Generalized susceptibilities in the 3-d, 3-state Potts model

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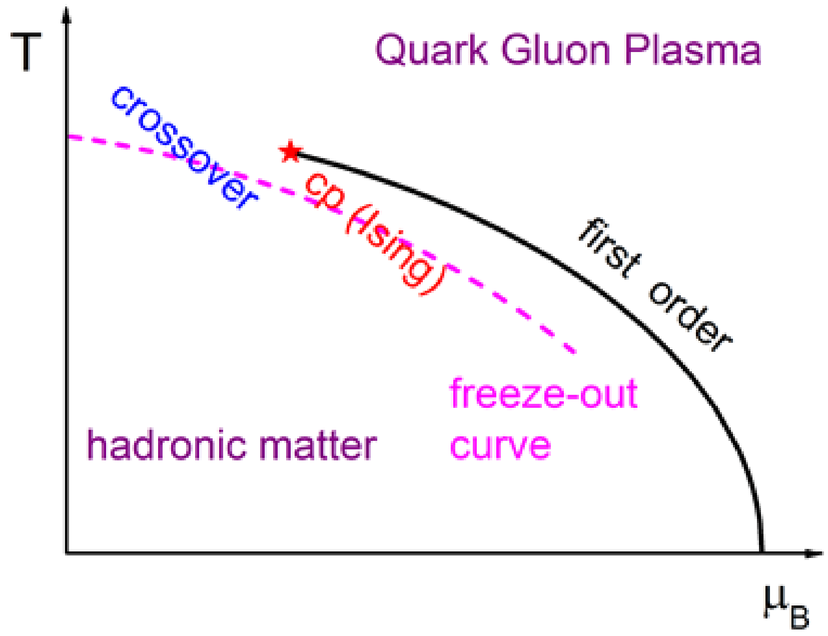
Outline

- Introduction
- Generalized susceptibilities in the Potts model
- Finite-size scaling of the susceptibilities
- Summary

Introduction: generalized susceptibilities



V. Koch, arXiv:0810.2520
M. Cheng et al, PRD 79 (2009) 074505
L. Adamczyk et al. ,
(STAR Collaboration)
Phys. Rev. Lett. 112 (2014) 032302

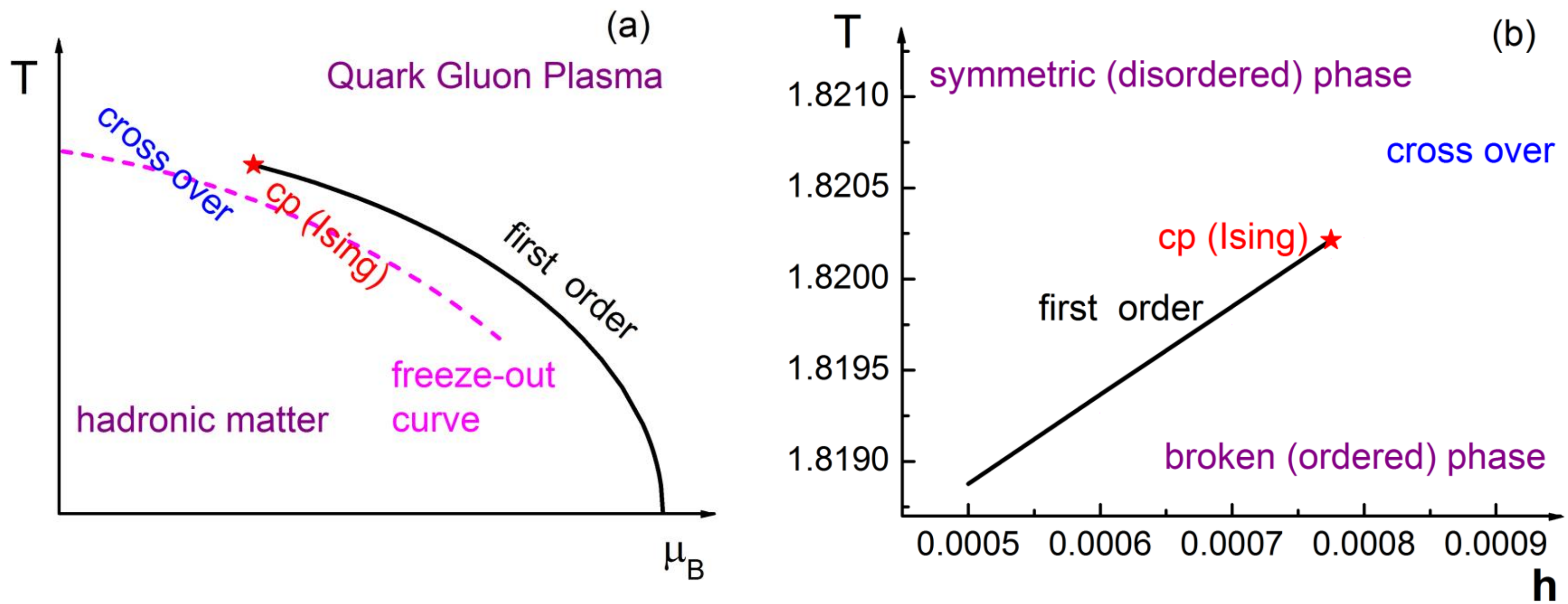
Theory Experiment

$$VT^3 \chi^{(2)}_{q,\mu} = \left\langle (\delta N_q)^2 \right\rangle$$

$$VT^3 \chi^{(4)}_{q,\mu} = \left\langle (\delta N_q)^4 \right\rangle - 3\left\langle (\delta N_q)^2 \right\rangle^2$$

$$\delta N_q = N_q - \left\langle N_q \right\rangle$$

A paradigm of QCD: the 3-d, 3-state Potts model



QCD

Decreasing μ_B

Potts

Increasing h

First-order → second-order (Ising) → crossover

critical point

?

