Azimuthal Anisotropy Distributions in High-Energy Collisions

Li Yan[1], Jean-Yves Ollitrault[1] and Arthur M. Poskanzer[2]

[1]CNRS, Institut de Physique Théorique, Saclay

[2]Lawrence Berkeley National Laboratory

22-26 Sep. 2014, CCNU, Wuhan, China

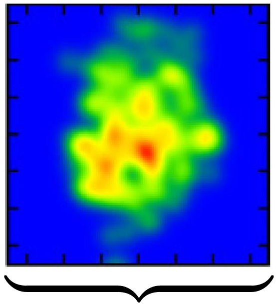
- arXiv: 1408.0921 [nucl-th]
- Phys. Rev. C (2014) 024903
- Phys. Rev. Lett. 112 (2014) 082301

Outline of this talk

- Introduction and motivation – why need to understand initial state fluctuations?
- Elliptic-Power and Power formulas – how to describe initial state fluctuations?
- Application – what can we learn from the analyses of fluctuations?
- Conclusions and outlook.

Introduction and motivation: fluctuations in a single event

- Fluctuating initial state and harmonic flow v_n:

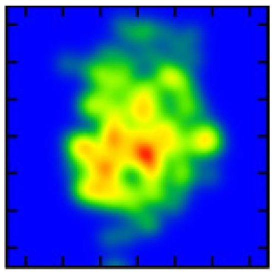


Glauber, KLN, IP-Glasma

$$\underbrace{\Longleftrightarrow}_{\text{medium exp.}} \qquad \frac{dN}{d\phi_p} \sim 1 + 2\sum_n v_n e^{in(\phi_p - \Psi_n)}$$

Introduction and motivation: fluctuations in a single event

- Fluctuating initial state and harmonic flow v_n:

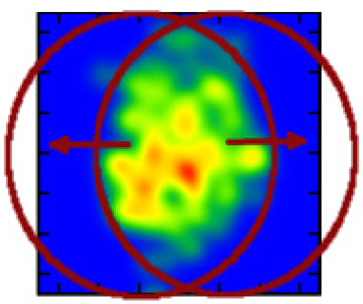


Glauber, KLN, IP-Glasma

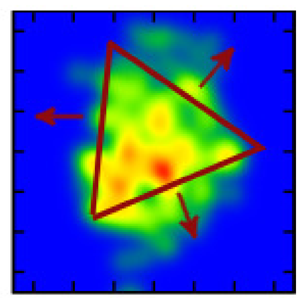
$\underbrace{\Longleftrightarrow}_{\text{medium exp.}}$

$$\frac{dN}{d\phi_p} \sim 1 + 2\sum_n v_n e^{in(\phi_p - \Psi_n)}$$

- Characterization of initial state azimuthal assymmtry: eccentricity



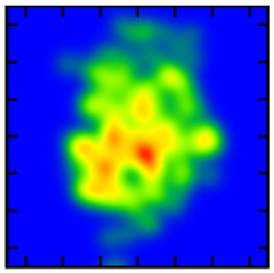
$\epsilon_2, \psi_2 \Rightarrow v_2$



$\epsilon_3, \psi_3 \Rightarrow v_3$

$+(\varepsilon_4, \psi_4) + \ldots$

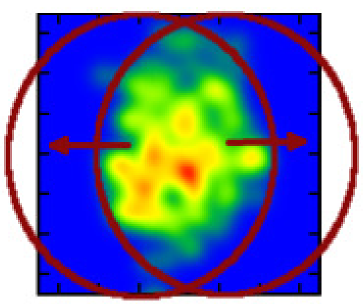
- Fluctuating initial state and harmonic flow v_n:



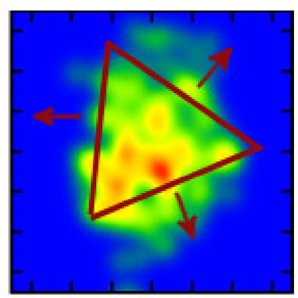
$\underbrace{}_{\text{Glauber, KLN, IP-Glasma}}$

$$\underbrace{\Longleftrightarrow}_{\text{medium exp.}} \qquad \frac{dN}{d\phi_p} \sim 1 + 2\sum_n v_n e^{in(\phi_p - \Psi_n)}$$

- Characterization of initial state azimuthal assymmtry: eccentricity



$\epsilon_2, \psi_2 \Rightarrow v_2$



$\epsilon_3, \psi_3 \Rightarrow v_3$

$+(\varepsilon_4, \psi_4) + \ldots$

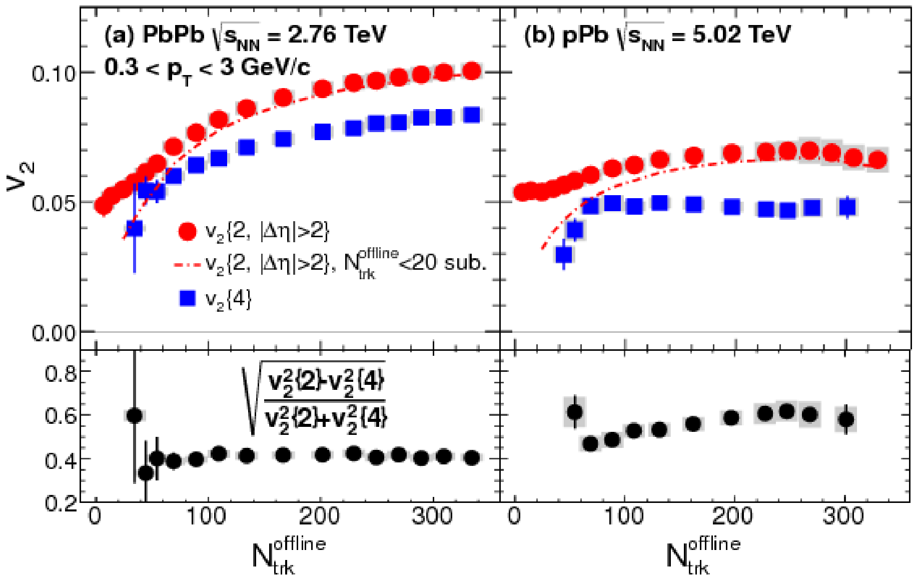
- Eccentricity: $\varepsilon_n e^{in\psi_n} = -\frac{\{r^n e^{in\phi_r}\}}{\{r^n\}} = \varepsilon_x + i\varepsilon_y$

$$\{\ldots\} = \int d^2\vec{x} \ldots \rho(\vec{x}). \quad \text{note that } |\varepsilon_n| < 1$$

Initial state fluctuations: fluctuations on an EbyE basis

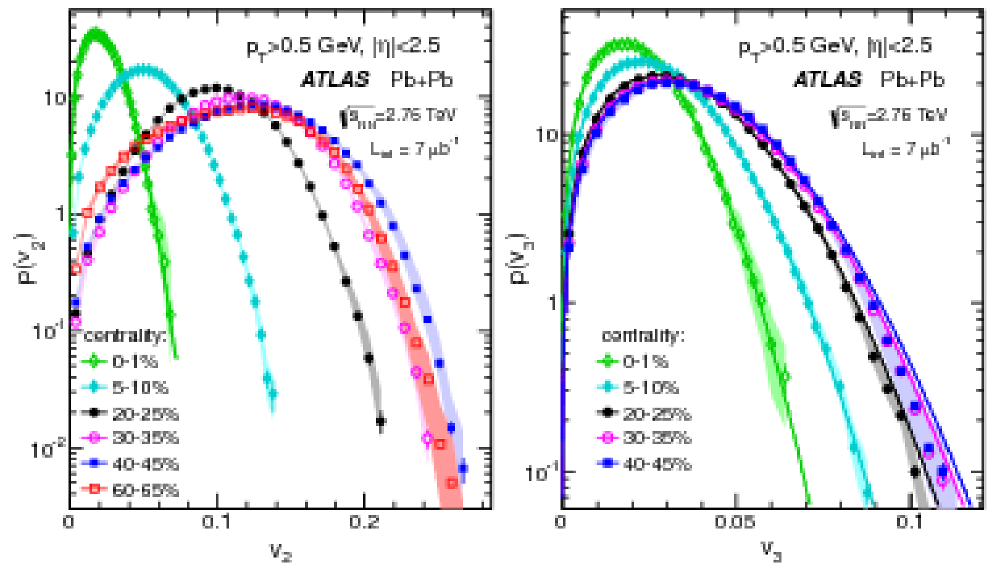
- Fluctuations of v_n in experiments: p-A and A-A

(CMS)PLB724 (2013)213-240



First 2 cumulants of v_2

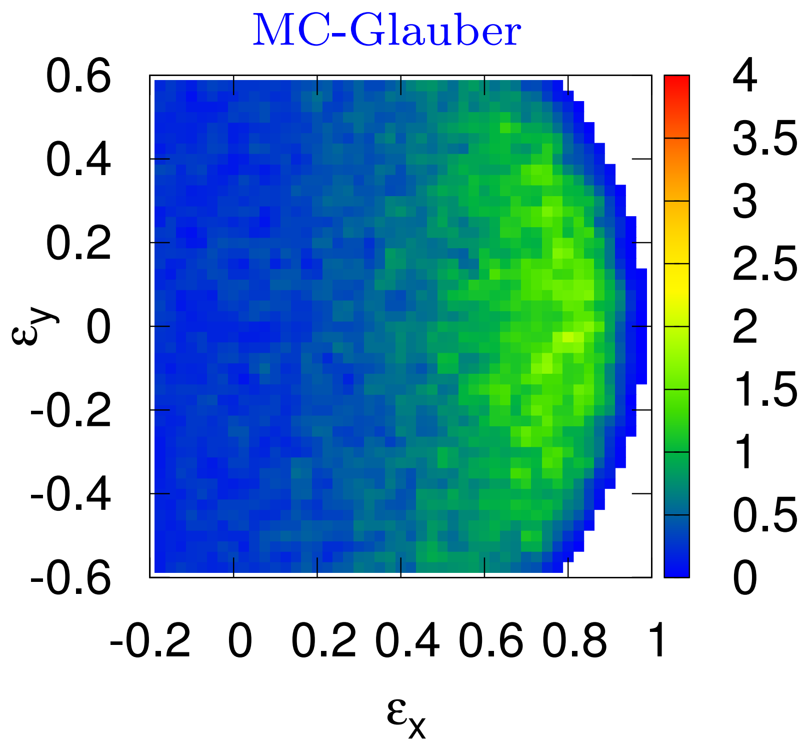
(ATLAS) JHEP 1311(2013)183



Probability distribution of v_n

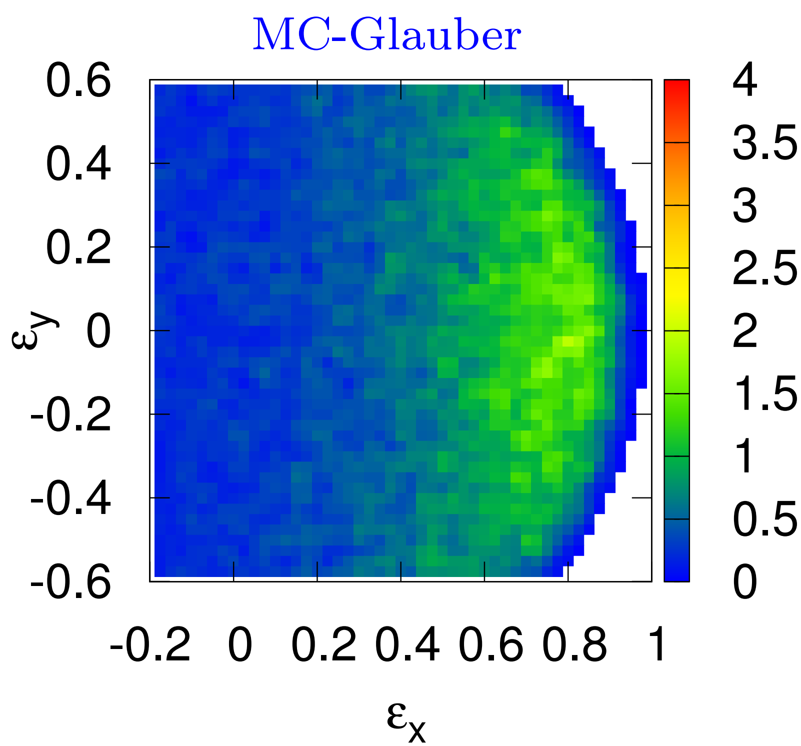
EbyE distribution of ε_2

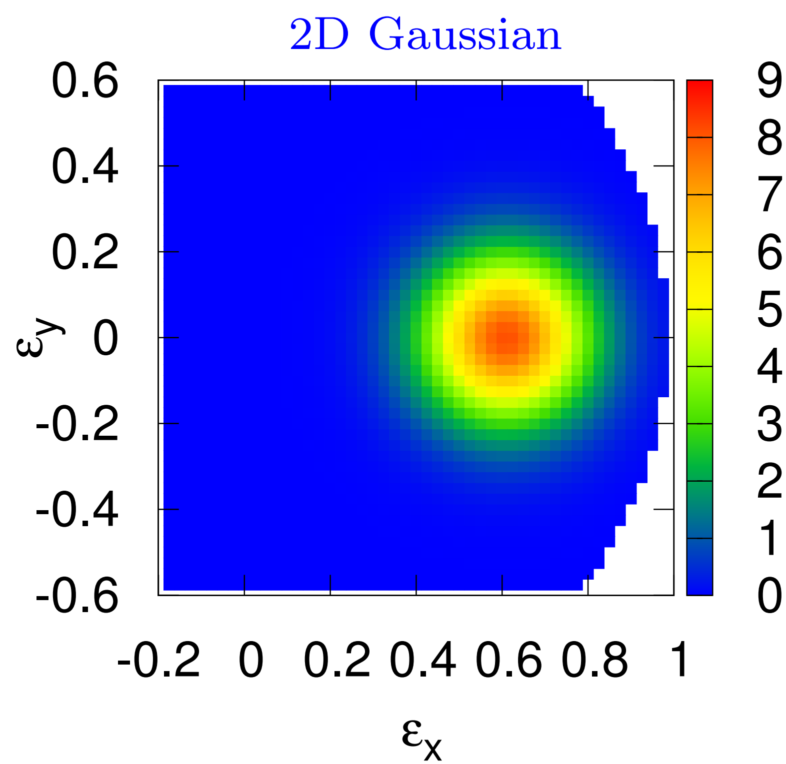
- ε_2 distribution (MC Glauber simulation of LHC PbPb 2.76TeV, centrality 75%-80%)



EbyE distribution of ε_2

- ε_2 distribution (MC Glauber simulation of LHC PbPb 2.76TeV, centrality 75%-80%)

