Precision physics with parton distributions and deep-inelastic structure functions

Andreas Vogt (University of Liverpool)

Results obtained with Sven Moch (DESY) and Jos Vermaseren (NIKHEF)

- Partons and deep-inelastic scattering in perturbative QCD
- Higher-order effects in the parton evolution, small-x logarithms
- Parton evolution in practice: evolution codes and benchmarks
- Structure functions F_L and F_2, perturbative accuracy for α_s

LHeC Workshop, Divonne, 02-09-08

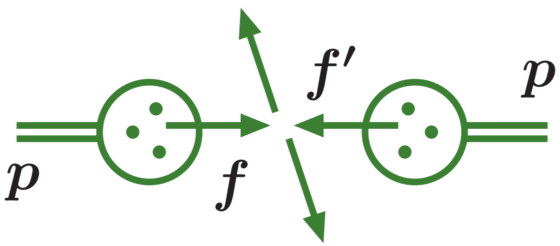
Perturbative QCD at the high-energy frontier

Search for Higgs Boson, new particles : highest possible energies

$\Rightarrow$ $p\bar{p}/pp$ colliders: Tevatron (2 TeV), LHC (14 TeV)

$\Downarrow$

$$\sigma^{pp} = \sum f^p * f'^p * \hat{\sigma}^{ff'}$$



Hard interactions of protons:

parton (q, g) distributions f^p

partonic cross sections $\hat{\sigma}^{ff'}$

Ideal process: lepton-proton scattering

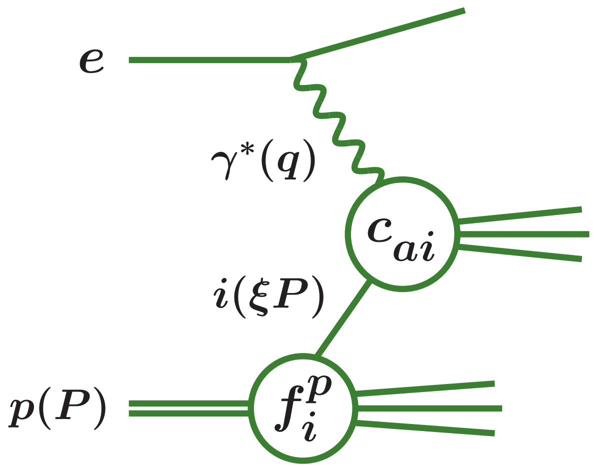
Fixed-target exp.: SLAC, . . . ; ep collider HERA; future EIC_{US}, LHeC?

$\Uparrow$

Quantitative understanding of QCD, precision determination of α_s

Parton densities and hard processes in pQCD

Example: inclusive photon-exchange deep-inelastic scattering (DIS)



Hard scale, Bjorken variable

$$Q^2 = -q^2$$
$$x = Q^2/(2P\cdot q)$$

Lowest order, quarks: $x = \xi$

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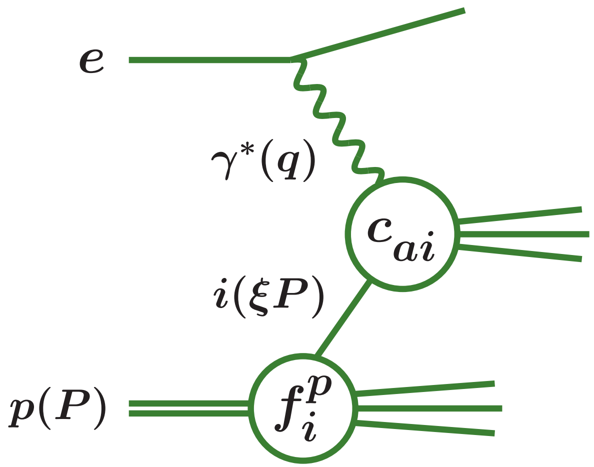

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Structure functions $F_{2,L}$ (at leading twist of operator-product exp.)

$$x^{-1} F_a^p(x, Q^2) = \sum_i \int_x^1 \frac{d\xi}{\xi}\, c_{a,i}\left(\frac{x}{\xi}, \alpha_s(\mu^2), \frac{\mu^2}{Q^2}\right) f_i^p(\xi, \mu^2)$$

Coefficient functions: scheme, scale $\mu = \mathcal{O}(Q)$, Mellin convolutions

$1/Q^2$ corrections ('higher twists'): extract or suppress by data cuts

Parton densities and hard processes in pQCD

Parton distributions f_i : renormalization-group evolution equations

$$\frac{d}{d\ln\mu^2} f_i(\xi,\mu^2) = \sum_k \left[P_{ik}(\alpha_s(\mu^2)) \otimes f_k(\mu^2)\right](\xi)$$

$\otimes$ **= Mellin convolution. Initial conditions incalculable in pert. QCD**

$\Rightarrow$ **predictions: fits of suitable reference processes, universality**

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Expansions in α_s: splitting functions P, coefficient functions c_a

$$\begin{aligned} P &= \alpha_s P^{(0)} + \alpha_s^2 P^{(1)} + \alpha_s^3 P^{(2)} + \ldots \\ c_a &= \underbrace{\alpha_s^{n_a} \Big[c_a^{(0)}}_{} + \alpha_s c_a^{(1)} + \alpha_s^2 c_a^{(2)} + \ldots \Big] \end{aligned}$$

LO: approximate shape, rough estimate of rate

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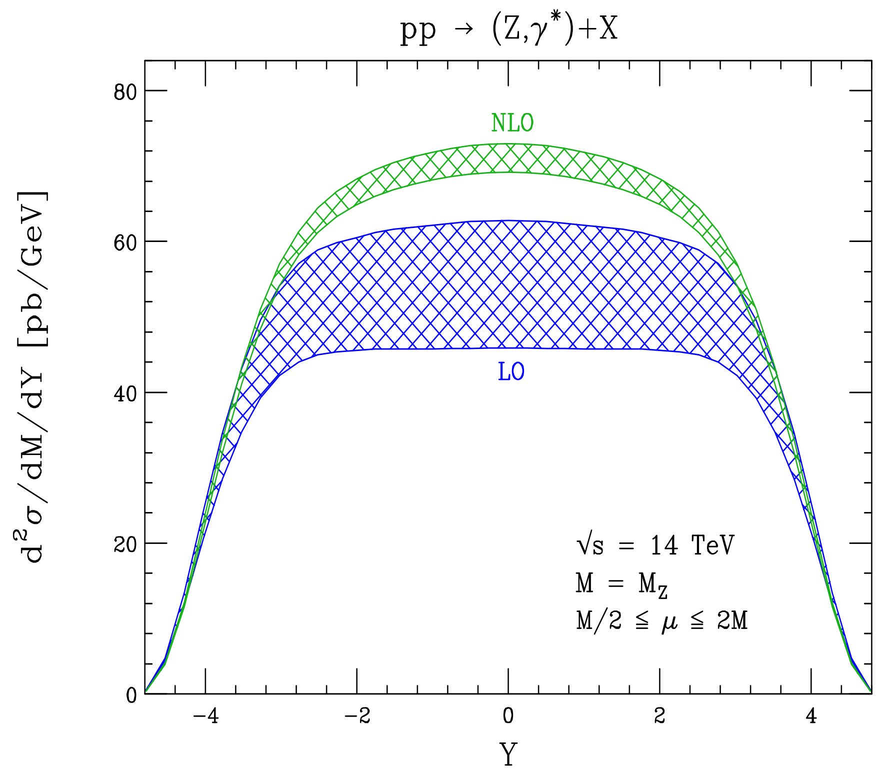
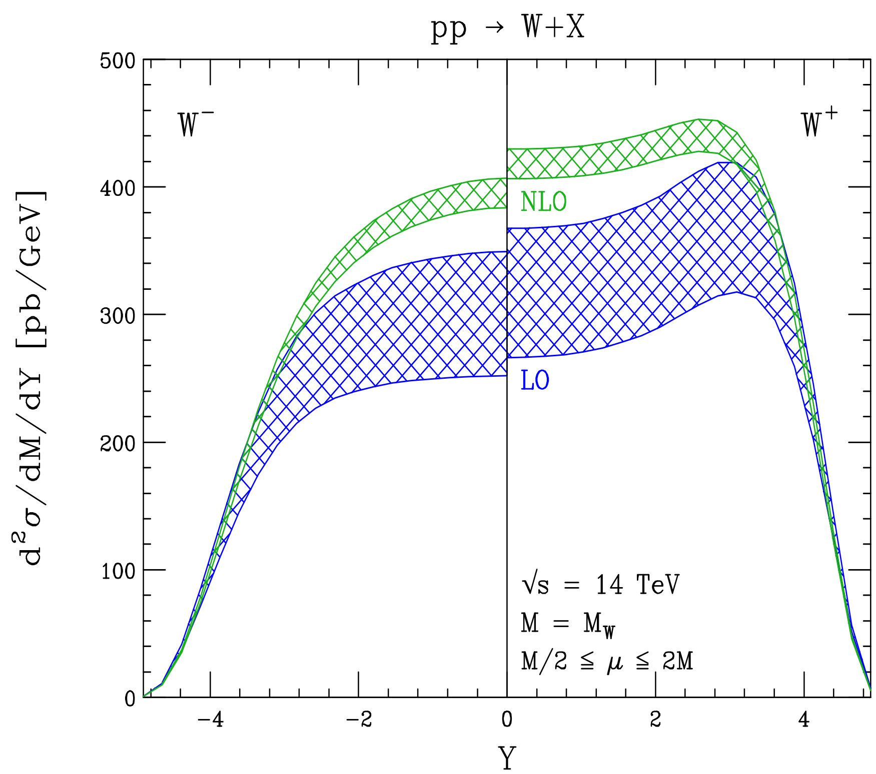
NLO: first real prediction of size of cross sections

NNLO, $P^{(2)}$, $c_a^{(2)}$: first serious error estimate $\Leftrightarrow$ precision physics

Example: gauge boson production at the LHC





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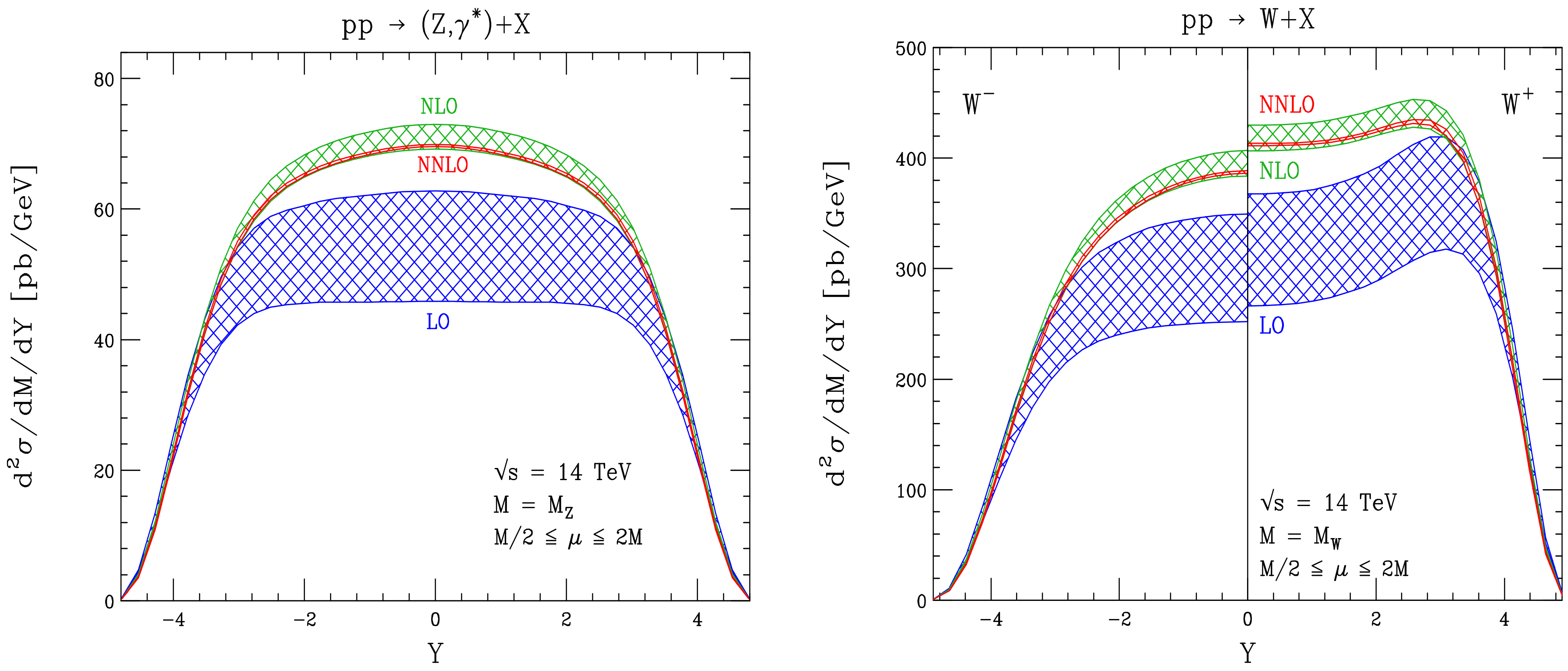


