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String theory in the bathtub

w/ S. Endlich, R. Penco	(EFT)
B. Horn, S. Gubser	(Strings)
W. Irvine, L. Stanzani	(Exp)
C. Barenghi	(Num)

(EFT for hydro: w/ L. Delacretaz, S. Dubovsky, T. Gregoire, S. Endlich, L. Hui, R. Penco, F. Piazza, R. Porto, R. Rattazzi, R. Rosen, S. Sibiryakov, D. T. Son, J. Wang.

JHEP 0603, JHEP 1104, JHEP 1206, PRD 85 (2012), PRL 110 (2013), JCAP 1310, PRD 88 (2013), JHEP 1311, PRD 89 (2014), PRD (2014) hep-th 1303.3289, 1310.2272, 1311.6491, …)

zero T super-fluid vs. ordinary fluid

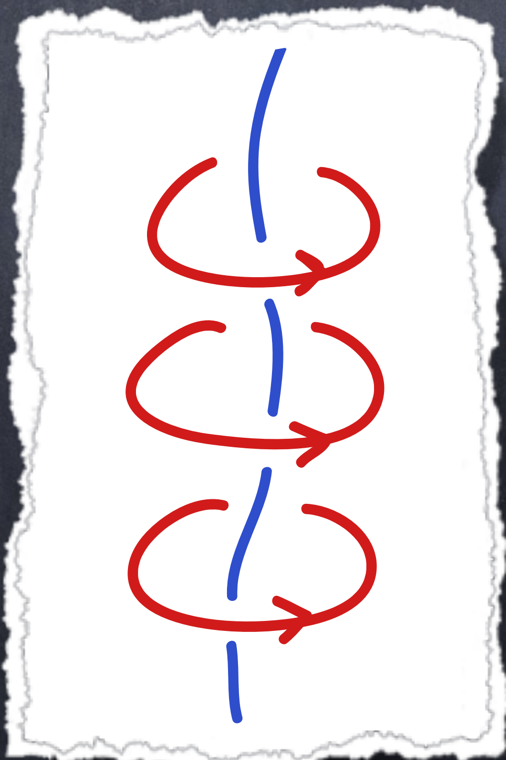
compressional (sound) sector

- Hydrodynamics
- Hydrodynamics

transverse (vortex) sector

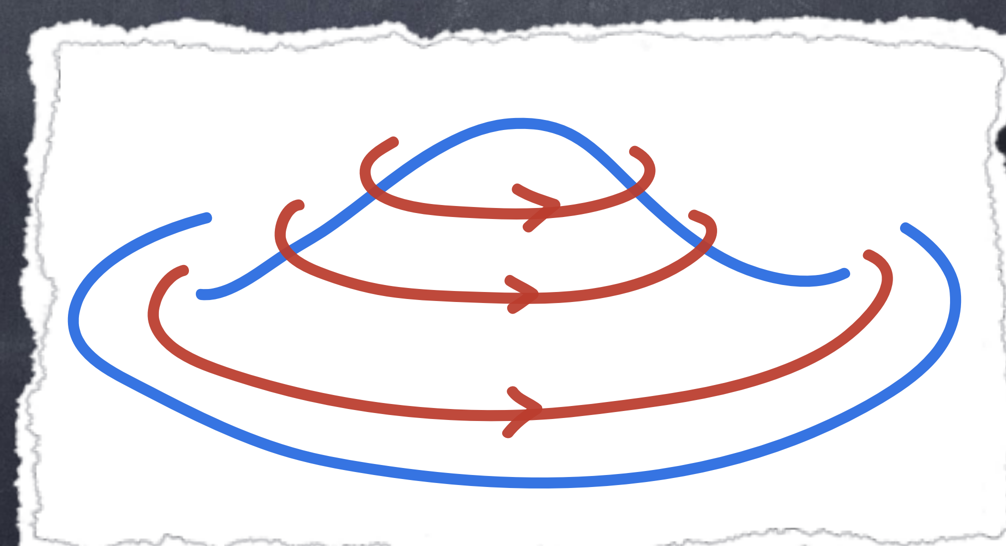
- Hard (gapped)

$$\vec{\nabla} \times \vec{v} = 0$$



- Soft (gapless)

$$\vec{\nabla} \times \vec{v} \neq 0$$

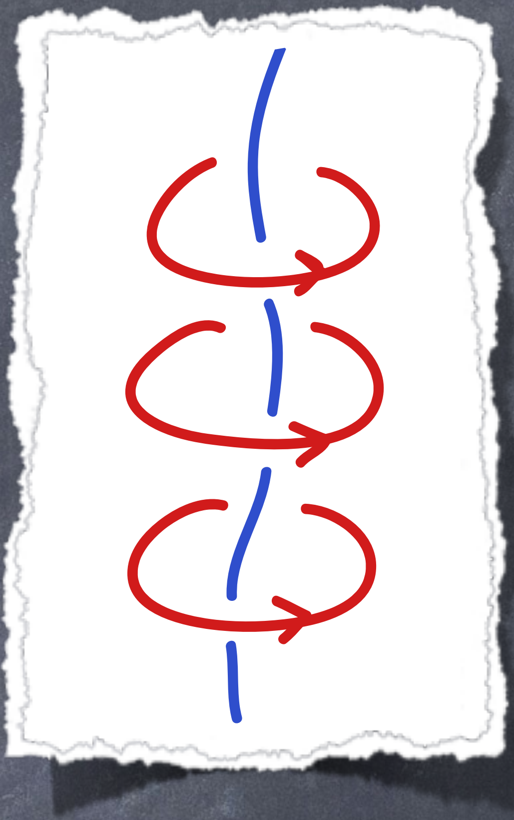


Vortex dynamics (incompressible limit)

For vortex lines

$$\Gamma = \oint \vec{v} \cdot d\vec{\ell} \quad \leftrightarrow \quad I$$

$$\vec{v} \quad \leftrightarrow \quad \vec{B}$$



Biot-Savart:

$$\vec{v}(\vec{x}) = -\frac{\Gamma}{4\pi} \int \frac{(\vec{x} - \vec{x}\,')}{|\vec{x} - \vec{x}\,'|^3} \times d\vec{x}\,'$$

1st order EOM!

