Momentum Broadening and Renormalization of $\hat{q}$

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Outline

- Introduction

  Radiative energy loss and $p_\perp$-Broadening

- Energy Loss in pA

- A Short Review

  Leading Order BDMPS-Z results

- $p_\perp$-broadening at NLO

  $\langle p_\perp^2 \rangle \sim \ln^2(L/l_0)$

- Radiative energy loss at NLO

  $\Delta E \sim L^{2+\gamma} \qquad \gamma = 2\sqrt{\alpha_s N_c/\pi}$

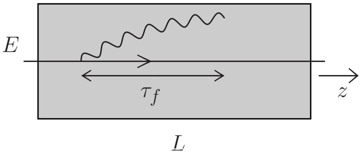
- Summary

In Medium Gluon Radiation

- $\boldsymbol{\tau_f < L}$, **BDMPS-Z** (Nucleus-Nucleus)

  Baier-Dokshitzer-Mueller-Peigné-Schiff-Zakharov (95' – 97')
  Landau-Pomeranchuk-Migdal (LPM) effect (53' – 56')

$$-\frac{dE}{dz} = \frac{1}{4}\alpha_s N_c \left\langle p_\perp^2 \right\rangle = \frac{1}{4}\alpha_s N_c \hat{q} L \qquad \text{(LO)}$$



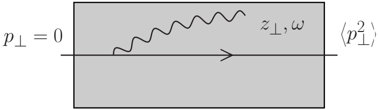
BDMPS-Z at NLO

- $\boldsymbol{\tau_f < L}$, $p_\perp$-broadening and radiative energy loss at NLO

$$\langle p_\perp^2 \rangle = \hat{q} L \Bigg[\underbrace{\; 1 \;}_{\text{LO}} + \underbrace{\alpha_s \left(\ln^2 \frac{L^2}{l_0^2} + \ln \frac{L}{l_0} + C \right)}_{\text{NLO}} \Bigg],$$

$$\text{NLO} \simeq 0.75 \, \text{LO}, \qquad \hat{q} \to \hat{q} \left(1 + \alpha_s \ln^2 \frac{L^2}{l_0^2} \right), \qquad \Delta E \sim L^{2+\gamma}$$

NLO: Wu 11'; Blaizot, Mehtar-Tani; Iancu; Wu, TL, Mueller (13' –14')

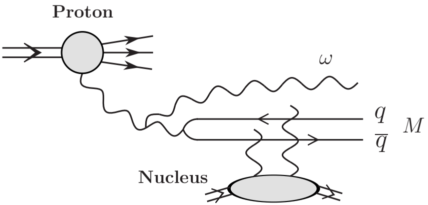


Radiative Energy Loss in pA

- $\tau_f \gg L$, $\boxed{\Delta E \sim E}$ (Proton-Nucleus)
  - **Arleo & Peigné** (11' – 13')

    J/ψ suppression in pA
  - **Liou & Mueller** 14'

    Dijet and J/ψ energy loss in pA



Radiative Energy Loss in pA

$$\omega \frac{dI}{d\omega}\bigg|_{q\bar{q}} = \frac{1}{\sigma_{pA}^{q\bar{q}}} \frac{\omega d\sigma_{pA}^{q\bar{q}g}}{d\omega}\bigg|_{\text{Nucleus}} - \frac{1}{\sigma_{pp}^{q\bar{q}}} \frac{\omega d\sigma_{pp}^{q\bar{q}g}}{d\omega}\bigg|_{\text{Proton}} = \frac{2\alpha_s N_c}{\pi} \ln \frac{EQ_s}{2\omega M}$$

$$\int_{\bar{\omega}} d\omega \, \frac{dI}{d\omega}\bigg|_{q\bar{q}} = \frac{1}{2}$$

