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Thermalization of Gluons with Bose-Einstein Condensation

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with

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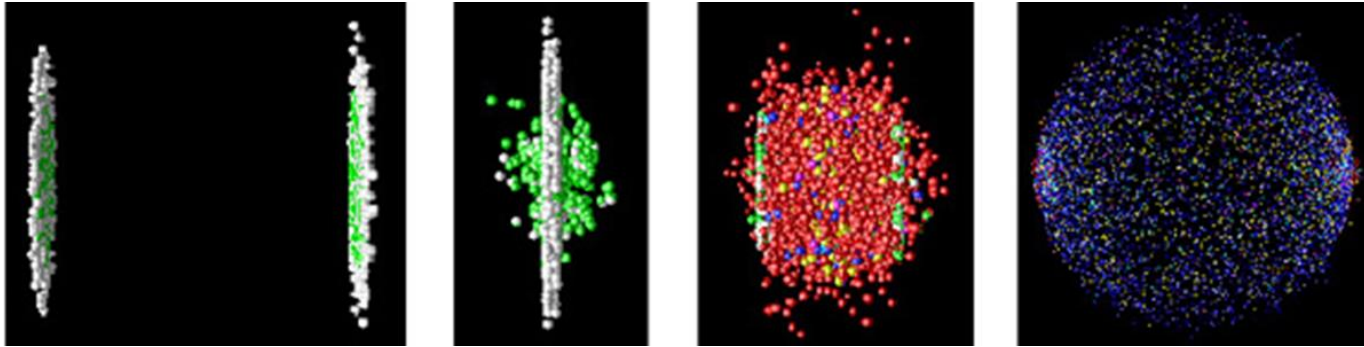
Tsinghua University



Outline

- Motivations
- Transport equation for Bose-Einstein condensation
- Bose statistics in BAMPS
- Recent results
- Summary and Outlook

Motivations



Ultrarelativistic
Heavy Ion
Collisions

CGC leads to overpopulated QGP

Blaizot, Gelis, Liao, McLerran, Venugopalan, NPA 873 (2012)

For CGC $\epsilon_0 \sim Q_s^4 / \alpha_s$, $n_0 \sim Q_s^3 / \alpha_s$; $n_0 \epsilon_0^{-3/4} \sim \alpha_s^{-1/4}$

In equilibrium $f_{eq} = \frac{1}{\exp(p/T) - 1}$, $n_{eq} \epsilon_{eq}^{-3/4} \sim 1$

For small α_s CGC leads to overpopulated QGP with BEC assuming number conservation

$$f = f_{eq} + (2\pi)^3 n_c \delta^{(3)}(\vec{p})$$

Motivations

- Studies within the kinetic theory

Onset of BEC, BUT no BEC

Blaizot, Gelis, Liao, McLerran, Venugopalan, NPA 873 (2012)

Blaizot, Liao, McLerran, NPA 920 (2013)

Huang, Liao, arXiv:1303.7214

Blaizot, Wu, Yan, arXiv:1402.5049

Scardina, Perricone, Plumari, Ruggieri, Greco, arXiv:1408.1313

- Studies within the classical field theory (no number conservation)

Epelbaum, Gelis, NPA 872 (2011)

Berges, Sexty, PRL 108 (2012)

Kurkela, Moore, PRD 86 (2012)

Berges, Boguslavski, Schlichting, Venugopalan, arXiv:1408.1670

Motivations

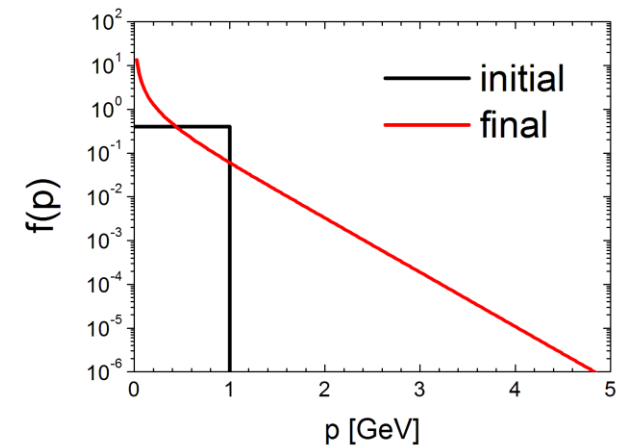
We study the thermalization of gluons with BE condensation by using a transport model **BAMPS**.

Assumptions:

- Static case: homogenous in space
- Gluon number conservation by considering only elastic scatterings
- Initial distribution: $f_{init}(\vec{p}) = f_0 \theta(Q_s - p)$
the condensation occurs when $f_0 > 0.154$

We know the final distribution

$$f_{eq}(\vec{p}) = \frac{1}{e^{p/T} - 1} + (2\pi)^3 n_c \delta^{(3)}(\vec{p})$$



We want to know how fast the thermalization occurs.

Transport Equation for BEC

Boltzmann Equation

$$\begin{aligned} \left(\partial_t + \frac{\vec{p}_1}{E_1} \cdot \vec{\nabla} \right) f_1(x, p_1) &= \frac{1}{2E_1} \int \frac{d^3 p_2}{(2\pi)^3 2E_2} \frac{d^3 p_3}{(2\pi)^3 2E_3} \frac{d^3 p_4}{(2\pi)^3 2E_4} \frac{1}{v} |\mathcal{M}_{12 \rightarrow 34}|^2 \\ &\times (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_4) \\ &\times [f_3 f_4 (\mathbf{1} + f_1)(\mathbf{1} + f_2) - f_1 f_2 (\mathbf{1} + f_3)(\mathbf{1} + f_4)] \end{aligned}$$

$$f = f^{gas} + f^c, \quad f^c = (2\pi)^3 n_c \delta^{(3)}(\vec{p})$$

g : gas particle

c : condensate particle

Included

$$g + g \leftrightarrow g + g$$

$$g + g \leftrightarrow g + c$$

Not included

$$g + c \leftrightarrow g + c$$

$$c + c \leftrightarrow c + c$$

Transport Equation for BEC

For gas particles:

$$\begin{aligned}
 \left(\partial_t + \frac{\vec{p}_1}{E_1} \cdot \vec{\nabla} \right) f_1^{gas}(x, p_1) &= \frac{1}{2E_1} \int \frac{d^3 p_2}{(2\pi)^3 2E_2} \frac{d^3 p_3}{(2\pi)^3 2E_3} \frac{d^3 p_4}{(2\pi)^3 2E_4} |\mathcal{M}_{12 \rightarrow 34}|^2 \\
 &\quad \times (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_4) \\
 &\quad \times \left[\frac{1}{2} f_3^{gas} f_4^{gas} (1 + f_1^{gas})(1 + f_2^{gas}) - \frac{1}{2} f_1^{gas} f_2^{gas} (1 + f_3^{gas})(1 + f_4^{gas}) \right. \\
 &\quad \left. + f_3^{gas} f_4^{gas} (1 + f_1^{gas}) f_2^c - \frac{1}{2} f_1^{gas} f_2^c (1 + f_3^{gas})(1 + f_4^{gas}) \right. \\
 &\quad \left. + \frac{1}{2} f_3^c f_4^{gas} (1 + f_1^{gas})(1 + f_2^{gas}) - f_1^{gas} f_2^{gas} f_3^c (1 + f_4^{gas}) \right]
 \end{aligned}$$

Transport Equation for BEC

For condensate particles:

$$\begin{aligned}\partial_t f_1^c(x, p_1) &= \frac{1}{2E_1} \int \frac{d^3 p_2}{(2\pi)^3 2E_2} \frac{d^3 p_3}{(2\pi)^3 2E_3} \frac{d^3 p_4}{(2\pi)^3 2E_4} |\mathcal{M}_{12 \rightarrow 34}|^2 \\ &\times (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_4) \\ &\times \left[f_3^{gas} f_4^{gas} f_1^c (1 + f_2^{gas}) - \frac{1}{2} f_1^c f_2^{gas} (1 + f_3^{gas})(1 + f_4^{gas}) \right]\end{aligned}$$

$$f^c = (2\pi)^3 n_c \delta^{(3)}(\vec{p})$$

$$\int \frac{d^3 p_1}{(2\pi)^3} \partial_t f_1^c = \frac{\partial n_c}{\partial t} = R_c^{gain} - R_c^{loss}$$

A small phase space volume competes a δ function.

Transport Equation for BEC

Whether the condensation occurs?

$$R_c^{gain} = \frac{n_c}{(4\pi)^3} \int dp_3 dp_4 f_3 f_4 (1 + f_2) \frac{p_3 p_4}{E_3 E_4} E \left\{ \frac{|\mathcal{M}_{34 \rightarrow 12}|^2}{s} \right\}_{E - \sqrt{p^2 + m^2} = m}$$

$$E = E_3 + E_4, \quad p = |\vec{p}_3 + \vec{p}_4|, \quad s = E^2 - p^2$$

m : particle mass

The kinematic constraint $E - \sqrt{p^2 + m^2} = m$ leads to $s = 2Em$.

For massless particles $s = 0$, i.e., \vec{p}_3 and \vec{p}_4 are parallel.

$\left\{ \frac{|\mathcal{M}_{34 \rightarrow 12}|^2}{s} \right\}_{s=0}$ can be zero, infinity, or has a finite value, which depends on the form of the scattering matrix.

Transport Equation for BEC

Whether the condensation occurs?

$$F = \left\{ \frac{|\mathcal{M}_{34 \rightarrow 12}|^2}{s} \right\}_{s=0} = ?$$

Case 1: interactions with the isotropic distribution of the collision angle.

$$|\mathcal{M}_{34 \rightarrow 12}|^2 \sim s\sigma, \quad F \text{ can be a finite value, if the cross section is not diverge at } s = 0.$$

Case 2: interactions with the pQCD cross section of gluons

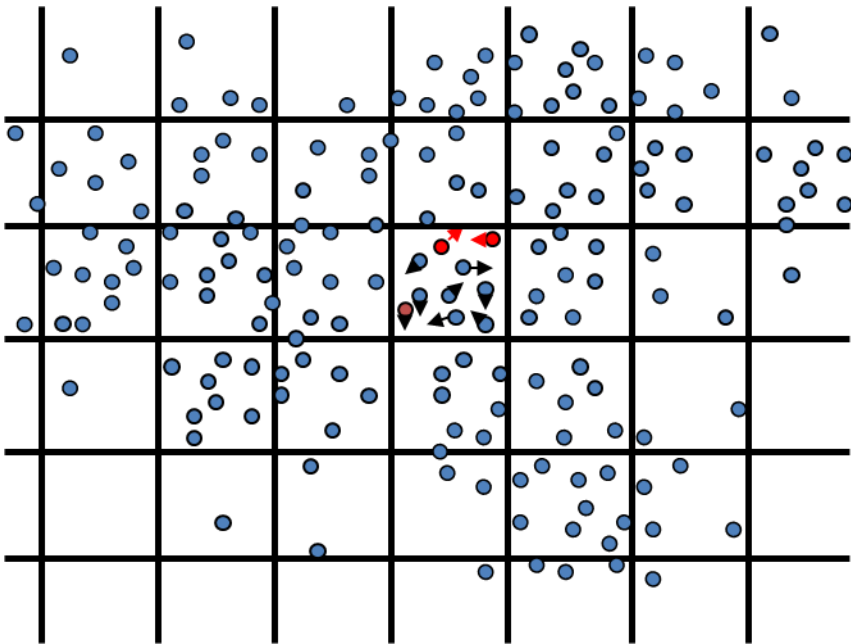
$$|\mathcal{M}_{34 \rightarrow 12}|^2 \sim \frac{s^2}{t^2} \approx \frac{s^2}{(t - m_D^2)^2}, \quad F = 0 \text{ at } s = 0.$$

$$|\mathcal{M}_{34 \rightarrow 12}|^2 \sim \frac{s^2}{t^2} \approx \frac{s^2}{t(t - m_D^2)}, \quad F \text{ has a finite value at } s = 0.$$

Bose statistics in BAMPS

BAMPS: Boltzmann Approach of MultiParton Scatterings
solves the **semi-classical**, **relativistic** Boltzmann equation
in the framework of **pQCD** by **Monte Carlo** simulations.

