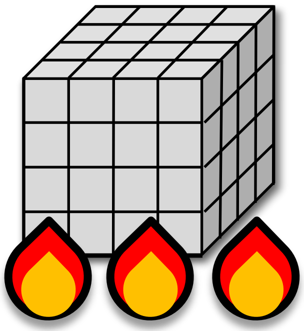
Magnetism and Rotation on the Lattice

Arata Yamamoto

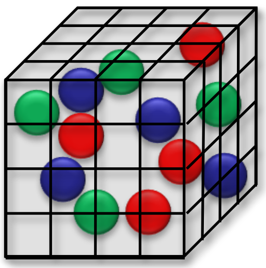
(University of Tokyo)

Extreme QCD

Finite temperature

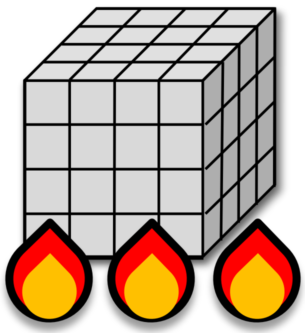


Finite density

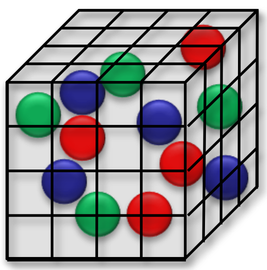


Extreme QCD

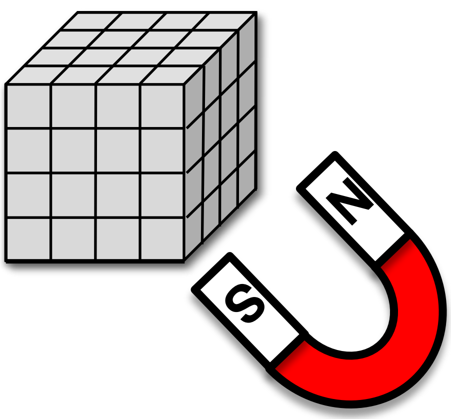
Finite temperature



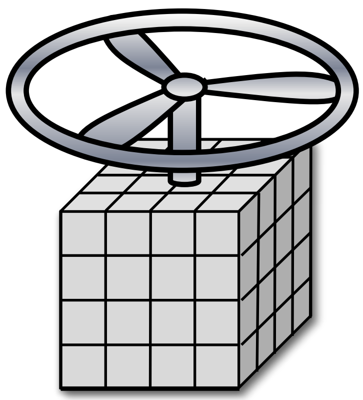
Finite density



Magnetic field

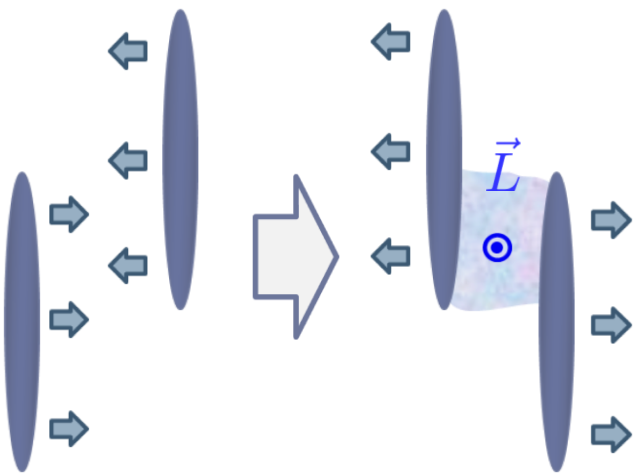


Rotation

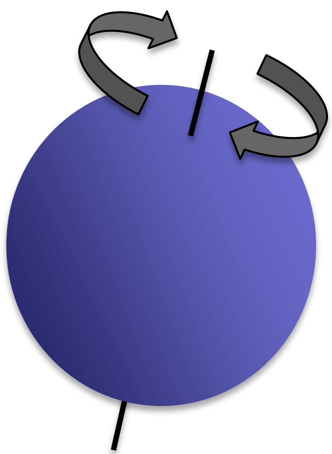


Magnetism and Rotation

peripheral heavy-ion collision



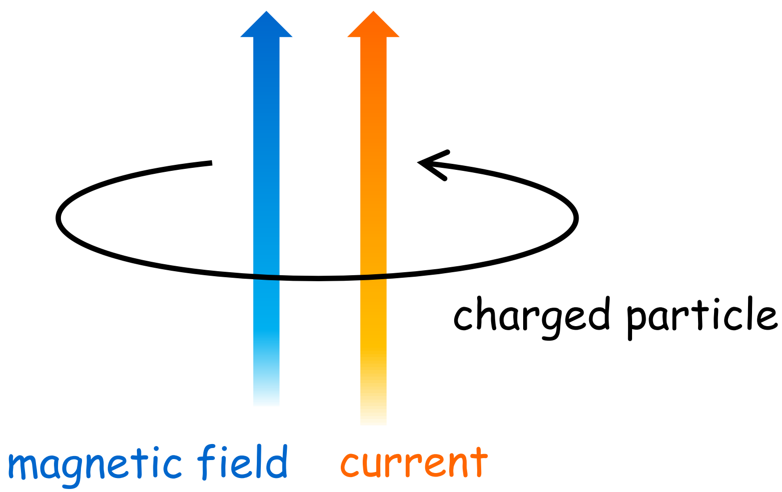
rotating compact star



Magnetism and Rotation

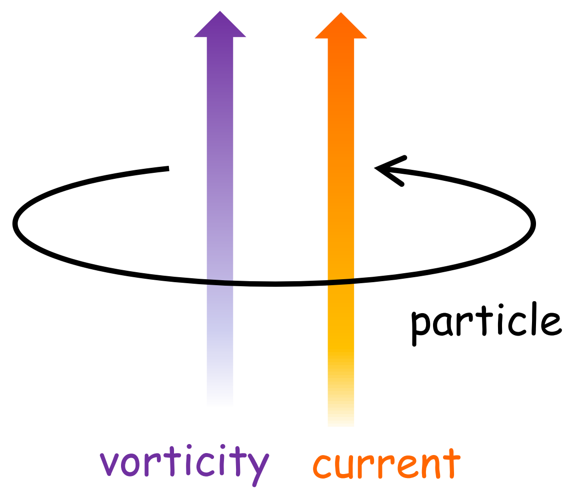
chiral magnetic effect

[Kharzeev McLerran Warringa (2007)]



chiral vortical effect

[Son Surowka (2009)]



What is lattice QCD?

lattice QCD ~ statistical mechanics

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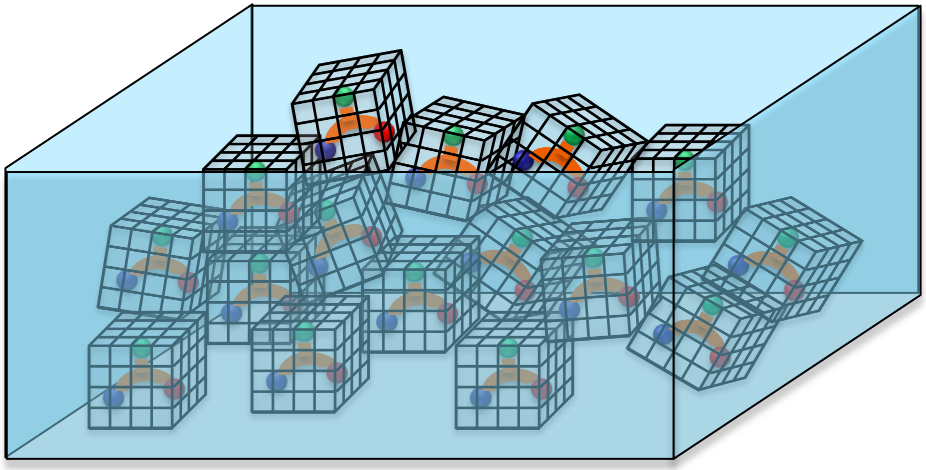
$$Z = \int e^{-S}$$

What is lattice QCD?

