Gluon saturation and Factorization Issues at small-x

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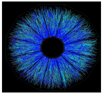
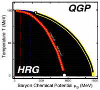


High Energy Strong Interactions:

A School for Young Asian Scientists

[http://conf.ccnu.edu.cn/~quadrangle2014/]



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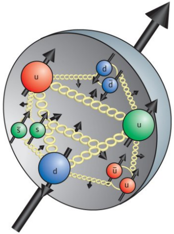
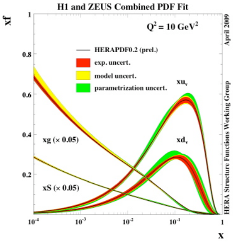
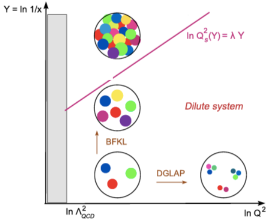
Outline

1. Introduction
2. Gluon distributions
3. k_t Factorization at One-loop Order
4. Phenomenological Application
5. Summary

Deep into small-x region



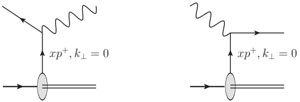




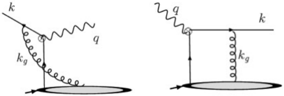
- Partons in the low-x region is dominated by gluons. See HERA data.
- Gluon splitting functions ($g \to gg$ and $q \to qg$) have $1/x$ singularities $\to$ Gluon density rises at low x. (Small-x gluon radiation is favored.)
- BFKL equation $\Rightarrow$ Resummation of the $\alpha_s \ln \frac{1}{x}$.
- When too many gluons squeezed in a confined hadron, gluons start to overlap and recombine $\Rightarrow$ Non-linear dynamics $\Rightarrow$ BK (JIMWLK) equation
- Use $Q_s(x)$ to separate the saturated dense regime from the dilute regime.
- Core ingredients: Multiple interactions + Small-x (high energy) evolution

Collinear Factorization vs $k_\perp$ Factorization

Collinear Factorization



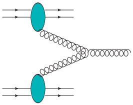
$k_\perp$ Factorization (Spin physics (TMD) and saturation physics)



- The incoming partons carry no $k_\perp$ in the Collinear Factorization.
- In general, there is intrinsic $k_\perp$. It can be negligible for partons in protons.
- When gluon density is large ($\frac{1}{\alpha_s}$), the resummation of multiple interactions becomes important.
- In collinear factorization, PDFs are universal.

k_t factorization

k_t factorization for gluon productions [Kaharzeev, Levin, Nardi, 03] surprisingly works.

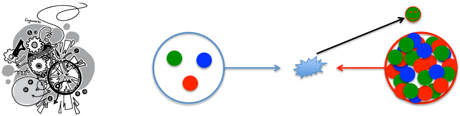


$$\frac{d\sigma}{d^2p_T dy} = \frac{2\alpha_s}{C_F\, p_T^2} \times$$
$$\times \int d^2k_{A,T}\, f_A(x_A, k_{A,T})\, f_B(x_B, p_T - k_{A,T}).$$

- Extension to quark pair productions. [Fujii, Gelis, Venugopalan, 06; Fujii, Watanabe, 13]
- Factorization and NLO correction? Only proved for DY and Higgs !
- Important requirements: Hard scattering and color neutral final states.
- Violations of k_t factorizations. Quantitative study [Fujii, Gelis, Venugopalan, Lappi $\cdots$].
- Dijet processes[Collins, Qiu, 08],[Rogers, Mulders; 10].
- k_t factorization $\simeq$ TMD factorization ? Yes, but more complicated.

Dilute-Dense factorizations

Dilute-Dense factorizations [Dumitru, Jalilian-Marian, 02; Hayashigaki, 06]



projectile: $x_1 \sim \frac{p_\perp}{\sqrt{s}} e^{+y} \sim 1$ valence

target: $x_2 \sim \frac{p_\perp}{\sqrt{s}} e^{-y} \ll 1$ gluon

- R. Feynman: Scattering protons on protons is like banging two fine Swiss watches to find out how they are built. Same analogy applies to AA collisions.
- The search for parton saturation is much easier in dilute-dense scatterings.
- Protons and virtual photons are dilute probes of the dense target hadrons.
- For dijet productions in forward pA collisions, effective k_t factorization:

$$\frac{d\sigma^{pA\to ggX}}{d^2P_\perp d^2q_\perp dy_1 dy_2} = x_p g(x_p, \mu) x_A g(x_A, q_\perp) \frac{1}{\pi} \frac{d\hat{\sigma}}{d\hat{t}}.$$