ZX and C. Greiner, PRC 71, 064901 (2005)



$$\left(\partial_t + \frac{\vec{p}_1}{E_1} \cdot \vec{\nabla} \right) f_1(x, p_1) = C$$

test particle representation of f

stochastic interpretation of the
collision rates

Bose statistics in BAMPS

Collision scheme: stochastic method

$$\text{For } \begin{array}{l} g + g \rightarrow g + g \\ g + c \rightarrow g + g \end{array} \quad P_{22} = v_{rel} \frac{\sigma'_{22}}{N_{test}} \frac{\Delta t}{\Delta V}$$

$$\begin{aligned} \sigma'_{22} &= \frac{1}{4s} \int \frac{d^3 p_1}{(2\pi)^3 2E_1} \frac{d^3 p_2}{(2\pi)^3 2E_2} |\mathcal{M}_{34 \rightarrow 12}|^2 (2\pi)^4 \delta^{(4)}(\dots) (\mathbf{1} + \mathbf{f}_1)(\mathbf{1} + \mathbf{f}_2) \\ &= \int d\Omega \frac{d\sigma_{22}}{d\Omega} (\mathbf{1} + \mathbf{f}_1)(\mathbf{1} + \mathbf{f}_2) = \int d\Omega \frac{d\sigma'_{22}}{d\Omega} \end{aligned}$$

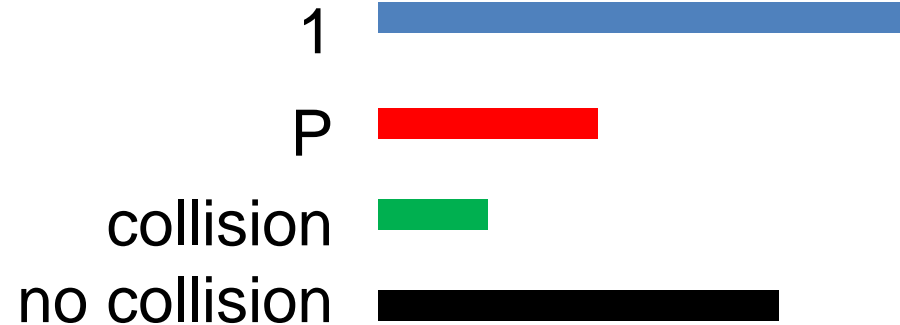
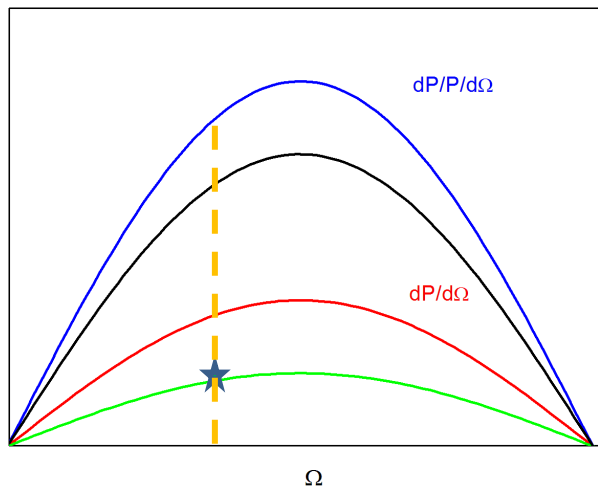
used by Scardina, Perricone, Plumari, Ruggieri, Greco, arXiv:1408.1313

Disadvantages:

1. Uncertainties from the extraction of f , and the integration in σ'_{22}
2. Time consuming due to the integrations in σ'_{22} for each particle pairs, whether or not they collide.

Bose statistics in BAMPS

$$\frac{dP_{22}}{d\Omega} = v_{rel} \frac{1}{N_{test}} \frac{d\sigma'_{22}}{d\Omega} \frac{\Delta t}{\Delta V}$$



New Scheme:

1. sample first the collision angle Ω
2. sample a random number between 0 and $dP/P / d\Omega$. If this number is smaller than $dP/d\Omega$, a collision occurs.

Bose statistics in BAMPS

For $g + g \rightarrow g + c$ $P_{22} = v_{rel} \frac{\sigma_{22}^c}{N_{test}} \frac{\Delta t}{\Delta V}$

$$\begin{aligned} \sigma_{22}^c &= \frac{1}{2s} \int \frac{d^3 p_1}{(2\pi)^3 2E_1} \frac{d^3 p_2}{(2\pi)^3 2E_2} |\mathcal{M}_{34 \rightarrow 12}|^2 (2\pi)^7 \delta^{(4)}(\dots) n_c \delta^{(3)}(\vec{p}_1) (1 + f_2) \\ &= \frac{\pi}{2} n_c (1 + f_2) \frac{1}{p} \frac{|\mathcal{M}_{34 \rightarrow 12}|^2}{s} \delta[(E - p)^2] \end{aligned}$$

Approximation:

Particles with $p < \varepsilon$ are regarded as condensate particles.

ε should be small to reduce the numerical uncertainty.

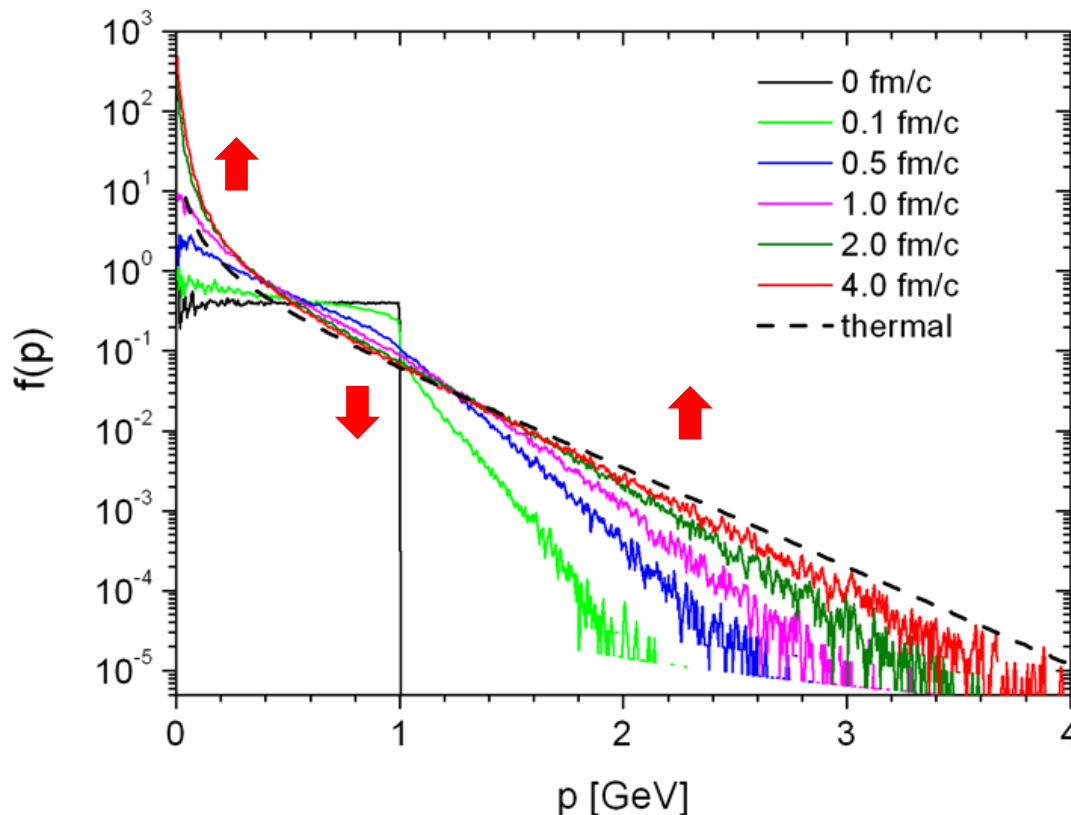
The rate for producing BEC does not depend on ε .

$$\begin{aligned} \delta^{(3)}(\vec{p}_1) &\approx \frac{\theta(\varepsilon - p_1)}{4\pi p_1^2 \varepsilon} \quad \Rightarrow \quad \sigma_{22}^c \approx \frac{\pi}{2} n_c (1 + f_2) \frac{1}{p} \frac{|\mathcal{M}_{34 \rightarrow 12}|^2}{s} \\ &\quad \times \frac{1}{4\varepsilon} \left(\frac{2}{E - p} - \frac{1}{\varepsilon} \right) \theta \left(\varepsilon - \frac{E - p}{2} \right) \end{aligned}$$

Recent results

$$|\mathcal{M}_{34 \rightarrow 12}|^2 \approx (12\pi)^2 \alpha_s^2 \frac{s^2}{t(t - m_D^2)}, \quad m_D^2 = 16\pi N_c \alpha_s \int \frac{d^3p}{(2\pi)^3} \frac{1}{p} f^{gas}$$

Onset of BEC: without $g + g \rightarrow g + c$ (BUT, there are still particles with energy smaller than ε .)



$$f_0 = 0.4$$

The distribution is frozen out at **4 fm/c** (onset of BEC happens earlier) and is far from the equilibrium one.

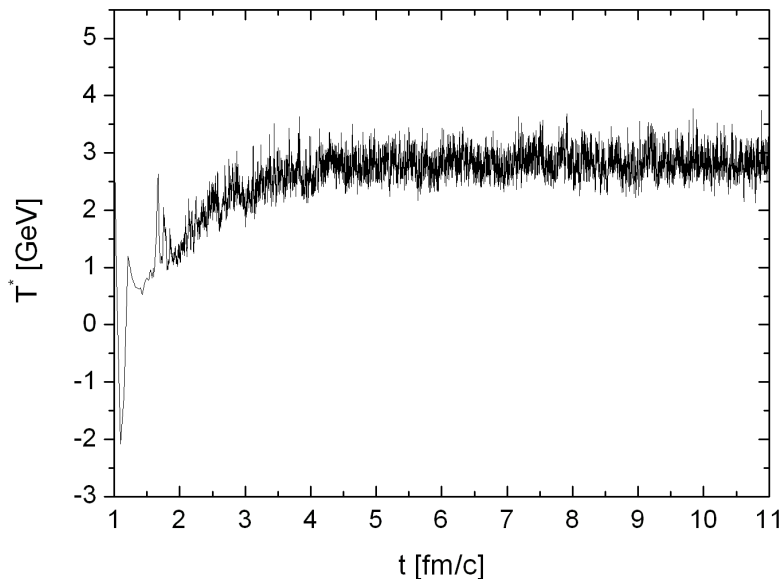
Recent results

Onset of BEC: without $g + g \rightarrow g + c$

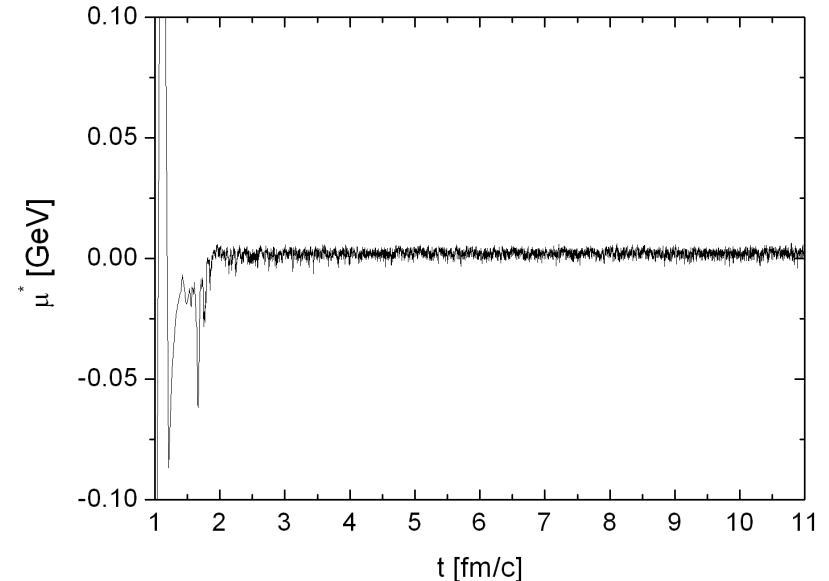
Assume: for small p ,

$$f \approx \frac{1}{\exp\left(\frac{p - \mu^*}{T^*}\right) - 1} \approx \frac{T^*}{p - \mu^*}$$

