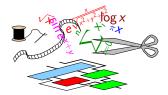
# Arithmetic for FPGAs (all the operators you will never see in a processor)

#### Florent de Dinechin







## Outline

Introduction: FPGAs for computing?

The FloPoCo project

One example of operator fusion

One example of operator specialization

One example of open-ended operator

Conclusion

# Introduction: FPGAs for computing?

#### Introduction: FPGAs for computing?

The FloPoCo project

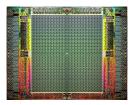
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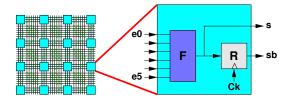
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## Finest Programmable Granularity Around



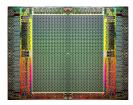


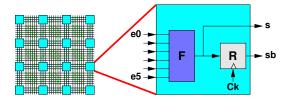
#### Field-Programmable Gate Array (FPGA)

Mass-produced chips, universally programmable, but:

- programming model: the circuit
- granularity: the bit

# Finest Programmable Granularity Around





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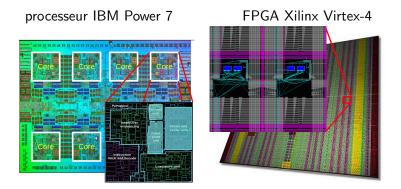
Mass-produced chips, universally programmable, but:

- programming model: the circuit
- granularity: the bit

#### Applications

- Rapid prototyping of VLSI circuits
- Small series, where a dedicated chip would be too expensive
- Universal computing accelerators?

Here are two universally programmable chips.



Who's best for (insert your computation here) ?

# Are FPGAs any good at floating-point?

Long ago (1995), people ported the basic operations:  $+,-,\times$ 

- Versus the highly optimized FPU in the processor,
- each operator 10x slower in an FPGA

This is the inavoidable overhead of programmability.

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If you lose according to a metric, change the metric.

Peak marketing lies for double-precision floating-point exponential:

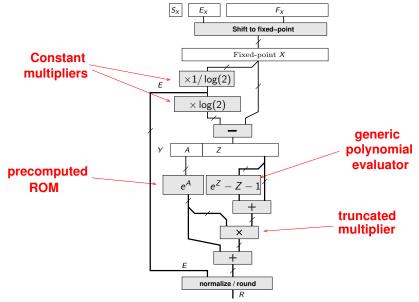
- AVX core: 40 cycles / 4 DPExp @ 4GHz: 400 MDPExp/s
- FPExp in FPGA: 1 DPExp/cycle @ 500MHz: 500 MDPExp/s
- Chip vs chip: 8 Pentium cores vs 200 FPExp/FPGA
- Energy/DPExp also much better on FPGA
- Single precision comparison even better for FPGA

(Intel MKL vector libm, vs FPExp in FloPoCo version 2.0.0)

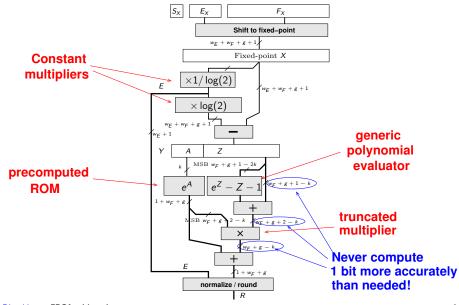
#### SPICE Model-Evaluation, cut from Kapre and DeHon (FPL 2009)

Table 2.      Verilog-AMS Compiler Output						
Models	Instruction Distribution					
	Add	Mult.	Div.	Sqrt.	Exp.	Log
bjt	22	30	17	0	2	0
diode	7	5	4	0	1	2
hbt	112	57	51	0	23	18
jfet	13	31	2	0	2	0
mosl	24	36	7	1	0	0
vbic	36	43	18	1	10	4