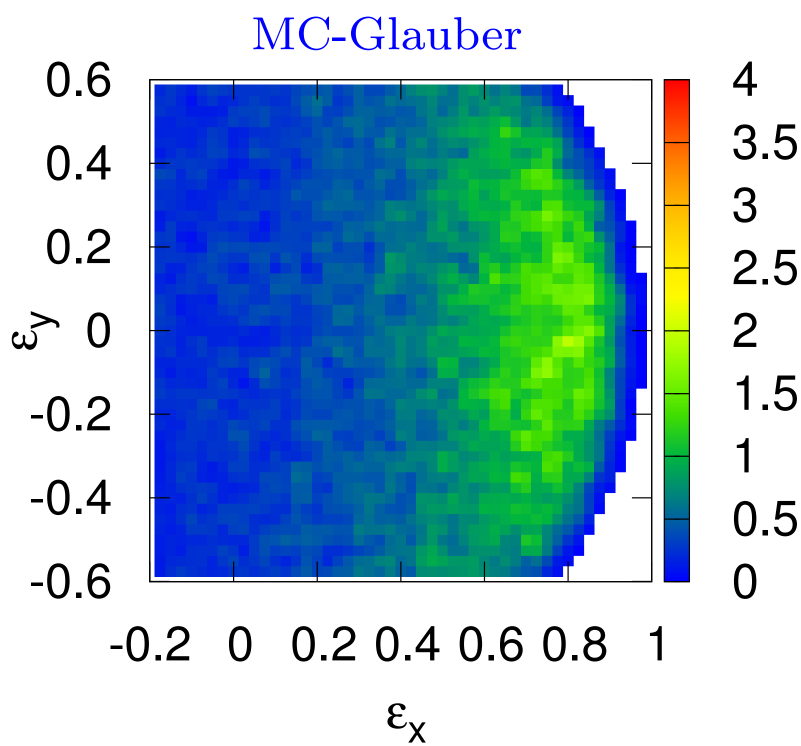


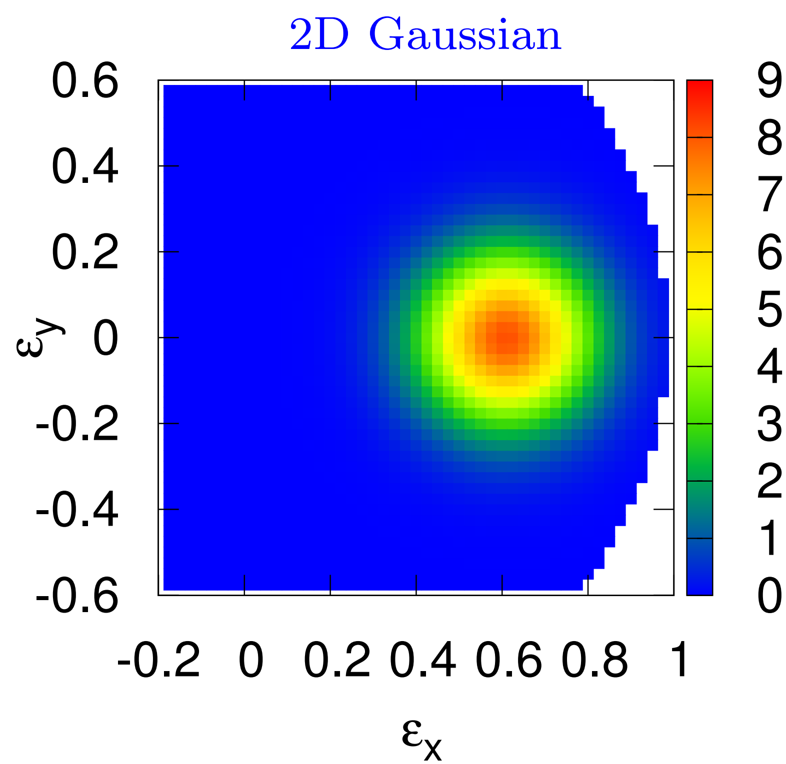


* 2D Gaussian with non-zero mean.

EbyE distribution of ε_2

- ε_2 distribution (MC Glauber simulation of LHC PbPb 2.76TeV, centrality 75%-80%)





* 2D Gaussian with non-zero mean. $\Leftrightarrow$ Bessel-Gaussian. *(S. Voloshin et al, PLB 659)*

$$P_{\mathrm{BG}}(\varepsilon_n) = \frac{\varepsilon_n}{\sigma^2} \exp\left(-\frac{\varepsilon_n^2 + \varepsilon_0^2}{2\sigma^2}\right) I_0\left(\frac{\varepsilon_n \varepsilon_0}{\sigma^2}\right), \qquad \text{with } \varepsilon_n \in [0, \infty)$$

ε_0 : Mean eccentricity in RP, σ : Characterizes fluctuations around ε_0.

Elliptic Power distribution and Power distribution

- **Elliptic Power distribution** : (e.g. assuming N independent point-like sources)

$$P_{\mathrm{EP}}(\varepsilon_x, \varepsilon_y) = \frac{\alpha}{\pi}(1-\varepsilon_0^2)^{\alpha+\frac{1}{2}} \frac{(1-\varepsilon_x^2-\varepsilon_y^2)^{\alpha-1}}{(1-\varepsilon_0\varepsilon_x)^{2\alpha+1}}, \qquad \text{with } \varepsilon_x^2+\varepsilon_y^2<1$$

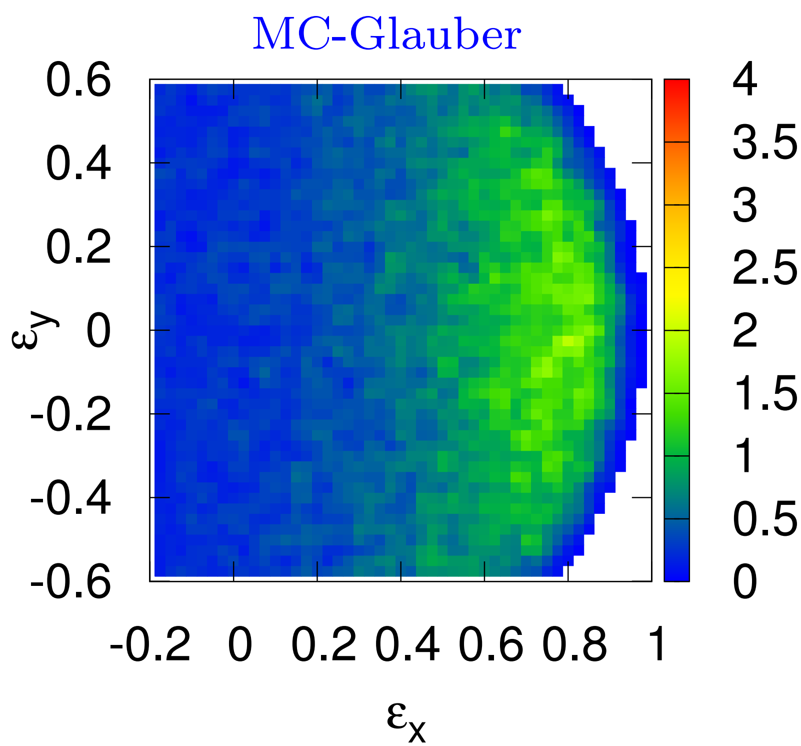
$\alpha \sim N \Rightarrow$ fluctuations, $\quad \varepsilon_0 \Rightarrow$ average RP eccentricity (roughly)

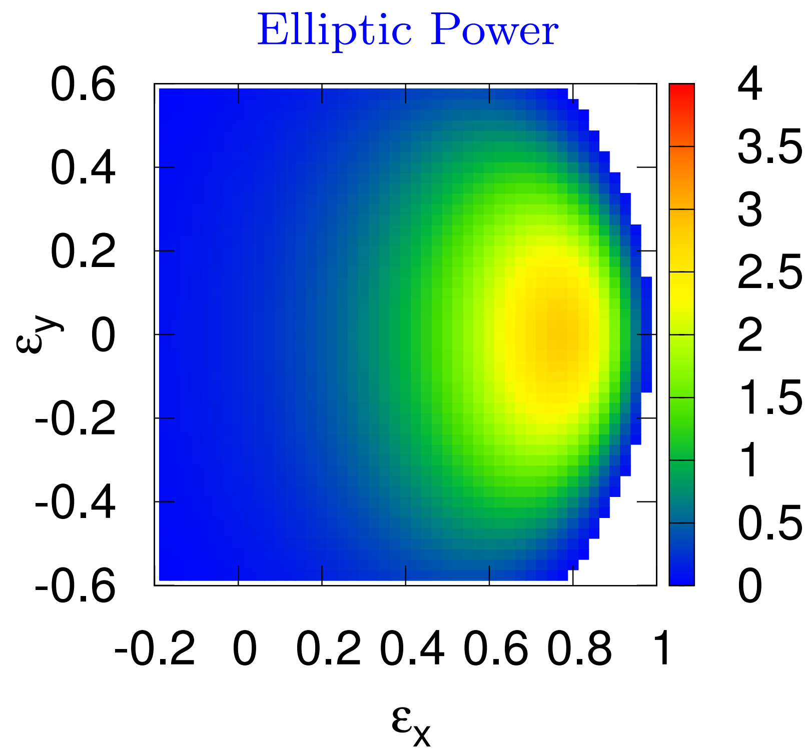
Elliptic Power distribution and Power distribution

- **Elliptic Power distribution** : (e.g. assuming N independent point-like sources)

$$P_{\rm EP}(\varepsilon_x,\varepsilon_y)=\frac{\alpha}{\pi}(1-\varepsilon_0^2)^{\alpha+\frac{1}{2}}\frac{(1-\varepsilon_x^2-\varepsilon_y^2)^{\alpha-1}}{(1-\varepsilon_0\varepsilon_x)^{2\alpha+1}},\qquad \text{with } \varepsilon_x^2+\varepsilon_y^2<1$$

$\alpha\sim N \Rightarrow$ fluctuations, $\quad \varepsilon_0 \Rightarrow$ average RP eccentricity (roughly)



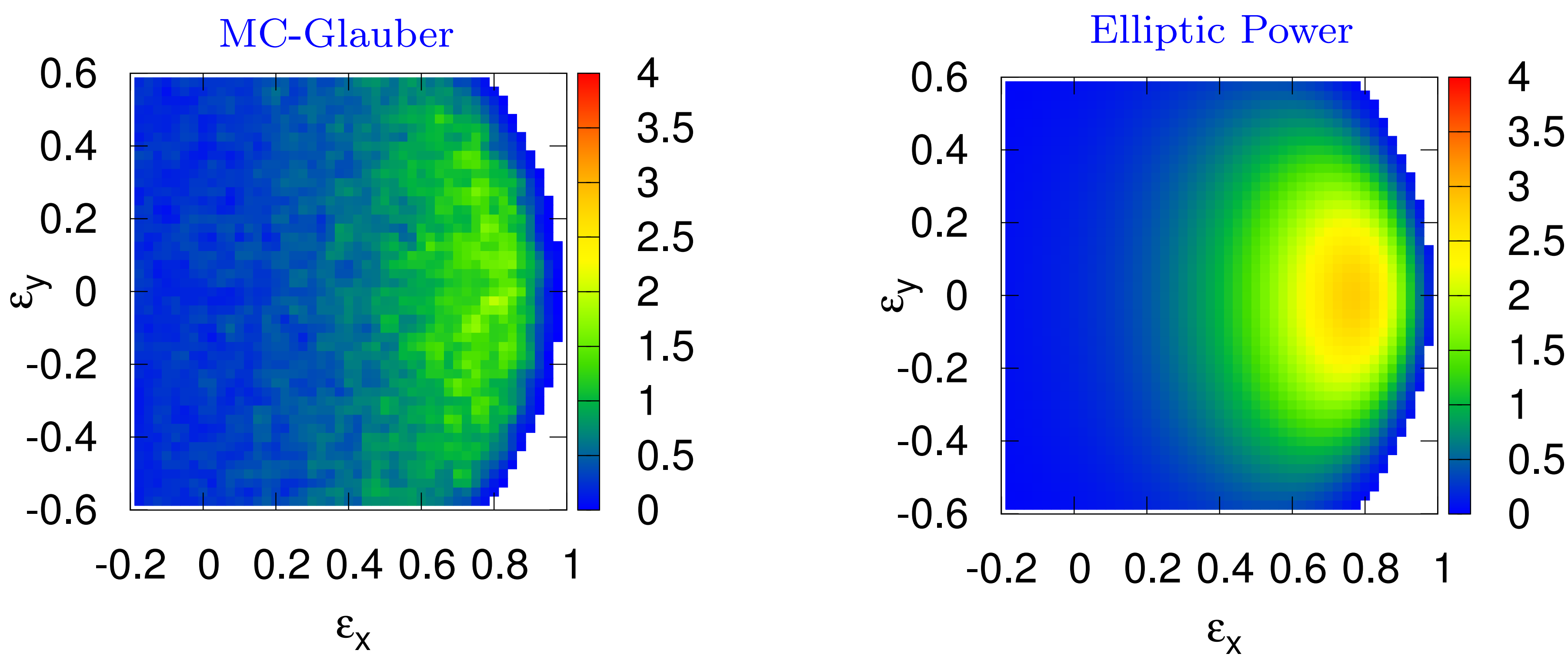


Elliptic Power distribution and Power distribution

- **Elliptic Power distribution** : (e.g. assuming N independent point-like sources)

$$P_{\rm EP}(\varepsilon_x,\varepsilon_y) = \frac{\alpha}{\pi}(1-\varepsilon_0^2)^{\alpha+\frac{1}{2}}\frac{(1-\varepsilon_x^2-\varepsilon_y^2)^{\alpha-1}}{(1-\varepsilon_0\varepsilon_x)^{2\alpha+1}}, \qquad \text{with } \varepsilon_x^2+\varepsilon_y^2<1$$

$\alpha \sim N \Rightarrow$ fluctuations, $\varepsilon_0 \Rightarrow$ average RP eccentricity (roughly)