effective temperature

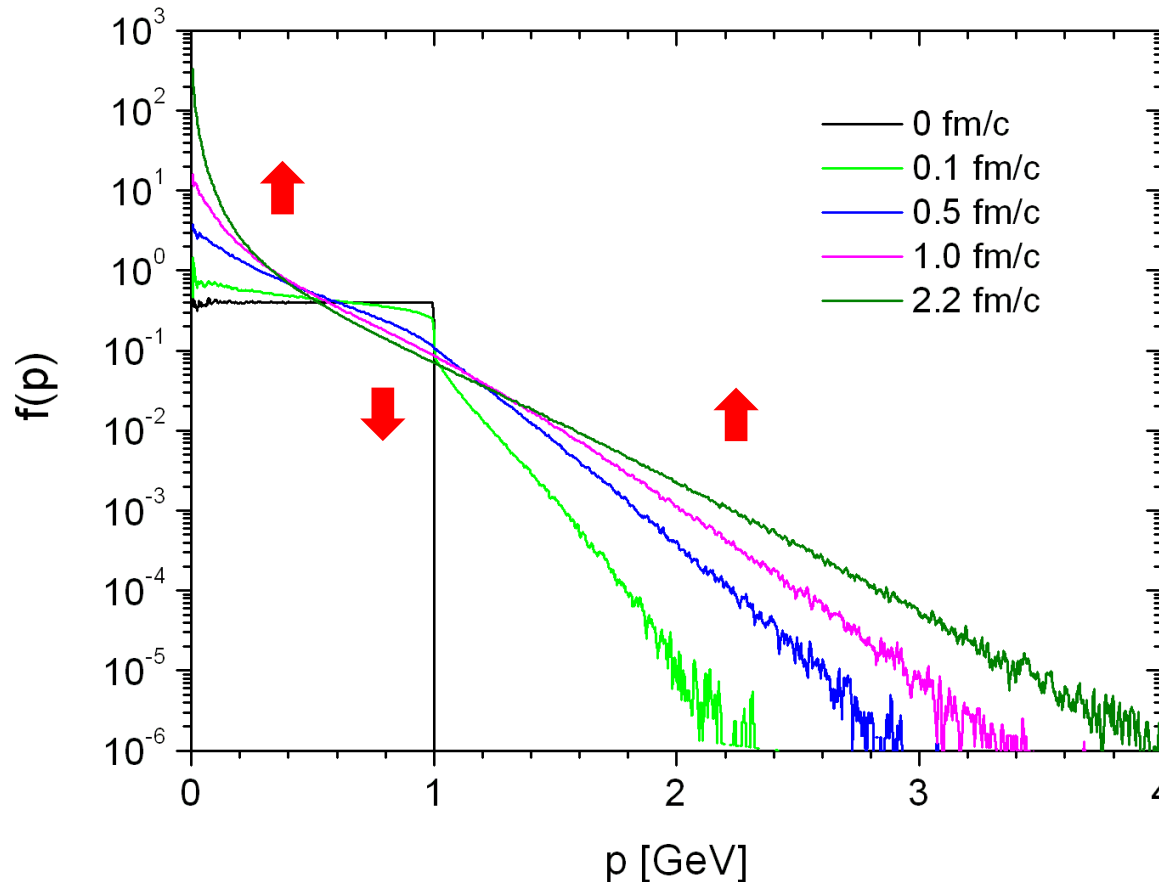


effective chemical potential



Recent results

BE Condensation: with $g + g \rightarrow g + c$, when μ^* is sufficiently close to zero and is **positive** due to numerical fluctuation.

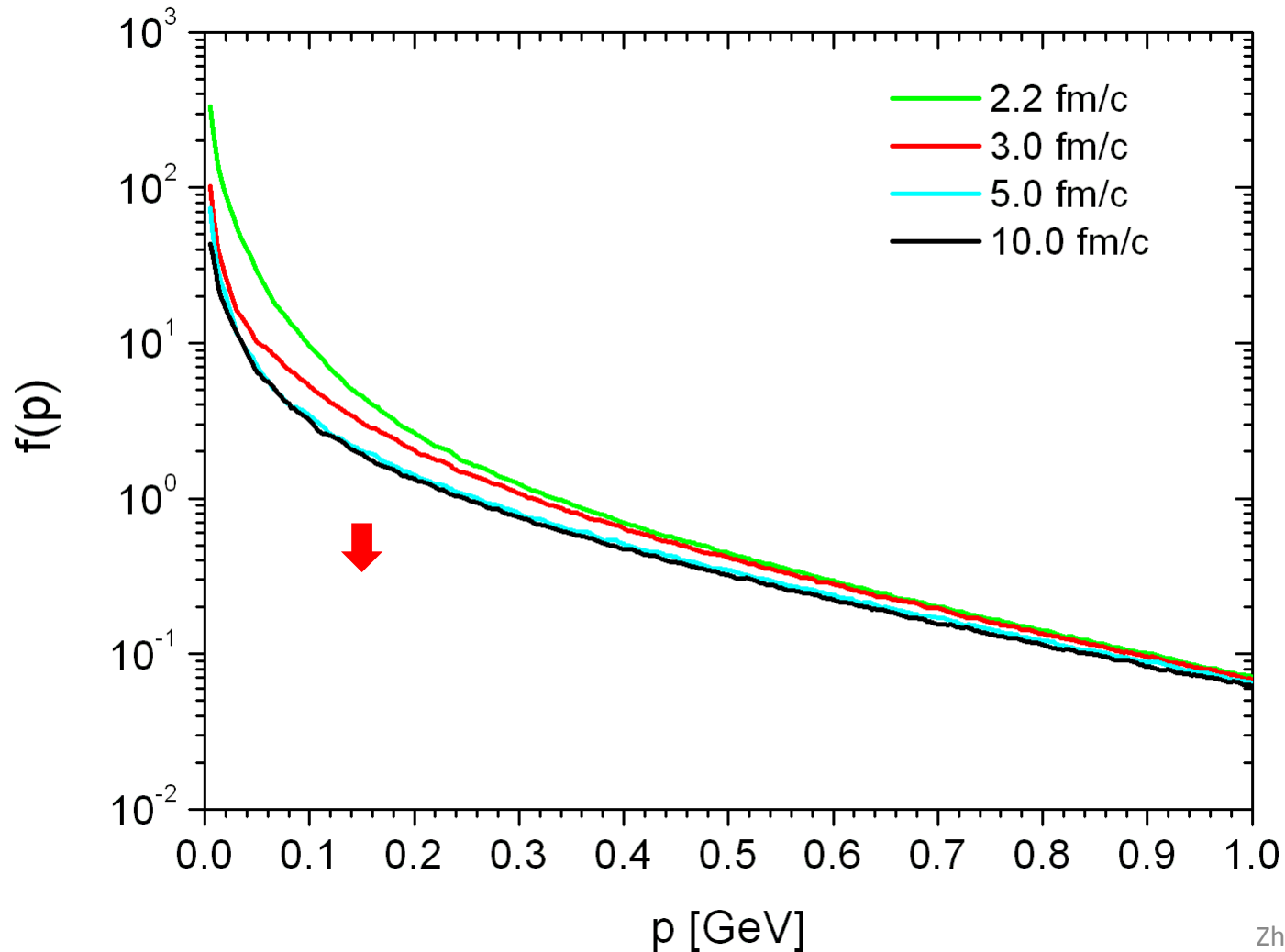


$$f_0 = 0.4$$

The condensation begins at **1.6 fm/c**. The distribution at small p is increasing until **2.2 fm/c**.

