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# Standard Model Theory

or

Theoretical uncertainties and their prospects for the HL-LHC

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IPPP, Durham University



ECFA High Luminosity LHC Experiments Workshop  
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# Theoretical Uncertainties

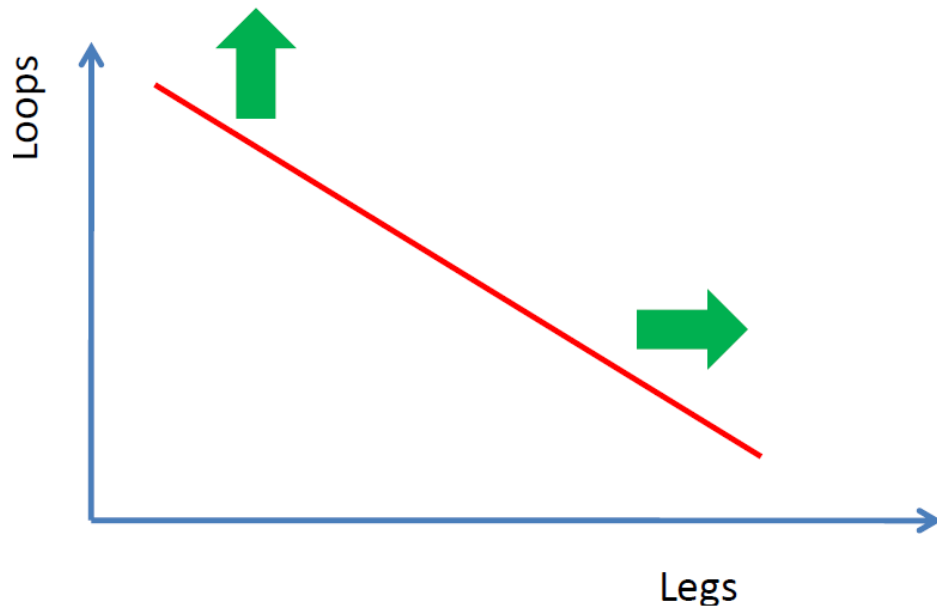
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- **Missing Higher Order corrections (MHO)**
  - truncation of the perturbative series
  - often estimated by scale uncertainties - renormalisation/factorisation
  - ✓ systematically improvable by inclusion of higher orders
- **Uncertainties in input parameters**
  - parton distributions
  - masses, e.g.,  $m_W$ ,  $m_h$ ,  $[m_t]$
  - couplings, e.g.,  $\alpha_s(M_Z)$
  - ✓ systematically improvable by better description of benchmark processes
- **Uncertainties in parton/hadron transition**
  - fragmentation (parton shower)
  - ✓ systematically improvable by matching/merging with higher orders
  - hadronisation (model)
  - underlying event (tunes)

**Goal:** To fully exploit LHC, need to reduce theory uncertainties by a **factor of two** compared to where we are now in next decade

# What is the hold up?

Rough idea of complexity of process  $\sim$  #Loops + #Legs (+ #Scales)



- loop integrals are ultraviolet/infrared divergent
- complicated by extra mass/energy scales
- loop integrals often unknown
  - ✓ completely solved at NLO
- real (tree) contributions are infrared divergent
- isolating divergences complicated
  - ✓ completely solved at NLO
- currently far from automation
  - ✓ mostly solved at NLO

**Current standard: NLO**

# 1. Estimating uncertainties of MHO

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- ✓ Consider a generic observable  $\mathcal{O}$  (e.g.  $\sigma_H$ )

$$\mathcal{O}(Q) \sim \mathcal{O}_k(Q, \mu) + \Delta_k(Q, \mu)$$

where

$$\mathcal{O}_k(Q, \mu) \equiv \sum_{n=0}^k c_n(Q, \mu) \alpha_s(\mu)^n, \quad \Delta_k(Q, \mu) \equiv \sum_{n=k+1}^{\dots} c_n(Q, \mu) \alpha_s(\mu)^n$$

