \text{ GeV}^2 \quad Q^2 = 2.7 \text{ GeV}^2 \quad Q^2 = 3.5 \text{ GeV}^2 \quad Q^2 = 4.5 \text{ GeV}^2 \]

\[ Q^2 = 6.5 \text{ GeV}^2 \quad Q^2 = 8.5 \text{ GeV}^2 \quad Q^2 = 10 \text{ GeV}^2 \quad Q^2 = 12 \text{ GeV}^2 \]

\[ Q^2 = 15 \text{ GeV}^2 \quad Q^2 = 18 \text{ GeV}^2 \quad Q^2 = 22 \text{ GeV}^2 \quad Q^2 = 27 \text{ GeV}^2 \]

\[ Q^2 = 35 \text{ GeV}^2 \quad Q^2 = 45 \text{ GeV}^2 \quad Q^2 = 60 \text{ GeV}^2 \quad Q^2 = 70 \text{ GeV}^2 \]

\[ Q^2 = 90 \text{ GeV}^2 \quad Q^2 = 120 \text{ GeV}^2 \]

\[ 10^{-3} \quad 10^{-1} \quad 10^{-3} \quad 10^{-3} \]

\[ 10^{-1} \quad 10^{-1} \quad 10^{-1} \quad 10^{-1} \]

\[ x \]

- HERA NC $e^+p$ (prel.) 0.5 fb$^{-1}$
- $\sqrt{s} = 318 \text{ GeV}$
- HERAPDF2.0 (prel.)
- NLO, $Q_{\text{min}}^2 = 3.5 \text{ GeV}^2$

NLO
$Q^2 > 3.5 \text{ GeV}^2$
H1 and ZEUS preliminary

NLO
$Q^2 > 3.5$ GeV$^2$
New since HERAPDF1.0
NLO
$Q^2 > 3.5 \text{ GeV}^2$

New since HERAPDF1.0
Now let’s take a look at the PDFs

Compare to HERAPDF 1.0—more in experimental less in param because of 15 parameters not 10 parameters.
Note that the extra parameters in the 10p+parametrisation variation are exactly those which now come into 15p.
Now make the same comparison by overlaying NLO $Q^2 > 3.5$ GeV$^2$

These figures have all the model/param variations included in the blue bands

- HERAPDF1.0 had a rather hard high-$x$ sea, harder than the gluon (within large uncertainties). This is no longer the case and uncertainties are reduced
- The valence shapes are also somewhat different - new high-$x$ data in the fit
The Sea is a lot softer at high-x
Whereas the gluon is harder

So the q-qbar luminosity at high-x comes down
And the g-g luminosity a high-x goes up
Compare to $Q^2 > 3.5$ results ----- $Q^2 > 10$ cut increases uncertainty of low-x gluon – obviously
We could also compare the $Q^2 > 10$ and $Q^2 > 3.5$ fits overlaid on each other.

As well as greater uncertainty at low-$x$ for Sea and glue there is also some small shift of gluon and sea shape at low-$x$.

At large $x$ gluon and sea and valence are all similar.
The model dependence on the gluon (yellow) is mostly the $Q^2$ cut – similar but not quite as extreme as before.
Reduction in gluon uncertainty both at low-x and high-x.
A lot of this reduction is because the effect of the Q2 cuts is not as dramatic now that we have more data.
(NOTE: HERAPDF2.0 has smaller uncertainty on the high-x g-g luminosity and hence on top predictions!)

Now make the same comparison by overlaying

**NNLO Q^2 > 3.5 GeV^2**
This uncertainty on the gluon decreases.

So this uncertainty on the g–g luminosity will also decrease.
Compare to $Q^2>3.5$ results -----$Q^2>10$ cut increases uncertainty of low-$x$ gluon-- obviously –BUT

there is not just an increase of uncertainty at low-$x$ as there was at NLO,

there is also a systematic shape difference, both in low-$x$ sea and gluon
We could also compare the $Q^2>10$ and $Q^2>3.5$ fits overlaid on each other.

Fits are VERY compatible at high-$x$ ---like in NLO case
BUT the difference in shape for low-$x$ Sea and gluon-- has now become pronounced
Summary

- HERA provides a clean determination of the proton’s PDFs based solely on ep collider data.
- New preliminary combined HERA I+II+low energy measurements improves precision of PDFs.
- $Q^2$ dependence of fit observed and two sets, $Q^2 > 3.5$ GeV$^2$ and $Q^2 > 10$ GeV, provided.

HERAPDF2.0(prel) at NLO and NNLO with full uncertainties.