Quarkonium formation time in relativistic heavy-ion collisions

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in collaboration with

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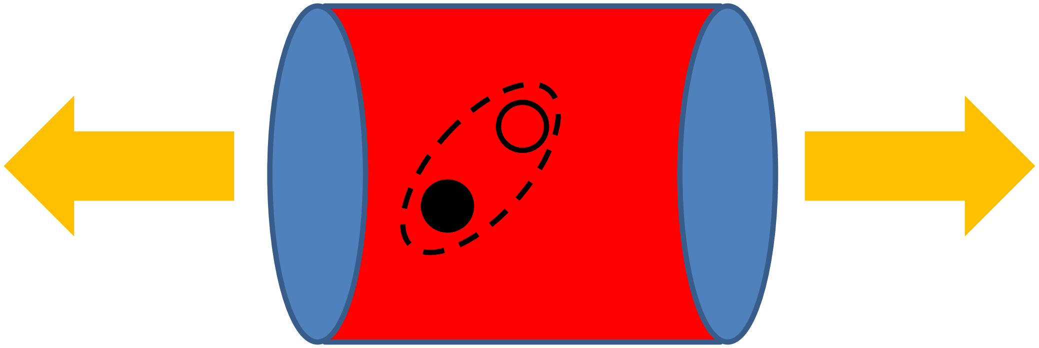
1. Motivation

- Quarkonium ($c\bar{c}$, $b\bar{b}$ bound states) is an important probe searching for the properties of the hot dense nuclear matter created in relativistic heavy-ion collisions.
- There are lots of experimental data on quarkonium production in relativistic heavy-ion collisions.
- In order to understand and interpret the experimental data, it is necessary to know the formation time of quarkonium in hot dense nuclear matter.



2. Quarkonium formation time in vacuum

D. Kharzeev and R. L. Thews, PRC 60 (1999) 041901

Space-time correlator of currents

- D. Kharzeev and R. L. Thews, PRC 60 (1999) 041901 -

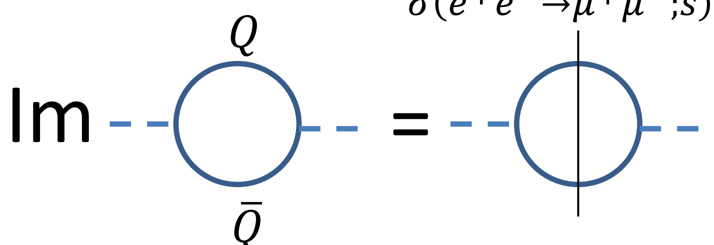
- $\Pi(x) = \langle 0|T\{j(x), j(0)\}|0\rangle = \frac{1}{\pi}\int \frac{d^4q}{(2\pi)^4} e^{-iq\cdot x}\Pi(q^2) = \frac{1}{\pi}\int ds\, Im\Pi(s) \int \frac{d^4q}{(2\pi)^4}\frac{e^{-iq\cdot x}}{s-q^2} = \frac{1}{\pi}\int ds\, Im\Pi(s) \mathrm{D}(\sqrt{s}, x)$

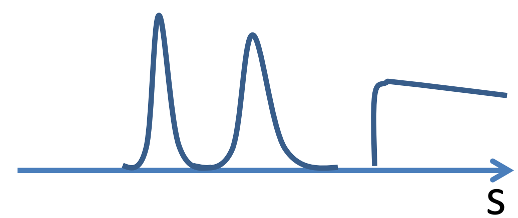
Fourier transformation

dispersion relation

- $\mathrm{D}(\sqrt{s}, \tau^2 = -x^2) = \frac{\sqrt{s}}{4\pi^2\tau} K_1(\sqrt{s}\tau)$: propagator in the coordinate representation
- $Im\Pi(s) = \frac{s}{(4\pi\alpha)^2}\sigma(e^+e^- \to \bar{Q}Q; s) \sim \frac{\sigma(e^+e^-\to\bar{Q}Q;s)}{\sigma(e^+e^-\to\mu^+\mu^-;s)} \equiv R(s)$

$j = \bar{Q}\Gamma Q$





- The correlator is a superposition of propagators of physical states, each with the weight proportional to the probability of their production in a hard process.

