Nuclear Symmetry Energy in QCD degree of freedom

Phys. Rev. C87 (2013) 015204 ([arXiv:1209.0080]())
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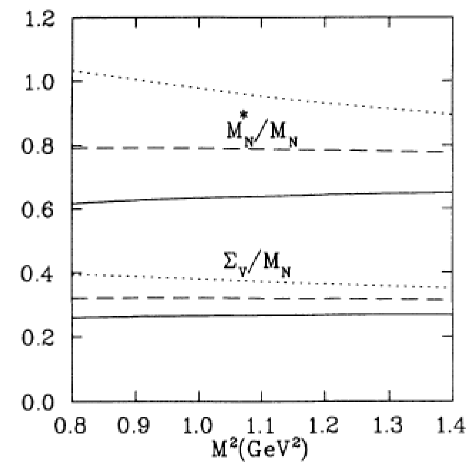
Motivation and Outline

- Nuclear phenomenology – QCD sum rule

$$U \simeq S + V\gamma^0$$

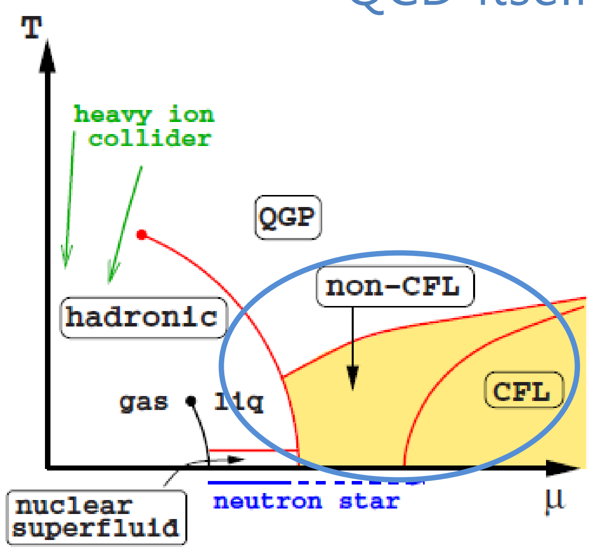
$$G(q) = \frac{1}{\not{q} - M_n - \Sigma(q)} \to \lambda^2 \frac{\not{q} + M^* - \not{u}\Sigma_v}{(q_0 - E_q)(q_0 - \bar{E}_0)}$$

$$M^* < M_n \qquad \Sigma_v > 0$$



Physical Review C49, 464 (1993)
(Thomas Cohen et al.)

- Extremely high density matter?
  – QCD itself is main dynamics



Rev. Mod. Phys. 80, 1455 (2008) (M. G. Alford et al.)

Cold matter Symmetry Energy from

$$\mathcal{Z}_\Omega = \mathrm{Tr}\exp\left[-\beta(\hat{H} - \vec{\mu}\cdot\hat{\vec{N}})\right]$$

$$= \int [D(\text{fields})] \exp\left[-\int_0^\beta d\tau \int x^3 \mathcal{L}_E(\text{fields})\right]$$

Hard Dense Loop resummation
Color BCS pairing

Nuclear Symmetry Energy

- From equation of state

Bethe-Weisaker formula

$$m_{tot} = Nm_n + Zm_p - E_B/c^2$$

$$E_B = a_V A - a_S A^{\frac{2}{3}} - a_C(Z(Z-1))A^{-\frac{1}{3}} - a_A I^2 A + \delta(A,Z)$$

$$I = (N-Z)/A$$

In continuous matter

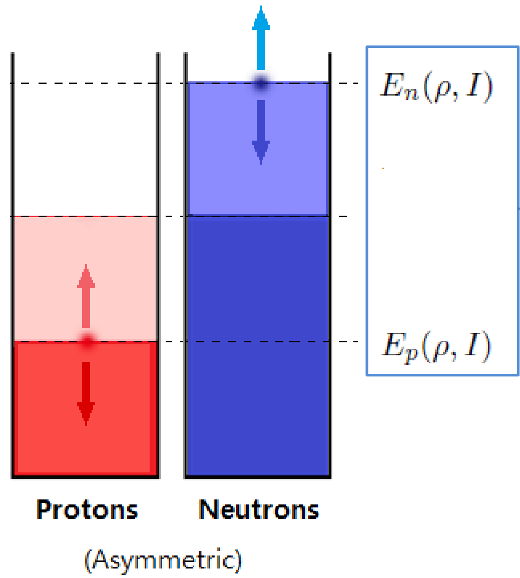
$$\bar{E}(\rho_N, I) = \bar{E}(\rho_N) + \bar{E}_{sym}(\rho_N) I^2 + \cdots$$

$$I = (\rho_n - \rho_p)/\rho$$

$$\overline{E} = \frac{1}{\int d^3k_n d^3k_p} \int d^3k_n d^3k_p E(\rho_n, \rho_p)$$

$$\Rightarrow E_{sym} = \frac{1}{2I} \cdot (\bar{E}_n - \bar{E}_p)$$ (Up to linear density order)

- Quasi-nucleon on the asymmetric Fermi sea



- RMFT propagator

$$G(q) = -i \int d^4x e^{iqx} \langle \Psi_0 | \mathrm{T}[\psi(x)\bar{\psi}(0)] | \Psi_0 \rangle = \frac{1}{\not{q} - M_n - \Sigma(q)} \to \lambda^2 \frac{\not{q} + M^* - \not{u}\Sigma_v}{(q_0 - E_q)(q_0 - \bar{E}_0)}$$

QCD Sum Rule

- Correlation function

$$\Pi(q) \equiv i \int d^4x\, e^{iqx} \langle \Psi_0 | \mathrm{T}[\eta(x)\bar{\eta}(0)] | \Psi_0 \rangle$$
$$= \Pi_s(q^2, q\cdot u) + \Pi_q(q^2, q\cdot u)\not{q} + \Pi_u(q^2, q\cdot u)\not{u}$$

$$\eta(x) = \epsilon_{abc}[u_a^T(x) C\gamma_\mu u_b(x)]\gamma_5\gamma^\mu d_c(x)$$

Ioffe's interpolating field for proton

- Energy dispersion relation and OPE

$$\Pi_i(q_0, |\vec{q}|) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} d\omega \frac{2\mathrm{Im}\Pi_i(\omega, |\vec{q}|)}{\omega - q_0} + \text{polynomials}$$