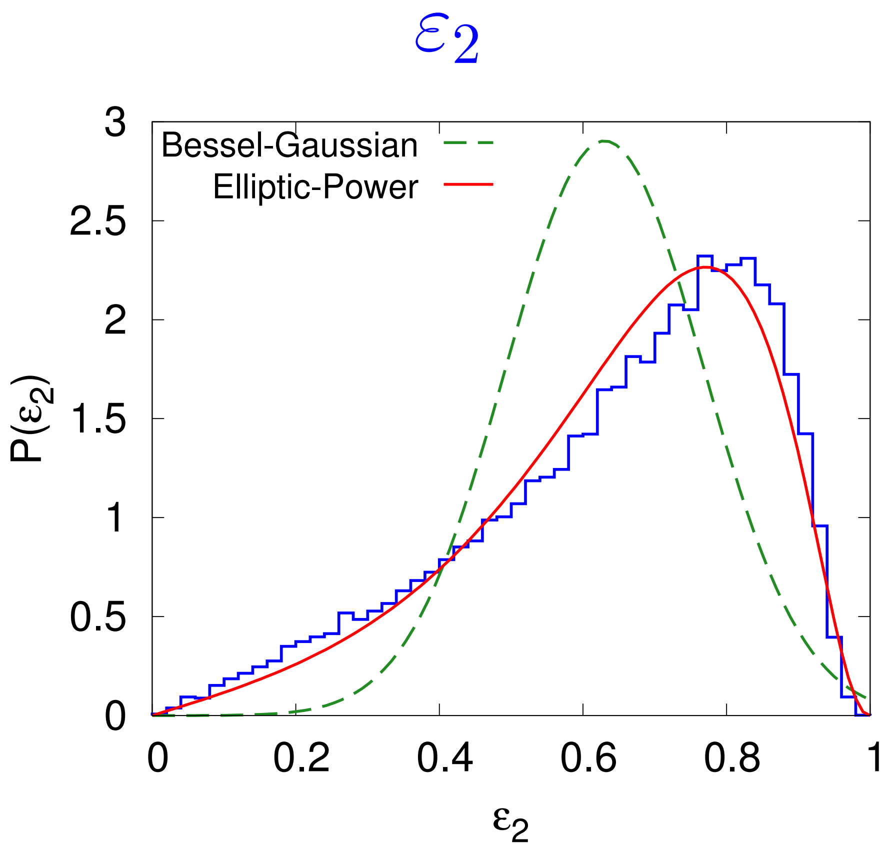


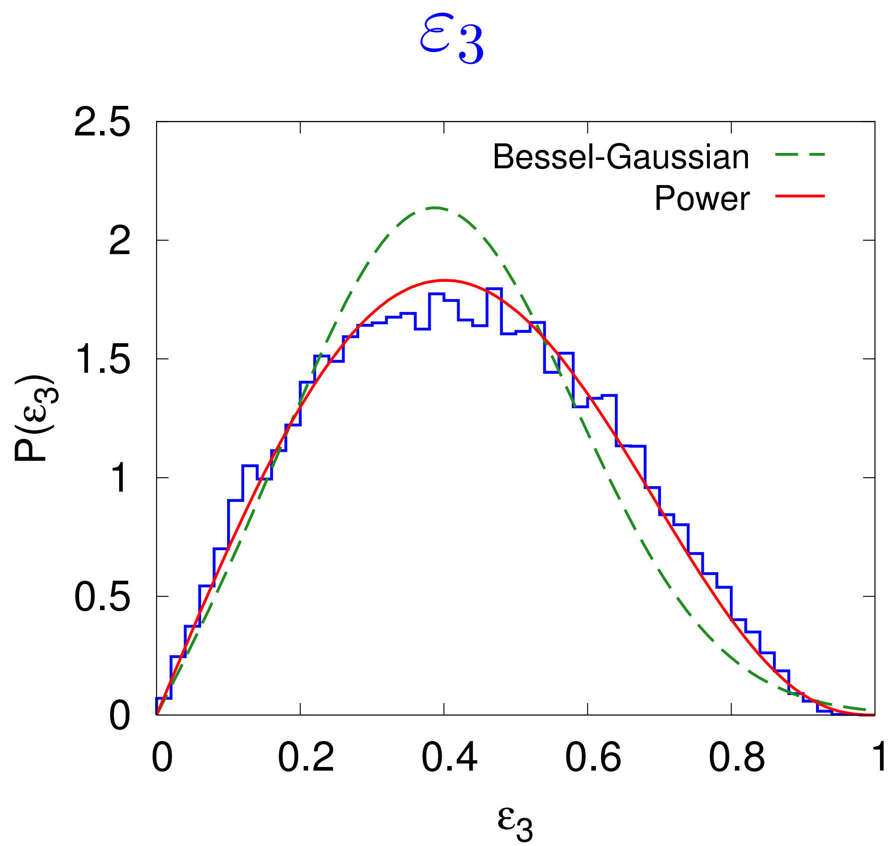
- **Power distribution** (e.g. ε_3 in AA, ε_n in p-Pb) : fluctuation-driven with $\varepsilon_0 = 0$

$$P_{\rm Power}(\varepsilon_x,\varepsilon_y) = \frac{\alpha}{\pi}(1-\varepsilon_x^2-\varepsilon_y^2)^{\alpha-1} \qquad \Leftarrow \qquad P_{\rm EP}(\varepsilon_0 \to 0)$$

Test of Elliptic Power and Power

- Test of Elliptic Power parameterization: MC-Glauber PbPb with centrality 75%-80%.



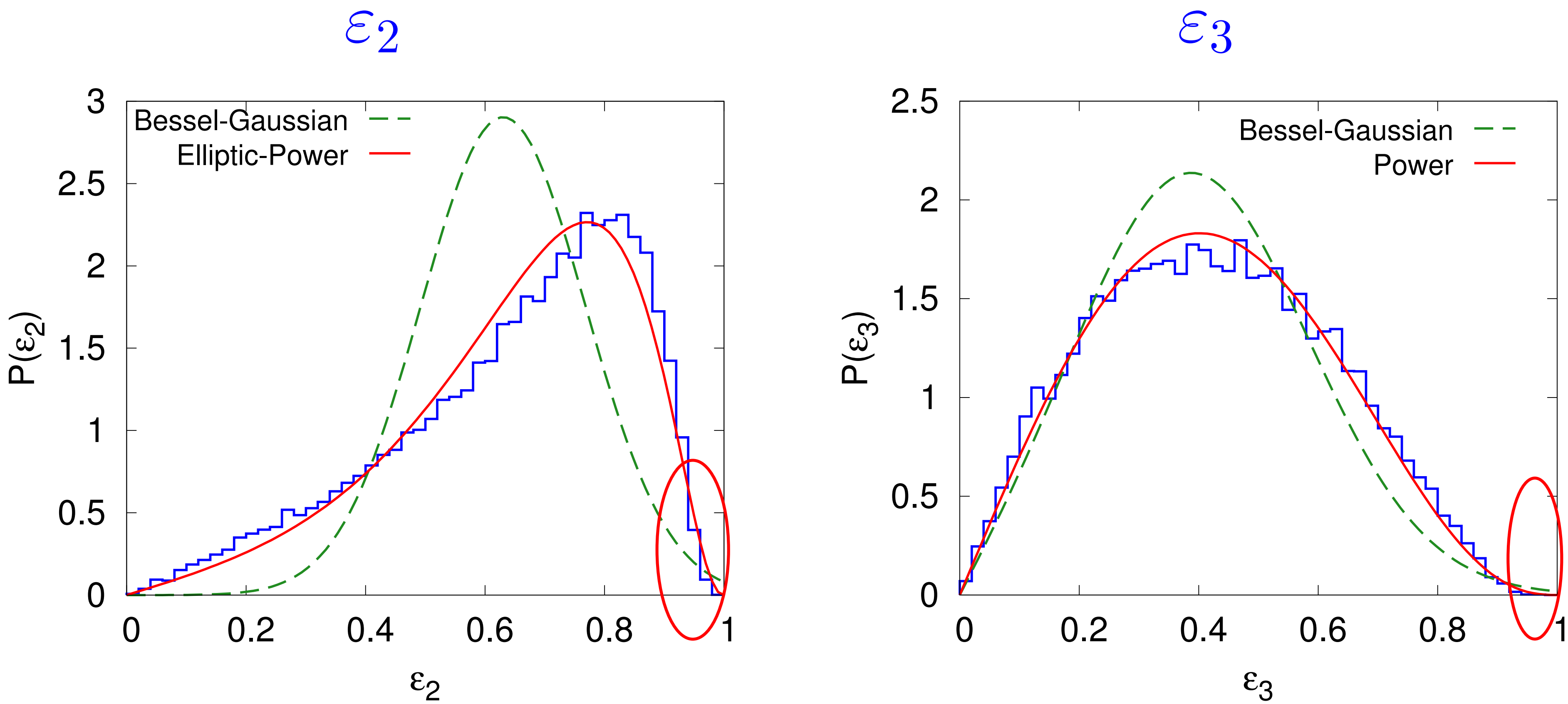


- Significant improvement with new parameterizations.

  * ε_2 fit $\rightarrow \chi^2/\mathrm{dof} \sim 8$(Elliptic Power) and 88(Bessel-Gaussian),
    ε_3 fit $\rightarrow \chi^2/\mathrm{dof} \sim 4$(Power) and 36(Bessel-Gaussian).

Test of Elliptic Power and Power

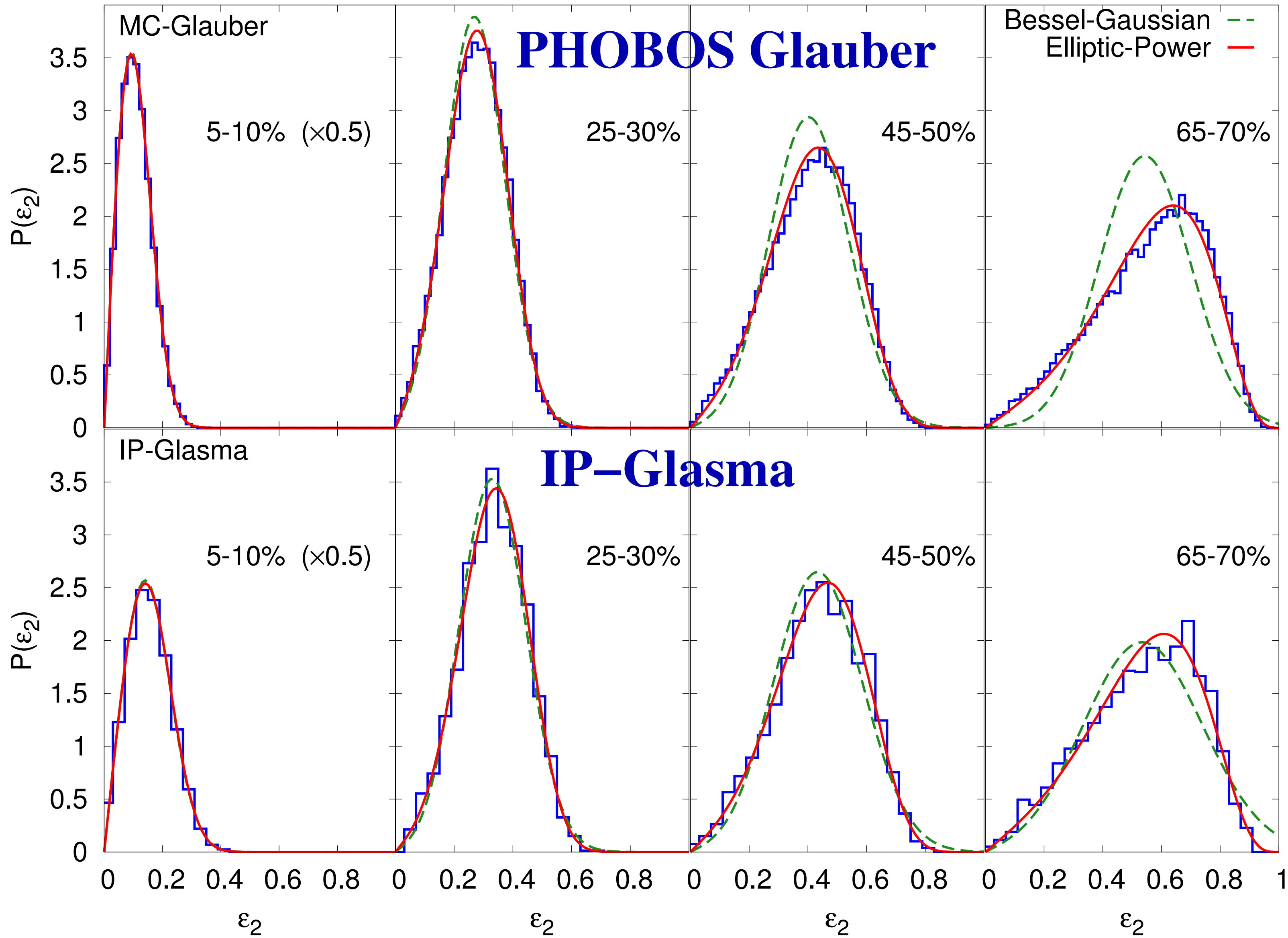
- Test of Elliptic Power parameterization: MC-Glauber PbPb with centrality 75%-80%.



- Significant improvement with new parameterizations.

  * ε_2 fit $\rightarrow \chi^2/\text{dof} \sim 8$(Elliptic Power) and 88(Bessel-Gaussian),
    ε_3 fit $\rightarrow \chi^2/\text{dof} \sim 4$(Power) and 36(Bessel-Gaussian).

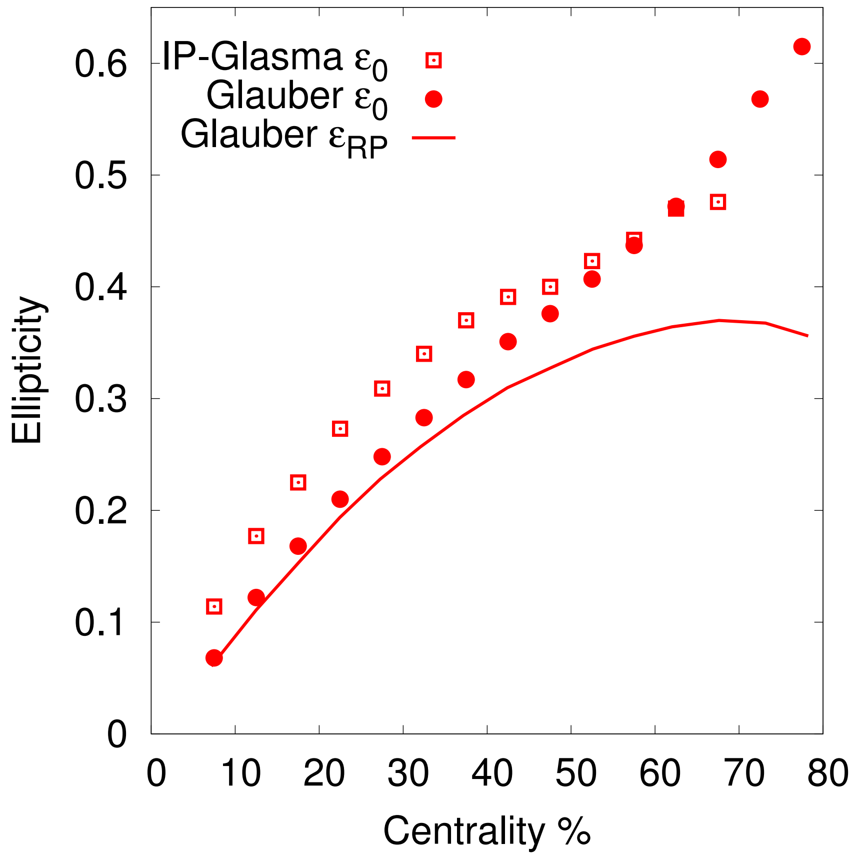
Elliptic Power distribution: test of universality



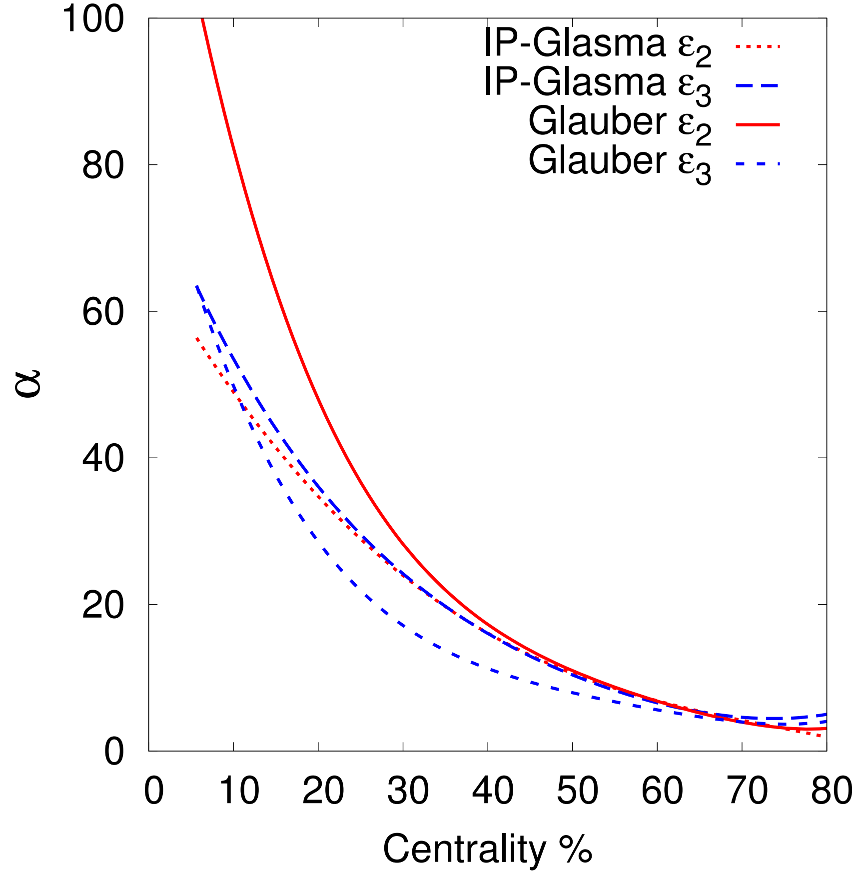
PHOBOS Glauber (B.Alver et al, arXiv:0805.4411) *IP-Glasma (B.Schenke et al, arXiv:1312.5588)*

* Elliptic Power (and Power) universally parameterizes fluctuations of ε_n.