Formation time

- If $\mathrm{Im}\Pi(s) \sim \delta(s - m^2)$, $\quad \Pi(x) \sim \tau^{-3/2} e^{-m\tau}$.
- The formation time is defined as the (Euclidean) time when the correlator approaches the asymptotic form, $\Pi(x) \sim \tau^{-3/2} e^{-m\tau}$ as $\tau \to \tau_f$.
- For example, if $\mathrm{Im}\Pi(s) \sim \delta(s - m_1^2) + \delta(s - m_2^2)$,

$$\Pi(\tau) \sim \tau^{-3/2} e^{-m_1\tau} + \tau^{-3/2} e^{-m_2\tau}$$

$$= \tau^{-3/2} e^{-m_1\tau} \left[1 + e^{-(m_2 - m_1)\tau}\right].$$

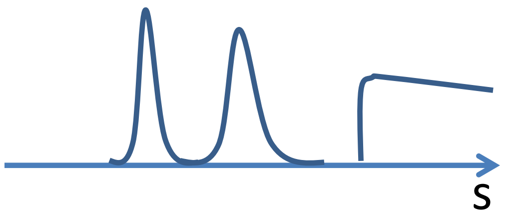
So the formation time of m_1, $\tau_f \sim \frac{1}{m_2 - m_1}$.

Correlator of heavy quark vector currents

- $\Pi(x) \equiv \Pi^{\mu}_{\mu}(x) = \frac{1}{4\pi^2}\int ds\, sR(s)\mathrm{D}(\sqrt{s}, x^2)$
- $R(s) = \sum_i \; R_i(s) + R_{cont}(s)$,

$$R_i(s) = \frac{72\pi e_Q^2}{M_i^2}|\psi_i(0)|^2 \frac{\Gamma_i/2}{(\sqrt{s}-M_i)^2+\Gamma_i^2/4}$$

$$R_{cont}(s) = 3e_Q^2\theta(s - s_{th})$$



$$\Pi_{res}(\tau) = \frac{9e_Q^2 M_i^2}{\pi^2\tau}|\psi_i(0)|^2 K_1(M_i\tau)$$

$$\Pi_{cont}(\tau) = \frac{3e_Q^2}{8\pi^4\tau^6}\int_{\sqrt{s_{th}}\tau}^{\infty} x^4 K_1(x)dx$$

$$\Gamma_i^{e^+e^-} = \frac{16\pi\alpha^2 e_Q^2}{M_i^2}|\psi(r=0)|^2$$

$\Gamma_i^{e^+e^-}$, M_i are given from experimental data.
$\sqrt{s_{th}}$ is taken to be the twice of open flavor mass.

Fraction of ground state

- $\Pi(\tau) = \Pi_0(\tau) + \Pi_1(\tau) + \cdots + \Pi_{cont}(\tau)$
- $F(\tau) = \frac{\Pi_0(\tau)}{\Pi(\tau)}$
- Initially Π_{cont} dominates.
- Later Π_0 dominates.

At large τ,

- $\Pi_0 \sim \exp(-M_0\tau)$
- $\Pi_{cont} \sim \exp\left(-\sqrt{s_{th}}\tau\right)$

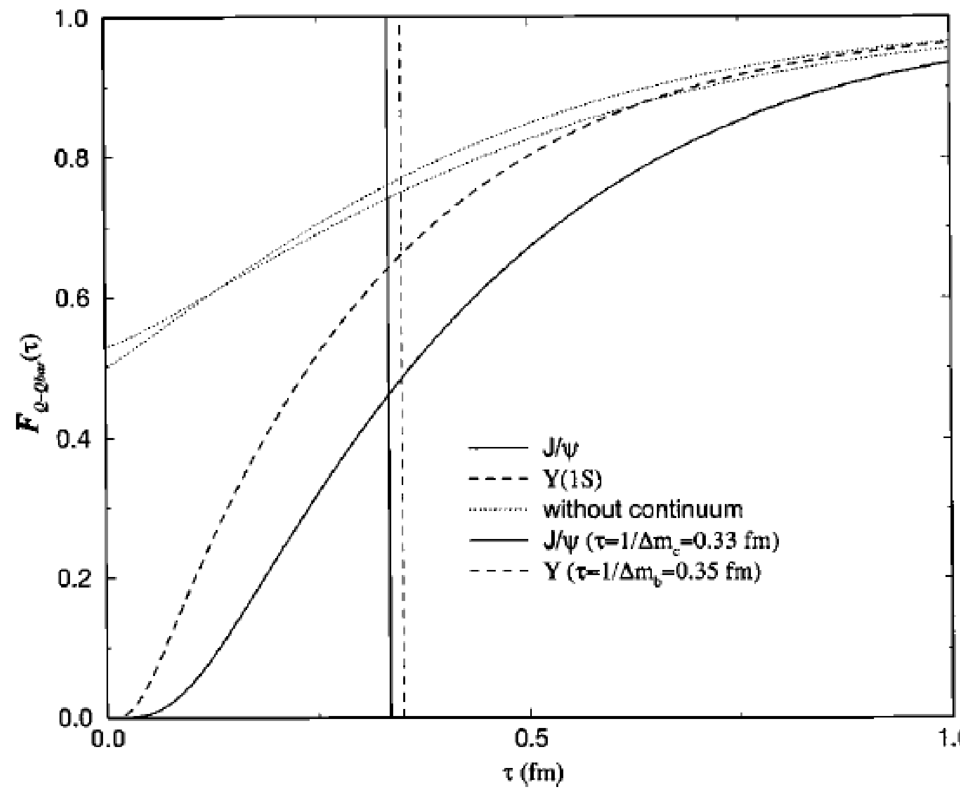


FIG. 1. Formation times for the ground states of quarkonium in $e^+ - e^-$ annihilation.