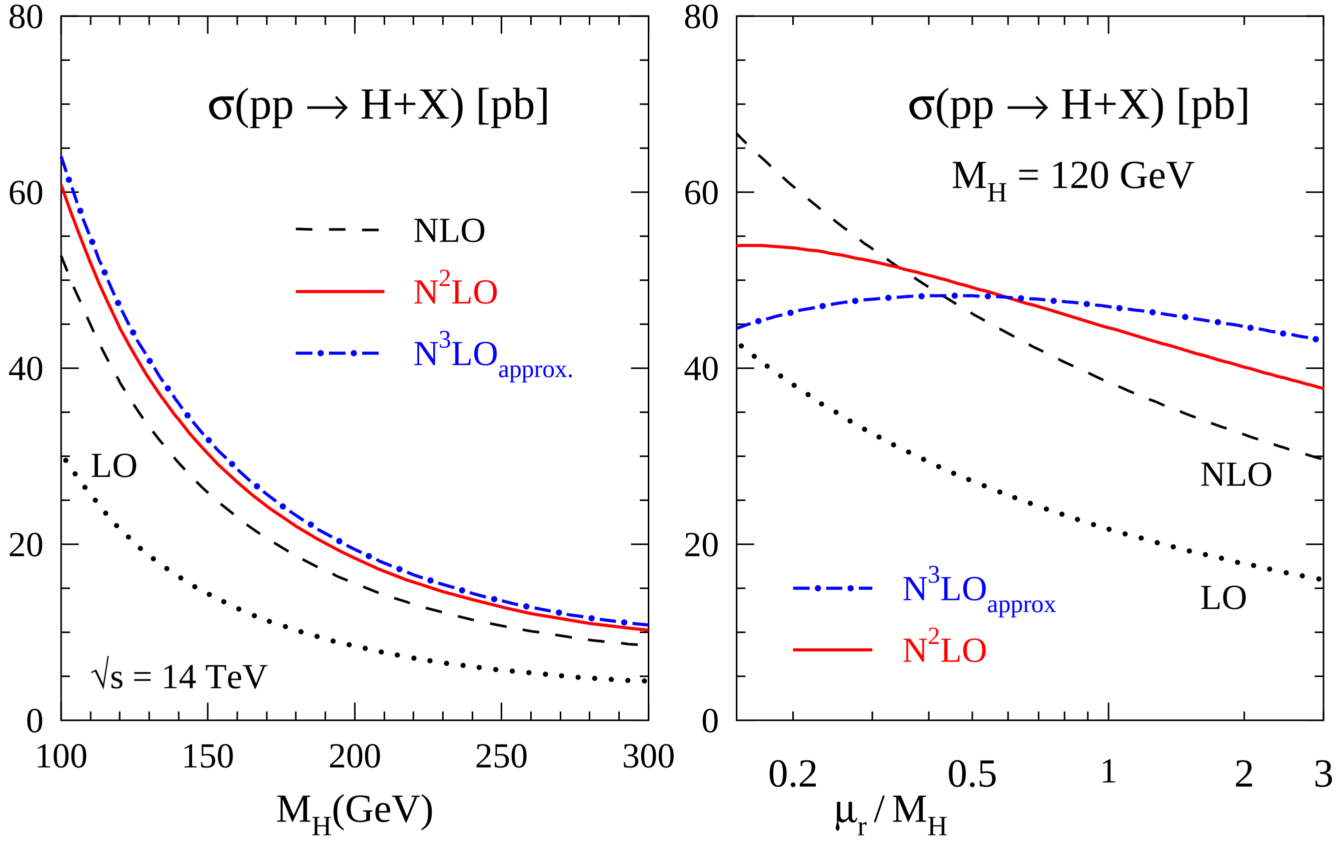
Rapidity-dependent $\hat{\sigma}_{\mathrm{NNLO}}$: Anastasiou, Dixon, Melnikov, Petriello (03)

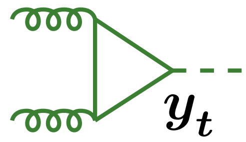
'Gold-plated' processes: NNLO perturbative accuracy better than 1%

$\Rightarrow$ potential for constraining parton densities (fixed in plots above)

Disclaimer (I): much lower accuracy possible

Example: total cross section for Higgs boson production at the LHC



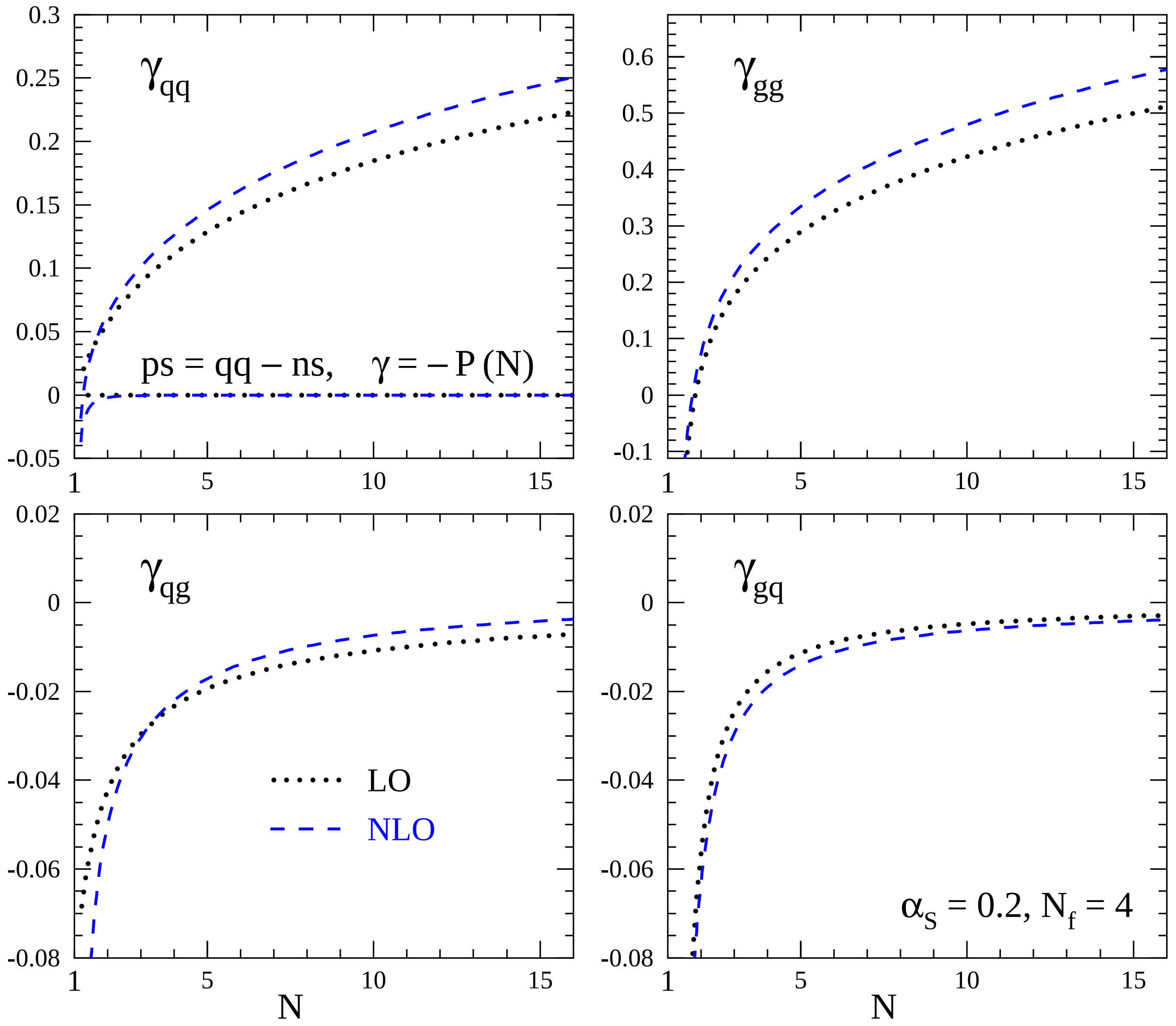


Moch, A.V. (05)

$\hat{\sigma}_{\text{NNLO}}$ **: Harlander, Kilgore (02); Anastasiou, Melnikov (02, 05 [σ_{diff}])**

Higher-order uncertainties: $\sim 15\%$ at NNLO, $\sim 5\%$ at approx. N^3LO

Mellin-N space splitting functions to NNLO



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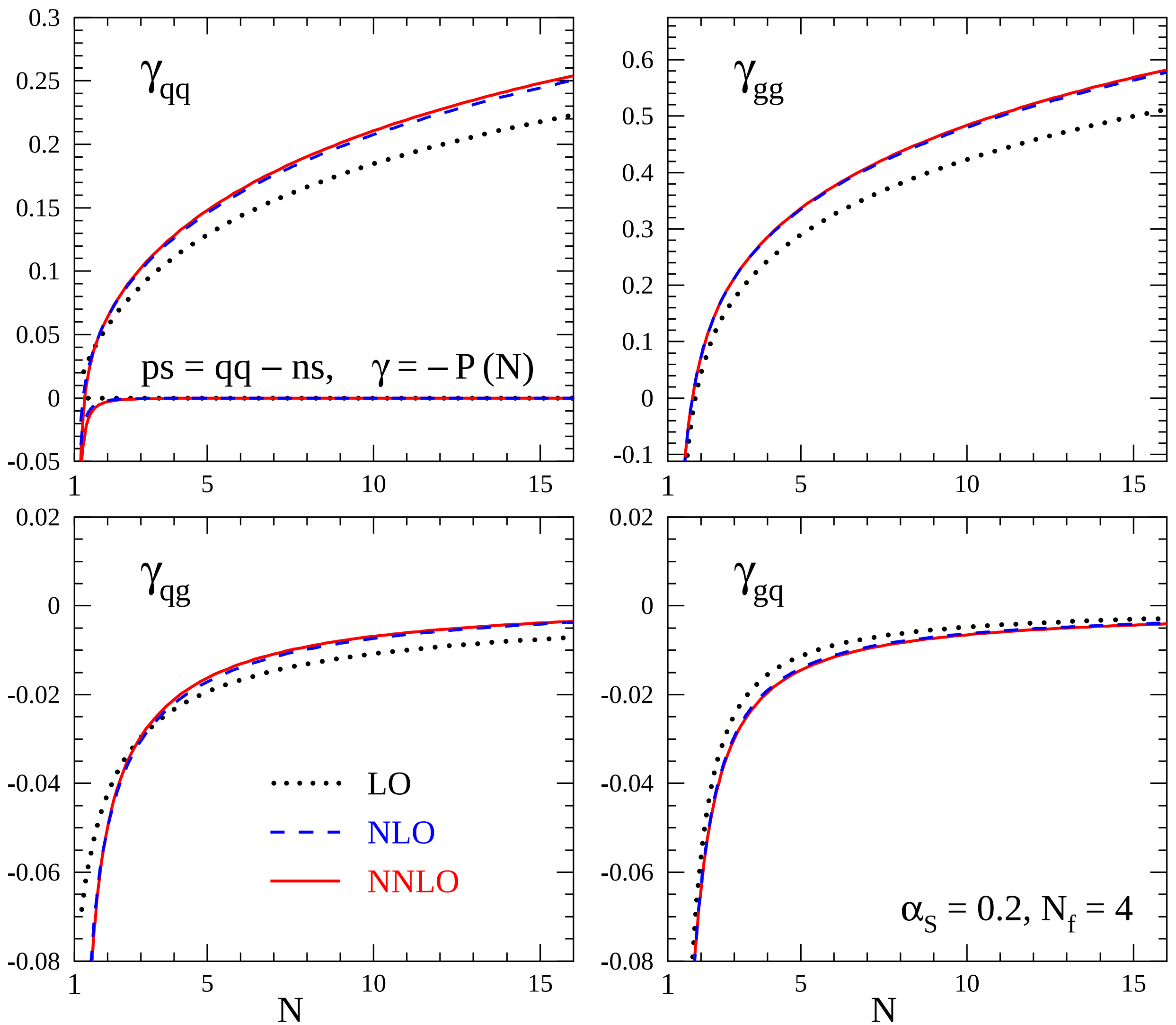
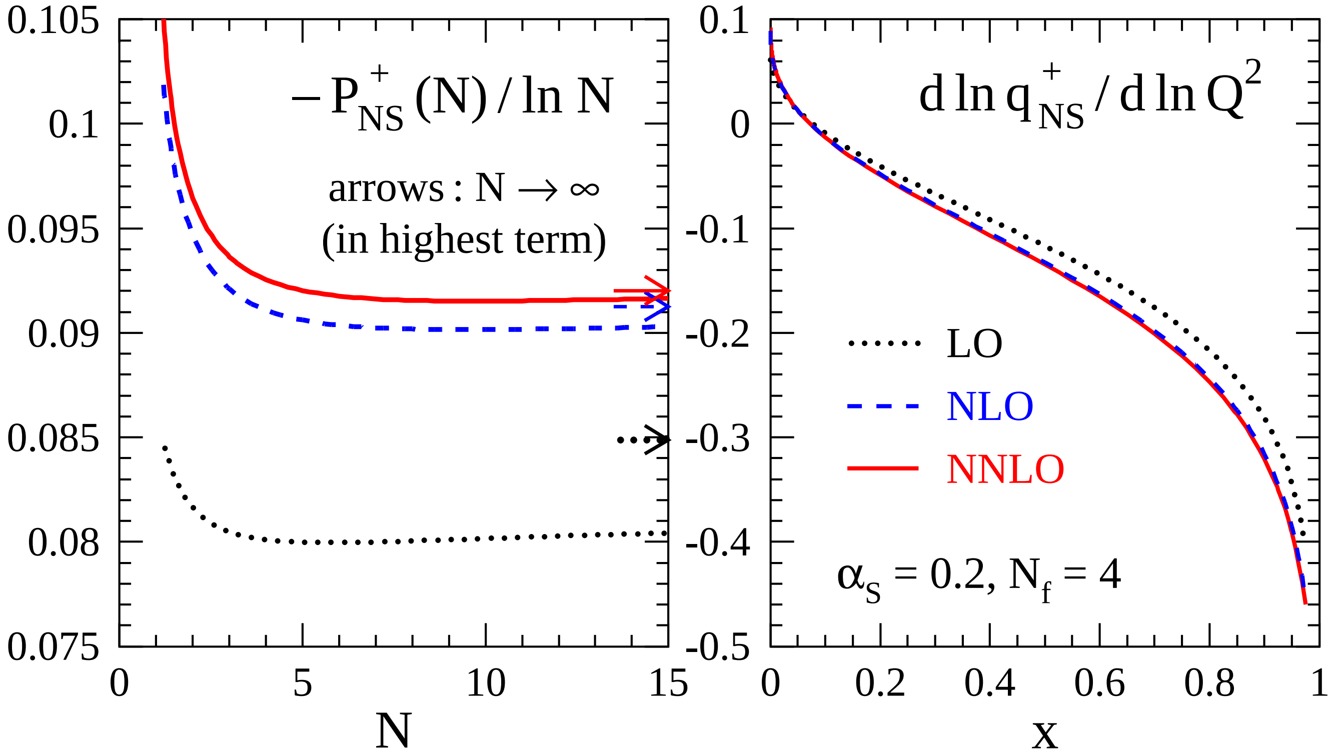