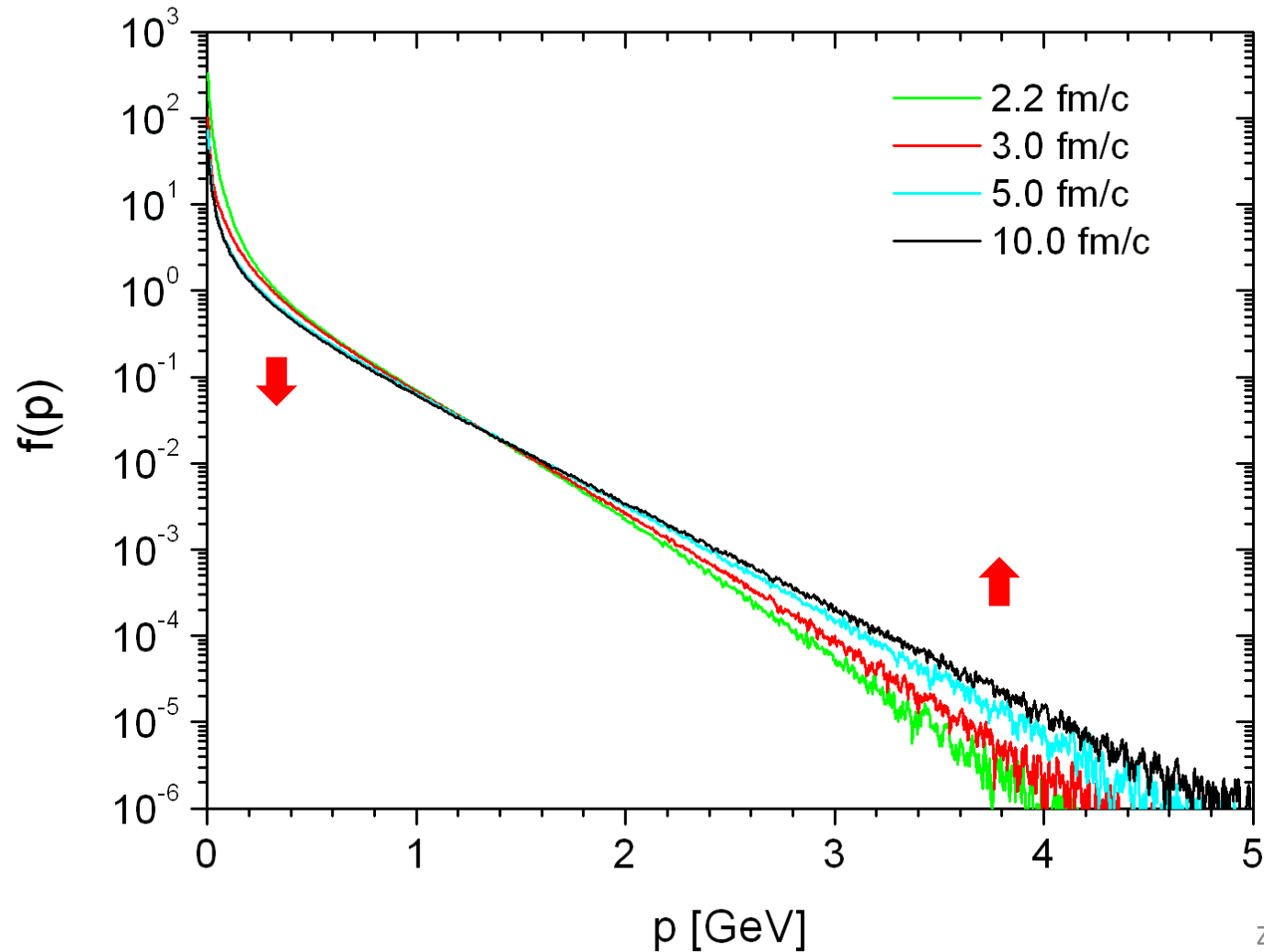
Recent results

Thermalization with BE Condensation



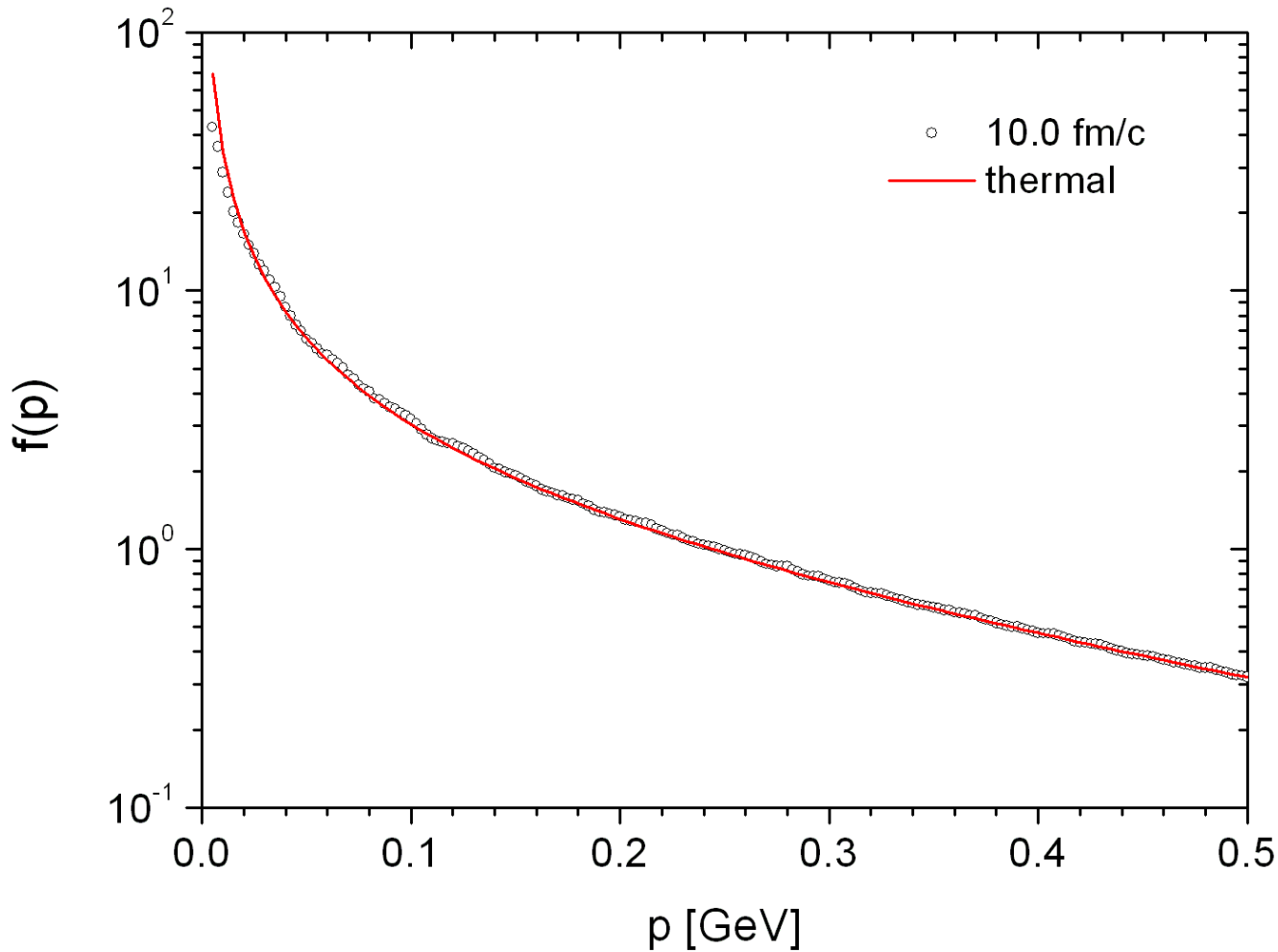
Recent results

Thermalization with BE Condensation



Recent results

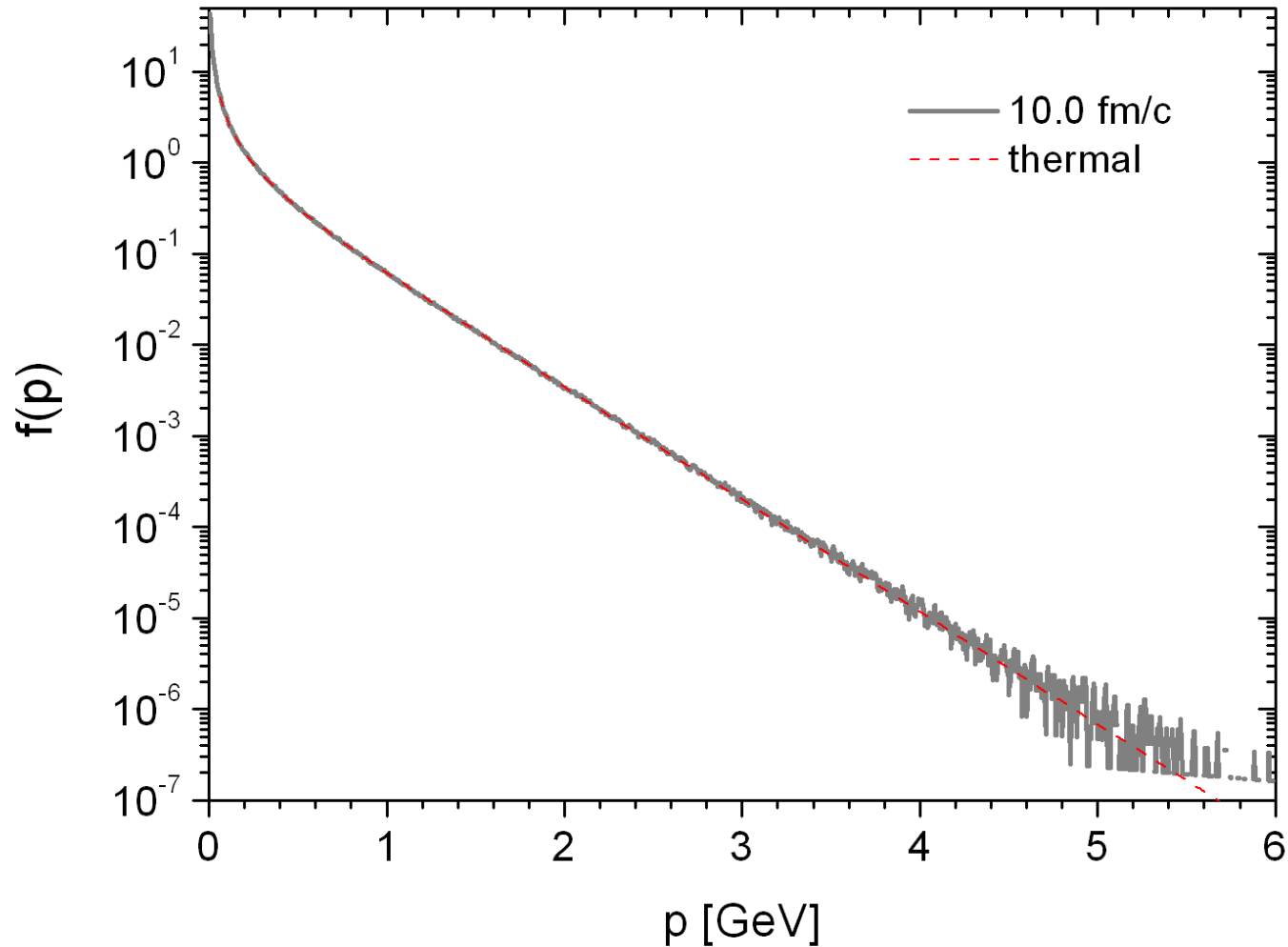
Thermalization with BE Condensation



perfect agreement !

Recent results

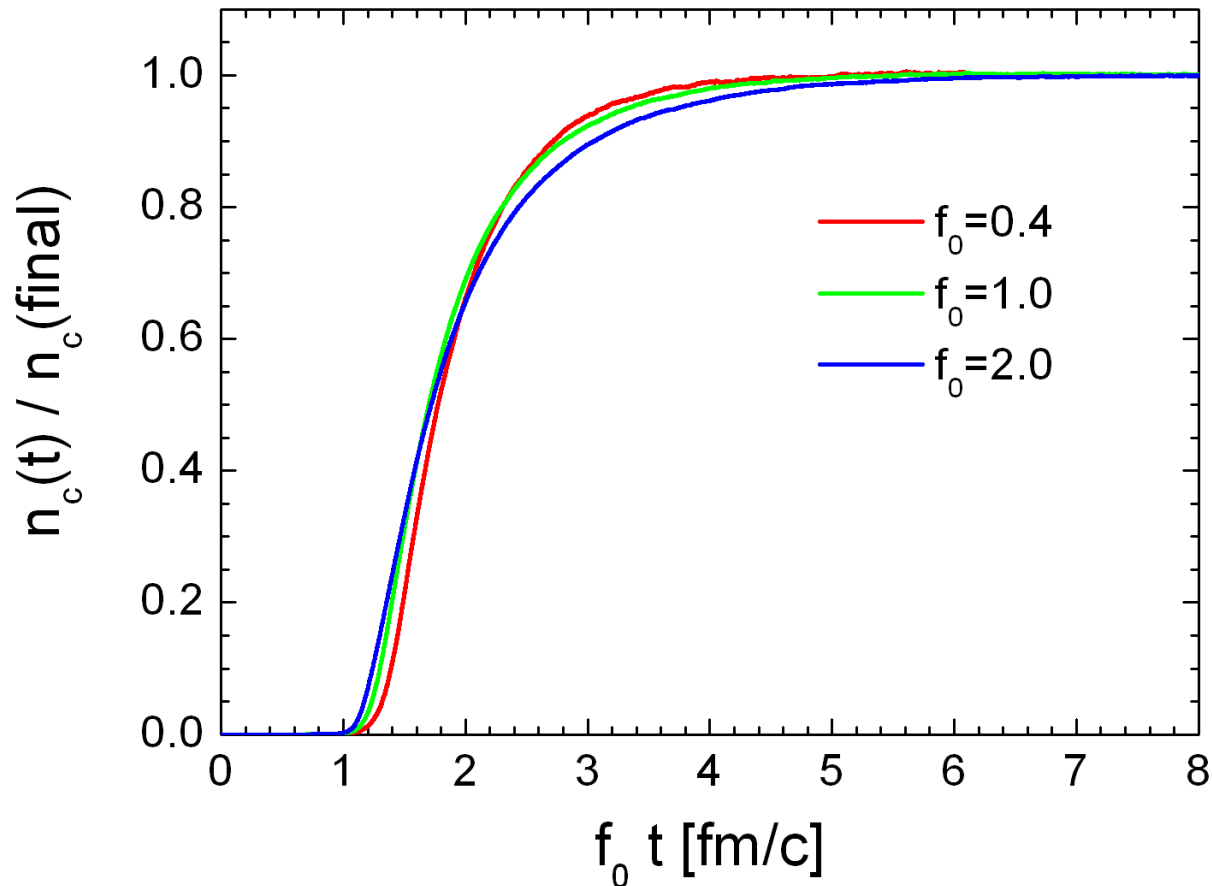
Thermalization with BE Condensation



perfect
agreement !