Factorization and NLO Calculation

- Factorization is about separation of short distant physics (perturbatively calculable hard factor) from large distant physics (Non perturbative).

$$\sigma \sim xf(x) \otimes \mathcal{H} \otimes D_h(z) \otimes \mathcal{F}(k_\perp)$$

- NLO (1-loop) calculation always contains various kinds of divergences.
  - Some divergences can be absorbed into the corresponding evolution equations.
  - The rest of divergences should be cancelled.
- Hard factor

$$\mathcal{H} = \mathcal{H}^{(0)}_{\text{LO}} + \frac{\alpha_s}{2\pi}\mathcal{H}^{(1)}_{\text{NLO}} + \cdots$$

  should always be finite and free of divergence of any kind.
- NLO vs NLL Naive α_s expansion sometimes is not sufficient!

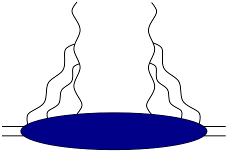
	LO	NLO	NNLO	$\cdots$
LL	1	$\alpha_s L$	$(\alpha_s L)^2$	$\cdots$
NLL		α_s	$\alpha_s (\alpha_s L)$	$\cdots$
$\cdots$			$\cdots$	$\cdots$

- Evolution $\rightarrow$ Resummation of large logs.
  LO evolution resums LL; NLO $\Rightarrow$ NLL.

A Tale of Two Gluon Distributions

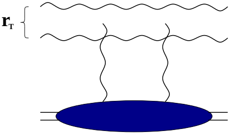
In small-x physics, two gluon distributions are widely used:[Kharzeev, Kovchegov, Tuchin; 03]
I. Weizsäcker Williams gluon distribution ([KM, 98'] and MV model):

$$xG^{(1)}(x, k_\perp) \quad \Leftarrow$$



II. Color Dipole gluon distributions:

$$xG^{(2)}(x, k_\perp) \quad \Leftarrow$$



Remarks:

- The WW gluon distribution simply counts the number of gluons.
- The Color Dipole gluon distribution often appears in calculations.
- Does this mean that gluon distributions are non-universal? Answer: Yes and No!

Two Different Gauge Invariant Operator Definitions

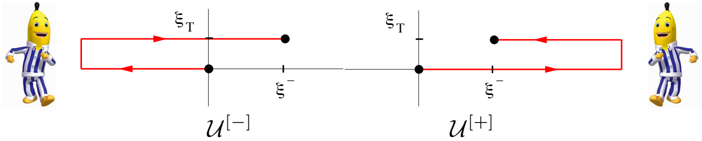
[F. Dominguez, BX and F. Yuan, Phys.Rev.Lett. 11]

I. Weizsäcker Williams gluon distribution: Gauge Invariant definitions

$$xG^{(1)} = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \mathrm{Tr}\langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[+]\dagger}F^{+i}(0)\mathcal{U}^{[+]}|P\rangle.$$

II. Color Dipole gluon distributions: Gauge Invariant definitions

$$xG^{(2)} = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \mathrm{Tr}\langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[-]\dagger}F^{+i}(0)\mathcal{U}^{[+]}|P\rangle.$$

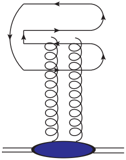


- The WW gluon distribution is the conventional gluon distributions.
- The dipole gluon distribution has no such interpretation.

A Tale of Twin Gluon Distributions

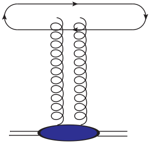
I. Weizsäcker Williams gluon distribution (never been measured)

$$xG^{(1)}(x, k_\perp) \qquad \Leftarrow$$



II. Color Dipole gluon distribution:

$$xG^{(2)}(x, k_\perp) \qquad \Leftarrow$$



- Quadrupole $\Rightarrow$ Weizsäcker Williams gluon distribution;
- Dipole $\Rightarrow$ Color Dipole gluon distribution;

A Tale of Twin Gluon Distributions

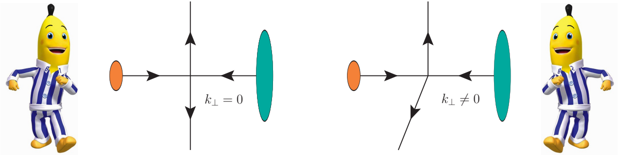
In terms of operators, we find these two gluon distributions can be defined as follows:
[F. Dominguez, C. Marquet, BX and F. Yuan, 11]