$$\beta_c = 1/T_c = 0.54938(2), h_c = 0.000775(10)$$

F. Karsch, S. Stickan, Phys. Lett. B 488 (2000) 319

Generalized susceptibilities in the 3-d, 3-state Potts model

- Definition

$$Z(\beta,h)=\sum_{\{s_i\}} e^{-(\beta E-hM)}, s_i \in \{1,2,3\}, \beta=1/T, h=\beta H$$

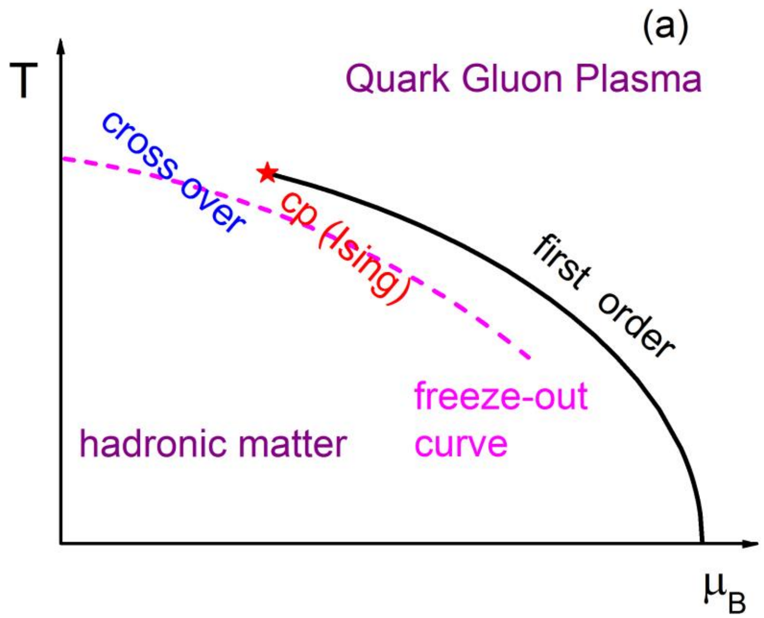
- Energy and Magnetization

$$E=-J\sum_{\langle i,j\rangle}\delta(s_i,s_j), M=\sum_i \delta(s_i,s_g)$$

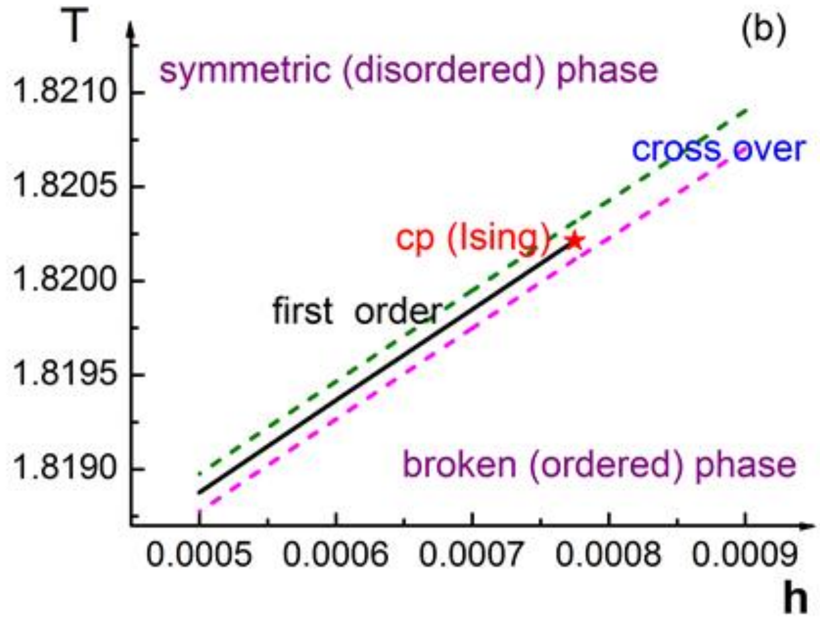
- Generalized susceptibility of Magnetization

$$f=-\frac{1}{V}\ln Z \quad \begin{cases} \chi_2=-\left.\dfrac{\partial^2 f}{\partial h^2}\right|_T=\dfrac{1}{V}\left\langle(\delta M)^2\right\rangle, \quad \delta M=M-\langle M\rangle \\ \chi_4=-\left.\dfrac{\partial^4 f}{\partial h^4}\right|_T=\dfrac{1}{V}\left(\left\langle(\delta M)^4\right\rangle-\left\langle(\delta M)^2\right\rangle^2\right) \end{cases}$$