Recent results

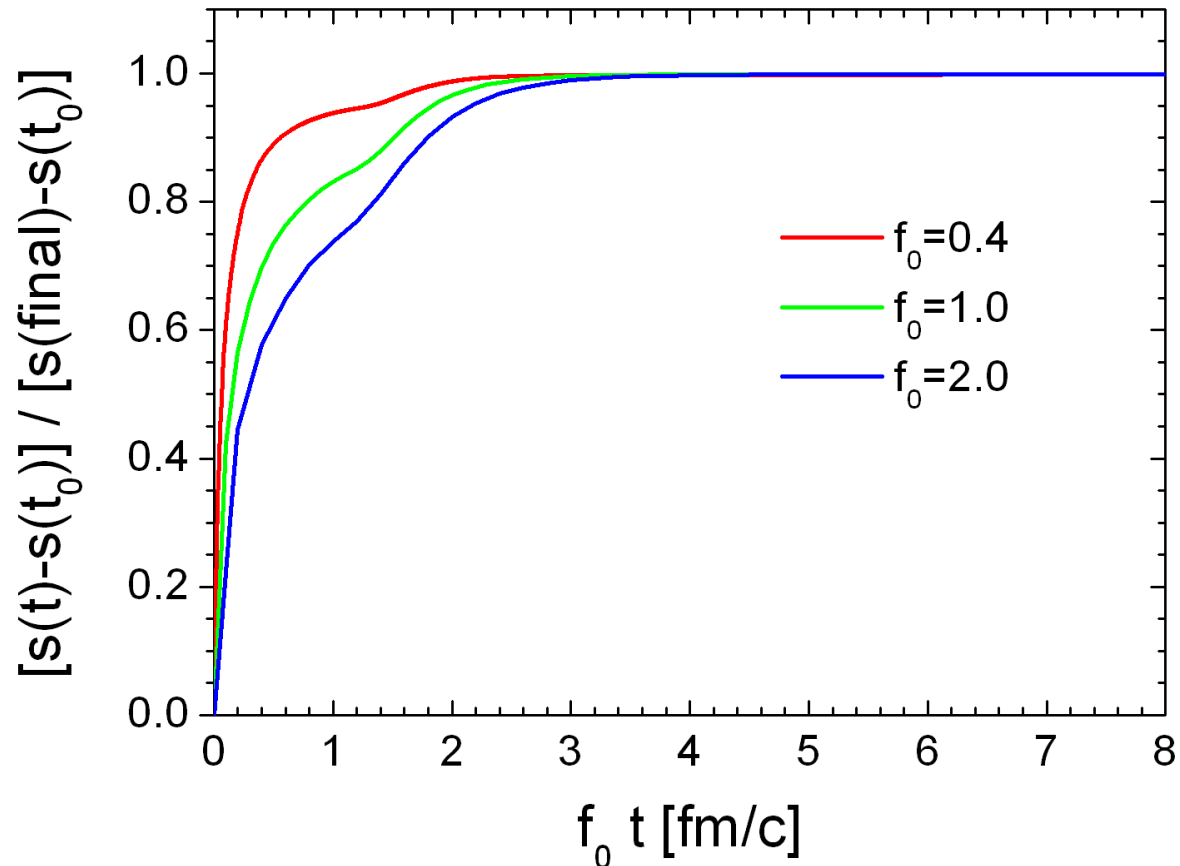
Scaling behaviour of BE condensation



BEC completion
at $f_0 t \approx 6$ fm/c

Recent results

Scaling behaviour of entropy production **during** BE condensation

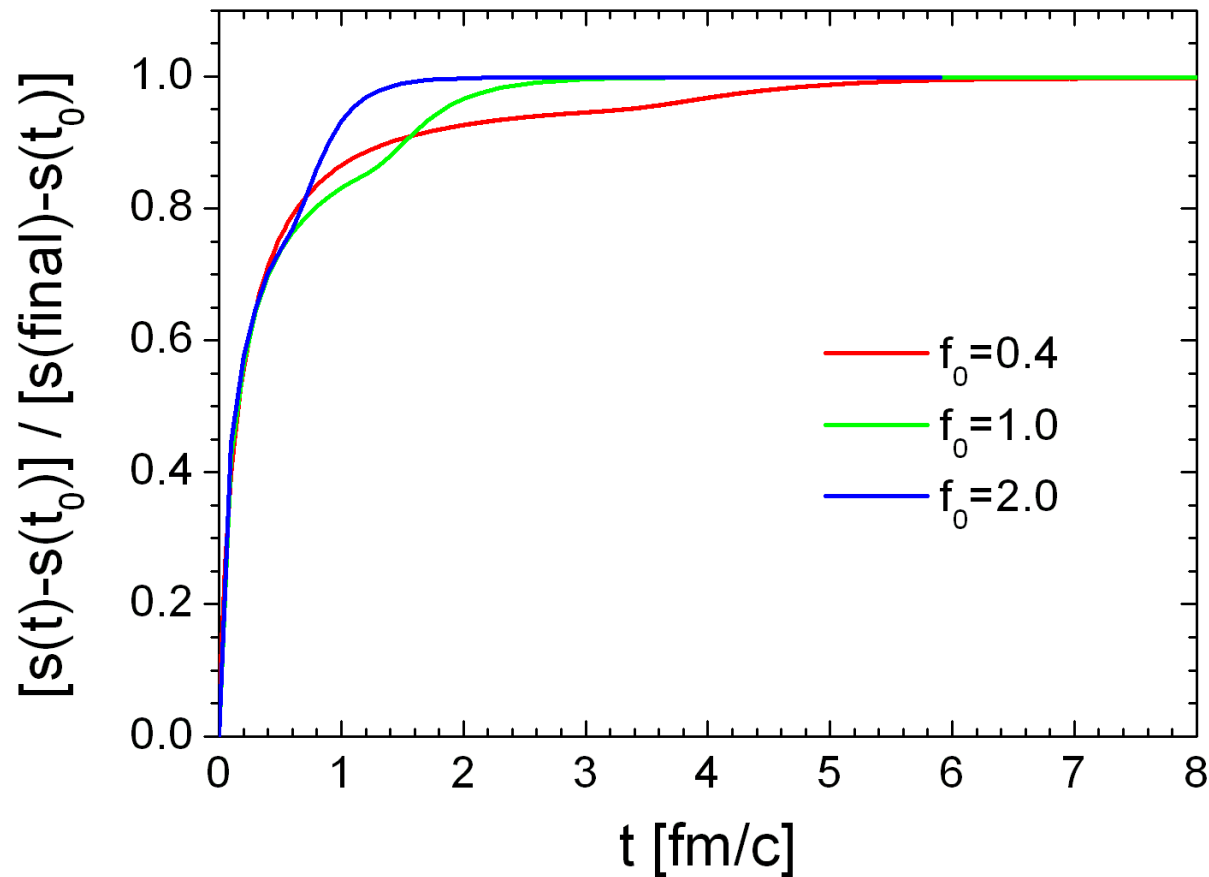


Two step entropy production

Full thermalization
at $f_0 t \approx 3 \text{ fm/c}$

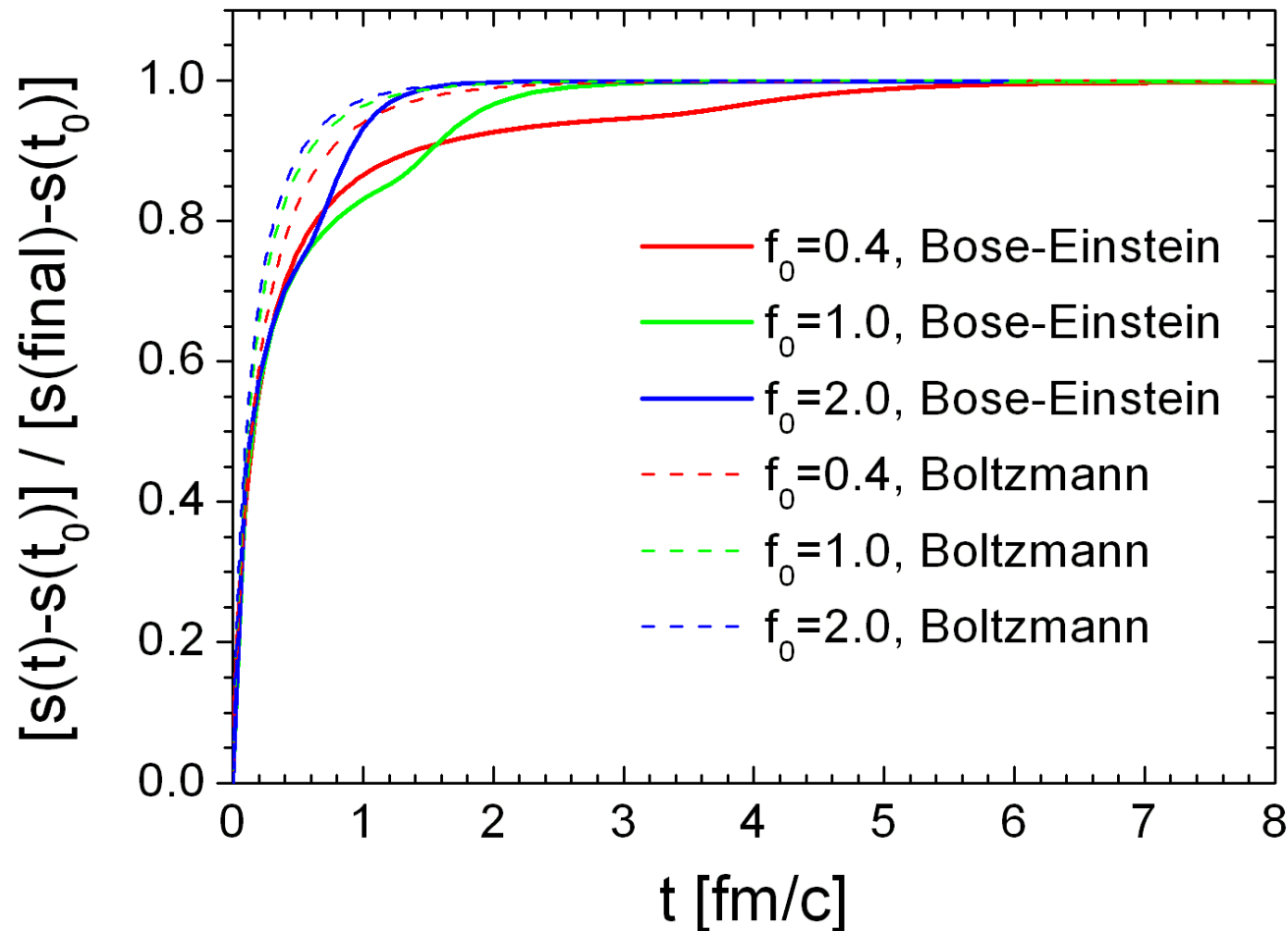
Recent results

Entropy production **before** BE condensation



Recent results

BE condensation **slows down** the entropy production.



Summary and Outlook

- show the onset and full process of BE condensation in BAMPS
- BE condensation is complete at $f_0 t \approx 6 \text{ fm}/c$
- almost full thermalization at $f_0 t \approx 3 \text{ fm}/c$
- BE condensation slows down the entropy production.