Contains **all possible hadronic resonance states** in **QCD degree of freedom**

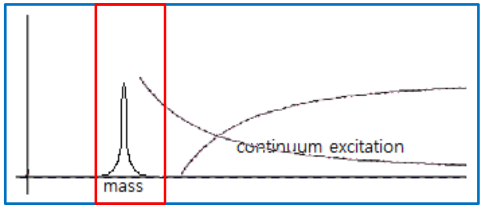
- Phenomenological ansatz in **hadronic degree of freedom**

$$\Pi(q_0, |\vec{q}|) \sim \frac{1}{(q^\mu - \tilde{\Sigma}_v^\mu)\gamma_\mu - M_N^*}$$

Equating both sides, hadronic quantum number can be expressed in QCD degree of freedom

- Weighting - Borel transformation

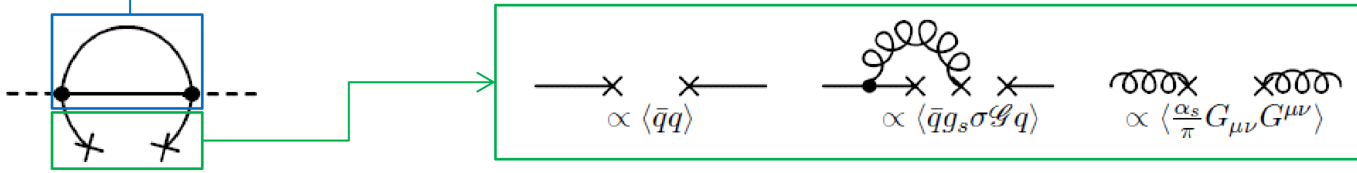
$$\mathcal{B}[\Pi_i(q_0, |\vec{q}|)] \equiv \lim_{\substack{-q_0^2, n\to\infty \\ -q_0^2/n = M^2}} \frac{(-q_0^2)^{n+1}}{n!}\left(\frac{\partial}{\partial q_0^2}\right)^n \Pi_i(q_0, |\vec{q}|)$$



QCD Sum Rule

- Operator Product Expansion

$$\Pi_i(q_0,|\vec{q}|) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} d\omega \frac{2\mathrm{Im}\Pi_i(\omega,|\vec{q}|)}{\omega - q_0} + \text{polynomials} = \sum_n C_n^i(q^2,q_0^2)\langle \hat{O}_n\rangle_{\rho,I}$$



Non-perturbative contribution

- In-medium condensate near normal nuclear density

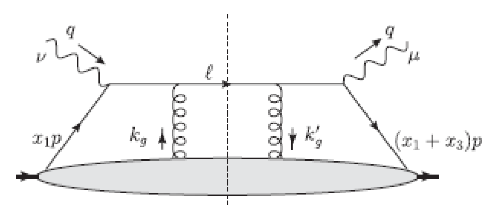
$$\langle \hat{O}_{u,d}\rangle_{\rho,I} = \langle \hat{O}_{u,d}\rangle_{\text{vac}} + (\langle p|\hat{O}_0|p\rangle \mp \langle p|\hat{O}_1|p\rangle I)\rho$$

Medium property can be accounted by nucleon expectation value x density

- Multi-quark operators (twist-4)

$$\langle(\bar{q}_1\gamma^\alpha\gamma_5 t^A q_1)(\bar{q}_2\gamma^\beta\gamma_5 t^A q_2)\rangle_p$$
$$\langle(\bar{q}_1\gamma^\alpha t^A q_1)(\bar{q}_2\gamma^\beta t^A q_2)\rangle_p$$

can be estimated from DIS experiments data

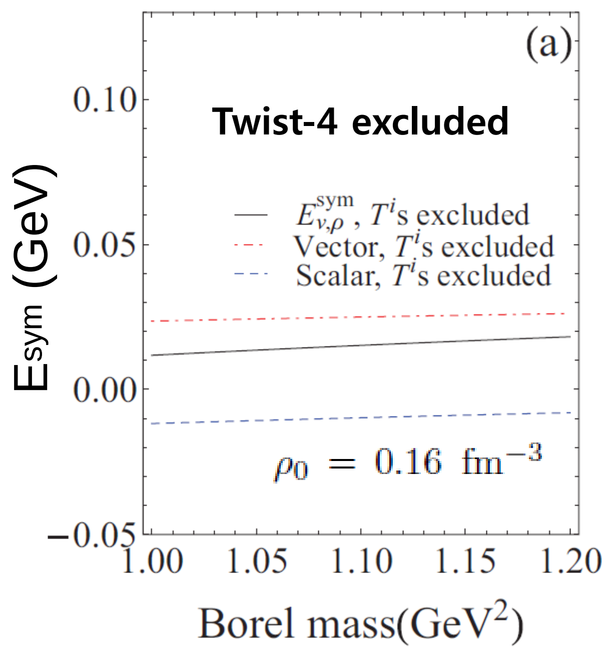


(a)

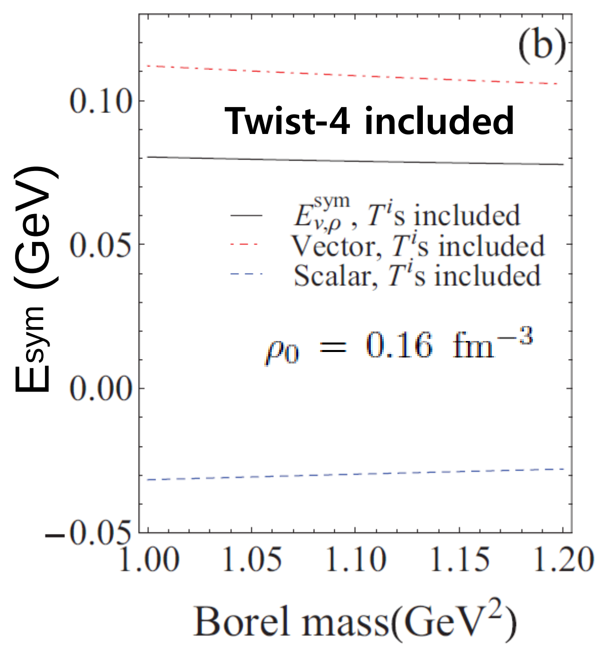
Nuclear Symmetry Energy from QCD SR

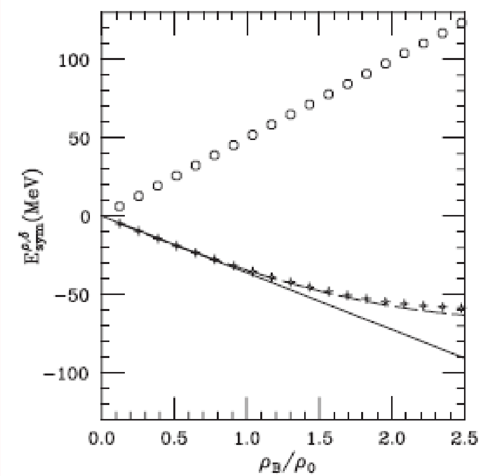
- **Iso-vector** scalar / vector decomposition

$\langle \bar{d}d \rangle - \langle \bar{u}u \rangle \quad \langle d^\dagger d \rangle - \langle u^\dagger u \rangle$



$\langle (\bar{d}\Gamma^\mu d)(\bar{d}\Gamma^\nu d) \rangle - \langle (\bar{u}\Gamma^\mu u)(\bar{u}\Gamma^\mu u) \rangle$





Physics Report, 410, 335 (2005)
(V. Baran et al.)