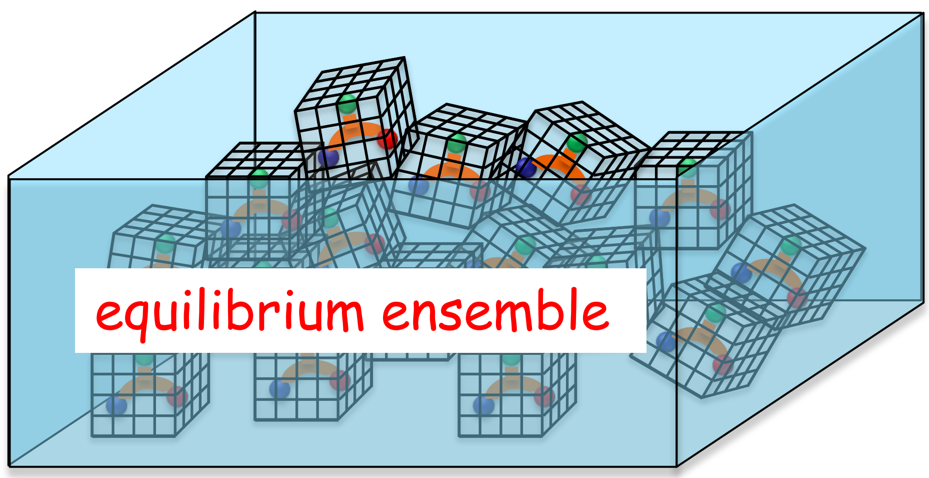
lattice QCD ~ statistical mechanics

$Z =$  with weight e^{-S}

What is lattice QCD?

lattice QCD ~ statistical mechanics

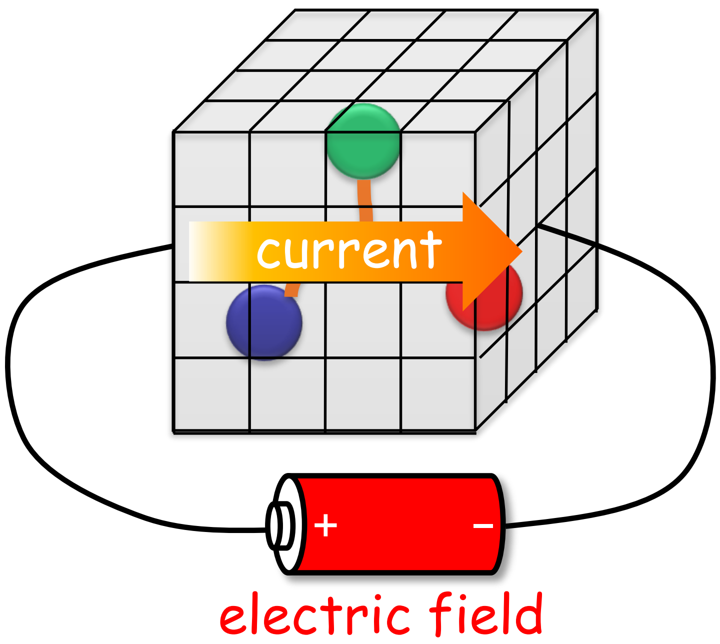
$Z =$



with weight e^{-S}

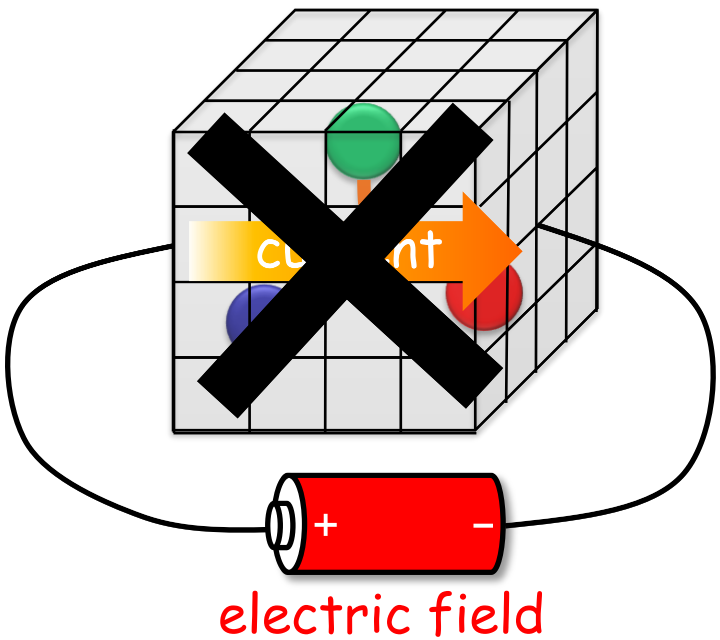
What is lattice QCD?

[AY (2013)]



What is lattice QCD?

[AY (2013)]



Part I

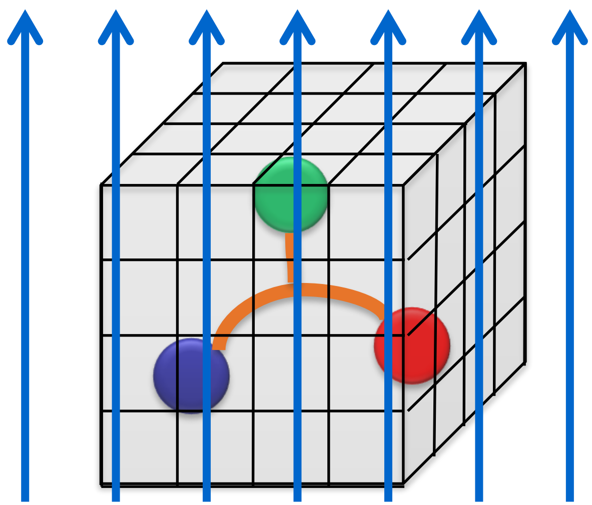
Magnetism

Simulation of magnetism

lattice QCD

+

background magnetic field



Simulation of magnetism

QCD action:

$$S = S_{\mathrm{YM}}[U] + S_{\mathrm{quark}}[\bar{\psi}, \psi, U]$$

Simulation of magnetism

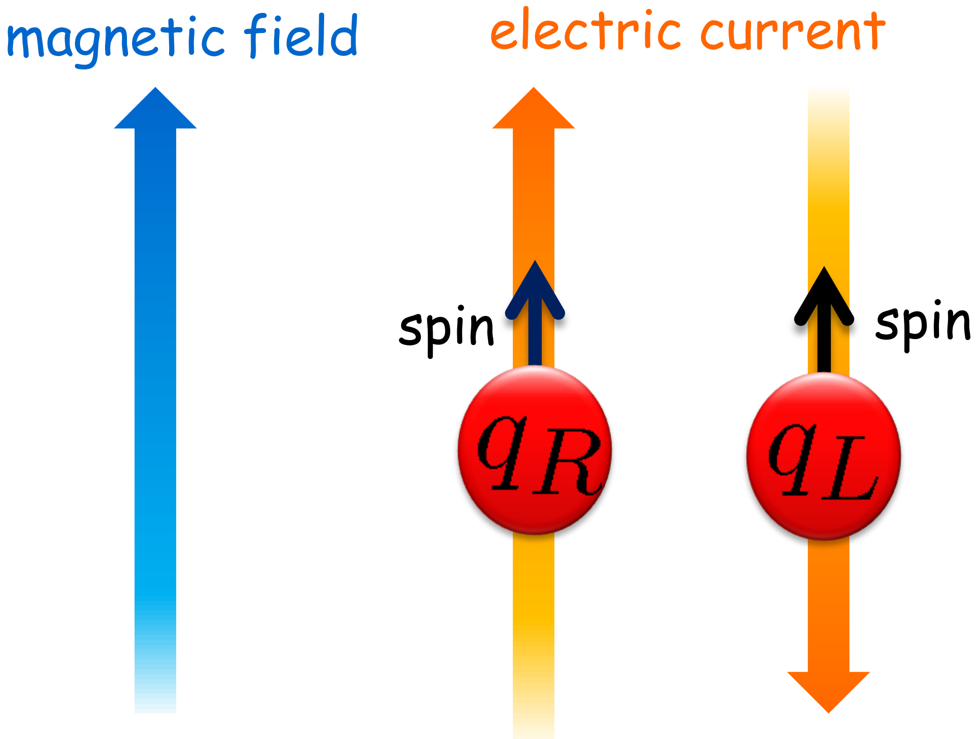
QCD + background U(1) gauge action:

$$S = S_{\rm YM}[U] + S_{\rm quark}[\bar{\psi}, \psi, U, u] + S_{\rm EM}[u]$$

not dynamical

Chiral magnetic effect

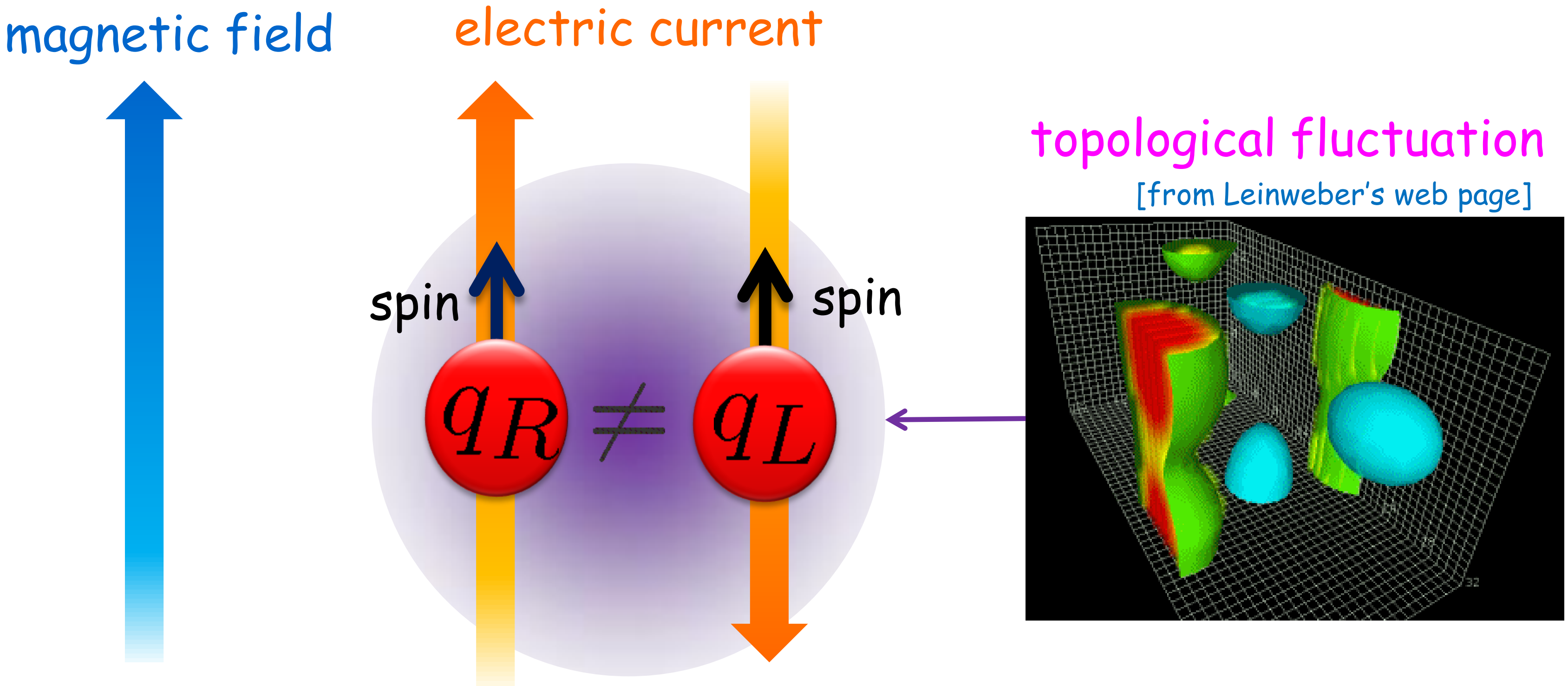
[Kharzeev McLerran Warringa (2007)]



Globally, $\langle J \rangle = 0$

Chiral magnetic effect

[Kharzeev McLerran Warringa (2007)]



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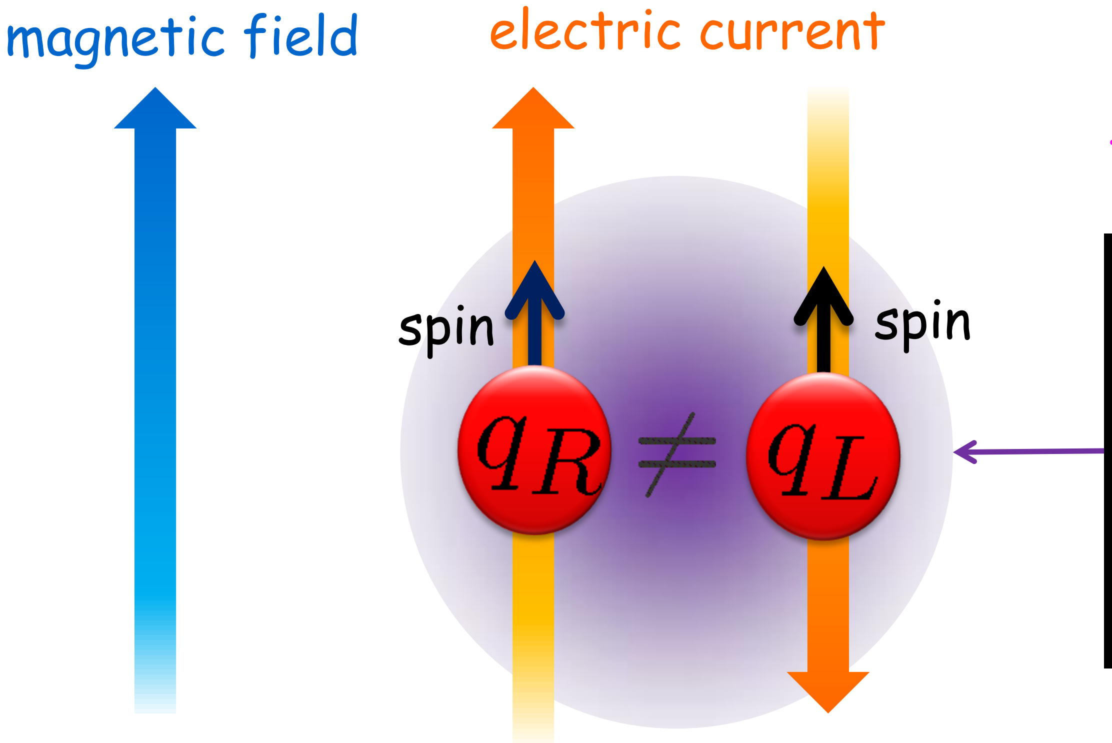
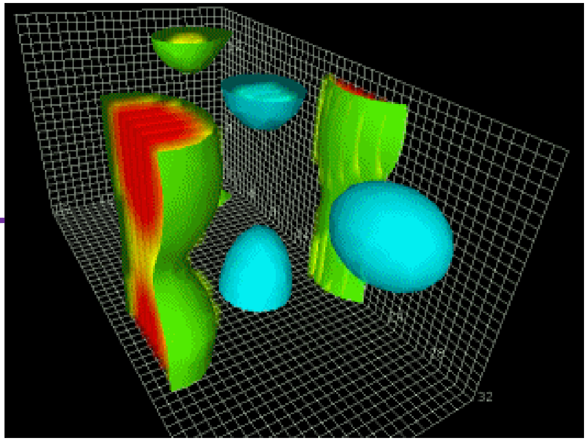


