Effect of anisotropy on HBT radii using leptonpair interferometry

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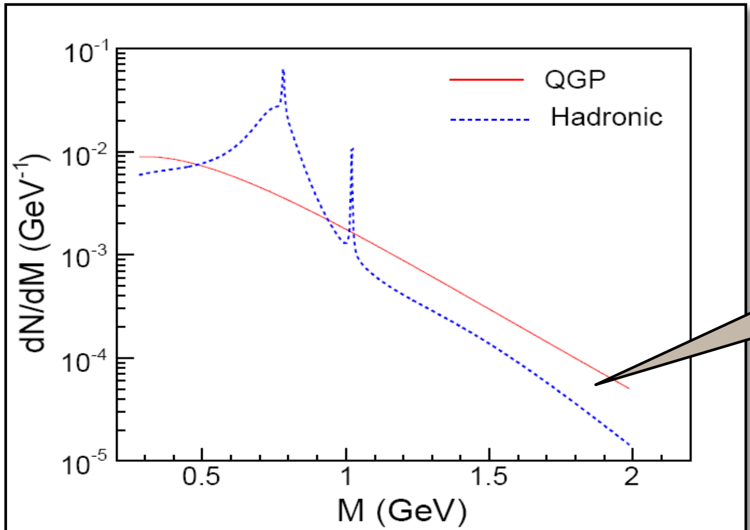
Importance of leptonpairs (dileptons) in HIC…

- Dileptons are the key observable to study dynamics of HIC and have been proposed as one of the most clear signals to identify the formation of QGP in HIC.
- They can couple only electromagnetically to the plasma, thus they have large mean free path ($\lambda = 1/n\sigma$) compared to system size.
- As a result they don't suffer further interaction in the medium and carry undistorted information of plasma.
- Unlike hadrons which produce at freeze-out surface, leptonpairs produce from every stages of collision and thus they probe the entire space-time history of system formed in HIC.
- One of the main advantages of dileptons is the fact that they possess finite pair mass and therefore, there are two kinematic variables that characterize dilepton spectra, the invariant mass M and transverse momentum pT. These two parameters can be varied to investigate the different stages of the collisions.



Above φ peak the dileptons from QGP dominates over its hadronic counterpart.

A judicious choice of pT and M window can disentangle the QGP & hadronic phase separately.

Importance of Interferometry in HIC ………..

Interferometry is the technique of diagnosing the properties of two or more waves by studying the pattern of interference created by their superposition.

- ✓ Two particle intensity interferometry is commonly called HBT interferometry.
- ✓ 1st time formulated and exploited by Hanbury, Brown and Twiss to correlate the intensity of EM radiations arriving from extra terrestrial objects and thus measured the angular diameter of stars and other astronomical objects

Though it has its origin in astrophysics but has significant theoretical development and wide spread application in HIC.

- ✓ This technique is one of the efficient ways to obtain direct experimental information on the space time structure of particle emitting source created in relativistic nuclear collision.

- ✓ The goal of this method is to extract space-time structure of the source from momentum spectra, making use of quantum statistical correlation between the pairs of identical particles.

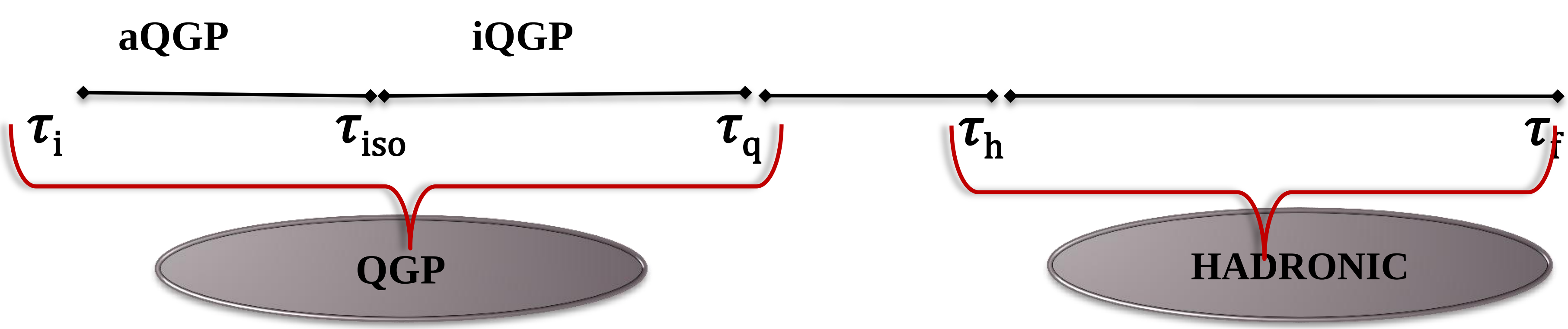
Let's understand the system …

- Thermalization in HIC is a debatable issue.

