Stefan Liebler

Report on the work of the MSSM subgroup: Neutral Higgs production

on behalf of the MSSM subgroup

Meeting of the LHC Higgs XS WG

Geneva - 13 June 2014

University of Hamburg
Outline

1. Status of neutral Higgs production in YR3
2. New developments since YR3
3. Transverse momentum distributions
4. Conclusions
Status of neutral Higgs production in YR3

MSSM Higgs production

Neutral Higgs production in the real MSSM:

Production cross sections according to YR3 [arXiv:1307.1347]

Bottom-quark annihilation:

Gluon fusion:
Gluon fusion: Calculation of MSSM Higgs cross sections in YR1:

$$\sigma(pp \rightarrow \phi + X) = (g^\phi_t)^2 \left( \sigma_{t,SM}^{NLO} + \Delta \sigma_{t,SM,0}^{NNLO} \right) + (g^\phi_b)^2 \sigma_{b,SM}^{NLO} + (g^\phi_t)(g^\phi_b) \sigma_{tb,SM}^{NLO}$$

**Couplings including resummation from FeynHiggs:**

$$g^h_t = \frac{\cos \alpha}{\sin \beta} \quad g^h_b = -\frac{\sin \alpha}{\cos \beta} \frac{1}{1 + \Delta_b} \left( 1 - \frac{\Delta_b}{\tan \alpha \tan \beta} \right)$$

Higgs mixing angle $\alpha$, $\tan \beta = v_u/v_d$, Resummation of sbottom effects in $\Delta_b$

**Improvements in YR3:**

- Inclusion of NLO third generation squark contributions (on top of $\Delta_b$)
- Inclusion of electroweak contributions by light quarks

$$\sigma(pp \rightarrow \phi + X) = \sigma_{MSSM}^{NLO} (1 + \delta_{lq}^{EW}) + (g^\phi_t)^2 \left( \Delta \sigma_{t,SM,0}^{NNLO} \right)$$

$$\Rightarrow \text{XS for } \phi \in \{h, H, A\} \text{ with SusHi (linked to FeynHiggs for } m_\phi \text{ and } \alpha\)$$

(SusHi also provides the $bb\phi$ XS in 5FS!)

Higlu [Spira '95], ggh@nnlo [Harlander Kilgore ‘02], SusHi [Harlander Liebler Mantler ‘12]

FeynHiggs [Hahn Heinemeyer Hollik Rzehak Weiglein]
Inclusion of NLO squark contributions:

- **gluon-quark**: known analytically (higher orders)
  
  [Spira Djouadi Graudenz Zerwas '95; Harlander Kant '05; ...]

- **gluon-squark**: known analytically/numerically

  [Anastasiou Beerli Bucherer Daleo Kunszt '06; 
  Aglietti Bonciani Degrassi Vicini '06; 
  Mühlleitner Spira '06; 
  Bonciani Degrassi Vicini '07]

- **gluino-squark-quark contributions**: 
  semi-analytically known

  [Anastasiou Beerli Daleo '08; Mühlleitner Spira Rzehak '10]

Challenge for gluino-quark-squark contributions:

**Five different masses**: $m_q, m_{\tilde{q}_1}, m_{\tilde{q}_2}, m_{\tilde{g}}, p^2 = m_\phi^2$

- **Taylor expansion in small Higgs mass**:
  
  \[ \rightarrow \text{top-stop-gluino contribution } m_\phi \ll m_t, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{g}} \]

  [Harlander Steinhauser '03 '04 + Hofmann '05; Degrassi Slavich '08]

  (NNLO top-stop-gluino contr. [Pak Steinhauser Zerf '10 '12])

- **Expansion in heavy SUSY masses**: $m_\phi, m_q \ll m_{\tilde{q}_1}, m_{\tilde{q}_2}, m_{\tilde{g}}$
  
  \[ \rightarrow \text{quark-squark-gluino} \text{ [Harlander Hofmann Mantler '10; Degrassi Slavich '10 + Di Vita '11 '12]} \]
✓ Inclusion of elw. contributions by light quarks: [Aglietti Bonciani Degrassi Vicini '04 '10]

Relevant diagrams with $V \in \{W, Z\}$:

Definition of SUSY electroweak correction factor:

$$
\delta_{lq}^{\ellq} = \frac{\alpha_{\text{EW}}}{\pi} 2 \text{Re}(A^\phi A^{\phi, \text{EW}}) / |A^\phi|^2
$$

$$
A^{\phi, \text{EW}} = -\frac{3}{8} x_W \frac{s_W^2}{c_W^2} \left[ \frac{2}{c_W^2} \left( \frac{5}{4} - \frac{7}{3} s_W^2 + \frac{22}{9} s_W^4 \right) A[x_Z] + 4 A[x_W] \right] g^\phi_V
$$

Complex mass scheme: $x_V = (m_V - i \frac{\Gamma_V}{2})^2 / m_\phi^2$

Supersymmetry enters $g^\phi_V$:

$$
g^h_V = \sin(\beta - \alpha), \quad g^A_V = 0, \quad g^H_V = \cos(\beta - \alpha)
$$

For moderate masses of SM-like Higgs results in similar correction as SM electroweak correction factor [Actis Passarino Sturm Uccirati '08].
Status of neutral Higgs production in YR3

Benchmark scenarios

Usage of YR3 setup for new benchmark scenarios (compatible with Run 1):

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<td>200</td>
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Root files provided on the webpage of the LHC Higgs XS WG as a function of $m_A$ and $\tan \beta$ for $\phi \in \{h, H, A\}$: [Vazquez Acosta Frensch]

- Higgs masses $m_\phi$ ($h$ mostly compatible with SM Higgs $\sim 125$ GeV)
- Gluon fusion XS (in accordance to YR3)
- Bottom-quark annihilation XS in 4FS/5FS and Santander-matched XS
- Branching ratios (with FeynHiggs and HDECAY - see A. Mück’s talk)
- Scale and PDF+$\alpha_s$ uncertainties

For other scenarios: Easy to use setup. Get in contact!
Plots for the new benchmark scenarios on the webpage (Picture Gallery), e.g.:
New developments since YR3

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Bottom-quark annihilation in 4FS: [Dittmaier Krämer Spira]

- Completion of grids for \( \sqrt{s} = 7, 8, 13 \) and 14 TeV for \( H/A \)
- Addition of complete grids for the \( Y_b \cdot Y_t \) interference terms (interferences of \( bbH/A \) in 4FS and \( ggH/A \) production)
- Update of Santander-matched XS for SM Higgs on webpage
New developments since YR3

Detailed uncertainty estimation for gluon fusion:

Study includes approx. NNLO stop contributions (incl. in SusHi).

Adopt vanishing-Higgs-mass limit (VHML) for top-stop sector to obtain NNLO:

\[ \mathcal{L}_{ggH} = -\frac{1}{4v} C(\alpha_s) H G_{\mu\nu} G^{\mu\nu} \Rightarrow C(\alpha_s) = C^{(0)} + \frac{\alpha_s}{\pi} C^{(1)} + \left( \frac{\alpha_s}{\pi} \right)^2 C^{(2)} \]

\[ \sigma_{\text{NNLO}} = |A_{t\bar{t}}^{1\ell}|^2 \Sigma^{(0)} + \frac{\alpha_s}{\pi} \left( |A_{t\bar{t}}^{1\ell}|^2 \Sigma^{(1)} + 2 C^{(1)} \Sigma^{(0)} \text{Re} A_{t\bar{t}}^{1\ell} \right) + \left( \frac{\alpha_s}{\pi} \right)^2 \left[ |A_{t\bar{t}}^{1\ell}|^2 \Sigma^{(2)} + 2 \left( C^{(1)} \Sigma^{(1)} + C^{(2)} \Sigma^{(0)} \right) \text{Re} A_{t\bar{t}}^{1\ell} + (C^{(1)})^2 \Sigma^{(0)} \right] \]

Approximation: \( C^{(2)} = C^{(2)}_t \leftrightarrow \text{Uncertainty } [0, 2C^{(2)}_t] \)

Discussion of XS and uncertainties for the light stop scenario:

- \( m_{\tilde{t}_1} = 324 \text{ GeV} \) and \( m_{\tilde{t}_2} = 672 \text{ GeV} \)
- \( m_{\tilde{b}_1} > 450 \text{ GeV} \) and \( m_{\tilde{b}_2} < 550 \text{ GeV} \)
New developments since YR3

Detailed uncertainty estimation for gluon fusion

Cross section predictions:

Electroweak contr.

Squark contr.