Question 1



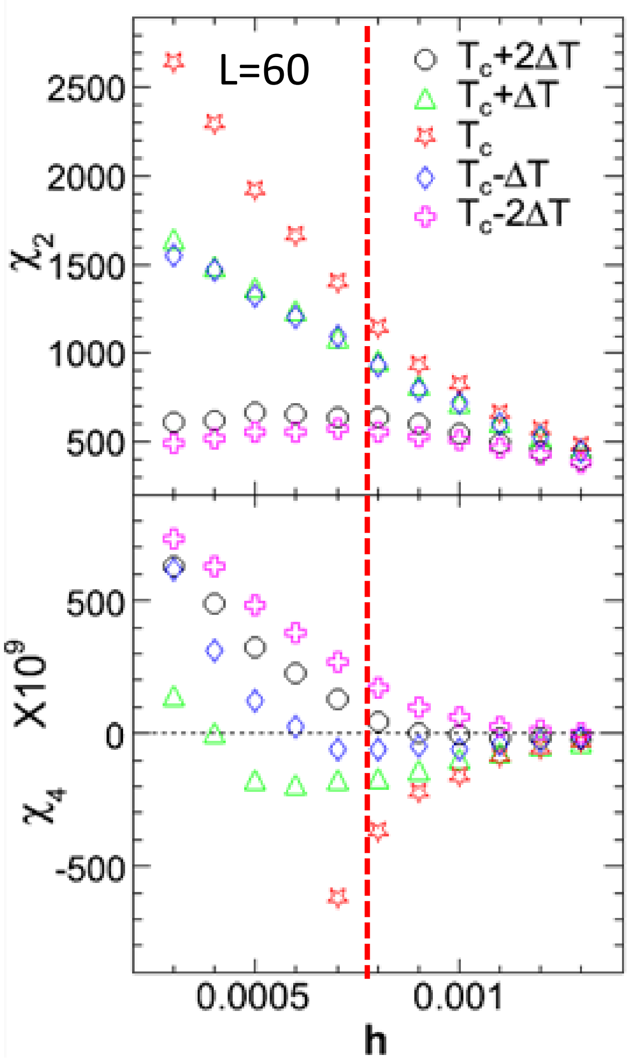
QCD

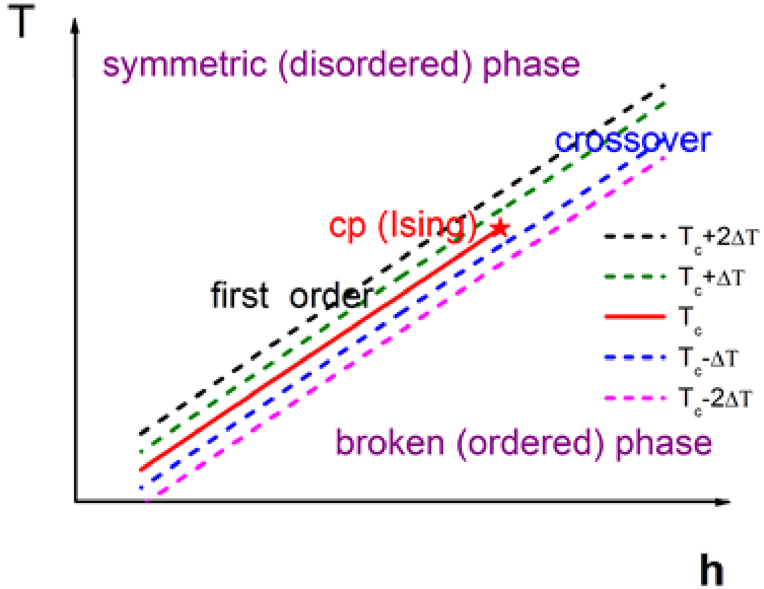


Potts

- In the 3-d, 3-state Potts model, how will the 2nd and 4th order susceptibilities behave from the first order phase transition side to crossover side?