- RMFT result

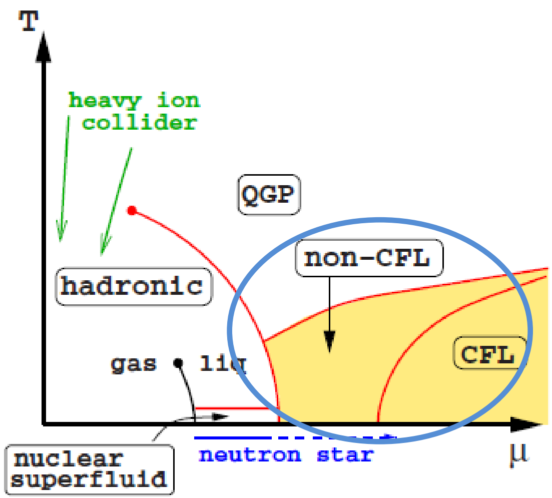
$$E_V^{sym} = \frac{1}{2}\left[f_\rho - f_\delta \left(\frac{m^*}{E_F^*} \right) \right] \rho_B$$

Iso-vector meson exchange

High density behavior
-> high density dependence of condensates

At extremely high density?

- QCD phase transition



- In $1/\mu \ll 1/\Lambda_{\text{QCD}}$ region, **QCD** can be immediately applicable
- Statistical partition function for dense QCD

$$\mathcal{Z}_\Omega = \mathrm{Tr}\exp\left[-\beta(\hat{H} - \vec{\mu}\cdot\hat{\vec{N}})\right] = \int [D(\text{fields})]\exp\left[-\int_0^\beta d\tau \int x^3 \mathcal{L}_E(\text{fields})\right]$$

- Normal QM phase - BCS paired phase

- Euclidean Lagrangian for dense QCD **at normal phase**

$$\mathcal{L}_E = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \frac{1}{2\xi}(\partial_\mu A^a_\mu)^2 + \bar{\eta}^a(\partial^2\delta_{ab} + g f_{abc}\partial_\mu A^c_\mu)\eta^b + \sum_f^{n_f}\left[\psi_f^\dagger\partial_\tau\psi_f + \bar{\psi}_f(-i\gamma^i\partial_i + m_f)\psi_f - \mu_f\psi_f^\dagger\psi_f - g\bar{\psi}_f \not{A}\psi_f\right]$$

$$\int\frac{d^4Q}{(2\pi)^4} \equiv T\sum_n\int\frac{d^3q}{(2\pi)^3}, \quad Q_\mu = (-\omega, \vec{q})$$

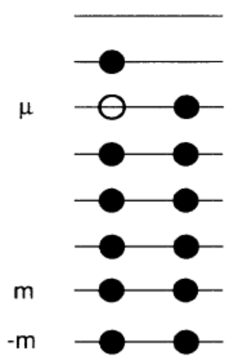
$\omega_n = (2n+1)\pi/\beta$ (For fermion)

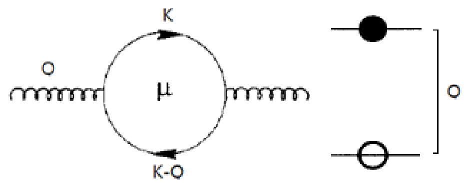
$\omega_n = 2n\pi/\beta$ (For boson)

Continuous energy integration -> Discrete sum over Matsubara frequency

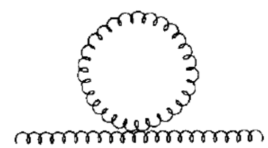
Hard Dense Loop resumation

- Quark-hole excitation is dominant ($Q \sim T \leq g\mu$)





Quark-anti hole contributes to transverse mode



Gluon/Ghost contribution is negligible at $Q \sim T \leq g\mu$

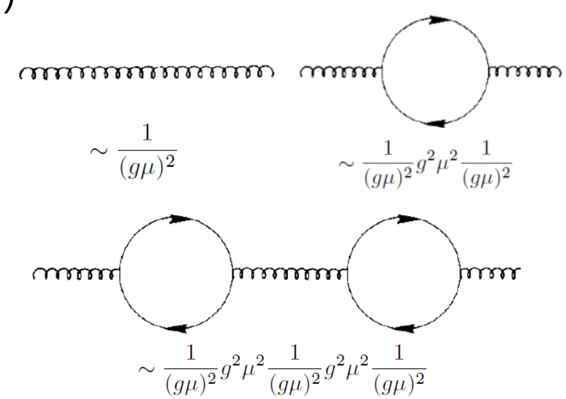
- Gluon self energy in cold matter ($Q \sim T \leq g\mu$)

$$\Pi_{\mu\nu}(Q) = g^2 \mathrm{Tr}\left[\gamma_\mu S_F(K)\gamma_\nu S_F(K-Q)\right]$$

$$= \frac{1}{2}g^2\left(\sum_f \frac{\mu_f^2}{\pi^2}\right)\int \frac{d\Omega}{4\pi}\left(\delta_{\mu 4}\delta_{\nu 4} + \hat{K}_\mu \hat{K}_\nu \frac{i\omega}{Q\cdot\hat{K}}\right)$$

$\sim g^2\mu^2$ order

Phys. Rev. D.53.5866 (1996) C. Manuel
Phys. Rev. D.48.1390 (1993) J. P. Blaizot and J. Y. Ollitrault



All equivalent 1PI diagrams should be resumed!