\( \sigma_{tot} \) \([pb]\), light-stop scenario

\( \frac{\sigma_{gg}^{QCD+EW}}{\sigma_{gg}^{QCD} \text{ light-stop scenario}} \)

\( \frac{\sigma_{gg}^{q+q \sim}}{\sigma_{gg}^{q \text{ light-stop scenario}}} \)

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New developments since YR3

Detailed uncertainty estimation for gluon fusion

Uncertainties in the gluon fusion XS prediction:

- Renormalization and factorization scale uncertainties $\sim \pm (5 - 25)\%$

Consider sets $C_\mu$ of pairs $(\mu_R, \mu_F)$ with

$\mu_R = \{m_\phi/4, m_\phi/2, m_\phi\}$, $\mu_F = \{m_\phi/4, m_\phi/2, m_\phi\}$,

with the additional constraint $1/2 \leq \mu_R/\mu_F \leq 2$

$$\sigma^- \equiv \min_{(\mu_R, \mu_F) \in C_\mu} \{\sigma(\mu_R, \mu_F)\}, \quad \sigma^+ \equiv \max_{(\mu_R, \mu_F) \in C_\mu} \{\sigma(\mu_R, \mu_F)\}$$

$$\Delta_\mu \equiv \Delta^+ - \Delta^- \quad \text{with} \quad \Delta^\pm_\mu \equiv \frac{\sigma^\pm - \sigma(\mu_0^0, \mu_F^0)}{\sigma(\mu_R^0, \mu_F^0)}$$
Uncertainties in the gluon fusion XS prediction:

- Renormalization and factorization scale uncertainties $\checkmark \sim \pm (5 - 25)\%$
- PDF + $\alpha_s$ uncertainties $\checkmark \sim \pm (2 - 5)\%$

Performed study according to PDF4LHC recipe:
Outcome: main dependence on the Higgs mass $m_\phi$
$\leftrightarrow$ can be taken from SM uncertainty
New developments since YR3

Detailed uncertainty estimation for gluon fusion

Uncertainties in the gluon fusion XS prediction:

- Renormalization and factorization scale uncertainties
- PDF + $\alpha_s$ uncertainties
- Uncertainty from heavy SUSY masses expansion

+ of approximate NNLO stop contributions

Multiply the two-loop $\tilde{t} + \tilde{b}$ contributions by test factors

$$\frac{A_{q_1}^{1\ell}}{A_{q_1}^{1\ell, \text{exp}}}$$

with $\tilde{q} = \{\tilde{t}, \tilde{b}\}$, $A_{q_1}^{1\ell, \text{exp}}$ includes only $\mathcal{O}(m_{\tilde{q}_1}^{-2})$

Approximative NNLO stop contributions uncertainty: $< 1\%$

(*) for $m_{\phi}$ close to the SUSY threshold, i.e. $2m_{\tilde{t}_1}$
New developments since YR3

Detailed uncertainty estimation for gluon fusion

Uncertainties in the gluon fusion XS prediction:

- **Renormalization and factorization scale uncertainties** ✓ ~ ±(5 − 25)%
- **PDF + $\alpha_s$ uncertainties** ✓ ~ ±(2 − 5)%
- **Uncertainty from heavy SUSY masses expansion** + of approximate NNLO stop contributions ✓ ~ ±(5 − 20)%
- **Uncertainty from missing contributions to $\Delta_b$** ✓ ~ ±10(25)%

Variation of $\Delta_b$ by ±10%

→ for positive $\mu$: Uncertainty < 10%
→ for negative $\mu$: Uncertainty up to ~ 25%

Future work: Inclusion of NNLO contributions
[Noth Spira ’08 ’10, Mihaila Reisser ’10]

(*) for $m_\phi$ close to the SUSY threshold, i.e. $2m_{\tilde{t}_1}$
(***) in case of large coupling $b\bar{b}\phi$ / large $\tan\beta$
Uncertainties in the gluon fusion XS prediction:

- Renormalization and factorization scale uncertainties ✓ \( \sim \pm (5 - 25)\% \)
- PDF + \( \alpha_s \) uncertainties ✓ \( \sim \pm (2 - 5)\% \)
- Uncertainty from heavy SUSY masses expansion + of approximate NNLO stop contributions ✓ \( \sim \pm (5 - 20)\% (**) \)
- Uncertainty from missing contributions to \( \Delta_b \) ✓ \( \sim \pm 10(25)\% (**) \)
- Uncertainty in renormalization of \( Y_b \) ✓ \( \sim -(0 - 80)\% (**) \)

