Decoherence between the initial and final state radiation in a dense QCD medium

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N. Armesto, H. Ma, Y. Mehtar-Tani and C. A. Salgado,

JHEP 1312 (2013) 052

3rd Workshop on Jet Modification in the RHIC and LHC Era

Wayne State Univ., Detroit MI, USA

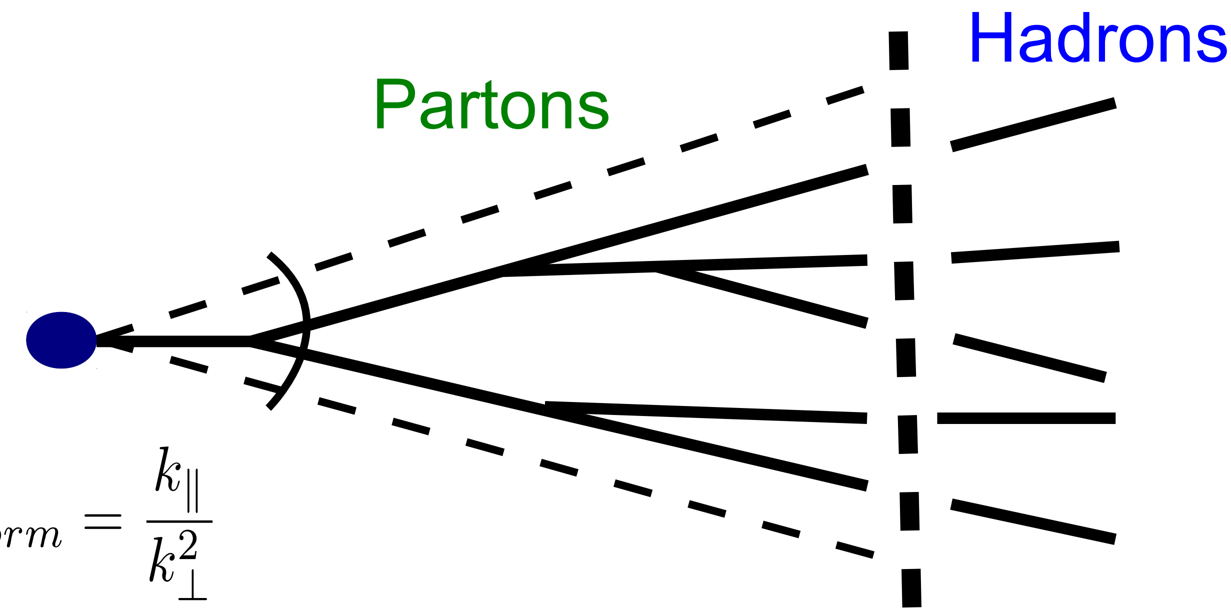
18-20 August, 2014

Outline

- **Brief review of color coherence in pQCD**
- **Medium induced radiation and color decoherence**
- **Color (de)coherence between the initial and final state radiation**
- **Generalization to pA collisions**

Brief review of color coherence in pQCD

What is a jet?



$$\tau_{form} = \frac{k_{\parallel}}{k_{\perp}^{2}}$$

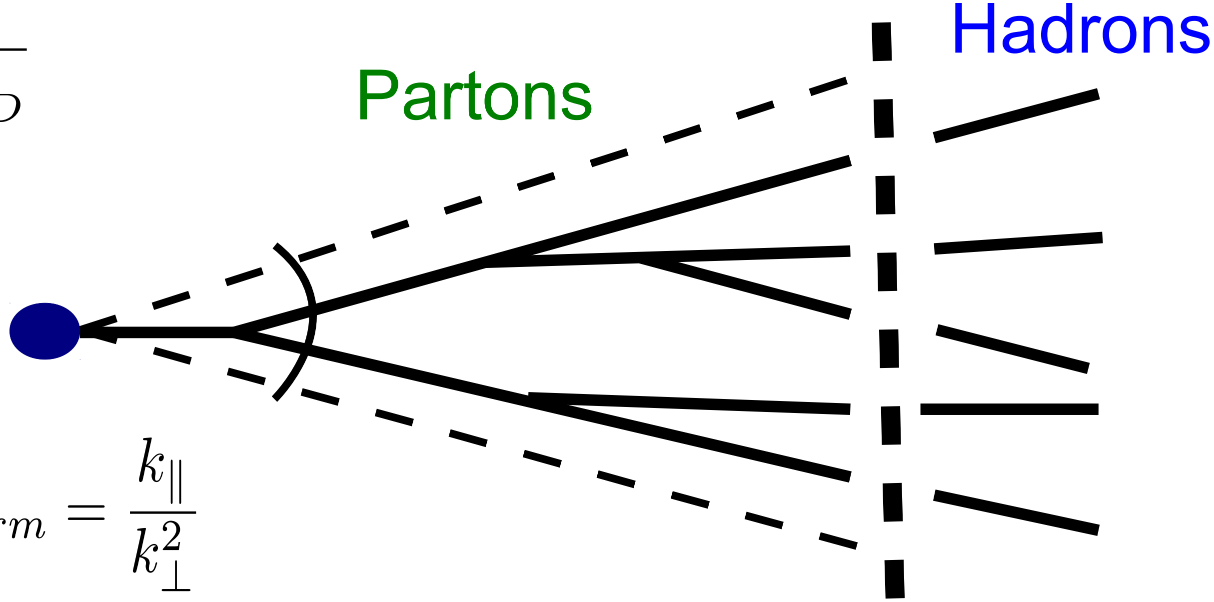
$$\tau_{hadr} = \frac{k_{\parallel}}{\Lambda_{QCD}^{2}}$$

- A jet is born as a *hard parton* which fragments into many partons when the time goes by decreasing virtuality down to a non-perturbative scale where *hadronization* happens.
- Parton shower is well described within pQCD
- *LPHD:* hadronization does not affect exclusive observables: jet shape, energy distribution, etc.

What is a jet?

Large domain of pQCD

$$\frac{1}{E} < \tau < \frac{E}{\Lambda_{QCD}^2}$$



$$\tau_{form} = \frac{k_\parallel}{k_\perp^2}$$

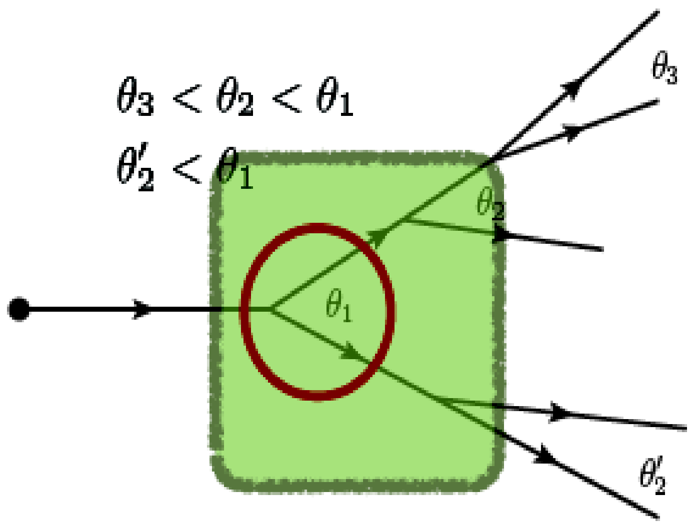
$$\tau_{hadr} = \frac{k_\parallel}{\Lambda_{QCD}^2}$$

- Inclusive jet observables are determined by two scales

Jet transverse mass $M_\perp = E\,\theta_{jet}$

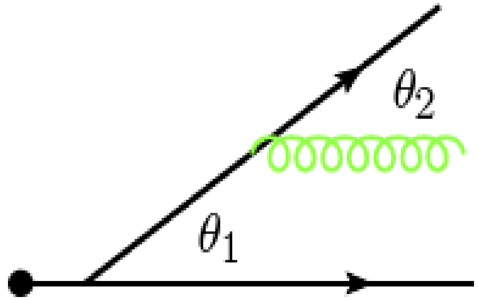
Non perturbative scale $Q_0 \sim \Lambda_{QCD}$

Parton shower: basic building blocks



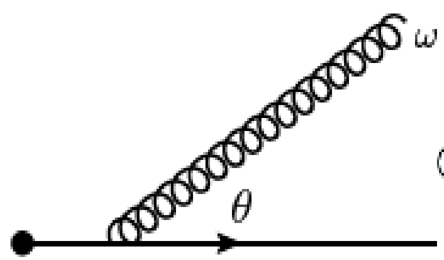
- ✓ Markovian Process
- ✓ Building block for QCD evolution ➙ MC implementation

Angular ordering



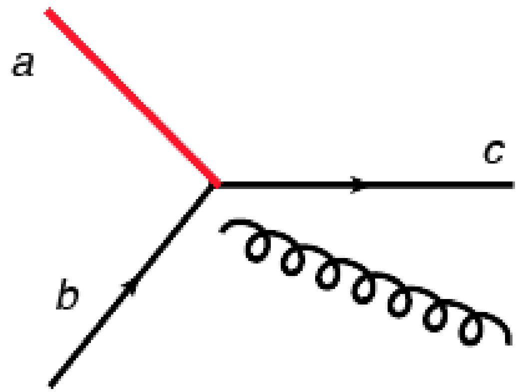
$\propto \Theta(\theta_1 - \theta_2)$

Leading singularities



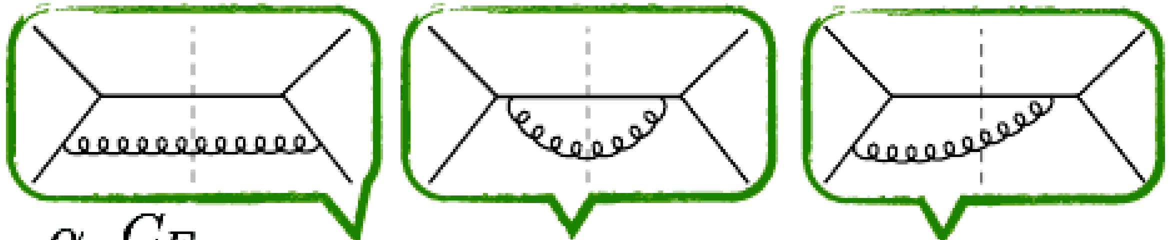
$$\propto \frac{\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{d\theta}{\theta}$$

Interference pattern between Initial-Final State



$Q_a = 0 \Rightarrow Singlet$

$Q_a \neq 0 \Rightarrow Octet$

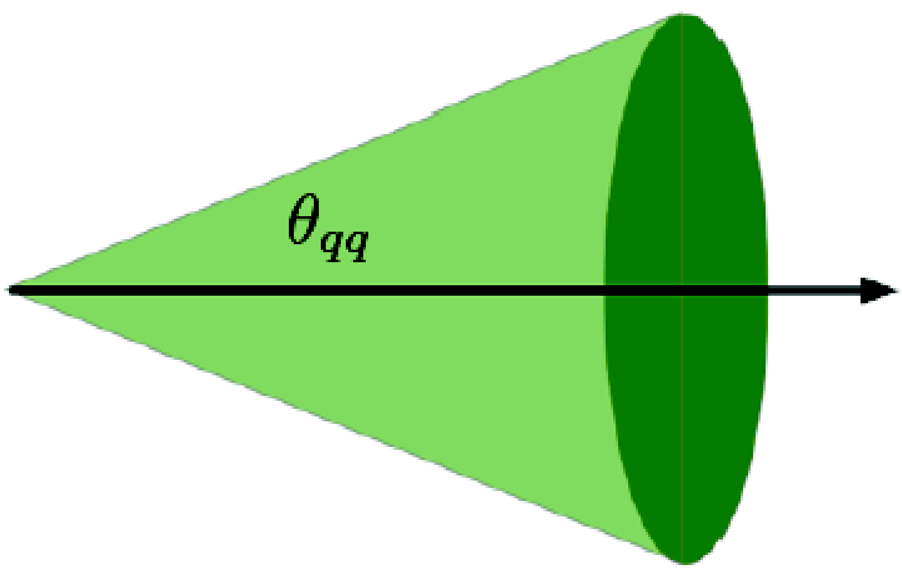


$$\omega \frac{dN}{d^3\vec{k}} = \frac{\alpha_s C_F}{(2\pi)^2 \omega^2} \left[Q_b^2 \mathcal{R}_b + Q_c^2 \mathcal{R}_c + 2\, Q_b \cdot Q_c \mathcal{J} \right]$$

$$= \frac{\alpha_s C_F}{(2\pi)^2 \omega^2} \left[Q_b^2 (\mathcal{R}_b - \mathcal{J}) + Q_c^2 (\mathcal{R}_c - \mathcal{J}) + Q_a^2 \mathcal{J} \right]$$

Coherent radiation

Total charge

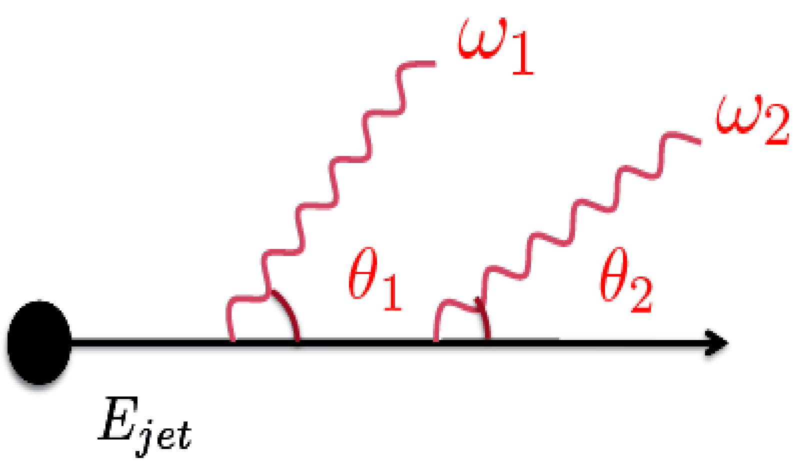


Coherent spectrum

$$\langle dN_i \rangle_\phi = \frac{\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{d\theta_i}{\theta_i} \Theta(\theta_{qq} - \theta_i)$$

Collinear and soft divergence

Multiplicity and color coherence



i) $E_{jet} \gg \omega_1 \gg \omega_2$

ii) $\theta_1 \gg \theta_2$

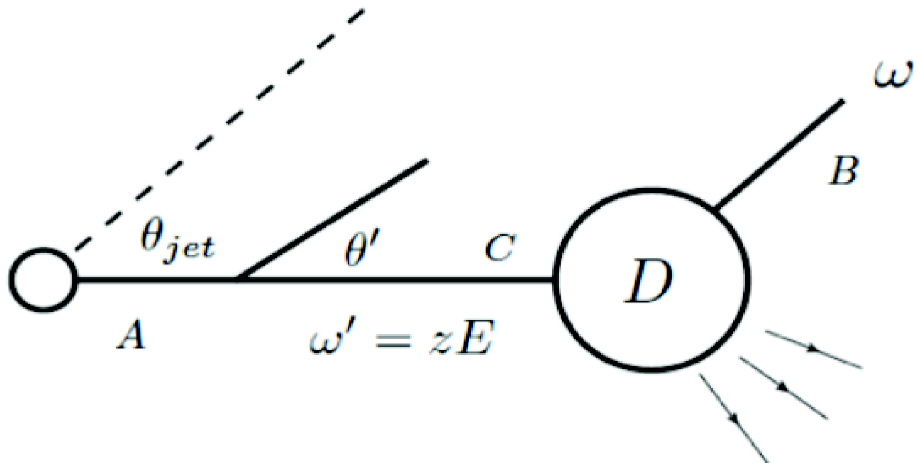
$$k_{2\perp} = \omega_2\theta_2 > \mu$$

$$N^{coh} \propto \alpha_s^2 \int_0^{E_{jet}} \frac{d\omega_1}{\omega_1} \int_{\mu/\omega_1}^{1} \frac{d\theta_1}{\theta_1} \int_0^{\omega_1} \frac{d\omega_2}{\omega_2} \int_{\mu/\omega_2}^{\theta_1} \frac{d\theta_2}{\theta_2} = \frac{1}{4!}\alpha_s^2 \log^4 E_{jet}/\mu$$

$$N^{incoh} \propto \alpha_s^2 \int_0^{E_{jet}} \frac{d\omega_1}{\omega_1} \int_{\mu/\omega_1}^{1} \frac{d\theta_1}{\theta_1} \int_0^{\omega_1} \frac{d\omega_2}{\omega_2} \int_{\mu/\omega_2}^{1} \frac{d\theta_2}{\theta_2} = \frac{2}{4!}\alpha_s^2 \log^4 E_{jet}/\mu = 2N^{coh}$$

QCD coherence leads to depletion of soft gluons!