Unlike $m\vec{a} = \vec{F}_{\text{ext}}$

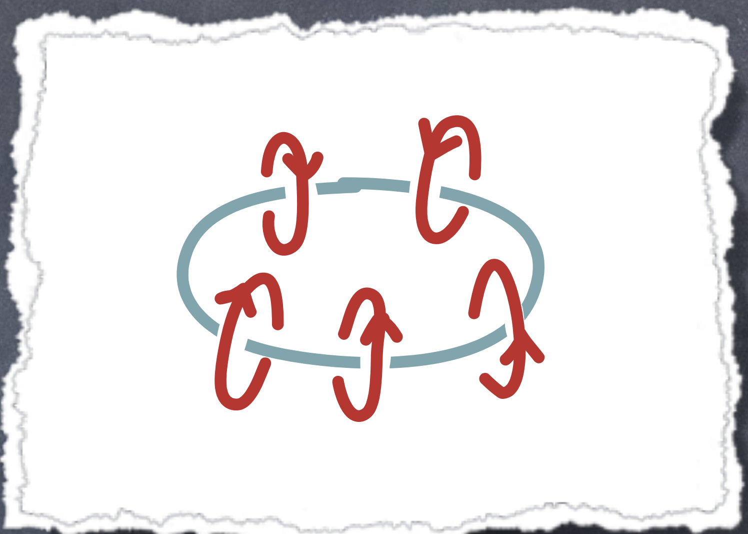
⇒ No room for “forces”

⇒ No free initial condition for v

⇒ Instantaneous v determined by geometry

For vortex rings

$$\vec{v} = \frac{\Gamma}{4\pi R} \log(R/a)\, \hat{n}$$



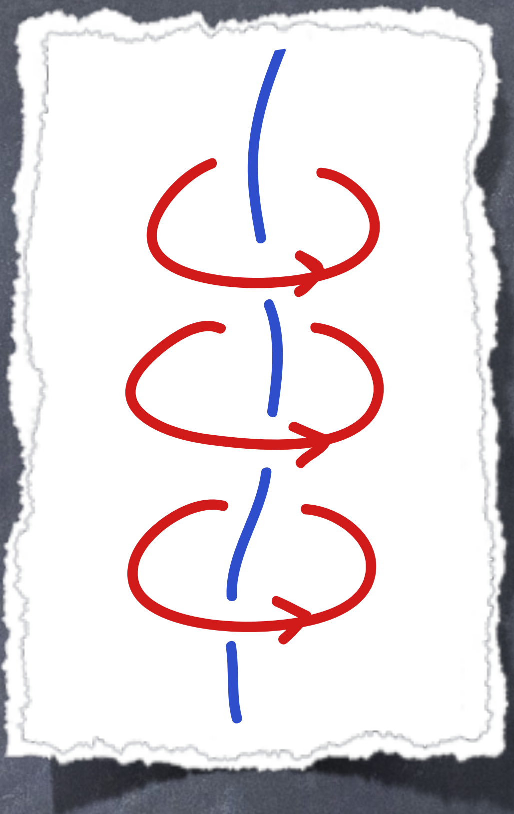
Far away:

$$\vec{v}(\vec{x}) = \vec{B}_{\rm dipole} \qquad \text{with} \qquad \vec{\mu} = (\pi R^2)\Gamma\, \hat{n}$$

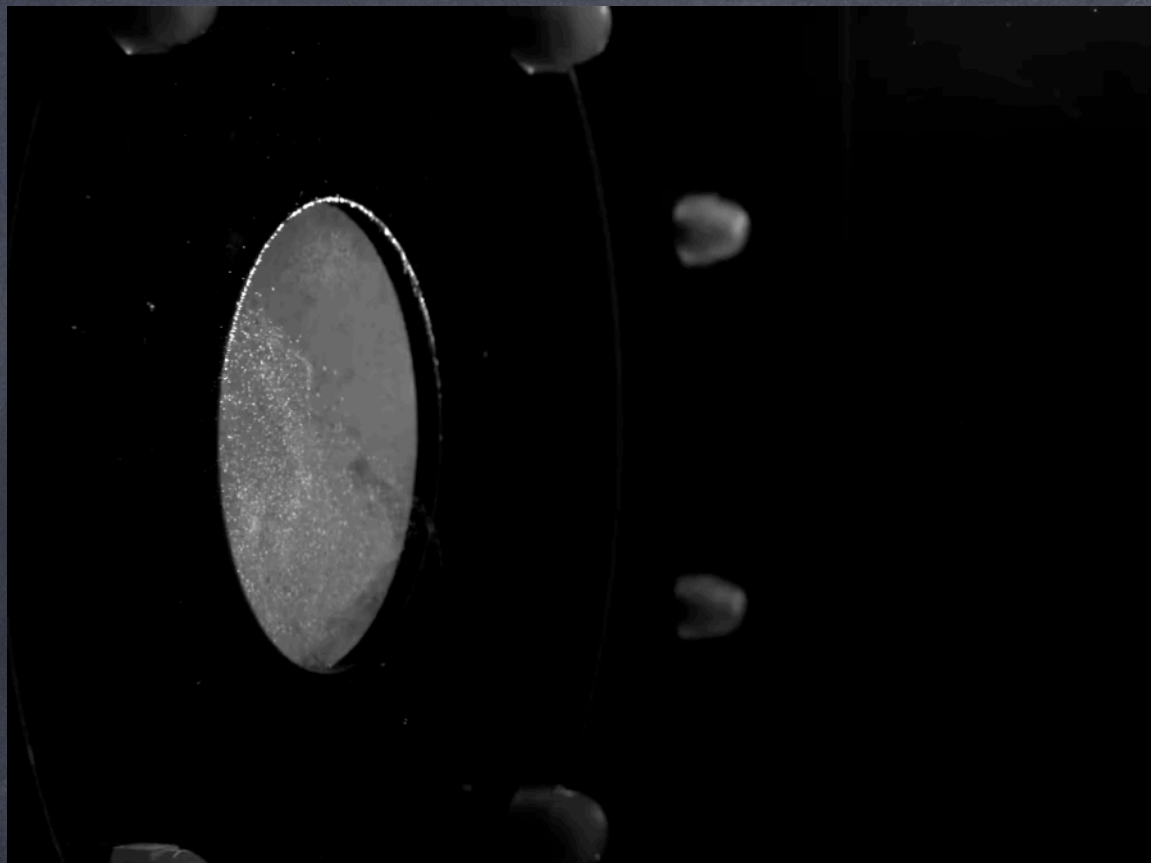
Excitations: Kelvin waves

Two modes overall ($\neq 2+2$)