Mean eccentricity: ε_0



Fluctuations: α



$$\varepsilon_{RP} = \int d\varepsilon_x \int d\varepsilon_y \varepsilon_x P(\varepsilon_x, \varepsilon_y)$$

Cumulants

- For any distribution function $P(x)$,

  Moments of n-th order :

$$\langle x^n \rangle = \int dx x^n P(x) \Rightarrow \langle \varepsilon_2^n \rangle.$$

  Cumulants:

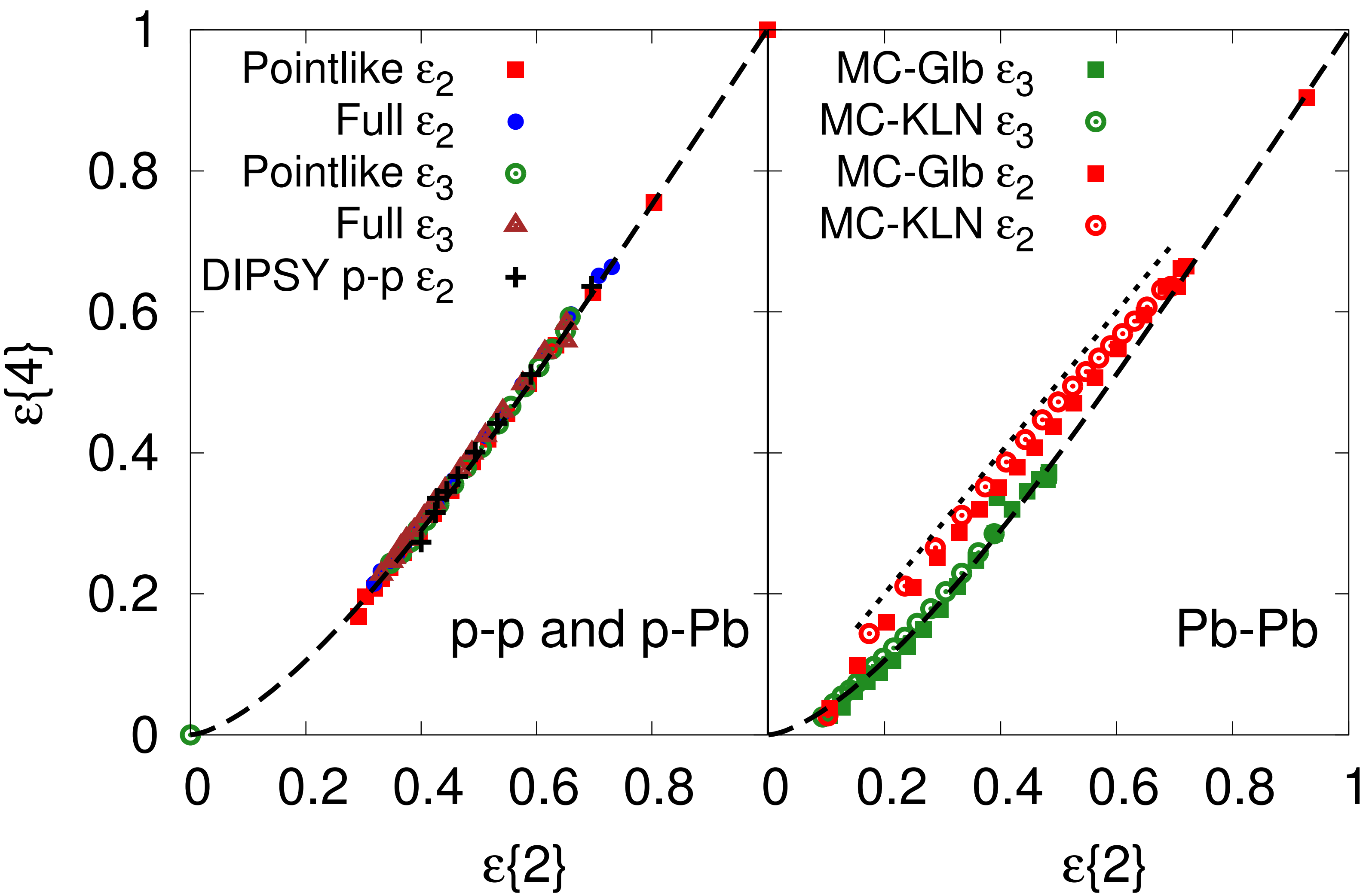
$$\varepsilon_2\{2\} = \langle \varepsilon_2^2 \rangle^{1/2}$$
$$\varepsilon_2\{4\} = \left[2\langle \varepsilon_2^2 \rangle^2 - \langle \varepsilon_2^4 \rangle\right]^{1/4}$$
$$\varepsilon_2\{6\} = \left[\frac{\langle \varepsilon_2^6 \rangle - 9\langle \varepsilon_2^2 \rangle\langle \varepsilon_2^4 \rangle + 12\langle \varepsilon_2^3 \rangle^2}{4}\right]^{1/6}$$
$$\ldots = \ldots$$

- For Elliptic Power (Power) distribution function: analytical function of α and ε_0.

	Gaussian	Bessel-Gaussian	Power law
$\varepsilon\{2\}$	σ	$\sqrt{\sigma^2 + \bar{\varepsilon}^2}$	$1/\sqrt{1+\alpha}$
$\varepsilon\{4\}$	0	$\bar{\varepsilon}$	$[2/(1+\alpha)^2(2+\alpha)]^{1/4}$
$\varepsilon\{6\}$	0	$\bar{\varepsilon}$	$[6/(1+\alpha)^3(2+\alpha)(3+\alpha)]^{1/6}$
$\varepsilon\{8\}$	0	$\bar{\varepsilon}$	$[48(1+(5\alpha/11))/(1+\alpha)^4(2+\alpha)^2(3+\alpha)(4+\alpha)]^{1/8}$

Power distribution: test of university

Analytical relation: $\varepsilon\{4\} = \varepsilon\{2\}^{3/2}\left(\frac{2}{1+\varepsilon\{2\}^2}\right)^{1/4}$



Applications to v_n fluctuations

- With Elliptic Power and Power characterizing initial state fluctuations,

- Linear eccentricity scaling: ($n = 2$ and 3) *H.Niemi et al.,Phys.Rev. C87 (2013) 054901*

$$v_n = \underbrace{\kappa_n(\eta/s)}_{\text{medium resp.}} \times \underbrace{\varepsilon_n(\alpha, \varepsilon_0)}_{\text{Elliptic Power or Power}}$$

- Ignore fluctuations in the medium response, i.e., κ_n does not fluctuate.

Applications to v_n fluctuations

- With Elliptic Power and Power characterizing initial state fluctuations,

- Linear eccentricity scaling: $(n = 2$ and $3)$ *H.Niemi et al.,Phys.Rev. C87 (2013) 054901*

$$v_n = \underbrace{\kappa_n(\eta/s)}_{\text{medium resp.}} \times \underbrace{\varepsilon_n(\alpha, \varepsilon_0)}_{\text{Elliptic Power or Power}}$$

- Ignore fluctuations in the medium response, i.e., κ_n does not fluctuate.

- Distribution of v_n is 'rescaled' Elliptic Power or Power distribution.

$$P(\varepsilon_n)d\varepsilon_n = P(\varepsilon_n(v_n, \kappa_n)) \left| \frac{\partial \varepsilon_n}{\partial v_n} \right| dv_n \quad \rightarrow \quad P(v_n/\kappa_n)/\kappa_n dv_n \quad \rightarrow \text{fit } v_n \text{ distribution}$$

$$\rightarrow \quad v_2\{n\} = \kappa_2 \varepsilon_2\{n\} \quad \rightarrow \text{solve cumulants}$$

Applications to v_n fluctuations

- With Elliptic Power and Power characterizing initial state fluctuations,

- Linear eccentricity scaling: $(n = 2$ and $3)$ *H.Niemi et al.,Phys.Rev. C87 (2013) 054901*

$$v_n = \underbrace{\kappa_n(\eta/s)}_{\text{medium resp.}} \times \underbrace{\varepsilon_n(\alpha, \varepsilon_0)}_{\text{Elliptic Power or Power}}$$

- Ignore fluctuations in the medium response, i.e., κ_n does not fluctuate.

- Distribution of v_n is 'rescaled' Elliptic Power or Power distribution.

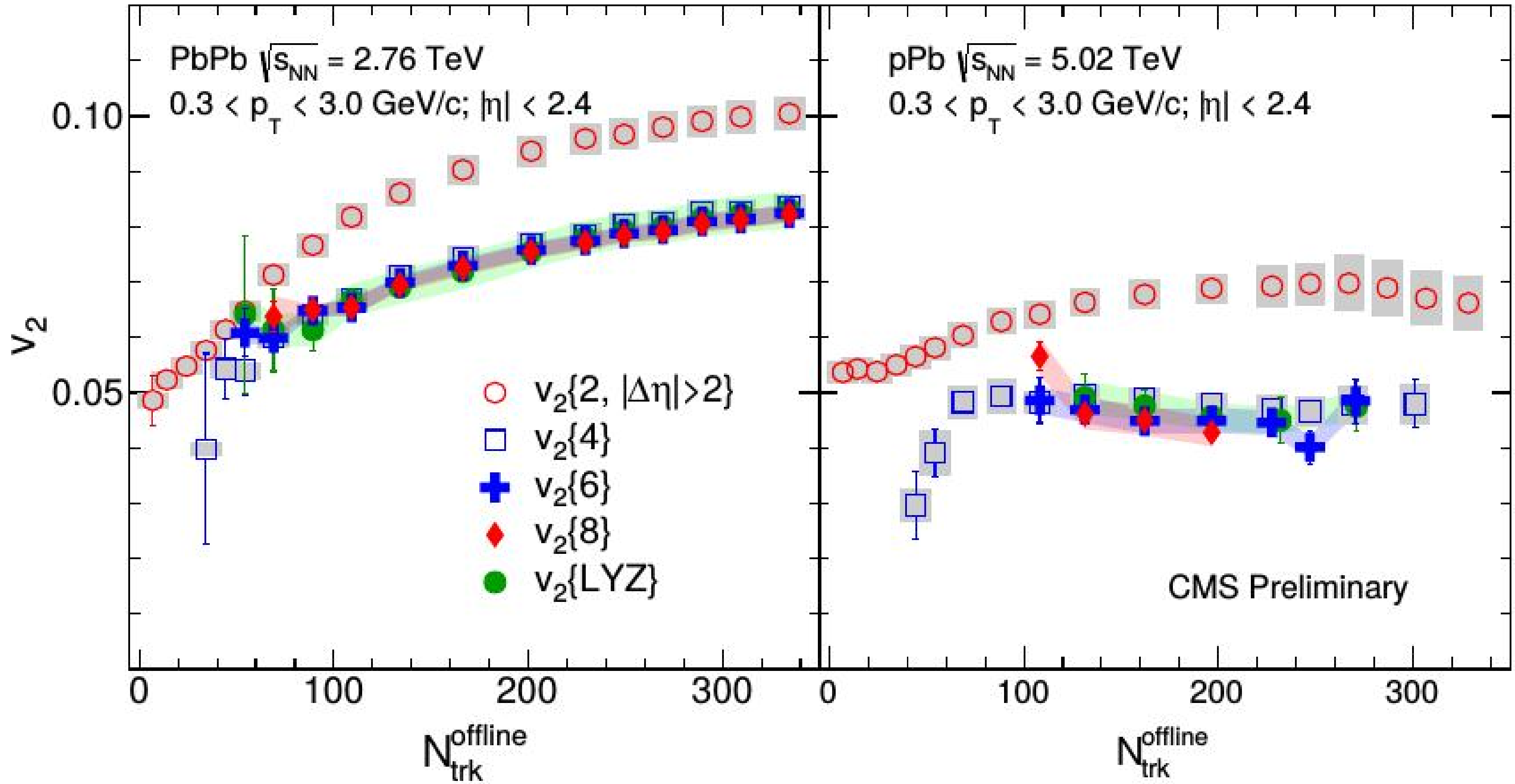
$$P(\varepsilon_n)d\varepsilon_n = P(\varepsilon_n(v_n, \kappa_n)) \left|\frac{\partial \varepsilon_n}{\partial v_n}\right| dv_n \quad \rightarrow \quad P(v_n/\kappa_n)/\kappa_n dv_n \quad \rightarrow \text{fit } v_n \text{ distribution}$$

$$\rightarrow \quad v_2\{n\} = \kappa_2 \varepsilon_2\{n\} \quad \rightarrow \text{solve cumulants}$$

The procedure is independent of effective modeling of initital state.

Fluctuations in p-Pb system

CMS collaboration, CMS-PAS-HIN-14-006



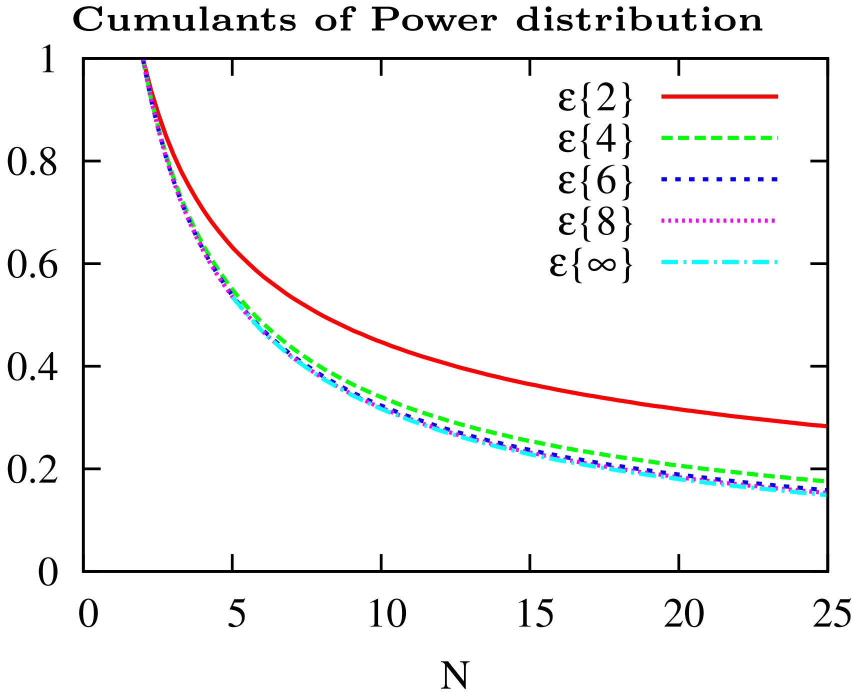
$$0 < v_2\{8\} \lesssim v_2\{6\} \lesssim v_2\{4\} < v_2\{2\}$$

Fluctuations in p-Pb system

- Generic feature of cumulants of Power distribution: $\varepsilon_n\{m\} \neq 0$