Multiplicity and color coherence



Recall DGLAP (no-angular ordering)

$$\frac{d}{d\ln M_\perp} D_A^B(x, M_\perp) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} P_A^C(z)\, D_C^B(x/z, M_\perp)$$

MLLA (angular ordering)

$$\frac{d}{d\ln M_\perp} D_A^B(x, M_\perp) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} P_A^C(z)\, D_C^B(x/z, z\, M_\perp)$$

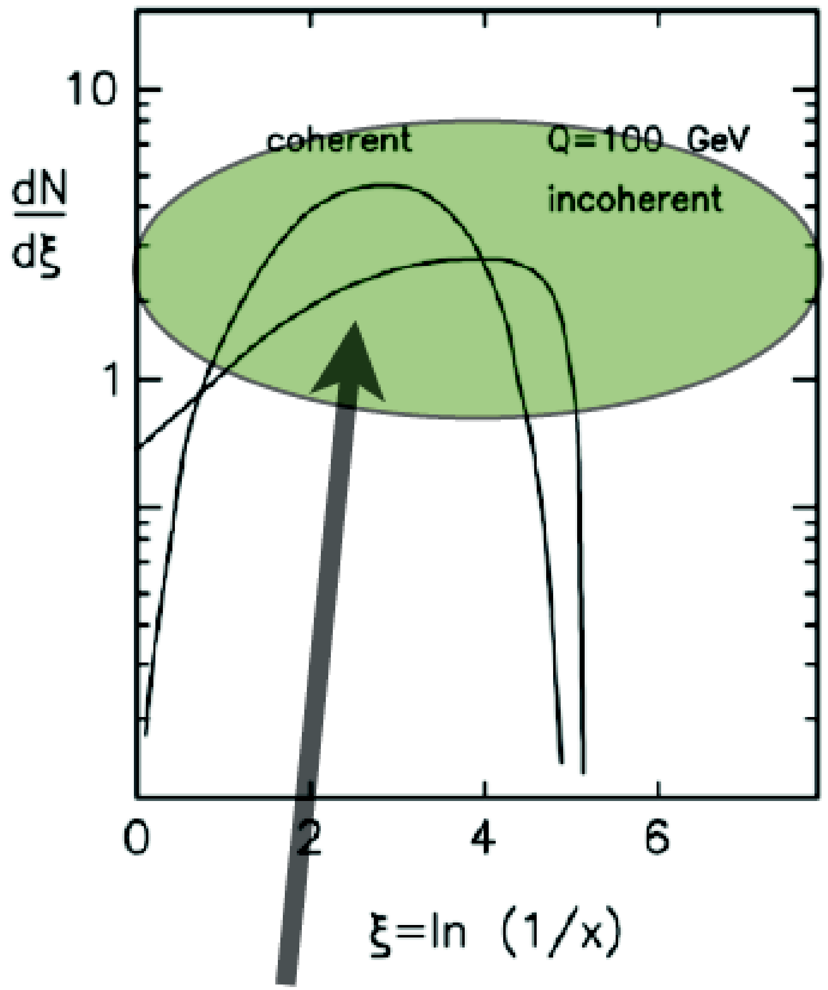
$$z \sim \frac{1}{\sqrt{Q}}$$

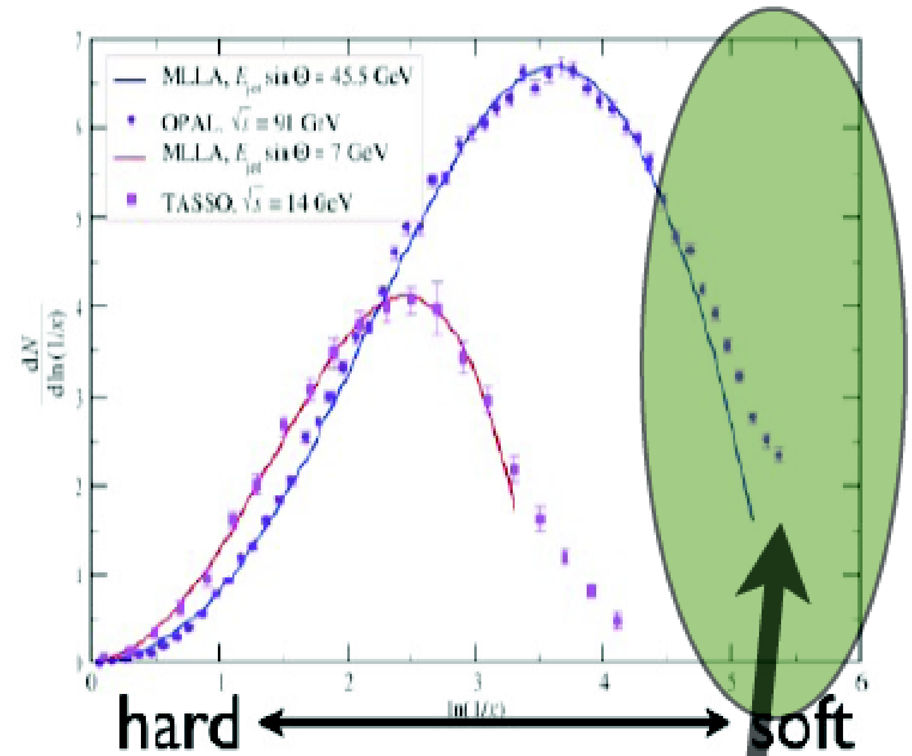
$$\theta' \sim \theta_{jet} \rightarrow M'_\perp = \omega'\theta' \sim \omega'\theta_{jet} = zM_\perp$$

Color coherence consequences

MC implementation

Experimental evidence





TASSO Collaboration, Z. Phys. C 47 (1990) 187
OPAL Collaboration, Phys. Lett. B 247 (1990) 617

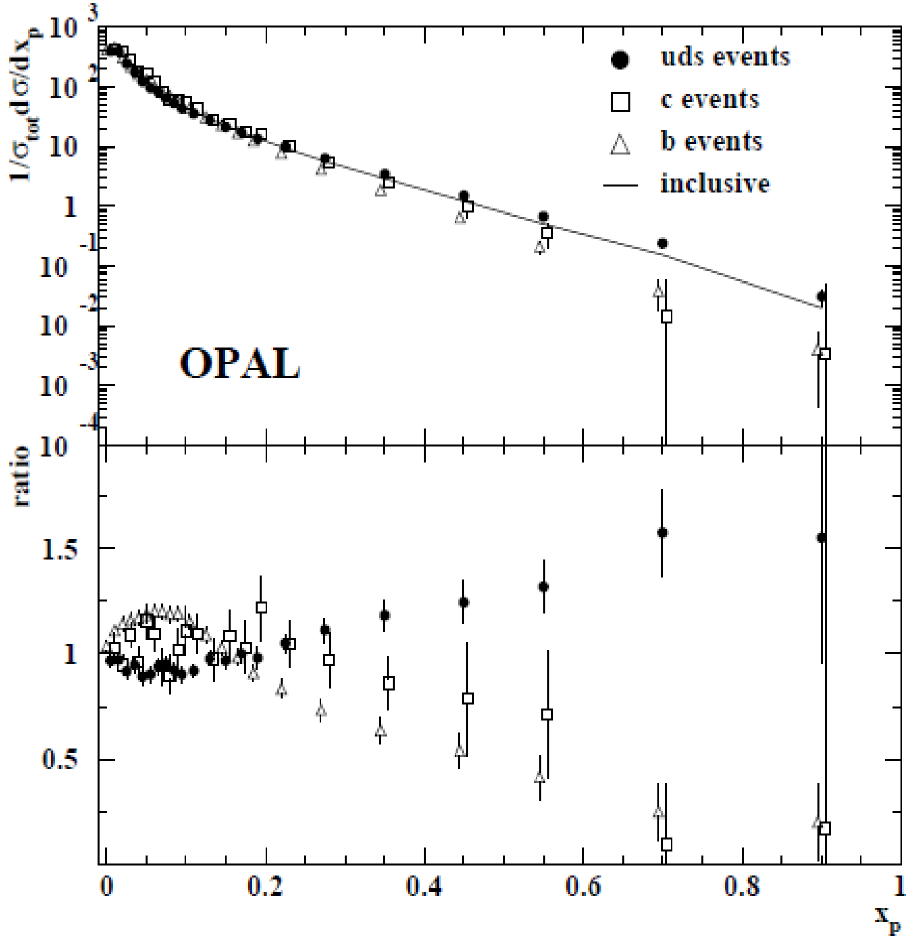
Sizable differences

Suppression of soft gluons

Khoze, Ochs, Wosiek, hep-ph/0009298

Color coherence consequences

Fragmentation function becomes softer for heavy quarks than for light quarks in the forward region



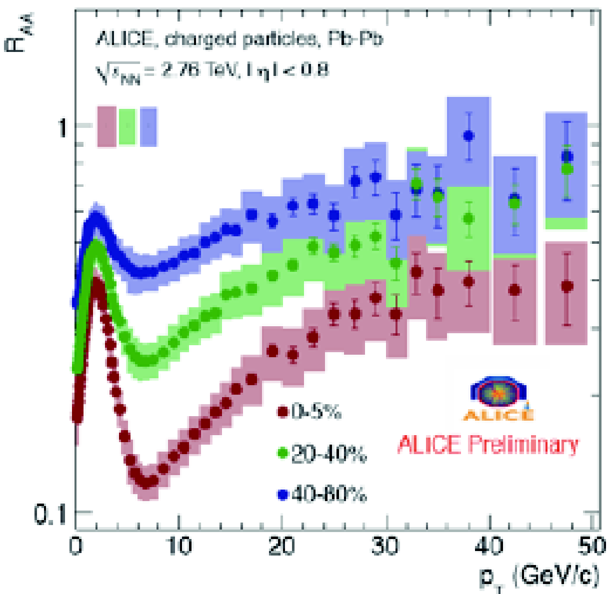
Measurements of flavor dependent fragmentation functions for charged particles in $Z_0 \rightarrow q\,\bar{q}$

OPAL Collaboration, Eur. Phys. J. C7 (1999) 369, Dokshitzer, Khoze, Muller and Troyan, Basics of pQCD

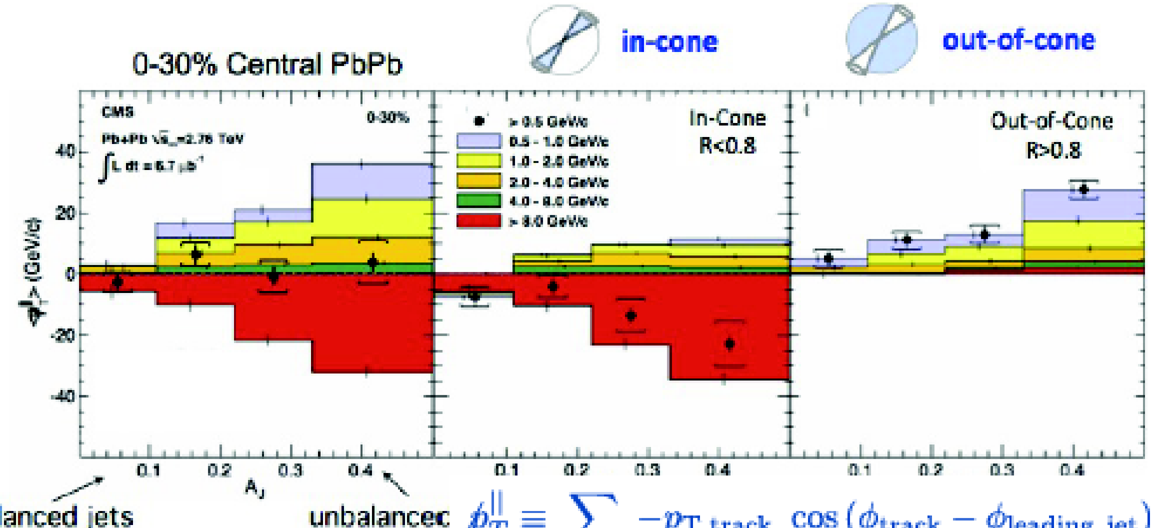
Medium induced radiation and color decoherence

Jets in Nucleus-Nucleus Collisions

(i) Suppression of high-p_T hadrons



(ii) Soft large angle emissions

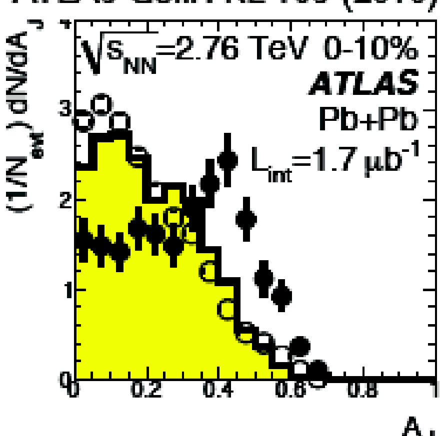


$$\not{p}_T^{\parallel} \equiv \sum_{\text{tracks}} -p_{T,\text{track}} \cos\left(\phi_{\text{track}} - \phi_{\text{leading jet}}\right)$$

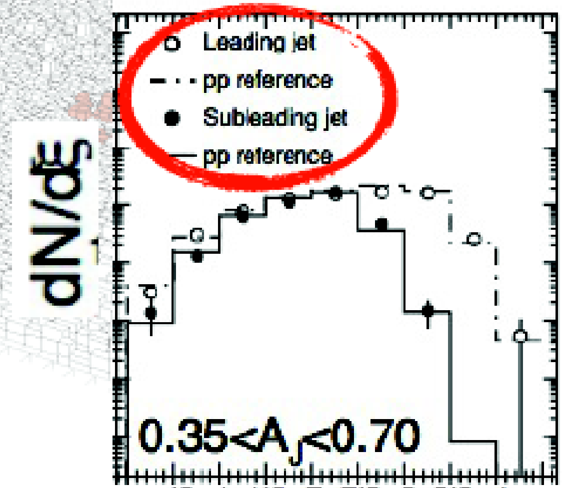
$$\Delta\phi > \pi/2$$

$$A_J = \frac{E_{T1} - E_{T2}}{E_{T1} + E_{T2}}$$

ATLAS Coll. PRL 105 (2010)