#### Custom arithmetic (not your Pentium's)

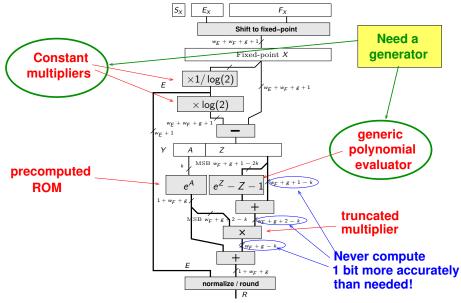


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F. de Dinechin FPGA arithmetic

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 Should a processor include hardware elementary functions ? Yes (Paul&Wilson, 1976), No since the transition to RISC

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- Should a processor include a multiplier by log(2)? No of course.

- Elementary functions ? Yes iff your application needs it
- Divider or square root? Yes iff your application needs it
- Decimal hardware? Yes iff your application needs it
- A multiplier by log(2)? Yes iff your application needs it

In FPGAs, useful means: useful to one application.

Arithmetic operators useful to at least one application:

- Elementary functions (sine, exponential, logarithm...)
- Algebraic functions (  $\frac{x}{\sqrt{x^2 + y^2}}$ , polynomials, ...)
- Compound functions (log<sub>2</sub>(1  $\pm$  2<sup>x</sup>),  $e^{-Kt^2}$ , ...)
- Floating-point sums, dot products, sums of squares
- Specialized operators: constant multipliers, squarers, ...
- Complex arithmetic
- LNS arithmetic
- Decimal arithmetic
- Interval arithmetic

Θ ...

#### What do we call arithmetic operators?

• An arithmetic operation is a *function* (in the mathematical sense)

- few well-typed inputs and outputs
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  - IEEE-754 FP standard: operator(x) = rounding(operation(x))
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The operator as a *circuit*...

- ... is a direct acyclic graph (DAG):
  - easy to build and pipeline
  - easy to test against its mathematical specification

# The FloPoCo project

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Initial goal: FPGA arithmetic the way it should be

that is: open-ended, unlike processor arithmetic

- an open-ended list of custom operators
- open-ended data formats: all operators fully parameterized
- open-ended performance trade-off: flexible pipeline

General philosophy: computing just right

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General philosophy: computing just right

#### Beyond the plan

- the FloPoCo framework was successfully used to design the FPU of the Kalray processor
- FloPoCo provides the floating-point back-end to the PandA project (politecnico de Milano)

FloPoCo is freely available from

http://flopoco.gforge.inria.fr/

- Command line syntax: a sequence of operator specifications
- Options: target frequency, target hardware, ...
- Output: synthesizable VHDL.
- Written in C++

# Don't trust this operator, it was written by an underpaid computer

FloPoCo is already able to generate an infinite number of operators. We haven't tested them all.

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Two operators, TestBench and TestBenchFile, generate test benchs for the operator preceding them on the command line

• flopoco FPExp 8 23 TestBenchFile 10000

generates 10000 random tests

• flopoco IntConstDiv 16 3 -1 TestBenchFile -2

generates an exhaustive test

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Specification-based test bench generation

Not by simulation of the generated architecture!

Operator-specific random generator overloading

- FPExp: bias towards the small interval on which it is defined
- FPAdder: bias towards catastrophic cancellation

# One example of operator fusion

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$$x^2 + y^2 + z^2$$

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  - half the hardware required

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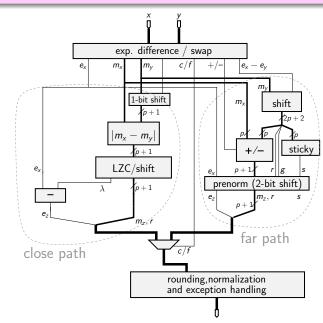
(not a toy example but a useful building block)