Hard Dense Loop resumation

- Projection along polarization

Euclidean propagator

$$*D_{\mu\nu} = \frac{1}{Q^2+\delta\Pi^L}P^L_{\mu\nu} + \frac{1}{Q^2+\delta\Pi^T}P^T_{\mu\nu} + \frac{1}{f_e}\frac{Q_\mu Q_\nu}{Q^2}$$

$$P^T_{ij} = \delta_{ij} - \hat{q}_i\hat{q}_j, P^T_{44} = P^T_{4i} = 0$$
$$P^L_{\mu\nu} = \delta_{\mu\nu} - \frac{Q_\mu Q_\nu}{Q^2} - P^T_{\mu\nu}$$

Longitudinal and transverse part

$$\delta\Pi^L(Q) = 2\sum_f \left(\frac{1}{2}g^2\frac{\mu_f^2}{\pi^2}\right)\frac{Q^2}{q^2}\left(1-\left(\frac{i\omega}{q}\right)Q_0\left(\frac{i\omega}{q}\right)\right)$$

In w->0 limit $\Rightarrow 2m_g^2 = g^2\frac{\mu_f^2}{\pi^2}$

$$\delta\Pi^T(Q) = \sum_f \left(\frac{1}{2}g^2\frac{\mu_f^2}{\pi^2}\right)\left(\frac{i\omega}{q}\right)\left[\left(1-\left(\frac{i\omega}{q}\right)^2\right)Q_0\left(\frac{i\omega}{q}\right)+\left(\frac{i\omega}{q}\right)\right]$$

$\Rightarrow 0$

- Debye mass and effective Lagrangian

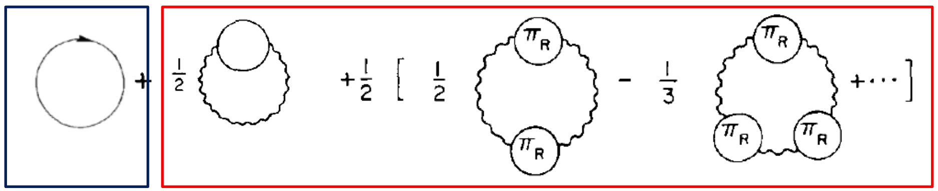
Effective Lagrangian for soft gluon in cold dense matter

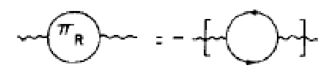
$$\mathcal{L} = -\frac{1}{4}F^2 \rightarrow \frac{1}{2}A_\mu(-Q^2 g^{\mu\nu} + 2m_g^2 P_L^{\mu\nu} + O(\omega/q)P_T^{\mu\nu} + \cdots)A_\nu$$

Debye mass from hard(dense) quark loop

HDL resumed thermodynamic potential

- Relevant ring diagrams



Where 
in $T \lesssim g\mu$ limit

$$\Omega(\mu) = -\left[\frac{1}{4}\sum_f \frac{\mu_f^4}{\pi^2} + \alpha_s^2 \frac{2}{\pi}\left(\sum_f \frac{\mu_f^2}{\pi^2}\right)^2 \left[\Lambda_1 - \Lambda_2 - \alpha \ln 2 - \ln\left(\sum_f \frac{\mu_f^2}{\pi^2}\frac{1}{\bar{\mu}^2}\right)\Lambda_1\right] - \alpha_s^2 \ln \alpha_s \frac{2}{\pi}\left(\sum_f \frac{\mu_f^2}{\pi^2}\right)^2 \Lambda_1\right]$$

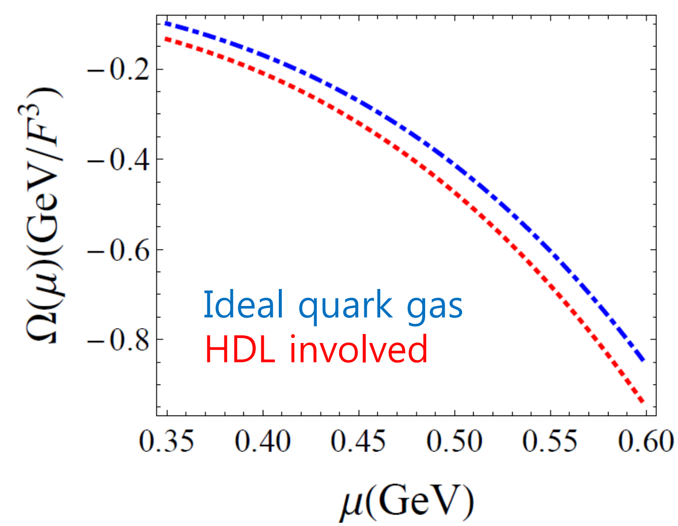
- Thermodynamic quantities can be obtained from $\Omega(\mu)$

$$\Omega(\mu) = \langle \hat{H} \rangle - \vec{\mu} \cdot \langle \hat{\vec{N}} \rangle = -\frac{1}{\beta V} \ln \mathcal{Z}_\Omega,$$

$$\rho_i(\mu) = \frac{\langle \hat{N}_i \rangle}{V} = \frac{1}{\beta V}\frac{\partial}{\partial \mu_i} \ln \mathcal{Z}_\Omega,$$

$$\epsilon(\mu) = \frac{\langle \hat{H} \rangle}{V} = -\frac{1}{V}\left(\frac{\partial}{\partial \beta} - \frac{1}{\beta}\vec{\mu} \cdot \frac{\partial}{\partial \vec{\mu}}\right) \ln \mathcal{Z}_\Omega,$$

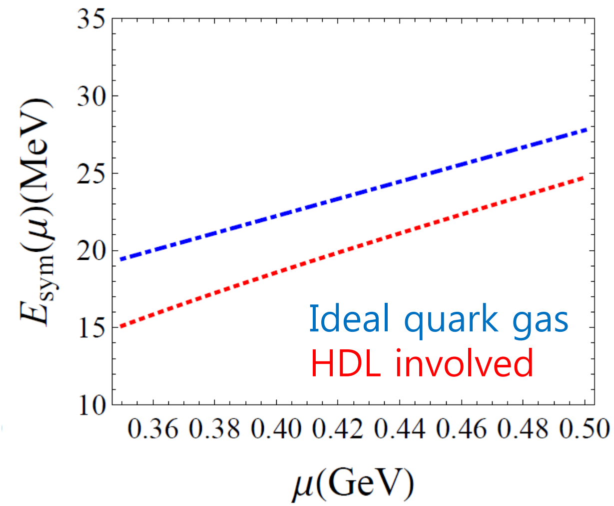
$$I_B = \frac{\rho_3}{\rho_B} = 3\frac{\rho_u - \rho_d}{\rho_u + \rho_d} \qquad \mu_d^u = \mu\left(1 \pm \frac{1}{3} I_B\right)^{\frac{1}{3}}$$