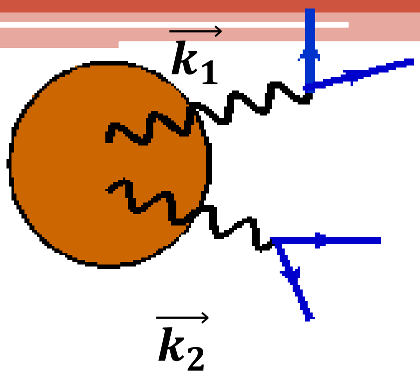
- Due to poor knowledge of isotropization time **(τ_{iso})** and thermalization time **(τ_{therm})** one need not to assume the hydrodynamic behavior from the very beginning.

- In realistic scenario, due to rapid longitudinal expansion at the onset of QGP phase, **anisotropy arises** in p_T–p_L plane with $\langle p^2_L \rangle << \langle p^2_T \rangle$ in the local rest frame.

- With time such asymmetry dies out with secondary partonic interactions, after which system considered to be isotropic and thermalized at proper time $\boldsymbol{\tau_{iso}}$ and beyond $\boldsymbol{\tau \geq \tau_{iso}}$ the system can be treated hydrodynamically.



Two (virtual) photon Correlation Function :



$$\boxed{C_2(\vec{k}_1,\vec{k}_2)=\frac{P_2(\vec{k}_1,\vec{k}_2)}{P_1(\vec{k}_1)P_1(\vec{k}_2)}}$$

$$P_1(\vec{k}_i)=\int d^4x_i\,\omega(x_i,k_i)$$

Source Function or
Static emission rate per unit four volume

and

$$P_2(\vec{k}_1,\vec{k}_2)=P_1(\vec{k}_1)P_1(\vec{k}_2)+\tfrac{1}{\lambda}\int d^4x_1d^4x_2\,\omega(x_1,K)\,\omega(x_2,K)\cos(\Delta x^\mu\Delta k_\mu)$$

$$\boxed{C_2(\vec{k}_1,\vec{k}_2)=1+\frac{1}{3}\left[\frac{\int d^4x_1d^4x_2\,\omega(x_1,K)\,\omega(x_2,K)\cos(\Delta x^\mu\Delta k_\mu)}{P_1(\vec{k}_1)P_1(\vec{k}_2)}\right]}$$

$\vec{\mathrm{k}}_{\mathrm{i}}=(\mathrm{k}_{\mathrm{iT}}\cos\psi_{\mathrm{i}},\mathrm{k}_{\mathrm{iT}}\sin\psi_{\mathrm{i}},\mathrm{k}_{\mathrm{iT}}\sinh y)\rightarrow$ three momentum of identical particles; i=1,2

$K=(k_1+k_2)/2$, $\Delta k_\mu = k_{1\mu} - k_{2\mu} = q_\mu$ and $\Delta x_\mu = x_{1\mu} - x_{2\mu}$

Source function ω(x,K) * :

$$\omega\,(x,K) = \int_{M_1^2}^{M_2^2} dM^2 \left[\frac{dR}{dM^2 d^2k_T dy}\right]$$

→ aQGP

PRL 100 (2008) 102301,
PRC 78(2008),034917

→ iQGP

→ Hadronic

THERMAL

PRD 35(1987)2153, PR 273(1996) 243, Ann. Phys.286(2001),159, Adv. Nucl. Phys. 25 (2000)1, Mod. Phys 65 (1993)1

Source function ω(x,K) * in aQGP :

$$\frac{dR^{l^+l^-}}{d^4P} = \int \frac{d^3\mathbf{p}_1}{(2\pi)^3}\frac{d^3\mathbf{p}_2}{(2\pi)^3} f_q(\mathbf{p}_1)\, f_{\bar{q}}(\mathbf{p}_2) v_{q\bar{q}}\, \sigma_{q\bar{q}}^{l^+l^-} \times \delta^{(4)}(P - p_1 - p_2)$$

PRC 78(2008),034917

We have assumed a system with highly momentum anisotropy where particle distribution function is obtained by either compressing $(\xi > 0)$ or stretching $(\xi < 0)$ along one direction in momentum space ,

$$\boxed{f_i(p, p_{hard}, \xi) = f_{iso}(\sqrt{p^2 + \xi(p.n)^2}, p_{hard})}$$

P. Romatschke & M. Strickland PRD 2004

- **n** is direction of anisotropy.
- $\xi = \frac{\langle p_T^2 \rangle}{2\langle p_L^2 \rangle} - 1$ is the momentum space anisotropy
- p_{hard} is the hard momentum scale which is directly related to average momentum of partons and has direct relevance with temperature (T).

Source function $\omega(x,K)$ * in thermal medium:

Beyond $\tau \geq \tau_{iso}$, leptonpairs emerge just after the system thermalizes from both **iQGP** and the hot **hadronic matter**.