- A square is simpler than a multiplication
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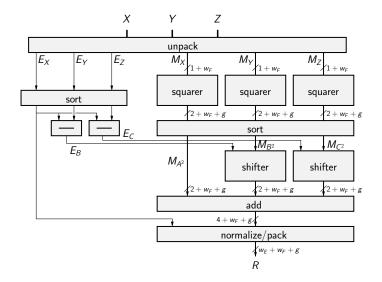
#### The FloPoCo Recipe

- Floating-point interface for convenience
- Clear accuracy specification for computing just right
- Fixed-point internal architecture for efficiency

#### A floating-point adder



### A fixed-point architecture



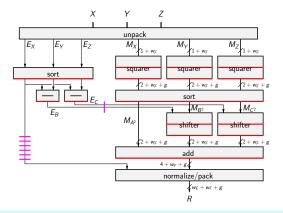
#### A few results for floating-point sum-of-squares on Virtex4:

Simple Precision	area	performance
LogiCore classic	1282 slices, 20 DSP	43 cycles @ 353 MHz
FloPoCo classic	1188 slices, 12 DSP	29 cycles @ 289 MHz
FloPoCo custom	453 slices, 9 DSP	11 cycles @ 368 MHz

Double Precision	area	performance
FloPoCo classic	4480 slices, 27 DSP	46 cycles @ 276 MHz
FloPoCo custom	1845 slices, 18 DSP	16 cycles @ 362 MHz

- all performance metrics improved, FLOP/s/area more than doubled
- Plus: custom operator more accurate, and symmetrical

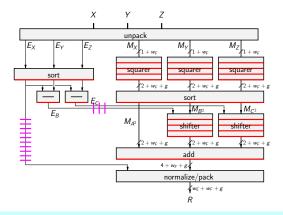
#### Custom also means: custom pipeline



#### One operator does not fit all

• Low frequency, low resource consumption

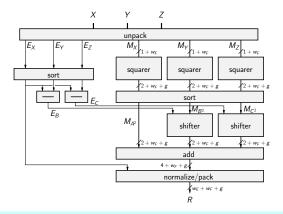
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### Custom also means: custom pipeline



#### One operator does not fit all

- Low frequency, low resource consumption
- Faster but larger (more registers)
- Combinatorial

# One example of operator specialization

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### Division by small integer constants

Motivation

divisions by 3 and by 9 in stencil applications (Jacobi)



# Division by small integer constants

Motivation

divisions by 3 and by 9 in stencil applications (Jacobi)



Also

- fancy address generation
- division by 10 for decimal/binary conversion,
- exponent processing in floating-point cubic root,
- coefficients 1/6 and 1/24 in sine/cosine Taylor formula

Ο ...

#### Implementation options

Floating-point, single precision, Virtex 5

• as a divider, with one input tied to 3: 1122 Reg + 945 LUT; 17 cycles @ 290 MHz

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- as an FP "multiplier by a constant": 149 Reg + 318 LUT; 4 cycles @ 439MHz
- the same, but exploiting the periodicity of the constant (1/3 = 0.010101010101010...)

107 Reg + 197 LUT; 4 cycles @ 422MHz

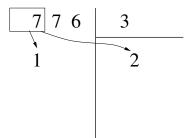
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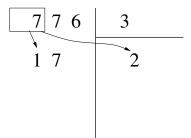
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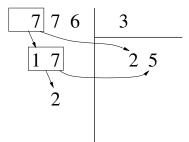
• this work:

105 Reg + 83 LUT ; 3 cycles @ 411 MHz

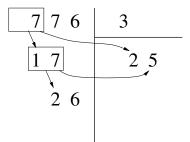
... and correctly rounded (equivalent to using a divider)



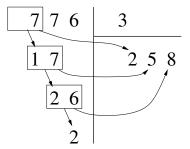




- iteration body: Euclidean division of a 2-digit decimal number by 3
- The first digit is a remainder from previous iteration: its value is 0, 1 or 2
- Possible implementation as a look-up table.