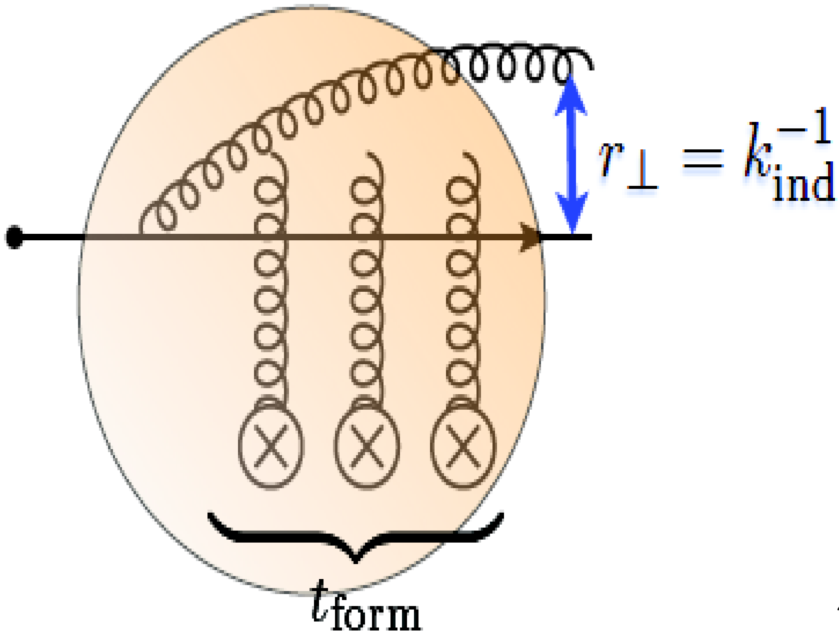
(iii) Significant dijet asymmetry



$$\xi = \ln(p_T^{\text{Jet}}/p_T^{\text{Track}})$$

(iv) Vacuum-like fragmentation function

Medium induced gluon radiation



$$\lambda \ll \tau_{form} < L$$

$$\tau_{form} \sim L \Rightarrow Q_s^2 = \hat{q} L$$

Minimal angle of emission

$$\theta_c = \frac{1}{\sqrt{\hat{q} L^3}}$$

Longitudinal coherence

- Landau-Pomeranchuk-Migdal effect
- induces a characteristic formation time larger than mean free path

$$k_{ind}^2 = \hat{q}\, \tau_{form}$$

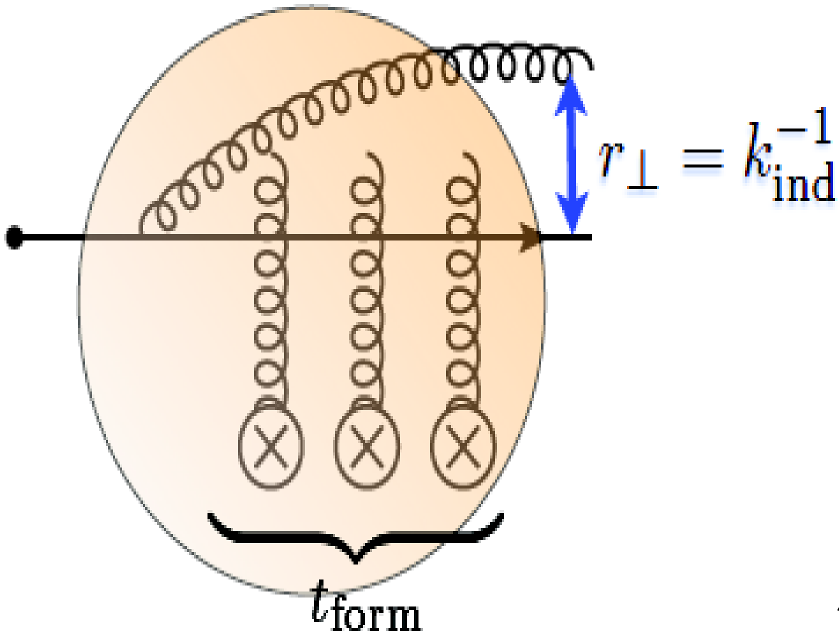
$$\tau_{form} = \frac{\omega}{k_{ind}^2}$$

$$\Rightarrow \quad \tau_{form} = \sqrt{\frac{w}{\hat{q}}}$$

$$k_{ind}^2 = \sqrt{\hat{q}\omega}$$

Baier, Dokshitzer, Mueller, Peigné, Schiff (1997-2000), Zakharov (1996), Wiedemann (2000), Gyulassy, Levai, Vitev (2000), Arnold, Moore, Yaffe (2001)

Medium induced gluon radiation



$$\lambda \ll \tau_{form} < L$$

$$\tau_{form} \sim L \Rightarrow Q_s^2 = \hat{q} L$$

Angle of emission

$$\underbrace{\frac{1}{\sqrt{\hat{q} L^3}}}_{\equiv \theta_c} \leq \theta \leq \frac{Q_s}{w}$$

Longitudinal coherence

- Landau-Pomeranchuk-Migdal effect
- induces a characteristic formation time larger than mean free path

$$k_{ind}^2 = \hat{q}\, \tau_{form} \qquad \tau_{form} = \frac{\omega}{k_{ind}^2} \quad \Rightarrow \quad \tau_{form} = \sqrt{\frac{w}{\hat{q}}} \qquad k_{ind}^2 = \sqrt{\hat{q}\omega}$$

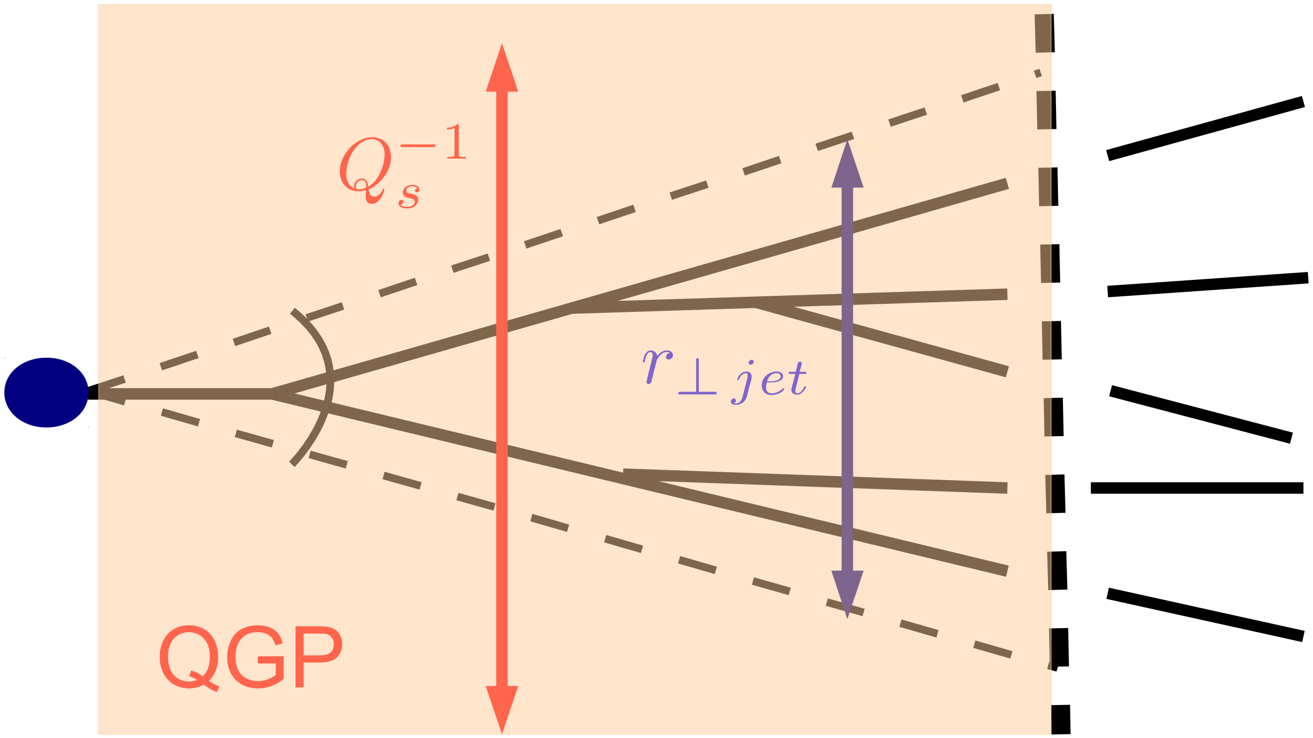
Coherent spectrum

$$\omega \frac{dN}{d\omega} \propto \alpha_s \frac{L}{t_{\rm form}} = \alpha_s \sqrt{\frac{\hat{q} L^2}{\omega}}$$

Energy loss: $\Delta E \propto \hat{q} L^2$

Baier, Dokshitzer, Mueller, Peigné, Schiff (1997-2000), Zakharov (1996), Wiedemann (2000), Gyulassy, Levai, Vitev (2000), Arnold, Moore, Yaffe (2001)

Jets in a QCD medium: Multiscale problem



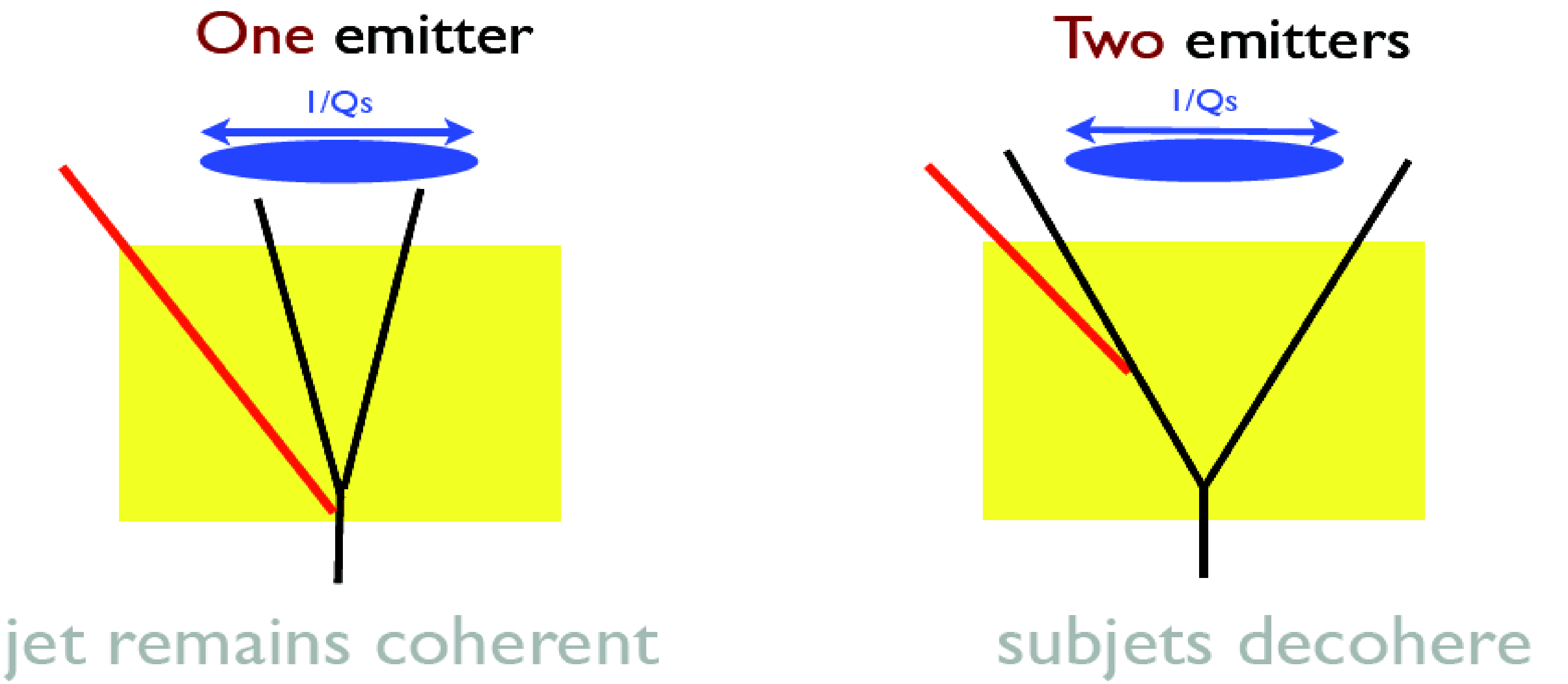
New scales:

$$M_\perp \equiv E\,\theta_{jet}$$
$$Q_0 \sim \Lambda_{\rm QCD}$$

+

$$Q_s \equiv \sqrt{\hat{q}L} \equiv m_D\sqrt{N_{\rm scat}}$$
$$r_{\perp\,jet}^{-1} \equiv (\theta_{jet}L)^{-1}$$

The antenna inside the QCD medium



The scale Q_s^{-1} determines the number of independent color sources that can are resolved by the medium.

:: medium induced radiation (BDMPS spectrum)

Armesto, Milhano, Salgado, Mehtar-Tani, Tywoniuk, Iancu, Casalderrey (2010-2013).

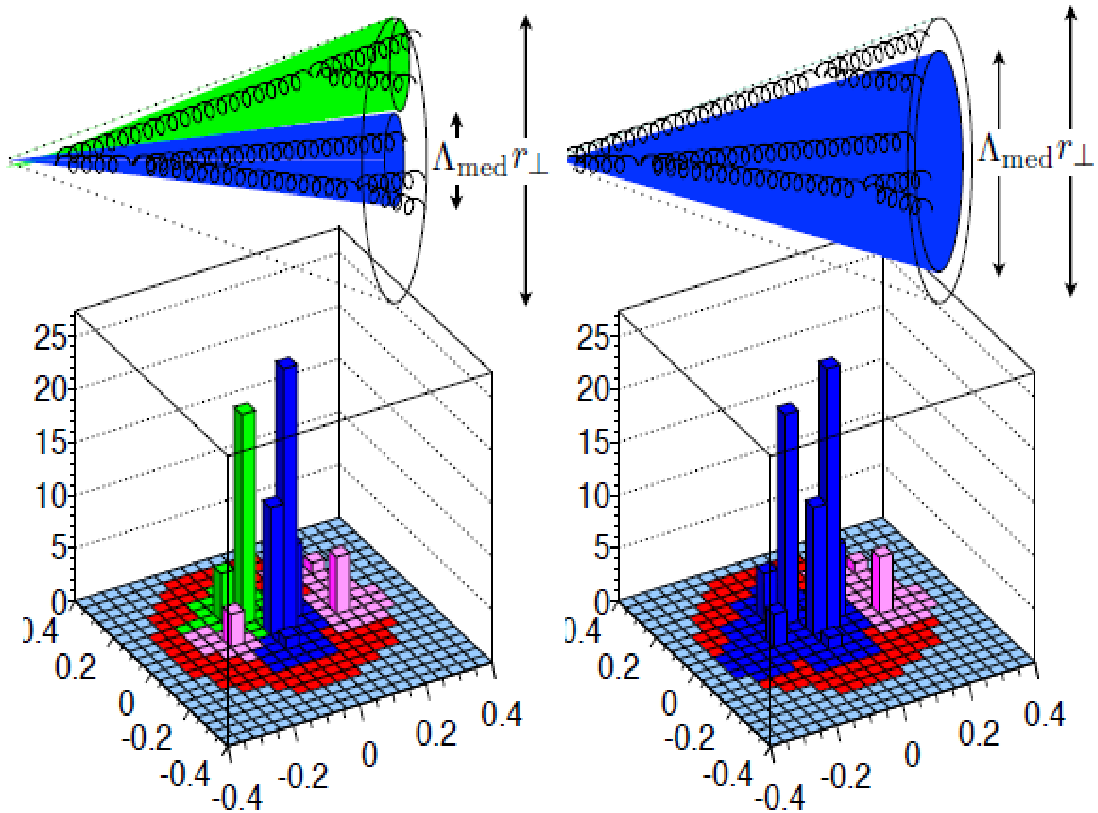
Implications of QCD decoherence

Survival probability

$$\Delta_{\rm med} = 1 - e^{-\Theta_{\rm jet}^2/\theta_c^2}$$

$$\theta_c = 1/\sqrt{\hat{q}L^3}$$

jet definition (Θ_{jet}=R)!