Fluctuations in p-Pb system

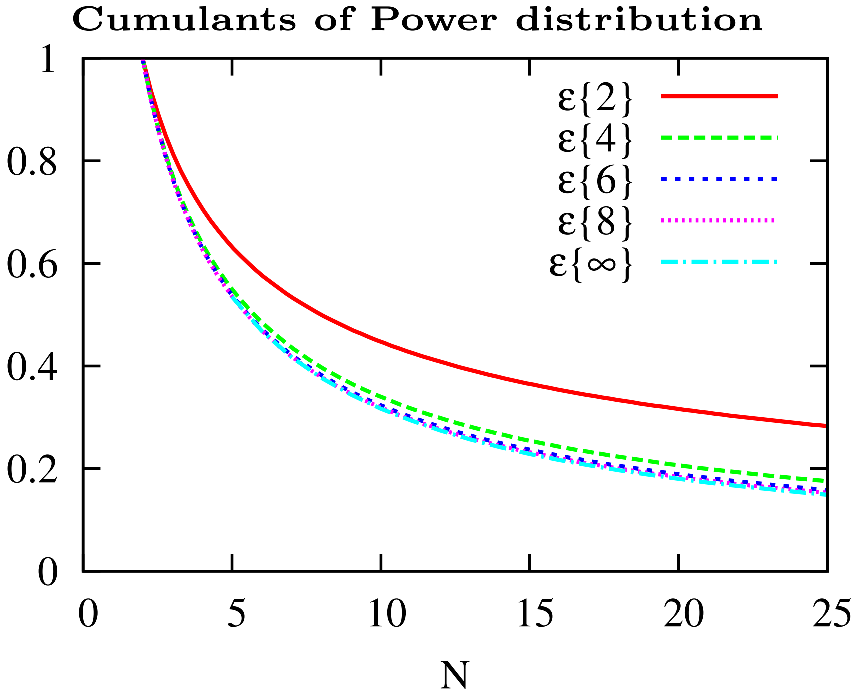
- Generic feature of cumulants of Power distribution: $\varepsilon_n\{m\} \neq 0$



(Seen also in MC-Glauber, Bozek and Broniowski arXiv:1304.3044 and Bzdak et al arXiv:1311.7325)

Fluctuations in p-Pb system

- Generic feature of cumulants of Power distribution: $\varepsilon_n\{m\} \neq 0$

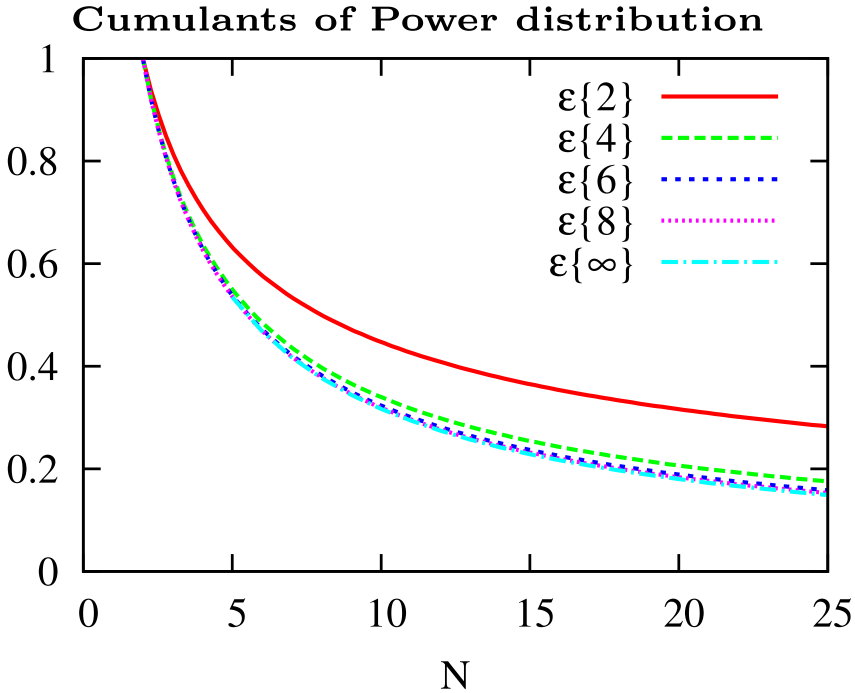


(Seen also in MC-Glauber, Bozek and Broniowski arXiv:1304.3044 and Bzdak et al arXiv:1311.7325)

- Large $v_2\{4\}$ etc. is natural in small system if:.
- Fluctuating ε_2 (follows Power distribution)
- Linear eccentricity scaling EbyE (hydro.)

Fluctuations in p-Pb system

- Generic feature of cumulants of Power distribution: $\varepsilon_n\{m\} \neq 0$



(Seen also in MC-Glauber, Bozek and Broniowski arXiv:1304.3044 and Bzdak et al arXiv:1311.7325)

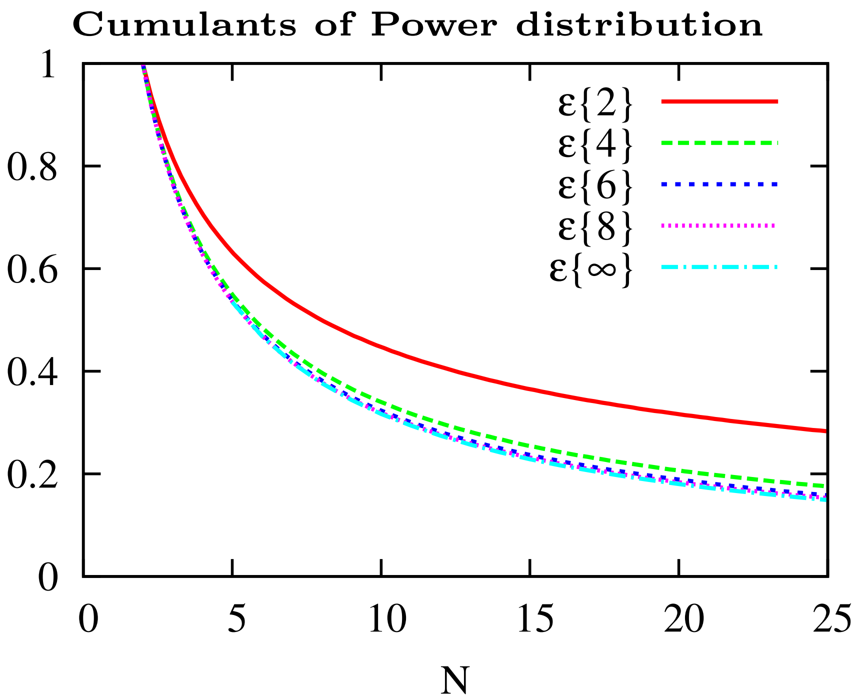
- Large $v_2\{4\}$ etc. is natural in small system if:.

- Fluctuating ε_2 (follows Power distribution)
- Linear eccentricity scaling EbyE (hydro.)

- Collective expansion (hydro.) of LHC p-Pb?

Fluctuations in p-Pb system

- Generic feature of cumulants of Power distribution: $\varepsilon_n\{m\} \neq 0$



(Seen also in MC-Glauber, Bozek and Broniowski arXiv:1304.3044 and Bzdak et al arXiv:1311.7325)

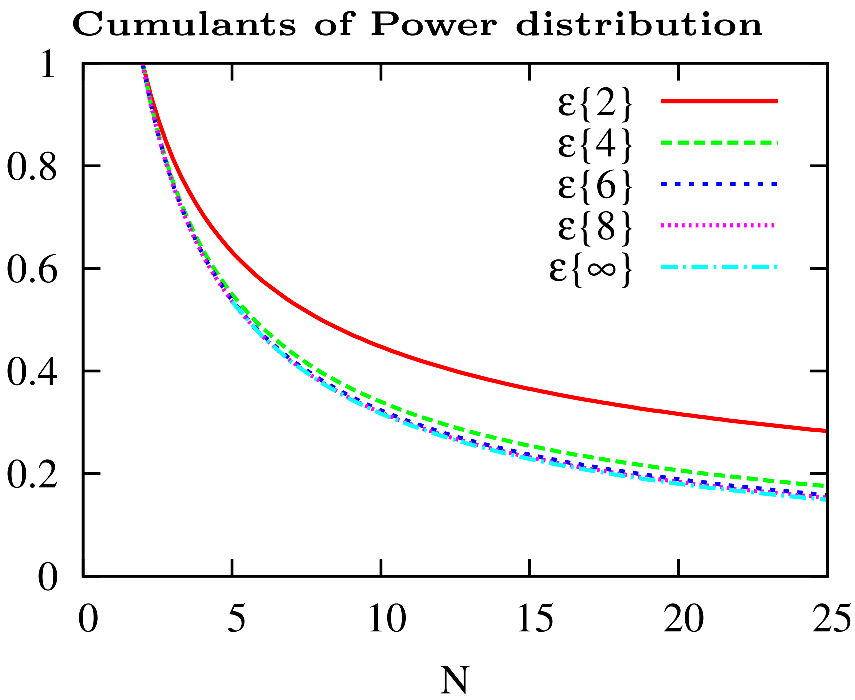
- Large $v_2\{4\}$ etc. is natural in small system if:.
  - Fluctuating ε_2 (follows Power distribution)
  - Linear eccentricity scaling EbyE (hydro.)
- Collective expansion (hydro.) of LHC p-Pb?
  - Other effects, e.g., coherence of initial fields:

$$0 < v_2\{4\} = v_2\{6\} = v_2\{8\} = \ldots$$

(See for instance, M. Gyulassy et al. arXiv:1405.7825)

Fluctuations in p-Pb system

- Generic feature of cumulants of Power distribution: $\varepsilon_n\{m\} \neq 0$



(Seen also in MC-Glauber, Bozek and Broniowski arXiv:1304.3044 and Bzdak et al arXiv:1311.7325)

- Large $v_2\{4\}$ etc. is natural in small system if:.

- Fluctuating ε_2 (follows Power distribution)
- Linear eccentricity scaling EbyE (hydro.)

- Collective expansion (hydro.) of LHC p-Pb?

- Other effects, e.g., coherence of initial fields:

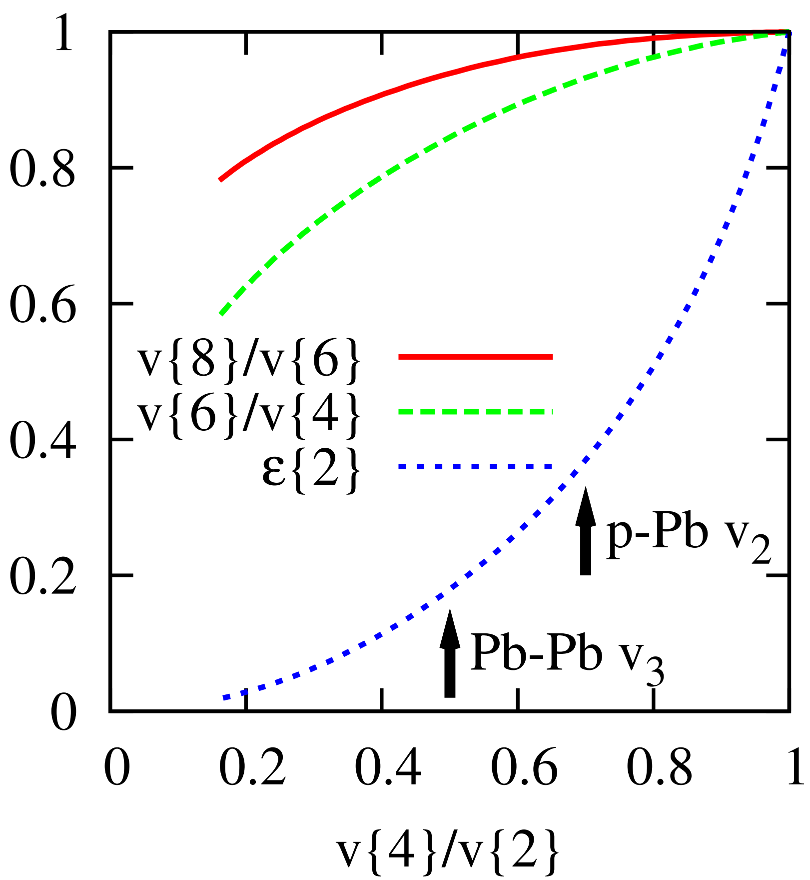
$$0 < v_2\{4\} = v_2\{6\} = v_2\{8\} = \ldots$$

(See for instance, M. Gyulassy et al. arXiv:1405.7825)

- From Power distribution (hydro.) quantify:

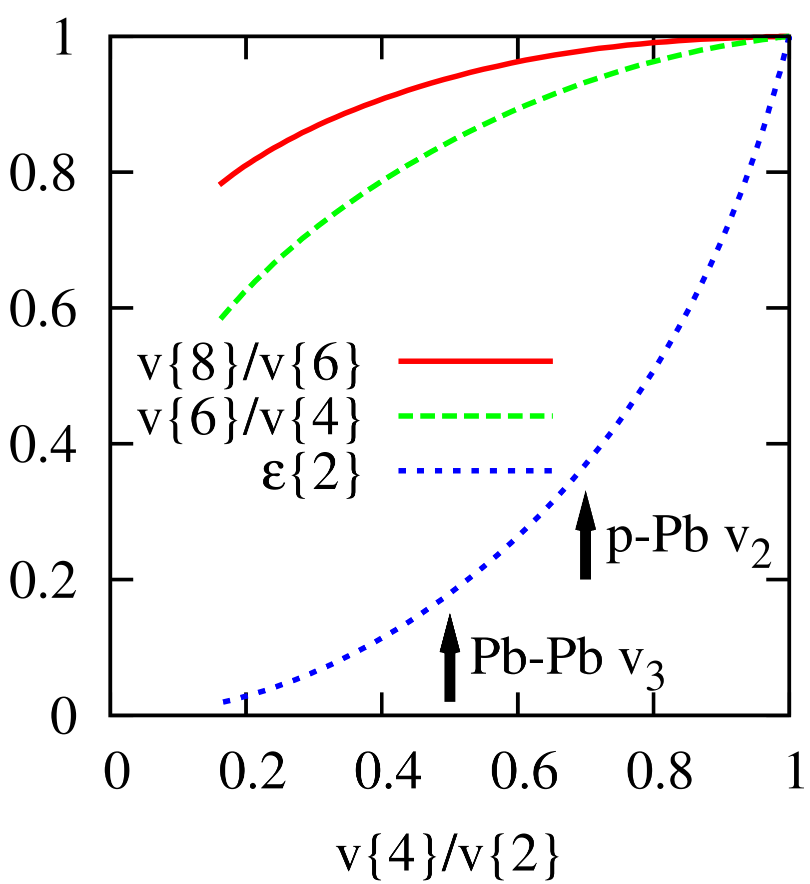
$$0 < v_2\{8\} < v_2\{6\} < v_2\{4\} \ldots$$