Further studies:

Influence of momentum anisotropy, 2 \leftrightarrow 3 processes, quarks, and expansion on BE condensation

Measurable effects

Backup

Transport Equation for BEC

$$R_c^{gain} = \frac{n_c}{(4\pi)^3} \int dp_3 dp_4 f_3 f_4 (1 + f_2) \frac{p_3 p_4}{E_3 E_4} E \left\{ \frac{|\mathcal{M}_{34 \rightarrow 12}|^2}{s} \right\}_{s=0}$$

$$R_c^{loss} = \frac{n_c}{(4\pi)^3} \int dp_3 dp_4 (1 + f_3)(1 + f_4) f_2 \frac{p_3 p_4}{E_3 E_4} E \left\{ \frac{|\mathcal{M}_{12 \rightarrow 34}|^2}{s} \right\}_{s=0}$$

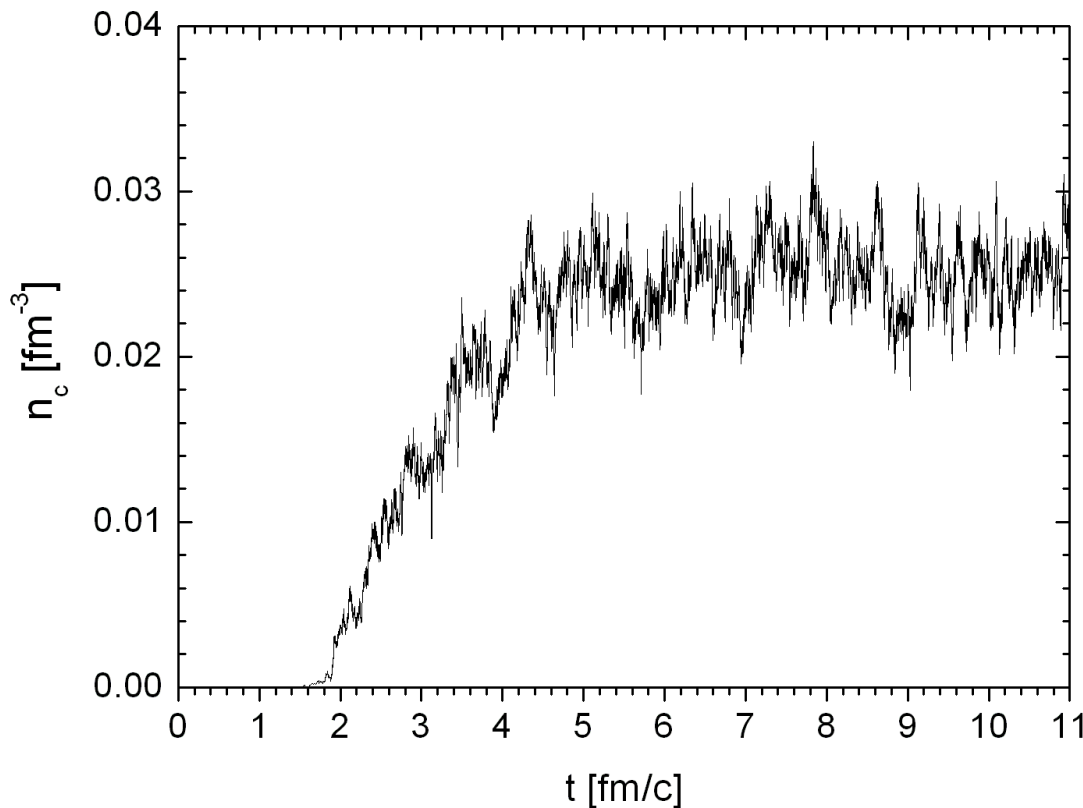
- $\frac{\partial n_c}{\partial t} \sim n_c$ need an initial seed for the condensation
- At thermal equilibrium, $f_{2,3,4} = \frac{1}{\exp\left(\frac{p_{2,3,4}}{T}\right) - 1}$

In this case $R_c^{gain} - R_c^{loss} = 0$, but, both R_c^{gain} and R_c^{loss} are divergent.

The divergence is logarithmically with a lower cutoff ε for p .

Recent results

Onset of BEC: without $g + g \rightarrow g + c$ (BUT, there are still particles with energy smaller than ε .)



$$\varepsilon = 0.0025 \text{ GeV}$$

Density of particles with energy smaller than ε , about 1% of the density of the real condensate Particles ($n_c = 2.98 \text{ fm}^{-3}$).

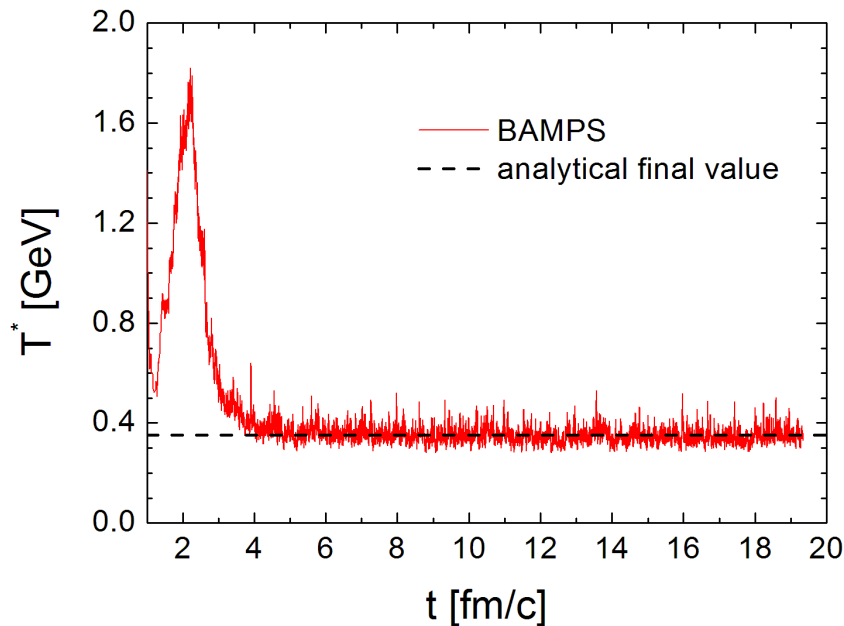
Recent results

BEC: with $g + g \rightarrow g + c$

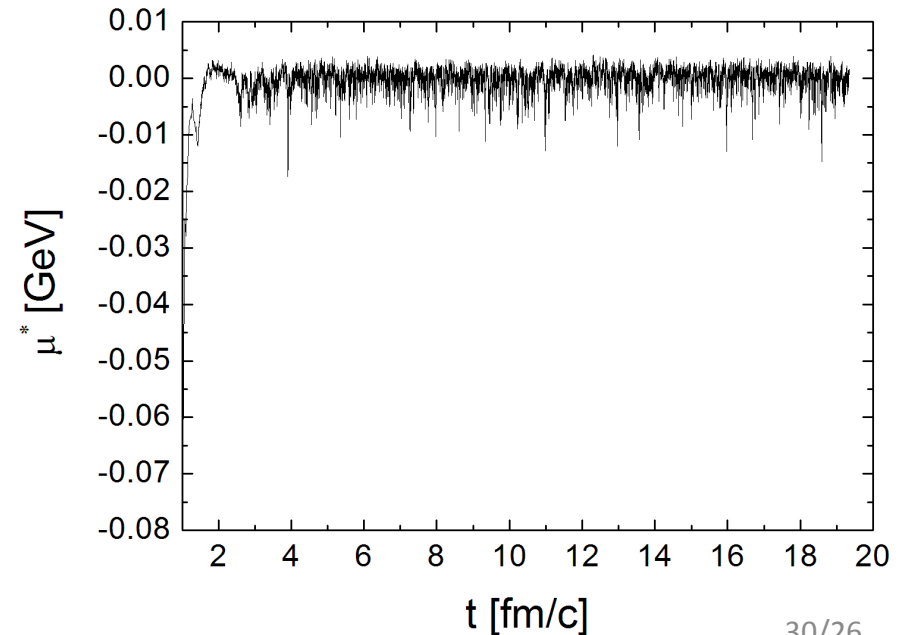
Assume: for small p ,

$$f \approx \frac{1}{\exp\left(\frac{p - \mu^*}{T^*}\right) - 1} \approx \frac{T^*}{p - \mu^*}$$