$$\omega_\pm = \frac{\Gamma}{2\pi} k^2 \log(1/ka)$$

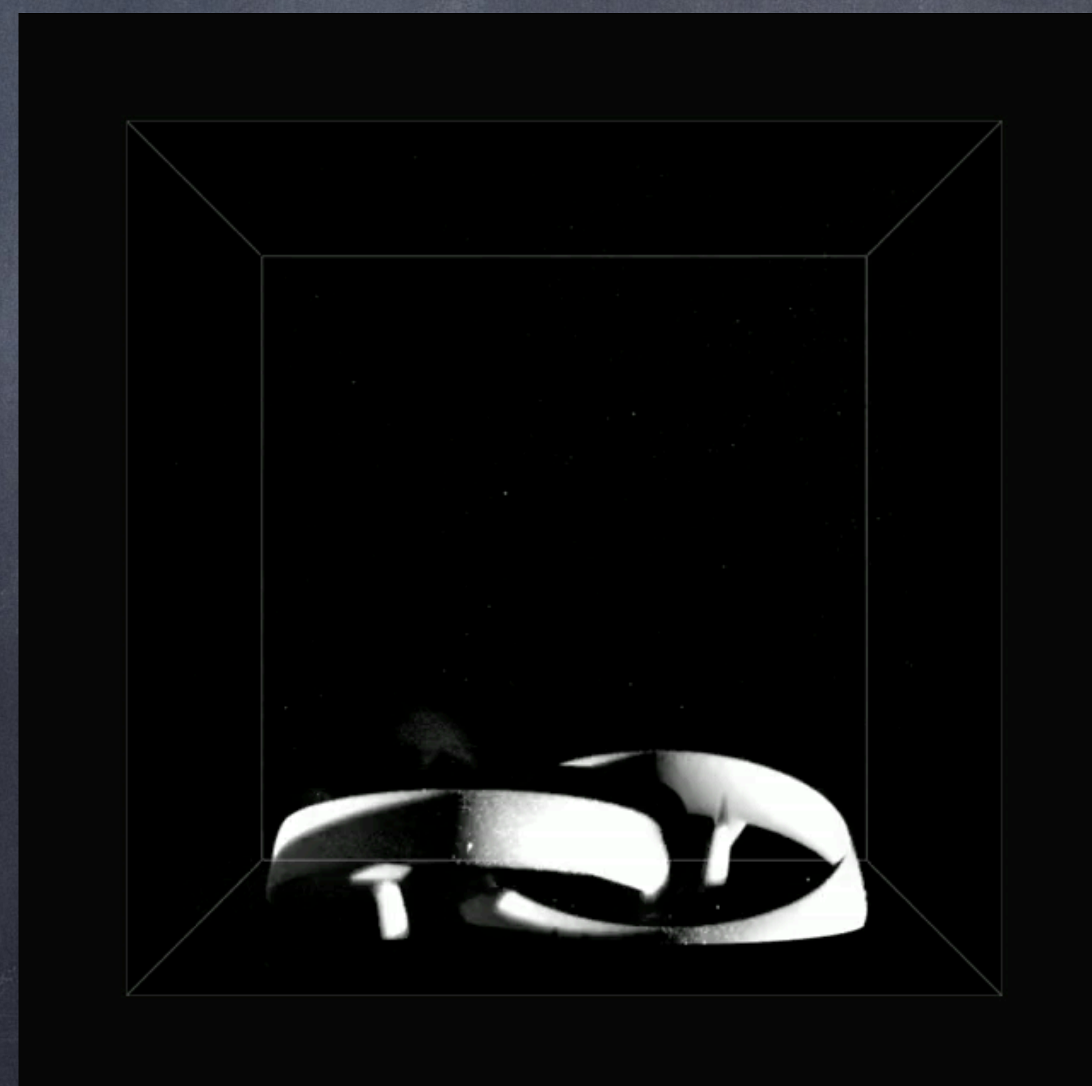


- fewer modes than 2-derivative eom
- ``non-local'' dispersion relation



William Irvine

U. of Chicago



other groups...

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Journal of Comparative Psychology
2000, Vol. 114, No. 1, 98–106

0735-7036/00/$5.00 DOI: 10.1037//0735-7036.114.1.98

Bubble Ring Play of Bottlenose Dolphins (*Tursiops truncatus*): Implications for Cognition

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Lori Marino
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Erik Vance and Leah Walke
Six Flags Marine World

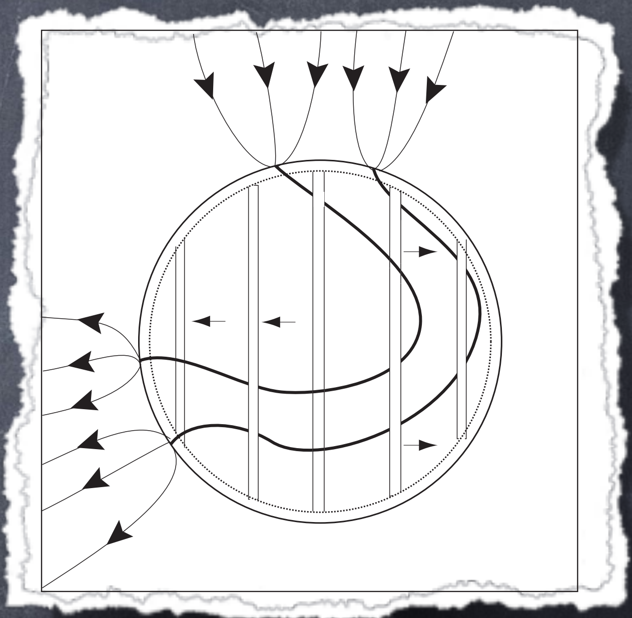
Diana Reiss
New York Aquarium

Research on the cognitive capacities of dolphins and other cetaceans (whales and porpoises) has importance for the study of comparative cognition, particularly with other large-brained social mammals, such as primates. One of the areas in which cetaceans can be compared with primates is that of object manipulation and physical causality, for which there is an abundant body of literature in primates. The authors supplemented qualitative observations with statistical methods to examine playful bouts of underwater bubble ring production and manipulation in 4 juvenile male captive bottlenose dolphins (*Tursiops truncatus*). The results are consistent with the hypothesis that dolphins monitor the quality of their bubble rings and anticipate their actions during bubble ring play.

In superfluids

- Only allowed vortices = quantized vortex lines w/ $\Gamma = 2\pi\hbar/m$ ($\sim .1\ \mathrm{mm}^2/\mathrm{s}$ for He)
- Superfluid turbulence = tangled mess of vortex lines. Decay?
- In pulsars: strong vortex line-flux tube interactions. Glitches?
- Observed in unitary Fermi gas

(Bulgac et al., PRL 2014)



(Ruderman 2009)

How to make sense of their dynamics?