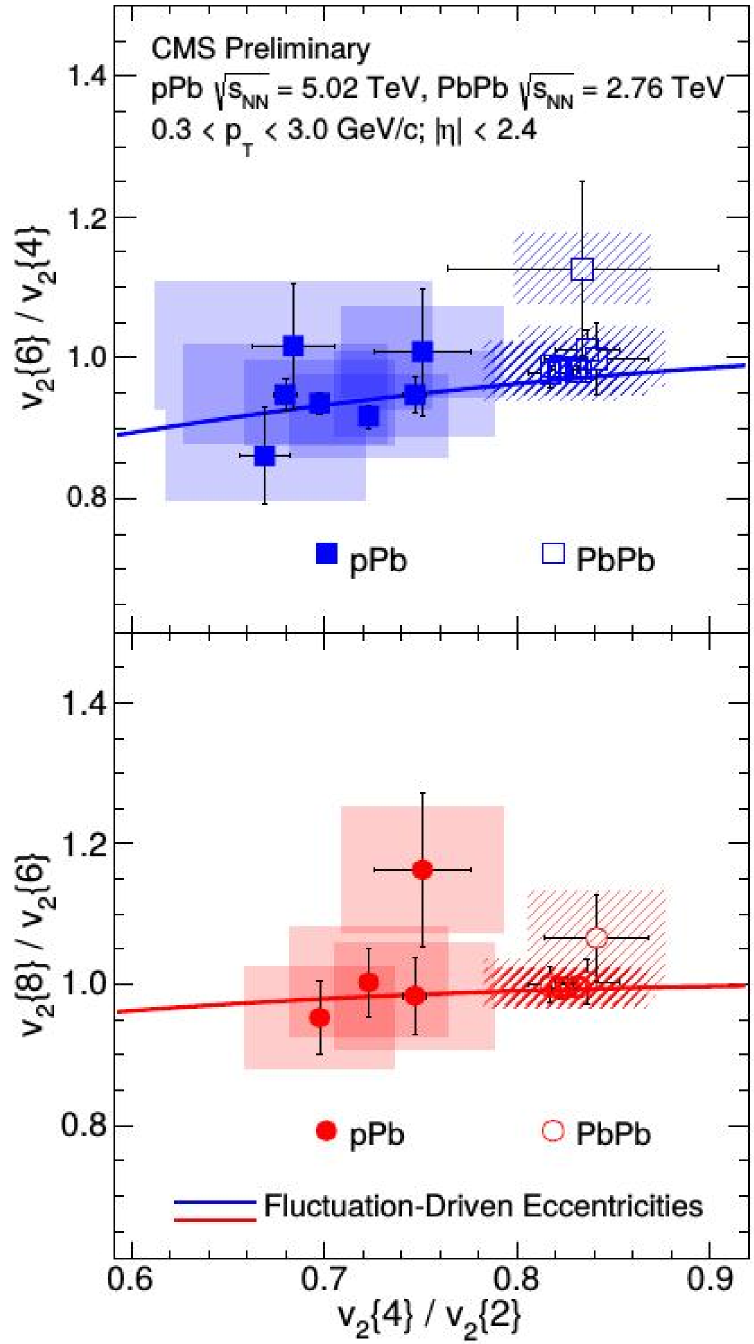
Analytical relations between cumulants:



Fluctuations in p-Pb system

Analytical relations between cumulants:

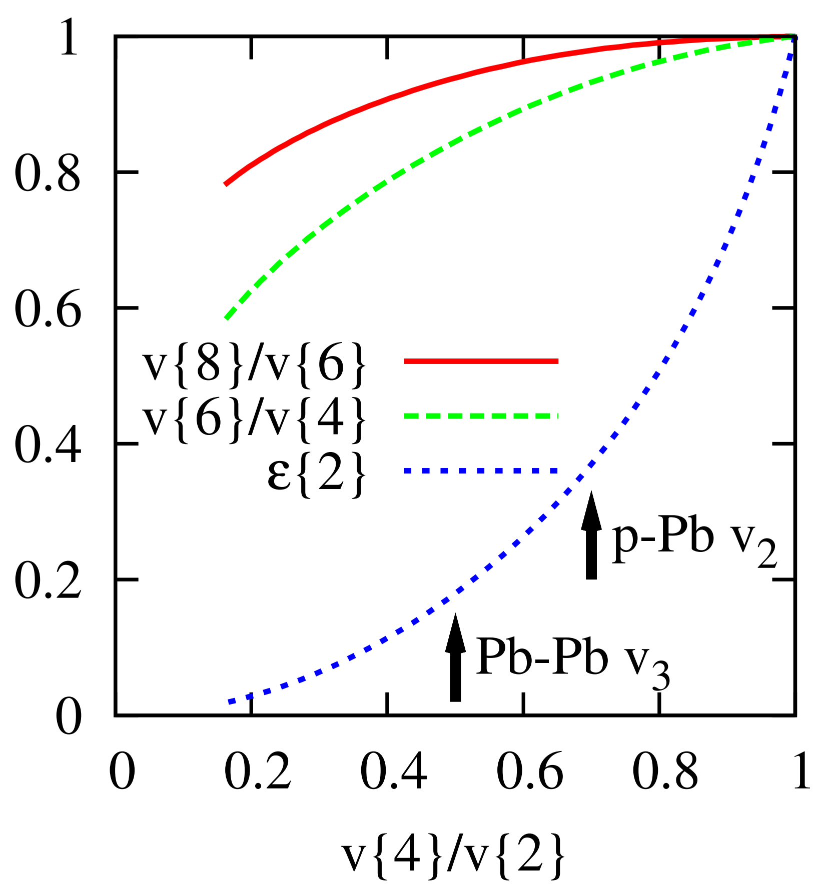


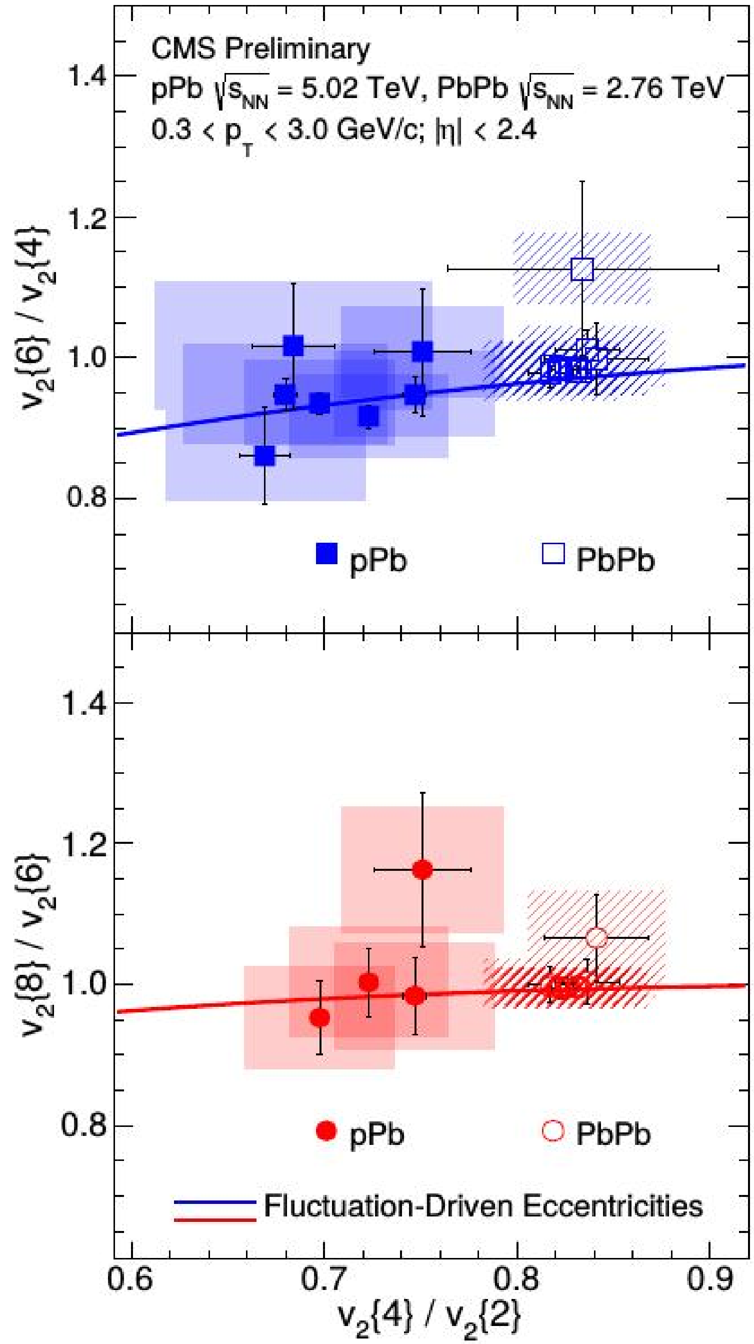


CMS collaboration, CMS-PAS-HIN-14-006

Fluctuations in p-Pb system

Analytical relations between cumulants:



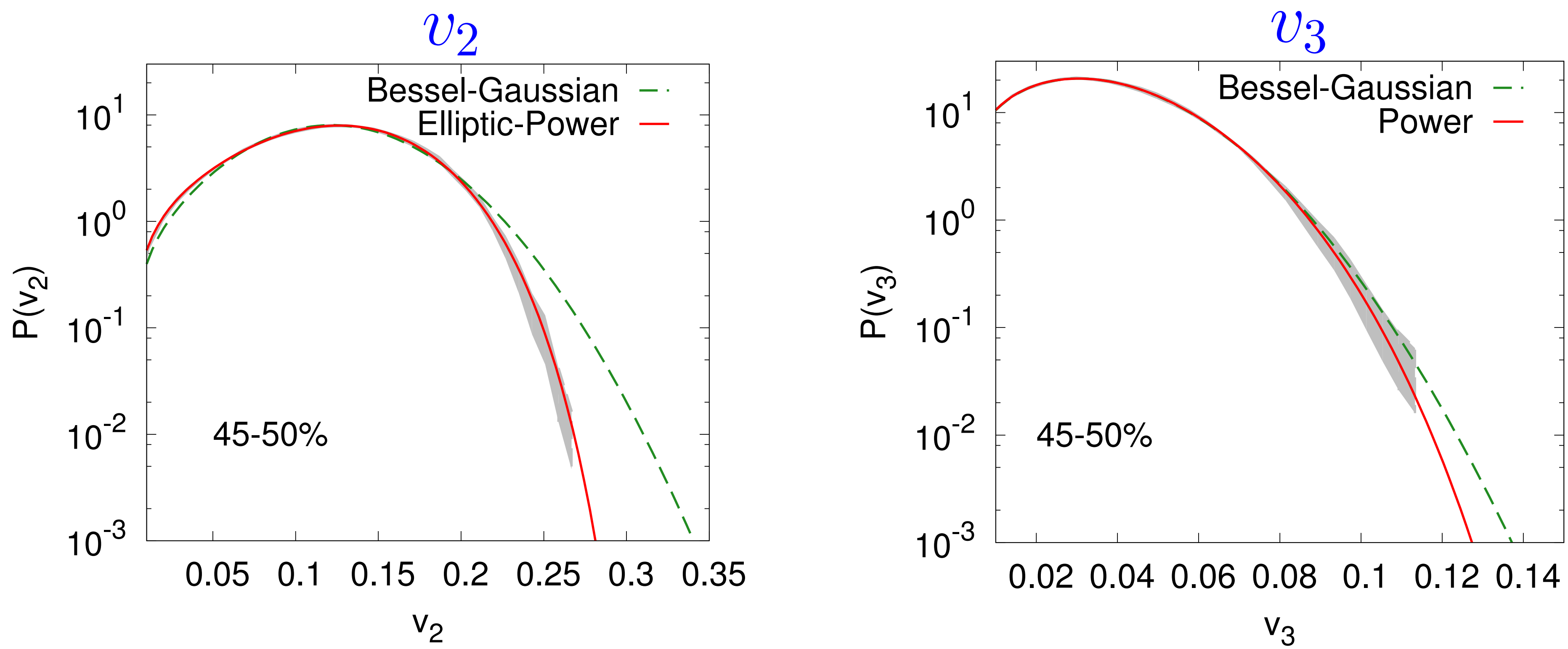


Accurate measurements are necessary.

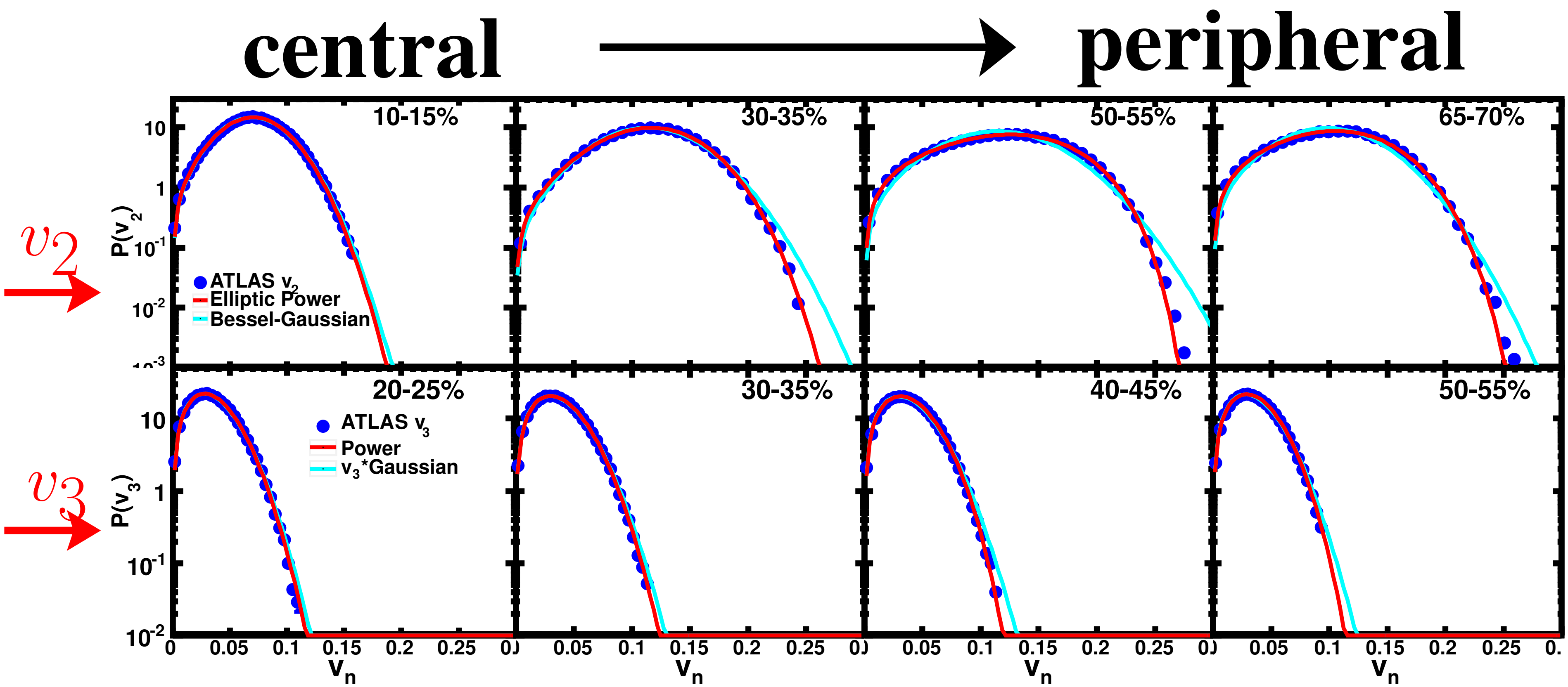
CMS collaboration, CMS-PAS-HIN-14-006

Fluctuations in Pb-Pb system: fit ATLAS EbyE v_n distribution

- Rescaled Elliptic Power (or Power) parameterization: ATLAS v_2 and v_3 at 45-50%.