I. Weizsäcker Williams gluon distribution:

$$xG^{(1)} = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \mathrm{Tr}\langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[+]\dagger}F^{+i}(0)\mathcal{U}^{[+]}|P\rangle.$$

II. Color Dipole gluon distributions:

$$xG^{(2)} = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \mathrm{Tr}\langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[-]\dagger}F^{+i}(0)\mathcal{U}^{[+]}|P\rangle.$$

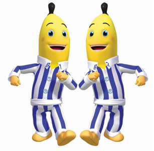


Questions:

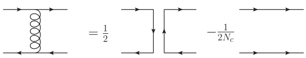
- Can we distinguish these two gluon distributions?
- How to measure $xG^{(1)}$ directly? DIS dijet.
- How to measure $xG^{(2)}$ directly? Direct γ+Jet in *pA* collisions.
- What happens in gluon+jet production in *pA* collisions? Need both gluon distributions.

Dijet processes in pA collisions in the large N_c limit

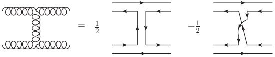
Need both gluon distributions in the effective factorization



The Fierz identity:

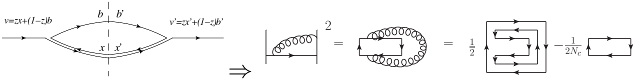


and

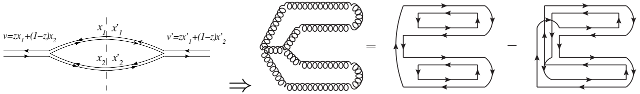


Graphical representation of dijet processes

$q \to qg$



$g \to gg$

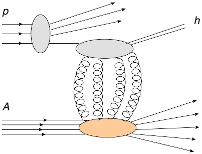


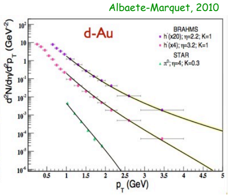
Forward hadron production in *pA* collisions

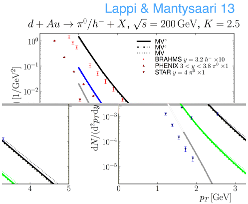
[Dumitru, Jalilian-Marian, 02] Inclusive forward hadron production in *pA* collisions

$$\frac{d\sigma_{\text{LO}}^{pA\to hX}}{d^2p_\perp dy_h} = \int_\tau^1 \frac{dz}{z^2}\left[\sum_f x_p q_f(x_p,\mu)\mathcal{F}(k_\perp)D_{h/q}(z,\mu) + x_p g(x_p,\mu)\tilde{\mathcal{F}}(k_\perp)D_{h/g}(z,\mu)\right].$$

$$p + A \to h(y, p_\perp) + X$$

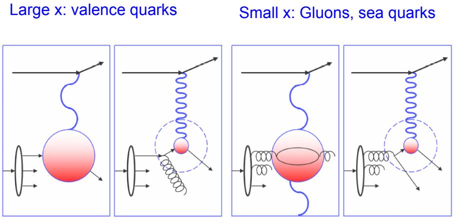


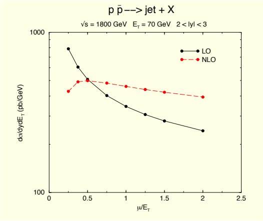




- $\mathcal{F}(k_\perp)$ is related to the dipole gluon distribution.
- Caveats: arbitrary choice of the renormalization scale μ and K factor.
- NLO correction? [Dumitru, Hayashigaki, Jalilian-Marian, 06; Altinoluk, Kovner 11] [Chirilli, Xiao and Yuan, 12]

Why do we need NLO calculations?

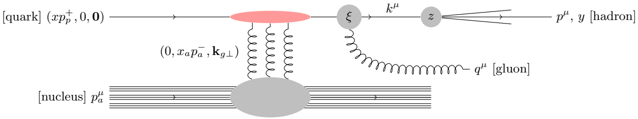




- Due to quantum evolution, $xf(x)$ and $D(z)$ change with scale. This introduces large theoretical uncertainties. Choice of the scale at LO requires information at NLO.
- LO cross section is a monotonic function of μ, thus it is order of magnitude estimate.
- NLO calculation significantly reduces the scale dependence. More reliable.
- $K = \frac{\sigma_{\rm LO}+\sigma_{\rm NLO}}{\sigma_{\rm LO}}$ is not a good approximation.
- NLO is vital in establishing the QCD factorization in saturation physics.