Effective field theory, quick way

$$\mathcal{L} = -\rho\Big[\Gamma \int d\lambda\, \epsilon^{ijk}\, X^i\, \partial_t\, X^j\, \partial_\lambda X^k + \Gamma^2 \int d\lambda d\lambda' \, \frac{\partial_\lambda \vec{X} \cdot \partial_{\lambda'} \vec{X}'}{|\vec{X} - \vec{X}'|}\Big]$$

$$\int d^3x\, (\partial_i A_j)^2 - \Gamma \int d\lambda\, \partial_\lambda \vec{X} \cdot \vec{A}(\vec{X}, t)$$

EOM: $\vec{v}(\vec{x}) = -\frac{\Gamma}{4\pi} \int \frac{(\vec{x} - \vec{x}\,')}{|\vec{x} - \vec{x}\,'|^3} \times d\vec{x}\,'$

Magnetostatics	Incompressible Hydro
current $\vec{J}$	vorticity $\vec{\omega}$
magnetic field $\vec{B}$	velocity field $\vec{v}$
vector potential $\vec{A}$	hydrophoton $\vec{A}$

Note: no Lorentz force

$$\vec{B} \to \vec{v}$$

Effective field theory, responsible way

DOF: $\vec{X}(\lambda, t)$

$\vec{v}(\vec{x}, t) = \vec{\nabla} \times \vec{A}$

Symmetries: translations, rotations

reparametrizations $\lambda \to \lambda'(\lambda, t)$

spontaneously broken Galilei

$$\vec{X} \to \vec{X} + \vec{v}_0 t$$

$$\vec{v} \to \vec{v} + \vec{v}_0$$

$$\vec{A} \to \vec{A} + \tfrac{1}{2}\vec{v}_0 \times \vec{x}$$

Invariants:

$$\begin{cases} (\partial_i A_j)^2 \\ \epsilon^{ijk} X^i \partial_t X^j \partial_\lambda X^k - \partial_\lambda \vec{X} \cdot \vec{A}(\vec{X}, t) \end{cases}$$

Running tension

One more term allowed by symmetries: $-T\int d\lambda \left|\partial_\lambda \vec{X}\right|$

Why not there?

Needed as counterterm:

$$\frac{dE}{dz} = \rho\Gamma^2 \int dz' \, \frac{\partial_z \vec{X} \cdot \partial_{z'} \vec{X}'}{\left|\vec{X} - \vec{X}'\right|} \sim \rho\Gamma^2 \log R/a$$

$$\Longrightarrow \qquad T(\mu) \sim \rho\Gamma^2 \log \mu$$

Many computations now simplified

Kelvin waves

Perturbing $$\epsilon X \partial_t X \partial_\lambda X + \partial_\lambda X \cdot A(X) + (\nabla A)^2_{\rm bulk}$$

(grad. energy from mixing w/ A)

vs.

$$\epsilon X \partial_t X \partial_\lambda X + T(\mu)\sqrt{1 + \partial_\lambda \pi_\perp^2}$$

(no mixing)

Exact NL waves: $$\vec{\pi}_\perp(z,t) = (\hat{x} + i\hat{y}) \times \phi_0 \, e^{ik(z - vt)}$$

$$v = \frac{\Gamma}{4\pi} k \log(1/ka) \times \frac{1}{\sqrt{1 + k^2 \phi_0^2}}$$

Point-particle limit for vortex loops

$$\mathcal{L} = \sum_n \left[\vec{\mu}_n \cdot \dot{\vec{x}}_n + \vec{\mu}_n \cdot (\vec{\nabla} \times \vec{A})\right] - \int d^3x \, \left(\partial_i A_j\right)^2$$

$$\to \sum_n \left(\vec{\mu}_n \cdot \dot{\vec{x}}_n - \mu_n^{3/2} \log \mu_n\right) - \sum_{n \neq m} \frac{\vec{\mu}_n \cdot \vec{\mu}_m - 3(\vec{\mu}_m \cdot \hat{r})(\vec{\mu}_n \cdot \hat{r})}{r^3}$$

Peculiar conservation laws: $(\mu_n = \pi R_n^2 \Gamma_n)$

$$\vec{P} = \sum_n \vec{\mu}_n$$

$$\vec{L} = \sum_n \vec{x}_n \times \vec{\mu}_n$$

$$E = \sum_n \mu_n^{3/2} \log \mu_n + \sum_{n \neq m} \frac{\vec{\mu}_n \cdot \vec{\mu}_m - 3(\vec{\mu}_m \cdot \hat{r})(\vec{\mu}_n \cdot \hat{r})}{r^3}$$

Coupling to sound/phonons

Subsonic regime: $v \ll c_s$

(Endlich, Nicolis 2013)