- **In isotropic QGP (iQGP):** Thermal leptonpairs are produced from iQGP via quark-anti-quark annihilation processes.

- **In hot hadronic matter :** The leptonpairs from hot hadrons are dominantly produced by decay of light vector mesons.

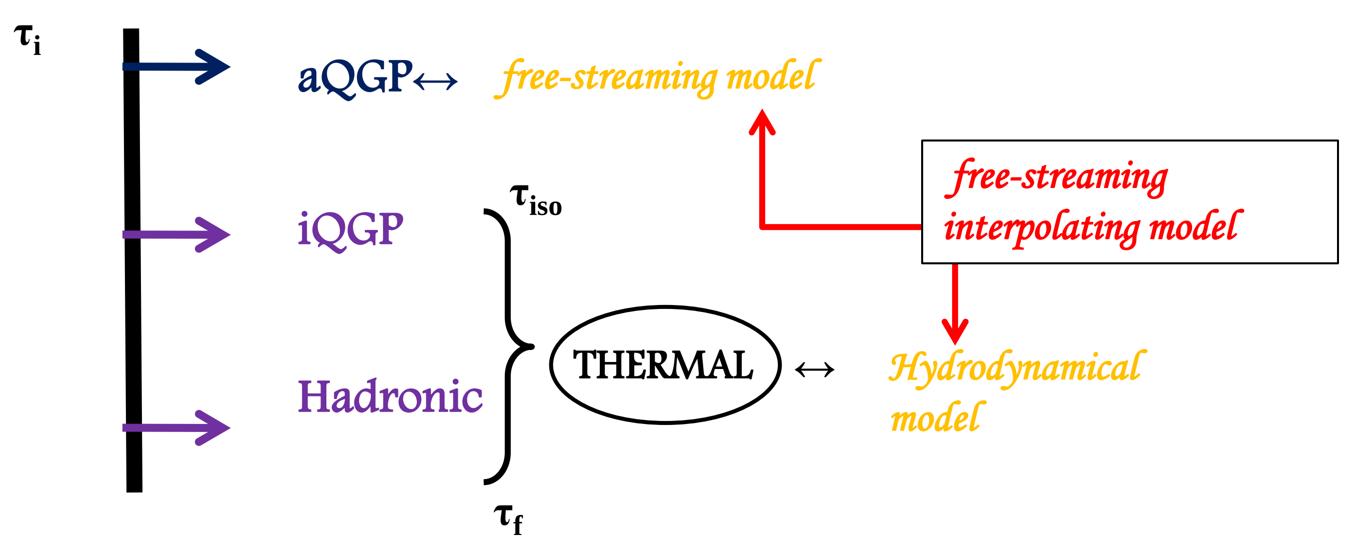
Contribution of thermal leptonpairs	
Quark Matter	**Hadronic Matter**
$q\bar{q} \rightarrow \gamma^* \rightarrow \ell^+\ell^-$ *Also* $\mathcal{O}(\alpha^2\alpha_s)$ *contribution are taken.* $q\bar{q} \rightarrow g\gamma^*$ $q(\bar{q})g \rightarrow q(\bar{q})\,\gamma^*$	*Decay of light vector mesons* $V\,(\rho,\omega,\phi)$ $\rho \rightarrow \ell^+\ell^-$ $\omega \rightarrow \ell^+\ell^-$ $\phi \rightarrow \ell^+\ell^-$

Space-time Evolution:

- The system evolves anisotropically from τ_i to τ_{iso} , where one needs to know time dependence of $p_{hard}(\tau)$and $\xi(\tau)$. We follow M. Martinez & M. Strickland PRL 2008 for evolution in aQGP.

- For $\tau \geq \tau_{iso}$, the system become thermalized and evolves hydrodynamically with energy density and velocity as a function of space and time and is described by (1+2)d ideal hydrodynamics model with cylindrical symmetry and boost invariance along the longitudinal direction.

The entire evolution is categorized as;



τ_{iso} is treated as free parameter which controls the transition to hdyrodynamic scenario.

Space-time Evolution in anisotropic medium:

M. Martinez & M. Strickland PRL 2008

According to the model there are 3 possible scenarios………..

i) $\tau_{iso} = \tau_i$ (system evolves hydrodynamically so $\xi=0$ and $p_{hard} \sim T$)

ii) $\tau_{iso} \longrightarrow \infty$ (system never comes to equilibrium ($\xi \neq 0$))

iii) $\tau_{iso} \geq \tau_i$ (smoothly interpolates between anisotropy and hydrodynamics)

For present work we follow the scenario (iii), which interpolates between anisotropic and hydrodynamic evolution.

