Partial restoration of chiral symmetry inside color flux

Takumi Iritani

Yukawa Institute of Theoretical Physics

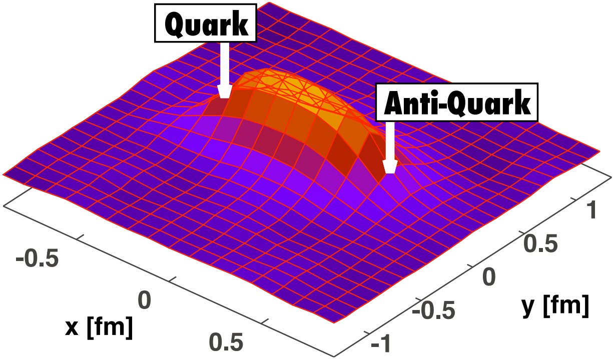
G. Cossu and S. Hashimoto (KEK)

High Energy Strong Interactions: A School for Young Asian Scientists,
September 22-26, 2014

Reference

- TI, G. Cossu, and S. Hashimoto, PoS (Hadron 2013) 159, arXiv:1401.4293.

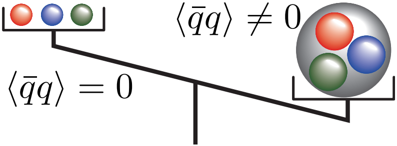
1. Introduction: Color Flux Structure in QCD
2. Chiral Symmetry Breaking in Color Flux
3. Chiral Symmetry Restoration in “Baryon”
4. Summary



Chiral Symmetry Breaking and QCD Vacuum

Chiral Symmetry Breaking

$$\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \to \mathrm{SU}(N_f)_V$$

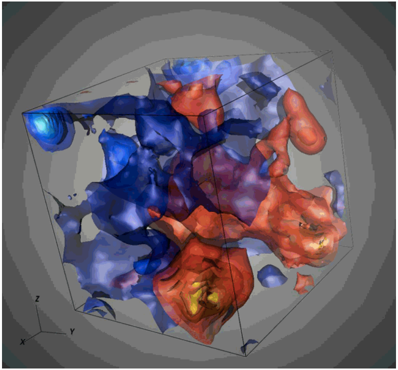


- symmetry breaking is modified by
  - High temperature
    - Quark Gluon Plasma
  - Finite/High density
    - nuclear matter
    - neutron star
  - Strong fields
    - heavy ion collision

topological structure of QCD vacuum

$\Longrightarrow$ important for χSB

a snapshot of QCD vacuum



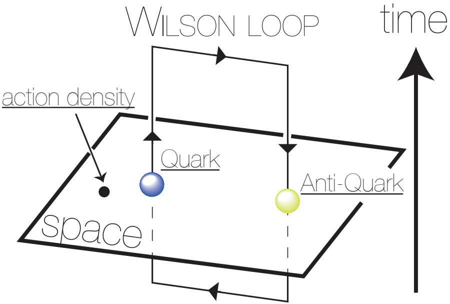
by JLQCD Coll. '12

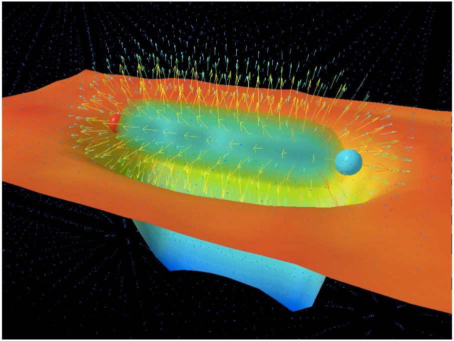
Color Sources in QCD — Color Flux Structure

color flux appears between color sources, i.e., quarks

color flux can be *observed* by spatial distribution of action density $\rho(\vec{x})$ or chromo fields around Quark-Antiquark

$$\langle \rho(\vec{x}) \rangle_W \equiv \frac{\langle \rho(\vec{x}) W \rangle}{\langle W \rangle} - \langle \rho \rangle_{\text{vac.}} \qquad W: \text{ Wilson loop}$$