Nearly incompressible

⇘

sound waves difficult to excite

⇙ ⇘

treat vortices non-linearly

treat sound perturbatively

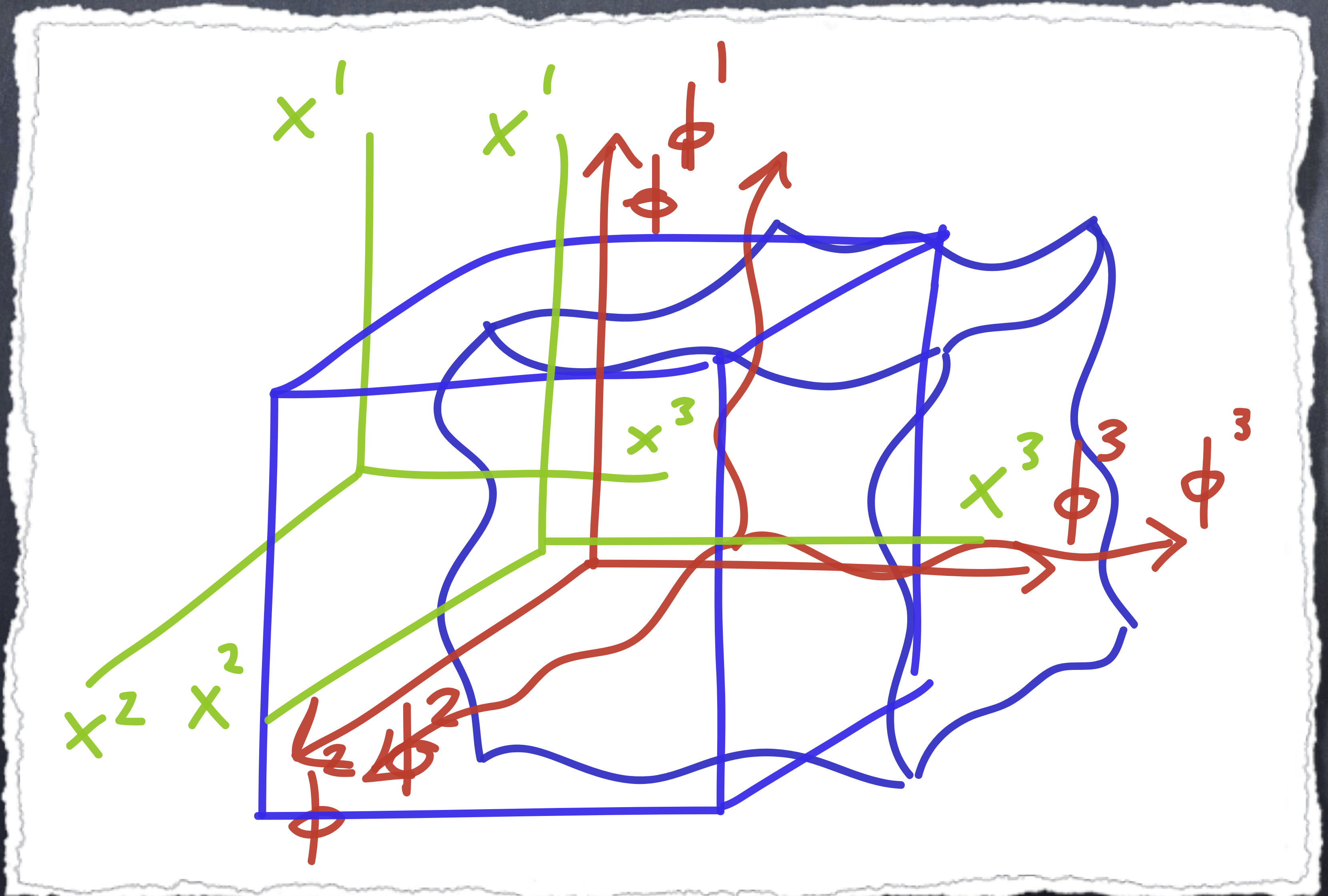
⇓

integrate it out

Deformations of the medium

Dof: volume elements' positions

$\phi^I(\vec{x}, t) \qquad I = 1, 2, 3$



$\langle \phi^I \rangle_{\text{eq}} = x^I$

Symmetries: Poincaré + internal

$$\left.\begin{aligned} \phi^I &\to \phi^I + a^I \\ \phi^I &\to SO(3)\,\phi^I \end{aligned}\right\}$$ recover homogeneity/isotropy

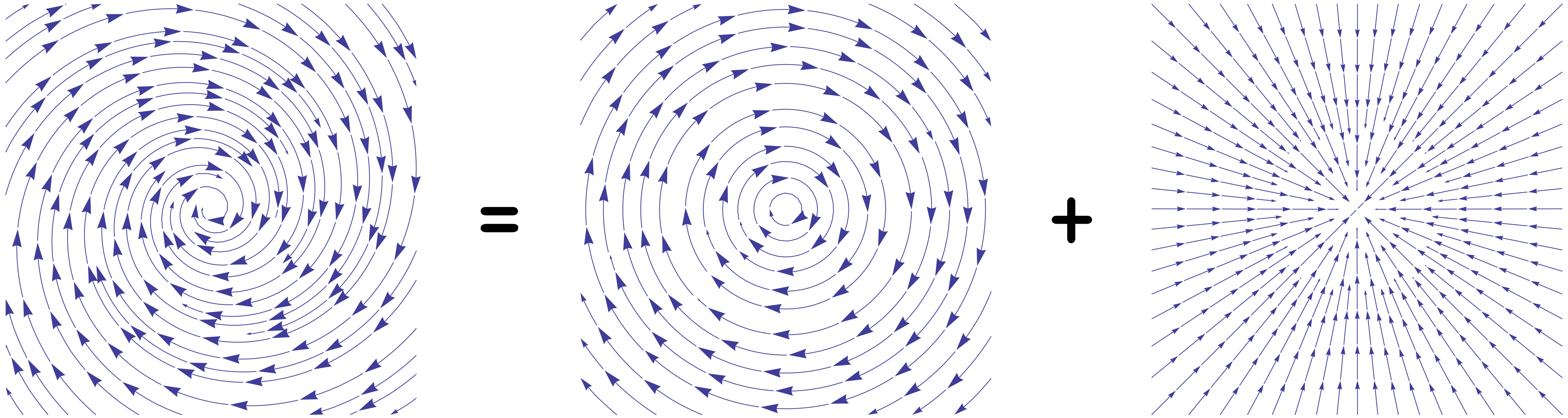
($\langle \phi^I \rangle_{\rm eq} = x^I$ preserves diagonal combinations)

$$\phi^I \to \xi^I(\phi) \qquad \det \frac{\partial \xi^I}{\partial \phi^J} = 1$$ fluid vs solid

Action: $$S = \int d^4x\, F(b) \qquad b = \sqrt{\det \partial_\mu \phi^I \partial^\mu \phi^J}$$

(Dubovsky, Gregoire, Nicolis, Rattazzi 2005)

Vortex-sound decomposition



$$\phi^I(\vec{x},t) = \phi_0^I(\vec{x},t) + \underbrace{\psi^I}_{\text{compression}}(\vec{x},t)$$

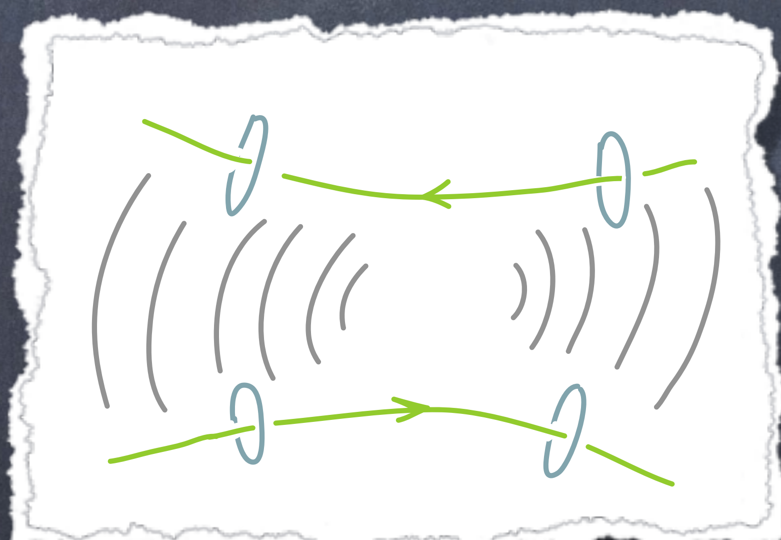
$$\det \frac{\partial \phi_0^I}{\partial x^j} = 1$$