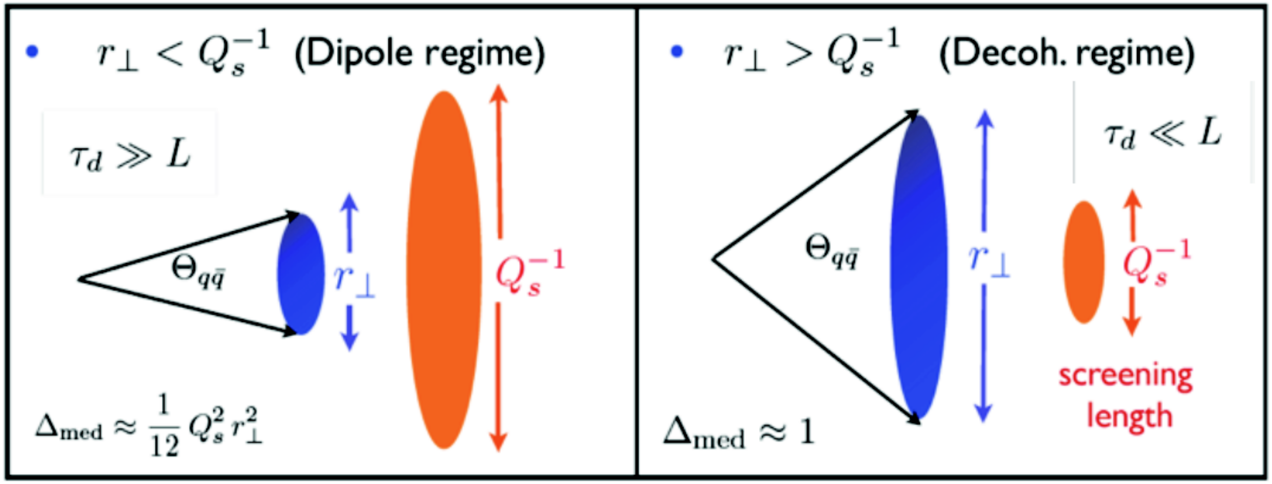
$$\Lambda_{med} = Q_s^{-1}$$

$$r_\perp = \theta\, L$$

Casalderrey et. Al

- $\theta=\theta$jet < θc: unresolved jet constituent, fragment as in vacuum, no medium effect for z> Qo/Eθc.
- $\theta=\theta$jet > θc: jet constituents resolved, soft decoherent radiation at large angles.

Further theoretical advances

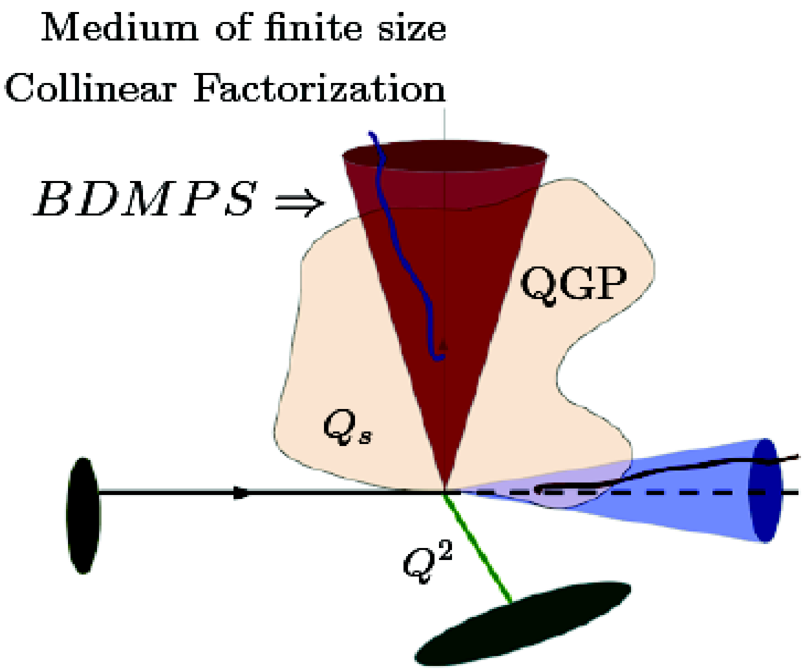


Probabilitistic parton picture: Blaizot, Dominguez, Mehtar-Tani, Iancu
Democratic branching and turbulence: Blaizot, Mehtar-Tani, Iancu
Renormalization of the jet quenching parameter: Iancu, Mehtar, Blaizot
Running coupling effects: Iancu, Triantafyllopoulos
Jet decoherence in PbPb @LHC: Tywoniuk, Mehtar-Tani
Angular structure of in medium parton cascade @LHC:
Blaizot, Mehtar, Torres
Gluon radiation and color coherence beyond gluon approximation:
Apolinario, Armesto, Milhano, Salgado

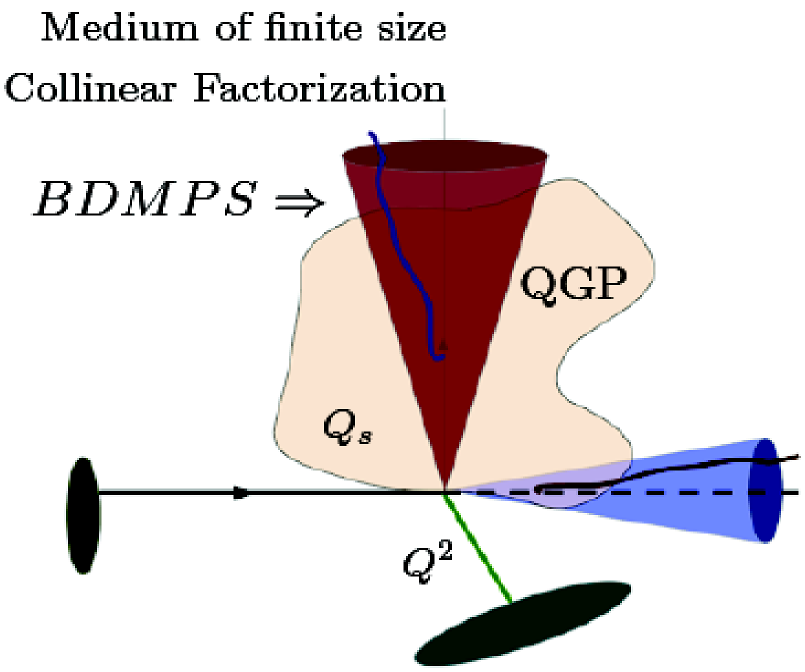
What about the *dilute*-*dense* regime?

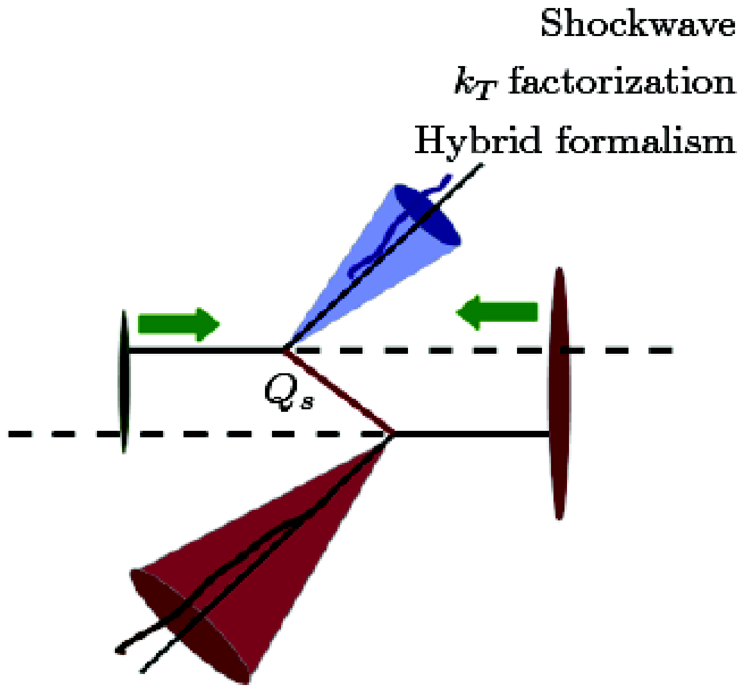
Color (de)coherence between the initial and final state radiation

A relevant configuration to study coherence

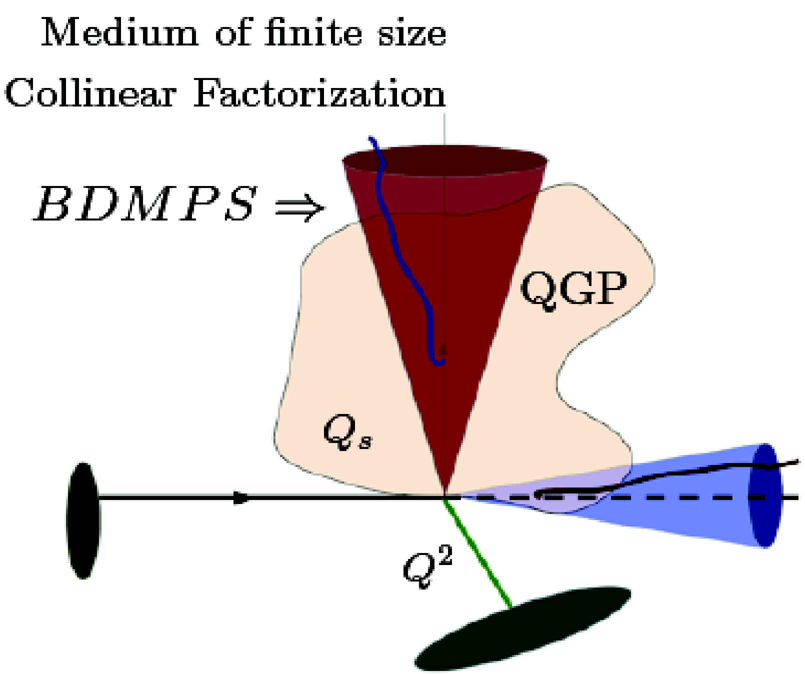


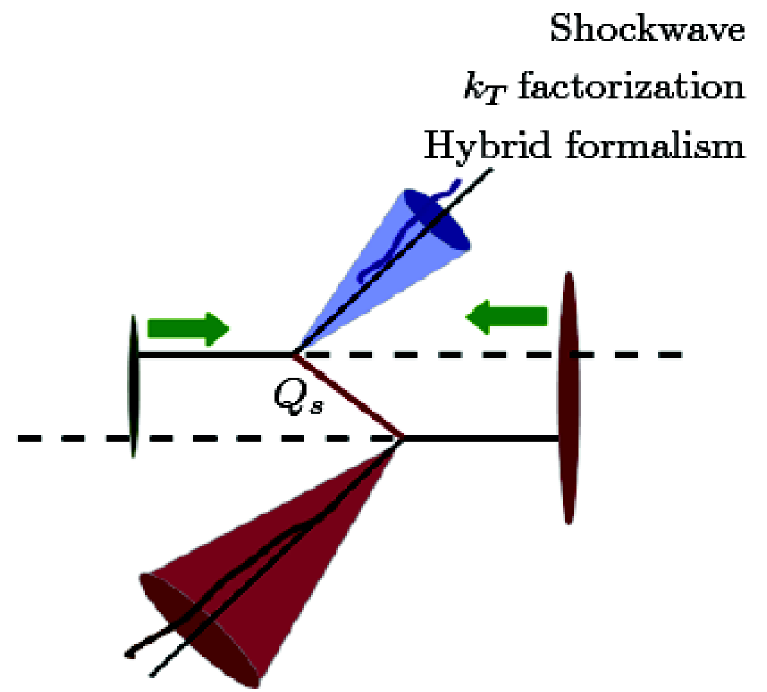
A relevant configuration to study coherence





A relevant configuration to study coherence



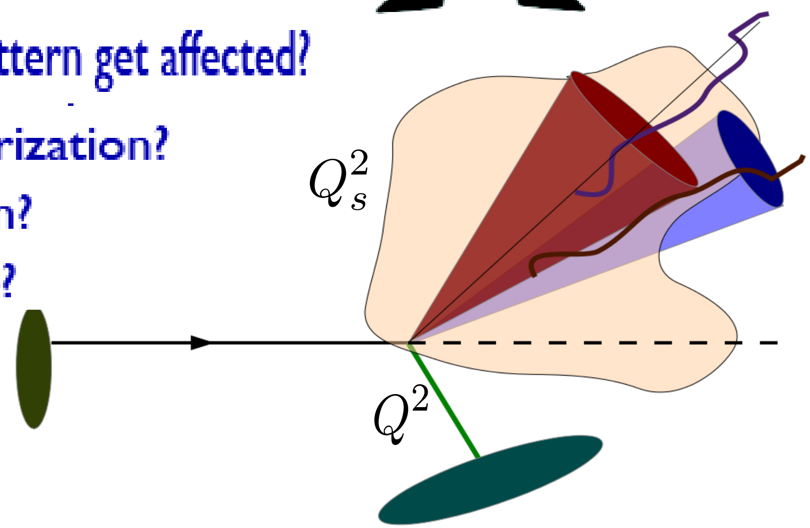


How is the vacuum coherence pattern get affected?

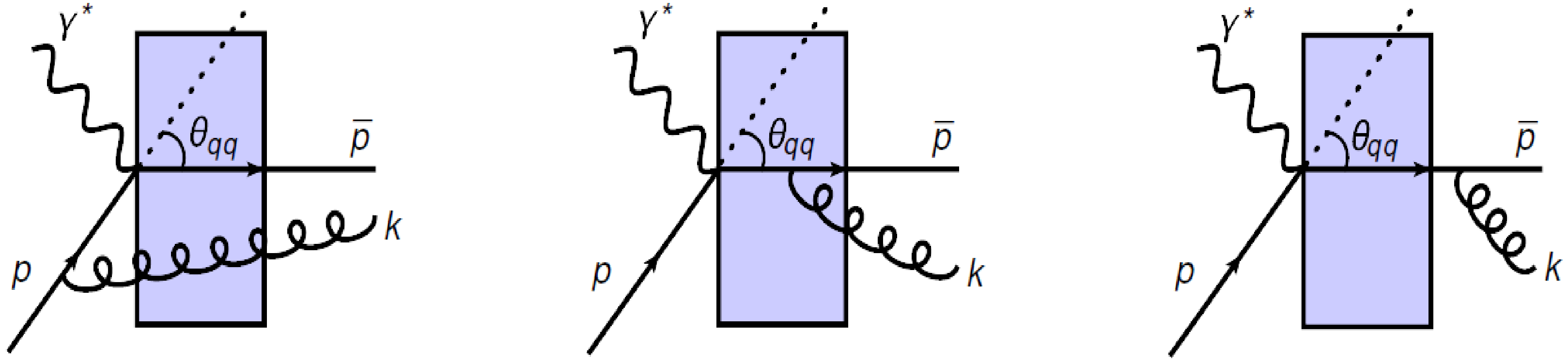
Do we have: Collinear factorization?

k_T Factorization?