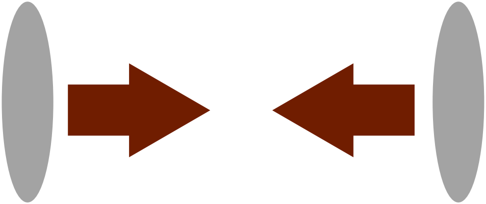


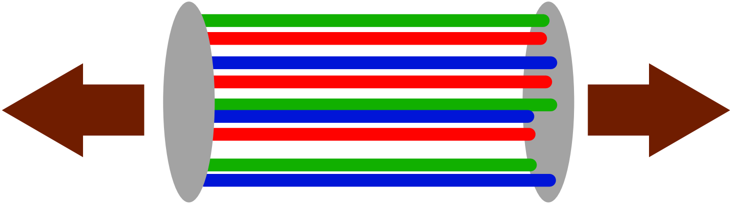


Leinweber, et al. '03
from www.physics.adelaide.edu.au/theory/staff/leinweber/VisualQCD/Novel/

Color Flux in Heavy Ion Collision

After relativistic heavy ion collision, gluons interact to form color flux, which is an important initial stage of QGP production (glasma)

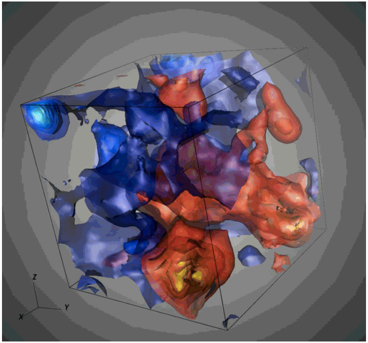


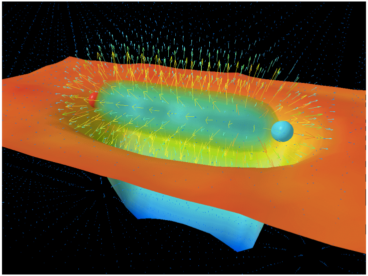


Aim of This Work

1. chiral condensate $\langle \bar{q}q \rangle$ characterizes
   spontaneous breaking of chiral symmetry *in the QCD vacuum*
2. color sources produce color flux
   chromo fields would modify non-perturbative properties of QCD
3. we analyze chiral condensate *in the color flux* from Lattice QCD

$\Longrightarrow$ chiral symmetry breaking inside hadrons and chromo fields





Chiral Symmetry Breaking and Dirac Eigenvalue

∘ chiral condensate $\langle\bar{q}q\rangle$ is given by

$$\langle\bar{q}q\rangle = -\mathrm{Tr}\frac{1}{\not{D}+m} = -\frac{1}{V}\sum_{\lambda}\frac{1}{i\lambda+m}$$

with Dirac eigenvalues λ $\Leftarrow$ $\not{D}\psi_\lambda = i\lambda\psi_\lambda$

accumulation of near-zero mode

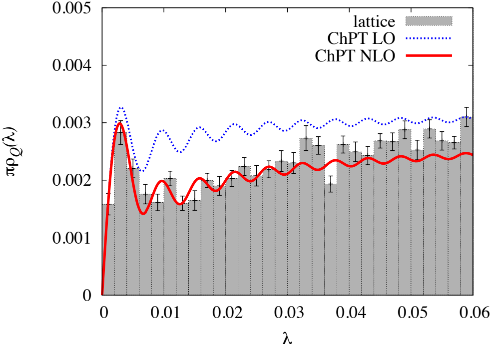
$\Longrightarrow$ chiral symmetry breaking

Banks-Casher Relation

$$\langle\bar{q}q\rangle = -\pi\langle\rho(0)\rangle \qquad m \to 0$$

but, besides eigenvalues λ, eigenfunctions $\psi_\lambda(x)$ also carry interesting information ...