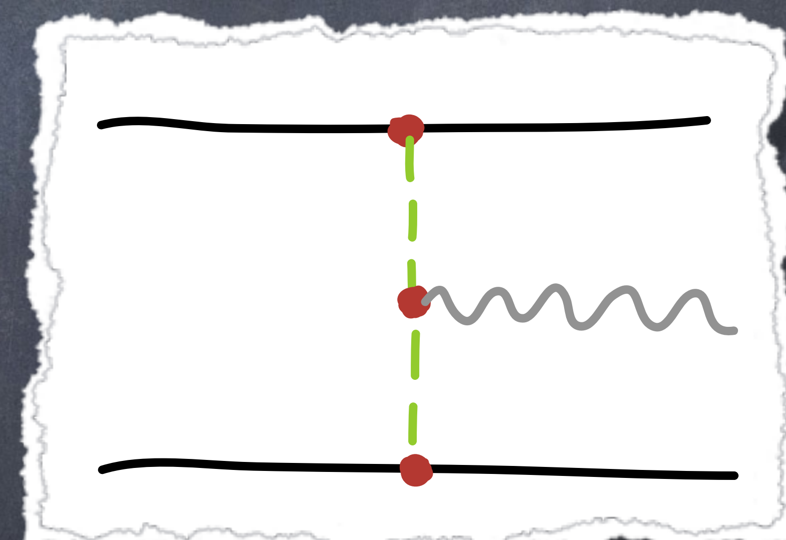
Expand the action in powers of ψ and v_0/c_s

Leading interaction

$$\mathcal{L} = \int d^3x \, (\vec{\nabla} \times \vec{A})^i ((\vec{\nabla} \times \vec{A}) \cdot \vec{\nabla}) \psi^i + \ldots$$

Ex: sound emission in vortex ring collisions

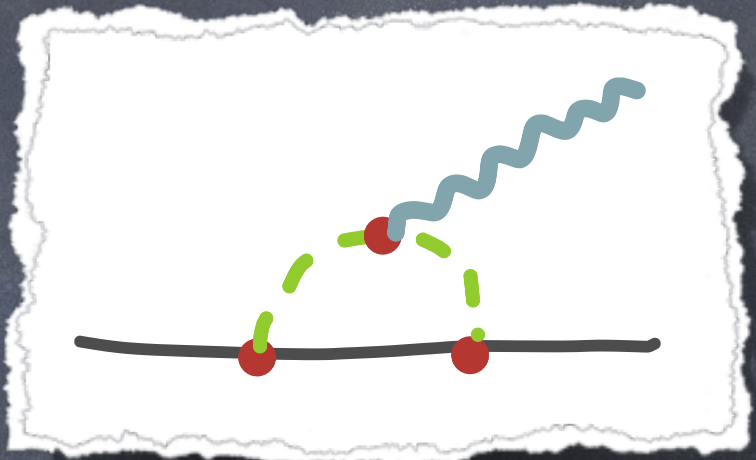




$$P = \frac{21}{2\pi} \frac{\rho (R_1^2 \Gamma_1)^2 (R_2^2 \Gamma_2)^2 v^4}{c_s^5 r^{10}(t)} \sim E_{\text{kin}} \omega \cdot (R/r)^{10} \cdot (v/c_s)^5$$

More promising (experimentally)

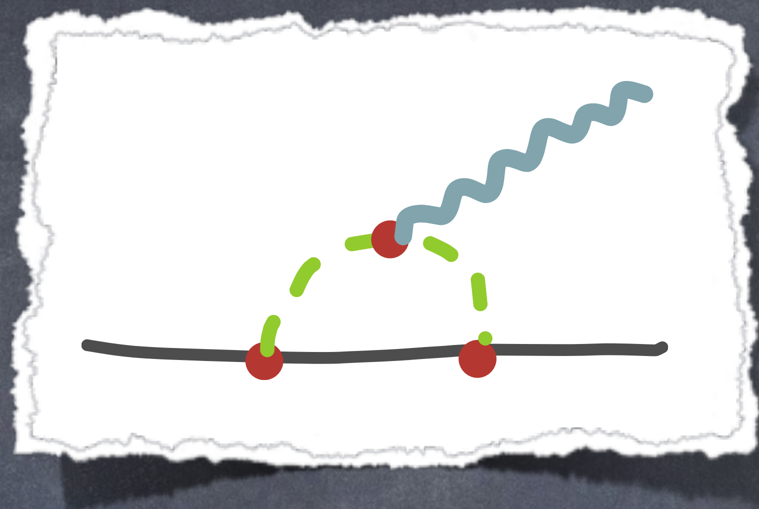
oscillating vortex ring



$$P = \frac{\rho\Gamma^4}{480\pi c_s^5}\left[\frac{1}{4}(|A_1|^2 + |A_{-1}|^2)\,\omega_1^2 + (|A_2|^2 + |A_{-2}|^2)\,\omega_2^2\right]\log^2 R/a$$

$$\sim E_{\text{kin}}\omega \cdot (A/R)^2 \cdot (v/c_s)^5$$

(Mitsou, Garcia-Saenz)

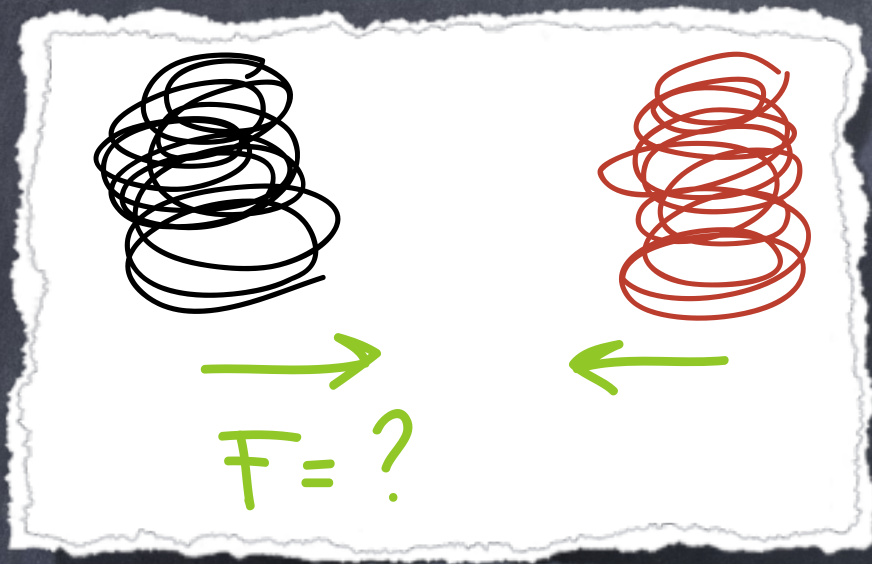
In fact:  log divergent

⇨ localized line-sound running coupling

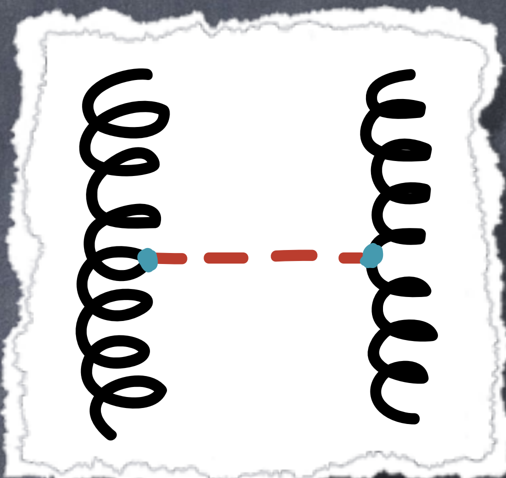
$$c(\mu)\int d\lambda \left|\partial_\lambda \vec{X}\right| \vec{\nabla}\cdot\vec{\psi}$$