χ_2 and χ_4 along PT line



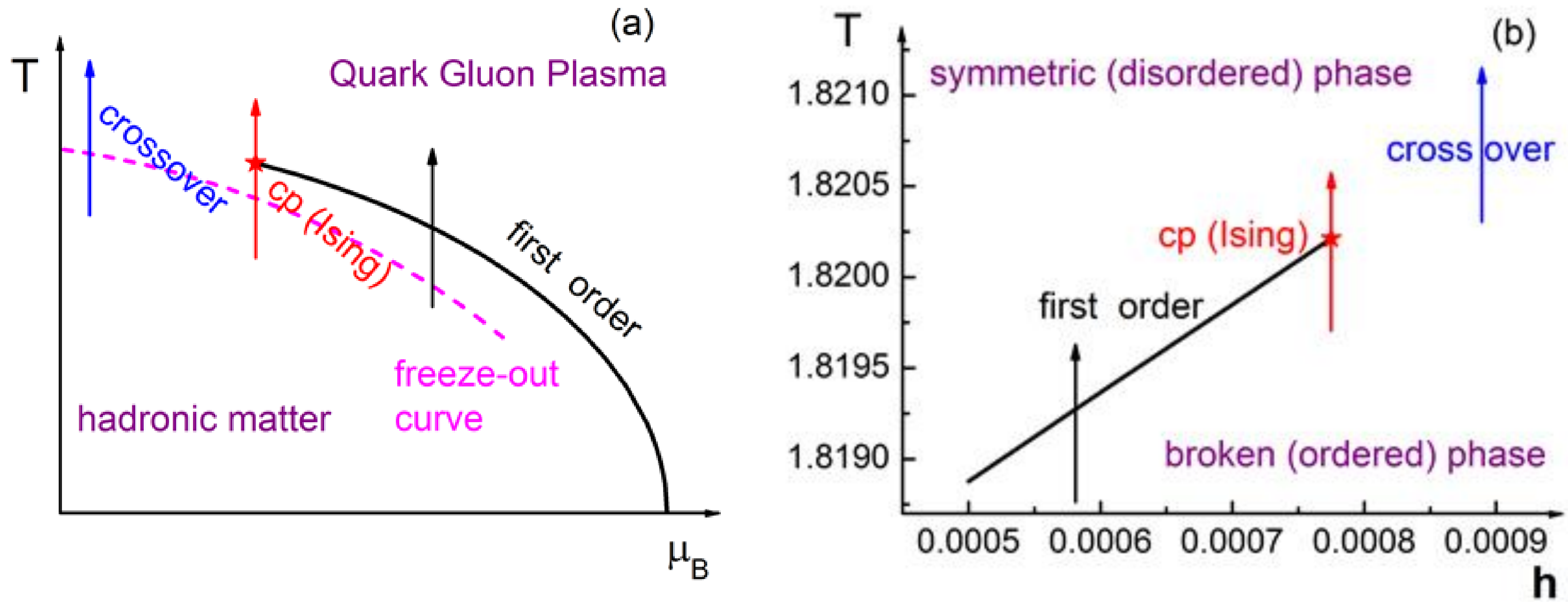


- From 1st order to crossover transition, χ_2 decreases monotonically

- The sign of χ_4 depends on the extent deviating from the PT line

1st order ⟶ cp ⟶ crossover

Question 2

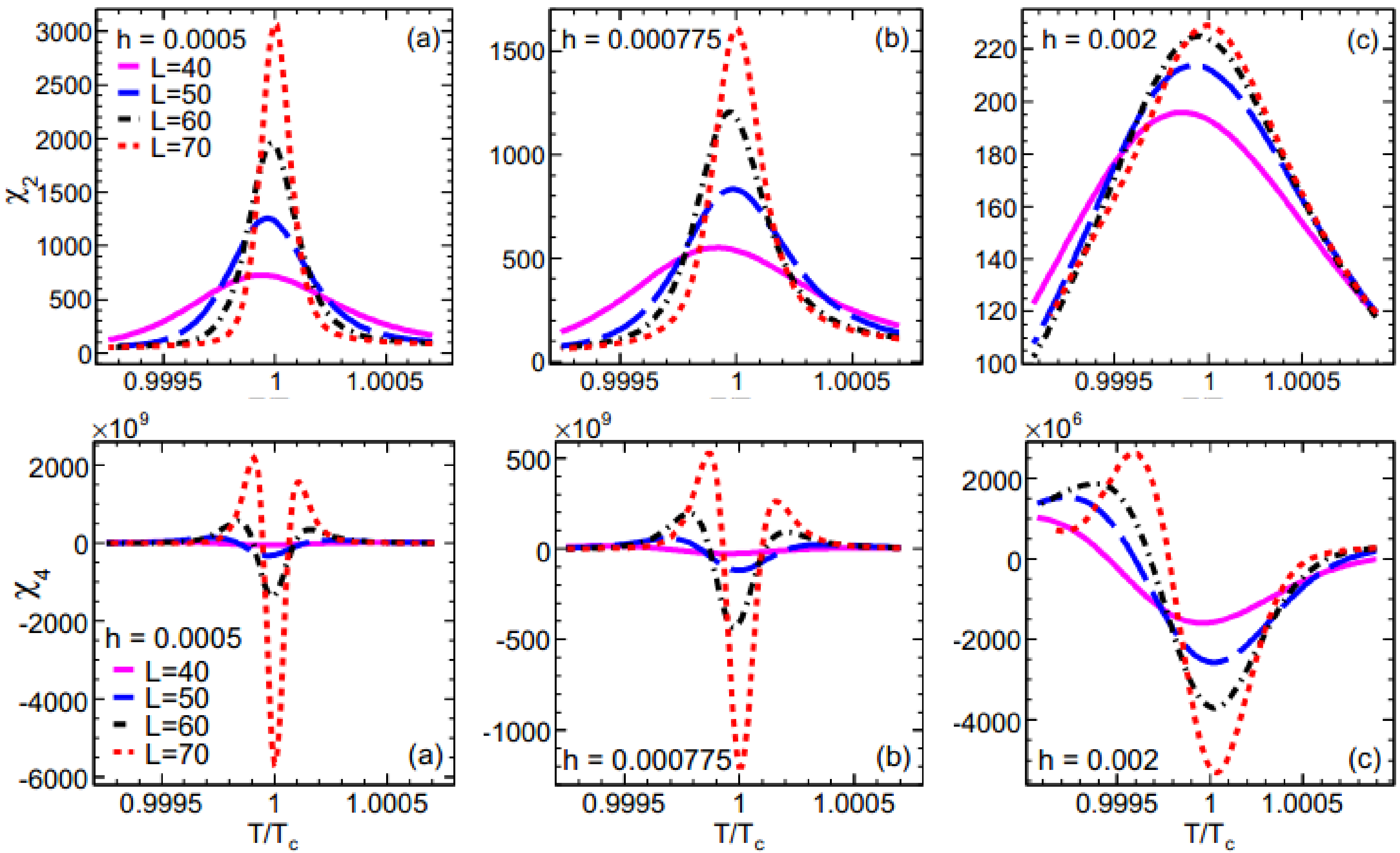


QCD　　　　　　　　Potts

- If it's not along the line from 1st order phase transition side to crossover side, but across the PT line at some fixed external fields of 1st, 2nd order PT and crossover, how will the 2nd and 4th order susceptibilities behave?
- Can we distinguish different type of PT from the 2nd and 4th order susceptibilities?