Dirac spectral density $\rho(\lambda)$



JLQCD Coll. '10

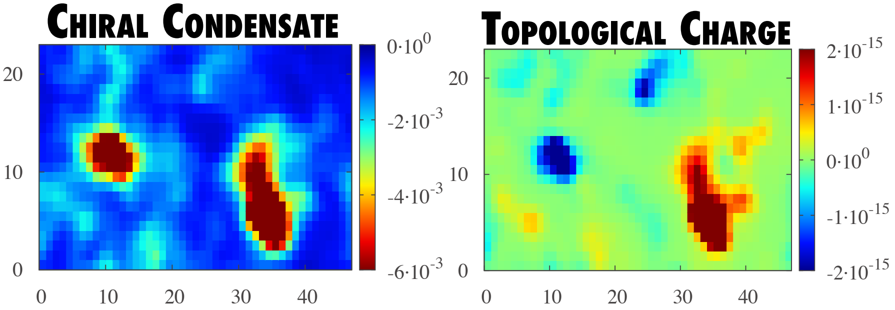
Local Structure of Chiral Condensate in QCD Vacuum

Using Dirac eigenfunction $\psi_\lambda(x)$, we define "local chiral condensate" $\bar{q}q(x)$

$$\langle \bar{q}q \rangle = -\mathrm{Tr}\frac{1}{\not{D}+m} = -\frac{1}{V}\sum_x \left[\sum_\lambda \frac{\psi_\lambda^\dagger(x)\psi_\lambda(x)}{i\lambda+m}\right] = \frac{1}{V}\sum_x \bar{q}q(x)$$

$\bar{q}q(x)$ forms clusters which correlate with topological charge, i.e., instanton

a snapshot of QCD vacuum [1]

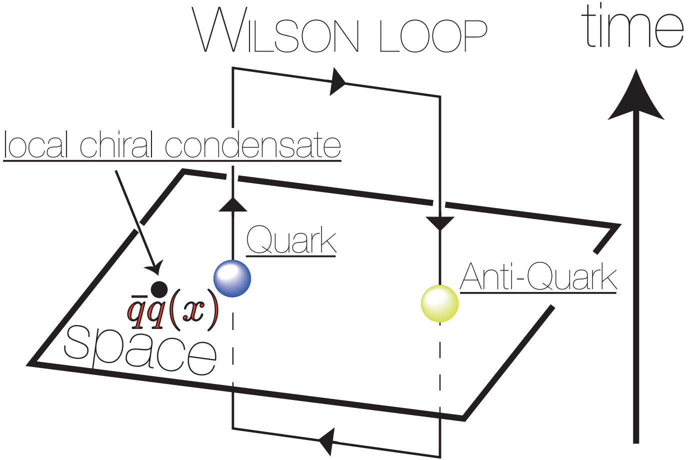


[1] both quantities are calculated by using low-lying 20 overlap-Dirac eigenmodes

Local Chiral Condensate around Quark-Antiquark

chiral condensate around color sources, i.e., Wilson loop $W(R,T)$

$$\langle \bar{q}q(\vec{x})\rangle_{\rm flux} \equiv \frac{\langle \bar{q}q(\vec{x}) W(R,T)\rangle}{\langle W(R,T)\rangle}$$



About Lattice QCD Setup

- 2+1 overlap-fermion configuration and eigenmode by JLQCD Coll
  - overlap-fermion keeps "exact chiral symmetry" on lattice

$$D_{\rm ov}(0) = m_0 \left[1 + \gamma_5 {\rm sgn} H_W(-m_0)\right]$$

with $H_W(-m_0)$: hermitian Wilson-Dirac operator (Neuberger '98)

- simulation parameter
  - pion mass $m_\pi \sim 300$ MeV, kaon mass $m_K \sim 500$ MeV
  - two lattice volume $24^3 \times 48$ and $16^3 \times 48$
  - fixed global topological charge at $Q = 0$
  - lattice spacing $a^{-1} = 1.759(10)$ GeV, i.e., $a \sim 0.11$ fm
- $W(R, T = 4)$ with APE smearing, and measure at $t = 2$ time slice
- use *low-mode truncated* chiral condensate

$$\bar{q}q(x) = -\sum_\lambda \frac{\psi_\lambda^\dagger(x)\psi_\lambda(x)}{m_q + \left(1 + \frac{m_q}{2m_0}\lambda\right)} \Rightarrow -\sum_\lambda^{N} \frac{\psi_\lambda^\dagger(x)\psi_\lambda(x)}{m_q + \left(1 + \frac{m_q}{2m_0}\lambda\right)}$$

about $N \sim \mathcal{O}(100)$ is enough to reproduce chiral condensate [2]