This model can be executed mathematically by generalizing anisotropic parameter $\xi(\tau)$ ……………

$$\boxed{\xi(\tau) = \left(\tau/\tau_i\right)^{\delta} - 1}$$

✓ $\delta=0$ and $\xi=0$ corresponds to scenario (i) where expansion is hydrodynamic (thus iQGP $\leftrightarrow$ $\delta=0$).

✓ $\delta\neq0$ and $\xi\neq0$ corresponds to scenario (ii) where system is highly anisotropy (thus aQGP $\leftrightarrow$ $\delta\neq0$).

free streaming interpolating model

In accordance with longitudinal ***free streaming*** model, $\xi(\tau) = \left(\tau/\tau_i\right)^{\delta} - 1\Big|_{\delta=2}$

- ✓ Two parameters;
  (i) plasma momentum space anisotropy - $\xi(\tau)$ (ii) hard momentum scale - $p_{hard}(\tau)$.
- ✓ Two time scale,
  (i) initial QGP formation time (τ_i) (ii) isotropization time (τ_{iso}). $[\tau_i \leq \tau_{iso}]$

M. Martinez & M. Strickland PRL 2008

The model used here interpolates between the longitudinal ***free streaming model*** ***(δ=2)*** and ***hydrodynamical model(δ=0)*** by introducing a smear step function,

$$\lambda(\tau)=\frac{1}{2}(\tanh[\gamma(\tau-\tau_{iso})/\tau_i]+1) \quad \begin{cases} \tau \ll \tau_{iso};\ \lambda=0 \rightarrow \textit{freestreming (FS)} \\ \tau \gg \tau_{iso};\ \lambda=1 \rightarrow \textit{hydrodynamics} \end{cases}$$

Time dependency of relevant quantities are as follows:

$$\xi(\tau) = a^{2[1-\lambda(\tau)]} - 1$$

$$\varepsilon(\tau) = \varepsilon_{FS}(\tau)[u(\tau)/u(\tau_i)]^{4/3}$$

$$p_{hard}(\tau) = T_i[u(\tau)/u(\tau_i)]^{1/3};$$

$$u(\tau) = [\mathcal{R}(a_{iso}{}^2 - 1)]^{3\lambda(\tau)/4}(a_{iso}/a)^{\lambda(\tau)}, a = \tau/\tau_i \text{ and } a_{iso} = \tau_{iso}/\tau_i$$

Space-time Evolution in isotropic medium:

❖In hydrodynamics prescription, P_i's from an expanding system can be calculated by convoluting the static thermal rate [$\omega(E^*,T)=EdR/d^3k$] with the expansion dynamics (d^4x) which depends on energy density [$\varepsilon(r,\tau)$] and radial velocity [$v_r(r,\tau)$] .

$$P_1^{hydro}(\vec{k}_i) = \sum_i \int d^4x_i\, \omega(E^*,T)$$

i =Q,M,H (iQGP, MIXED, Hadronic Phases)

$$P_2^{hydro}(\vec{k}_1,\vec{k}_2) = P_1^{hydro}(\vec{k}_1)P_1^{hydro}(\vec{k}_2) + \tfrac{1}{3}\sum_i \int d^4x_1 d^4x_2 \omega(E_1^*,T)\omega(E_2^*,T)\cos(\Delta x^\mu \Delta k_\mu)$$

$$p_i^{hydro} = p_i^{iQGP} + p_i^{mixed} + p_i^{hadron}$$

❖The energy E* should be replaced by $k^\mu u_\mu$ for an expanding medium with space time dependent four velocity u_μ . Assuming cylindrical symmetry and longitudinal boost invariance, $k^\mu u_\mu$ can be expressed as;

$$k^\mu u_\mu = \gamma_r[k_T \cosh(y-\eta) - v_r k_T \cos\phi] \quad ; \text{where} \quad \gamma_r(r,\tau) = [1 - v_r(r,\tau)]^{-1/2}$$

❖The system produced at QGP at $\tau = \tau_{iso}$ ($T_i^{hydro} = p_{hard}(\tau_{iso}), \tau_i^{hydro} = \tau_{iso}$) and reverts back to hadronic phase at T~Tc. Thermal equilibrium may be maintained until mfp remain comparable to the system size at $T \sim T_f$.

Results:

$\sqrt{s_{NN}}$	2.76 TeV
T_i	646 MeV
τ_i	0.08 fm
T_c	175 meV
T_f	120 MeV
EoS	Lattice+HRG

- For different τ_{iso} values the C2 is calculated for 2.76 TeV @LHC at mid rapidity for different M windows (for M =0.3, 0.5, 0.77, 1.02, 1.6 and 2.5 GeV) as function of q_{side} and q_{out} which are related to the transverse momentum of individual pair as follows:

$$q_{side} = \left| \vec{q_T} - q_{out} \frac{\vec{K_T}}{K_T} \right| = \frac{2k_{1T}k_{2T}\sqrt{1-\cos^2(\psi_1-\psi_2)}}{\sqrt{k_{1T}^2+k_{2T}^2+2k_{1T}k_{2T}\cos(\psi_1-\psi_2)}}$$