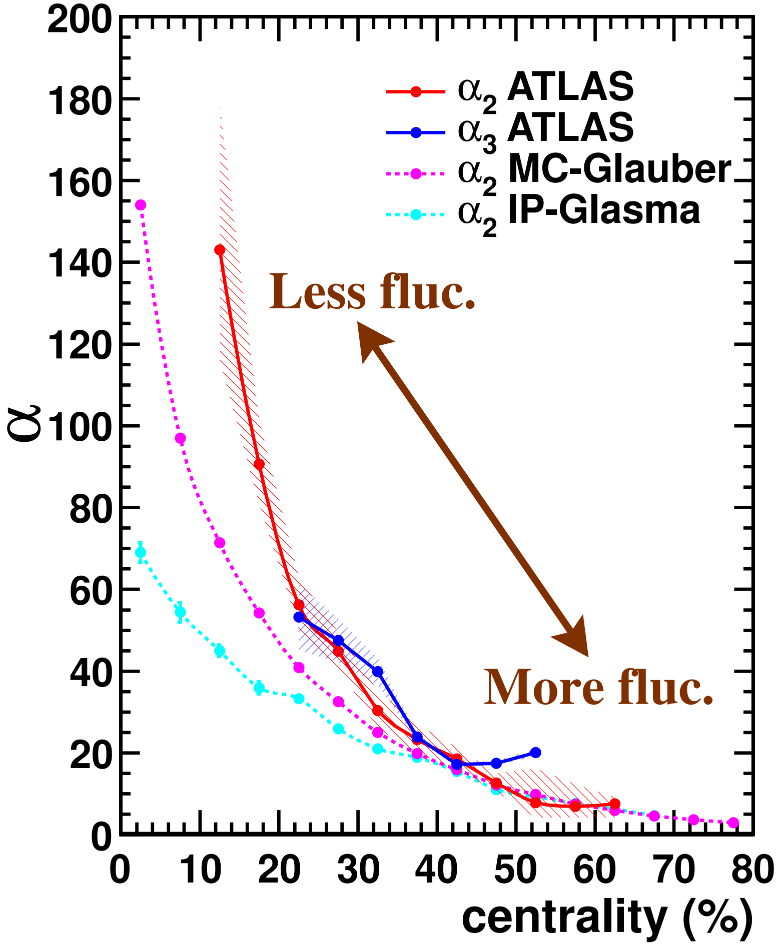
- Fits much better than Bessel-Gaussian.
- Deviations of Bessel-Gaussian fixed at tails are mostly due to $\varepsilon_n < 1$.
- $\kappa_n \Rightarrow$ flow resp. $\varepsilon_0 \Rightarrow$ average RP eccentricity $\alpha \Rightarrow$ magnitude of fluctuations



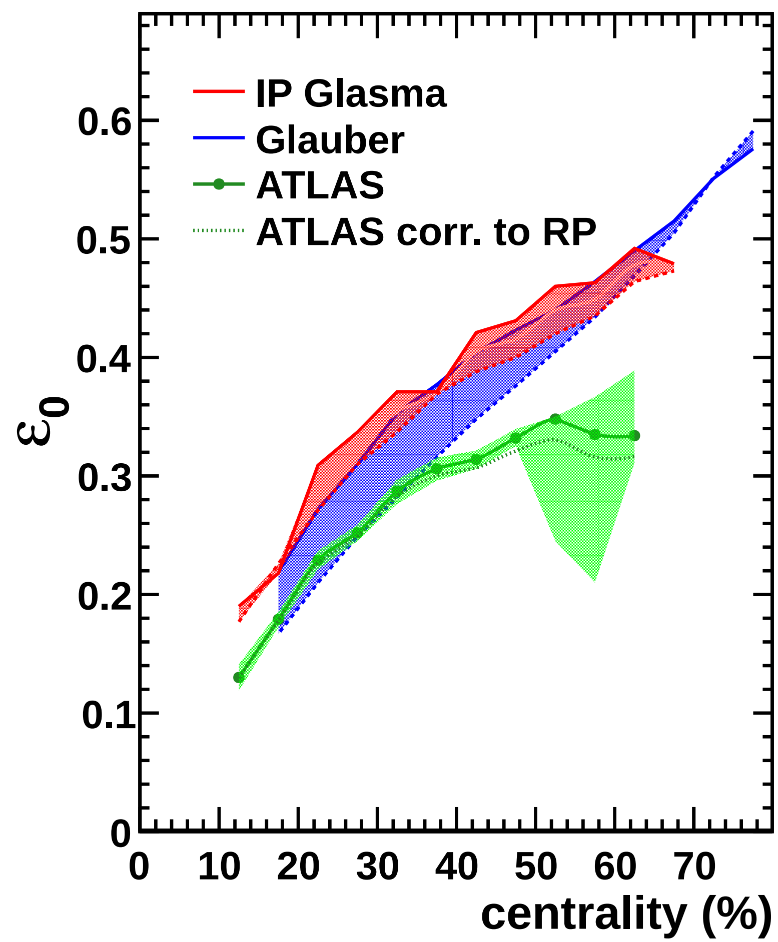
- Significant improvement with Elliptic Power and Power parameterization.
- Error of v_2 fit is dominated by systematic errors on $\sigma_v / < v >$ from ATLAS results.
- Error of v_3 fit is from statistical error of v_3 only. Systematic errors are too large.

Extract information of initial state from the fit

Fluctuations



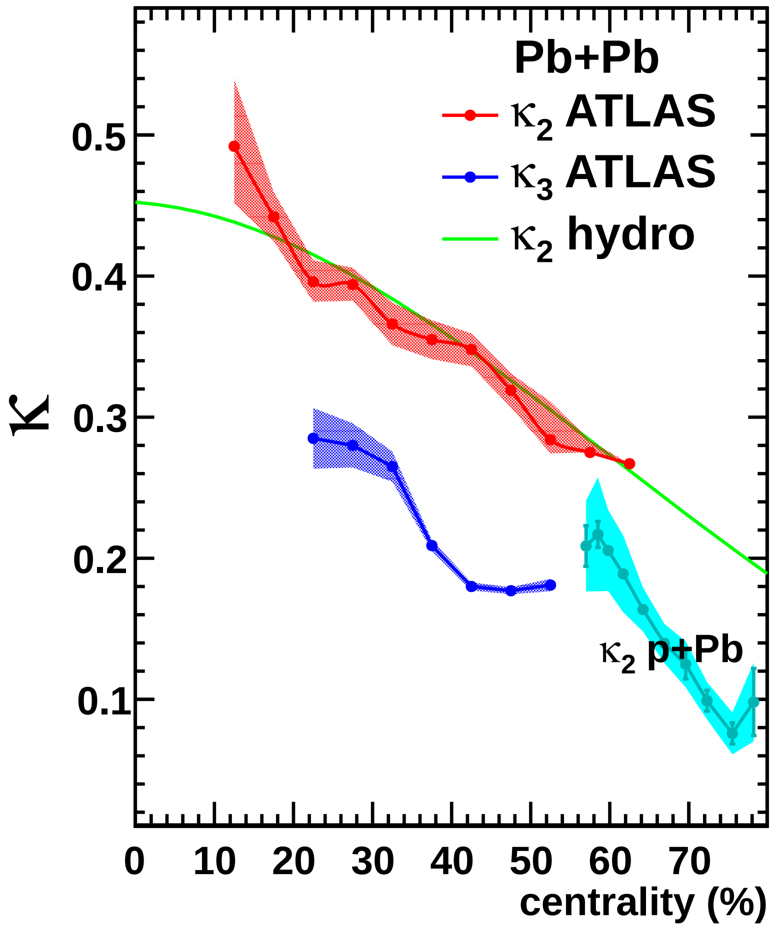
Average RP eccentricity



- Fluctuations become stronger for peripheral collisions.
- ε_0 grows with centrality percentage.

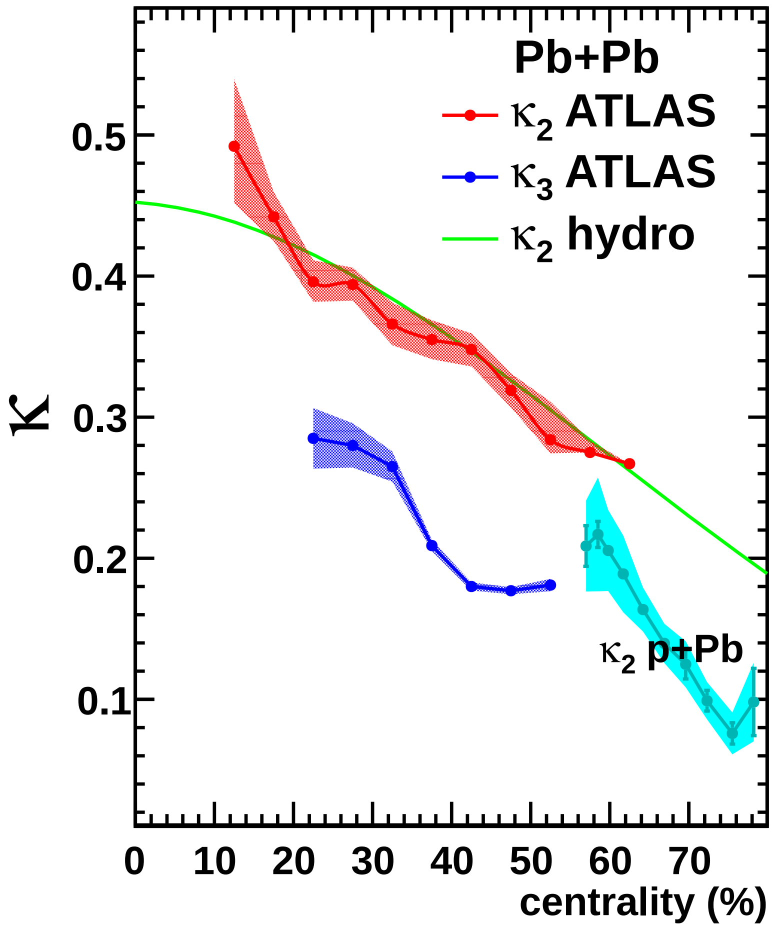
κ_n and extracting η/s in hydrodynamic response

- Flow response coefficient $\kappa_n = v_n/\varepsilon_n$ vs centrality



- $\kappa_2 > \kappa_3$.
- κ decreases from central to peripheral.

- Flow response coefficient $\kappa_n = v_n/\varepsilon_n$ vs centrality



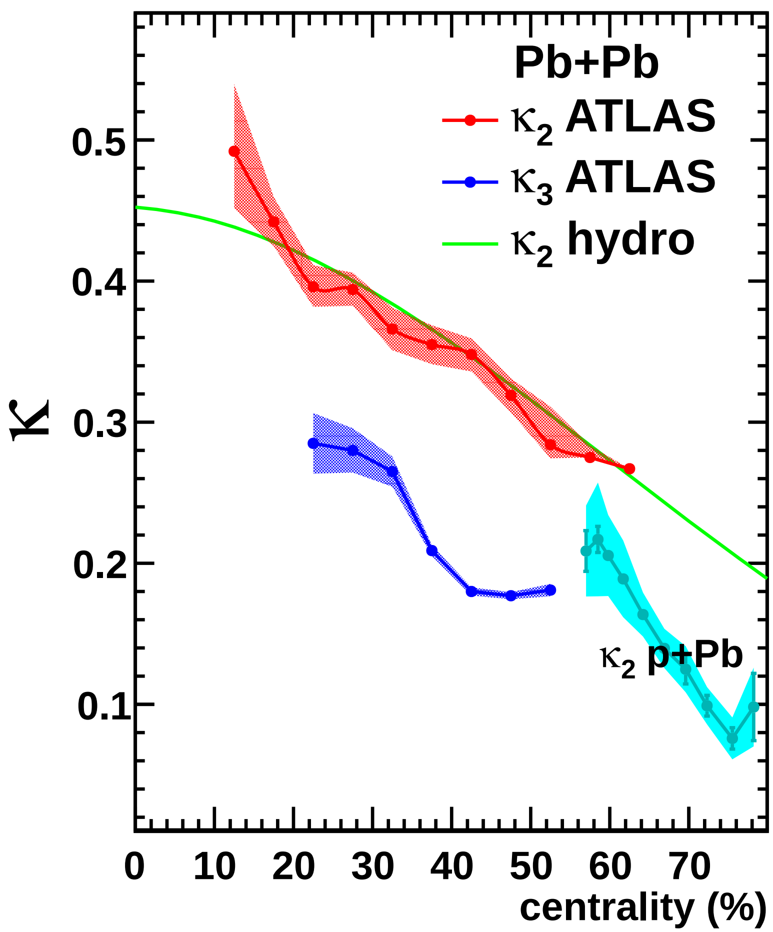
- $\kappa_2 > \kappa_3$.
- κ decreases from central to peripheral.
  - Since η/s determines κ_n
- Fit by hydro.: $\delta\kappa = -\frac{\kappa^{\text{visc.}} - \kappa^{\text{ideal}}}{\eta/s}$

$$\kappa\left(\frac{\eta}{s}\right) = C_0\left[\kappa^{\text{ideal}} - \frac{\eta}{s}\delta\kappa\right],$$

$\kappa^{\text{visc.}}$ and κ^{ideal} are given by hydro.

κ_n and extracting η/s in hydrodynamic response

- Flow response coefficient $\kappa_n = v_n/\varepsilon_n$ vs centrality



- $\kappa_2 > \kappa_3$.
- κ decreases from central to peripheral.
  - Since η/s determines κ_n
- Fit by hydro.: $\delta\kappa = -\frac{\kappa^{\text{visc.}} - \kappa^{\text{ideal}}}{\eta/s}$

$$\kappa\left(\frac{\eta}{s}\right) = C_0\left[\kappa^{\text{ideal}} - \frac{\eta}{s}\delta\kappa\right],$$

$\kappa^{\text{visc.}}$ and κ^{ideal} are given by hydro.

$$\text{Fit of } v_2 \quad \Rightarrow \quad \frac{\eta}{s} \sim 0.18$$

$$\Rightarrow \quad C_0 \sim 1.68$$

Our ideal hydro. needs to be rescaled to match data.

Conclusions and outlook

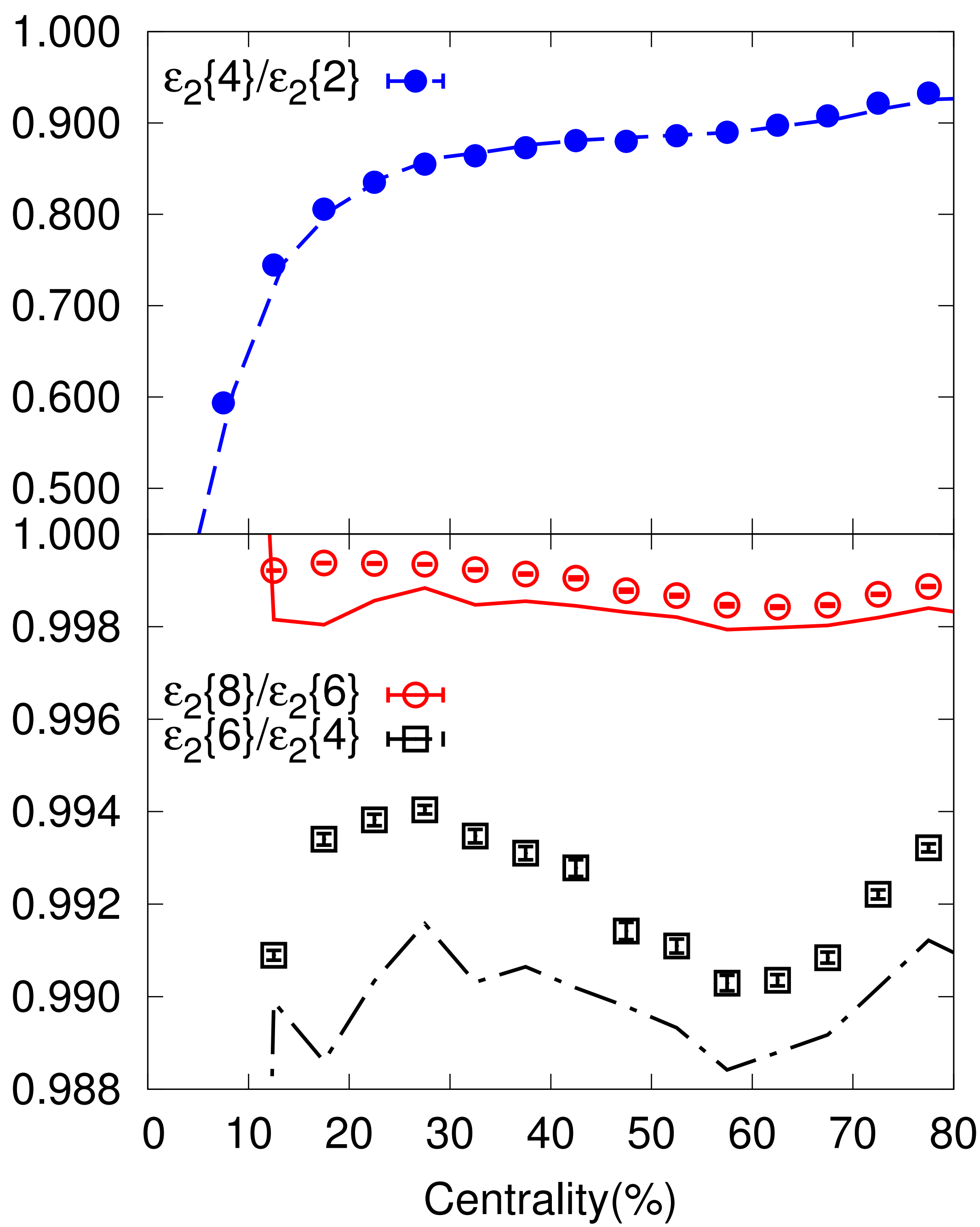
- New parameterizations of eccentricity fluctuations: Elliptic Power and Power
  1. Implement the condition $|\varepsilon_n| < 1$: large anisotropies are correctly modeled
  2. Fit all models of the initial state (Glauber, KLN, IP-Glasma, etc.)
  3. Reveals physical information of initial state: fluctuations (α) and average shape (ε_0).