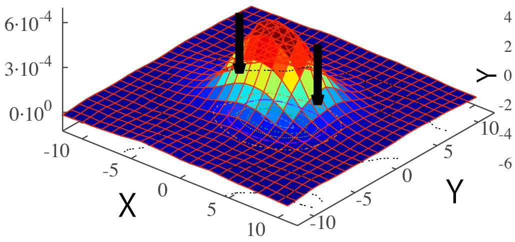
[2] N-dependence can be removed by $\langle \bar{q}q \rangle^{(N)} = \langle \bar{q}q^{(\rm subt)} \rangle + c_1^{(N)} m_q/a^2 + c_2^{(N)} m_q^3$
reference Noaki, et al., JLQCD Coll. '09

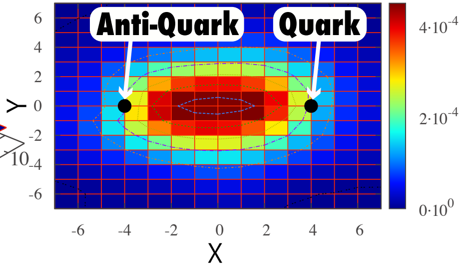
Chiral Condensate between Quark-Antiquark

Change of chiral condensate

$$\langle \bar{q}q(\vec{x}) \rangle_W \equiv \langle \bar{q}q(\vec{x}) \rangle_{\text{flux}} - \langle \bar{q}q \rangle_{\text{vac.}}$$

- a tube structure of local chiral condensate
- "POSITIVE" change $\langle \bar{q}q(\vec{x}) \rangle_W > 0 \Longrightarrow |\langle \bar{q}q(\vec{x}) \rangle_{\text{flux}}| < |\langle \bar{q}q \rangle_{\text{vac.}}|$
- chiral symmetry is PARTIALLY RESTORED between quark-antiquark





lattice unit $a \sim 0.11$ fm

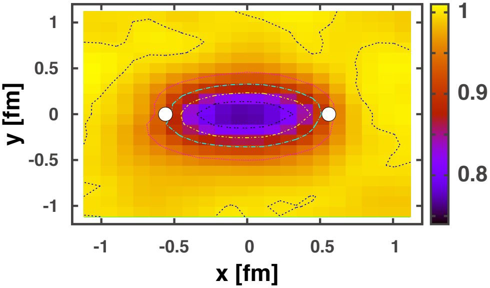
Ratio of chiral condensate

$$r(\vec{x}) \equiv \frac{\langle \bar{q}q(\vec{x})\rangle_{\text{flux}}}{\langle \bar{q}q\rangle_{\text{vac.}}} < 1$$

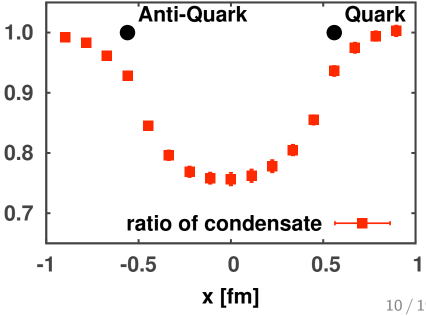
- about 20% reduction of chiral condensate