Choose \( Y_b \propto m_b^{\text{pole}} \) or \( m_b^{\text{MS}} (\mu_R \sim m_\phi/2) \)

\[ \delta Y_b, \text{light-stop scenario} \]

\[ h \]

\[ \tan \beta \]

\[ m_A [\text{GeV}] \]

\[ \delta Y_b, \text{light-stop scenario} \]

\[ H \]

\[ \tan \beta \]

\[ m_A [\text{GeV}] \]

\( (*) \) for \( m_\phi \) close to the SUSY threshold, i.e. \( 2m_{\tilde{t}_1} \)

\( (**) \) in case of large coupling \( b\bar{b}_\phi \) / large \( \tan \beta \)

\( \leftrightarrow \) large, where \( bb_\phi \) dominates
Gluon fusion: Status in YR3
Example: light stop scenario with $m_A = 130 \text{ GeV}$, $\tan \beta = 40$

Red: SusHi fixed order, Black: POWHEG fixed order,
Blue: POWHEG method (Resummation of $\log(p_T/m_\phi)$ via Sudakov form factor and PS)

\[ R = \frac{d\sigma^{\text{SUSY}}/dp_T}{d\sigma^{\text{SM}}/dp_T} \]

[Bagnaschi Degrassi Slavich Vicini '11]

Light Higgs $h$

Heavy Higgs $H$
**Gluon fusion: Analytic resummation**

Treatment of heavy-quark masses in SM

[Stefan Liebler - MSSM group]

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**Bottom-quark annihilation:** $p_T$ resummation in 5FS at NNLO+NNLL

(NLO+NLL [Belyaev Nadolsky Yuan '06])

(*) SM recommendation: Scale choices: $\mu_R = \mu_F = \mu_0 = m_T = \sqrt{m_H^2 + p_T^2}$, $Q_t = m_H/2$, $Q_b = m_b$

Uncertainties: $m_T/2 < \mu_F/\mu_R < 2m_T$ (excluding $\mu_R/\mu_F = 4, 1/4$), $m_H/4 < Q_t < m_H$, $m_b/3 < Q_b < 3m_b$

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Our group is actively working on these topics, and we are collaborating with several other groups on the development of tools.

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**Ongoing: Translation to MSSM/2HDM - Detailed comparison** [Nikitenko]

**Public:** POWHEG-BOX (gg_H_MSSM, gg_H_2HDM) [Bagnaschi Vicini]

**On the way:** SusHi version with analytic resummation [Harlander Mantler Wiesemann]

SusHi-amplitudes to POWHEG-BOX/MG5_aMC@NLO [Mantler Wiesemann]

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For top in VHML: NNLO+NNLL [Bozzi Catani de Florian Grazzini '06]
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The MSSM subgroup was a **success**, similar to the whole LHC Higgs cross section working group.

Recently (2013-2014) the MSSM subgroup

- improved precision in Higgs production cross sections by including squark and electroweak contributions to $gg\phi$, adding more contributions to $bb\phi$.
- provided root files/figures for benchmark scenarios compatible with Run 1. (Check the webpage!)

**Are we done? Never ever!**

- We can improve the theoretical uncertainty estimation and/or work harder to calculate higher orders.
- Work on transverse momentum distributions is appreciated.
- Inclusion of bottom-quark annihilation to **POWHEG**.

Thank you for your attention!
The neutral components of the Higgs doublets $H_u = (H_u^+, H_u^0)^T$ and $H_d = (H_d^0, H_d^-)^T$ mix as follows

$$
\begin{pmatrix}
H_u^0 \\
H_d^0
\end{pmatrix} = \begin{pmatrix} v_u \\ v_d \end{pmatrix} + \frac{1}{\sqrt{2}} R_\alpha \begin{pmatrix} h \\ H \end{pmatrix} + \frac{i}{\sqrt{2}} R_\beta \begin{pmatrix} G \\ A \end{pmatrix}.
$$

The mixing matrix is expressed in terms of the “Higgs mixing angle $\alpha$”

$$
R_\alpha = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix}.
$$

The Higgs sector at LO is determined by fixing $\tan\beta = \frac{v_u}{v_d}$ and $m_A^2$:

$$
m_{h, H} = \frac{1}{2} \left( m_A^2 + m_Z^2 \mp \sqrt{(m_A^2 - m_Z^2)^2 + 4m_Z^2 m_A^2 \sin^2(2\beta)} \right)
$$

$$
\tan(2\alpha) = \tan(2\beta) \frac{m_A^2 + m_Z^2}{m_A^2 - m_Z^2}.
$$

The lightest Higgs $h$ mass obtains large corrections at higher orders:

FeynHiggs [Frank Degrassi Hahn Heinemeyer Hollik Rzehak Slavich Weiglein Williams]

3-loop [Martin ’07; Kant Harlander Mihaila Steinhauser ’08 ’10].
In the MSSM, Higgs couplings to the $b$-quark can be enhanced by $\tan \beta$:

Relative strength of the Higgs boson couplings $g^\phi_f$ with $\phi \in \{h, H, A\}$ to the SM fermions (with respect to the SM Higgs boson couplings):

$$
\begin{align*}
    g^h_u &= \frac{\cos \alpha}{\sin \beta}, & g^H_u &= \frac{\sin \alpha}{\sin \beta}, & g^A_u &= \frac{1}{\tan \beta}, \\
    g^h_d &= -\frac{\sin \alpha}{\cos \beta}, & g^H_d &= \frac{\cos \alpha}{\cos \beta}, & g^A_d &= \tan \beta.
\end{align*}
$$

In addition, the superpartners of the quarks, the squarks, are relevant

$$
\mathcal{L} \supset -(\tilde{q}_L^\dagger, \tilde{q}_R^\dagger) M^2_{\tilde{q}} \begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix}
$$

with the mass matrix:

$$
M^2_{\tilde{q}} = \begin{pmatrix} M^2_{qL} + m_q^2 + m^2_{E1} & m_q(A_q - \mu \kappa) \\ m_q(A_q - \mu \kappa) & M^2_{qR} + m_q^2 + m^2_{E2} \end{pmatrix}
$$

They form two mass eigenstates:

$$
\begin{pmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{pmatrix} = \begin{pmatrix} \cos \theta_{\tilde{q}} & \sin \theta_{\tilde{q}} \\ -\sin \theta_{\tilde{q}} & \cos \theta_{\tilde{q}} \end{pmatrix} \begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix}
$$

$$
m^2_{E1} = m^2_z \cos(2\beta) (T^3_q - Q_q \sin^2 \theta_W); \quad m^2_{E2} = m^2_Z \cos(2\beta) Q_q \sin^2 \theta_W; \quad \kappa = \tan \beta(d), \cot \beta(u)
$$
Gluon fusion at LO using $\tau_\phi = m_\phi^2 / s$:

$$\sigma(pp \rightarrow \phi + X) = \sigma_0^\phi \tau_\phi \frac{d\mathcal{L}^{gg}}{d\tau_\phi}$$

LO partonic cross section (XS):

$$\sigma_0^\phi = \frac{G_F \alpha_s^2}{288 \sqrt{2 \pi}} |A^\phi|^2$$

$$A^\phi = \sum_{q \in \{t, b\}} \left( a_q^{\phi,(0)} + a_{\tilde{q}}^{\phi,(0)} \right)$$

with

$$a_q^{\phi,(0)} = g_q^{\phi} \frac{3\tau_q}{2} \left( 1 + (1 - \tau_q^\phi) f(\tau_q^\phi) \right)$$

$$a_{\tilde{q}}^{\phi,(0)} = -\frac{3\tau_{\tilde{q}}^\phi}{8} \sum_{i=1}^2 \tilde{g}_{\tilde{q}ii}^{\phi} \left( 1 - \tau_{\tilde{q}i}^\phi f(\tau_{\tilde{q}i}^\phi) \right)$$

Partonic → Hadronic XS:

$$\frac{d\mathcal{L}^{gg}}{d\tau} = \int_{\tau}^1 \frac{dx}{x} g(x) g(\tau / x)$$

Quark contributions

Squark contributions

$$\tau_q^\phi = 4m_q^2 / m_\phi^2, \quad \tau_{\tilde{q}i}^\phi = 4m_{\tilde{q}i}^2 / m_\phi^2, \quad f(\tau) = \begin{cases} \arcsin^2 \frac{1}{\sqrt{\tau}} & \tau \geq 1 \\ -\frac{1}{4} \left( \log \frac{1 + \sqrt{1 - \tau}}{1 - \sqrt{1 - \tau}} - i\pi \right)^2 & \tau < 1 \end{cases}$$
Cancellation of logs - Bottom 2-loop contributions:

[Spira Djouadi Graudenz Zerwas ’95] in the notation of [Degrassi Slavich ’10]

\[
G_b^{2\ell} = C_F G_b^{(g, CF)} + C_A G_b^{(g, CA)}
\]

\[
2 m_b^2 G_b^{(g, CF)} = \mathcal{F}_{1/2}^{(2\ell, a)}(\tau_b) + \mathcal{F}_{1/2}^{(2\ell, b)}(\tau_b) \left( \ln \frac{m_b^2}{Q^2} - \frac{1}{3} \right)
\]

\[
2 m_b^2 G_b^{(g, CA)} = \mathcal{G}_{1/2}^{(2\ell, CA)}(\tau_b)
\]

In the limit \( \tau_b = 4m_b^2/m_\phi^2 \ll 1 \) the above expressions reduce to:

\[
\mathcal{F}_{1/2}^{(2\ell, a)}(\tau) = -\tau \left[ 9 + \frac{9}{5} \zeta_2 - \zeta_3 - (1 + \zeta_2 + 4 \zeta_3) \ln \left( -\frac{4}{\tau} \right) - (1 - \zeta_2) \ln^2 \left( -\frac{4}{\tau} \right) \right. \\
+ \frac{1}{4} \ln^3 \left( -\frac{4}{\tau} \right) + \frac{1}{48} \ln^4 \left( -\frac{4}{\tau} \right) \left. \right] + \mathcal{O}(\tau^2)
\]

\[
\mathcal{F}_{1/2}^{(2\ell, b)}(\tau) = 3 \tau \left[ 1 + \frac{1}{2} \ln \left( -\frac{4}{\tau} \right) - \frac{1}{4} \ln^2 \left( -\frac{4}{\tau} \right) \right] + \mathcal{O}(\tau^2)
\]

\[
\mathcal{G}_{1/2}^{(2\ell, CA)}(\tau) = -\tau \left[ 3 - \frac{8}{5} \zeta_2 - 3 \zeta_3 + 3 \zeta_3 \ln \left( -\frac{4}{\tau} \right) - \frac{1}{4} (1 + 2 \zeta_2) \ln^2 \left( -\frac{4}{\tau} \right) \\
- \frac{1}{48} \ln^4 \left( -\frac{4}{\tau} \right) \right] + \mathcal{O}(\tau^2)
\]

For \( Q = m_b \) the various logarithms accidentally cancel in case of gluon fusion.
Appendix
Choice of $Y_b$

Resummation of large $\tan \beta$-enhanced terms in the MSSM

$$\mathcal{L} \supset -Y_t H_u Q t_R + Y_b H_d Q b_R$$

Using $\langle H_u \rangle = v_u, \langle H_d \rangle = v_d$ and $v_d^2 + v_u^2 = v^2$, $\tan \beta = v_u/v_d$ we define

$$Y_t = \frac{m_t}{v_u} = \frac{m_t}{v \sin \beta}, \quad Y_b = \frac{m_b}{v_d} = \frac{m_b}{v \cos \beta}$$

The effective Lagrangian motivates:

$$m_b = Y_b v_d + \tilde{Y}_b v_u = Y_b v_d (1 + \epsilon \tan \beta)$$

$$\Rightarrow \quad Y_b = \frac{m_b}{v_d (1 + \Delta_b)}$$

$$\mathcal{L}^{\text{eff}} \supset Y_b H_d Q b_R - \tilde{Y}_b H_u^* Q b_R$$

$$\Delta_b = \frac{\tilde{Y}_b v_u}{Y_b v_d} = : \epsilon \tan \beta$$

This replacement implies a resumma-

[Hall Rattazzi Sarid '93, Hempfling '94, Carena Garcia Nierste Wagner '99, Guasch Häfliger Spira '03]
Bottom-quark annihilation:

\[ pp \rightarrow (b\bar{b})\phi + X \] for enhanced couplings to \( b \)-quarks relevant \( \rightarrow \) MSSM!

4 flavour scheme (4FS)
Collinear logarithms \( \propto \log\left(\frac{m_b}{m_\phi}\right) \)

5 flavour scheme (5FS)
Resummation of logarithms
\( b \) quarks as partons

Calculation of inclusive cross section at NNLO in the 5FS:
\[ bbh@nnlo \] [Harlander Kilgore '03]

Distributions for Higgs+jet(s) production in the 5FS
[Harlander Ozeren Wiesemann '10, Buehler Herzog Lazopoulos Mueller '12]