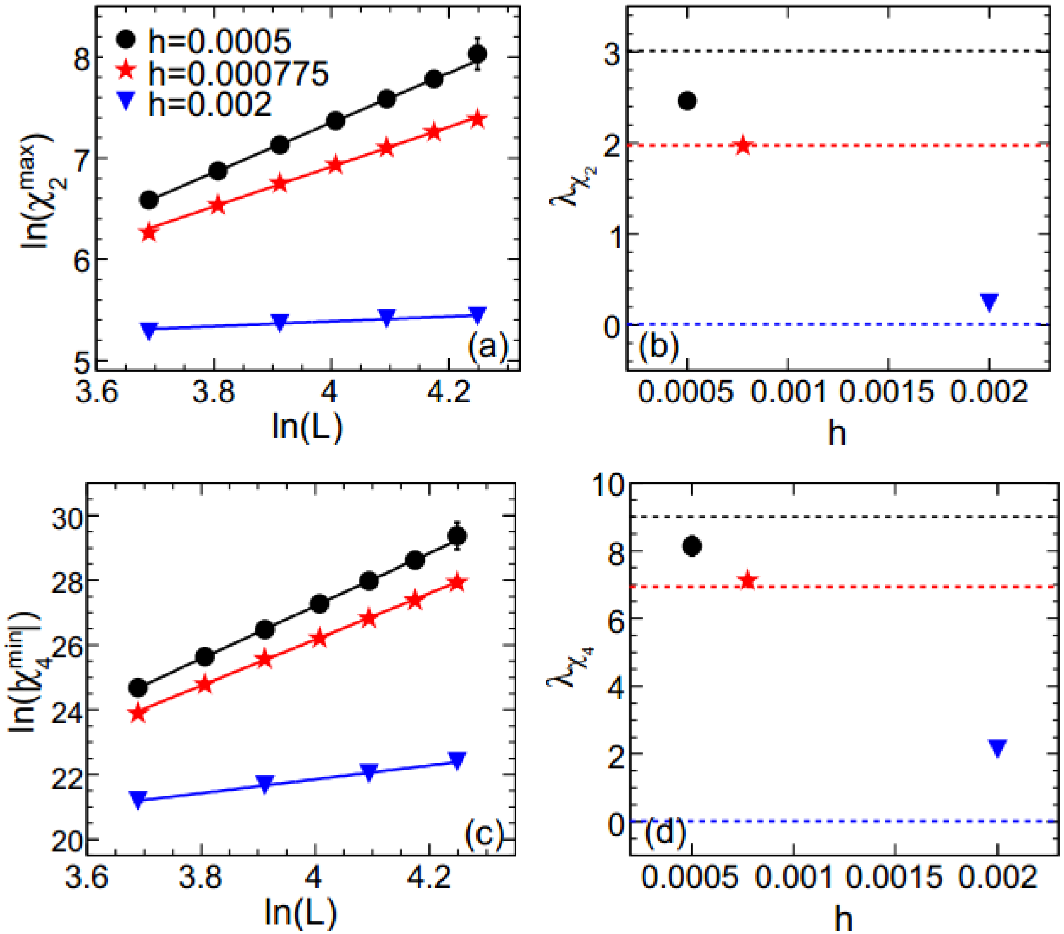
The 2nd and 4th order susceptibility

1st order PT | 2nd order PT | crossover



- Non-monotonic behavior are signatures of 2nd order PT, but also can be observed at 1st order PT and crossover
- Quantitatively, their size dependence is different

Finite-size scaling of χ_2



$$\chi_2^{\max}(L) \propto L^{\lambda_{\chi 2}}$$

$$\ln(\chi_2^{\max}) = \lambda_{\chi_2} \ln L + C_1$$

$$\left|\chi_4^{\min}(L)\right| \propto L^{\lambda_{\chi 4}}$$

$$\ln(\left|\chi_4^{\min}\right|) = \lambda_{\chi_4} \ln L + C_2$$

········ $\lambda_{\chi 2}$ or $\lambda_{\chi 4}$ of 1st order PT

········ $\lambda_{\chi 2}$ or $\lambda_{\chi 4}$ of 2nd order Ising

········ $\lambda_{\chi 2}$ or $\lambda_{\chi 4}$ of crossover

- $\chi_2^{\max}$ and $\left|\chi_4^{\min}\right|$ are fitted well by a straight line at 3 values of h.
- The exponent of finite-size scaling decreases with increasing h, characterizing the order of PT or crossover.

Summary

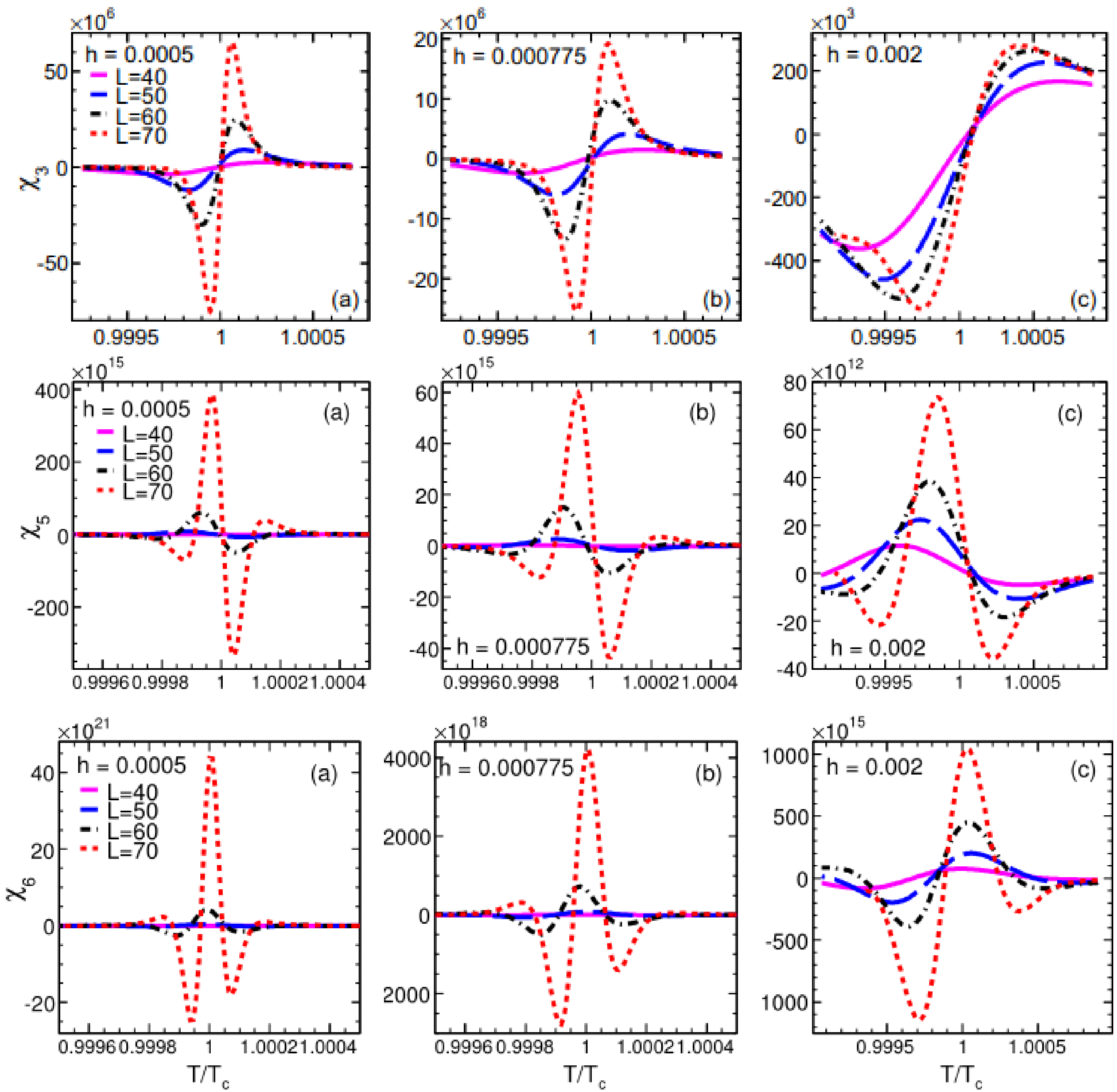
- We studied the 2nd and 4th order susceptibility along the phase transition line, and also in different type of phase transitions in the 3-d, 3-state Potts model .
- The sign of χ_4 is dependent on the extent deviating from the phase transition line. The negative value of χ_4 should be dealt carefully as signals of critical region in this model.
- Peak, oscillation, or sign change of generalized susceptibilities are signatures of 2nd order PT, but also can be observed at 1st order PT and crossover when crossing the phase transition line of a system of finite-size.
- Finite-size scaling exponents of the generalized susceptibilities characterize the order of the phase transitions.