  $\Rightarrow$ partial restoration of chiral symmetry inside the color flux-tube

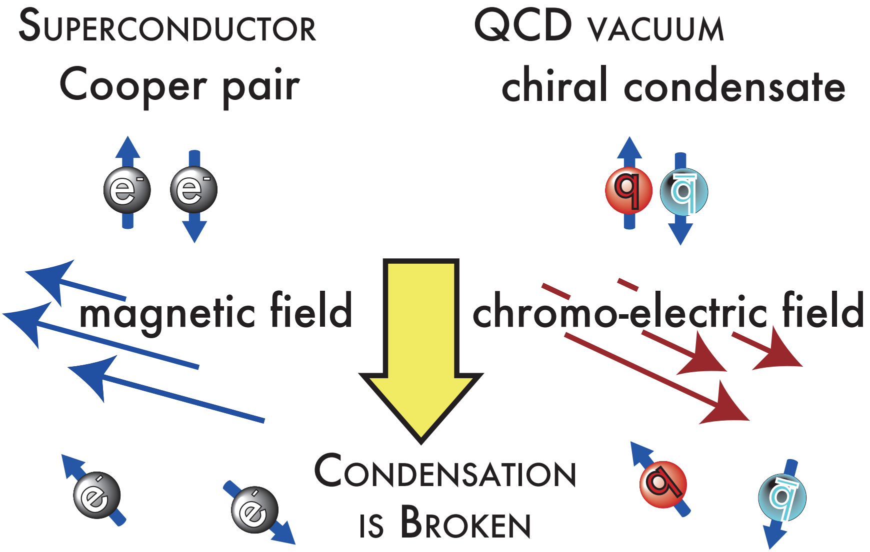
heat map of condensate



cross-section

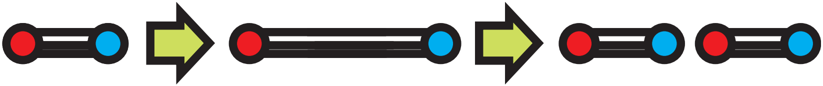


Analogy of Superconductivity



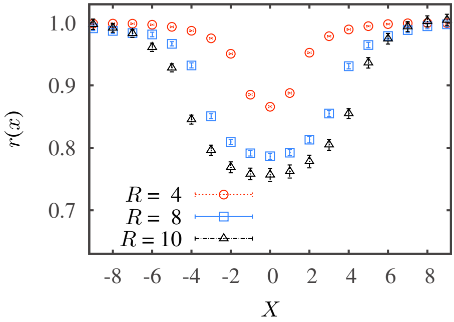
a schematic picture of an analogy of superconductivity

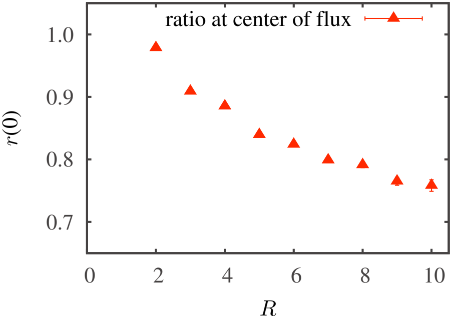
Distance between Color Sources and Chiral Condensate



By increasing the interquark separation R,
chiral symmetry restoration becomes LARGER until string breaking occurs

cross-section of $\langle \bar{q}q(\vec{x})\rangle_{\rm flux}/\langle \bar{q}q\rangle_{\rm vac.}$





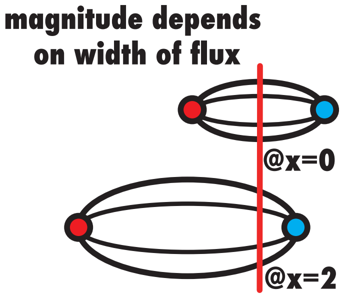
lattice unit $a \sim 0.11$ fm

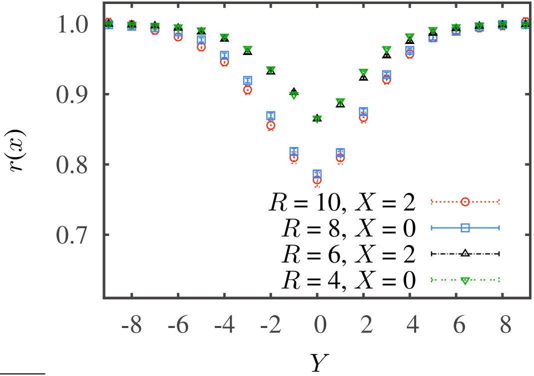
Details of Interquark Distance Dependence

In fact, a thickness of flux is known to grow as[3] $\Longleftarrow$ "roughening"

$$w^2 \sim w_0^2 \ln R/R_0$$

- separation $R \nearrow \Longrightarrow$ thickness w^2 grows $\Longrightarrow$ reduction becomes large
- magnitude of restoration correlates with a thickness of flux