topological fluctuation

[from Leinweber's web page]



Globally, $\langle J \rangle = 0$

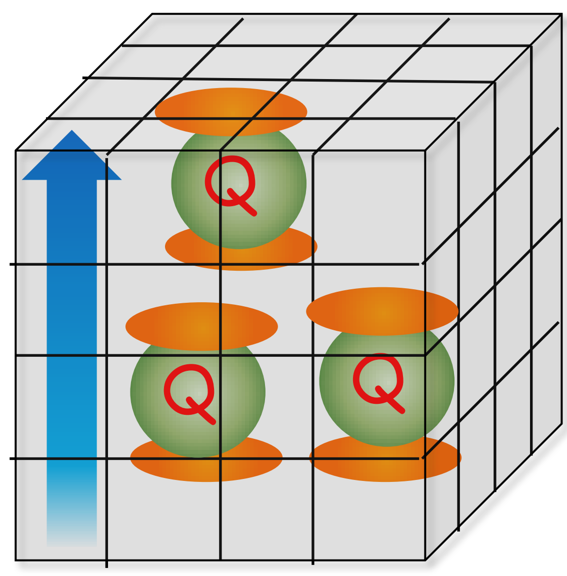
Locally, $\langle J \rangle_{\text{local}} \neq 0$ "event-by-event" electric current

Chiral magnetic effect in lattice QCD

1. topological fluctuation

Buividovich Chernodub Luschevskaya Polikarpov (2009)

Bali Bruckmann Endrödi Fodor Katz Schäfer (2014)

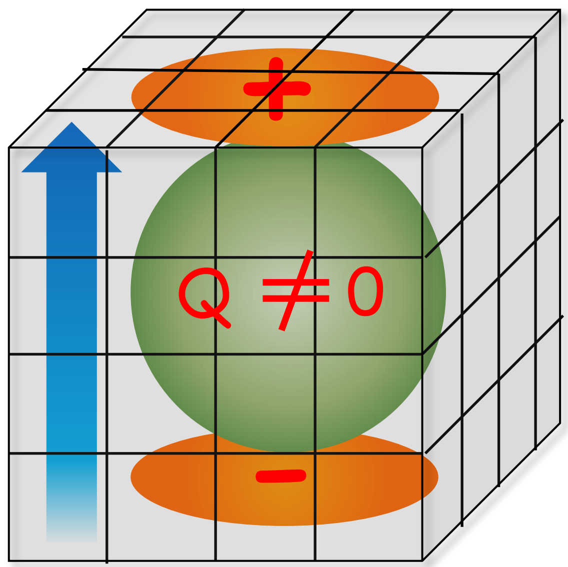


magnetic field current fluctuation

Chiral magnetic effect in lattice QCD

2. fixed topology

Abramczyk Blum Petropoulos Zhou (2009)

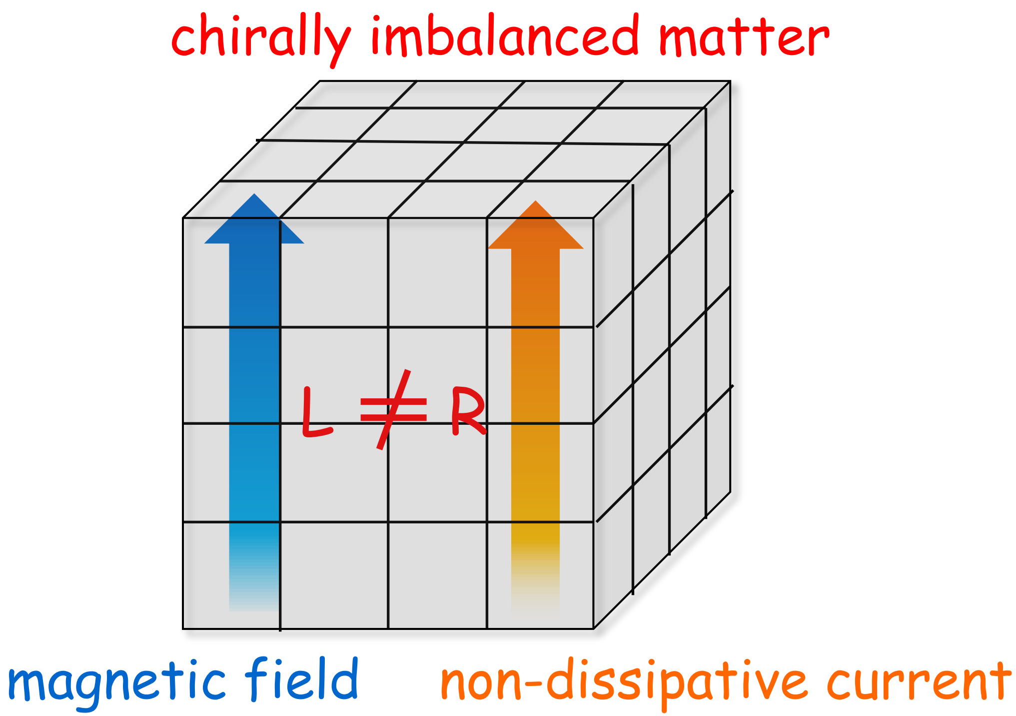


Chiral magnetic effect in lattice QCD

3. chiral chemical potential

AY (2011)