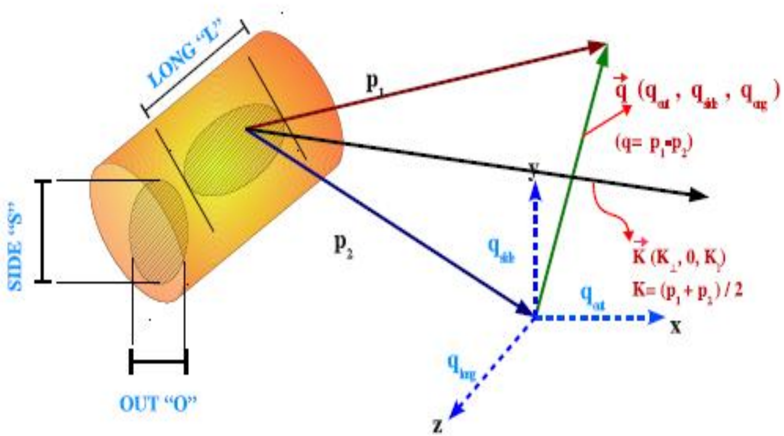
$\psi_1=0$, $\psi_1 \neq 0$, $y_1=y_2=0$, $k_{1T}=k_{2T}=2$ GeV,

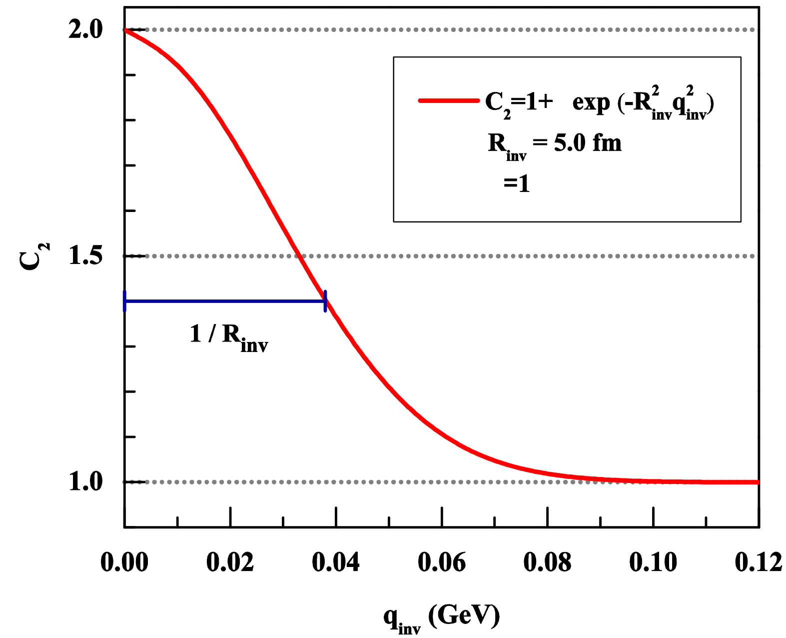
$$q_{out} = \frac{\vec{q_T}.\vec{K_T}}{|K_T|} = \frac{(k_{1T}^2-k_{2T}^2)}{\sqrt{k_{1T}^2+k_{2T}^2+2k_{1T}k_{2T}\cos(\psi_1-\psi_2)}}$$

$k_{1T}=2$ GeV, $\psi_1=\psi_2=0$, $y_1=y_2=0$

Basically we attempted to examine sensitivity of momentum anisotropy on C2 by controlling the variable τ_{iso} and extracted the HBT radii from it to obtain the source size.

Source Dimension (HBT radii):