[3] Hasenfratz-Hasenfratz-Hasenfratz '81, Lüscher-Münster-Weisz '81

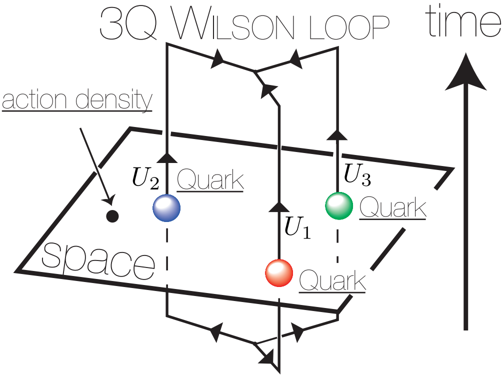
Three Quarks System

⇒ It is also possible to analyze chiral condensate inside "baryon"

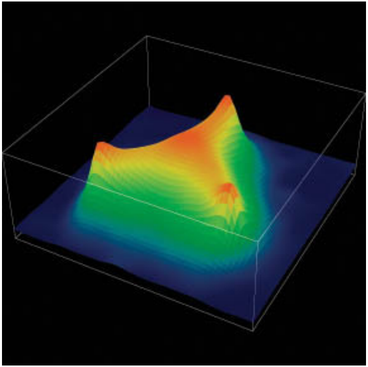
■ 3Q-Wilson loop cf. Takahashi-Suganuma '01

$$W_{3Q} \equiv \frac{1}{3!}\varepsilon_{abc}\varepsilon_{a'b'c'}U_1^{aa'}U_2^{bb'}U_3^{cc'} \qquad (a^{(\prime)}, b^{(\prime)}, c^{(\prime)} : \text{ color index})$$

⇒ color singlet products of 3 Wilson lines U_k



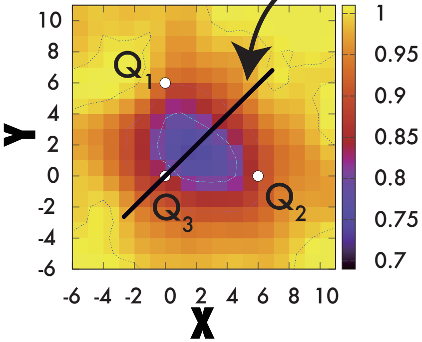
Y-type flux Ichie, et al. '03



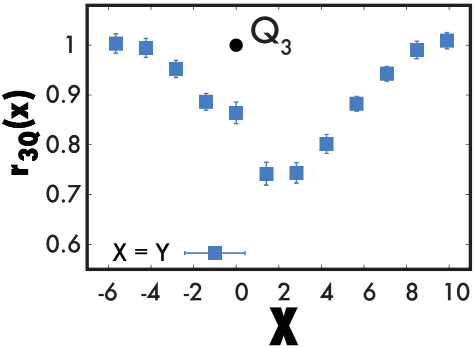
Ratio of Chiral Condensate among 3Q-system

$$r_{3Q}(\vec{x}) \equiv \frac{\langle \bar{q}q(\vec{x})\rangle_{3Q}}{\langle \bar{q}q\rangle_{\text{vac.}}} < 1 \qquad \text{with} \qquad \langle \bar{q}q(\vec{x})\rangle_{3Q} \equiv \frac{\langle \bar{q}q(\vec{x}) W_{3Q}\rangle}{\langle W_{3Q}\rangle}$$

- about 20 $\sim$ 30% reduction of chiral condensate inside "baryon"



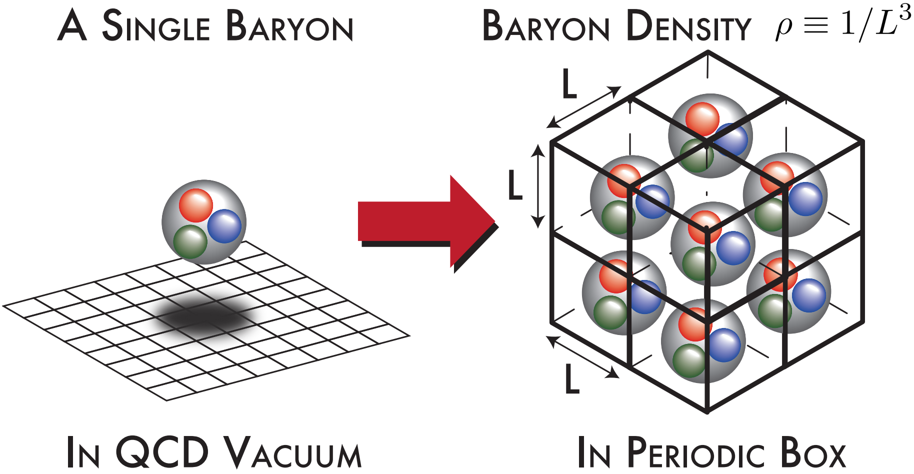
cross-section of ratio



lattice unit $a \sim 0.11$ fm

Chiral Symmetry Restoration at “Finite Density”

Considering a single “static” baryon in finite periodic box,
we discuss chiral symmetry restoration at “finite density”.