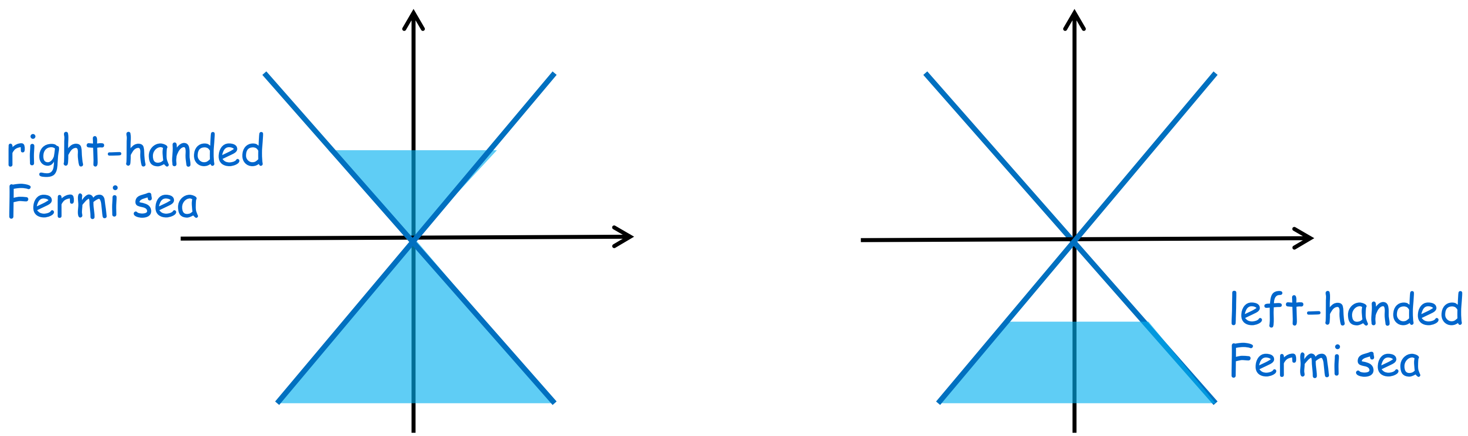
Buividovich (2013)



Chiral chemical potential

[Fukushima Kharzeev Warringa (2008)]

$$\mathcal{L}_F = \bar{\psi}(\gamma_\mu D_\mu + m + \mu_5 \gamma_4 \gamma_5)\psi$$



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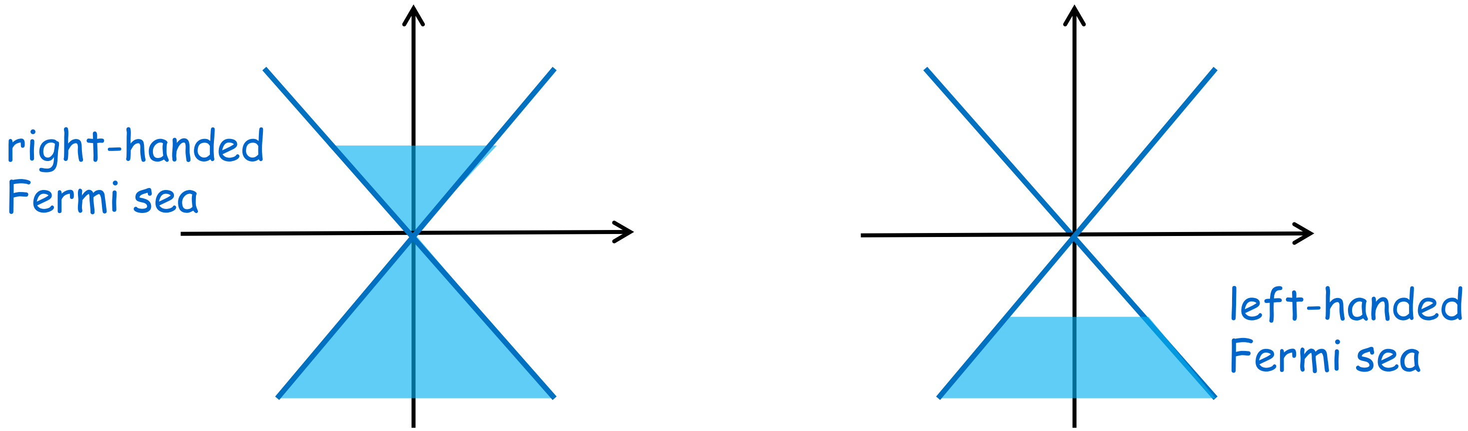
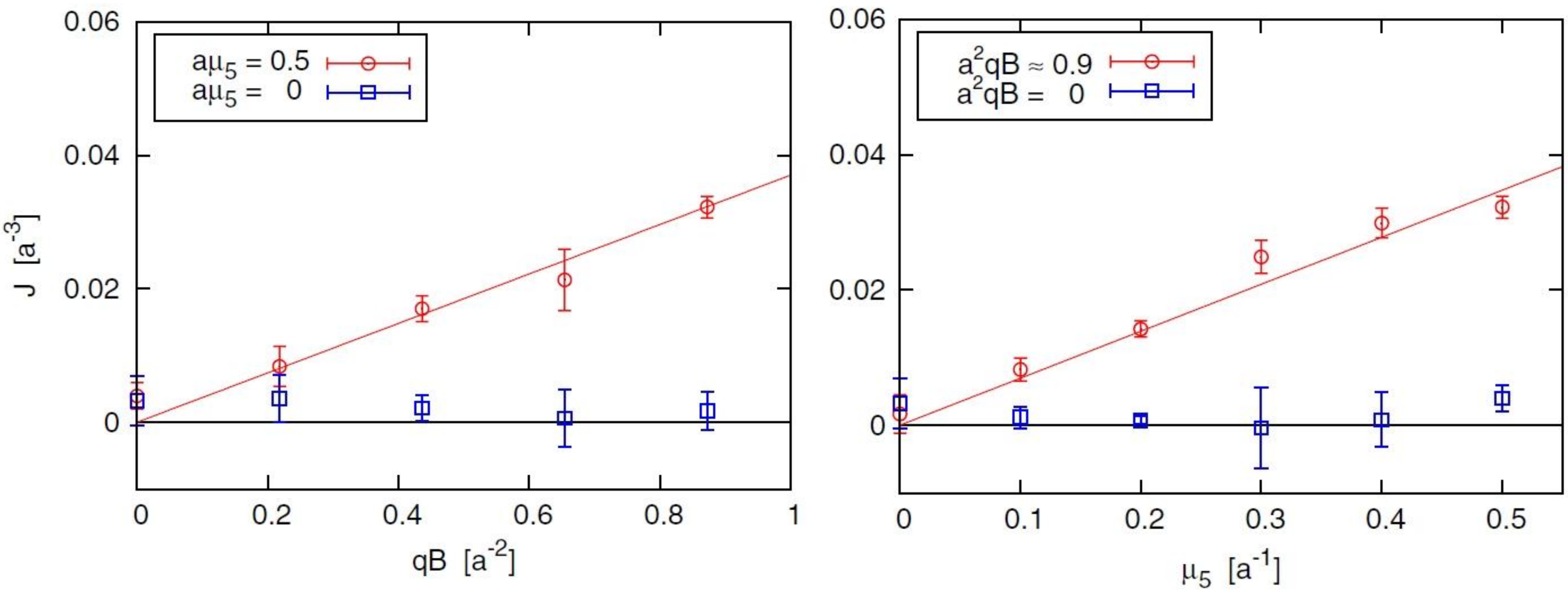


chiral magnetic effect: $J = \frac{q}{2\pi^2}\mu_5 B$

Simulation results

[AY (2011)]