Distribution of formation times of quarkonia

- $P(\tau) = \dfrac{dF\,(\tau)}{d\tau}$

where $F(\tau) = \dfrac{\Pi_0(\tau)}{\Pi(\tau)}$

- Average formation time

$$\langle\tau\rangle = \frac{\int d\tau P(\tau)\tau}{\int d\tau P(\tau)}$$

$\langle\tau_{J/\psi}\rangle$=0.44 fm/c

$\langle\tau_{\Upsilon}\rangle$=0.32 fm/c

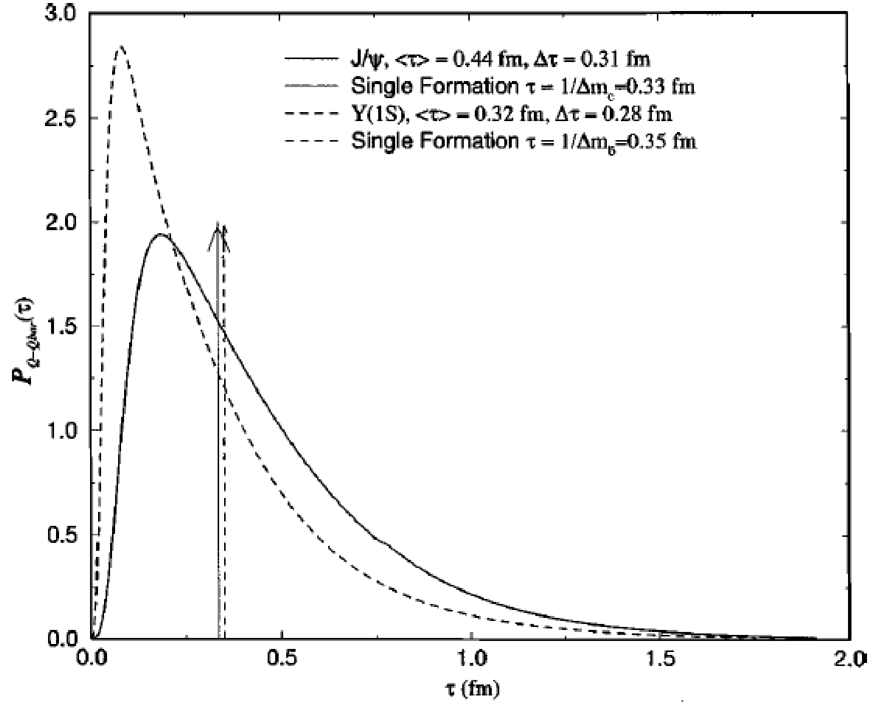


FIG. 2. Normalized distribution functions for formation times of J/ψ and Υ in e^+-e^- annihilation.