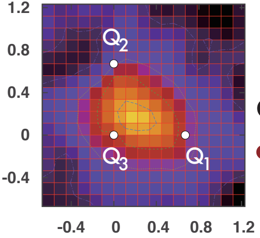


Chiral Symmetry Restoration in Finite Box

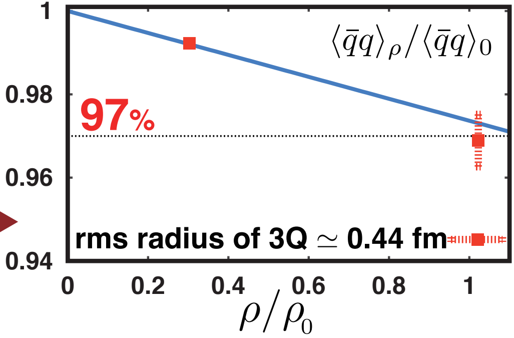
total change of chiral condensate with a single static baryon

$$\frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} \equiv \frac{1}{L^3} \sum_{\vec{x}} \frac{\langle \bar{q}q(\vec{x}) \rangle_{3Q}}{\langle \bar{q}q \rangle_{\text{vac.}}}$$

chiral condensate around 3Q



spatial average



- ◦ ρ_0: normal nuclear matter density
- ◦ cf. proton charge radius ~ 0.88 fm

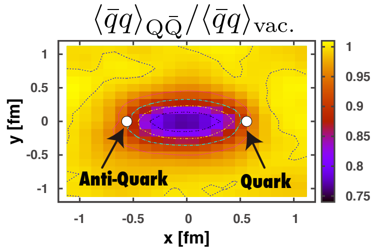
Summary

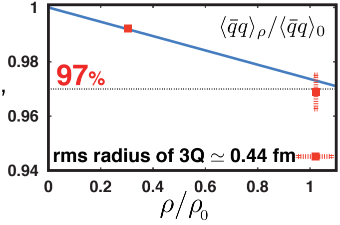
Using overlap-Dirac eigenmode, we discuss chiral condensate in color-flux.

- color flux modifies chiral sym. breaking
- magnitude of chiral condensate $\langle \bar{q}q \rangle$ is reduced inside the flux-tube,

$$\frac{\langle \bar{q}q \rangle_{\text{flux}}}{\langle \bar{q}q \rangle_{\text{vacuum}}} = 0.7 \sim 0.8$$

  until string breaking occurs
- considering a "static" baryon in finite box, we discuss the partial restoration of chiral symmetry at "finite density"

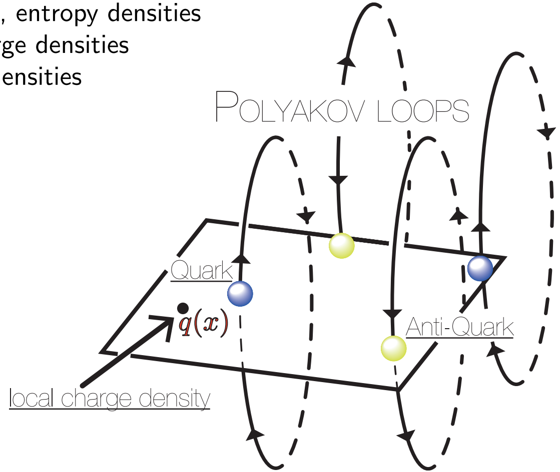




Outlook

In this work, we discuss "chiral condensate" inside "color flux" in vacuum.

- Using "Polyakov loop", we can discuss color source effects inside QGP
  - it is possible to set multi-body system of quarks and antiquarks
- in addition to chiral condensate, we can use various kinds of probes
  - energy densities, entropy densities
  - topological charge densities
  - quark number densities
  - axial charge
  - ...