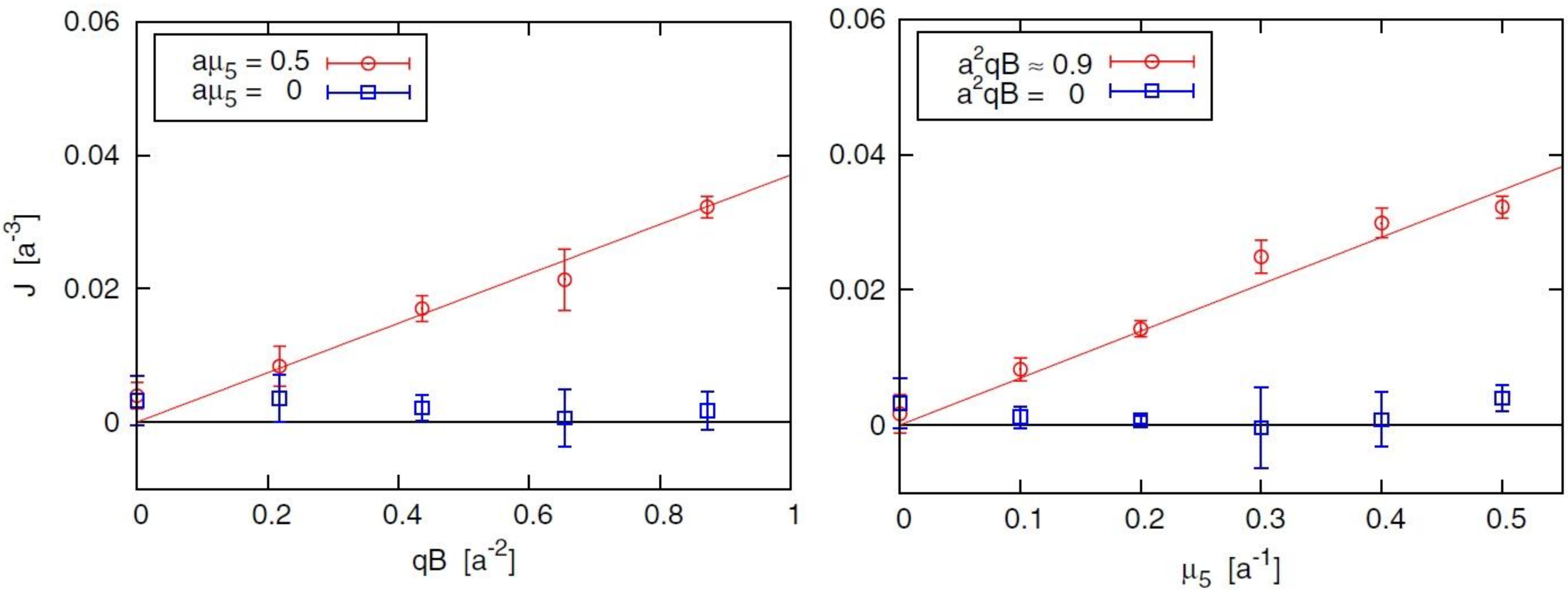
vector current: $J = \langle \bar{\psi} \gamma_\mu \psi \rangle$



Simulation results

[AY (2011)]

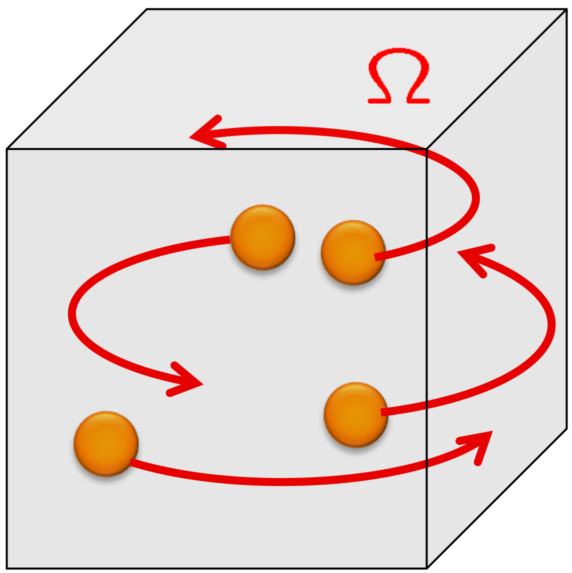
vector current: $J = \langle \bar{\psi}\gamma_\mu \psi \rangle \propto N_{\mathrm{dof}}\mu_5 qB$