Summary

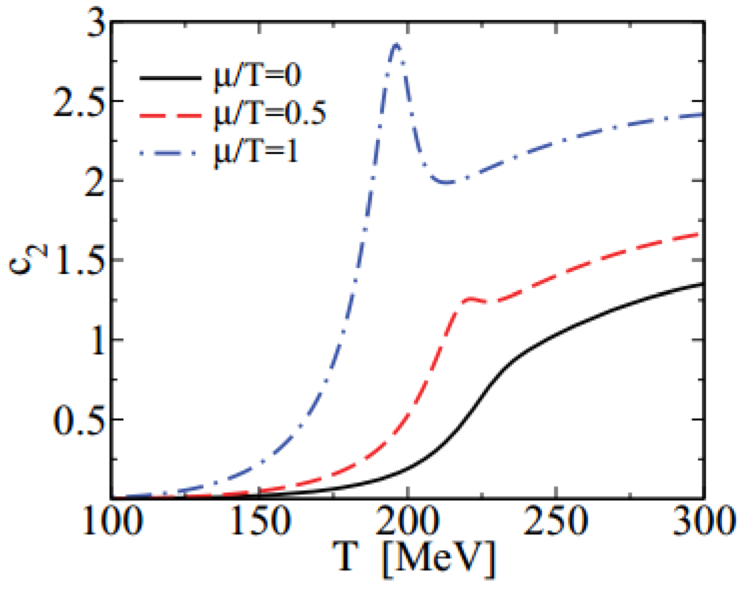
- We studied the 2nd and 4th order susceptibility along the phase transition line, and also in different type of phase transitions systematically in the 3-d, 3-state Potts model .
- The sign of χ_4 is dependent on the extent deviating from the phase transition line. The negative value of χ_4 should be dealt carefully as signals of critical region in this model.
- Peak, oscillation, or sign change of generalized susceptibilities are signatures of 2nd order PT, but also can be observed at 1st order PT and crossover when crossing the phase transition line of a system of finite-size.
- Finite-size scaling exponents of the generalized susceptibilities characterize the order of the phase transitions.

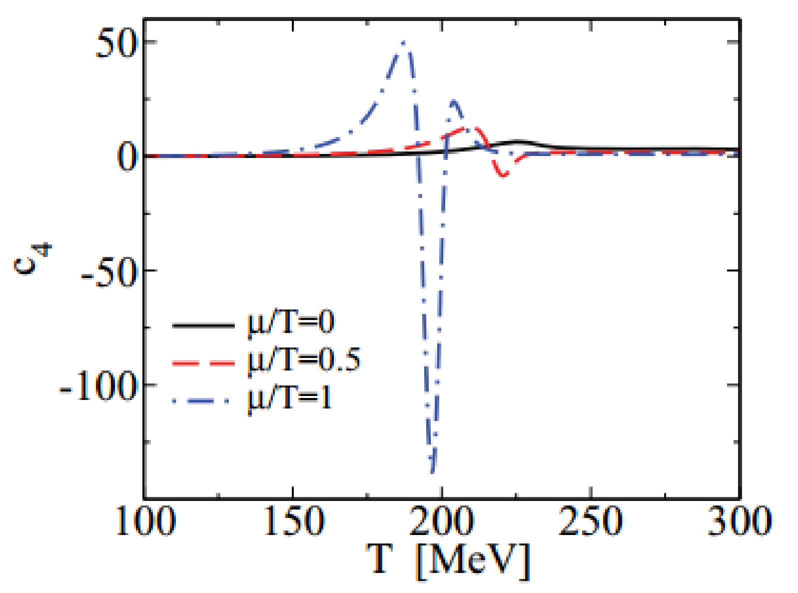
Thanks for your attention!