$N > 2$: off-diagonal (NNLO to about 5%) $\ll$ diagonal (NNLO to about 2%)

Parton evolution at large N / large x in $\overline{\text{MS}}$

Moments $A^N = \int_0^1 dx\, x^{N-1} A(x)$. **Non-singlet**$^+$: $u + \bar{u} - (d + \bar{d})$ etc



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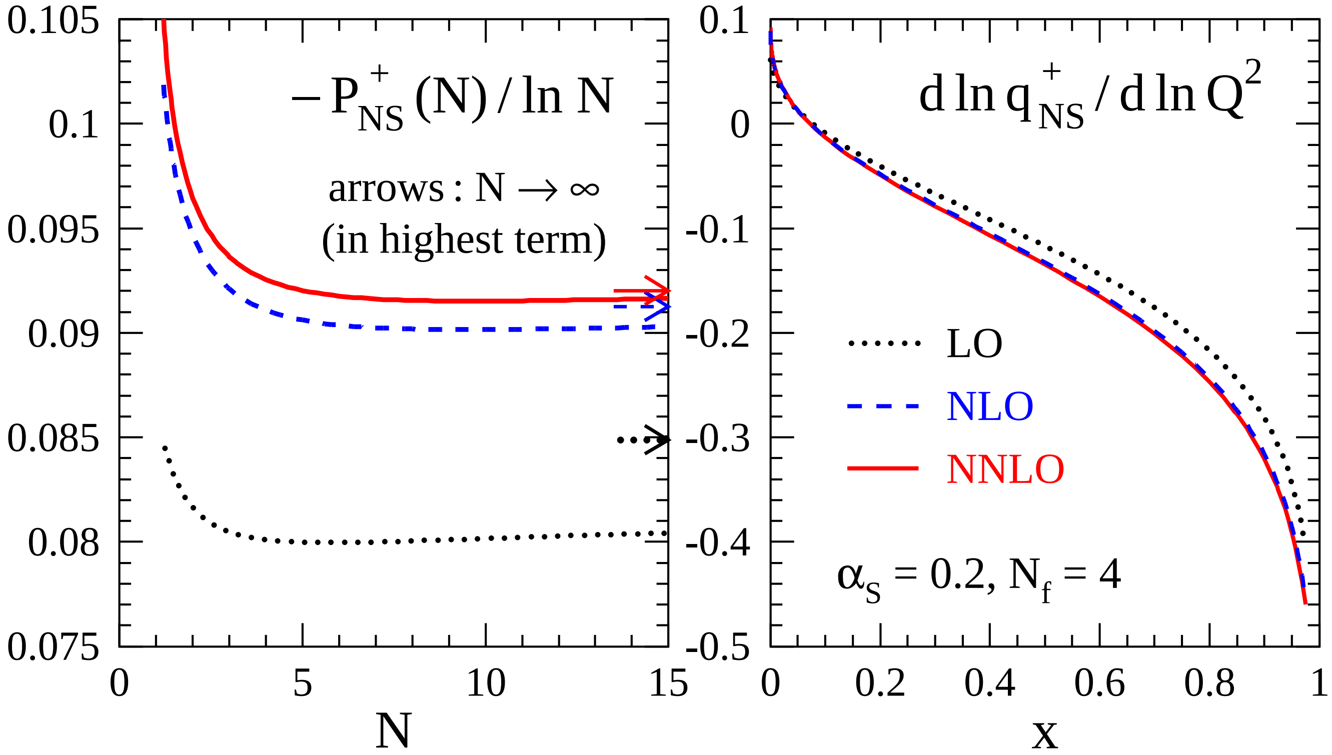


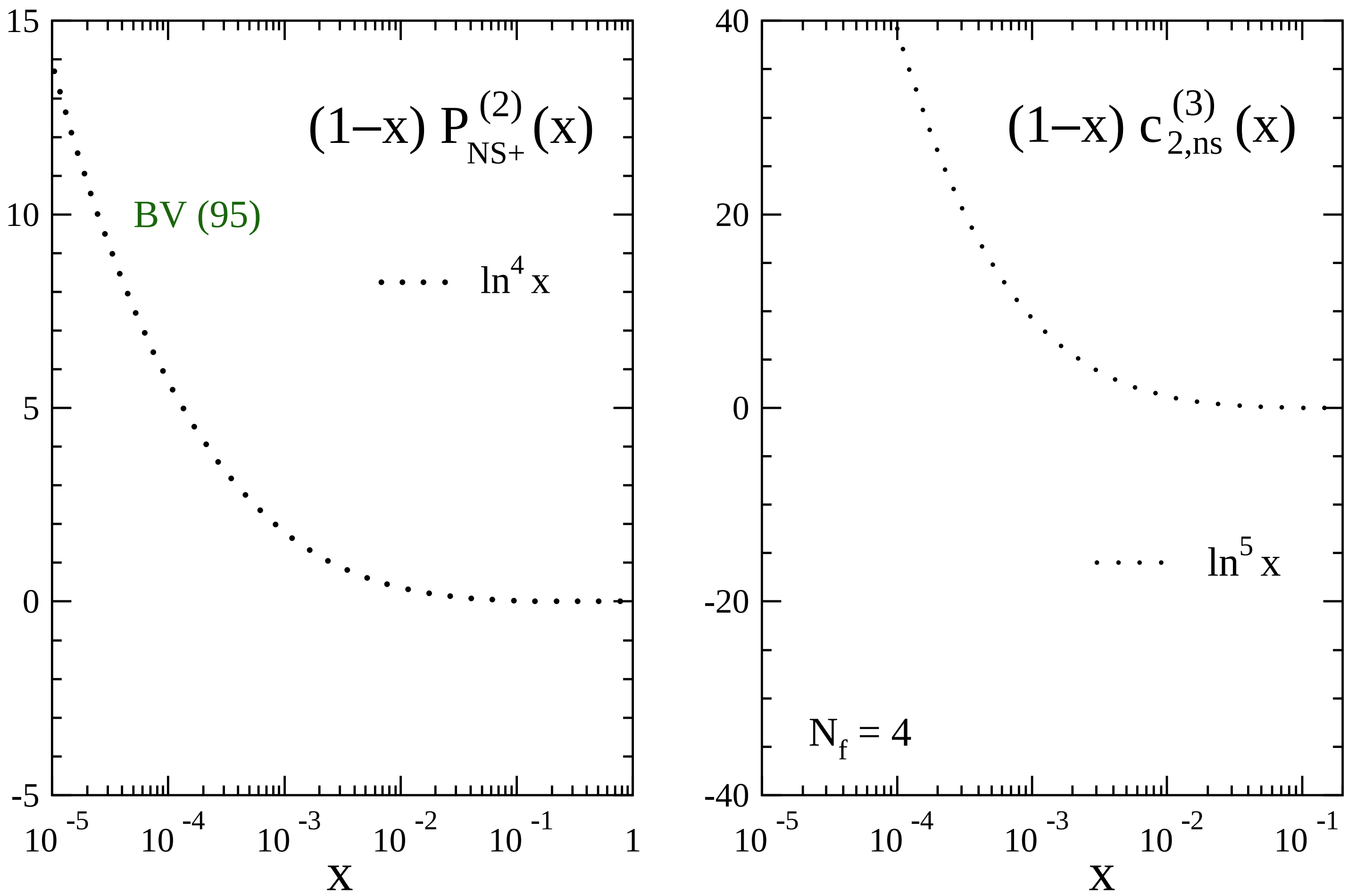
N^{3}LO: $P^+_{\rm ns}$ **computed for** $N = 2$, $n_f = 3$ **Baikov, Chetyrkin (06)**

$$P^+_{\rm ns} \;=\; -\,0.283\,\alpha_s\,[\,1 \,+\, 0.869\,\alpha_s \,+\, 0.798\,\alpha_s^2 \,+\, 0.926\,\alpha_s^3 \,+\, \ldots\,]$$

$N > 2$, $n_f > 3$: **similar / smaller** $\ln N$ **coeff's.** $\simeq$ **1% accuracy at** $\alpha_s \lesssim 0.25$

Non-singlet three-loop quantities at small x

Order α_s^n: small-x ‘double logs’ $\ln^{2k} x$ with $k \le n-1$ $(n-\frac{1}{2})$ in P (c)



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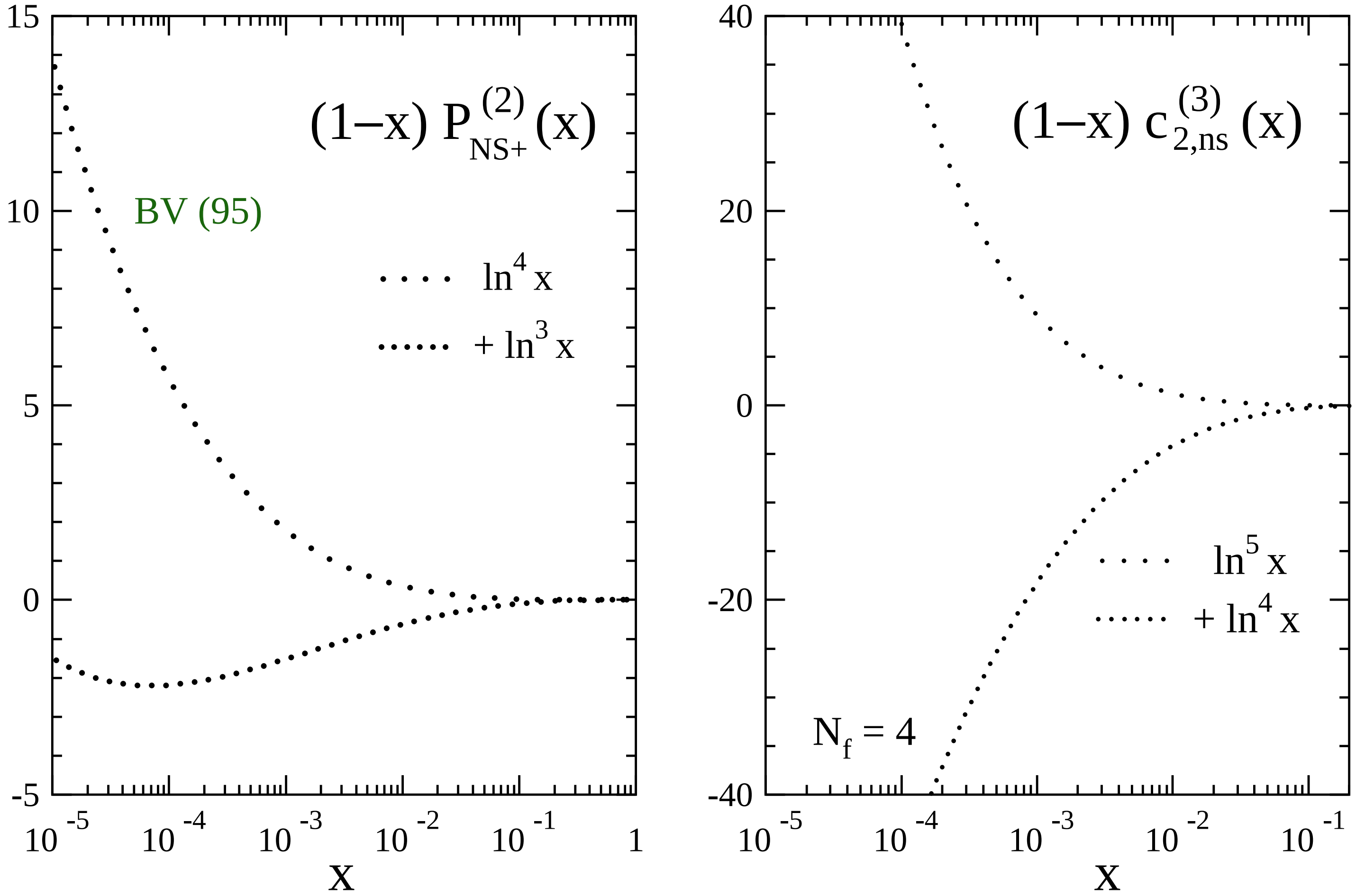