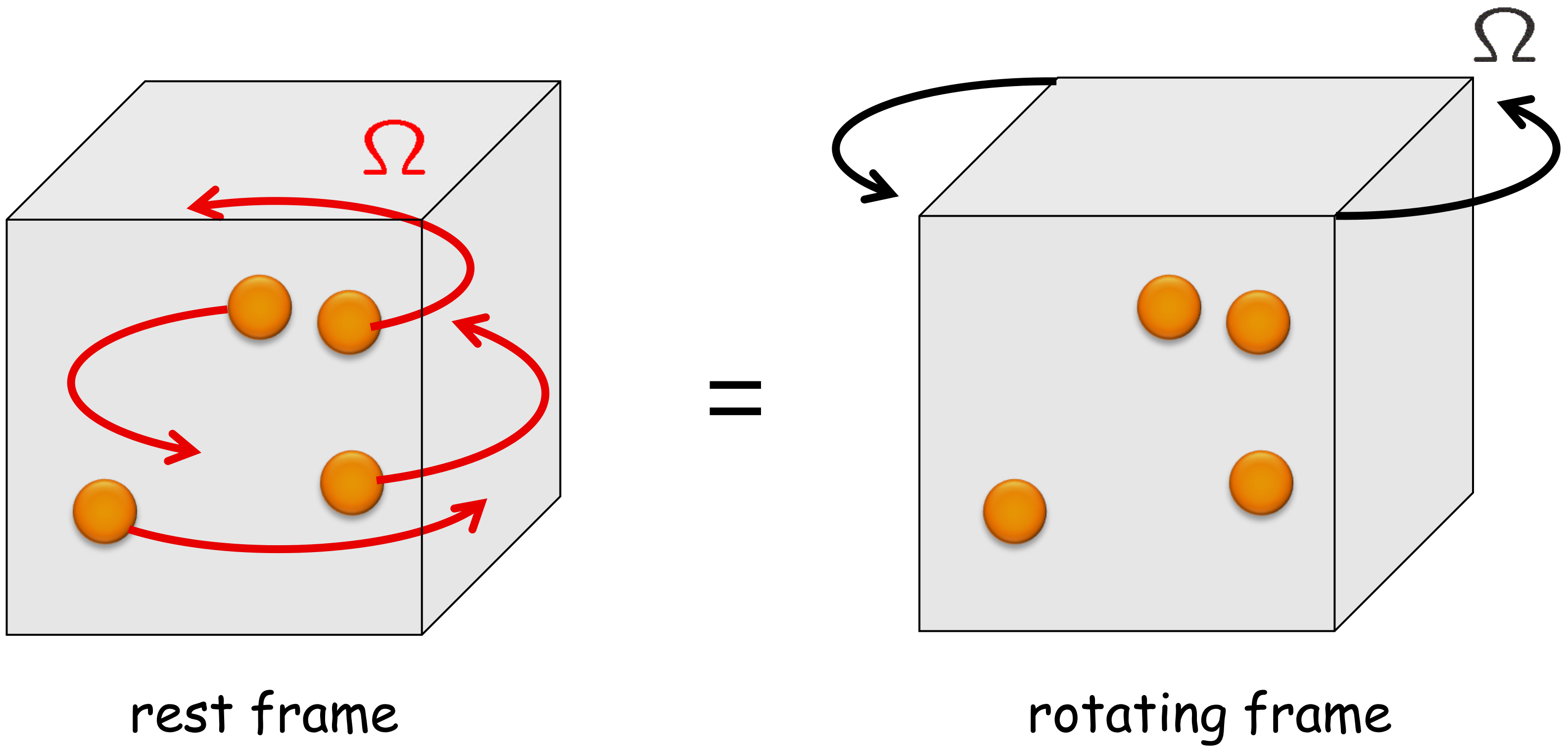
Part II

Rotation

Simulation of rotation



Simulation of rotation



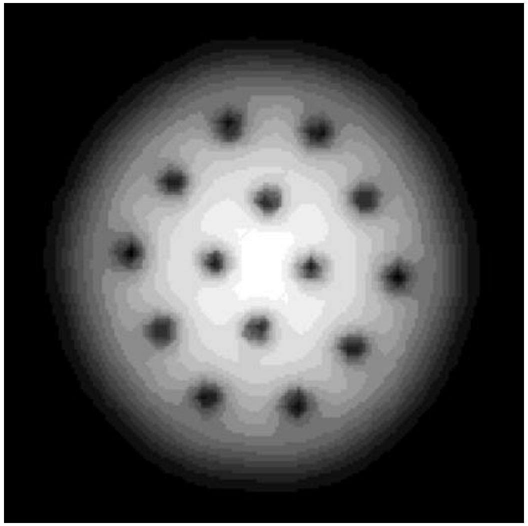
$$d\vec{x}_{\mathrm{rest}} = d\vec{x} - \vec{\Omega} \times \vec{x}\, dt$$

Simulation of rotation

cf.) rotating BEC in condensed matter physics

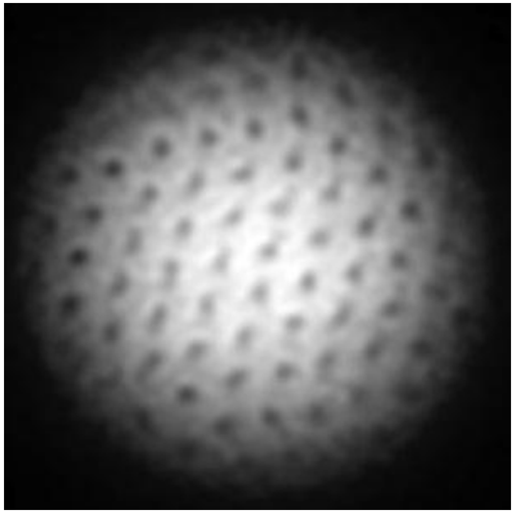
$$H \to H - \vec{L} \cdot \vec{\Omega}$$

simulation



[Kasamatsu Tsubota Ueda (2002)]

experiment

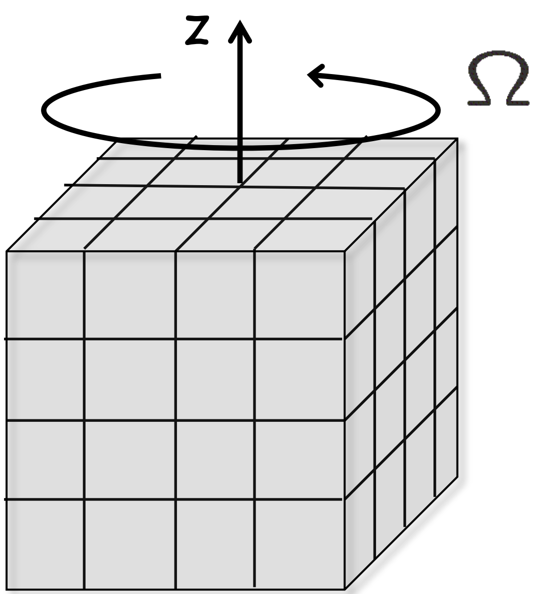


[Zwierlein et. al. (2005)]

Rotating lattice QCD

Euclidean rotation:

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & y\Omega \\ 0 & 1 & 0 & -x\Omega \\ 0 & 0 & 1 & 0 \\ y\Omega & -x\Omega & 0 & 1 + r^2\Omega^2 \end{pmatrix}$$



Gluon action

quadratic terms

$$
\begin{aligned}
S_G = \int d^4x \, \frac{1}{g_{\mathrm{YM}}^2} \mathrm{tr}\Big[& (1 + r^2\Omega^2) F_{xy} F_{xy} \\
& + (1 + y^2\Omega^2) F_{xz} F_{xz} + (1 + x^2\Omega^2) F_{yz} F_{yz} \\
& + F_{x\tau} F_{x\tau} + F_{y\tau} F_{y\tau} + F_{z\tau} F_{z\tau} \\
& + 2y\Omega F_{xy} F_{y\tau} - 2x\Omega F_{yx} F_{x\tau} \\
& + 2y\Omega F_{xz} F_{z\tau} - 2x\Omega F_{yz} F_{z\tau} + 2xy\Omega^2 F_{xz} F_{zy} \Big]
\end{aligned}
$$

cross terms

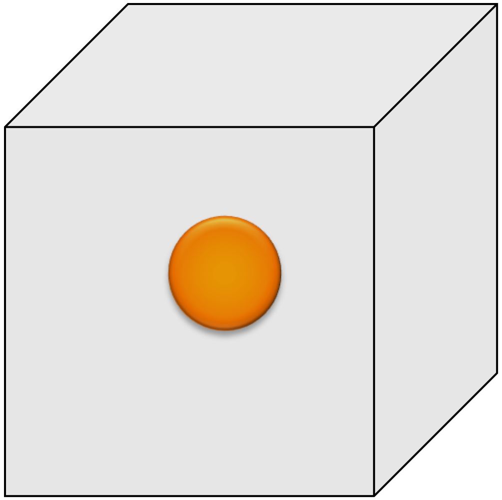
Fermion action

$$S_F = \int d^4x\, \bar{\psi} \Big[\gamma^1 D_x + \gamma^2 D_y + \gamma^3 D_z + \gamma^4 D_\tau + \boxed{\gamma^4 \Omega (x D_y - y D_x)} + \boxed{\gamma^4 i \Omega \frac{\sigma^{12}}{2}} \Big] \psi$$