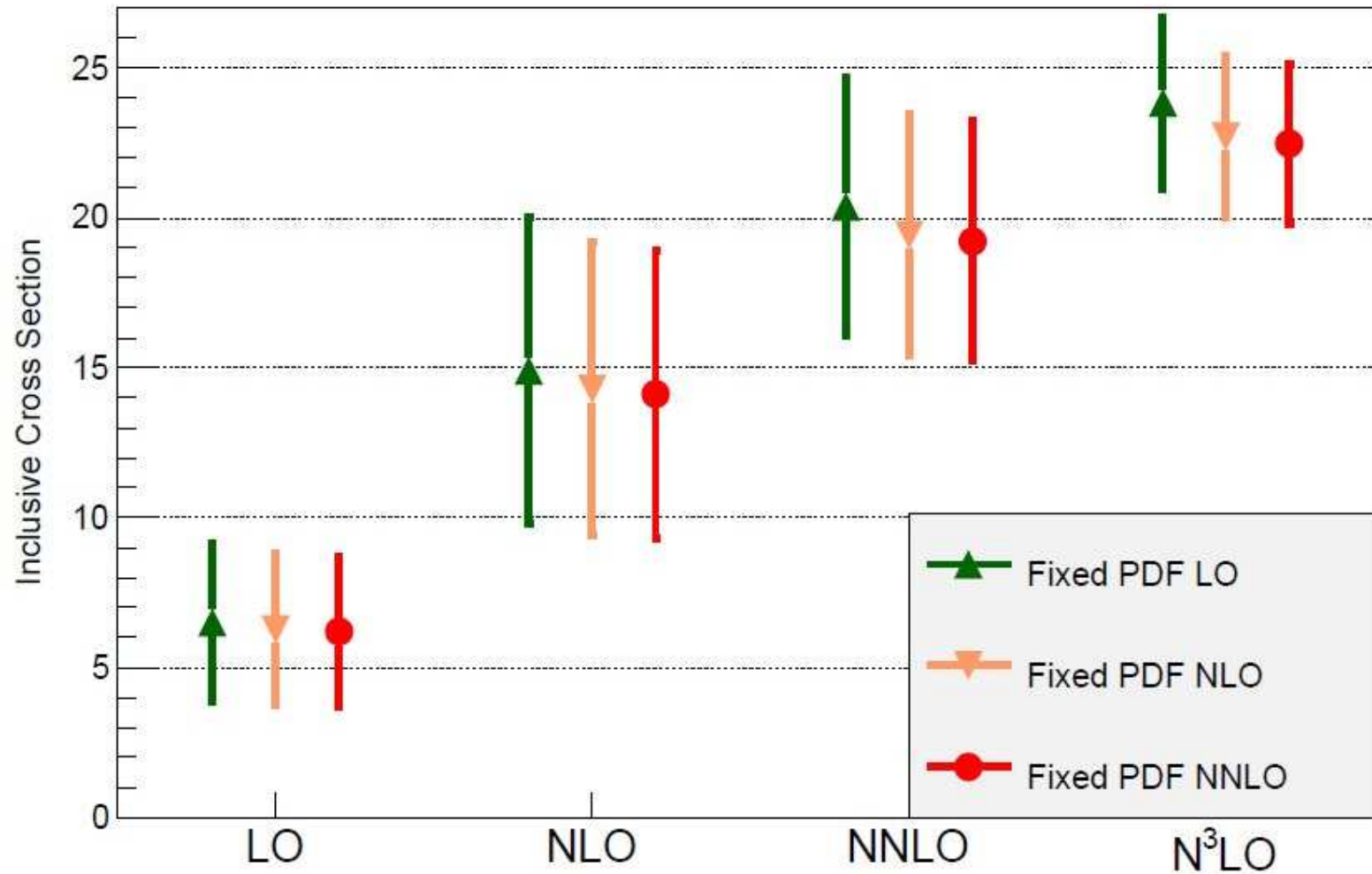
- ✓ Usual procedure is to use scale variations to estimate  $\Delta_k$ ,

$$\Delta_k(Q, \mu) \sim \max \left[ \mathcal{O}_k \left( Q, \frac{\mu}{r} \right), \mathcal{O}_k(Q, r\mu) \right] \sim \alpha_s(\mu)^{k+1}$$

where  $\mu$  is chosen to be a typical scale of the problem and typically  $r = 2$ .