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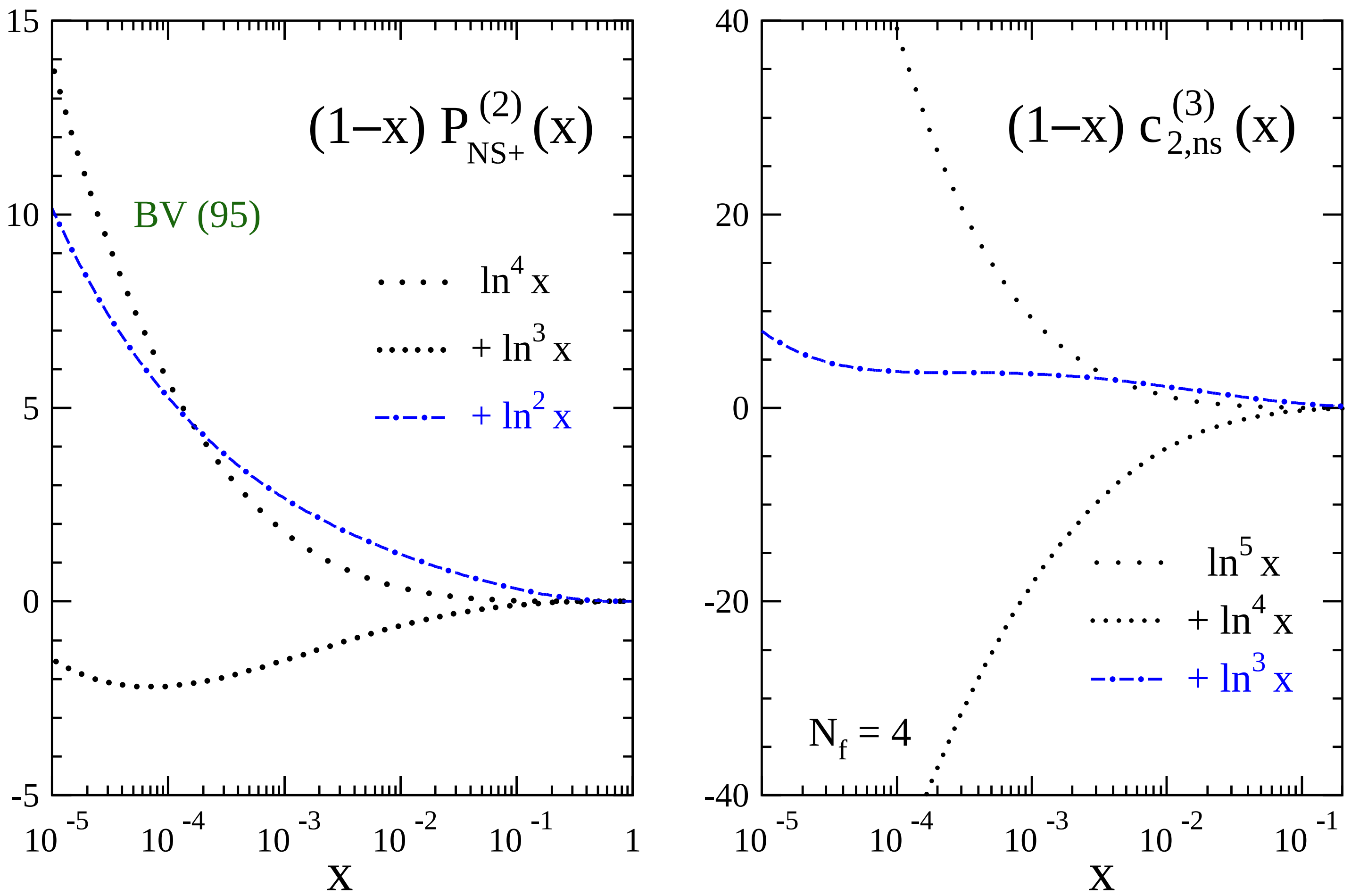

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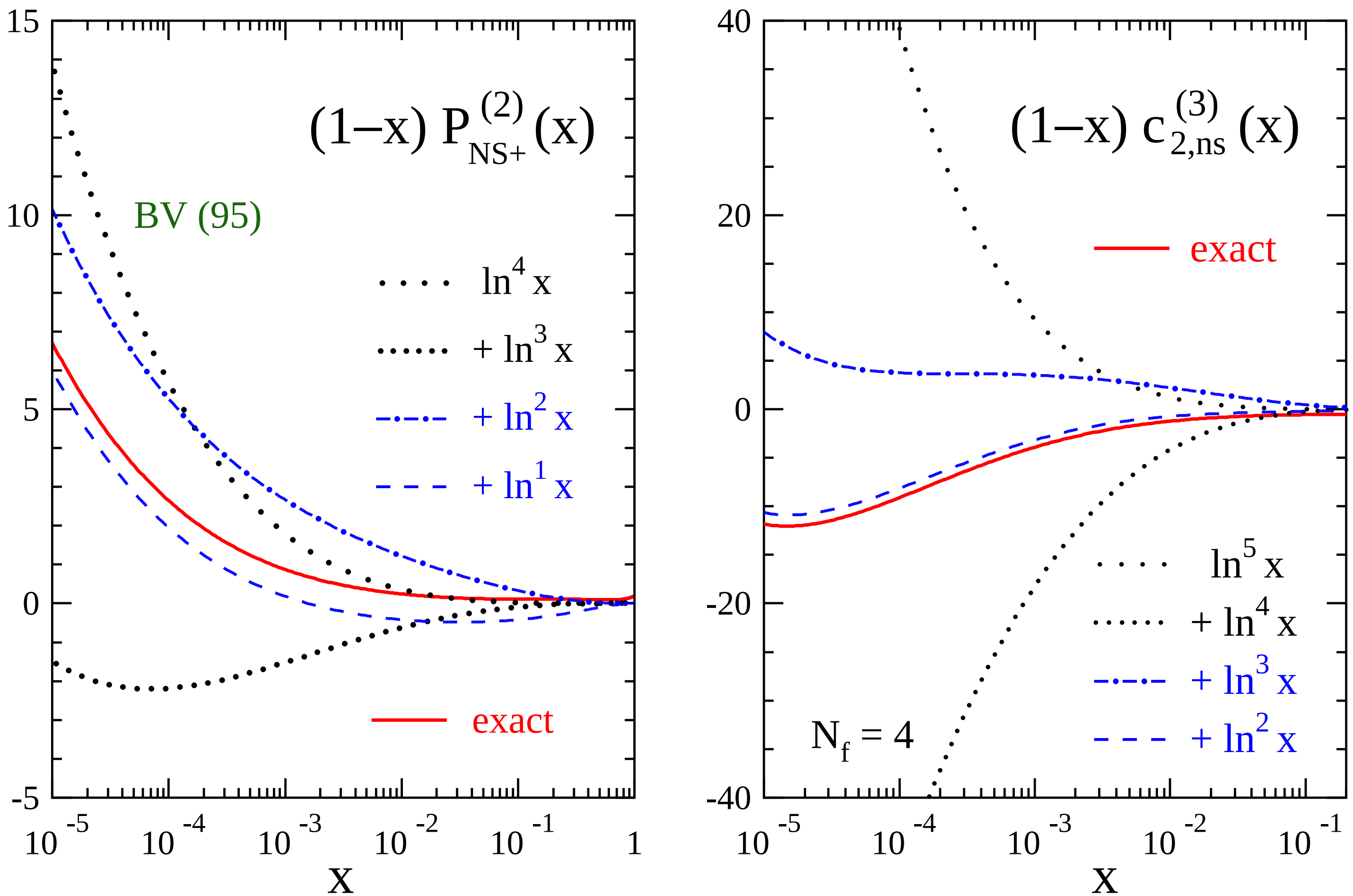
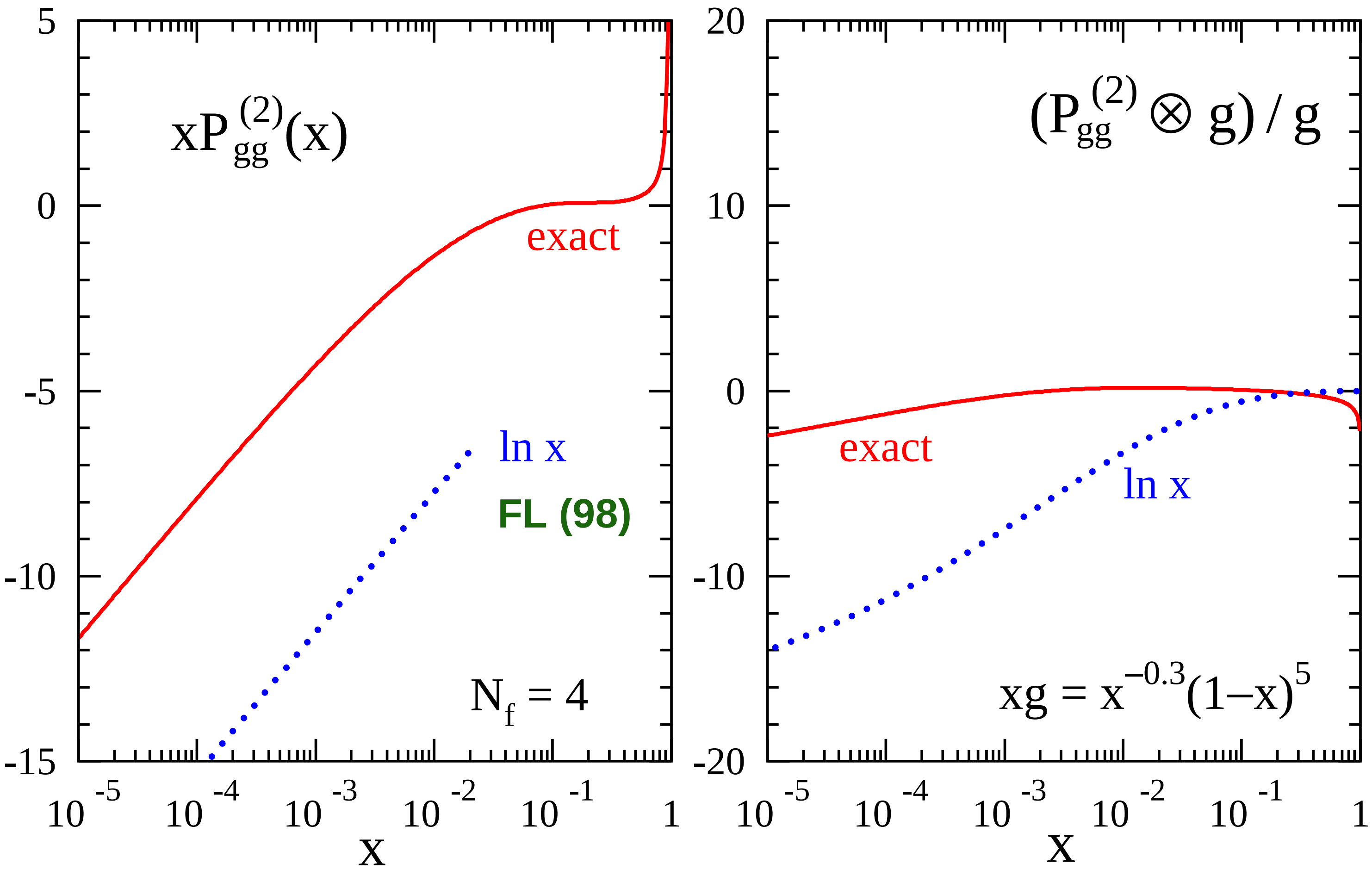


x-values for colliders: not even shape guaranteed by (next-to-) leading logs

Singlet splitting and evolution at small x

Splitting functions $\longrightarrow$ observables: **Mellin convolutions** $\int_x^1 \frac{dy}{y} P(y)\, f\left(\frac{x}{y}\right)$