(in progress…)

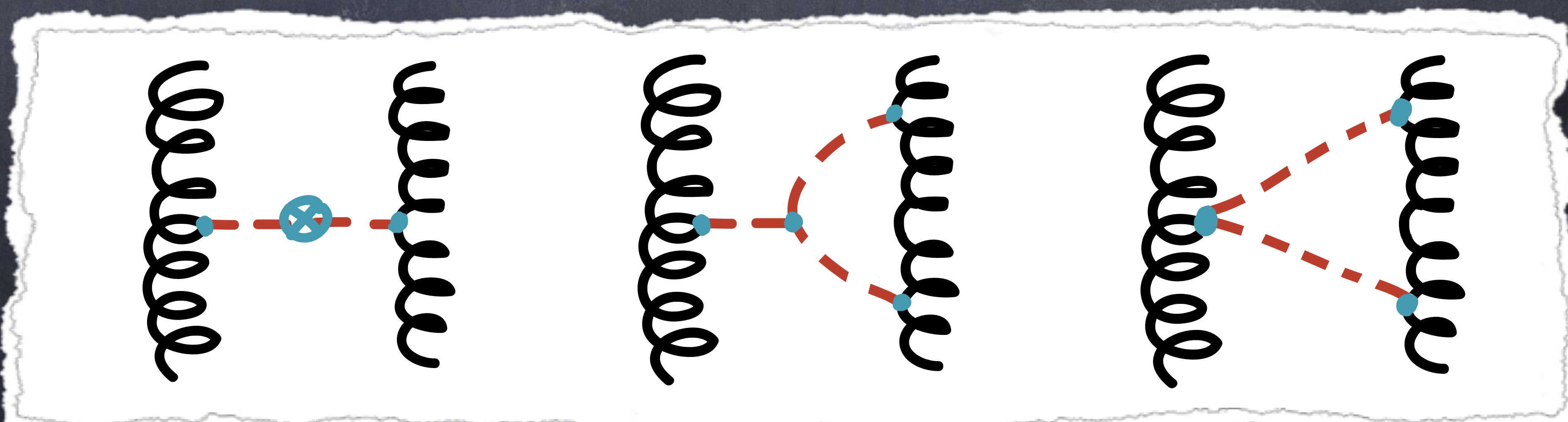
Sound mediated vortex-vortex potential



Leading order



Next to leading order



Long range potential:

$$V \sim \frac{\rho}{c_s^2} \cdot \frac{q_1 q_2}{r^3} \sim V_{\rm dipole} \times \left(v/c_s\right)^2$$

$$q \equiv \int_{\rm vortex} d^3x \, v^2$$

Detectable?

potential ⇨ force $F = -\partial_r V$, right?

NO. $V \to \Delta\mathcal{L} \to \Delta(\text{eom})$

potentially detectable for $V_{\rm dipole} \to 0$

Who is this hydrophoton anyway?

For $c_s \to \infty$: $\nabla^2 \vec{A} = \ldots$

$\Longrightarrow$ instantaneous propagation

For finite cs, propagation at cs ?

Hydrophoton = sound ?

Yet: $\vec{\nabla} \cdot \vec{A} = 0$ vs. $\vec{\nabla} \times \vec{\psi} = 0$

unsuppressed interactions vs. suppressed by $(v/c_s)^{\#}$

?

Relativistic generalization

superfluid $\phi(\vec{x}, t) = \mu t + \ldots$

$$\mathcal{L}_{\text{bulk}} = P\big((\partial\phi)^2 \big)$$

to couple to defects: $d\mathcal{A}_{(2)} \propto \star d\phi$

$$P(d\phi) \quad \leftrightarrow \quad F(d\mathcal{A})$$

in some gauge: $\mathcal{A}_{[0i]} = A_i$

$$\mathcal{A}_{[ij]} = \epsilon_{ijk}(x^k + \psi^k)$$

$$\vec{A}, \vec{\psi} \quad \to \quad \mathcal{A}_{[\mu\nu]}$$

$$S \to \int d\sigma d\tau\, \mathcal{A}_{\mu\nu}\, \partial_\sigma X^\mu \partial_\tau X^\nu + \int d^4x\, F(d\mathcal{A}) + S'_{\mathrm{NG}}$$

S'_{NG} more general than standard NG:

$$g_{\alpha\beta} = \eta_{\mu\nu}\, \partial_\alpha X^\mu \partial_\beta X^\nu \to g_{\alpha\beta}, h_{\alpha\beta}, \ldots$$

$$h_{\alpha\beta} = (\eta_{\mu\nu} + \gamma\, u_\mu u_\nu)\, \partial_\alpha X^\mu \partial_\beta X^\nu$$

$$(\, u \sim d\phi \sim \star d\mathcal{A}\,)$$

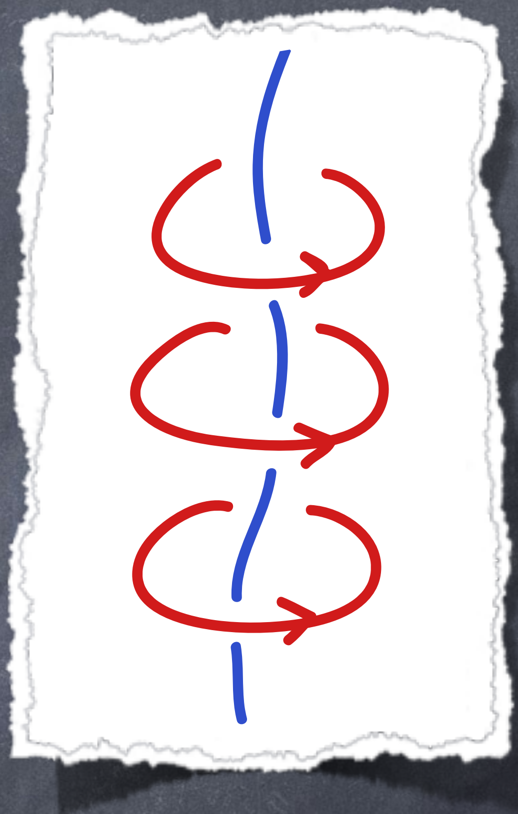
$$S'_{\mathrm{NG}} = \int d\sigma d\tau\, \sqrt{-h}\, G\Big(\frac{\sqrt{-g}}{\sqrt{-h}}, (d\mathcal{A})^2\Big)$$