**Choice of  $\mu$  and  $r = 2$  is convention**

# Theoretical uncertainty on $\sigma_H$



Forte, Isgro, Vita

**Warning:** Scale variation may not give an accurate estimate of the uncertainty in the cross section!!

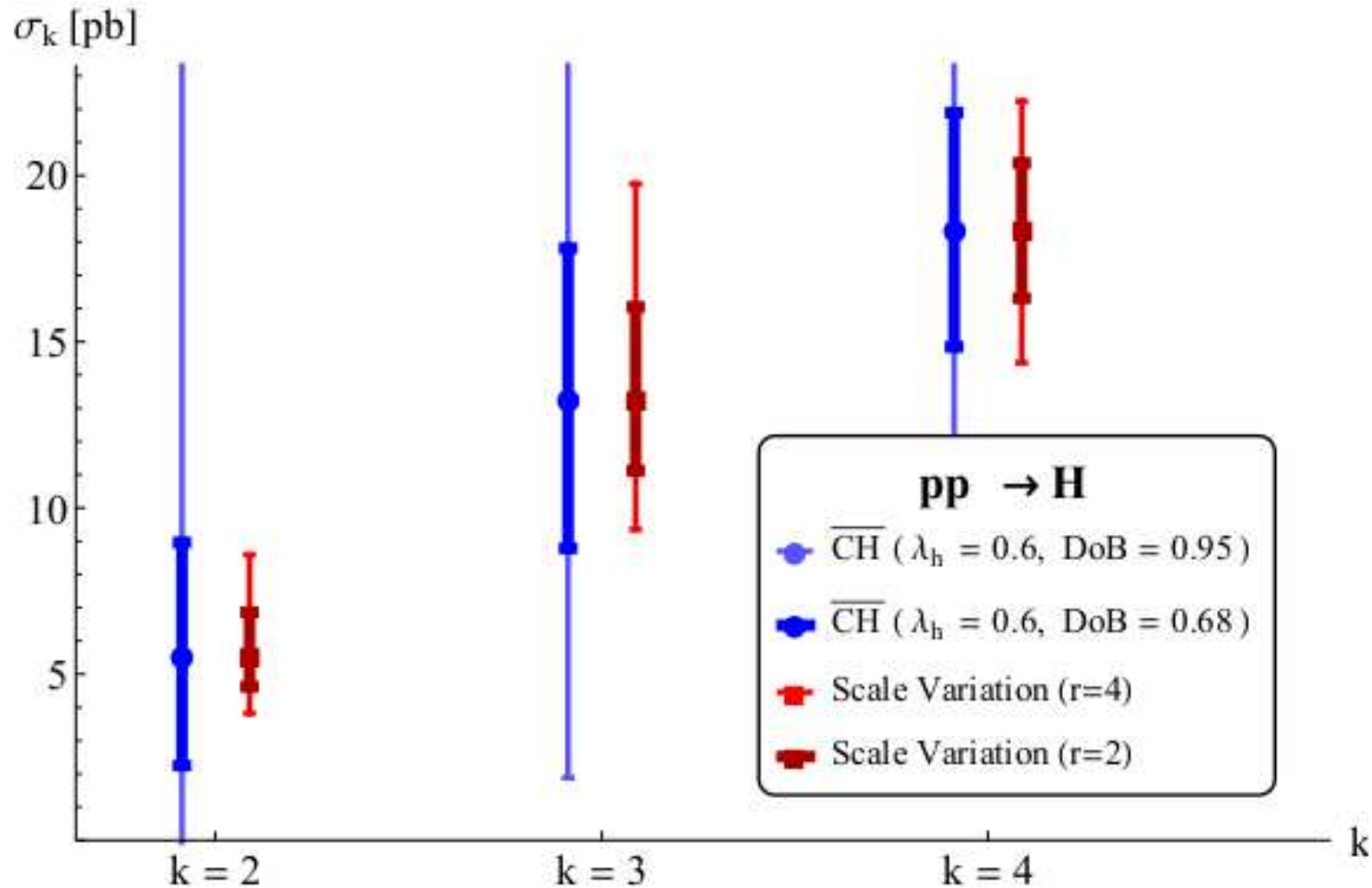
# Going beyond scale uncertainties

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- ✓ **Series acceleration** David, Passarino  
sequence transformations gives estimates of some of the unknown terms in series
- ✓ **Estimate coefficients using information on the singularity structure of the Mellin space cross section coming from all order resummation** Ball et al
  - large N (soft gluon, Sudakov)
  - small N (high energy, BFKL)
- ✓ **Bayesian estimate of unknown coefficients** Cacciari, Houdeau  
make the assumption that all the coefficients  $c_n$  share a (process dependent) upper bound  $\bar{c} > 0$  leading to density functions  $f(c_n|\bar{c})$  and  $f(\ln \bar{c})$   
recent refinement of method Bagnaschi, Cacciari, Guffanti, Jenniches
- ✓ **Accepting that scale variation does not give reliable error estimate, can predict the part of the N<sup>3</sup>LO cross section coming from scale variations.**

Pressure is building to better estimate MHO

# Theoretical uncertainty on $\sigma_H$ revisited



Bagnaschi, Cacciari, Guffanti, Jenniches

Errors in Modified CH approach larger but more realistic!!