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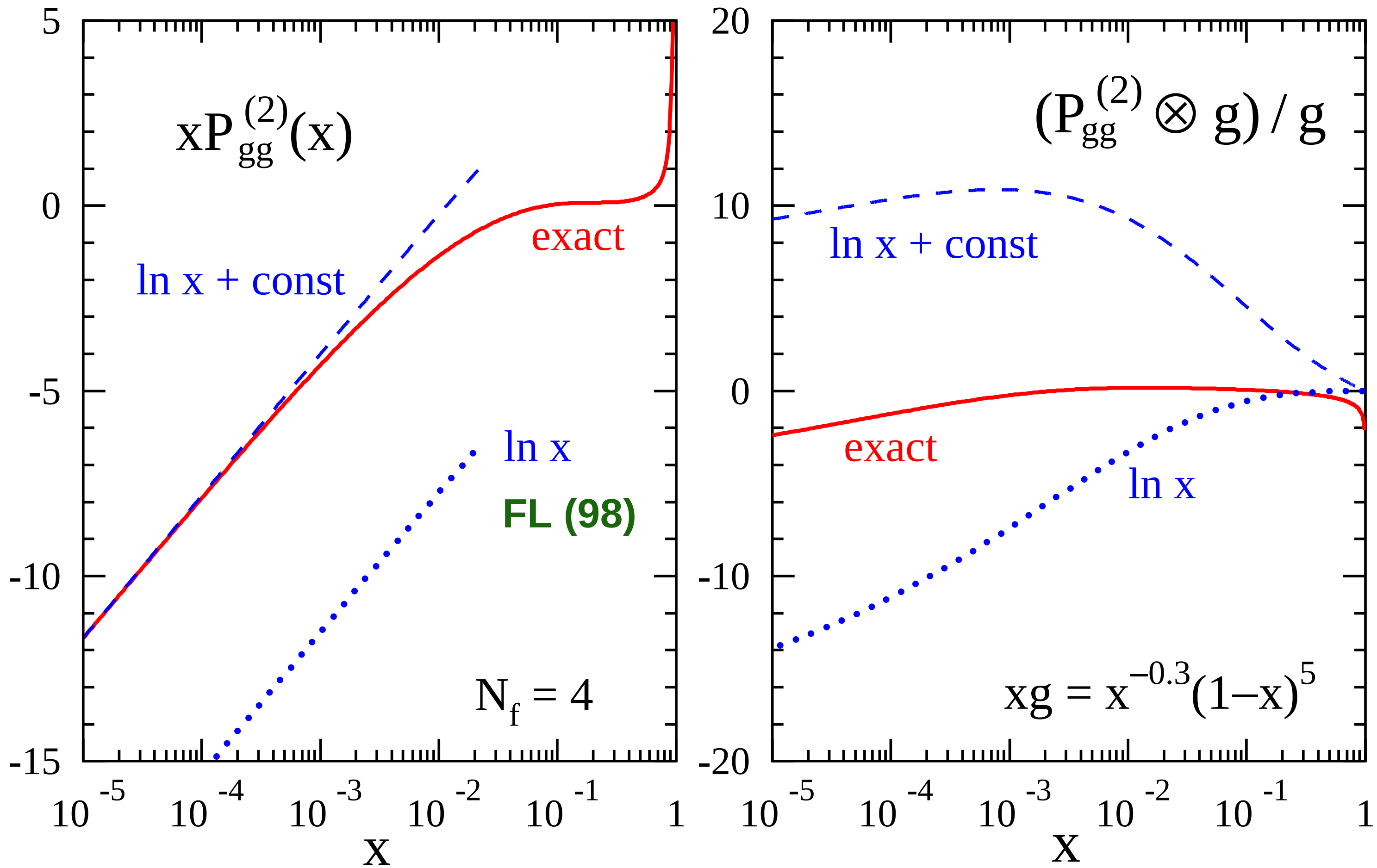
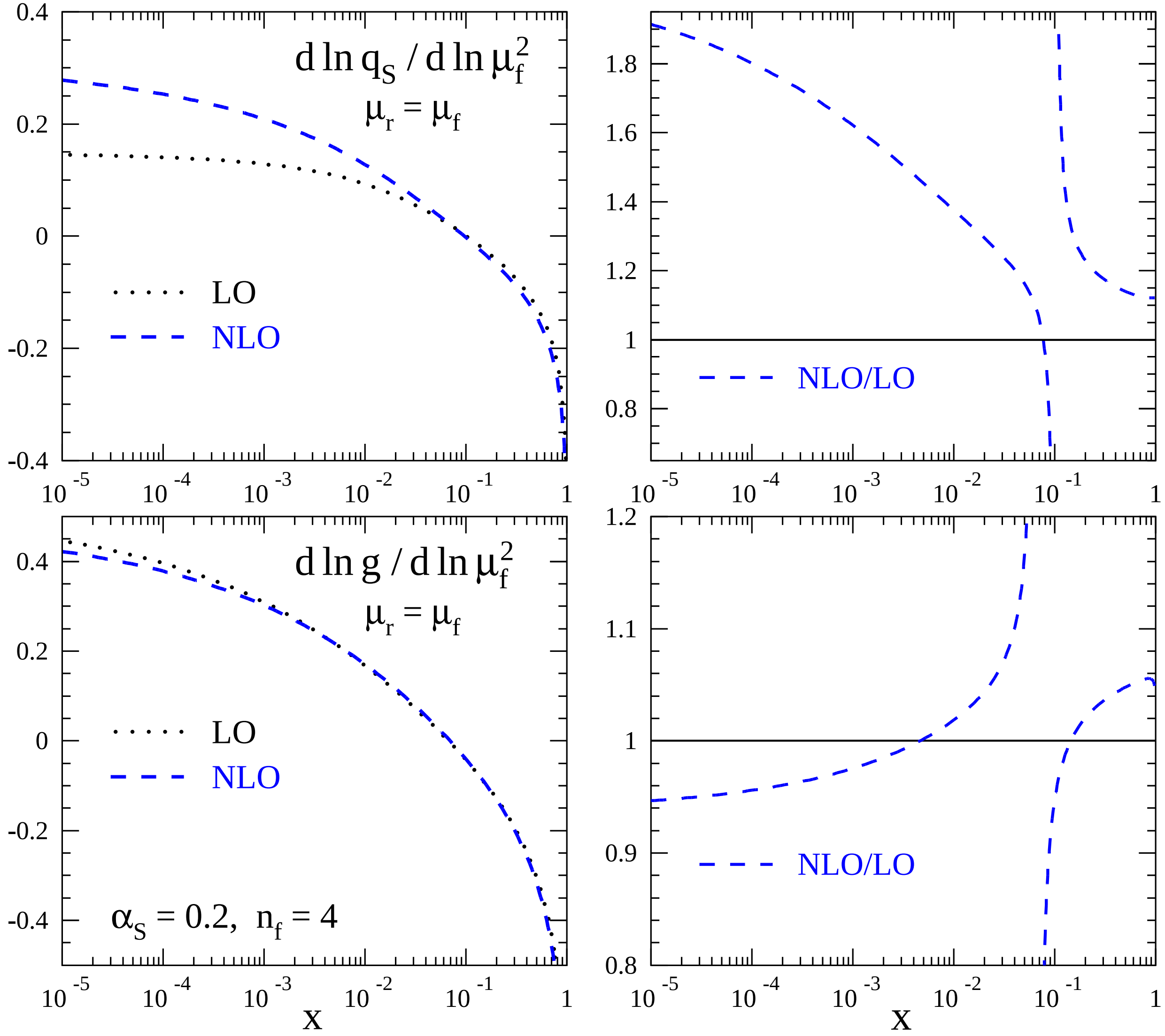


General: small-x limits of pQCD functions insufficient due to convolutions

Scale derivatives of singlet parton densities



scale ≈ 30 GeV^2

quark distrib'n

gluon distrib'n

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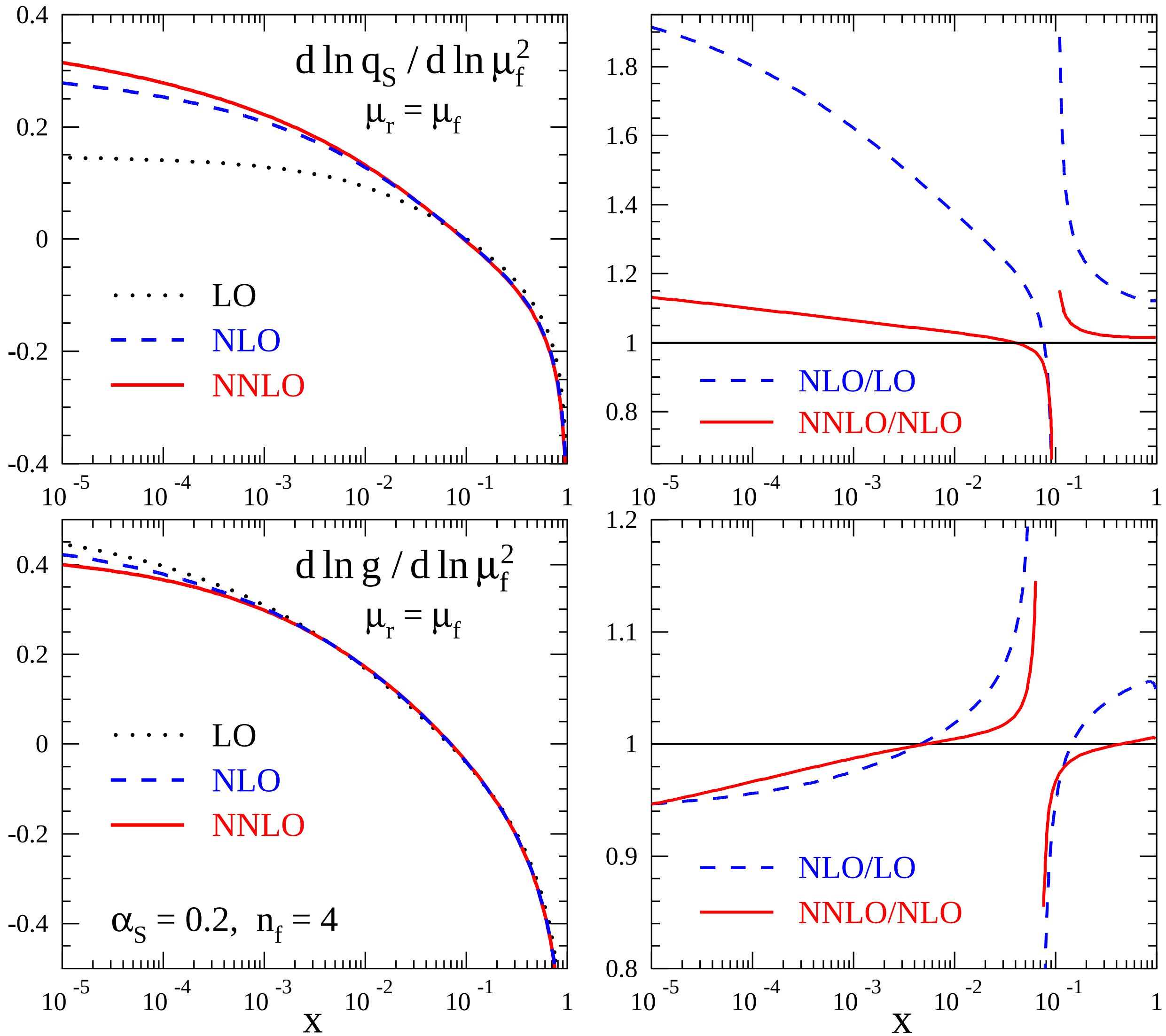

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Good convergence at collider-x – but NNLO is 10% for q_s at $x = 10^{-4}$

Available evolution codes including NNLO

x-space: discretization in x, $\mu_{\mathbf{f}}$ of coupled integro-differential equations

HOPPET (G. Salam, publ. 2008), http://hepforge.cedar.ac.uk/hoppet/ with J. Rojo

QCDNUM (M. Botje, now v.17β), http://www.nikhef.nl/˜h24/qcdnum/

N-space: ordinary diff. eqs., time-ordered exponential, $N \rightarrow x$ numerical