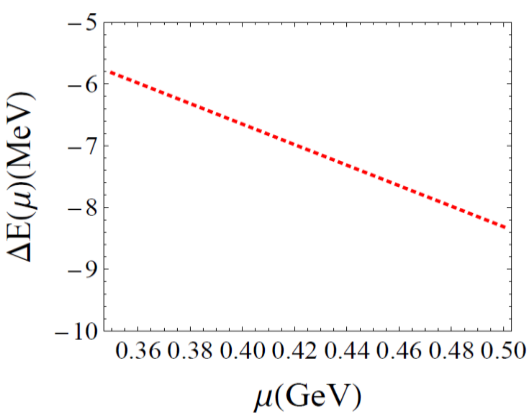
Symmetry Energy at normal phase

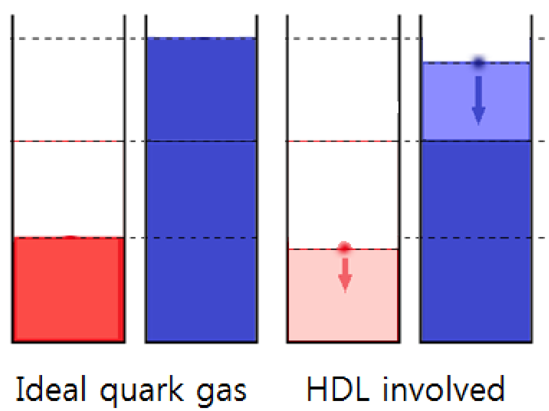
- **Symmetry Energy**

$$\frac{\epsilon(\mu,I_B)}{\rho(\mu,I_B)} = \frac{E(\mu,I_B)}{N_B}$$
$$= \bar{E}(\mu,I_B) = \bar{E}(\mu) + \bar{E}_{sym}(\mu)I_B^2 + \cdots$$
$$\bar{E}_{sym}(\mu) = \frac{1}{2!}\frac{\partial^2}{\partial I_B^2}\bar{E}(\mu,I_B).$$
$$= \boxed{\widetilde{E}^{q,0}_{sym}(\mu)} - \boxed{\widetilde{E}^{g,HDL}_{sym}(\mu)}$$



- HDL correction suppress Quasi-Fermi sea



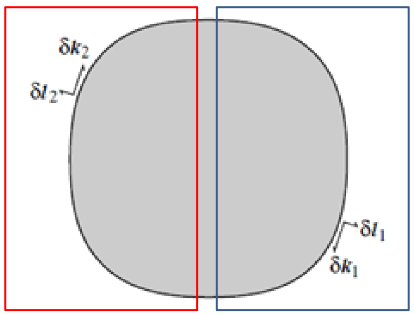


As density becomes higher,
suppression becomes stronger

The difference between quasi-Fermi seas becomes smaller
-> Costs less energy than ideal gas
-> **Reduced symmetry energy**

Color Superconductivity

- **BCS** Pairing near **Fermi sea**



- In terms of effective interaction near Fermi sea

  $$\mathcal{S}_{4q} \sim \left(\psi^\dagger(-p_f)\psi(-p_i)\right)\left(\psi^\dagger(p_f)\psi(p_i)\right)$$

  is marginal along to Fermi velocity
- Fermion – conjugated fermion interaction
- When $V<0$ two states form **a condensate** **(gap)**

- **Nambu-Gorkov Formalism**

$$\mathcal{S}_{BCS} \sim \frac{1}{2}\left[\psi(-p)^T C\Delta(p)\psi(p) + \psi(p)\tilde{\Delta}(p)C\bar{\psi}^T(-p)\right]$$

Inverse propagator of $\Psi = (\psi, \overline{\psi}^T)$

$$S^{-1}(q) = \begin{pmatrix} \not{q} + \not{\mu} - m & \overline{\Delta} \\ \Delta & (\not{q} - \not{\mu} + m)^T \end{pmatrix}$$

Diagrammatically described gapped quasi-state

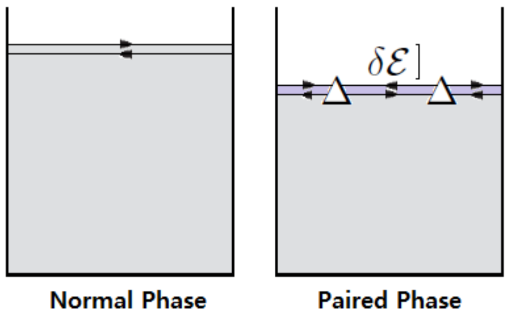
->

$$S_\Delta(l) = \frac{l_0 + l_v}{l_0^2 - l_v^2 - \Delta^2}\gamma_0$$

In Wilsonian high density effective formalism

Color BCS paired state

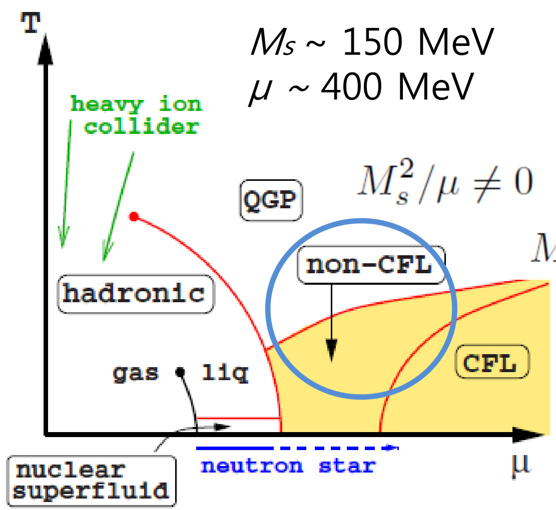
- **BCS** Pairing locks the gapped quasi-states