## 2. Improved calculations wishlist

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- ✓ From the 2013 Les Houches proceedings

arXiv:1405.1067

- ✓ Counting of orders is done relative to LO QCD independent of absolute power of  $\alpha_s$  in cross section
- ✓  $\alpha \sim \alpha_s^2$  so that NNLO QCD and NLO EW effects naively of same size
- ✓ List contains processes where improved precision motivated by experiment.
- ✓ Is ambitious, but doable over the remainder of LHC running

- LO  $\equiv O(1)$
- NLO QCD  $\equiv O(\alpha_s)$
- NLO EW  $\equiv O(\alpha)$
- NNLO QCD  $\equiv O(\alpha_s^2)$
- NNLO QCD+EW  $\equiv O(\alpha_s \alpha)$
- NNNLO QCD  $\equiv O(\alpha_s^3)$
- + plus matching to parton shower for fully exclusive final states
- ✓  $d\sigma$  indicates fully differential cross section
- ✓  $d\sigma$  NNLO QCD+NLO EW indicates a single code including both NNLO QCD and NLO EW effects. Where possible full resonance production including interference and background to be taken into account.



# Les Houches wishlist for Higgs

Process	State of the Art	Desired
$H$	$d\sigma$ @ NNLO QCD (expansion in $1/m_t$ ) full $m_t/m_b$ dependence @ NLO QCD and @ NLO EW NNLO+PS, in the $m_t \rightarrow \infty$ limit	$d\sigma$ @ NNNLO QCD (infinite- $m_t$ limit) full $m_t/m_b$ dependence @ NNLO QCD and @ NNLO QCD+EW NNLO+PS with finite top quark mass effects
$H + j$	$d\sigma$ @ NNLO QCD ( $g$ only) and finite-quark-mass effects @ LO QCD and LO EW	$d\sigma$ @ NNLO QCD (infinite- $m_t$ limit) and finite-quark-mass effects @ NLO QCD and NLO EW
$H + 2j$	$\sigma_{\text{tot}}(\text{VBF})$ @ NNLO(DIS) QCD $d\sigma(\text{VBF})$ @ NLO EW $d\sigma(gg)$ @ NLO QCD (infinite- $m_t$ limit) and finite-quark-mass effects @ LO QCD	$d\sigma(\text{VBF})$ @ NNLO QCD + NLO EW  $d\sigma(gg)$ @ NNLO QCD (infinite- $m_t$ limit) and finite-quark-mass effects @ NLO QCD and NLO EW
$H + V$	$d\sigma$ @ NNLO QCD $d\sigma$ @ NLO EW $\sigma_{\text{tot}}(gg)$ @ NLO QCD (infinite- $m_t$ limit)	with $H \rightarrow b\bar{b}$ @ same accuracy $d\sigma(gg)$ @ NLO QCD with full $m_t/m_b$ dependence
$tH$ and $\bar{t}H$	$d\sigma(\text{stable top})$ @ LO QCD	$d\sigma(\text{top decays})$ @ NLO QCD and NLO EW
$t\bar{t}H$	$d\sigma(\text{stable tops})$ @ NLO QCD	$d\sigma(\text{top decays})$ @ NLO QCD and NLO EW
$gg \rightarrow HH$	$d\sigma$ @ NLO QCD (leading $m_t$ dependence) $d\sigma$ @ NNLO QCD (infinite- $m_t$ limit)	$d\sigma$ @ NLO QCD with full $m_t/m_b$ dependence