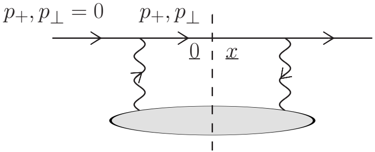
For the LHC, $M \simeq 7\,\mathrm{GeV}$, $Q_s \simeq 2.5\,\mathrm{GeV}$, $\alpha_s \simeq 1/3$

$$\frac{\bar{\omega}}{E} = \frac{Q_s}{2M} \exp\left(- \sqrt{\frac{\pi}{2\alpha_s N_c}} \right) \simeq 5\%$$

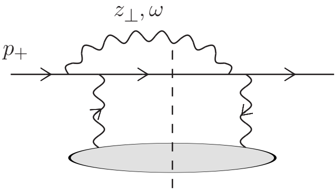
Frankfurt & Strikman, 07'

A Short Review: Leading Order

BDMPS-Z Formalism



Momentum Broadening



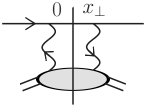
Energy Loss

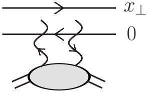
$$\langle p_\perp^2 \rangle = \int d^2 p_\perp p_\perp^2 \frac{dN}{d^2 p_\perp} = \hat{q} L$$

$$-\frac{dE}{dz} = \int \omega \frac{dI}{d\omega} d\omega \sim \hat{q} L$$

$$\langle p_\perp^2 \rangle \sim -\frac{dE}{dz} \quad (\text{LO})$$

$p_\perp$-broadening at LO

$S(x) =$ 

$=$ 

$= \exp(-\hat{q} L x_\perp^2/4)$

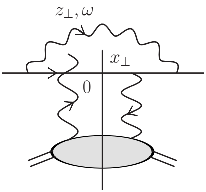
Transverse momentum distribution

$$\frac{dN}{d^2p_\perp} = \int \frac{d^2x_\perp}{(2\pi)^2} e^{-ip_\perp \cdot x_\perp} S(x_\perp)$$

$$\langle p_\perp^2 \rangle = \int d^2p_\perp \, p_\perp^2 \frac{dN}{d^2p_\perp} = -\nabla_{x_\perp}^2 S(x_\perp)\Big|_{x_\perp=0} = \hat{q} L$$

$S(x_\perp)$ at NLO ?

Extra Gluon Radiation

$S(x_\perp) =$ 

$S \sim$ (Gluon Emission) $\times$ Interaction

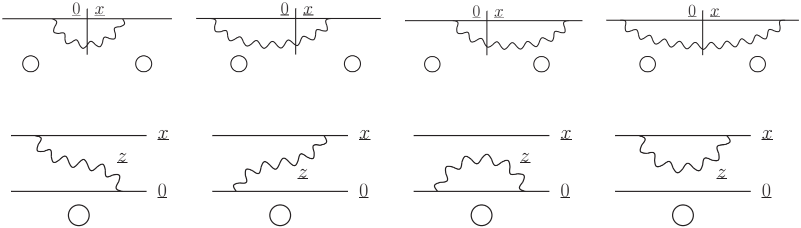
Gluon Emission: $\int \frac{d\omega}{\omega} d^2 z_\perp \frac{x_\perp^2}{z_\perp^2 (z_\perp - x_\perp)^2} \sim \int \frac{d\omega}{\omega} \frac{dz_\perp^2}{(z_\perp^2)^2}$

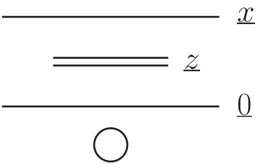
Interaction: Single Scattering $z_\perp^2$

$S \Longrightarrow$ Double Log ? Yes!