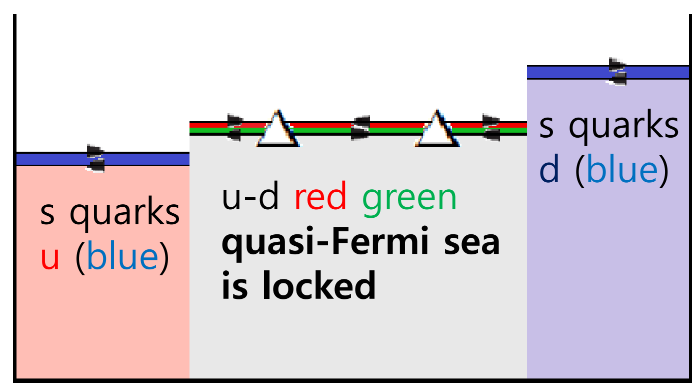
- In QCD, anti-triplet gluon exchange interaction is attractive (V<0)
- $\langle \psi_a^\alpha C\gamma_5 \psi_b^\beta \rangle \sim \Delta_1 \epsilon^{\alpha\beta 1}\epsilon_{ab1} + \Delta_2 \epsilon^{\alpha\beta 2}\epsilon_{ab2} + \boxed{\Delta_3 \epsilon^{\alpha\beta 3}\epsilon_{ab3}}$
- In non negligible M_s^2/μ, **2SC** state is favored

- **2 color superconductivity**

$M_s \sim$ 150 MeV
$\mu \sim$ 400 MeV



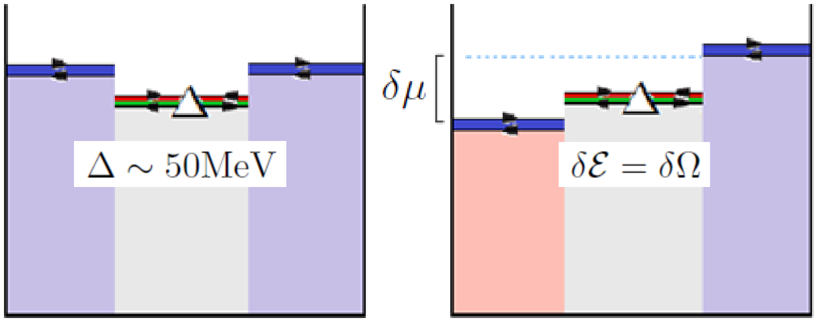
In 2SC phase, u-d red-green states are gapped



Only s quarks and u-d blue quarks are liberal

Asymmetrizing in 2SC phase

- **Only** **Blue state (1/3)** can affect iso-spin asymmetry

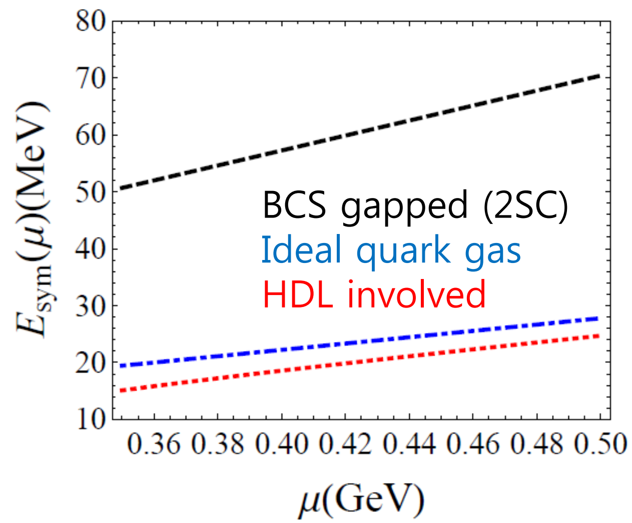


- BCS phase remains in $\delta\mu < (1/\sqrt{2})\Delta \sim \Lambda$
 (Phys. Rev. Lett. 9, 266 (1962) A. M. Clogston)
- Only u-d blue states can be asymmetrized
- The other 4 gapped quasi-states are **locked**

- In HDET formalism $\delta\mathcal{E} = \delta\Omega = -\sum_{ud,rg} \frac{\mu_f^2}{\pi^2}\left[\Lambda\sqrt{\Lambda^2+\Delta^2} + \Delta^2 \ln((\Lambda + \sqrt{\Lambda^2+\Delta^2})/\Delta)\right]$

- **Symmetry energy**

$$\frac{\epsilon(\mu,\Delta,I_{B_\Delta})}{\rho(\mu,\Delta,I_{B_\Delta})} = \frac{E(\mu,\Delta,I_{B_\Delta})}{N_{B_\Delta}}$$
$$= \bar{E}(\mu,\Delta) + \bar{E}_{sym}(\mu,\Delta) I_{B_\Delta}^2 + \cdots$$
$$\bar{E}_{sym}(\mu,\Delta) = \frac{1}{2}\frac{\partial^2}{\partial I_{B_\Delta}^2}\bar{E}_\Delta(\mu, I_{B_\Delta})$$
$$I_{B_\Delta} = 3\frac{\rho_d - \rho_u}{\rho_d + \rho_u} \times \frac{1}{3}$$



Quasi-fermion state in 2SC phase

- Meissner mass effect?

Quasi-fermion in gapped state 

$$S_\Delta(l) = \frac{l_0 + l_v}{l_0^2 - l_v^2 - \Delta^2}\gamma_0$$

Fermion in ungapped state 

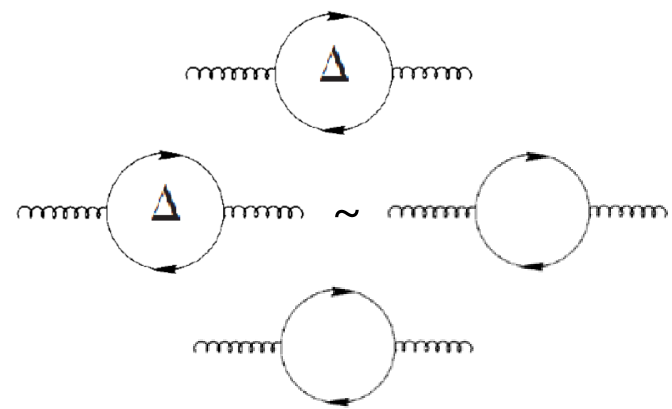
$$S(l) = \frac{l_0 + l_v}{l_0^2 - l_v^2}\gamma_0$$

- Gapped states and dense loop

1) If gap size is quite large $\Delta \sim g\mu$
-> matter loop do not have hard contribution
-> do not need resummation -> **reduction vanish**

2) But if gap size is quite small $\Delta < g\mu$
-> needs resummation -> **reduction remains**

3) For ungapped quark loop -> **reduction remains**



- High density effective Lagrangian

$$\mathcal{L}_D = \sum_{\vec{v}}{}' \left[\psi^\dagger iV\cdot D\psi - \psi^\dagger \frac{1}{2\mu + i\tilde{V}\cdot D} D_\perp^2 \psi\right]$$