Something else?



Color decoherence between the initial and final state radiation



N. Armesto, H. Ma, M. Martinez, Y. Mehtar-Tani, C. Salgado

N=1 Opacity expansion: PLB 717 (2012)280

Multiple soft scatterings: JHEP 1312(2013)052 → *Today!!*

Relevant configuration to investigate:

Medium modifications to the initial state radiation

Energy Loss problem in the dilute-dense regime

Finite length/energy effects in pA collisions

Semi-Classical methods of pQCD

Evolution of the gauge field: $\left[D_\mu, F^{\mu\nu}\right] = \mathcal{J}^\nu$

Color charge conservation: $\left[D_\mu, \mathcal{J}^\mu\right] = 0$

Linearizing around a background field: $\mathcal{A}^\mu = A^\mu_{med} + a^\mu$

$$\Box_x a^i - 2ig\left[\mathcal{A}^-_{med}, \partial_- a^i\right] = \mathcal{J}^i - \partial^i\left(\frac{\mathcal{J}^+}{\partial_-}\right)$$ **LC gauge**

Reduction formula: $\mathcal{M}^a_\lambda = \lim_{k^2 \to 0} \int d^4x e^{ik\cdot x} \Box_x \mathcal{A}^a_\mu(x) \epsilon^\mu_\lambda(\vec{k})$

Our goal is to obtain the single gluon spectrum

$$(2\pi)^3 2k^+ \frac{dN}{d^3\vec{k}} = \sum_{\lambda=1,2} \left\langle \left\langle |\mathcal{M}^a_\lambda(\vec{k})|^2 \right\rangle_p \right\rangle_A$$

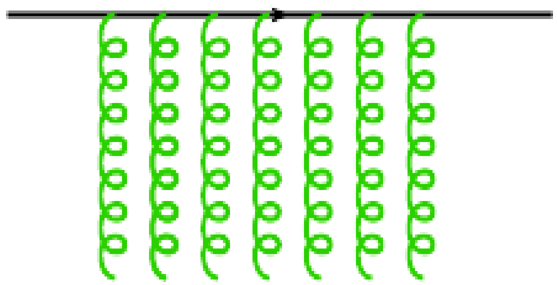
F. Gelis, R. Venugopalan, NPA 776 (2006), 135
J. Blaizot, F. Gelis, R. Venugopalan, NPA 743 (2004), 13

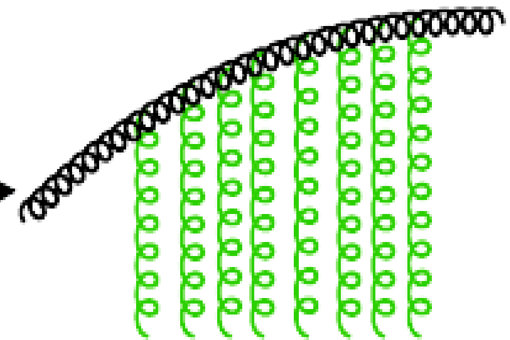
Propagators

Hard partons: eikonal trajectory in a background field

$$\mathcal{J}^\mu(x)_a = \mathcal{U}^{ab}(x^+, -\infty, \boldsymbol{x})\ \ \rho^b(\boldsymbol{x}-\boldsymbol{B})$$

$$\mathcal{U}^{ab}(x^+,y^+) = \mathcal{P}\exp\left[ig\int_{y^+}^{x^+} dz^+\, \mathcal{A}^-_{med}\left(z^+,\boldsymbol{r}(z^+)\right)\right]^{ab}$$



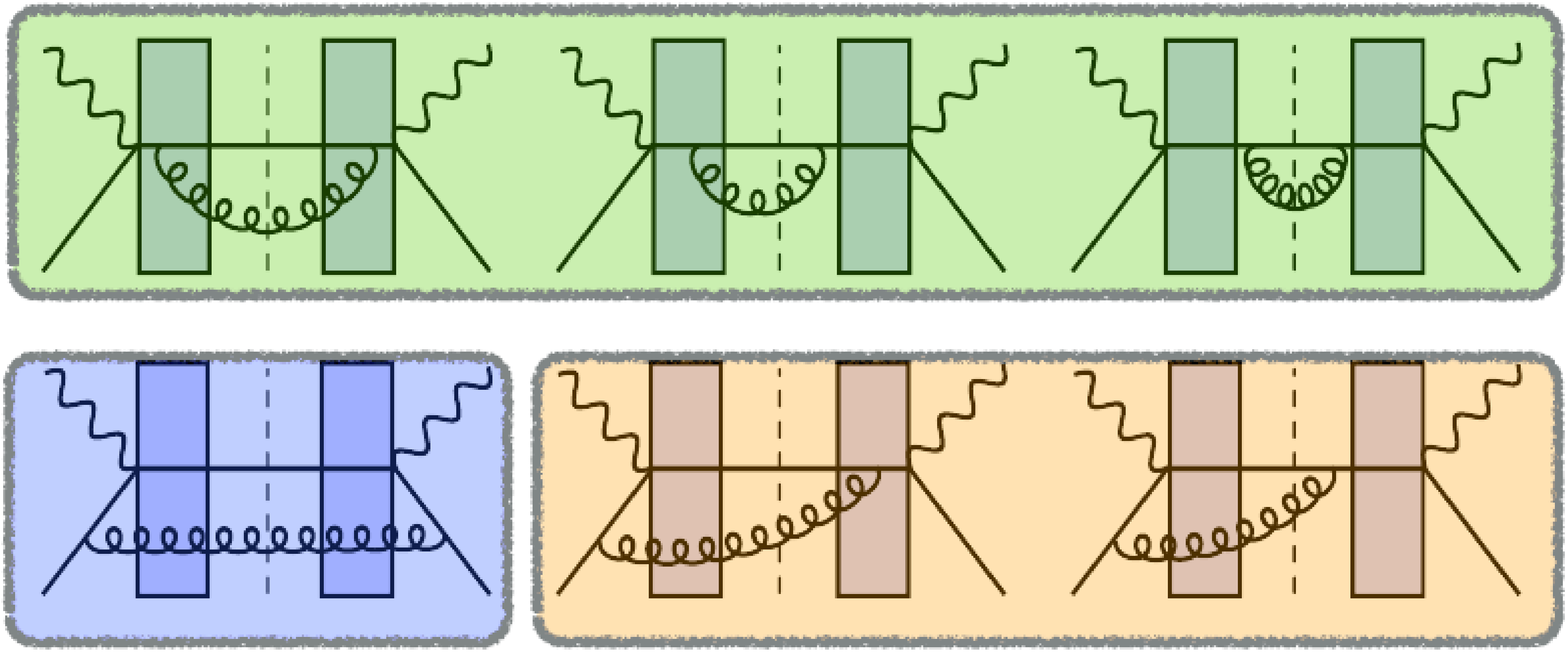


Soft gluon: Color rotation + Brownian motion

$$\mathcal{G}_{ab}(x^+,\mathbf{x};y^+,\mathbf{y}|k^+) = \int_{\mathbf{r}(y^+)=\mathbf{y}}^{\mathbf{r}(x^+)=\mathbf{x}} \mathcal{D}\mathbf{r}\exp\left(i\frac{k^+}{2}\int_{y^+}^{x^+} dz\,\dot{\mathbf{r}}^2(z)\right)\mathcal{U}_{ab}(x^+,y^+)$$

Gluon spectrum

BDMPS-Z + vacuum

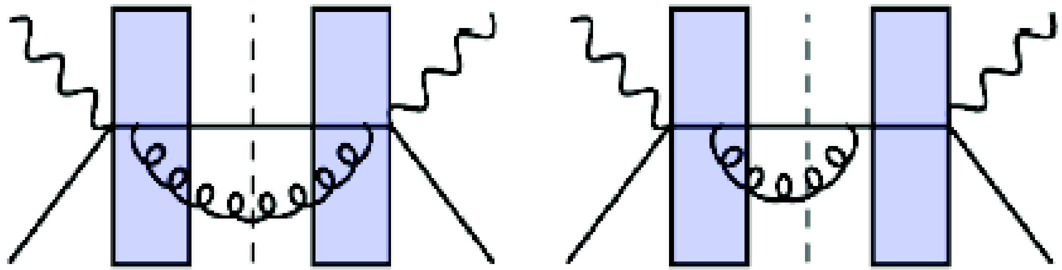


P_T broadening of ISR

Interferences in the medium: New!!

Armesto et. al, JHEP 1312 (2013) 052

Gluon spectrum I: direct emissions



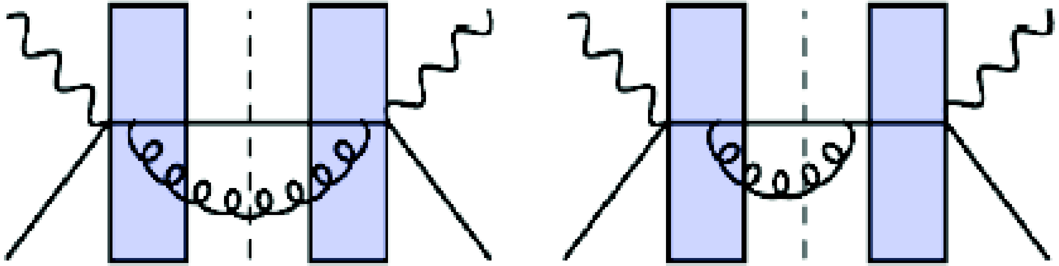
$$\sim \int_0^L dt' \int \frac{d^2 k'}{(2\pi)^2} \mathcal{P}(\boldsymbol{k} - \boldsymbol{k}', L - t') \sin\left(\frac{\boldsymbol{k}'^2}{2k_f^2}\right) e^{-\frac{\boldsymbol{k}'^2}{2k_f^2}}$$

$$\mathcal{P}(\boldsymbol{k}, \xi) = \frac{4\pi}{\hat{q}\xi} e^{-\frac{\boldsymbol{k}^2}{\hat{q}\xi}} \qquad k_f^2 = \sqrt{\hat{q}\omega}$$

- BDMPS-Z can be approximated by: quantum emission plus classical p_T broadening.

Armesto et. al, JHEP 1312 (2013) 052

Gluon spectrum I: direct emissions



$$\sim \int_0^L dt' \int \frac{d^2\boldsymbol{k}'}{(2\pi)^2} \mathcal{P}(\boldsymbol{k}-\boldsymbol{k}', L-t') \sin\left(\frac{\boldsymbol{k}'^2}{2k_f^2}\right) e^{-\frac{\boldsymbol{k}'^2}{2k_f^2}}$$

$$\mathcal{P}(\boldsymbol{k},\xi) = \frac{4\pi}{\hat{q}\xi} e^{-\frac{\boldsymbol{k}^2}{\hat{q}\xi}} \qquad k_f^2 = \sqrt{\hat{q}\omega}$$

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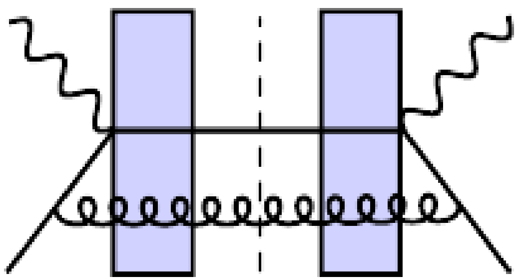


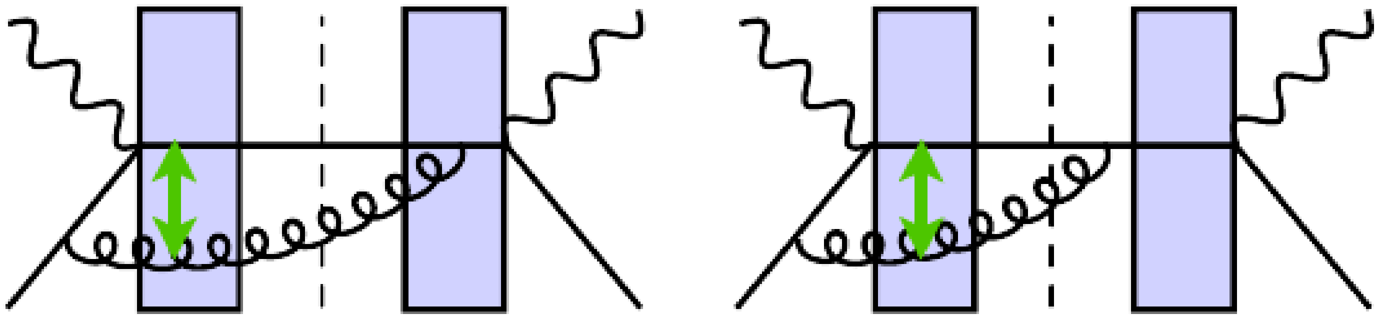
$$\sim \int \frac{d^2\boldsymbol{k}'}{(2\pi)^2} \frac{\mathcal{P}(\boldsymbol{k}'-\bar{\boldsymbol{\kappa}}, L^+)}{\boldsymbol{k}'^2}$$

- p_T broadening of ISR contains classical p_T broadening plus collinear divergence.

$$\langle k_T \rangle \sim Q_s = \sqrt{\hat{q}L^+}$$

Armesto et. al, JHEP 1312 (2013) 052

Guon spectrum II: Interferences



Transverse size of the Quark-gluon system

- If hard scattering is the largest scale:

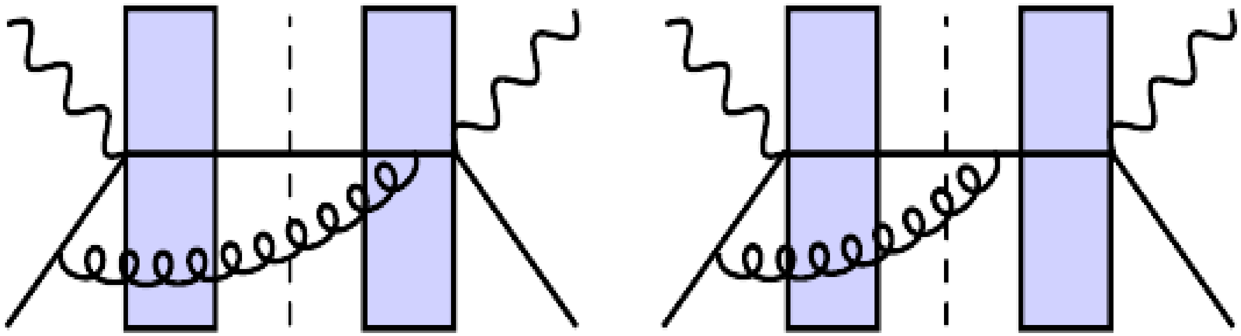
⇒ Insensitive to the medium

- If typical medium induced momentum is the largest scale

⇒ Medium is able to resolve the qg system

Armesto et. al, JHEP 1312 (2013) 052

Guon spectrum II: Interferences



The Color correlation of the Quark-gluon system is measured by

$$\mathcal{K}(x^+,\boldsymbol{x};y^+,\boldsymbol{y}|k^+) = \int_{\boldsymbol{r}(y^+)=\boldsymbol{y}}^{\boldsymbol{r}(x^+)=\boldsymbol{x}} \mathcal{D}\boldsymbol{r}\ \exp\left[\int_{y^+}^{x^+} d\xi \left(i\frac{k^+}{2}\dot{\boldsymbol{r}}^2(\xi) - \frac{1}{2}n(\xi)\sigma\left(\boldsymbol{r}(\xi)\right)\right)\right]$$