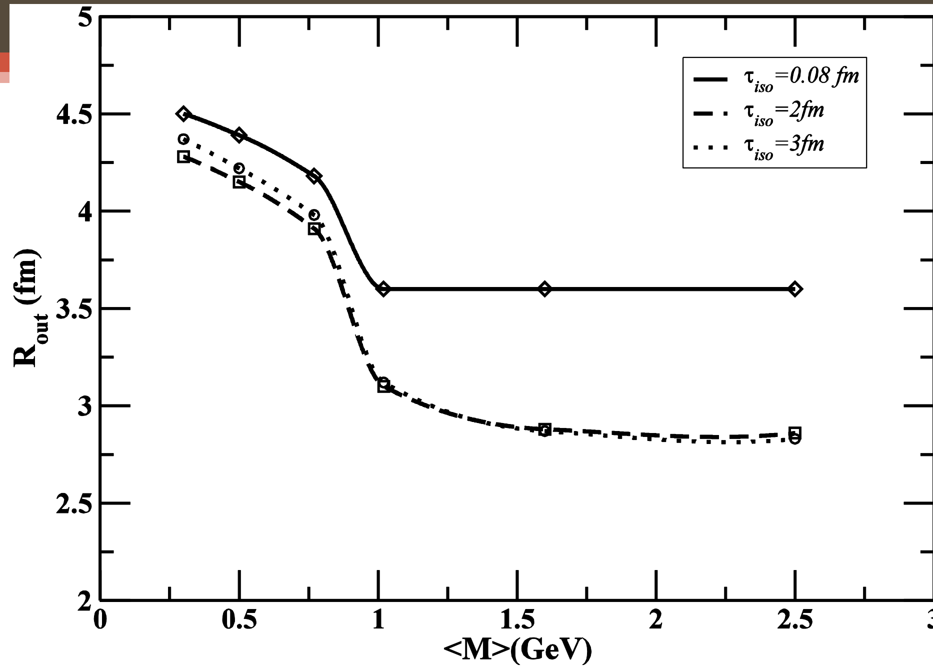




$$C_2(q,K) = 1 + \lambda \exp(-R_i^2 q_i^2)$$
$$= \quad 1 + \lambda \exp(-R_{side}^2 q_{side}^2 - R_{out}^2 q_{out}^2)$$

For a static system, HBT radii are the geometric size of the source. But for evolving system, HBT radii are smaller than the geometric size

R_{out} vs. M



$\psi_1=\psi_2=0$, $y_1=y_2=0$, k_{1T}=2 GeV and varying k_{2T}

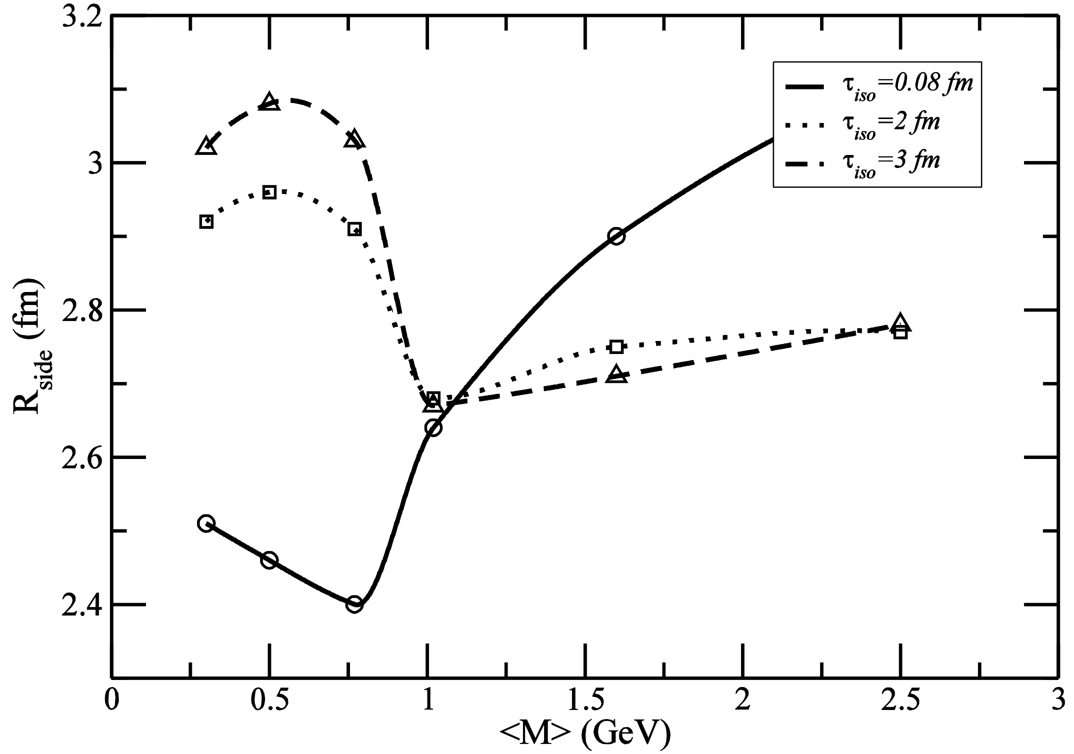
⌑ R_{out} probes both the transverse size and the duration of emission.

⌑ The effect of flow is small in the initial stage of collision which is dominant in large M region (corresponds to larger size) and the duration of emission is small - resulting in a small values of R_{out}.

⌑ Whereas M ~ m_ρ region suffers from larger flow effects which should have resulted in a minimum value in R_{out} in this M region. However, R_{out} probes the duration of emission too, which is large for hadronic phase because of the slower expansion due to softer EoS used in the present work for the hadronic phase.