3. Quarkonium formation time in QGP

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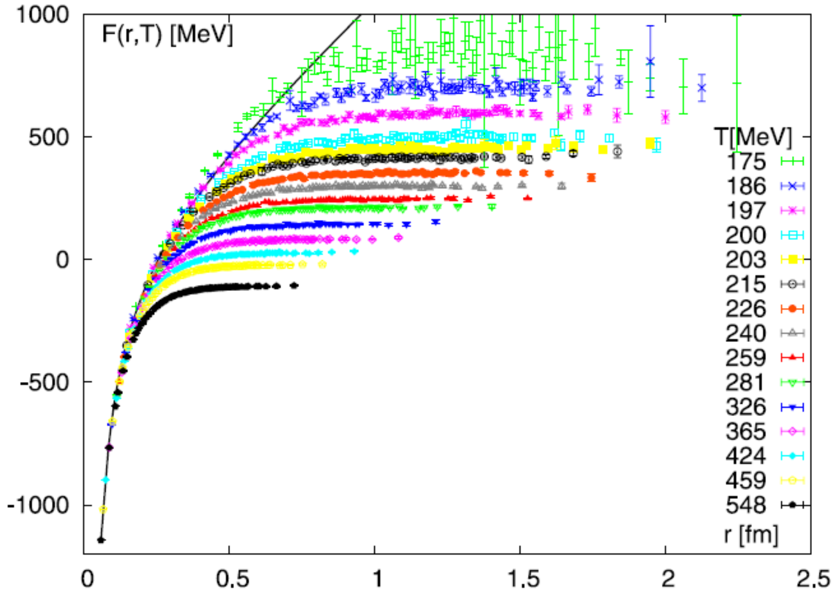
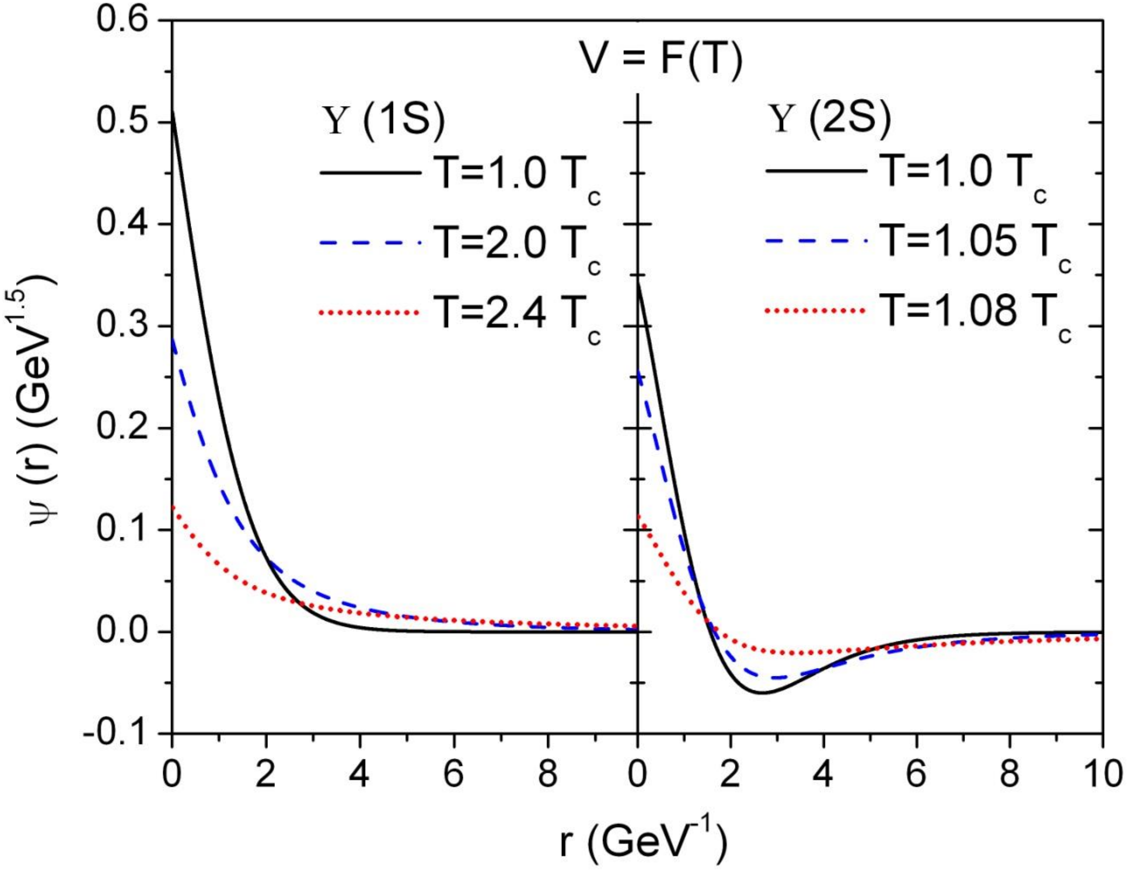
Formation time in QGP

- We don't know exactly the spectral function of quarkonia in QGP.
- We use the lattice free energy for the potential between heavy quark and heavy antiquark and solve Schrodinger equation.
- Eigenvalue → quarkonium mass
- Eigenfunction →|ψ(0)|
- $\sqrt{s_{th}} = 2m_Q + V(\infty)$