QCD-Pegasus (A.V., publ. 2004), http://www.liv.ac.uk/˜avogt/pegasus.html

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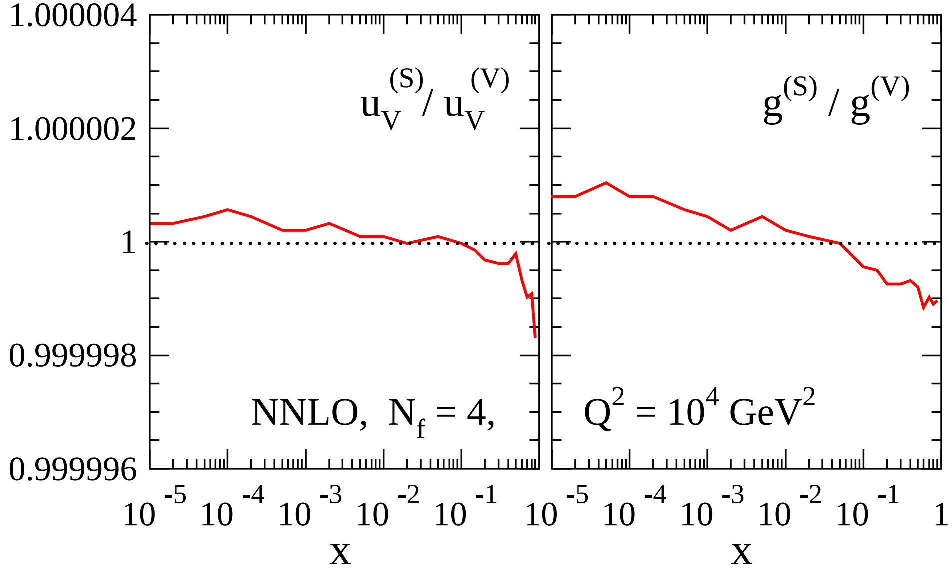
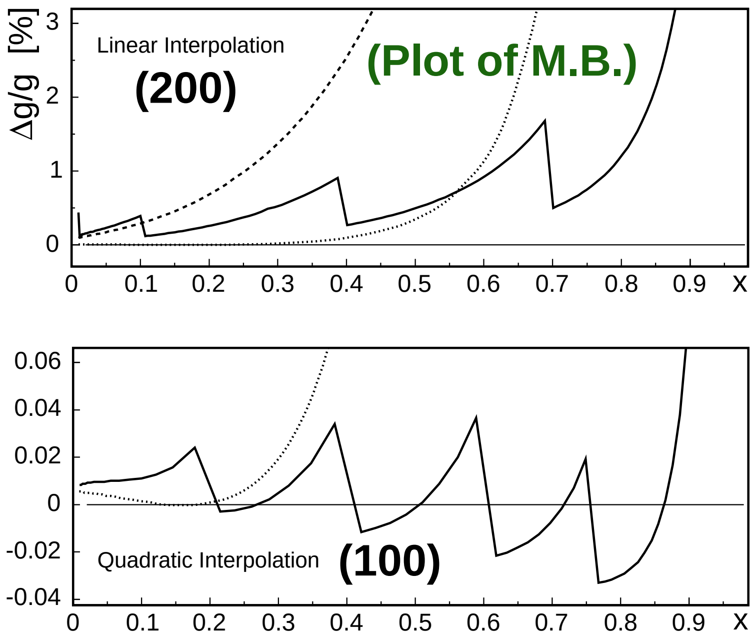
Sample comparisons





Benchmark tables for the parton evolution

Evolution of Les Houches (2001) reference input at scale $\mu_{\mathrm{f},0}^2 = 2$ GeV^2

$$xu_v(x, \mu_{\mathrm{f},0}^2) \;=\; 5.1072\; x^{0.8}\; (1-x)^3 \quad , \quad \ldots$$
$$xg\,(x, \mu_{\mathrm{f},0}^2) \;=\; 1.7000\, x^{-0.1}\, (1-x)^5$$

with

$$\alpha_s(\mu_r^2 = 2\ \text{GeV}^2) \;=\; 0.35$$

at LO, NLO and NNLO, for $\mu_r = \{0.5,\ 1,\ 2\}\,\mu_f$, with fixed and variable N_f

Use of two completely different codes. G. Salam, A.V. (2002, 05)

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Five-digit agreement over wide range in x, μ_f^2 $\Rightarrow$ reference tables

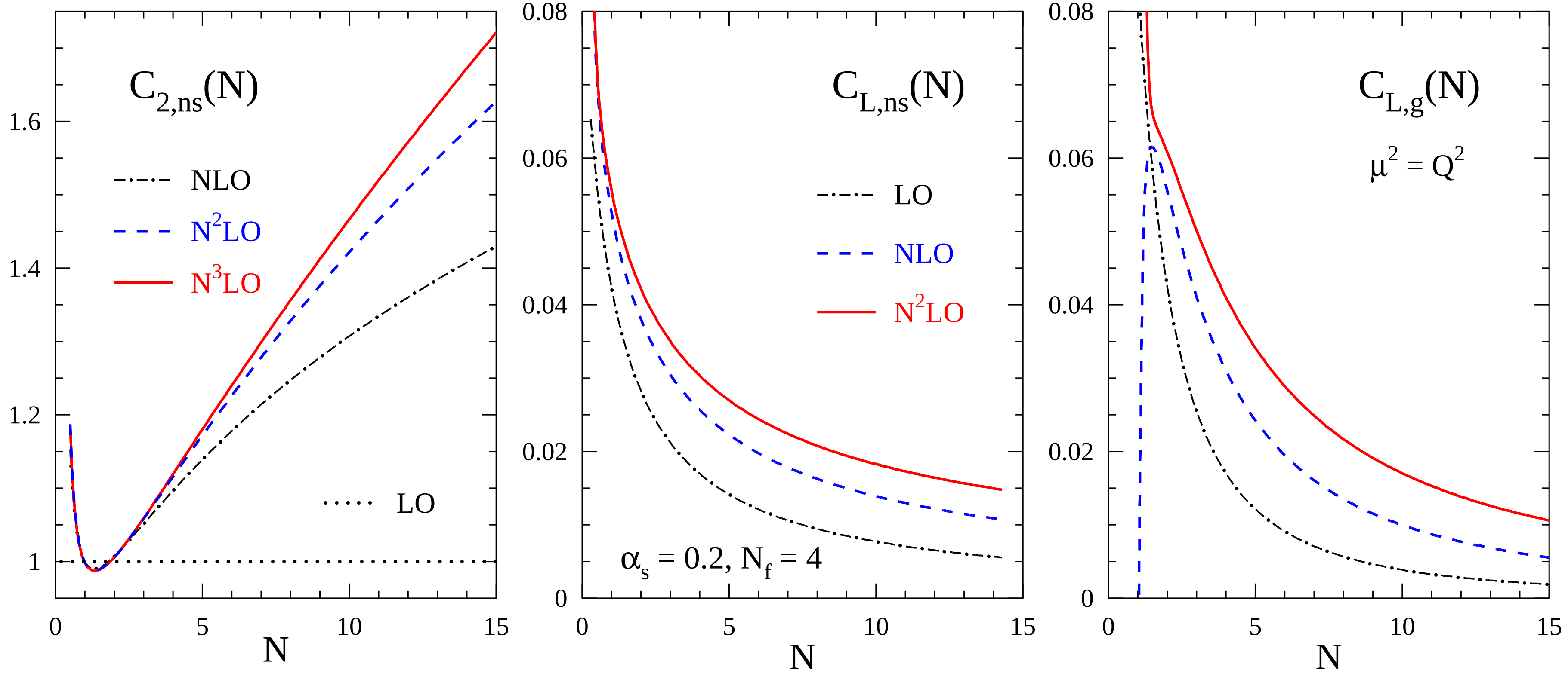
Example: (iterated) NNLO results, $\mu_r = 2\mu_f$, $N_f = 4$ at $\mu_f^2 = 10^4$ GeV^2

$$x = 10^{-5}\ , \quad xu_v = 2.9032\cdot 10^{-3}\ , \quad \ldots\ , \quad xg = 2.2307\cdot 10^{2}$$
$$\ldots x = \ 0.9\ \ , \quad xu_v = 3.6527\cdot 10^{-4}\ , \quad \ldots\ , \quad xg = 1.2489\cdot 10^{-6}$$

Full tables in hep-ph/0204316 (Les Houches), hep-ph/0511119 (HERA-LHC)

Coefficient functions for $F_{2,L}$ in N-space

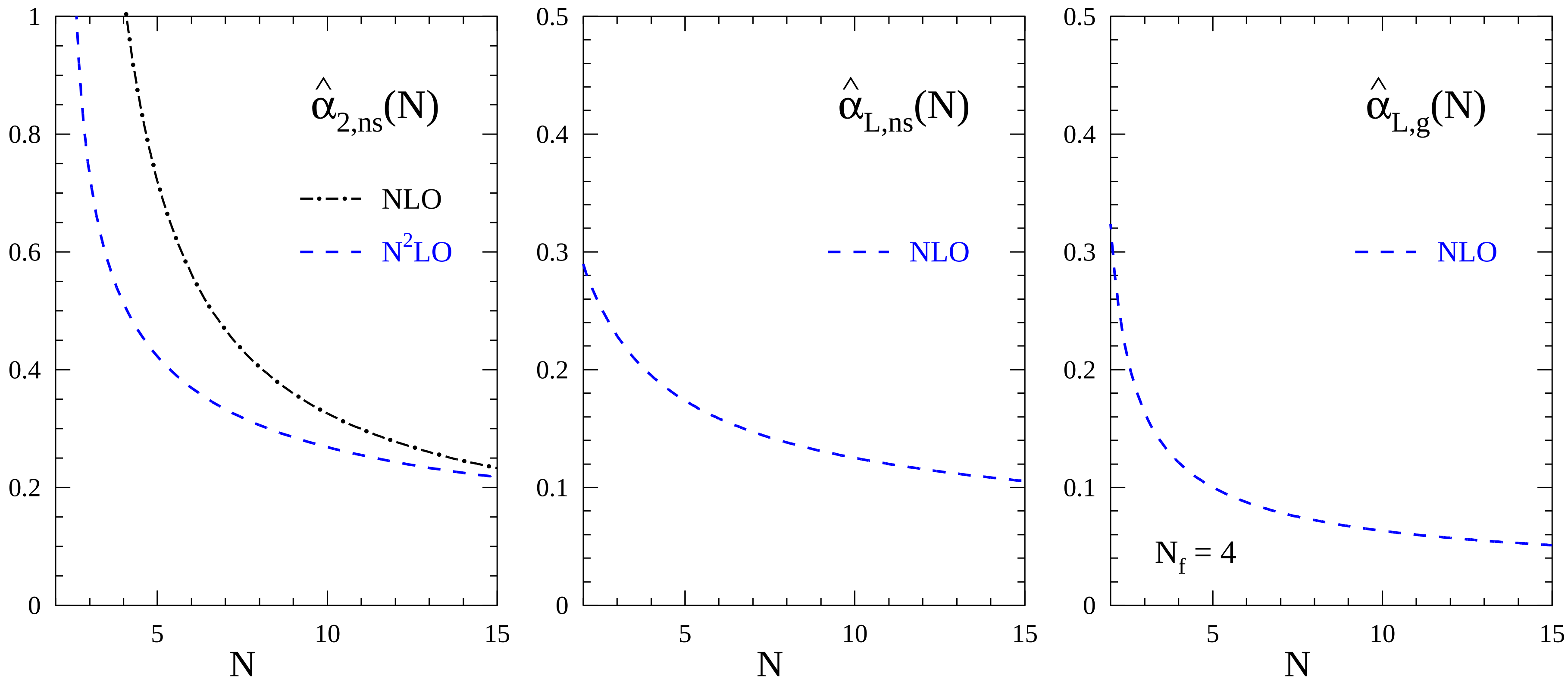
Recall: contributions of order α_s^n are of N^{n}LO for F_2, but N^{n-1}LO for F_L



$N > 2$: corrections for $F_L \gg$ corrections for $F_2 \gg$ corr's for parton evolution

Indications that the expansion is asymptotic?

$\widehat{\alpha}_a^{(n)}(N)$: α_s **for which effect of** $c_a^{(n)}(N)$ **is half that of** $c_a^{(n-1)}(N)$