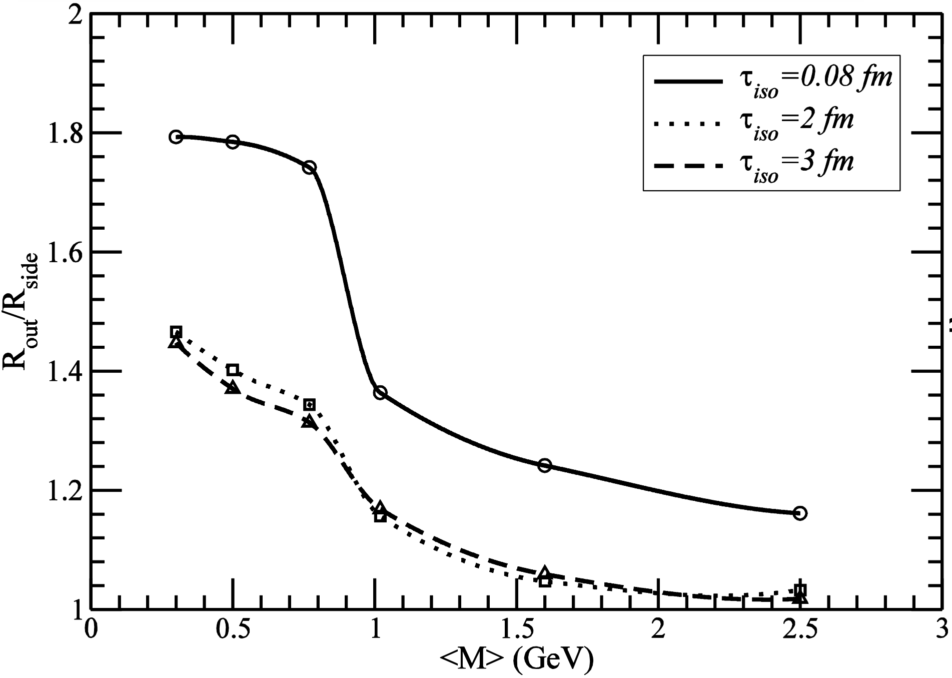
⌑Again by increasing iso, the duration of particle emission shortens and results in smaller value of R_{out} .

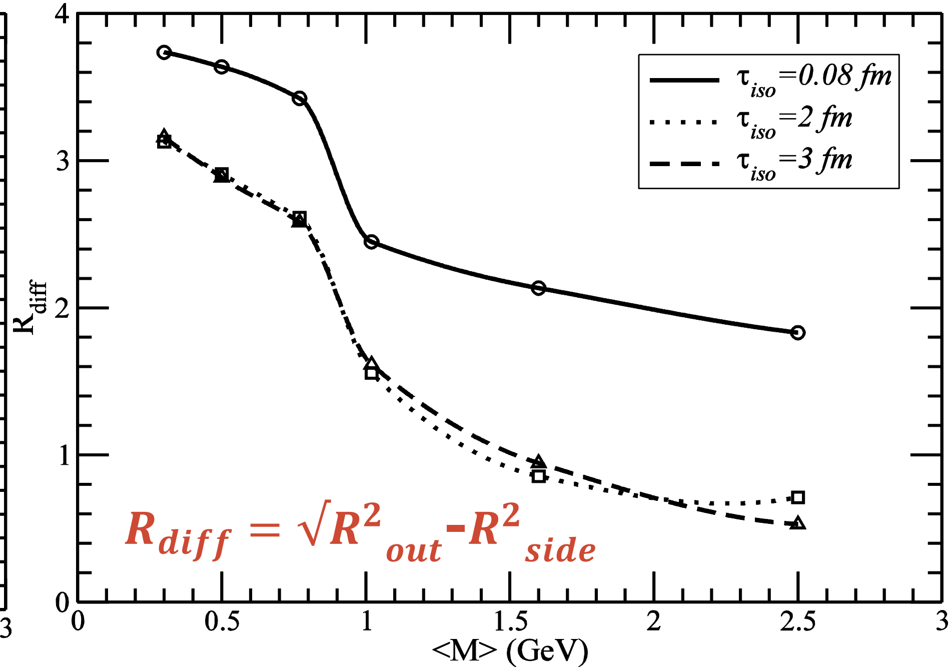
R_{side} vs. M



- $R_{side} \sim 1/(1+E_{collective}/E_{thermal})$
- There is a qualitative as well as quantitative change is observed.
- By increasing iso, the flow is reduced resulting in larger values of R_{side} in anisotropic scenario in hadronic regime ($M \sim m_\rho$).

Temporal Information





- However, by increasing the values of τ_{iso} we observe quantitative change in the magnitude of both of these quantities.

- This is because the duration of particle emission reduces as the system takes more time to become thermalized (by increasing τ_{iso}). Hence both these quantities have smaller value in the anisotropic scenario compared to the isotropic one.

SUMMARY

- We have studied the Bose-Einstein Correlation Function (BECF) for two identical virtual photons decaying to leptonpairs at most central collision of LHC energy having fixed transverse momentum of one of the virtual photons (k_{1T}= 2 GeV).