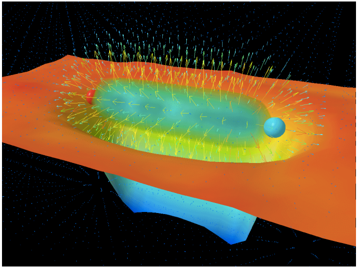


5 Appendix

Cross-section of Flux-tube

it is also possible to investigate gluonic components of flux-tube
by using $G_{12}, G_{13}, \ldots$ instead of action density $\mathrm{Tr}\ G_{\mu\nu}G_{\mu\nu}$

$\Rightarrow$ tube is almost formed by "longitudinal chromo-electric fields" — E_z
other chromo-electric/magnetic components are almost zero



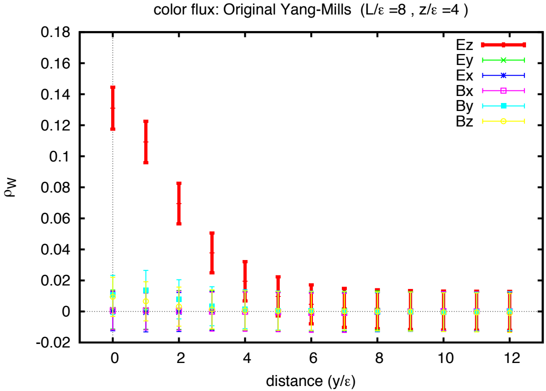
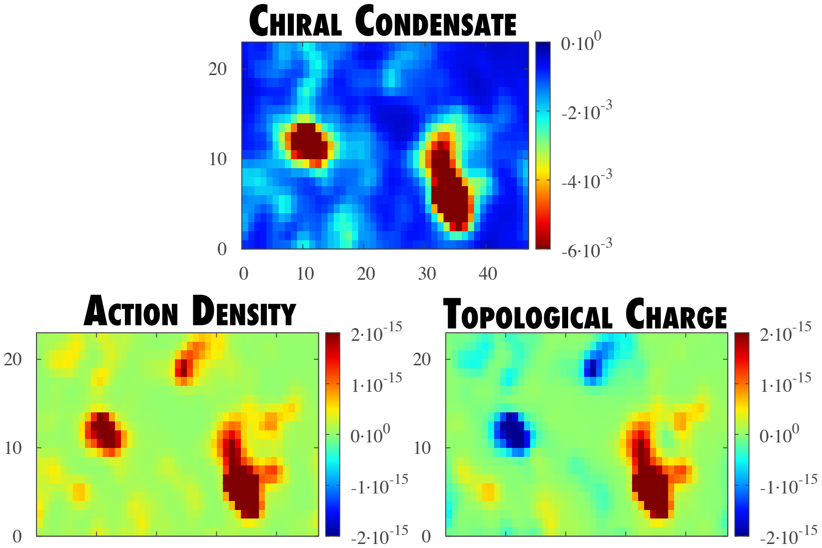


Fig. from Shibata-Kondo-Kato-Shinohara '12

Local Chiral Condensate and Instantons

local chiral condensate $\bar{q}q(x)$ correlates with (anti-)instantons.

a snapshot of QCD vacuum



Regularization of Chiral Condensate

Due to the exact chiral symmetry of overlap-Dirac fermion, Dirac-mode truncated chiral condensate is parameterized as [4]

$$\langle \bar{q}q \rangle^{(N)} = \langle \bar{q}q^{(\mathrm{subt})} \rangle + c_1^{(N)} m_q/a^2 + c_2^{(N)} m_q^3,$$

where $\langle \bar{q}q^{(\mathrm{subt})} \rangle$ is free from power divergence, these coefficients are determined by varying current quark mass m_q.

[4] reference Noaki, et al., for JLQCD Coll. '09

Quark Mass Dependence of Chiral Condensate Reduction

$16^3 \times 48$ lattice with low-lying 120 eigenmodes

- $m_{\rm ud} = 0.015$: $m_\pi \sim 0.30$ GeV
- $m_{\rm ud} = 0.050$: $m_\pi \sim 0.53$ GeV