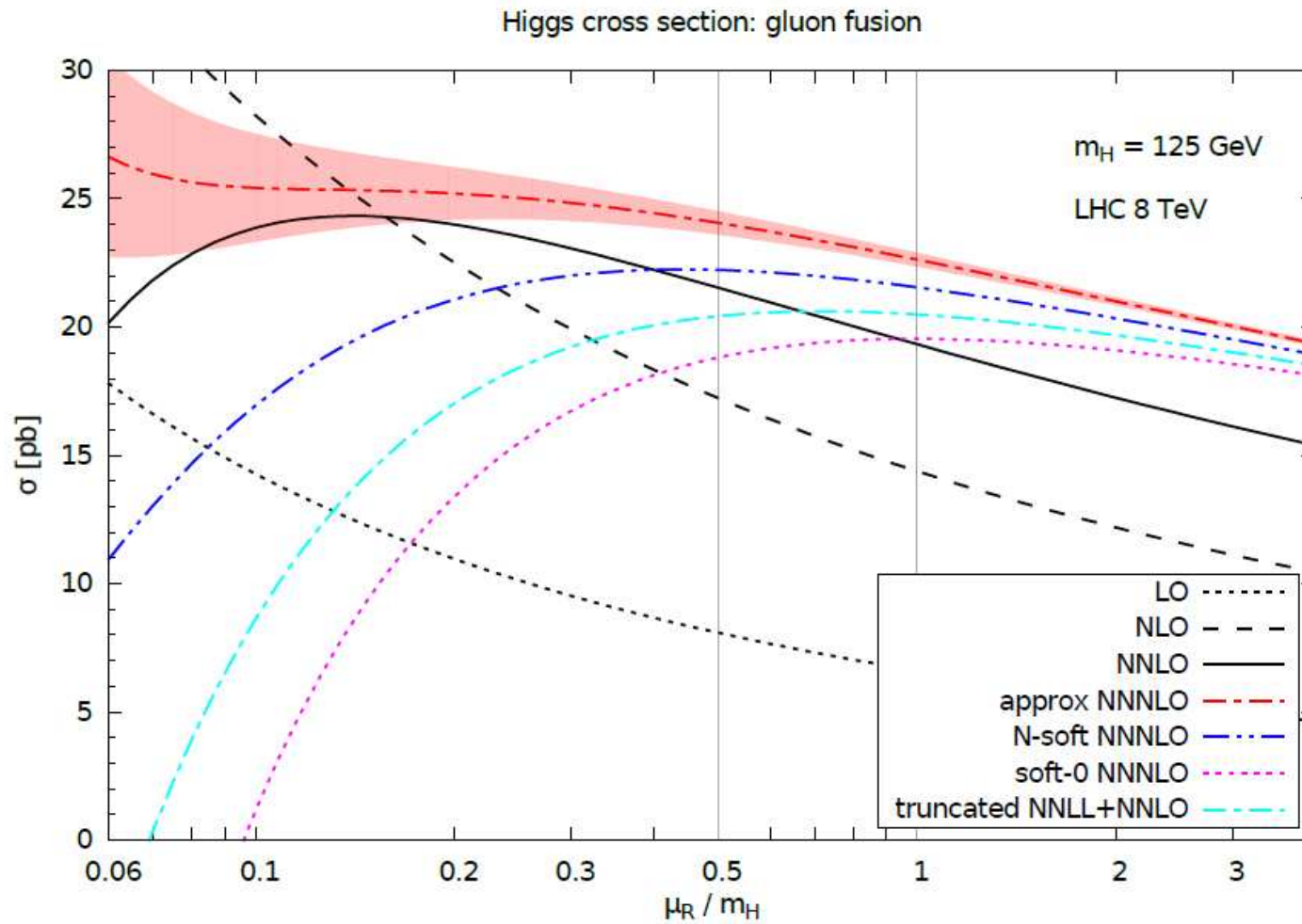
# First assault on Higgs production at N3LO $m_t \rightarrow \infty$

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- ✓ Currently,  $\sigma_H$  via gluon fusion known to NNLO QCD in infinite top mass limit, with finite quark mass effects at NLO QCD and NLO EW. Theoretical uncertainties  $\mathcal{O}(15\%)$  with HO and PDF+ $\alpha_s$  uncertainties roughly equal.
- ✓ Aim to reduce the theoretical error for the inclusive Higgs cross section via gluon fusion to  $\mathcal{O}(5\%)$  by computing NNNLO QCD effects.
- ✓ **Ingredients:** Three-loop H+0 parton, Two-loop H+1 parton, One-loop H+2 parton, Tree-level H+3 parton - all known as matrix elements for  $m_t \rightarrow \infty$ 
  - **key part is to extract the infrared singularities so the parts can be combined**
- ✓ **Threshold corrections: Major new result**

Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger
- ✓ Opens up the possibility of **full** N3LO corrections in future.

# Higgs cross section at approximate N3LO



Bonvini, Ball, Forte, Marzani and Ridolfi

# pp $\rightarrow$ H + jet production at NNLO $m_t \rightarrow \infty$

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- ✓ Key goal: Establish properties of the Higgs boson!
- ✓ experimental event selection according to number of jets
  - ✓ different backgrounds for different jet multiplicities
  - ✓ H+0 jet known at NNLO

Anastasiou, Melnikov, Petriello; Catani, Grazzini

- ✓ H+n jets (n=1,2,3) known at NLO
- ✓ H+0 jet and H+1 jet samples of similar size
- ✓ NNLO H+1 jet crucial, particularly for  $WW$  channel
  - ✓ gluons-only total cross section