Irrelevant high energy excitation has been integrated out

Incorporating Nambu-Gorkov formalism

- By introducing **Nambu-Gorkov** formalism (G. Nardulli et al, PLB524.144)

Invariant coupling and gap Lagrangian $\langle \psi_a^\alpha C\gamma_5 \psi_b^\beta \rangle \sim \Delta_1 \epsilon^{\alpha\beta 1}\epsilon_{ab1} + \Delta_2 \epsilon^{\alpha\beta 2}\epsilon_{ab2} + \Delta_3 \epsilon^{\alpha\beta 3}\epsilon_{ab3}$

$$\mathcal{L}_\Delta = -\frac{\Delta}{2}\psi^T C\epsilon\psi\epsilon \; - (L \to R) + \text{h.c.} \qquad \langle \psi_{\alpha i}^{L\,T} C \psi_{\beta j}^L \rangle = -\langle \psi_{\alpha i}^{R\,T} C \psi_{\beta j}^R \rangle = \frac{\Delta}{2}\epsilon_{\alpha\beta 3}\epsilon_{ij3}$$

2SC description by linear combination of Gellman matrices

$$\psi_{+,\alpha i} = \sum_{A=0}^{5} \frac{(\tilde\lambda_A)_{\alpha i}}{\sqrt{2}}\psi_+^A \qquad \tilde\lambda_0 = \frac{1}{\sqrt{3}}\lambda_8 + \sqrt{\frac{2}{3}}\lambda_0; \;\; \tilde\lambda_A = \lambda_A\,(A=1,2,3); \; \tilde\lambda_4 = \frac{\lambda_{4-i5}}{\sqrt{2}}; \; \tilde\lambda_5 = \frac{\lambda_{6-i7}}{\sqrt{2}}$$

With 2 component **Nambu-Gorkov** field

$$\chi = \begin{pmatrix} \psi_+ \\ C\psi_-^* \end{pmatrix}$$

Here + and – represents Fermi velocity, not energy eigenstate

Nambu-Gorkov type description can be written as

$$\mathcal{L}_D \;=\; \sum_{\vec v}\sum_{A,B=0}^{5} \chi^{A\dagger}\begin{pmatrix} iTr[\tilde T_A V\cdot D\tilde T_B] & \Delta_{AB} \\ \Delta_{AB} & iTr[\tilde T_A^* \tilde V\cdot D^* \tilde T_B^*] \end{pmatrix}\chi^B \;+\; (L\to R)\;.$$

$$\begin{aligned} \Delta_{AB} &= \frac{\Delta}{2}Tr[\epsilon\sigma_A^T\epsilon\sigma_B] \qquad (A,B=0,...3) \\ \Delta_{AB} &= 0 \qquad\qquad\qquad\quad (A,B=4,5)\;. \end{aligned} \qquad \tilde T_A = \frac{\tilde\lambda_A}{\sqrt{2}} \qquad (A=0,...,5)$$

Modification of gluon self energy

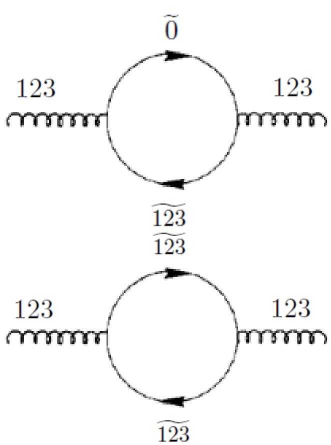
- Gluon self energy in HDET

$$i\Pi^{\mu\nu}_{ab}(p) = -2g^2 \sum_v \frac{4\pi\mu^2}{(2\pi)^4} \int d^2l \frac{1}{(p_0+l_0)^2 - (p_\parallel + l_\parallel)^2 - \Delta_C^2 + i\epsilon} \frac{1}{l_0^2 - l_\parallel^2 - \Delta_A^2 + i\epsilon}$$
$$\times \left(\bar{V}\cdot(l+p)\bar{V}\cdot l V_\mu V_\nu k_{CbA} k_{AbC} + \Delta_C \Delta_A V_\mu \bar{V}_\nu k^*_{CbA} k_{AbC} + (\bar{V} \leftrightarrow V)\right)$$

where $k_{AbC} = \frac{1}{2}\mathrm{Tr}[\tilde{\lambda}_A \tau_b \tilde{\lambda}_C]$ $\quad \tilde{\lambda}_0 = \frac{1}{\sqrt{3}}\lambda_8 + \sqrt{\frac{2}{3}}\lambda_0; \tilde{\lambda}_A = \lambda_A \ (A = 1,2,3);$ $\quad \tilde{\lambda}_4 = \frac{\lambda_{4-i5}}{\sqrt{2}}; \ \tilde{\lambda}_5 = \frac{\lambda_{6-i7}}{\sqrt{2}}$

Gapped $\qquad$ Ungapped

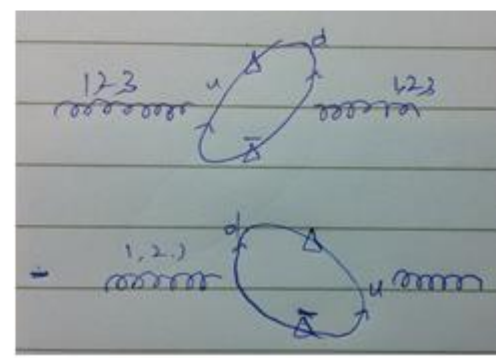
- Adjoint color 1,2,3 only couple with gapped states (0,1,2,3)



In p->0 limit, $\quad \Pi^{\mu\nu}_{ab}(0) = 0$

Gluon in adjoint 1,2,3 do not have rest masses (Debye and Meissner mass)

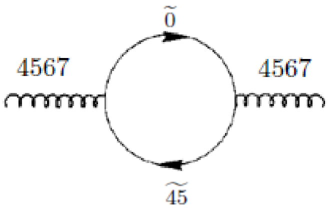
Symmetry energy do not reduced by HDL resummation