Backup



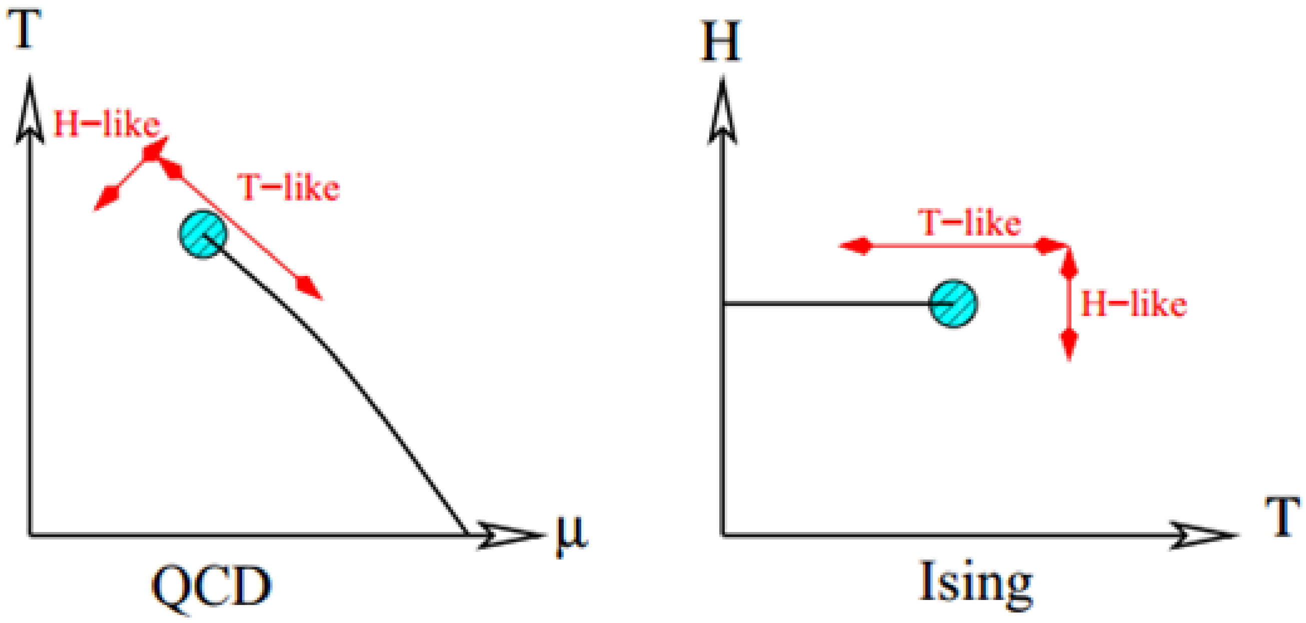
Quark number susceptibility





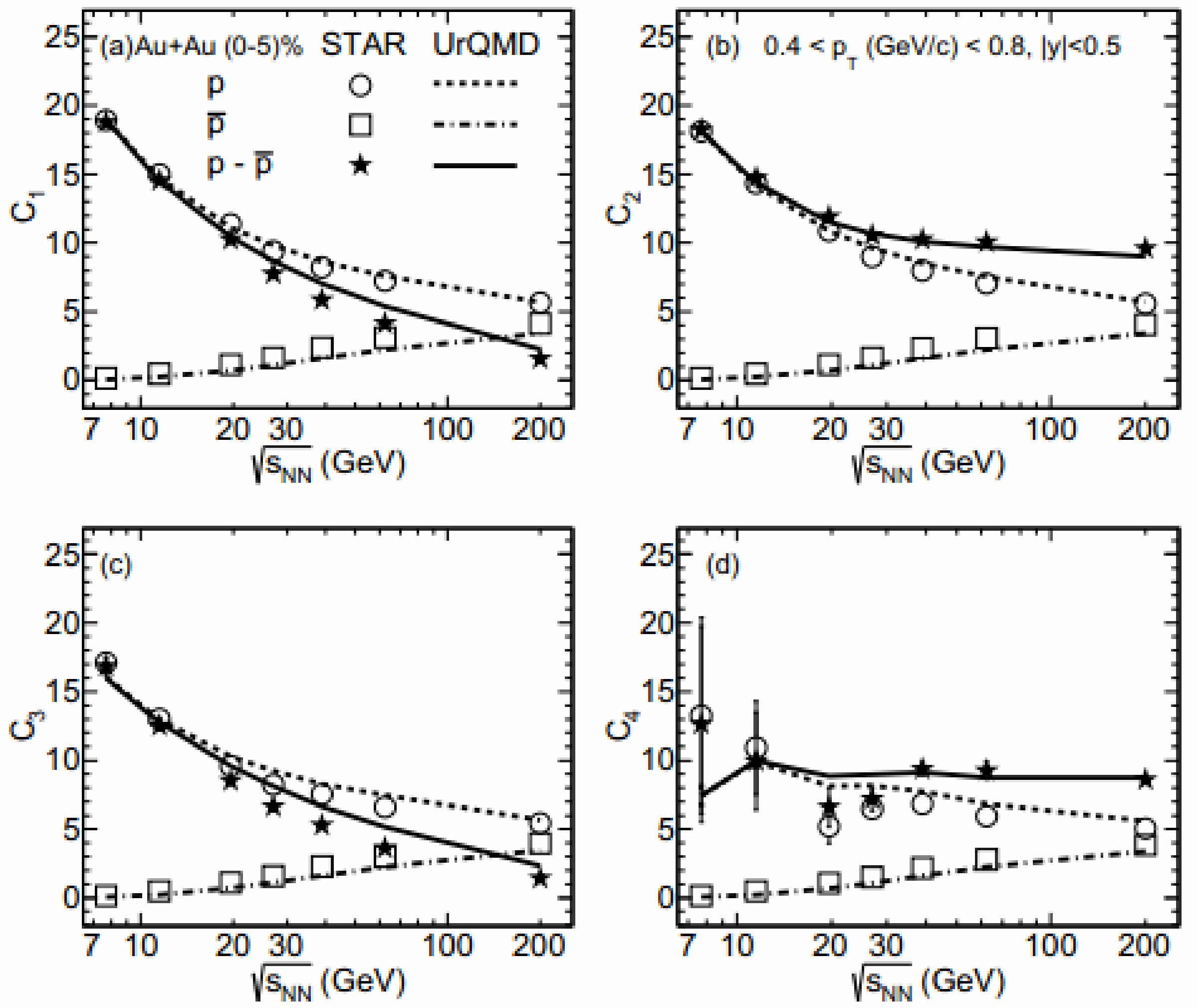
The 2nd and 4th order susceptibilities from PQM in the mean-field approximation