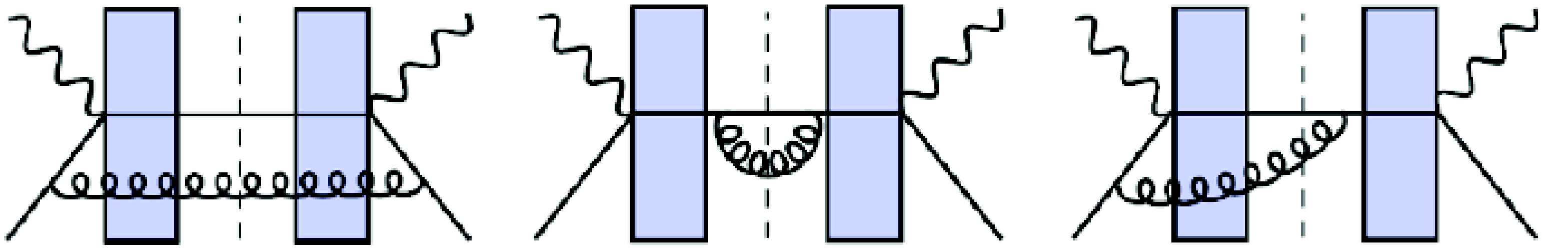
- Describes the Brownian motion of the gluon
- Harmonic oscillator approximation: $n\sigma(\mathbf{r}) \approx \hat{q}\mathbf{r}^2$
- Two extreme limits

$\Rightarrow$ High Energy Limit (Shockwave) $\tau_f \sim \sqrt{\omega/\hat{q}} \gg L^+$

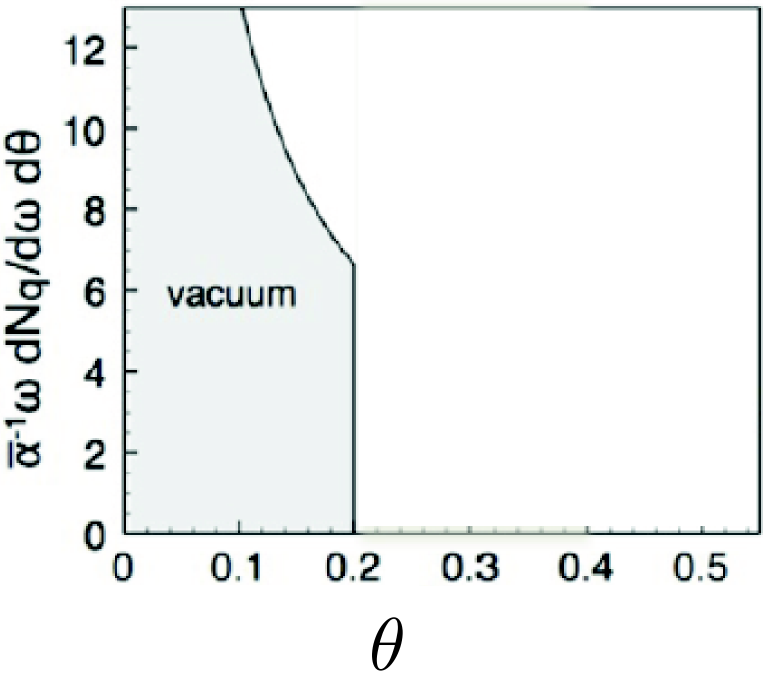
$\Rightarrow$ "Infinite" medium length $\tau_f \sim \sqrt{\omega/\hat{q}} \ll L^+$

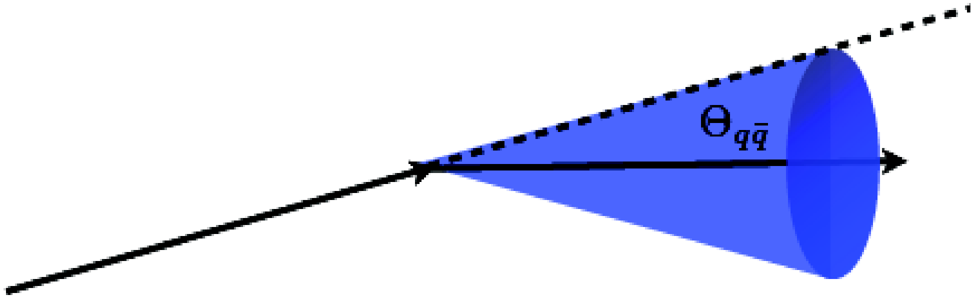
Armesto et. al, JHEP 1312 (2013) 052

Gluon spectrum: High energy limit (shockwave)



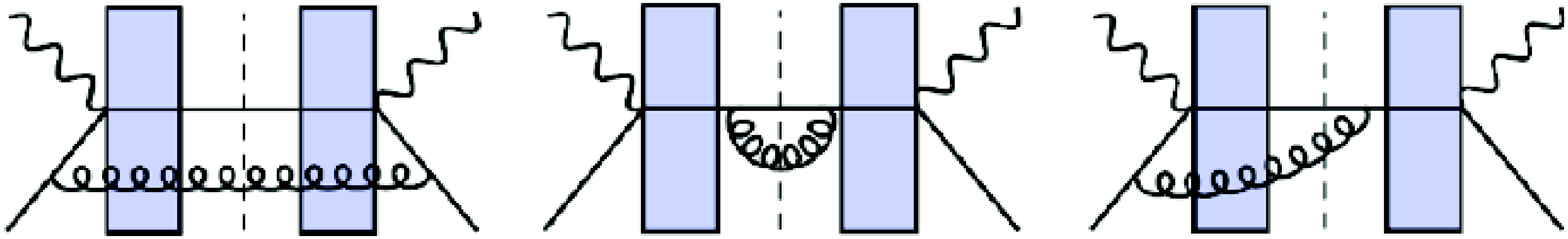
In the absence of a medium we recover the **vacuum** *coherence* pattern



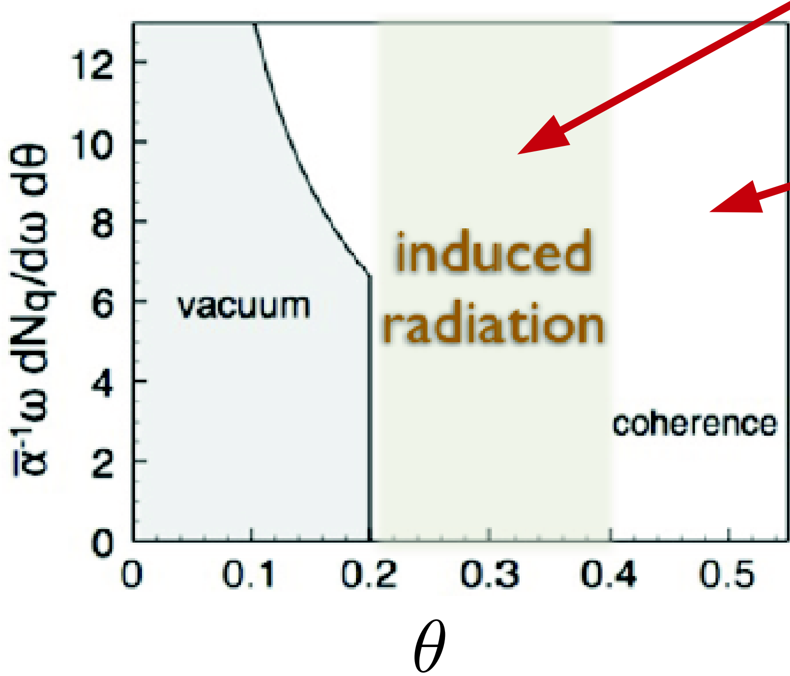


Armesto et. al, JHEP 1312 (2013) 052

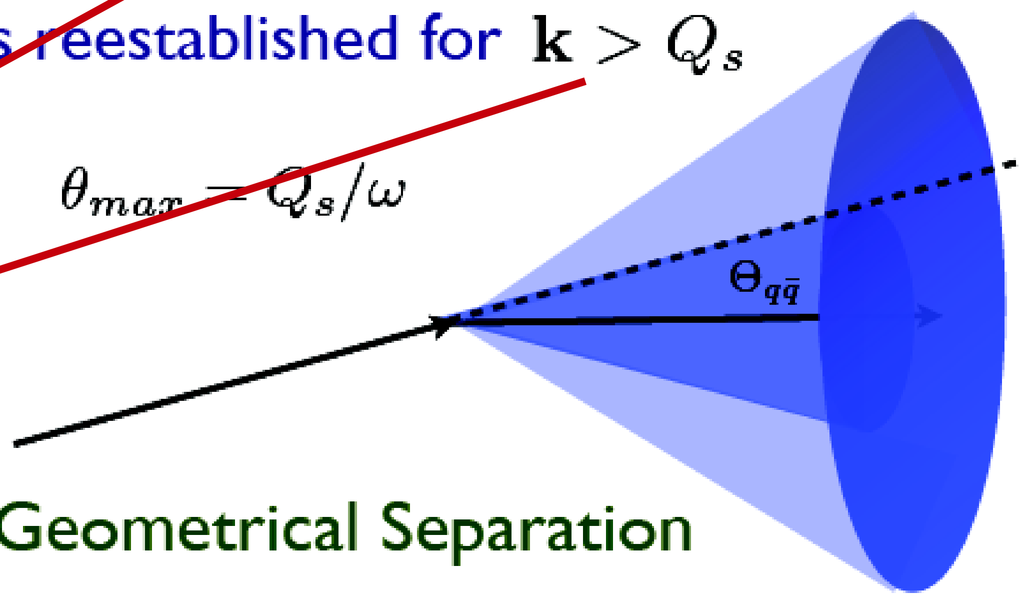
Gluon spectrum: High energy limit (shockwave)



- Medium acts as a unique scattering center
- Interferences are suppressed if $\mathbf{k} < Q_s$
- Vacuum color coherence is reestablished for $\mathbf{k} > Q_s$



$\theta_{max} = Q_s/\omega$

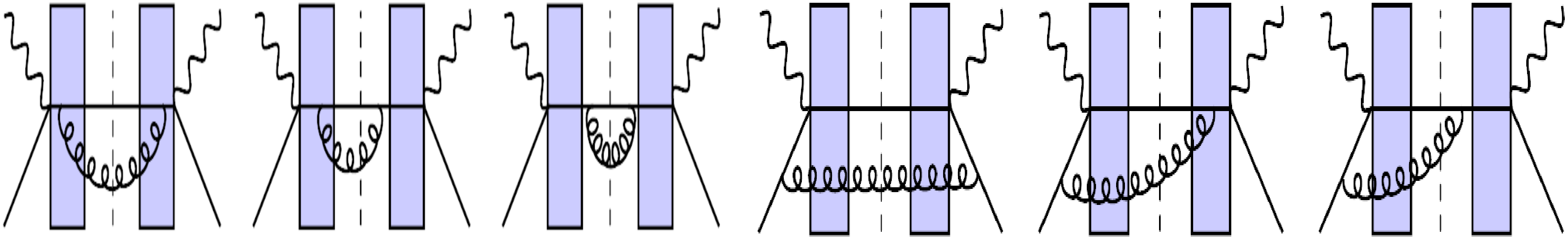


Geometrical Separation

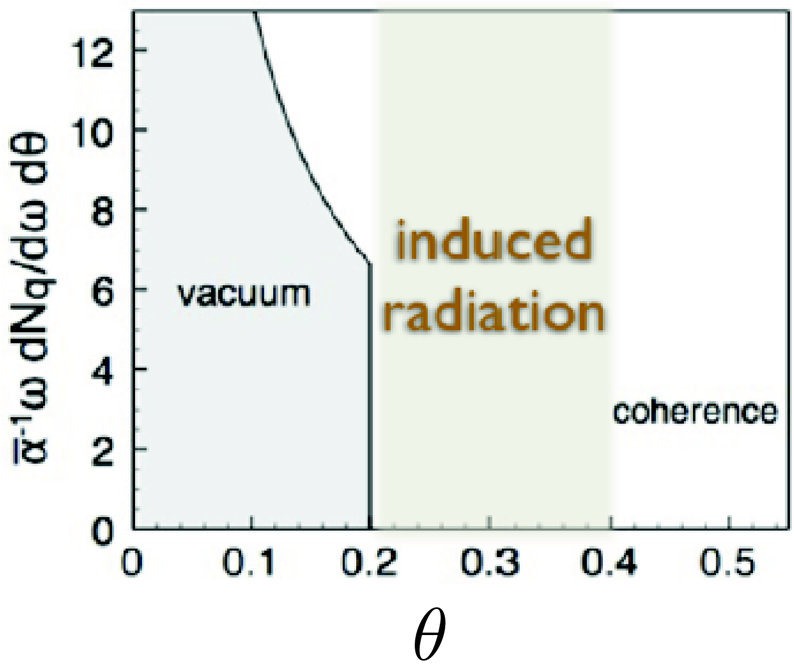
Contact with high energy limit: Kovchegov-Mueller (1998)

Armesto et. al, JHEP 1312 (2013) 052

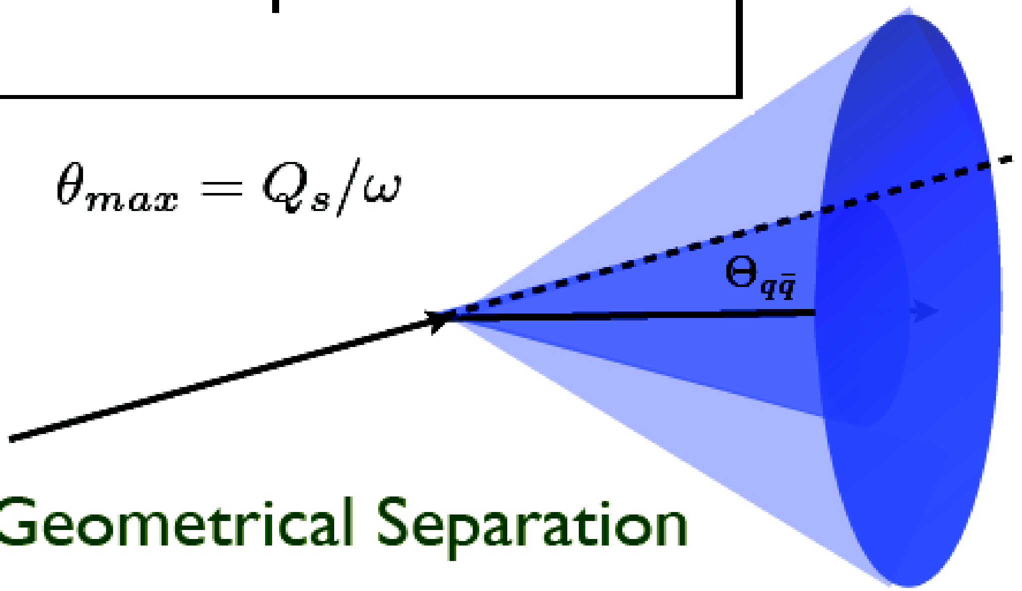
Gluon spectrum: Infinite medium lenght



- Similar results in the incoherent regime: medium opens the phase space of emissions up to a maximum angle.



$$\theta_{max} = Q_s/\omega$$



Geometrical Separation

Armesto et. al, JHEP 1312 (2013) 052

Generalizing to pA collisions: first results

- We perform a systematic eikonal expansion to the gluon propagator in the background field.
- We study soft gluon production in pA collisions beyond eikonal accuracy.

$$k^+ \frac{d\sigma}{dk^+ \, d^2\mathbf{k}} = \frac{1}{\mathbf{k}^2} \int \frac{d^2\mathbf{q}}{(2\pi)^2} \, \varphi_p(\mathbf{q}) \, (\mathbf{k}-\mathbf{q})^2 \int_{\mathbf{b},\mathbf{r}} e^{-i(\mathbf{k}-\mathbf{q})\cdot\mathbf{r}} \, S_A(\mathbf{r},\mathbf{b}) + O\left(\left(\frac{L^+}{k^+}\partial_\perp^2\right)^2\right)$$

Recover the k_T factorization formula

- Some spin asymmetries

$$A_N(\boldsymbol{k}) \equiv \frac{\frac{d\sigma^\uparrow}{d^2k\,dy} - \frac{d\sigma^\downarrow}{d^2k\,dy}}{\frac{d\sigma^\uparrow}{d^2k\,dy} + \frac{d\sigma^\downarrow}{d^2k\,dy}} = \frac{\frac{d\sigma^\uparrow}{d^2k\,dy}(\boldsymbol{k}) - \frac{d\sigma^\uparrow}{d^2k\,dy}(-\boldsymbol{k})}{\frac{d\sigma^\uparrow}{d^2k\,dy}(\boldsymbol{k}) + \frac{d\sigma^\uparrow}{d^2k\,dy}(-\boldsymbol{k})}$$

Polarized target: $p + A^\uparrow \to g + X$

Polarized projectile: $p^\uparrow + A \to g + X$

Polarized gluon production from unpolarized pA: $p + A \to g^\pm + X$

The eikonal contribution ***vanishes*** exactly while the leading dominant terms are the ***next to eikonal*** terms (finite size/medium effects)!!!