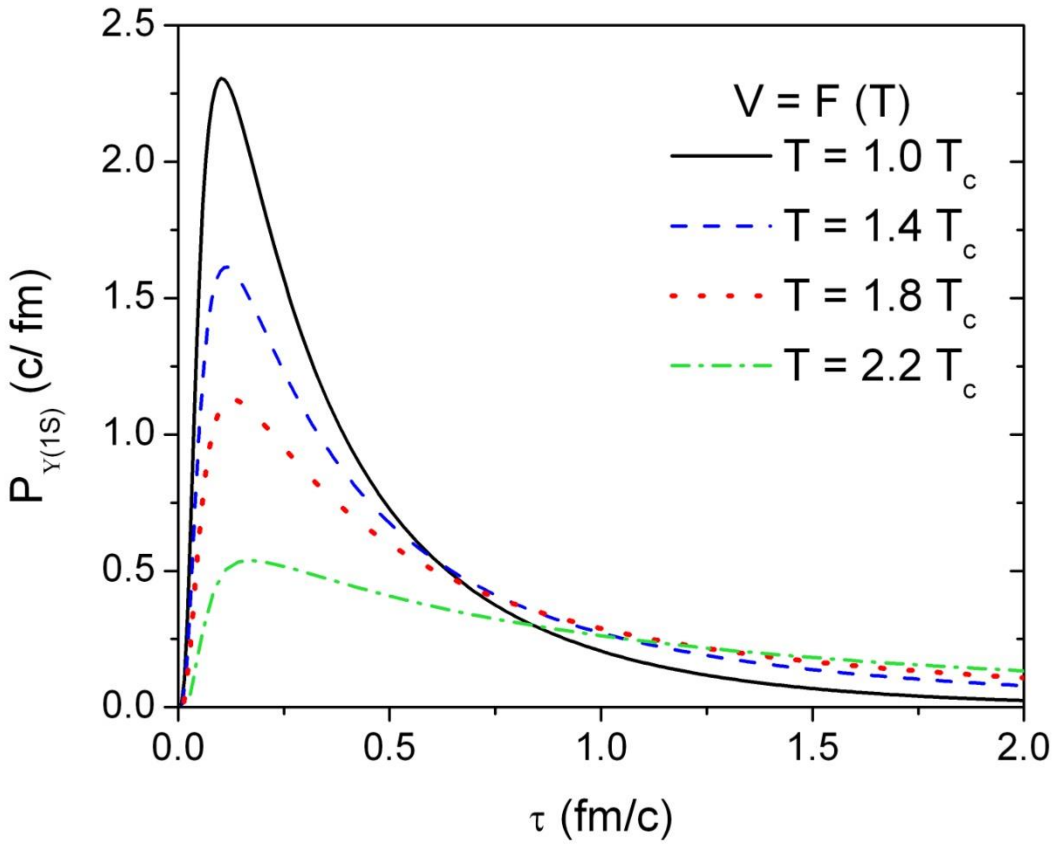


Wavefunctions of Υ(1S) and Υ(2S) in QGP for free energy potential



|ψ(0)| decreases with temperature

P(τ)=dF(τ)/dτ of $b\bar{b}$ 1S bound state for free energy potential



As temperature increases,
M_0 and $\sqrt{s_{th}}$ close to each other.