effective temperature

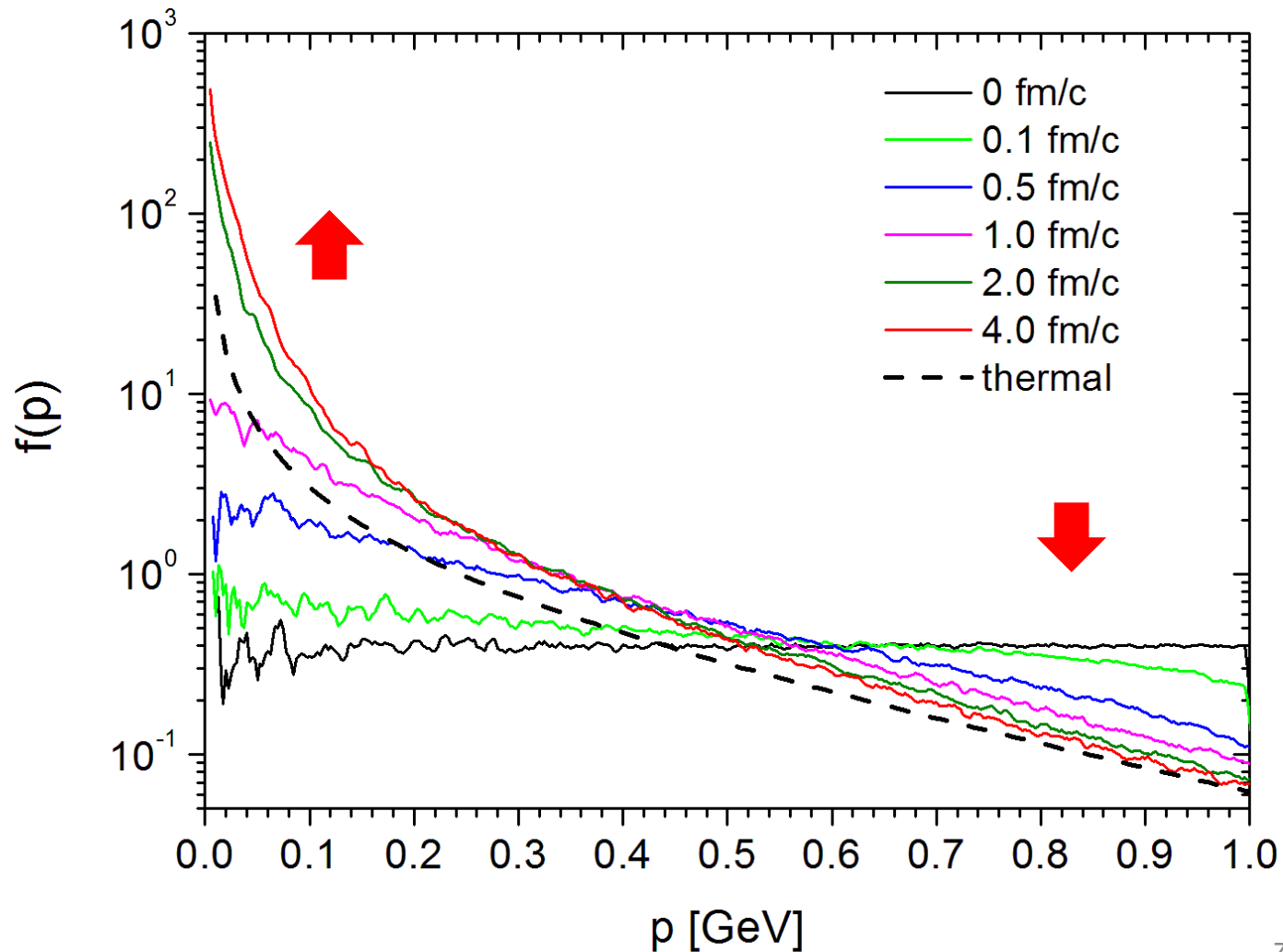


effective chemical potential



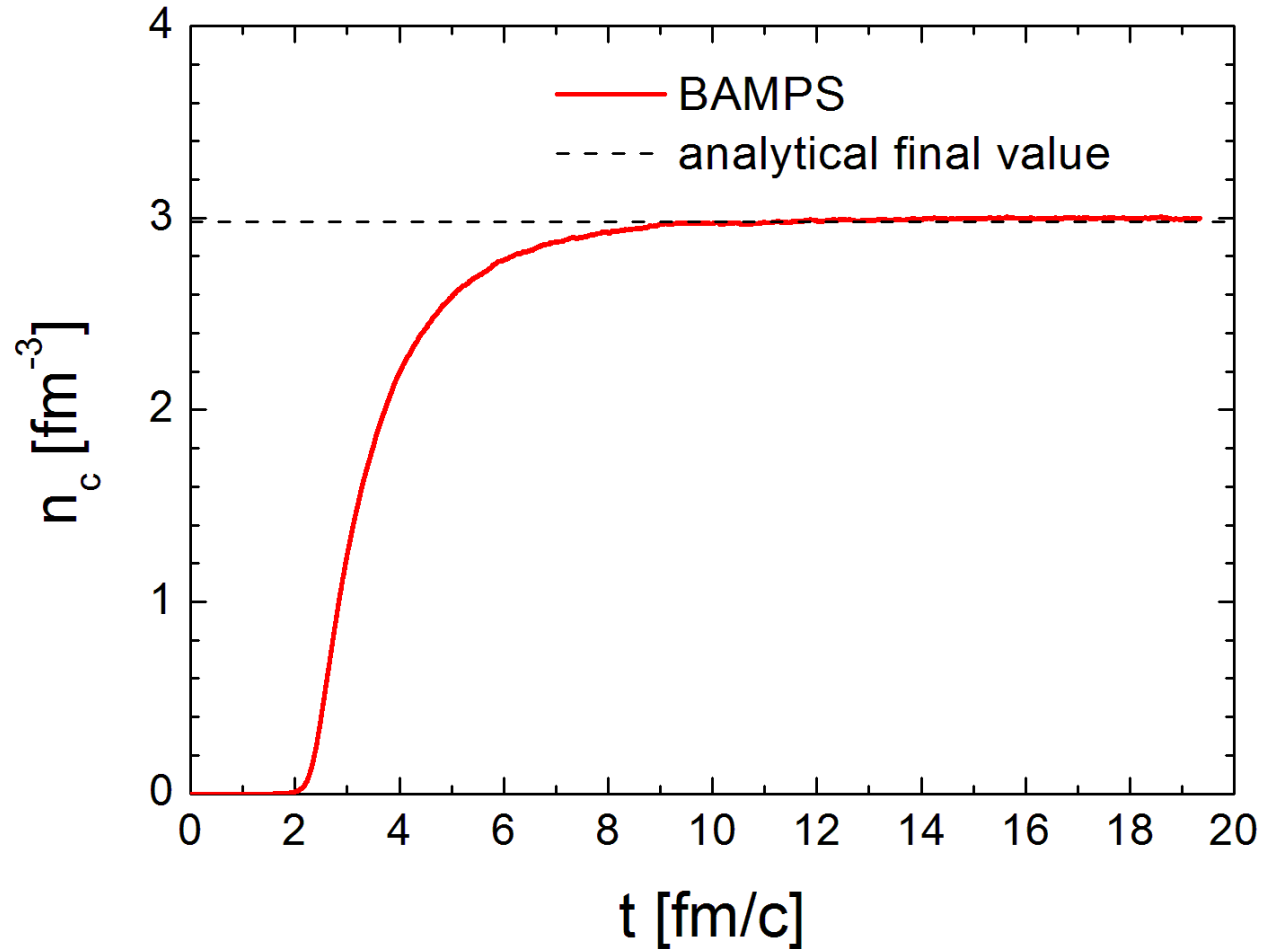
Recent results

Onset of BEC: without $g + g \rightarrow g + c$



Recent results

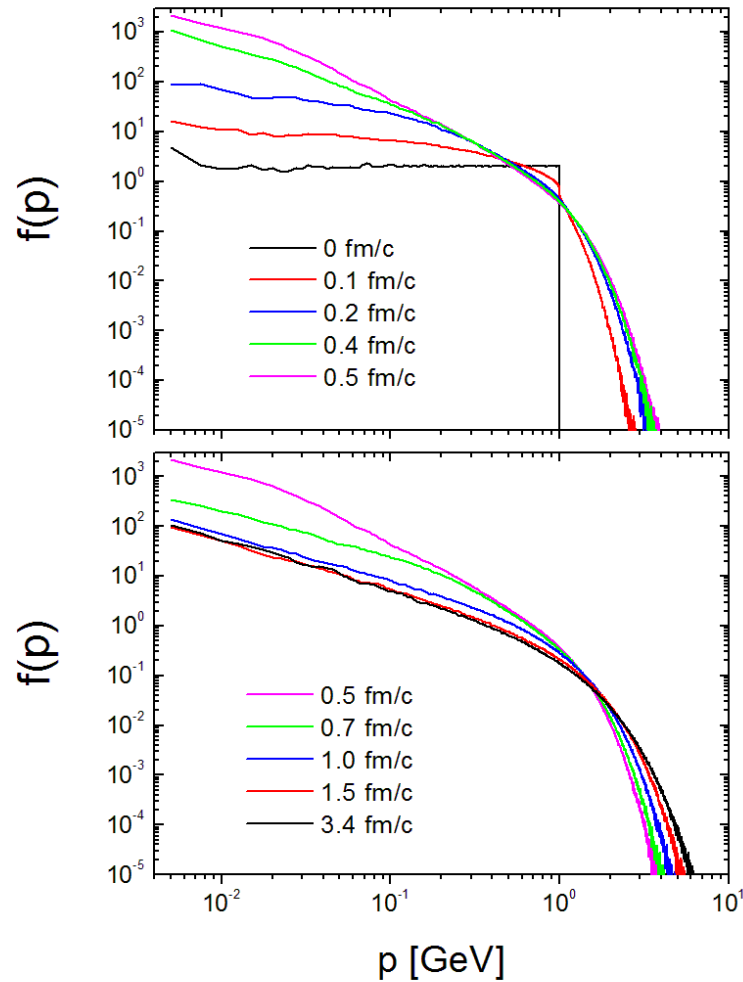
BE Condensation is complete at 10 fm/c for $f_0 = 0.4$.



perfect agreement !

Recent results

Thermalization with BE Condensation ($f_0 = 2$)



Recent results

Computation setups:

Box with volume: $3 \text{ fm} \times 3 \text{ fm} \times 3 \text{ fm}$

Cells with volume: $0.125 \text{ fm} \times 0.125 \text{ fm} \times 0.125 \text{ fm}$

Number of test particles per each real particle: $N_{test} = 6000$

Total particle number in computation: 2.23×10^6

MPI: 216 CPUs

Tests:

new scheme

collision rates

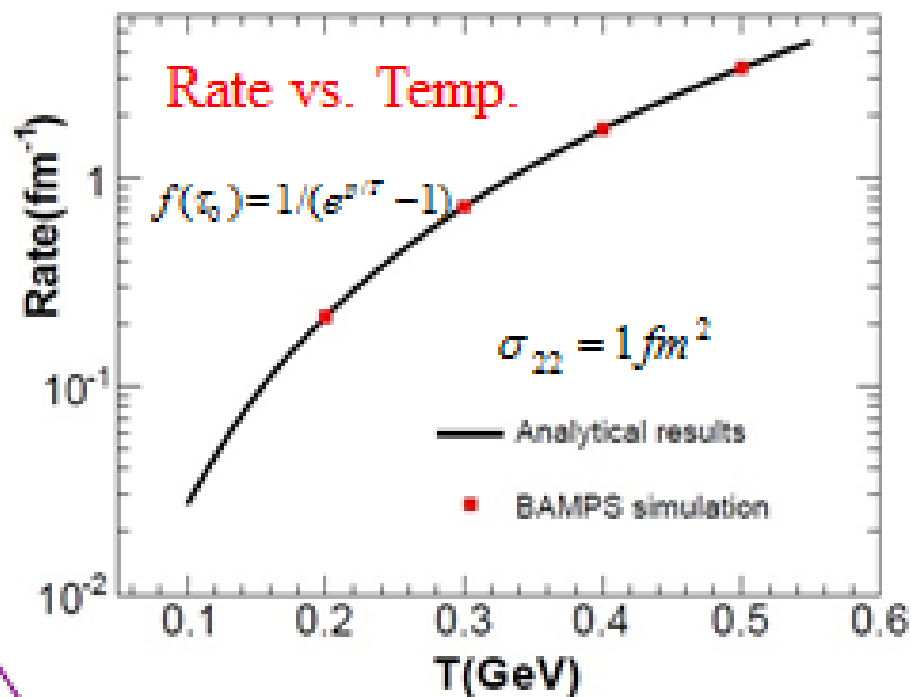
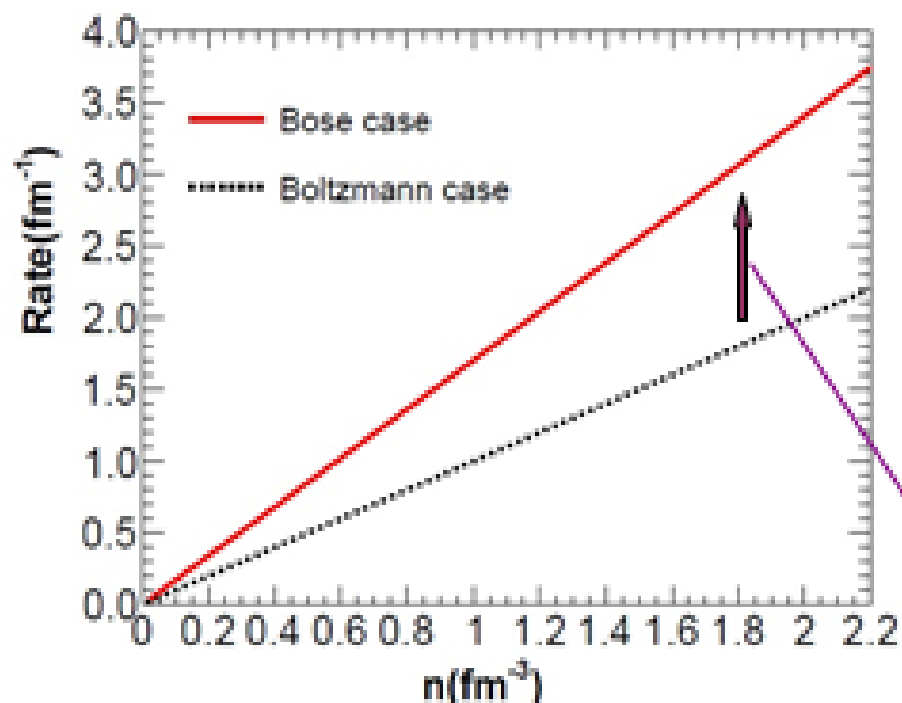
case of underpopulated gluons (negative chemical potential)

critical case (zero chemical potential and no condensation)