Dipole Form

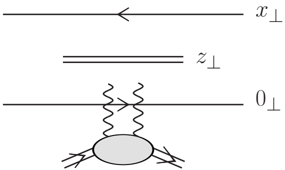
Gluon Emission Amplitude

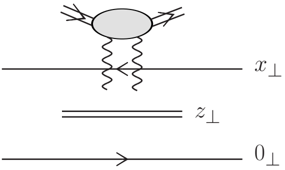


Dipole 

$$= \frac{\alpha_s N_c}{2\pi^2} \int d^2 z_\perp \frac{x_\perp^2}{z_\perp^2 (z_\perp - x_\perp)^2} \frac{d\omega}{\omega}$$

Interaction: Single Scattering



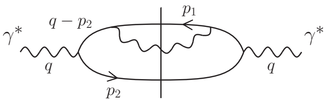


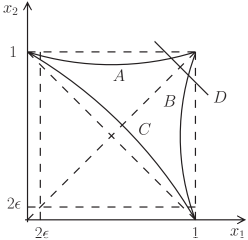
$$S(x_\perp) = \frac{\alpha_s N_c}{2\pi^2} \int d^2 z_\perp \underbrace{\frac{x_\perp^2}{z_\perp^2 (z_\perp - x_\perp)^2}}_{1/(z_\perp^2)^2} \frac{d\omega}{\omega} \left(- \frac{\hat{q} L}{4} \right) \underbrace{\left[z_\perp^2 + (z_\perp - x_\perp)^2 \right]}_{z_\perp^2}$$

$$= - \frac{x_\perp^2}{4} \frac{\alpha_s N_c}{\pi} \hat{q} L \int_{?} \frac{d z_\perp^2}{z_\perp^2} \frac{d\omega}{\omega} \qquad (z_\perp^2 > x_\perp^2)$$

Sterman-Weinberg jets

Sterman-Weinberg '77, Stevenson '78, Weeks '79





Jet cone 2δ, energy fraction ϵ

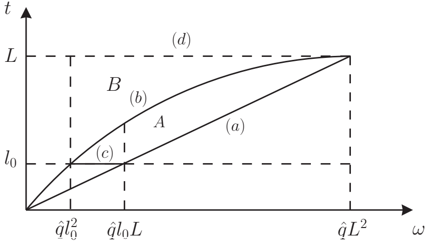
$$\frac{\sigma_{\text{2jet}}}{\sigma_{\text{tot}}} = 1 - \frac{2\alpha_s}{3\pi} \int_{\widehat{ABCD}} dx_1 dx_2 \frac{x_1^2 + x_2^2}{(1 - x_1)(1 - x_2)}$$

$$= 1 - \frac{4\alpha_s}{3\pi} \left[4 \ln \frac{1}{\delta} \ln \frac{1}{2\epsilon} - 3 \ln \frac{1}{\delta} + \frac{\pi^2}{3} - \frac{5}{2} \right]$$

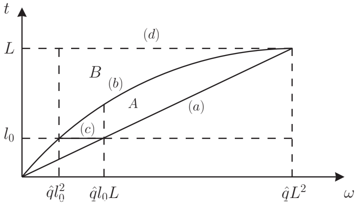
Double Logarithmic region

Gluon Formation Time $t = \omega z^2$

$$
\begin{aligned}
(a):&\quad \underline{z}^2 \gg \underline{x}^2 \approx \frac{1}{\hat{q}L}, &\quad t > \frac{\omega}{\hat{q}L}\\
(c)+(d):&\quad l_0 \ll \omega \underline{z}^2 \ll L, &\quad l_0 < t < L\\
(b):&\quad \omega \underline{z}^2 \hat{q} \ll \frac{1}{\underline{z}^2}, &\quad t < \sqrt{\frac{\omega}{\hat{q}}}
\end{aligned}
$$



Double Logarithmic Contribution



Region A: Single Scattering. Region B: Multiple Scattering.

$$\left\langle p_\perp^2 \right\rangle = -\nabla_{x_\perp^2} S(x_\perp)\Big|_{x_\perp=0}$$

$$= \frac{\alpha_s N_c}{\pi}\hat{q}L\int_{l_0}^{L}\frac{dt}{t}\int_{\hat{q}t^2}^{\hat{q}Lt}\frac{d\omega}{\omega} = \frac{\alpha_s N_c}{8\pi}\hat{q}L\ln^2\left(\frac{L}{l_0}\right)^2$$