At large τ,

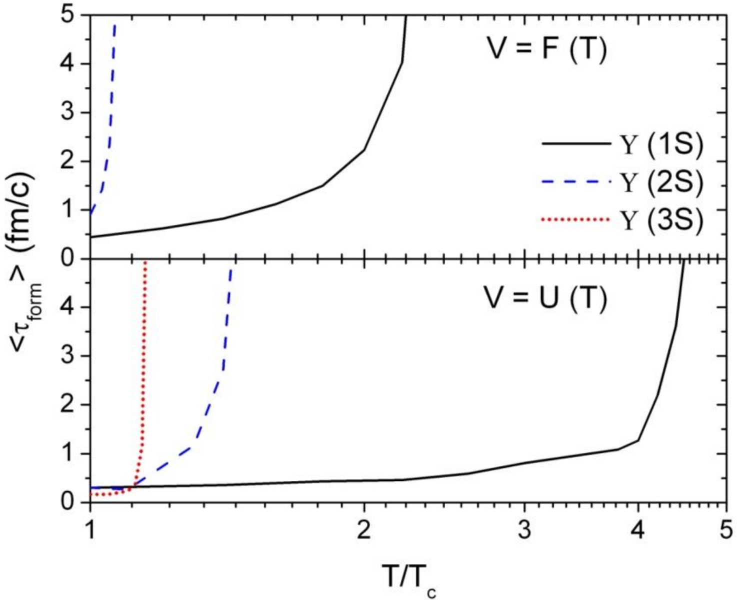
$$\Pi_0 \sim \exp(-M_0\tau)$$

$$\Pi_{cont} \sim \exp(-\sqrt{s_{th}}\tau)$$

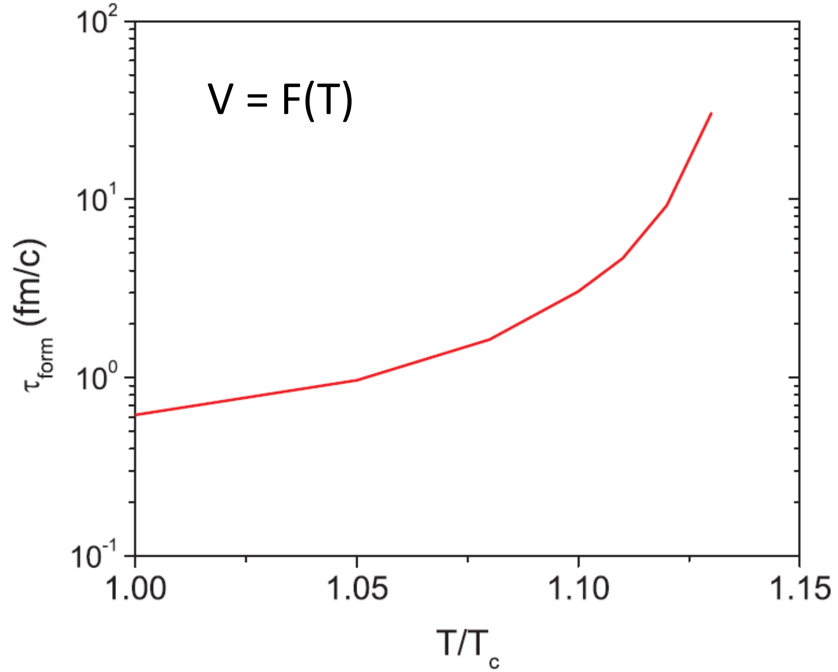
→ quarkonium formation delays

Formation times of quarknoia as functions of temperature

Bottomonia ($b\bar{b}$ bound state)

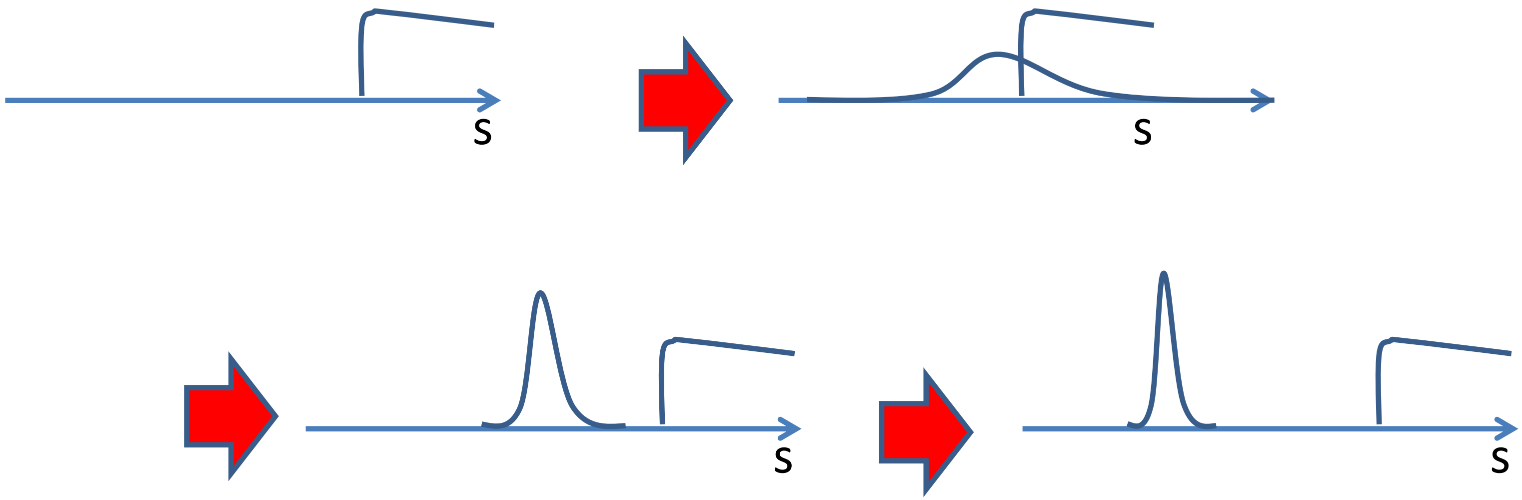


Charmonia ($c\bar{c}$ bound state)