- Applications to p-Pb and Pb-Pb without detailed modeling of initial state:
  1. Fluctuations in p-Pb: strong indications of collective expansion.
  2. Fluctuations in Pb-Pb: procedure to extract α, ε_0, and κ_2, from fits.

* Similar procedure can be generalized.
* Corrections from non-linear resp. and flow fluctuations.

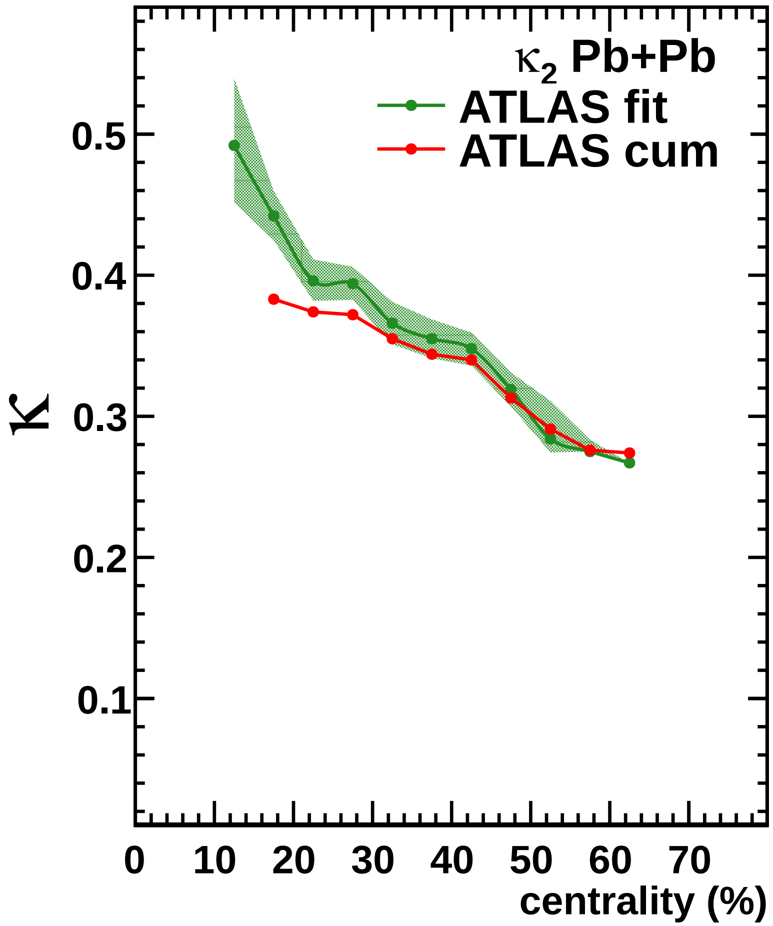
Back-up slides

Cumulants from Elliptic Power

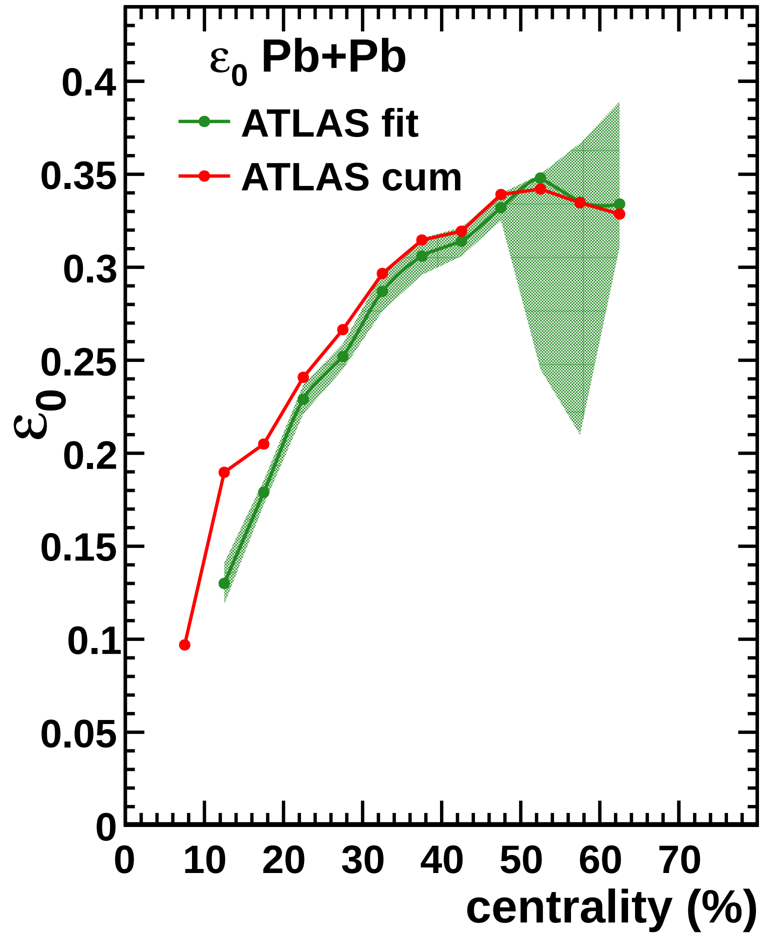


Solving cumulants of v_n

Resp. coefficient



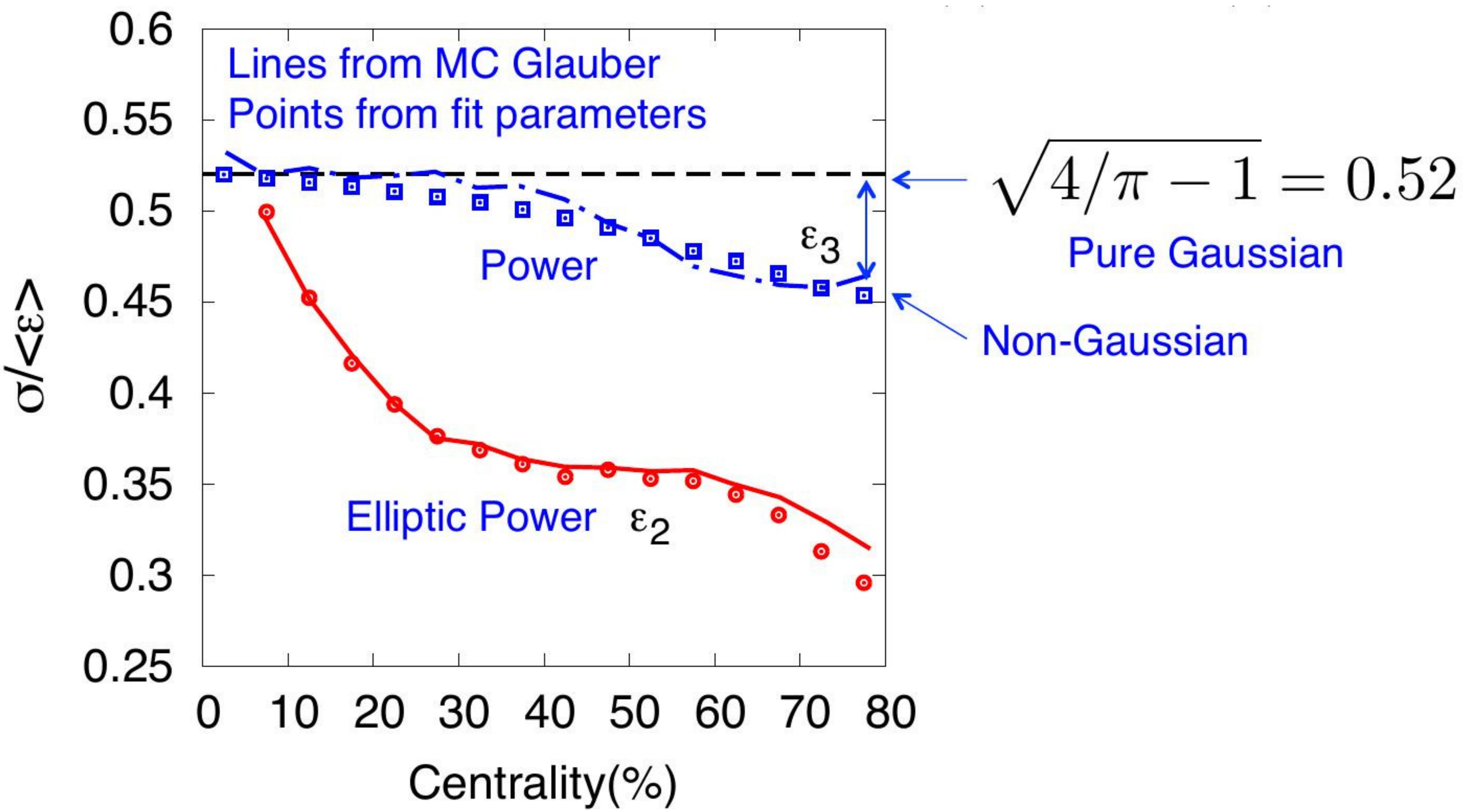
Average RP eccentricity



Some properties – Gaussian limit

- Large system with small mean eccentricity (or $\varepsilon_0 = 0$) and small fluctuations ($\alpha \gg 1$):

$$P_{\mathrm{EP}} \longrightarrow P_{\mathrm{BG}} \quad \text{and} \quad P_{\mathrm{Power}} \longrightarrow P_{\mathrm{Gaussian}}$$



- Remarks :

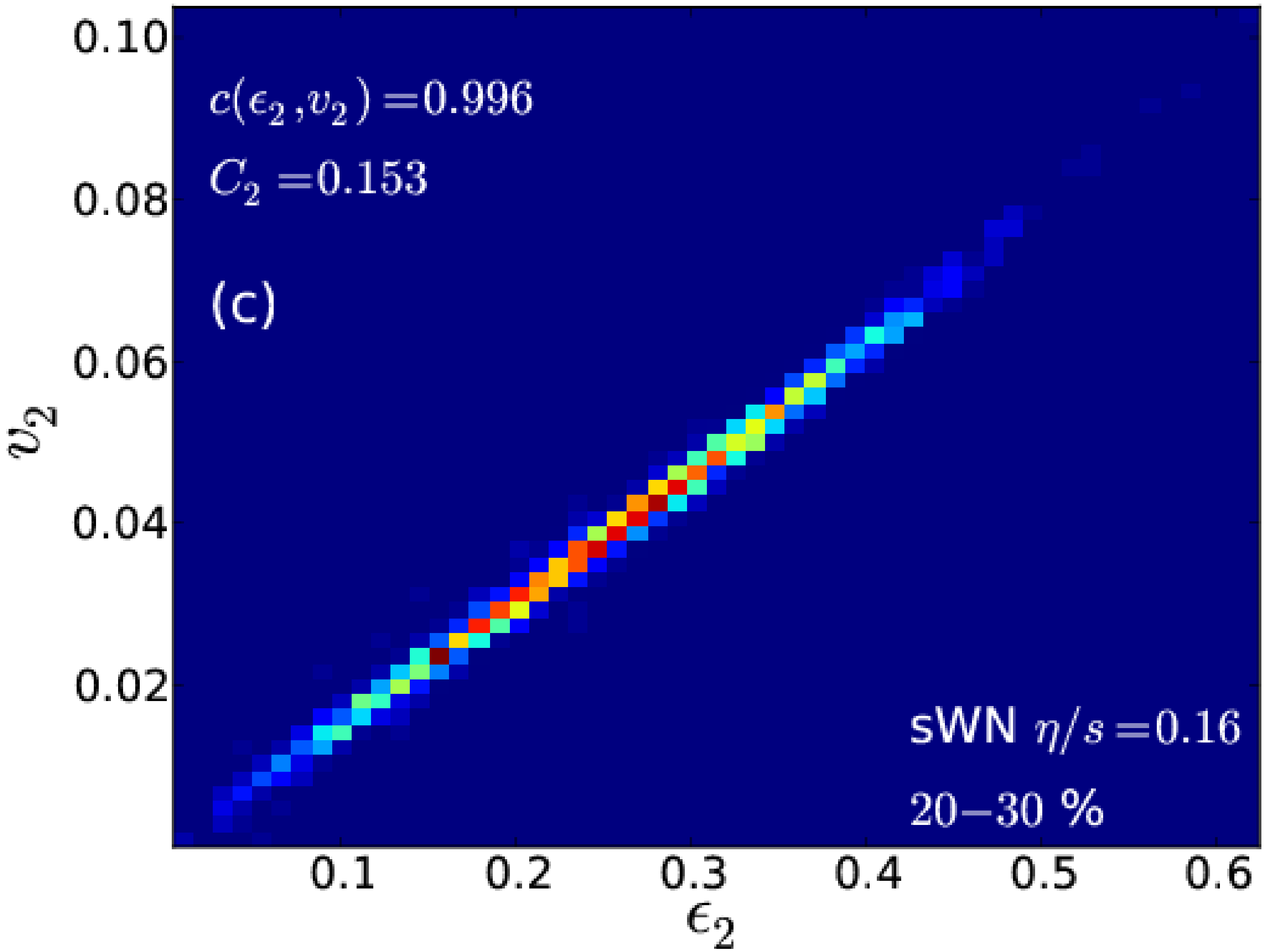
1. Non-Gaussianity is crucial for disentanglement of κ and ε_n.

$$P_{BG}(\varepsilon_n \to \varepsilon_n/\kappa_n) \equiv P_{BG}(\sigma \to \sigma\kappa)$$

2. Generalization accounting for non-linear corrections and fluctuations in flow response.

Linear eccentricity scaling

EbyE dissipative hydro. with shear viscosity = 0.16.



H.Niemi et al.,Phys.Rev. C87 (2013) 054901

Goodness of fit