Factorization for single inclusive hadron productions

Factorization for the $p + A \to H + X$ process
[Chirilli, BX and Yuan, Phys. Rev. Lett. 108, 122301 (2012)]



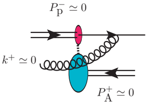
$$\frac{d^3\sigma^{p+A\to h+X}}{dyd^2p_\perp} = \sum_a \int \frac{dz}{z^2}\frac{dx}{x}\xi x f_a(x,\mu) D_{h/c}(z,\mu) \int [dx_\perp] S^Y_{a,c}([x_\perp]) \mathcal{H}_{a\to c}(\alpha_s, \xi, [x_\perp]\mu)$$

Collinear divergence: pdfs

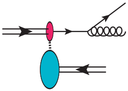
Collinear divergence: fragmentation functs

Rapidity divergence: BK evolution

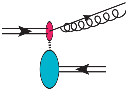
Finite hard factor



Rapidity Divergence



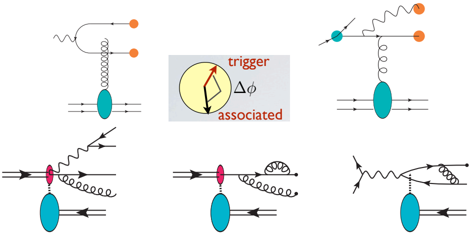
Collinear Divergence (P)



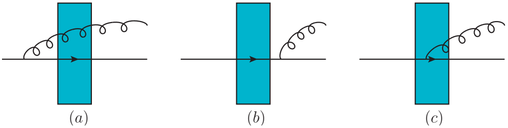
Collinear Divergence (F)

One-loop factorizations for other processes

- One-loop Calculation for Dijet processes, Higgs, Heavy-Quarkonium $\Rightarrow$ Demonstration of factorization and connection to TMD.
- Sudakov double logarithms in small-x physics. [Mueller, BX and Yuan, Phys. Rev. Lett. 13].



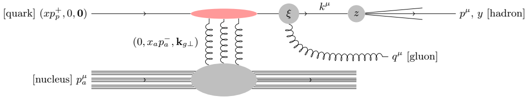
- Extension to jet quenching (energy loss) problem for large size medium?



Numerical implementation of the NLO result

Single inclusive hadron production up to NLO

$$
\begin{aligned}
d\sigma &= \int xf_a(x) \otimes D_a(z) \otimes \mathcal{F}_a^{x_g}(k_\perp) \otimes \mathcal{H}^{(0)} \\
&\quad + \frac{\alpha_s}{2\pi} \int xf_a(x) \otimes D_b(z) \otimes \mathcal{F}_{(N)ab}^{x_g} \otimes \mathcal{H}_{ab}^{(1)}.
\end{aligned}
$$

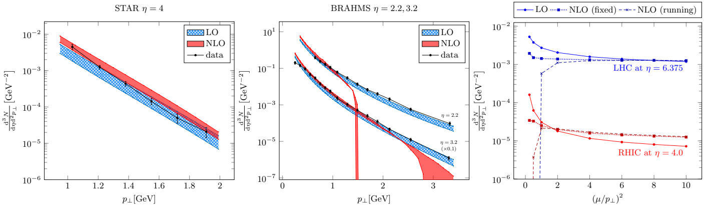


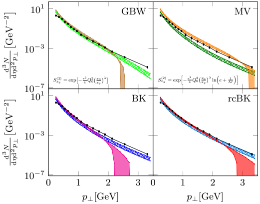
Consistent implementation should include all the NLO α_s corrections.

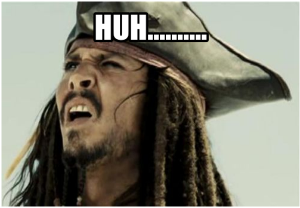
- NLO parton distributions. (MSTW or CTEQ)
- NLO fragmentation function. (DSS or others.)
- Use NLO hard factors. Partially by [Albacete, Dumitru, Fujii, Nara, 12]
- Use the one-loop approximation for the running coupling
- rcBK evolution equation for the dipole gluon distribution [Balitsky, Chirilli, 08; Kovchegov, Weigert, 07]. Full NLO BK evolution not available.
- Saturation physics at One Loop Order (SOLO). [Stasto, Xiao, Zaslavsky, 13]