4. Quarkonium formation time in relativistic heavy-ion collisions

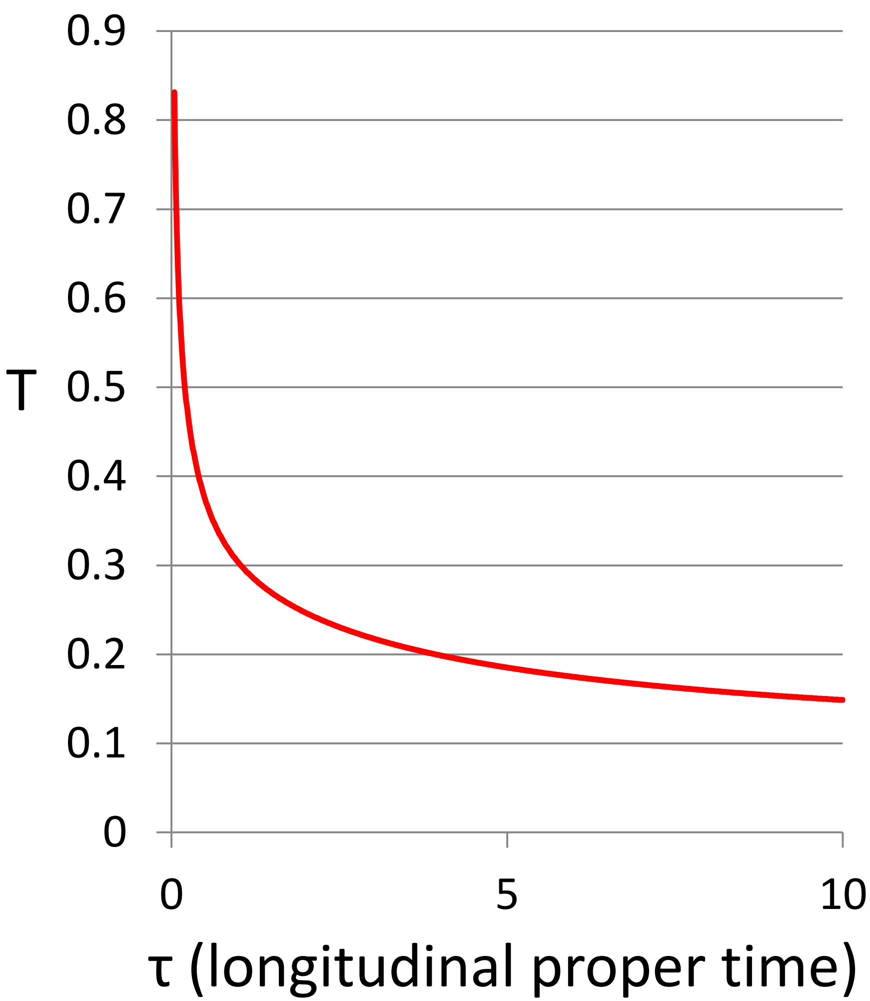
- Temperature changes with time



$$\Pi_{res}(\tau) = \frac{9e_Q^2 M_i^2(\tau)}{\pi^2\tau}|\psi_i(0)|^2(\tau)K_1(M_i\tau)$$

$$\Pi_{cont}(\tau) = \frac{3e_Q^2}{8\pi^4\tau^6}\int_{\sqrt{s_{th}(\tau)}\tau}^{\infty} x^4 K_1(x)dx$$

Time-evolution of temperature from 2+1 hydrodynamics (HLLE algorithm)



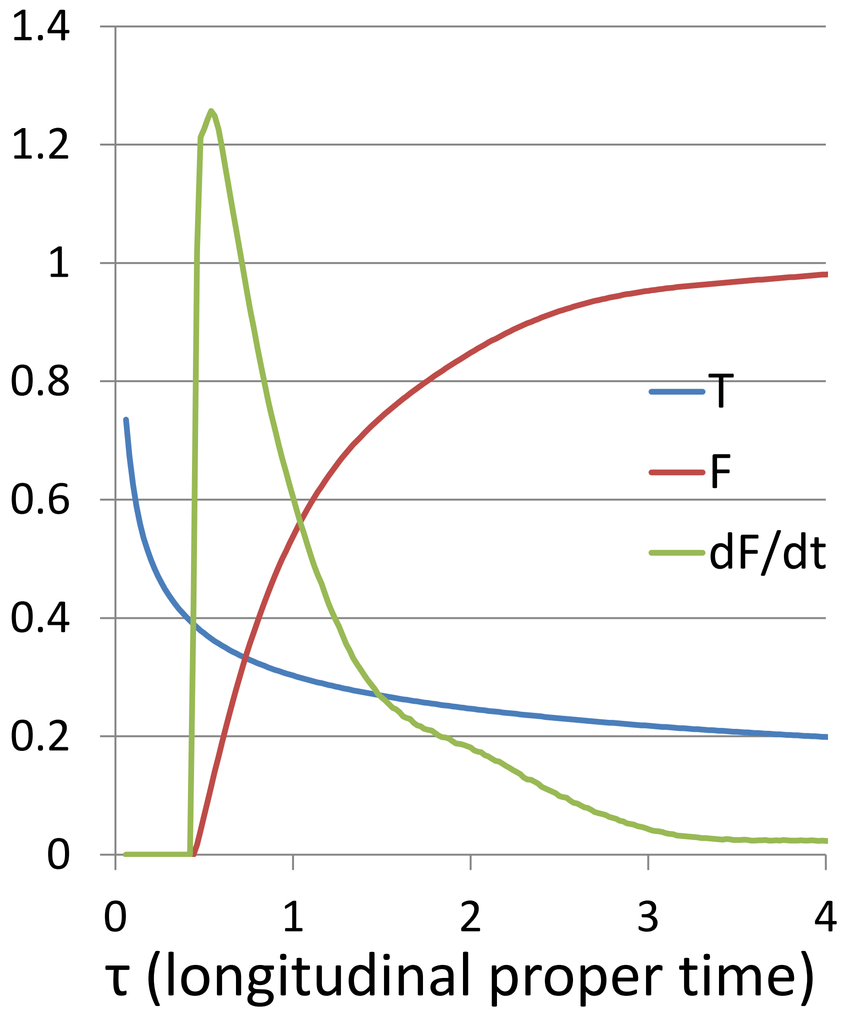
0~5 % central Au+Au collisions @ 200 GeV

$$\langle T(\tau)\rangle = \int dxdyT(x,y,\tau)\, n_{coll}(x,y)$$

Before initial thermalization
$(\tau < \tau_0 = 0.6\ \mathrm{fm/c})$
Only longitudinal expansion