Boughezal, Caola, Melnikov, Petriello, Schulze

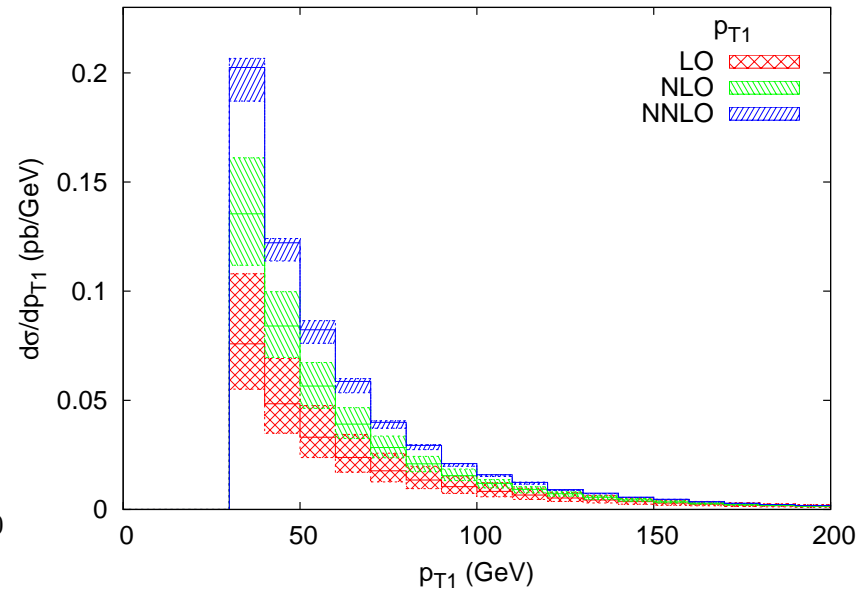
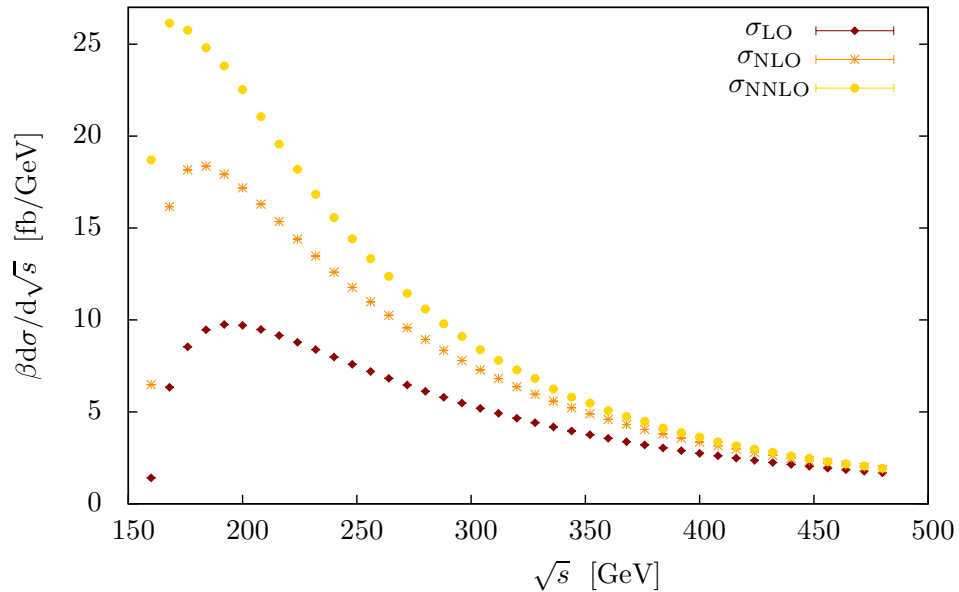
- ✓ gluons-only differential cross sections

Chen, Gehrmann, NG, Jacquier

- ✓ gg-channel dominant - at NLO gg(70%), qg (30%)