Transport model: Reaction Rate test

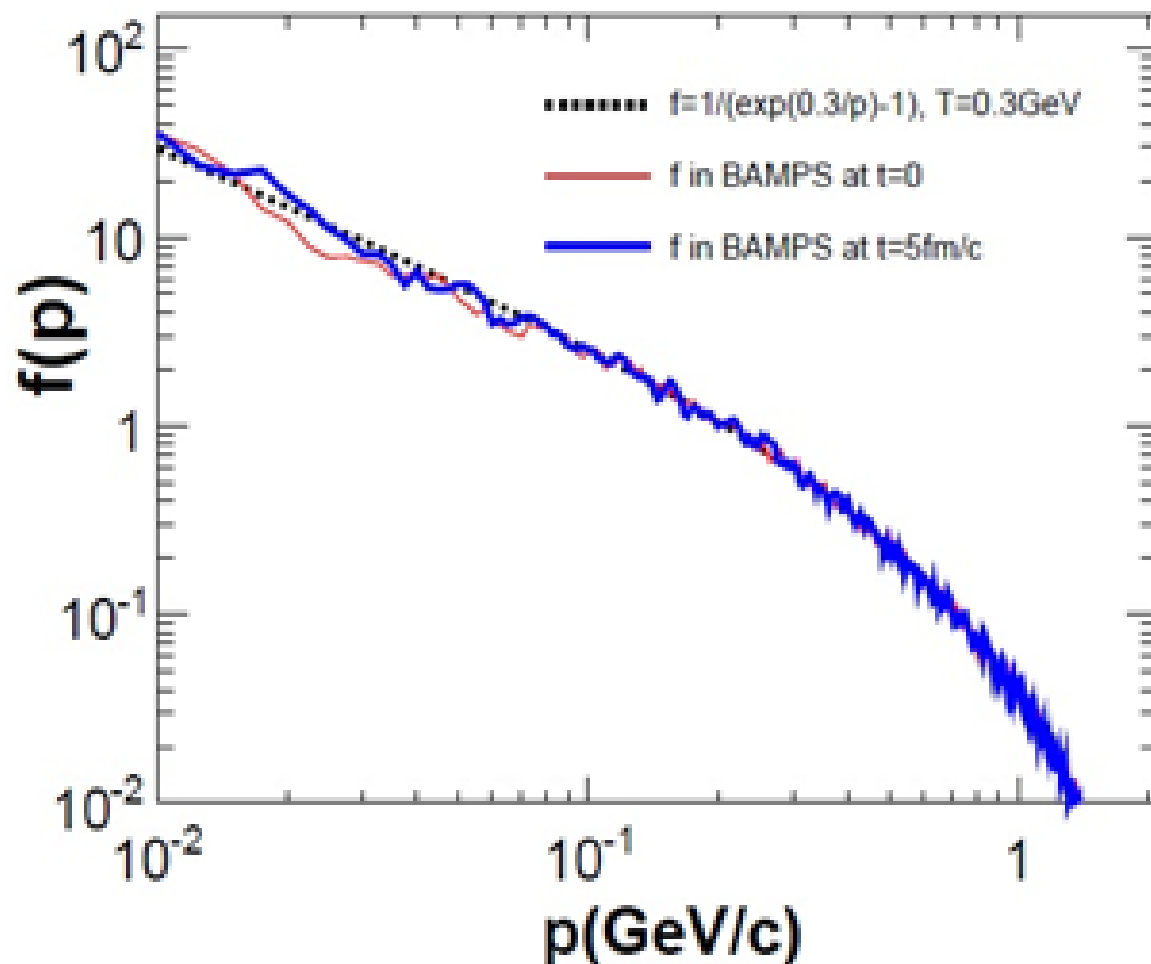
$$Rate = n \cdot \langle v_{rel} \sigma_{22} \rangle_{medium} = \frac{1}{n} \int \frac{d^3 \vec{p}_1}{(2\pi)^3} \frac{d^3 \vec{p}_2}{(2\pi)^3} f_1 f_2 \int d\Omega^* \frac{d\sigma_{22}}{d\Omega^*} v_{rel} (1 + f_3)(1 + f_4)$$

$$= N_{coll} / (t \cdot N_g / 2)$$



Bose Enhancement effect

Transport model: Equilibrium Test

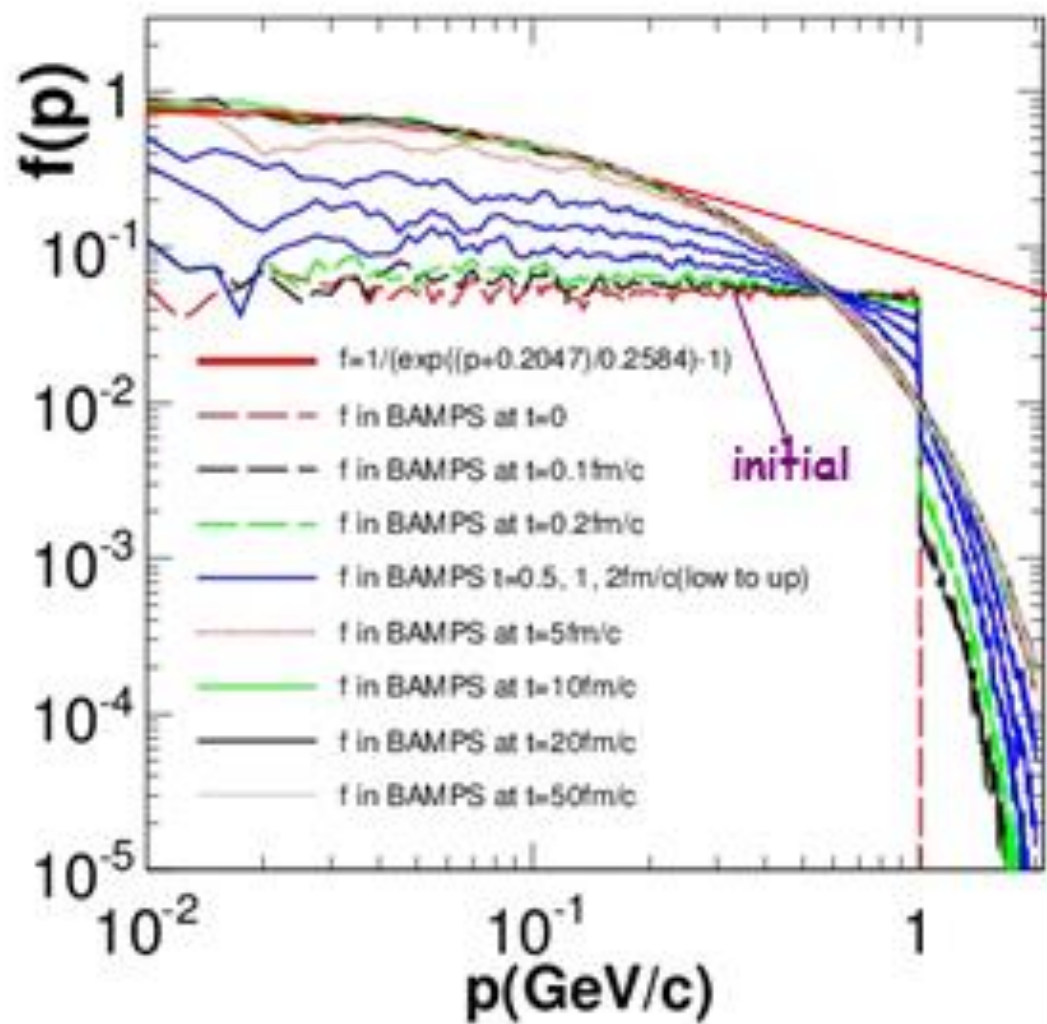


$$f(\tau_0) = 1/(e^{p/T} - 1)$$

$$T = 300\text{MeV}$$

within acceptable fluctuation, the equilibrium state can be maintained
<-> detailed balance ok and BE distribution is fix-point of the system from simulation

$f_0 = 0.05$ simulation results



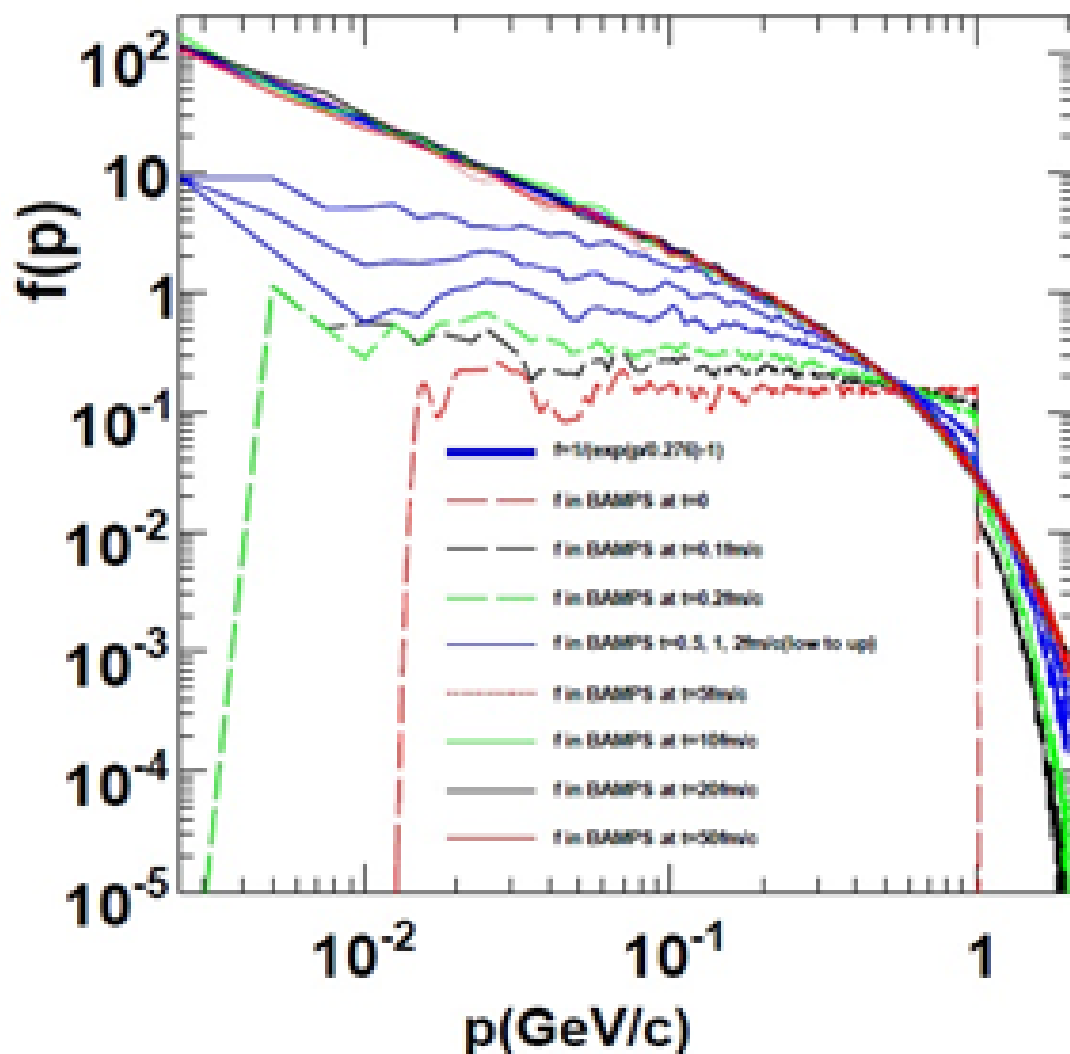
$3 \times 3 \times 3 \text{ fm}^3$ static box

$$f_0 = 0.05 < f_{critical}$$

$$\left\{ \begin{array}{l} T_{eq} = 0.258 \text{ MeV} \\ \mu_{eq} = -0.205 \text{ MeV} \end{array} \right.$$

the system thermalizes to the fix-point: thermal BE distribution

$f_0 = 0.154$ simulation results



$3 \times 3 \times 3 \text{ fm}^3$ static box

$$f_0 = 0.154 = f_{critical}$$

$$T_{eq} = 0.276 \text{ MeV}$$

$$\mu_{eq} = 0 \text{ MeV}$$

the system
thermalizes
to
the fix-point:
thermal BE
distribution