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#### Writing an integer x in radix $2^{\alpha}$



(split of the bits of x into chunks of  $\alpha$  bits)

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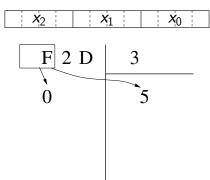
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Example: good old hexadecimal is  $\alpha = 4$ 

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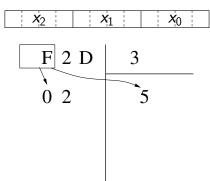
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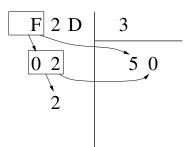
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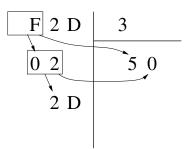
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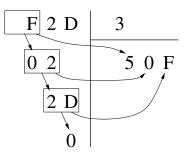
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And now for some mathematical obfuscation

procedure CONSTANTDIV(x, d) $r_k \leftarrow 0$ for i = k - 1 down to 0 do  $y_i \leftarrow x_i + 2^{\alpha} r_{i+1}$ (this + is a concatenation) $(q_i, r_i) \leftarrow (|y_i/d|, y_i \mod d)$ end for return  $q = \sum_{i=0}^{k} q_i \cdot 2^{-\alpha i}, r_0$ 

end procedure

(read from a table)

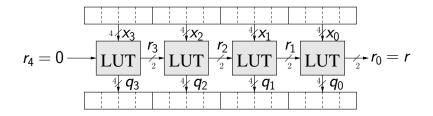
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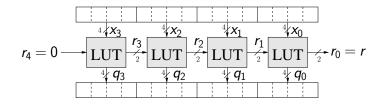
#### Each iteration

- consumes  $\alpha$  bits of x, and a remainder of size  $\gamma = \lceil \log_2 d \rceil$
- produces  $\alpha$  bits of  $\textbf{\textit{q}},$  and a remainder of size  $\gamma$
- implemented as a table with  $\alpha + \gamma$  bits in,  $\alpha + \gamma$  bits out

# Unrolled implementation



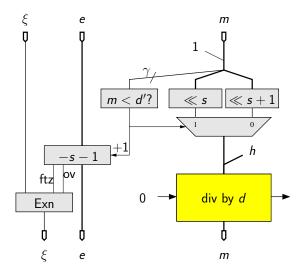
# Choice of parameters for a logic-based implementation



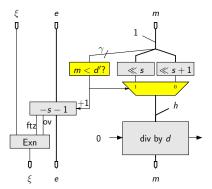
• FPGA logic is LUT-based (Example: LUT6 is a 6-input LUT)

- A 6-bit in, 6-bit out LUT consumes 6 LUT6
- Easy to pipeline (one register behind each LUT)
- Optimal  $\alpha$  s.t.  $\alpha + \gamma = 6$
- Efficient for small constants only (need small γ) Notice that 24 is actually 3...

# Floating-point has low overhead

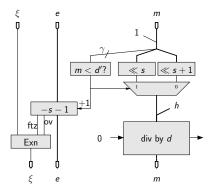


#### Floating-point has low overhead



• normalisation: small comparison, big mux;

#### Floating-point has low overhead



• normalisation: small comparison, big mux;

• rounding for free!  $\circ(z/d) = \lfloor z/d + \frac{1}{2} \rfloor$ 

$$\circ\left(\frac{2^{s+\epsilon}m}{d}\right) = \left\lfloor\frac{2^{s+\epsilon}m}{d} + \frac{1}{2}\right\rfloor = \left\lfloor\frac{2^{s+\epsilon}m + d/2}{d}\right\rfloor$$

and this + is again a concatenation (h on the picture)

F. de Dinechin FPGA arithmetic

# Synthesis results on Virtex-5 for pipelined floating-point division by 3

#### single precision

FF + LUT6	performance		
35 Reg + 69 LUT	1 cycle @ 217 MHz		
105 Reg + 83 LUT	3 cycles @ 411 MHz		
standard correctly rounded divider			
1122 Reg + 945 LUT	17 cycles @ 290 MHz		

#### double precision

FF + LUT6	performance		
122 Reg + 166 LUT	2 cycles @ 217 MHz		
$200 \ Reg + 214 \ LUT$	4 cycles @ 336 MHz		
using shift-and-add			
282 Reg + 470 LUT	5 cycles @ 307 MHz		

# Conclusion

#### Was it worth to waste your precious time on division by 3?

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(this slide intentionally left blank)

Two weeks from the first blackboard description of the algorithm to complete pipelined FloPoCo implementation + paper submission.