- ⚠ other channels in progress

# pp $\rightarrow$ H + jet at NNLO $m_t \rightarrow \infty$



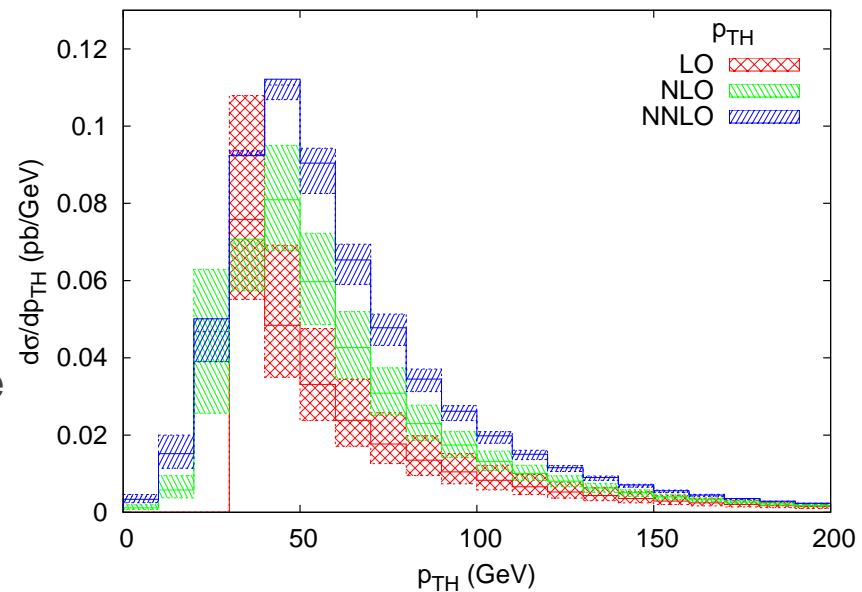
✓ large effects near partonic threshold

✓ large  $K$ -factor

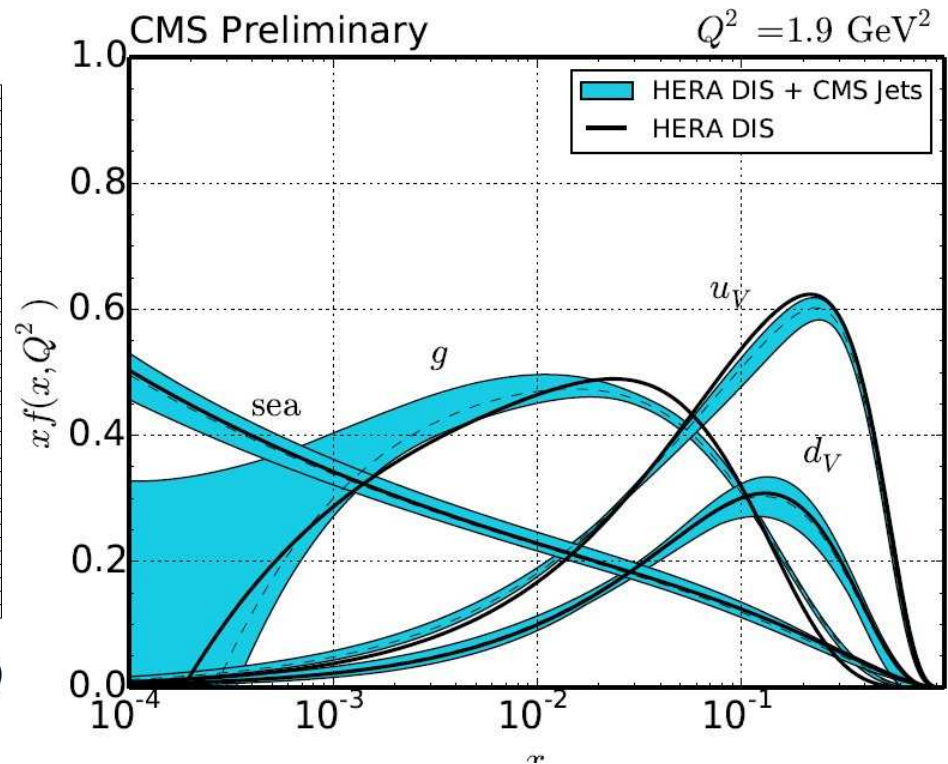
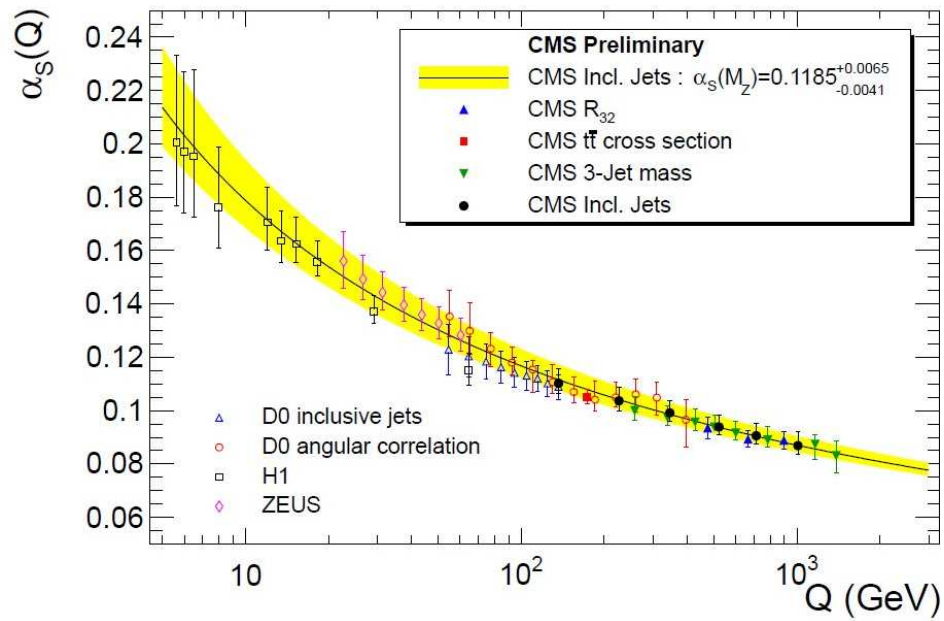
$$\sigma_{NLO}/\sigma_{LO} \sim 1.6$$

$$\sigma_{NNLO}/\sigma_{NLO} \sim 1.3$$

✓ significantly reduced scale dependence  $\mathcal{O}(4\%)$



# 3. Improved precision for input parameters



- ✓ More precise measurements of strong coupling
- ✓ Improved parton distributions