Within the EFT, perturbative expansion in t-derivatives:

$$v(r \sim a) \lesssim c_s < c$$

$$v(\ell) \sim \Gamma/\ell \sim c_s \cdot (a/\ell) \ll c_s$$

$$\partial_t X \ll c_s \cdot \partial_\lambda X$$



Quantum effects

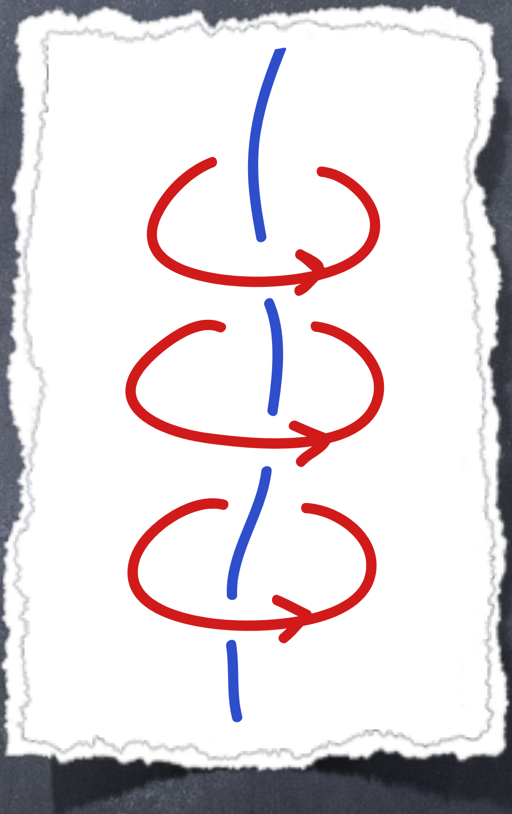
(work in progress)

Virtual phonons

Sound production/exchange suppressed by $(v/c_s)^{\#}$

For liquid helium: $c_s \sim 200\ \mathrm{m/s}$

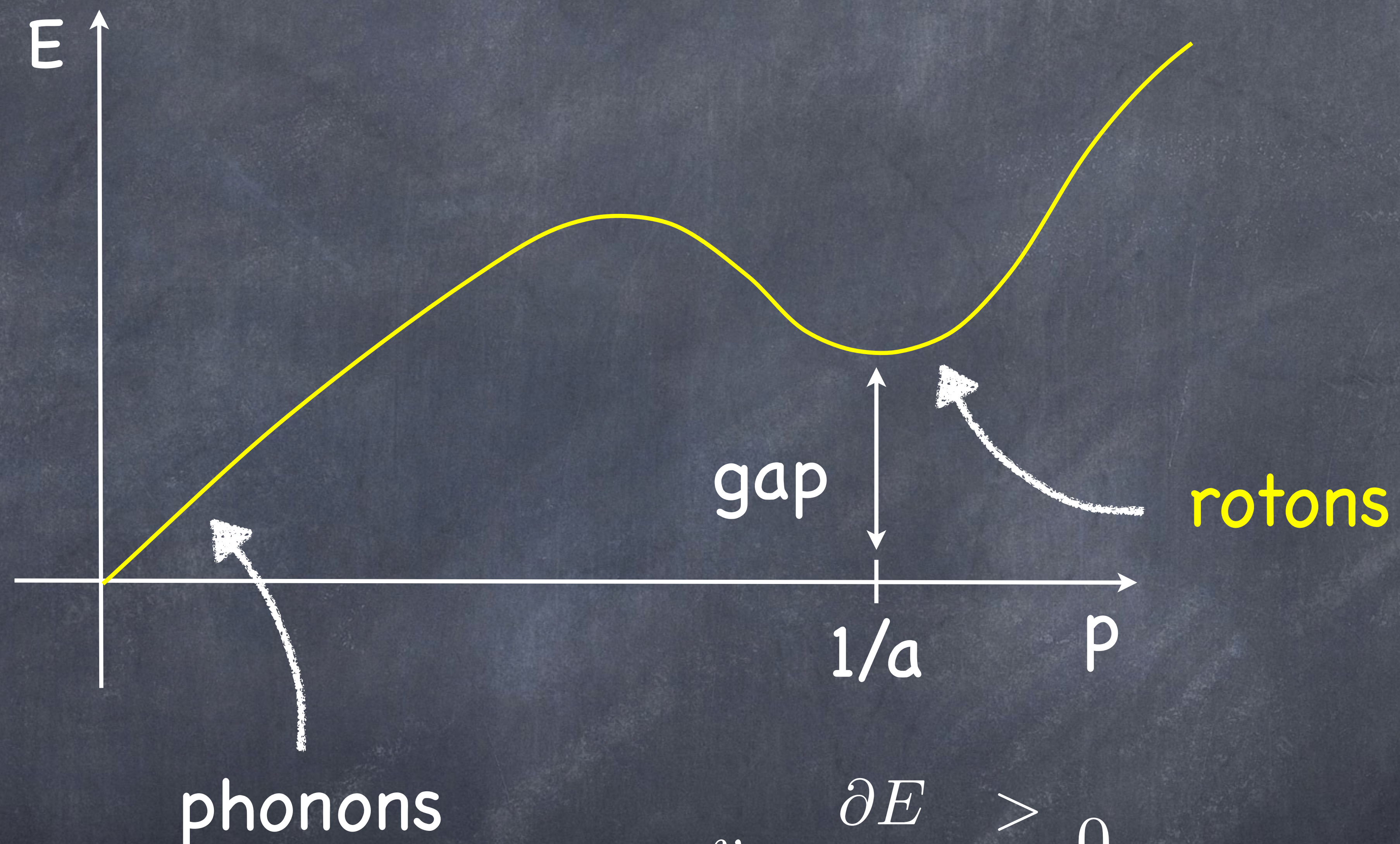
$\Gamma \sim .1\ \mathrm{mm}^2/\mathrm{s}$



$\Longrightarrow$ $v \sim c_s$ at $r \sim a$

Measurable perturbative effects at $r \gg a$?

Rotons in Helium 4



$$v = \frac{\partial E}{\partial p} \gtreqless 0$$

usually thought of as microscopic vortex rings.
can we check? what does it mean?

(Donnelly 1997)

Summary

- Vortex lines and rings: very unconventional mechanical systems
- Important degrees in freedom in superfluids
- EFT: efficient tool to understand them
- Only tool to couple them to sound
- Looking forward to experiments