#### Implementation time

- 10 minutes to obtain a testbench generator
- 1/2 day for the integer Euclidean division
- 20 mn for its flexible pipeline
- 1/2 day for the FP divider by 3

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This was advertising for the FloPoCo framework.

# One example of open-ended operator

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A polynomial evaluator for arbitrary functions

Example:

flopoco FunctionEvaluator "(sin(x\*Pi/2))^ 2" 32 32 4

- The string is a *mathematical* function
- 32-bit in, 32-bit out
- Last-bit accurate (all returned bits hold useful information)
- 4 is the degree of the polynomial, allows to express a memory/multiplier trade-off
- Works for the set of functions for which it works

Another one is HOTBM. Still work in progress...

## Conclusion

- Introduction: FPGAs for computing?
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#### Conclusion

### My current crusade

#### The evil (at least on FPGAs)

"A fast implementation of single-precision floating-point exponential (but accurate to  $2^{-8}$  only)"

Do you see why it is wrong?

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#### The evil (at least on FPGAs) "A fast implementation of single-precision floating-point exponential (but accurate to 2<sup>-8</sup> only)" Do you see why it is wrong?

## A line I shall have in each of my talks until the world is saved Save routing! Save power! Don't move useless bits around!

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A line I shall have in each of my talks until the world is saved Save routing! Save power! Don't move useless bits around!

Or maybe this one

Do you really need to compute this bit?

For 20 years, the FPGA community has been waiting for the "killer application".

(The widely useful application on which the FPGA is so much better)

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Theorem: we'll wait forever.

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(The widely useful application on which the FPGA is so much better)

Theorem: we'll wait forever.

Proof: When such an application pops up,

- either it is indeed widely useful, and next year's Pentium will do it in hardware 10x faster than the FPGA, so it won't be an FPGA killer app next year,
- or the FPGA remains competitive next year, but it means that it was not a killer app.

For 20 years, the FPGA community has been waiting for the "killer application".

(The widely useful application on which the FPGA is so much better)

#### Theorem: we'll wait forever.

Proof: When such an application pops up,

- either it is indeed widely useful, and next year's Pentium will do it in hardware 10x faster than the FPGA, so it won't be an FPGA killer app next year,
- or the FPGA remains competitive next year, but it means that it was not a killer app.

#### The killer feature of FPGAs is flexibility

To exploit it, we do need infinitely many arithmetic operators.

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the choice is between

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#### Save routing! Save power! Don't move useless bits around!

## An almost virgin land

#### Most of the arithmetic literature addresses the construction of SUVs.

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When they become as easy to program as processors?

(now that's a challenge)

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(or do we quietly wait for processors to become as messy to program as FPGAs?)

## Thanks for your attention

The following people have contributed to FloPoCo: S. Banescu, N. Brunie, S. Collange, J. Detrey, P. Echeverría, F. Ferrandi, M. Grad, K. Illyes, M. Iştoan, M. Joldeş, C. Klein, D. Mastrandrea, B. Paşca, B. Popa, X. Pujol, D. Thomas, R. Tudoran, A. Vasquez.

Introduction: FPGAs for computing?

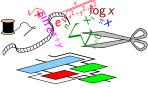
The FloPoCo project

One example of operator fusion

One example of operator specialization

One example of open-ended operator

Conclusion



http://flopoco.gforge.inria.fr/