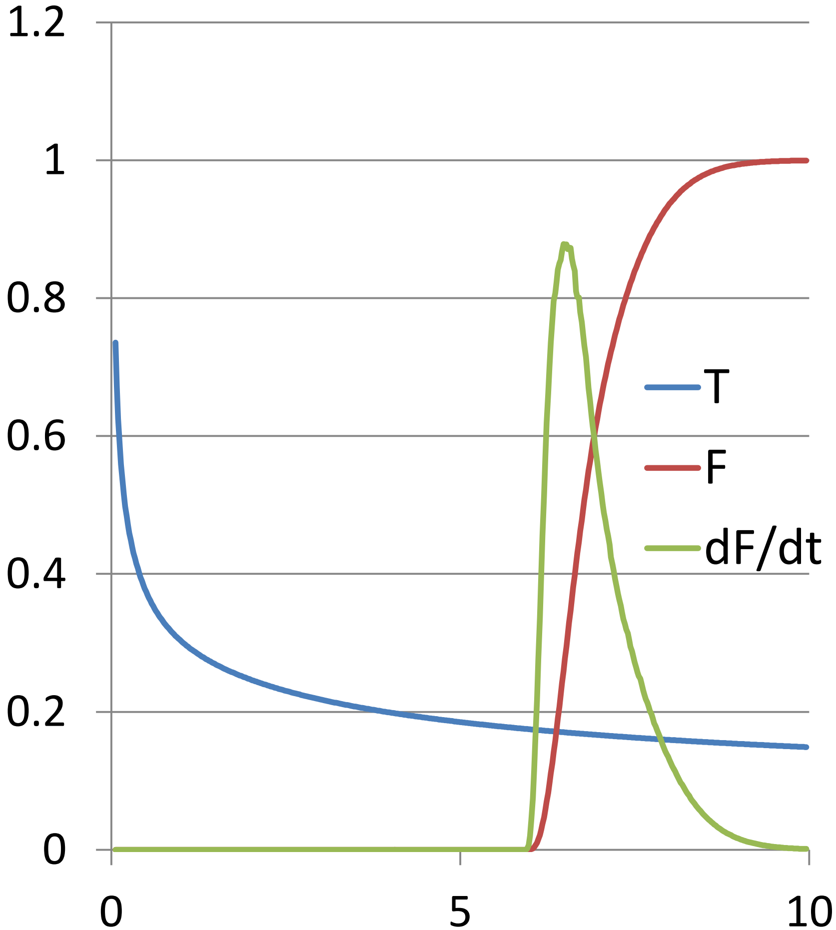
After initial thermalization
$(\tau > \tau_0 = 0.6\ \mathrm{fm/c})$
Hydrodynamics expansion

For bottomonium (1S $b\bar{b}$)



- 1S bottomonium begins to be formed at τ=0.45 fm/c when T equals the dissociation temperature of 1S state (2.5 Tc).
- 1S bottomonium is formed at τ=1.2 fm/c in average.

For regenerated J/Ψ (1S $c\bar{c}$)



- J/Ψ begins to be formed at τ=5.95 fm/c when T equals the dissociation temperature of J/Ψ (1.13 Tc).
- However, it is doubtful for initial ccbar correlation to survive up to τ≃6 fm/c.

For regeneration

$$\langle T(\tau)\rangle = \int dxdyT(x,y,\tau)\, n_{coll}^2(x,y)$$

- J/Ψ is regenerated at τ=6.9 fm/c in average.

(about 1.0 fm/c after its dissociation temperature)

5. Summary

- Quarkonium formation time is calculated from the space-time correlator of heavy quark vector currents and dispersion relation.
- In QGP, quarkonium spectral function is obtained by solving Schrodinger equation with lattice free energy potential.
- The results are applied to relativistic heavy-ion collisions (RHIC) by using hydrodynamics background.
- As a result, the formation times of 1S bottomonium and 1S charmonium in RHIC are longer than those in vacuum.

  Υ(1S) (0.32 fm/c → 0.7 fm/c after the dissociation T of Υ)

  J/ψ (0.44 fm/c → 1.0 fm/c after the dissociation T of J/ψ)

  using lattice free energy as heavy quark potential