Altinoluk, Armesto, Beuf, Martinez, Salgado, JHEP 1407 (2014) 068

Conclusions

- We investigate medium modifications to the color coherence pattern between the initial and final state radiation.
- There is a gradual onset of ***decoherence*** between both emitters due to multiple scatterings with the medium

  $\Rightarrow$ Opening of phase space for ***large angle*** emissions
- In pA collisions beyond the eikonal approximation:
  - *Soft gluon production:* non eikonal corrections to the CGC are suppressed.
  - *Some spin asymmetries:* non eikonal corrections are the dominant contribution.

Outlook (keep tuned!!!)

- **Energy loss in high energy forward processes in pA collisions:** Kopeliovich et. Al, Strickman et. al., Kaidalov et. al., Peigne & Arleo, Liou & Mueller.
- **Inclusion of quarks. Forthcoming**
- **Single inclusive gluon production in the hybrid formalism beyond eikonal accuracy. Forthcoming**

Backup

Semi-classical approach to gluon production I

Our goal is to obtain the single gluon spectrum

$$(2\pi)^3 2k^+ \frac{dN}{d^3\vec{k}} = \sum_{\lambda=1,2} \left\langle \left\langle |\mathcal{M}_\lambda^a(\vec{k})|^2 \right\rangle_p \right\rangle_A$$

Scattering amplitude

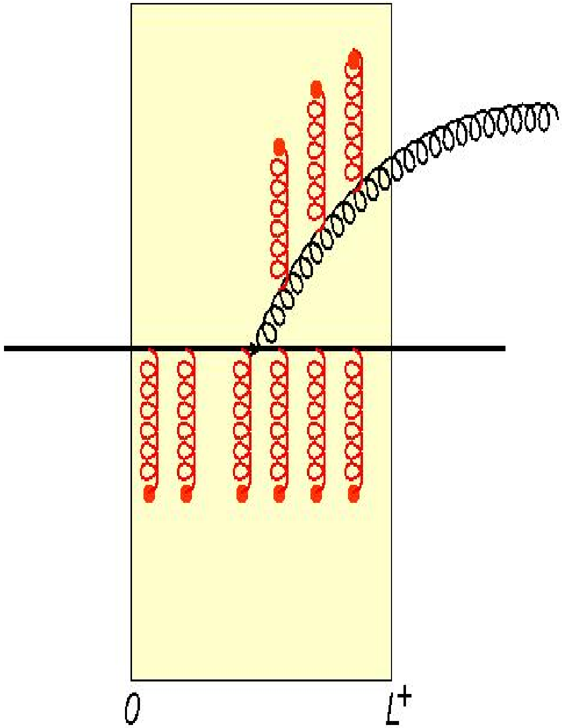
- ***Medium***: Classical background field $A^\mu_{(0)}$

$$-\partial_{\mathbf{x}} A^-_{a,(0)} = \rho_a(x)$$

$$\langle A^-_{a,(0)}(x^+,\mathbf{q}) A^{*,-}_{b,(0)}(x',\mathbf{q}')\rangle = \delta_{ab}\, n(x^+)\, \delta(x^+ - x'^+)\, \delta(\mathbf{q}-\mathbf{q}')\, \mathcal{V}^2(\mathbf{q})$$

Density of scattering centers

Debye potential



- ***Highly energetic particle:*** Classical current

$$\mathcal{J}^\mu_a(x) = g\, v^\mu\, \delta^{(3)}(\vec{x} - \vec{v}t)\, \mathcal{U}(x^+, 0, \boldsymbol{x})_{ab}\, Q_b$$

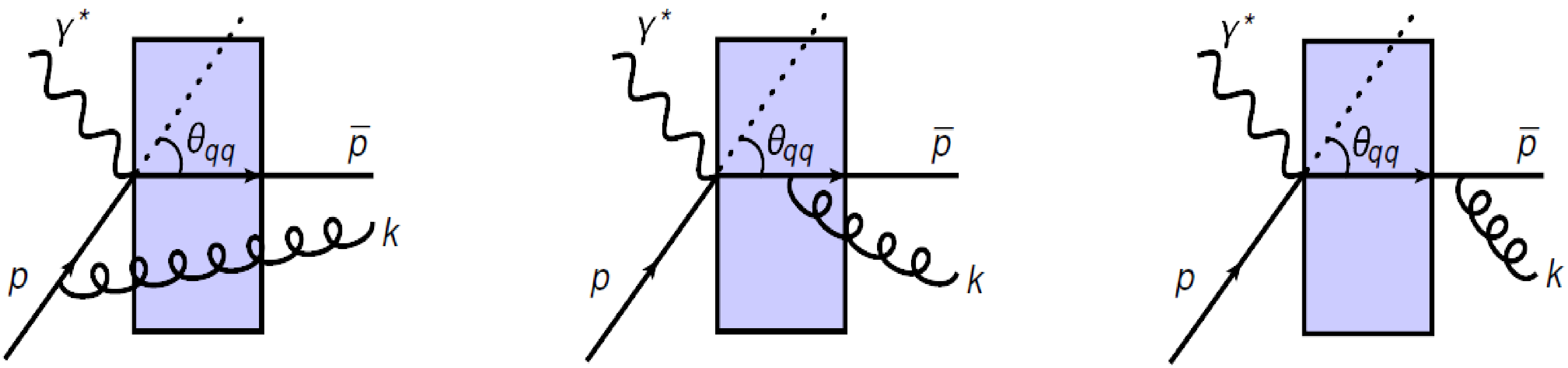
$$\mathcal{U}(x^+, 0, \boldsymbol{x}) = \mathcal{P}_+ \exp\left\{ig \int_0^{x^+} dz^+\, T \cdot A(z^+, \boldsymbol{x})\right\}^{ab}$$

Blaizot, Gelis, Venugopalan, NPA 743 (2004) 13-56

Semi-classical approach to gluon production II

Expand ***perturbatively*** the gluon field as $A_a^\mu \approx \underbrace{A_{(0)}^\mu}_{\mathcal{O}(g^{-1})} + \underbrace{a_a^\mu}_{\mathcal{O}(g)}$

a_a^μ is a *fluctuation* around the background field and it is a solution of the Classical Yang-Mills Eqs. with retarded boundary conditions

$\mathcal{M}_\lambda^a =$ 

via the LSZ reduction formula, the scattering amplitude in the LC gauge is

$$\mathcal{M}_\lambda^a = \lim_{x^+\to\infty} \int d^2\boldsymbol{x}\, d^4y\, e^{i(k^-x^+ - \boldsymbol{k}\cdot\boldsymbol{x})}\, e^{ik^+y^-} \mathcal{G}_{ab}(x^+,\boldsymbol{x};y^+,\boldsymbol{y})\, \mathcal{J}_b(y)\, \epsilon_\lambda$$

Retarded gluon propagator in a background field

Color rotated current

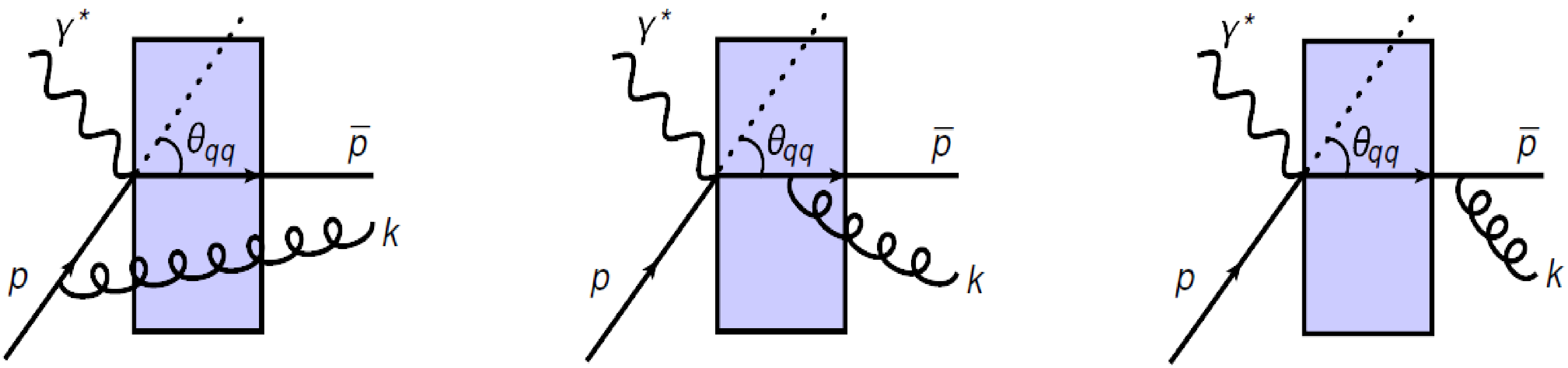
Blaizot, Gelis, Venugopalan, NPA 743 (2004) 13-56

Semi-classical approach to gluon production II

Expand ***perturbatively*** the gluon field as $A^\mu_a \approx \underbrace{A^\mu_{(0)}}_{\mathcal{O}(g^{-1})} + \underbrace{a^\mu_a}_{\mathcal{O}(g)}$

a^μ_a is a *fluctuation* around the background field and it is a solution of the Classical Yang-Mills Eqs. with retarded boundary conditions

$\mathcal{M}^a_\lambda =$



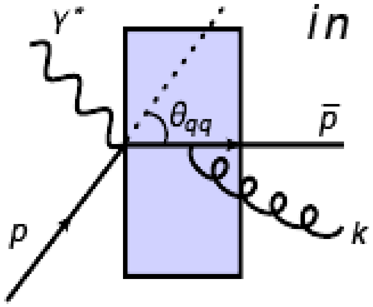
via the LSZ reduction formula, the scattering amplitude in the LC gauge is

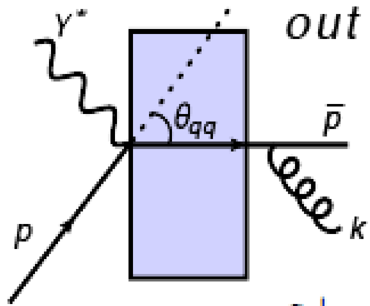
$$\mathcal{M}^a_\lambda = \lim_{x^+\to\infty} \int d^2\boldsymbol{x}\, d^4y\, e^{i(k^-x^+ - \boldsymbol{k}\cdot\boldsymbol{x})}\, e^{ik^+y^-} \mathcal{G}_{ab}(x^+, \boldsymbol{x}; y^+, \boldsymbol{y})\, \mathcal{J}_b(y)\cdot \epsilon_\lambda$$

$$\mathcal{G}_{ab}(x^+, \boldsymbol{x}; y^+, \boldsymbol{y}) = \int_{\boldsymbol{z}(y^+)=\boldsymbol{y}}^{\boldsymbol{z}(x^+)=\boldsymbol{x}} D\boldsymbol{z}(z^+) \exp\left[i\frac{k^+}{2}\int_{y^+}^{x^+} dz^+ \dot{\boldsymbol{z}}^2 \right] \mathcal{U}_{ab}(x^+, y^+, \boldsymbol{z}(z^+))$$

Blaizot, Gelis, Venugopalan, NPA 743 (2004) 13-56

Scattering amplitude from CYM Eqs.





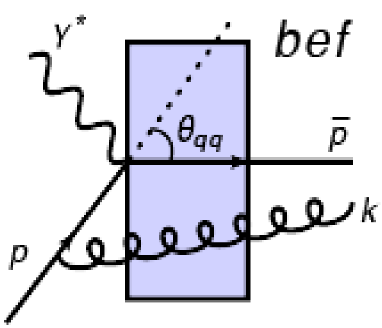
Outcoming parton

$$\mathcal{M}^a_{\lambda,in}(\vec{k}) = \frac{g}{k^+}\int d^2\boldsymbol{x} e^{i(k^- L^+ - \boldsymbol{k}\cdot\boldsymbol{x})}\int_0^{L^+} dy^+ e^{ik^+\bar{u}^- y^+}$$
$$\times\ \epsilon_\lambda\cdot\left(i\boldsymbol{\partial}_y + k^+\bar{\boldsymbol{u}}\right)\mathcal{G}_{ab}\left(L^+,\boldsymbol{x},y^+,\boldsymbol{y}=\bar{\boldsymbol{u}}y^+|k^+\right)\mathcal{U}_{bc}(y^+,0)Q^{out}_c$$

$$\mathcal{M}^a_{\lambda,out}(\vec{k}) = -2i\frac{\epsilon_\lambda\cdot\bar{\boldsymbol{\kappa}}}{\bar{\boldsymbol{\kappa}}^2}e^{i(\boldsymbol{k}\cdot\bar{\boldsymbol{u}})L^+}\mathcal{U}_{ab}(L^+,0)Q^{out}_b\ ,$$