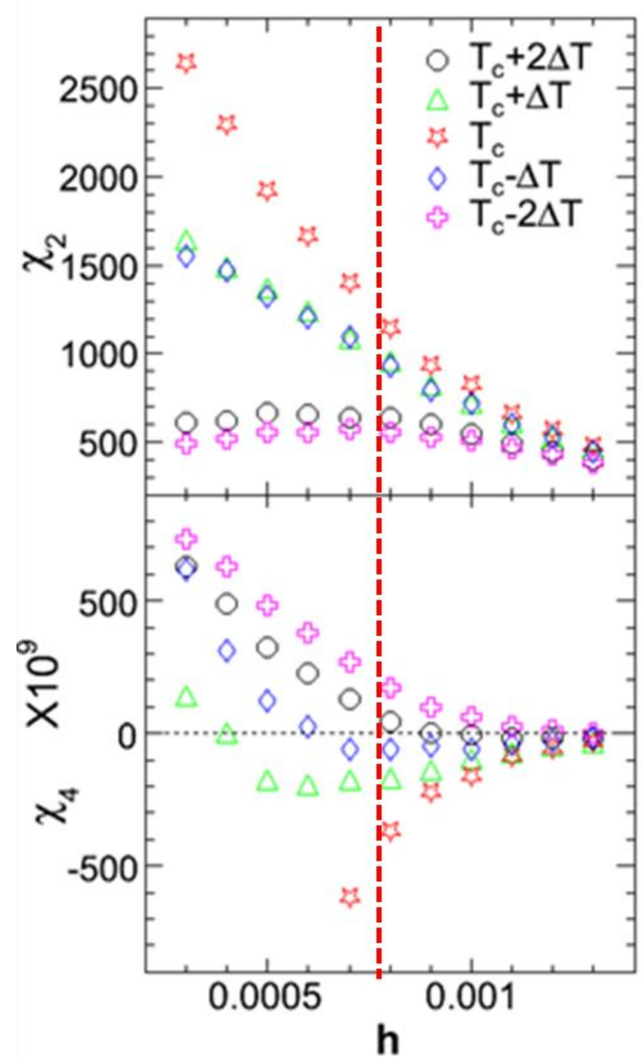
V. Skokov, B. Friman, and K. Redlich,
PHYSICAL REVIEW C 83, 054904 (2011)



$$t \approx \beta - \beta_{cp} + a(\mu - \mu_{cp}), h \approx \mu - \mu_{cp} + b(\beta - \beta_{cp})$$

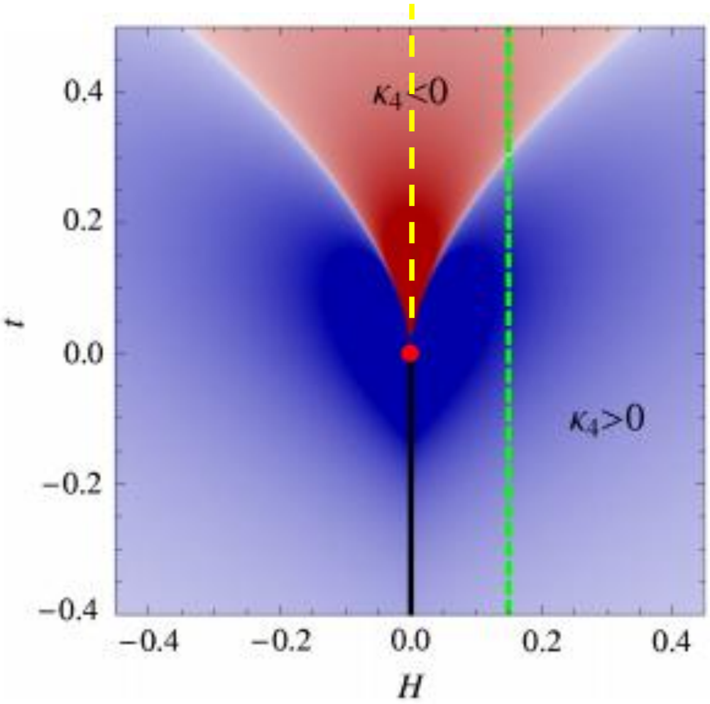
Cumulants of net-proton at RHIC



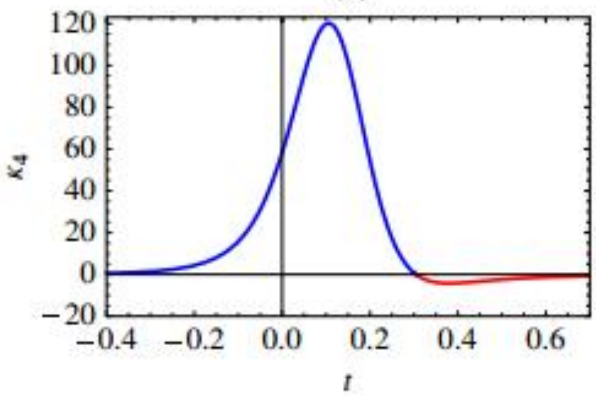


Critical fluctuations

The linear parametric Ising

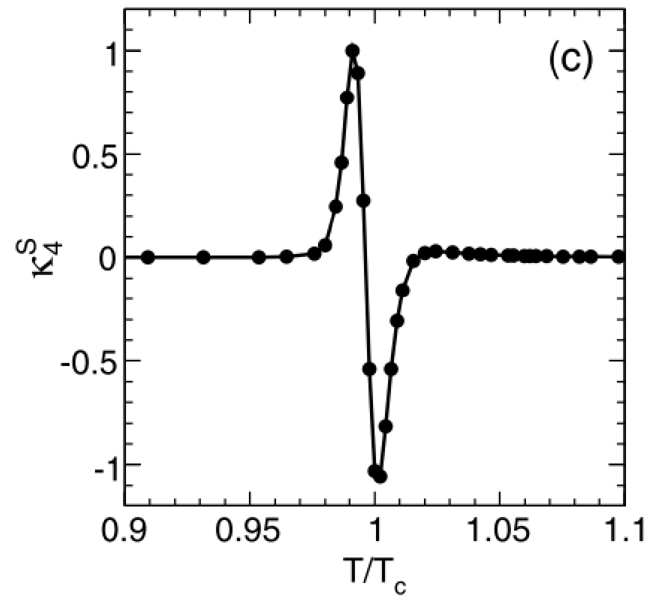


(a)



(b)

Monte Carlo simulation



- The fourth order cumulant is negative when the critical point is approached from the crossover side

M. A. Stephanov, PRL 107, 052301
Xue Pan et al, NPA 913, 206