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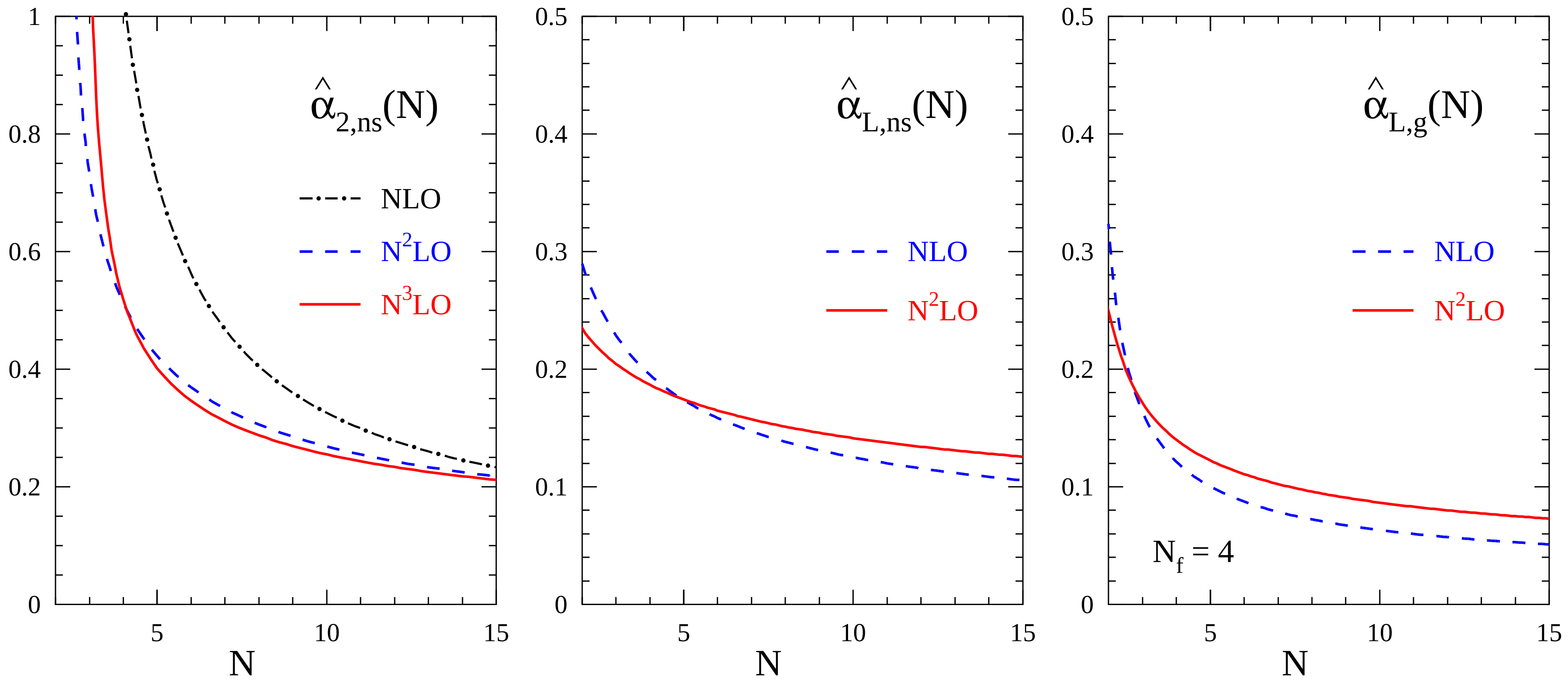
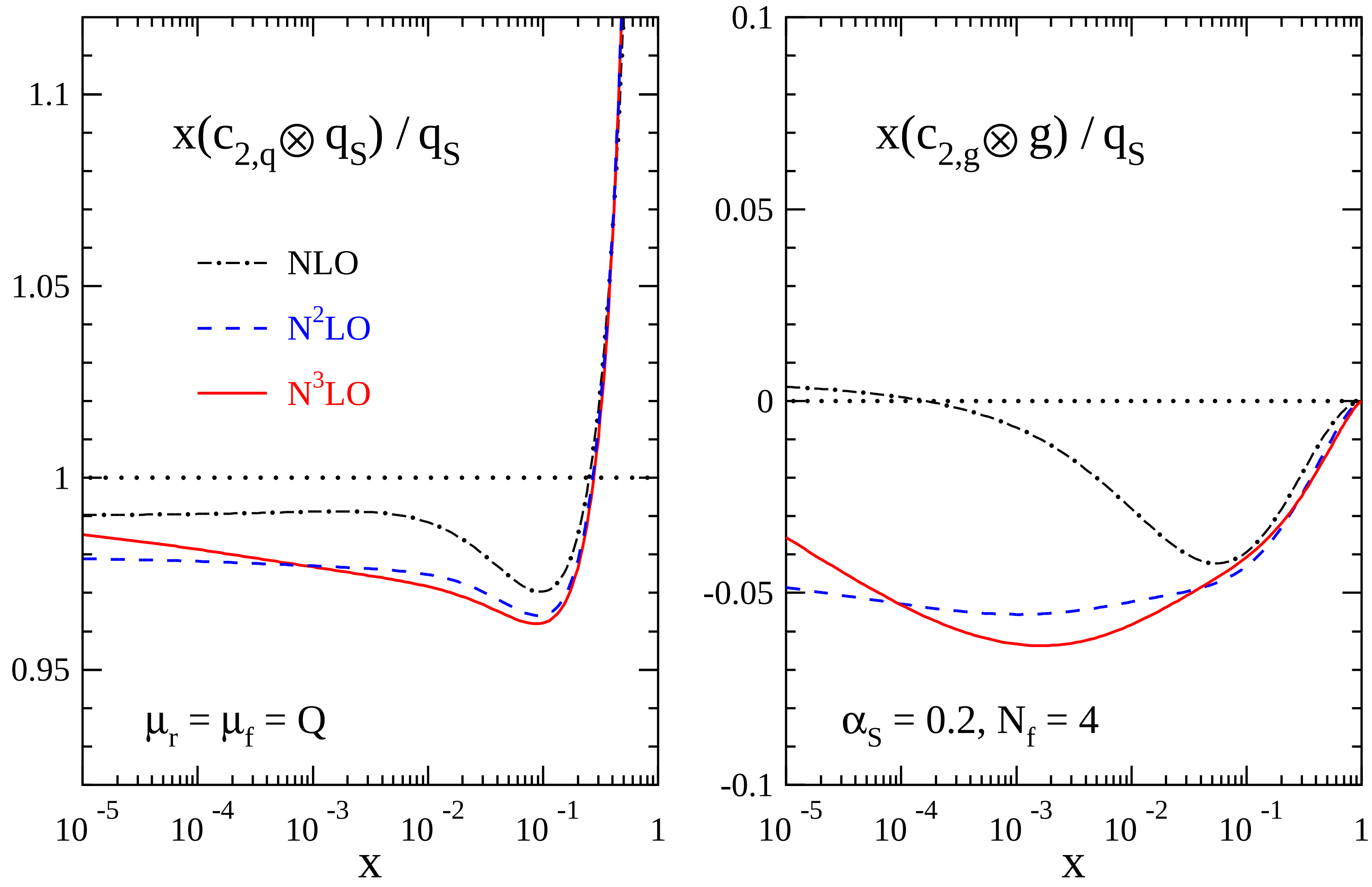


If coefficients grow factorially: $\widehat{\alpha}_a^{(n)}(N)$ decreasing for increasing order n

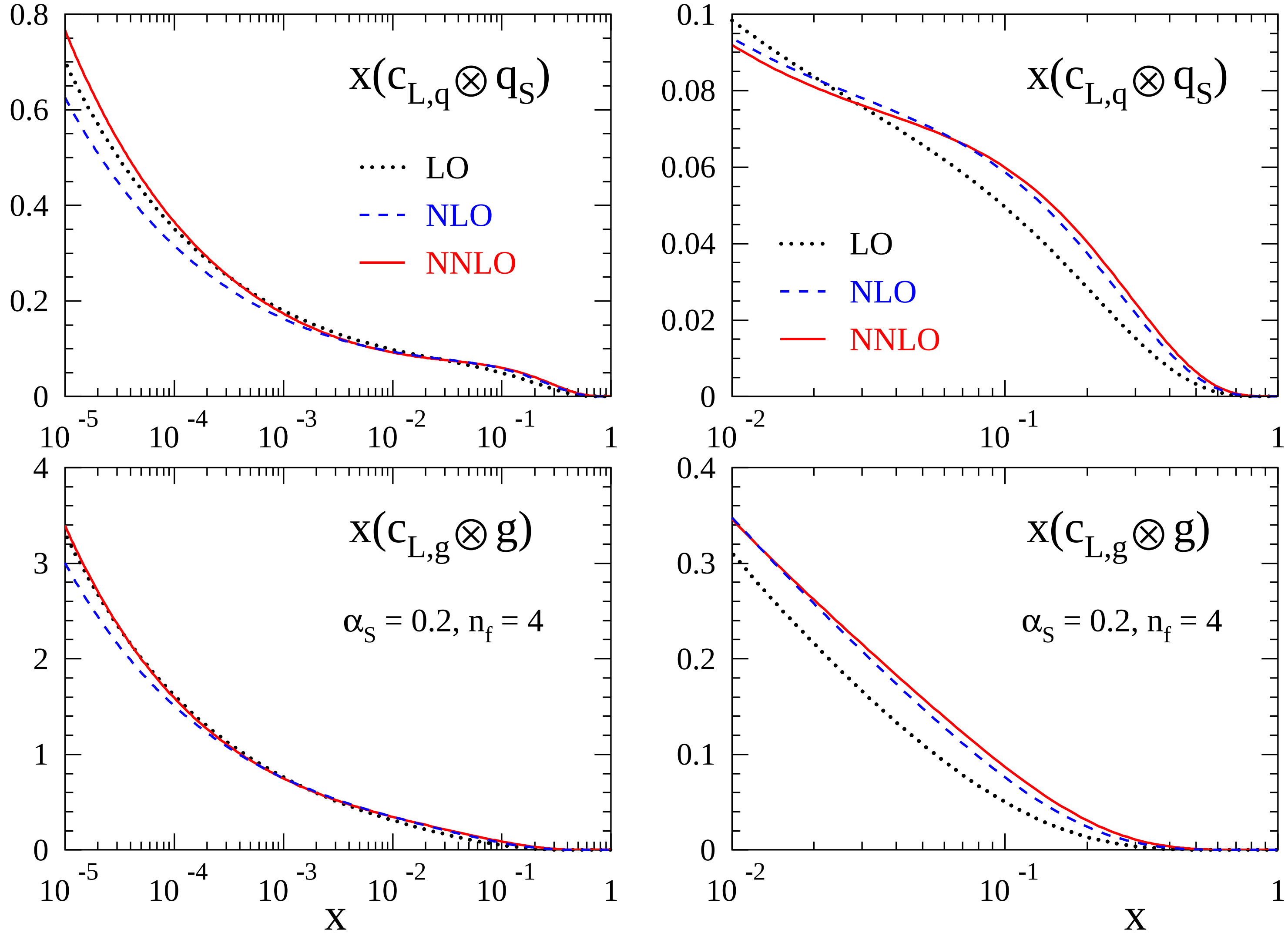
No sign of asymptotic character. Convergence for F_L: higher scales

Perturbative expansion of F_2 in x-space



Total N^{3}LO corr. $\leq 1\%$ at $4 \cdot 10^{-5} \leq x \leq 0.65$. N^{3}LO $>$ NNLO for $x \lesssim 10^{-8}$

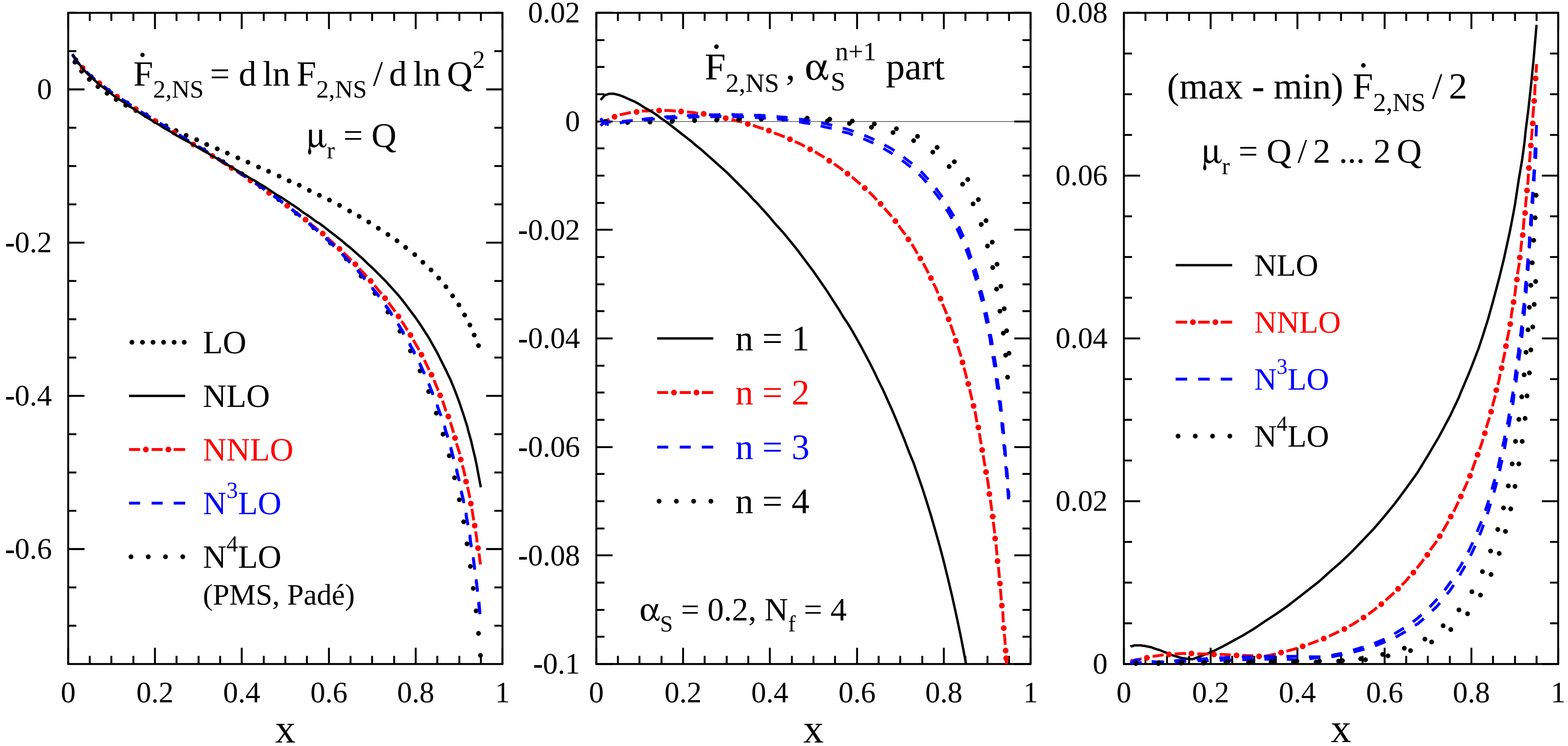
Perturbative expansion of F_L in x-space