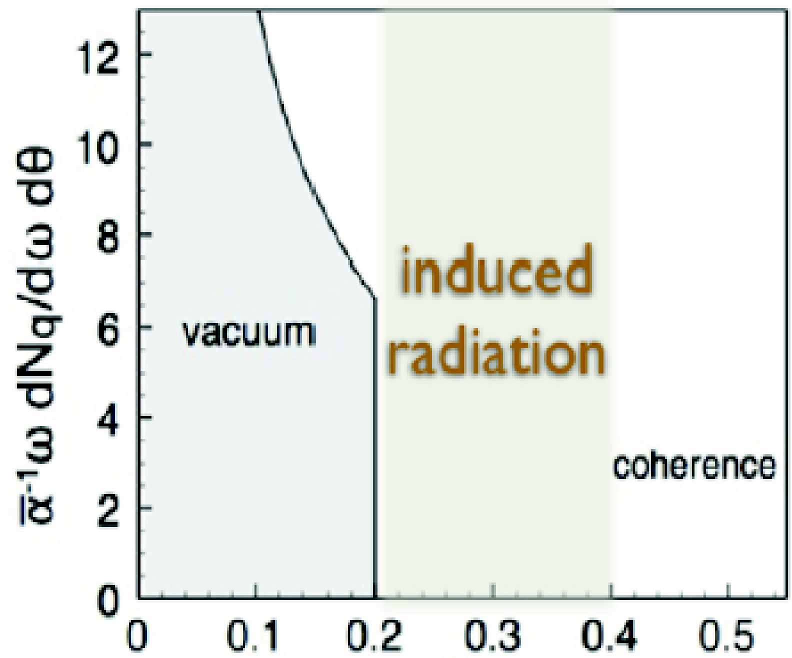
Incoming parton

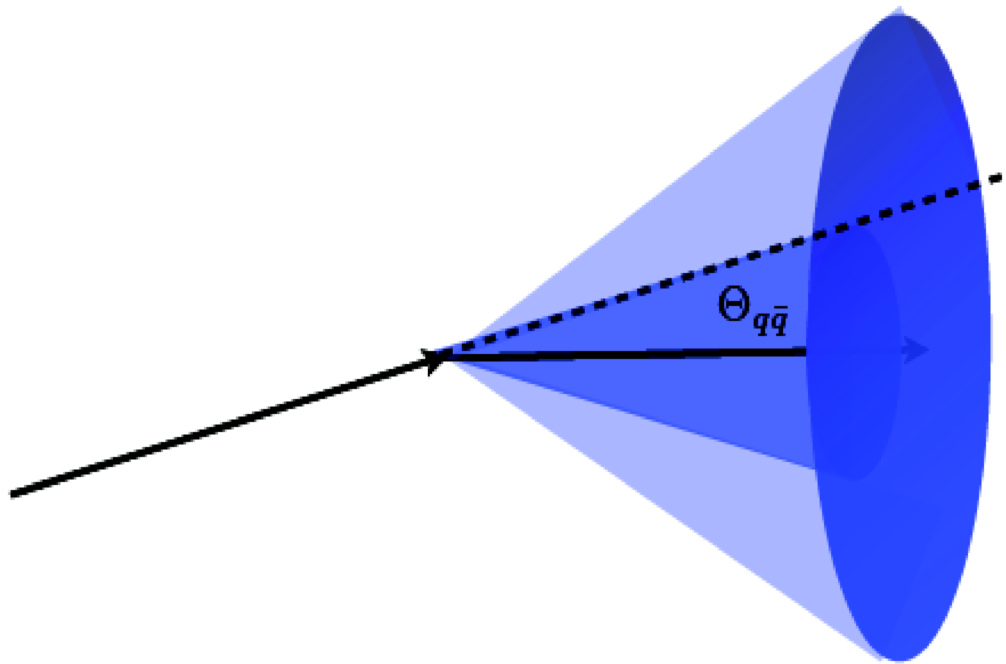
$$\mathcal{M}^a_{\lambda,bef}(\vec{k}) = \frac{g}{k^+}\int_{x^+=\infty} d^2\boldsymbol{x}\, e^{i(k^- x^+ - \boldsymbol{k}\cdot\boldsymbol{x})}\int_{-\infty}^{0} dy^+ e^{ik^+ u^- y^+}$$
$$\times\ \epsilon_\lambda\cdot\left(i\boldsymbol{\partial}_y + k^+\boldsymbol{u}\right)\mathcal{G}_{ab}\left(x^+,\boldsymbol{x},y^+,\boldsymbol{y}=\boldsymbol{u}y^+|k^+\right)Q^{in}_b$$



Armesto et. al, JHEP 1312 (2013) 052

Gluon spectrum: incoherent regime





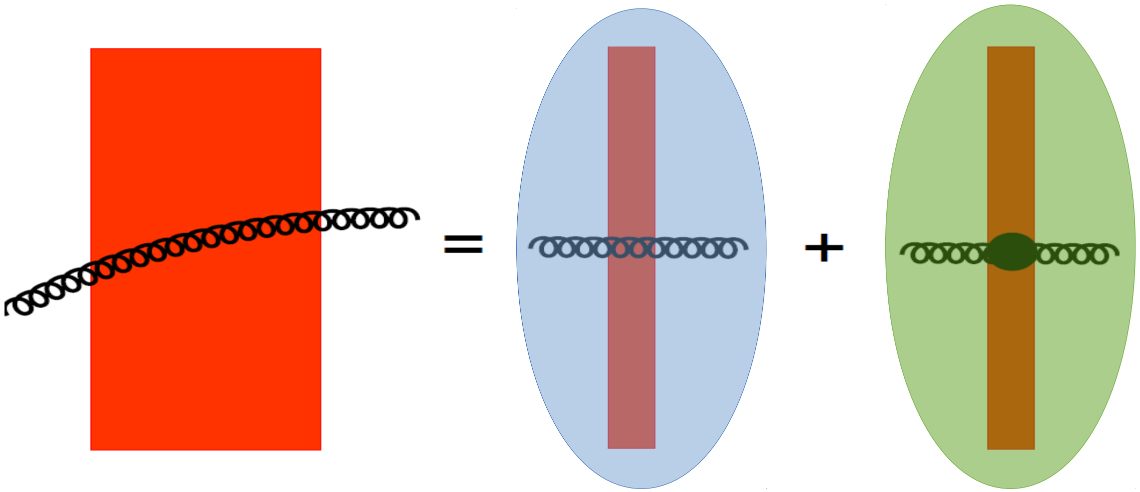
- Interferences play a role at early-times
- Gluon loses vacuum coherence

  ⇒ Open phase space at large angle emissions up to $\theta_{max} = Q_s/\omega$

- Typical ``medium induced'' gluon momentum ~ $Q_s = \hat{q}L$

Armesto et. al, JHEP 1312 (2013) 052

Next-to-Eikonal exp. to the gluon propagator



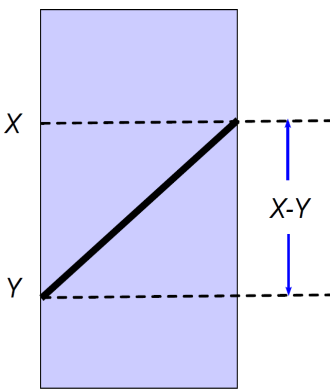
Wilson line
(shockwave)

$$\int d^2\mathbf{x}\; e^{-i\mathbf{k}\cdot\mathbf{x}}\; \mathcal{G}^{ab}_{k^+}(\underline{x};\underline{y}) = \theta(x^+-y^+)\; e^{-i\mathbf{k}\cdot\mathbf{y}}\; e^{-ik^-(x^+-y^+)} \Big\{ \mathcal{U}(x^+,y^+,\mathbf{y}) + \frac{(x^+-y^+)}{k^+}\mathbf{k}^i\, \mathcal{U}^i_{(1)}(x^+,y^+,\mathbf{y}) + i\,\frac{(x^+-y^+)}{2k^+}\, \mathcal{U}_{(2)}(x^+,y^+,\mathbf{y})$$

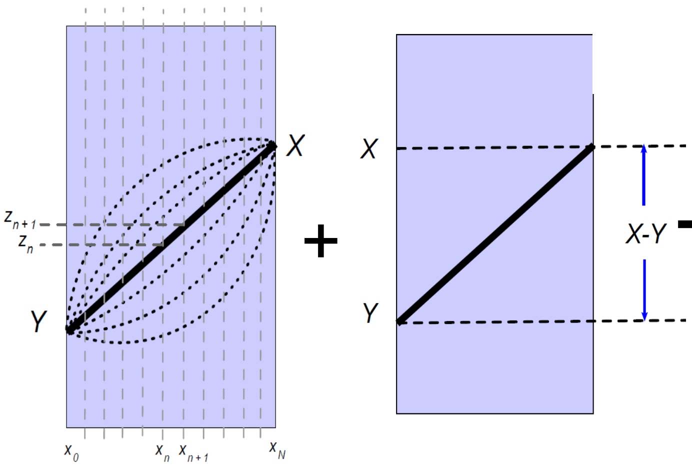
"Decorated operators"
Non Eikonal Corrections

Altinoluk, Armesto, Beuf, Martinez, Salgado, JHEP 1407 (2014) 068

Next-to-Eikonal exp. to the gluon propagator



$$\mathcal{U}^{i,ab}_{(1)}(x^+,y^+,\mathbf{y}) = \int_{y^+}^{x^+} dz^+ \frac{1}{(x^+-y^+)} \Big\{ [\partial_{\mathbf{y}^i}\mathcal{U}(x^+,z^+,\mathbf{y})]\mathcal{U}(z^+,\mathbf{y}^+,\mathbf{y}) \Big\}^{ab}$$



$$\mathcal{U}^{ab}_{(2)}(x^+,y^+,\mathbf{y}) = \int_{y^+}^{x^+} dz^+ \frac{1}{(x^+-y^+)} \Big\{ [\partial^2_{\mathbf{y}}\mathcal{U}(x^+,z^+,\mathbf{y})]\mathcal{U}(z^+,y^+,\mathbf{y}) \Big\}^{ab} .$$

Altinoluk, Armesto, Beuf, Martinez, Salgado, JHEP 1407 (2014) 068

K_T factorization beyond Eikonal accuracy

$$\frac{1}{N_c^2-1}\sum_{\lambda\,\text{phys.}}\left\langle \overline{\mathcal{M}}_\lambda^{ab}(\underline{k},\boldsymbol{q})^\dagger\,\overline{\mathcal{M}}_\lambda^{ab}(\underline{k},\boldsymbol{q})\right\rangle_A = \frac{1}{\boldsymbol{k}^2\,\boldsymbol{q}^2}\int d^2\boldsymbol{b}\int d^2\boldsymbol{r}\;e^{-i(\boldsymbol{k}-\boldsymbol{q})\cdot\boldsymbol{r}}$$

$$\times\Bigg\{4\,(\boldsymbol{k}-\boldsymbol{q})^2 S_A(\boldsymbol{r},\boldsymbol{b}) + 2\frac{L^+}{k^+}\left[(\boldsymbol{k}-\boldsymbol{q})^2\,\boldsymbol{k}^j + \boldsymbol{k}^2(\boldsymbol{k}^j-\boldsymbol{q}^j)\right]\left[\mathcal{O}^j_{(1)}(\boldsymbol{r},\boldsymbol{b}) + \mathcal{O}^j_{(1)}(-\boldsymbol{r},\boldsymbol{b})\right]$$

$$+2i\frac{L^+}{k^+}\boldsymbol{k}\cdot(\boldsymbol{k}-\boldsymbol{q})\left[\mathcal{O}_{(2)}(\boldsymbol{r},\boldsymbol{b}) - \mathcal{O}_{(2)}(-\boldsymbol{r},\boldsymbol{b})\right] + O\left(\left(\frac{L^+}{k^+}\partial^2_\perp\right)^2\right)\Bigg\}\,.$$

where

$$S_A(r,b) = \frac{1}{N_c^2-1}\left\langle \mathrm{tr}\left[\mathcal{U}^\dagger\left(L^+,0;b-\frac{r}{2}\right)\mathcal{U}\left(L^+,0;b+\frac{r}{2}\right)\right]\right\rangle_A ,$$

→ Dipole amplitude
Shockwave contribution

$$\mathcal{O}^j_{(1)}(r,b) = \frac{1}{N_c^2-1}\left\langle \mathrm{tr}\left[\mathcal{U}^\dagger\left(L^+,0;b-\frac{r}{2}\right)\mathcal{U}^j_{(1)}\left(L^+,0;b+\frac{r}{2}\right)\right]\right\rangle_A ,$$

$$\mathcal{O}_{(2)}(r,b) = \frac{1}{N_c^2-1}\left\langle \mathrm{tr}\left[\mathcal{U}^\dagger\left(L^+,0;b-\frac{r}{2}\right)\mathcal{U}_{(2)}\left(L^+,0;b+\frac{r}{2}\right)\right]\right\rangle_A .$$

→ Non Eikonal
Corrections

Altinoluk, Armesto, Beuf, Martinez, Salgado, JHEP 1407 (2014) 068

SSA: Polarized Target

$$k^+ \left(\frac{d\sigma^\uparrow}{dk^+\, d^2\mathbf{k}} - \frac{d\sigma^\downarrow}{dk^+\, d^2\mathbf{k}} \right) = \frac{2}{\mathbf{k}^2} \frac{L^+}{k^+} \int \frac{d^2\mathbf{q}}{(2\pi)^2}\, \varphi_p(\mathbf{q}; x_{\rm cut})$$

$$\times \Bigg\{ \left[(\mathbf{k}-\mathbf{q})^2 \mathbf{k}^j + \mathbf{k}^2 (\mathbf{k}^j - \mathbf{q}^j) \right] \int d^2\mathbf{r} \cos\Big(\mathbf{r}\cdot(\mathbf{k}-\mathbf{q})\Big) \int d^2\mathbf{b}\, \mathcal{O}^j_{(1)}(\mathbf{r},\mathbf{b},\mathbf{s})$$

$$+ \mathbf{k}\cdot(\mathbf{k}-\mathbf{q}) \int d^2\mathbf{r} \sin\Big(\mathbf{r}\cdot(\mathbf{k}-\mathbf{q})\Big) \int d^2\mathbf{b}\, \mathcal{O}_{(2)}(\mathbf{r},\mathbf{b},\mathbf{s}) \Bigg\} + O\left(\left(\frac{L^+}{k^+} \partial_\perp^2 \right)^2 \right)$$

- **Eikonal corrections cancel exactly due to the rotational symmetry around the center of the target.**
- **First subleading Non-Eikonal corrections turn out to be the dominant terms.**

Final interactions play an important role

- **Similar behavior observed with higher twist contributions**

Altinoluk, Armesto, Beuf, Martinez, Salgado, JHEP 1407 (2014) 068

SSA: Longitudinal Polarized Gluon Production

$$k^+ \frac{d\sigma^+}{dk^+ \, d^2\mathbf{k}} - k^+ \frac{d\sigma^-}{dk^+ \, d^2\mathbf{k}} = \frac{L^+}{k^+} \int \frac{d^2\mathbf{q}}{(2\pi)^2} \varphi_p(\mathbf{q}; x_{\rm cut}) \, \mathbf{q}^2 \int d^2\mathbf{b} \int d^2\mathbf{r} \, e^{-i(\mathbf{k}-\mathbf{q})\cdot\mathbf{r}}$$
$$\times \Bigg\{ -i \left[\left(\frac{\mathbf{k}^i}{\mathbf{k}^2} - \frac{\mathbf{q}^i}{\mathbf{q}^2} \right) \epsilon^{ij} - 2 \frac{\left(\epsilon^{il} \, \mathbf{k}^i \, \mathbf{q}^l\right)}{\mathbf{k}^2 \, \mathbf{q}^2} \mathbf{k}^j \right] \mathcal{O}^j_{(1)}(\mathbf{r}, \mathbf{b})$$
$$- \frac{\left(\epsilon^{ij} \, \mathbf{k}^i \, \mathbf{q}^j\right)}{\mathbf{k}^2 \, \mathbf{q}^2} \mathcal{O}_{(2)}(\mathbf{r}, \mathbf{b}) \Bigg\} + O\left(\left(\frac{L^+}{k^+} \partial_\perp^2 \right)^2 \right) .$$

- **Shockwave contribution vanishes exactly again!!!.**
- **Longitudinal polarization of the gluon (via polarized hadrons) is a good observable to study the structure of the next to eikonal corrections.**

Altinoluk, Armesto, Beuf, Martinez, Salgado, JHEP 1407 (2014) 068