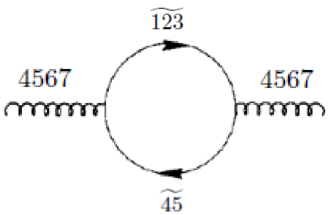


Adjoint 1,2,3 trapped in BCS gap

Modification of gluon self energy

- Adjoint color 4,5,6,7 partially couple with gapped state

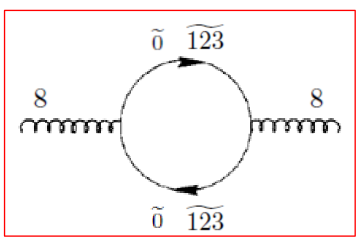


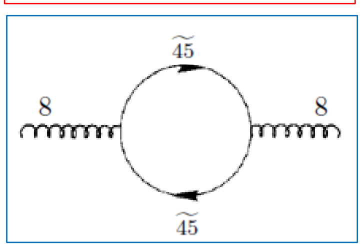


In p->0 limit, $\Pi^{00}_{ab}(0) = \frac{3}{2}m_g^2 \quad -\Pi^{ij}_{ab}(0) = \frac{1}{2}m_g^2$

Gluon in adjoint 4,5,6,7 have Debye and Meissner mass
$\Rightarrow$ Reduction from **HDL** remains
$\Rightarrow$ **HDL** from ungapped internal line reduces symmetry energy

- Adjoint color 8





In p->0 limit, $\Pi^{00}_{ab}(0) = 3m_g^2 \quad -\Pi^{ij}_{ab}(0) = \frac{1}{3}m_g^2$

Gluon in adjoint 8 have Debye and Meissner mass
$\Rightarrow$ **HDL** from gapped loop do not shift asymmetric Fermi sea
$\Rightarrow$ **HDL** from ungapped loop gives reduction

Reduction from HDL significantly dropped

Reported results (G. Nardulli et al, PLB524.144)

- Self energies have been evaluated in gradient expansion approximation

Gradient expansion

$$\Sigma^{0,\mu\nu} = k_1(V^\mu V^\nu) + k_2(V^\mu \tilde{V}^\nu) \ + \ (V \to \tilde{V}) \ ,$$

$$\Sigma^{\mu\nu} = aV^\mu V^\nu \frac{(\tilde{V}\cdot p)^2}{\Delta^2} + bV^\mu \tilde{V}^\nu \frac{V\cdot p\,\tilde{V}\cdot p}{\Delta^2} \ + \ (V \to \tilde{V}) \ .$$

Table 1
Debye and Meissner masses for the gluons in the 2SC phase

a	$\Pi^{00}(0)$	$-\Pi^{ij}(0)$
1–3	0	0
4–7	$\frac{3}{2}m_g^2$	$\frac{1}{2}m_g^2$
8	$3m_g^2$	$\frac{1}{3}m_g^2$

For example, adjoint color in 1,2,3

$$\Pi_{ij}^{00}(p) = \Pi_{ij}^{00}(0) + \delta\Pi_{ij}^{00}(p) = \delta\Pi_{ij}^{00}(p) = \delta_{ij}\frac{\mu^2 g^2}{18\pi^2\Delta^2}|\vec{p}\,|^2,$$

$$\Pi_{ij}^{kl}(p) = \Pi_{ij}^{kl}(0) + \delta\Pi_{ij}^{kl}(p) = \delta_{ij}\delta^{kl}\frac{\mu^2 g^2}{3\pi^2}\left(1 + \frac{p_0^2}{6\Delta^2}\right),$$

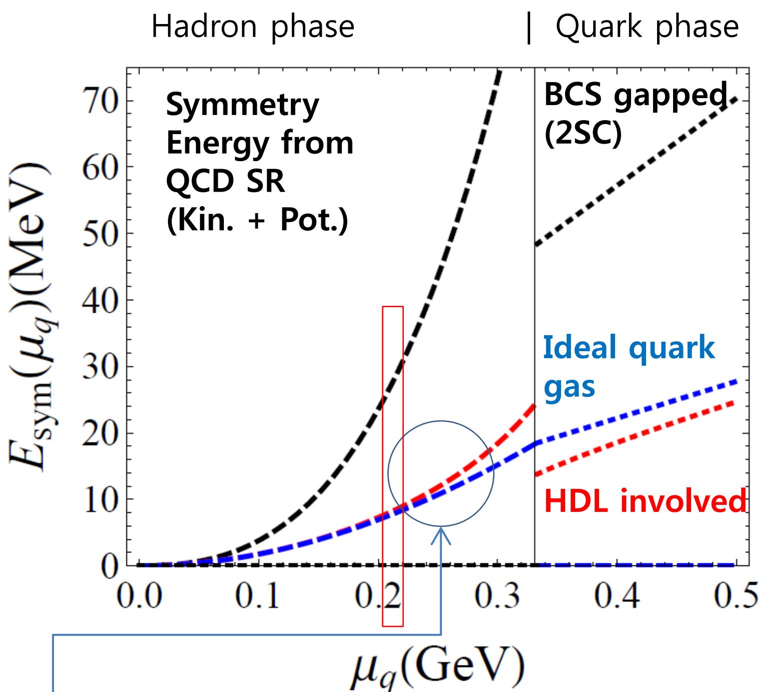
$$\Pi_{ij}^{0k}(p) = \delta\Pi_{ij}^{0k}(p) = \delta_{ij}\frac{\mu^2 g^2}{18\pi^2\Delta^2}p^0 p^k.$$

-> Has a quite large uncertainty in non-vanishing momentum

- It is needed **analytic description** for self energy to obtain correct statistics -> **Working in progress**

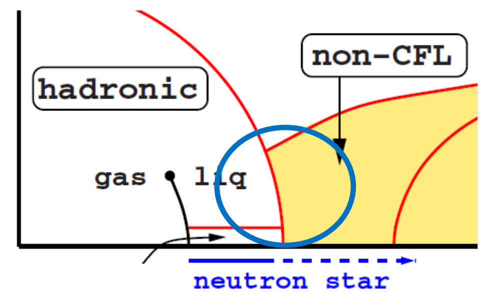
Future goals and Summary

- **Nuclear Symmetry Energy** in hadron and quark phase



Kinetic part **(Ideal nucleon gas)**
Kinetic part **(QCD SR based interaction involved)**

- Quark-hadron continuity?



Important quantum numbers? (e.g. strangeness)
-> High density behavior at hadron phase

- Including temperature

Symmetry energy in heated quark matter also may provide fruitful information

Conclusion

- For hadron phase, **Nuclear Symmetry Energy** can be described **in terms of quark and gluon condensate** via **QCD Sum rule**
- For quark phase (in T~0 limit), **Symmetry Energy of normal phase** can be calculated immediately via **thermal QCD.** The Debye mass from **HDL resummation** **reduces Symmetry Energy**
- **BCS paired states** lock the gapped quasi-states and favors symmetrized condition (**enhancing Symmetry Energy**)
- Color blocked gluon (in adjoint color 1,2,3) **do not reduce Symmetry Energy**