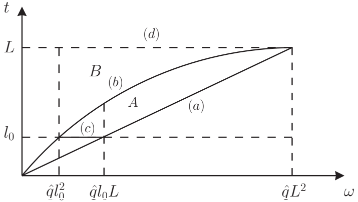
Full BDMPS-Z Calculation

LPM Effect (A+B)

$$\omega \frac{d}{d\omega} S(x_\perp, \omega) = -\frac{\alpha_s N_c}{2\omega^2} \mathrm{Re} \int_0^L dz_2 \int_0^{z_2} dz_1 \, \nabla_{B_{1\perp}} \cdot \nabla_{B_{2\perp}}$$

$$\left[e^{-\hat{q} x_\perp^2 (L - z_2)/4 - \hat{q} x_\perp^2 z_1 / 4} G(B_{2\perp}, z_2; B_{1\perp}, z_1) - G_0(B_{2\perp}, z_2; B_{1\perp}, z_1) \right] \Big|_{B_{2\perp}=0}^{B_{2\perp}=x_\perp} \Big|_{B_{1\perp}=0}^{B_{1\perp}=x_\perp}$$

In Single Scattering Region (A), Confirm the Double Log.
Additionally, Extract Single Log.



Full result

$$\langle p_\perp^2 \rangle_{\rm NLO} = \frac{\alpha_s N_c}{8\pi}\hat{q}L \ln^2\frac{L^2}{l_0^2} + \frac{\alpha_s N_c}{\pi}\hat{q}L\Bigg[\ln\frac{8ml_0}{\underline{x}^2\hat{q}L} - \frac{1}{3} + \int_0^1\frac{dx}{x}\left(\frac{x\mathcal{G}}{xG} - 1\right)\Bigg]\ln\frac{L}{l_0} + C \qquad (l_0 \to \lambda l_0)$$

Rough Estimation:

$L \simeq 5\,\text{fm}$, $m \simeq 300\,\text{MeV}$, $ml_0 \simeq 1$, $x\mathcal{G}/xG \simeq 1$, $x_\perp^2\hat{q}L \simeq 4$, $\alpha_s \simeq 1/3$

$$\langle p_\perp^2 \rangle_{\rm NLO} \simeq 0.75\hat{q}L$$

Double Log Resummation in Large N_c:

$$\langle p_\perp^2 \rangle = \hat{q}L\sqrt{\frac{4\pi}{\alpha_s N_c}}\frac{1}{\ln\frac{L^2}{l_0^2}}I_1\left[\sqrt{\frac{\alpha_s N_c}{\pi}}\ln\frac{L^2}{l_0^2}\right]$$

How to Interpret

$$\langle p_\perp^2 \rangle = \underbrace{\hat{q}\left(1 + \frac{\alpha_s N_c}{8\pi} \ln^2 \frac{L^2}{l_0^2}\right)}_{\text{New } \hat{q}} \; L$$

- Renormalization of $\hat{q}$ **?** (Blaizot, Mehtar-Tani — '14)
  New Scaling for Energy Loss (DL Resummation, Large L)

$$\Delta E \sim \hat{q}L^2 \text{ (LO)} \to \hat{q}L^{2+\gamma} \text{ (NLO-Res)}, \qquad \gamma = 2\sqrt{\frac{\alpha_s N_c}{\pi}}$$

- Evolution Equation for $\hat{q}$ **?** (Iancu — '14)

Summary

- $p_\perp$-broadening (BDMPS-Z) at NLO

$$\langle p_\perp^2 \rangle = \hat{q}L(1 + \ln^2)$$

- Renormalization of $\hat{q}$
- New Scaling $\Delta E \sim L^{2+\gamma}$
- Energy loss in pA, $\Delta E \sim E$