orbit-rotation coupling spin-rotation coupling

Angular momentum

rest frame

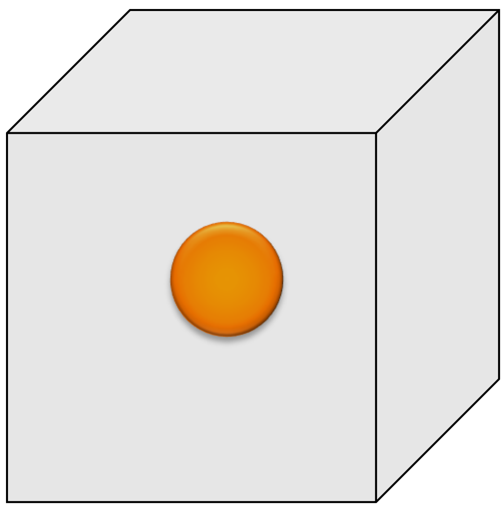


$$\mathcal{L} = \frac{1}{2} m r^2 \dot{\theta}^2_{\text{rest}}$$

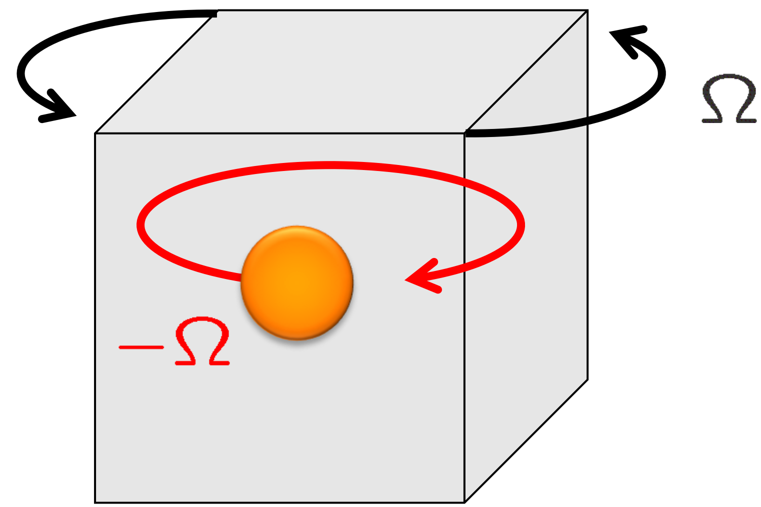
$$J = 0$$

Angular momentum

rest frame



rotating frame



$$\mathcal{L} = \frac{1}{2} m r^2 \dot{\theta}^2_{\text{rest}}$$

$$\mathcal{L} = \frac{1}{2} m r^2 (\dot{\theta} + \Omega)^2$$

$$J = 0$$

$$J = -m r^2 \Omega$$

Simulation results

[AY Hirono (2013)]

gluon :

$$J_G = \left\langle \frac{1}{g_{\mathrm{YM}}^2} \mathrm{tr}[2yF_{xy}F_{y\tau} - 2xF_{yx}F_{x\tau} + 2yF_{xz}F_{z\tau} - 2xF_{yz}F_{z\tau}] \right\rangle$$

fermion orbit :

$$J_{FL} = \langle \bar{\psi}\gamma^4 (xD_y - yD_x)\psi \rangle$$

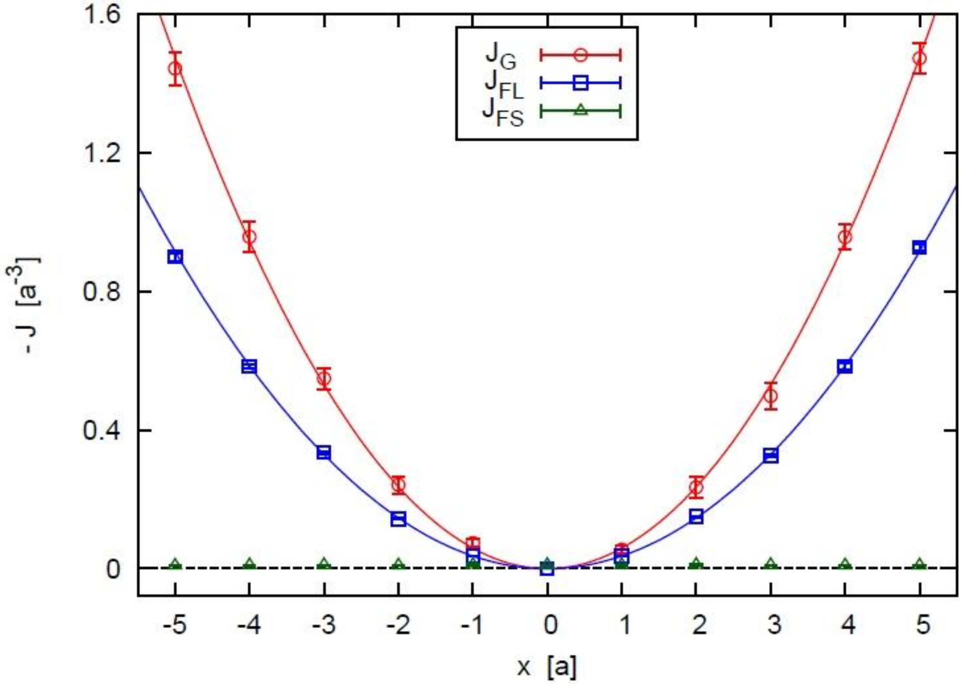
fermion spin :

$$J_{FS} = \left\langle i\bar{\psi}\gamma^4 \frac{\sigma^{12}}{2}\psi \right\rangle$$

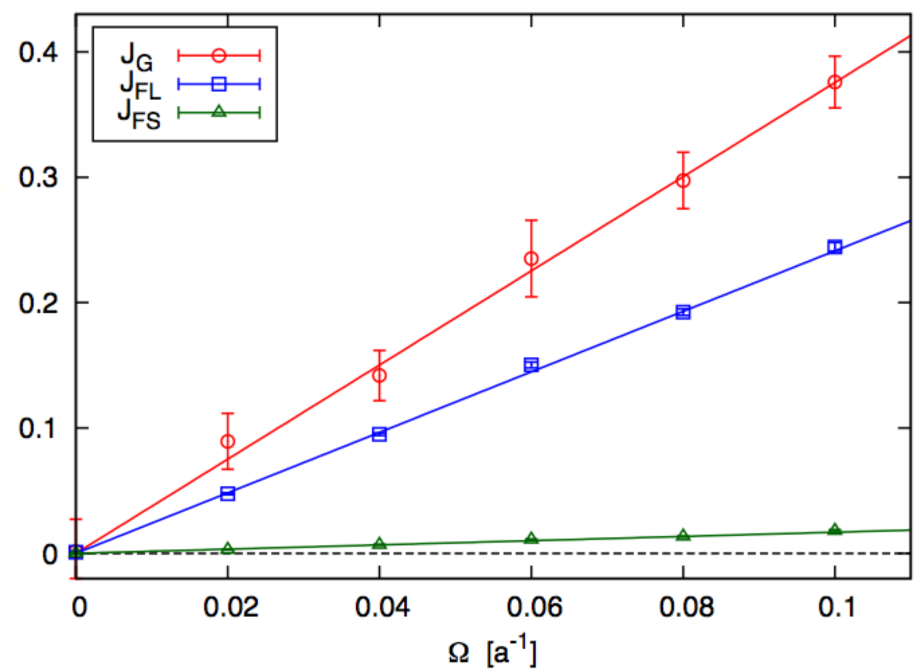
Simulation results

[AY Hirono (2013)]

radius dependence



angular-velocity dependence



Simulation results

[AY Hirono (2013)]

gluon : $$J_G = -(0.94 \pm 0.01)a^{-4} \times r^2\Omega$$

fermion orbit : $$J_{FL} = -(0.60 \pm 0.01)a^{-4} \times r^2\Omega$$

fermion spin : $$J_{FS} = -(0.17 \pm 0.01)a^{-2} \times \Omega$$

cf.) classical particle $$J = -I\Omega = -mr^2\Omega$$

Chiral vortical effect

No lattice QCD simulation of the chiral vortical effect

cf) free fermion at linear response [Buividovich (2013)]

Summary

Magnetism and rotation are frontiers of lattice QCD.

There are many topics:

- hadron property
- phase diagram
- chiral effects
- vortex nucleation