- The free streaming interpolating model with fixed initial condition has been used for the evolution in anisotropic Quark Gluon Plasma (aQGP) and the relativistic (1+2)d hydrodynamics model with cylindrical symmetry and longitudinal boost invariance has been used for both isotropic Quark Gluon Plasma (iQGP) and hadronic phases.

- We have extracted the HBT radii and found a significant change in the spatial and temporal dimension of the evolving system in presence of initial state momentum-space anisotropy.


Thank You!

BACK UP

- The initial condition are essential for ideal hydrodynamics ; $T_i^{hydro} = p_{hard}(\tau_{iso}), \quad \tau_i^{hydro} = \tau_{iso}$

- The initial conditions are given through the energy density and velocity profile;

$$\mathcal{E}(\tau_i, r) = \frac{\mathcal{E}_0\left(\xi = 0, T_i^{\text{hydro}}\right)}{1 + \exp\left(\frac{r - R_A}{\sigma}\right)},$$

$$v_r(\tau_i, r) = v_0\left(1 - \frac{\mathcal{E}_0\left(\xi = 0, T_i^{\text{hydro}}\right)}{1 + \exp\left(\frac{r - R_A}{\sigma}\right)}\right),$$

$\mathcal{E}_0(\xi=0, T_i^{hydro})$ is initial energy density related to initial temperature T_i^{hydro}.

- Tc= 175 MeV
- Tf=120 MeV.
- EOS: Lattice QCD (T>Tc ,QGP) and
  Hadron resonance gas (T<Tc, Hadronic phase)

- For transition region we have used following parametrisation;

$$s(T) = s_q(T) f_q(T) + [1 - f_q(T)] s_h(T),$$

$$f_q(T) = \frac{1}{2}\left(1 + \tanh\frac{T - T_c}{\Gamma}\right),$$

Γ= 25 MeV

Γ is the width parameter and assumes a finite value for the crossover transition and for 1st order transition this value can be turned to zero.

Results:

We calculated the C_2 as function of q_{out}, and q_{side} for different M windows.

$$q_{side} = \left| \vec{q_T} - q_{out} \frac{\vec{K_T}}{K_T} \right|$$

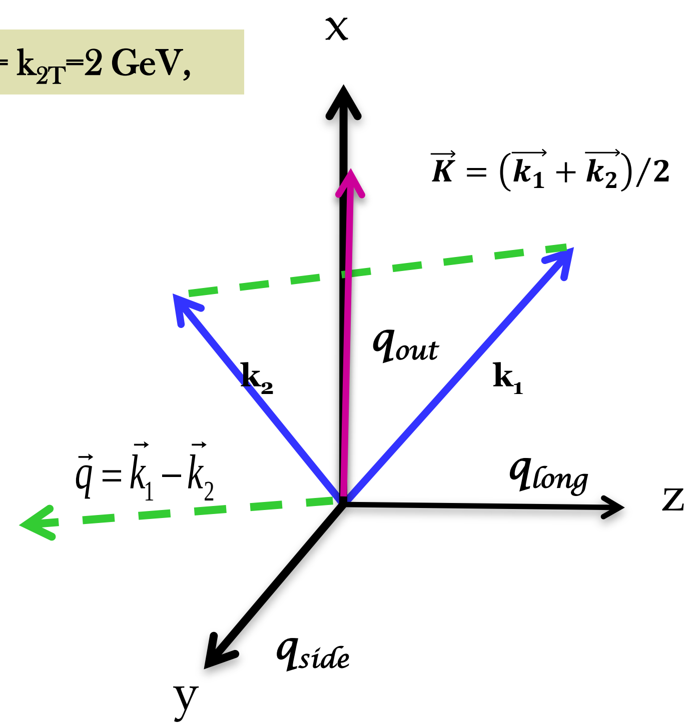
$$= \frac{2k_{1T}k_{2T}\sqrt{1-\cos^2(\psi_1-\psi_2)}}{\sqrt{k_{1T}^2+k_{2T}^2+2k_{1T}k_{2T}\cos(\psi_1-\psi_2)}}$$

$\psi_1=0$, $\psi_1 \neq 0$, $y_1=y_2=0$, $k_{1T}=k_{2T}=2$ GeV,

$$q_{out} = \frac{\vec{q_T}.\vec{K_T}}{|K_T|}$$

$$= \frac{(k_{1T}^2-k_{2T}^2)}{\sqrt{k_{1T}^2+k_{2T}^2+2k_{1T}k_{2T}\cos(\psi_1-\psi_2)}}$$

$k_{1T}=2$ GeV, $\psi_1=\psi_2=0$, $y_1=y_2=0$



- k_{iT} is the individual transverse momentum
- y_i is rapidity of each partons
- ψ_i is angles made by k_{iT} with x-axis of each partons