Numerical implementation of the NLO result

[Stasto, Xiao, Zaslavsky, Phys. Rev. Lett.14]



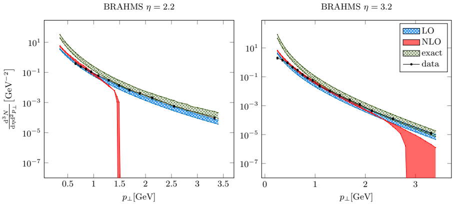


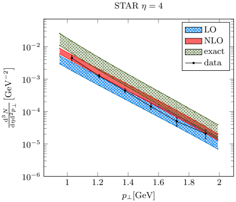


- Agree with data for $p_\perp < Q_s(y)$, and reduced scale dependence, no K factor.
- The abrupt drop of the NLO correction when $p_\perp > Q_s$ was really puzzling.
- For more forward rapidity, the agreement gets better and better.

The Old Empire Strikes Back

[Stasto, Xiao, Yuan, Zaslavsky, 14]

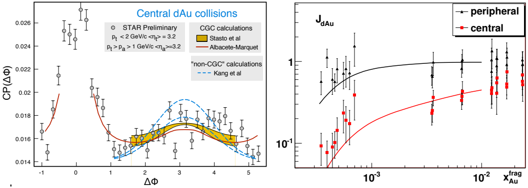




- Adopt exact kinematics and match with collinear factorization at high $p_\perp$. [G. Beuf]
- Systematic matching between the small-x and collinear factorization at high $p_\perp$.
- Saturation effects is elusive.
- The increment of the matching point implies the increase of Q_s.

Dihadron correlations in *dAu* collisions

$$C(\Delta\phi) = \frac{\int_{|p_{1\perp}|,|p_{2\perp}|} \frac{d\sigma^{pA\to h_1 h_2}}{dy_1 dy_2 d^2p_{1\perp} d^2p_{2\perp}}}{\int_{|p_{1\perp}|} \frac{d\sigma^{pA\to h_1}}{dy_1 d^2p_{1\perp}}} \qquad J_{dA} = \frac{1}{\langle N_{\rm coll}\rangle} \frac{\sigma_{dA}^{\rm pair}/\sigma_{dA}}{\sigma_{pp}^{\rm pair}/\sigma_{pp}}$$

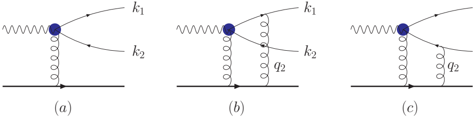


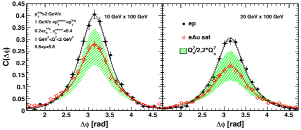
Comparing to STAR and PHENIX data

- Physics predicted by [C. Marquet, 09].
- Further calculated in [Marquet, Albacete, 10; Stasto, BX, Yuan, 11]
- Physical picture: de-correlation of dijets due to dense gluonic matter.

Dijet production in DIS

[L. Zheng, E. Aschenauer, J. H. Lee and BX, 14]





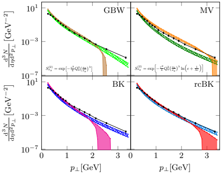
TMD factorization approach:

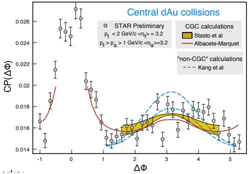
$$\frac{d\sigma^{\gamma_T^* A\to q\bar{q}+X}}{d\mathcal{P}.\mathcal{S}.} = \delta(x_{\gamma^*}-1)x_g G^{(1)}(x_g, q_\perp)H_{\gamma_T^* g\to q\bar{q}},$$

Remarks:

- For away side correlation $|k_{1\perp}| \simeq |k_{2\perp}| \gg q_\perp = k_{1\perp} + k_{2\perp}$.
- Unique golden measurement for the Weizsäcker Williams gluon distributions.
- EIC and LHeC will provide us perfect machines to study the strong gluon fields in nuclei. Important part in EIC and LHeC physics design. [arXiv:1212.1701; J.Phys. G39 (2012) 075001.]

Conclusion





- Effective k_t factorization for single and dihadron productions in pA collisions in the small-x saturation formalism at one-loop order.
- Towards the quantitative test of saturation physics beyond LL.
- Dijet (dihadron) correlation in pA collisions.
- Gluon saturation could be the next interesting discovery at the LHC and future EIC.