Total N^{3}LO corr. $\leq 10\%$ at $5 \cdot 10^{-5} \leq x \leq 0.1$. Lower x: N^{3}LO > NNLO

Scaling violations of non-singlet part of F_2

Large-x ($\gtrsim 10^{-2}$) convergence of P series: effect. N^{3}LO scaling violations



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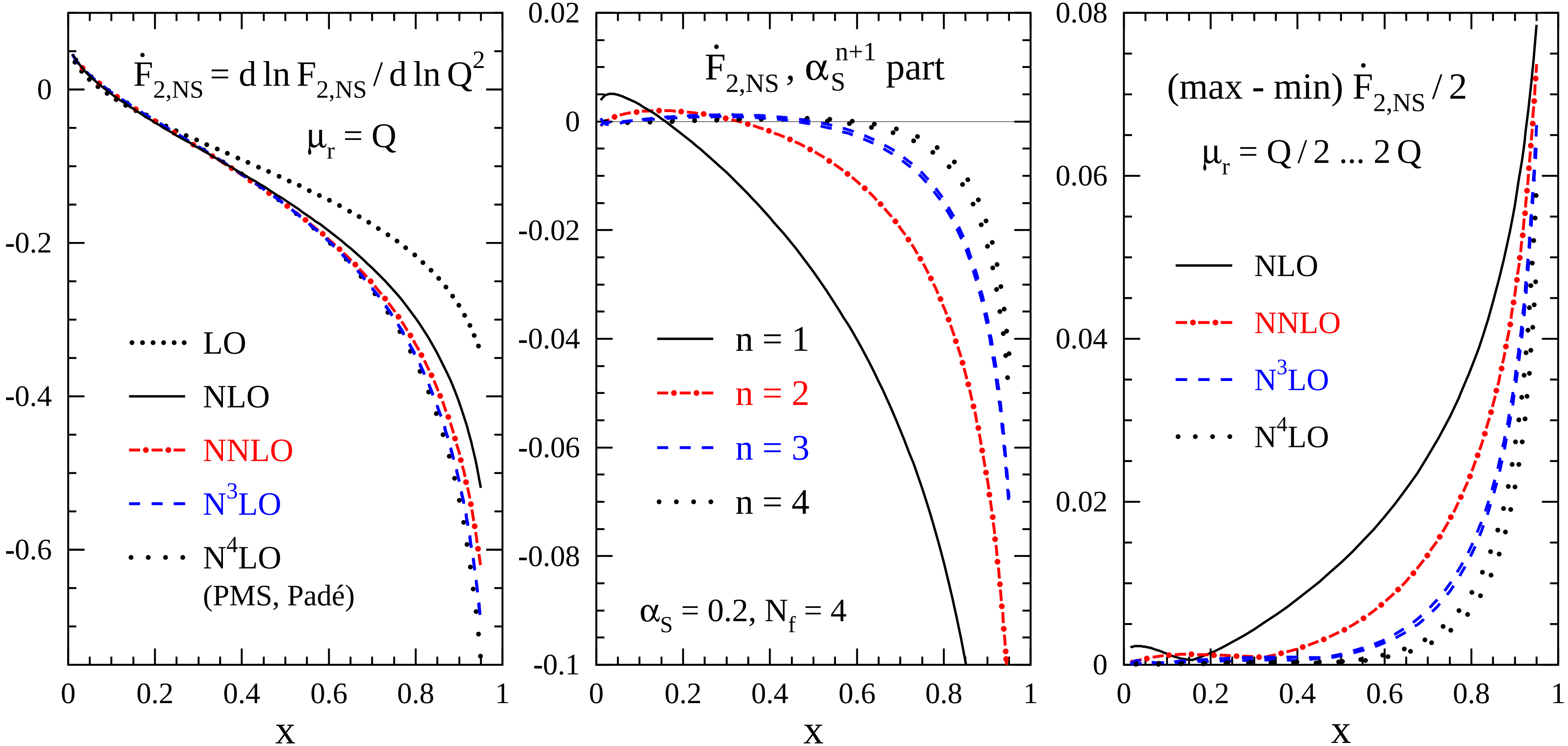


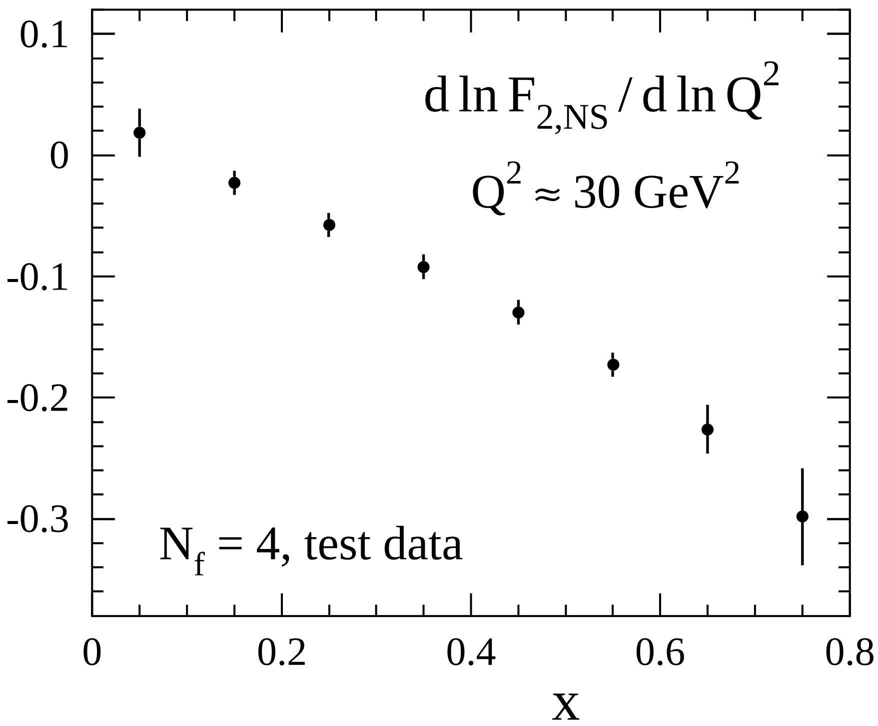
Potential for 'gold-plated' α_s determination from structure functions. LHeC ?

Very large x ($\gtrsim 0.8$): further improvement by soft-gluon resumm. possible

Accuracy of α_s from Q^2-dependence of $F_{2,\mathrm{ns}}$

Toy analysis: $F_{2,\mathrm{NS}}$ known at Q_0^2,

one-parameter fits to $\dfrac{dF_{2,\mathrm{NS}}}{d\ln Q^2}$ 'data'



Central values, scale uncertainties

$$\alpha_s(Q_0^2)_{\mathrm{NLO}} = 0.208 \begin{smallmatrix} +0.021 \\ -0.013 \end{smallmatrix}$$

$$\alpha_s(Q_0^2)_{\mathrm{NNLO}} = 0.201 \begin{smallmatrix} +0.008 \\ -0.0025 \end{smallmatrix}$$

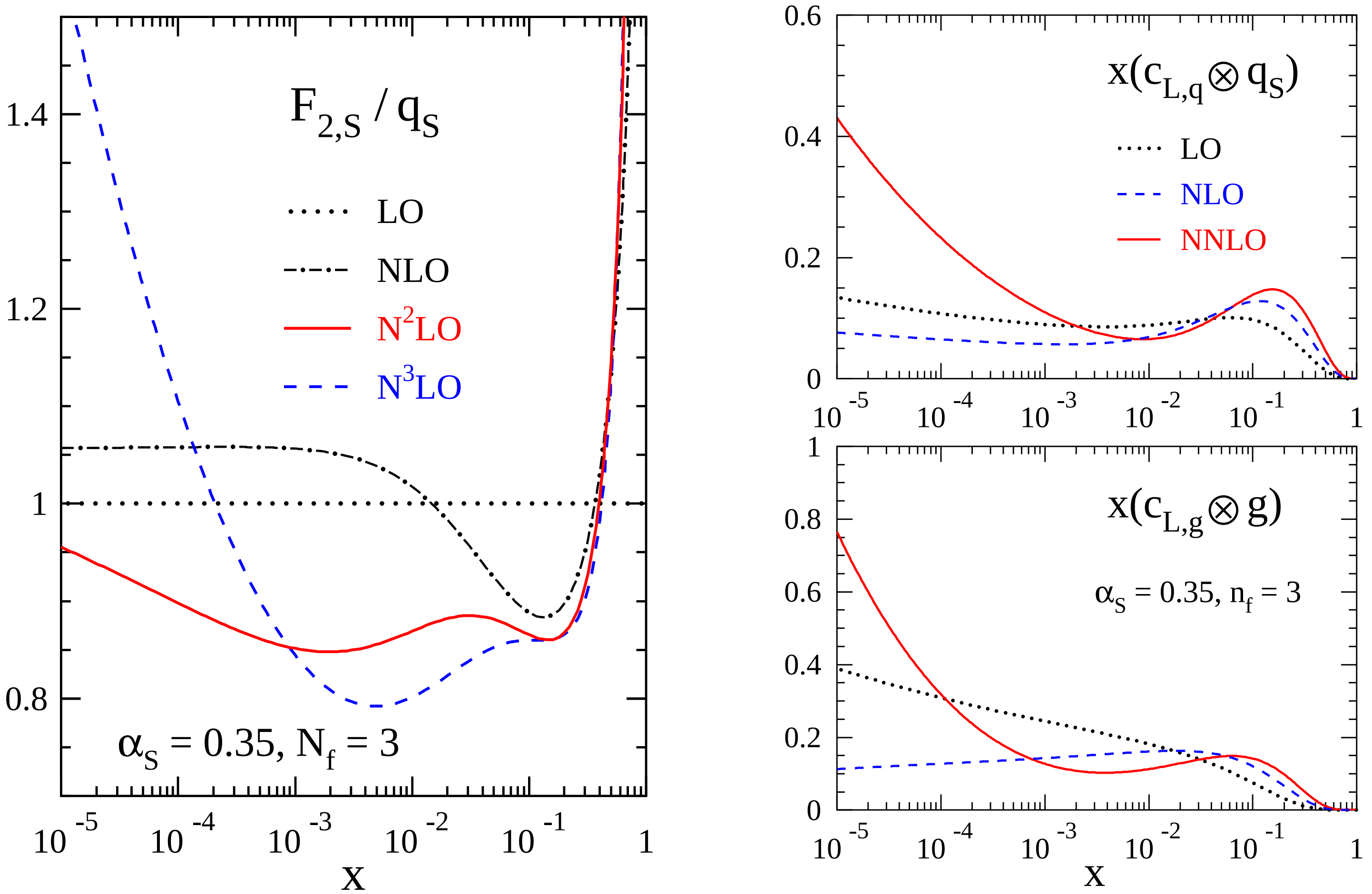
$$\alpha_s(Q_0^2)_{\mathrm{N^3LO}} = 0.200 \begin{smallmatrix} +0.003 \\ -0.001 \end{smallmatrix}$$

$$\alpha_s(Q_0^2)_{\mathrm{N^4LO}} = 0.200 \begin{smallmatrix} +0.0015 \\ -0.0005 \end{smallmatrix}$$

M_Z^2: Diff's, errors smaller by factor 3

Large x: $\Delta_{\mathrm{pert.}}\,\alpha_s(M_Z) < 1\%$ – but many other issues at such accuracy

Disclaimer (II): beware of small x at low Q^2



$Q^2 \approx 2$ GeV^2: expansion out of control at $x \lesssim 10^{-4}$ (F_2) and $x \lesssim 10^{-3}$ (F_L)

Cut at $Q^2 \simeq 5$ GeV^2 to be safe $\Rightarrow$ gluon at $x \approx 10^{-4}$ from DIS only at LHeC

Concluding remarks

- Last 18 years: tremendous progress in deep-inelastic scattering
  (collider measurements at HERA, higher-order calculations, . . .)
- Still many open questions (see yesterday's overview by Altarelli)
  $\Rightarrow$ physics at LHeC very worthwhile (disregarding cost aspects)

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- **NNLO perfectly fine down to at least $x \simeq 10^{-3}$ at scales $\gtrsim 5$ GeV^2**
  Don't trust resummers seeing sizable effects at $0.03 \gtrsim x \gtrsim 10^{-3}$
- **Eff. N^{3}LO for $F_{2,3}$ at $x \gtrsim 10^{-2}$ $\Rightarrow$ $\Delta_{\rm pert.}\, \alpha_s(M_Z) < 1\%$ from DIS**

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- NNLO presently restricted to few (DIS, Drell-Yan type) processes
  Jet progress (e^+e^- now) Gehrmann [-de Ridder], Glover, Heinrich (08)
- Hard work also needed on heavy quarks. Motivation by LHeC . . .