# Les Houches wishlist for Top/Jet processes

Process	State of the Art	Desired
$t\bar{t}$	$\sigma_{\text{tot}}$ (stable tops) @ NNLO QCD d $\sigma$ (top decays) @ NLO QCD d $\sigma$ (stable tops) @ NLO EW	d $\sigma$ (top decays) @ NNLO QCD + NLO EW
$t\bar{t} + j(j)$	d $\sigma$ (NWA top decays) @ NLO QCD	d $\sigma$ (NWA top decays) @ NNLO QCD + NLO EW
$t\bar{t} + Z$	d $\sigma$ (stable tops) @ NLO QCD	d $\sigma$ (top decays) @ NLO QCD + NLO EW
single-top	d $\sigma$ (NWA top decays) @ NLO QCD	d $\sigma$ (NWA top decays) @ NNLO QCD + NLO EW
dijet	d $\sigma$ @ NNLO QCD ( $g$ only) d $\sigma$ @ NLO EW (weak)	d $\sigma$ @ NNLO QCD + NLO EW
$3j$	d $\sigma$ @ NLO QCD	d $\sigma$ @ NNLO QCD + NLO EW
$\gamma + j$	d $\sigma$ @ NLO QCD d $\sigma$ @ NLO EW	d $\sigma$ @ NNLO QCD + NLO EW

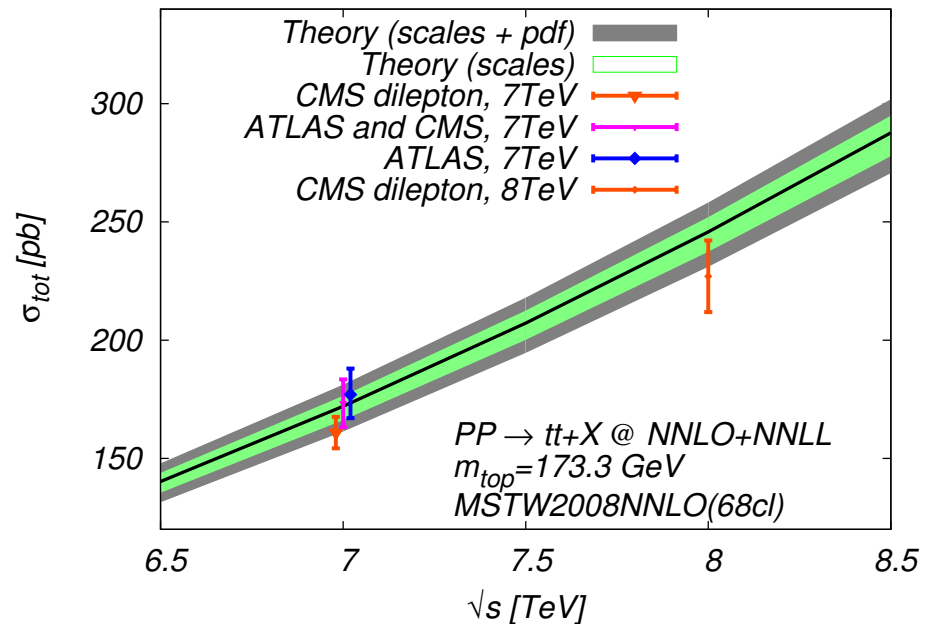
# pp $\rightarrow t\bar{t}$ at NNLO

- ✓ Top production important as possible venue for new physics **and** for determination of gluon PDF at large  $x$
- ✓ Experimental uncertainties  $\mathcal{O}(5\%)$
- ✓ NNLO total cross section known

Czakon, Fielder, Mitov

 distributions in progress

- ✓ NNLO theory uncertainty  $\mathcal{O}(4\%)$



Collider	$\sigma_{\text{tot}}$ [pb]	scales [pb]	pdf [pb]
Tevatron	7.009	+0.259(3.7%) -0.374(5.3%)	+0.169(2.4%) -0.121(1.7%)
LHC 7 TeV	167.0	+6.7(4.0%) -10.7(6.4%)	+4.6(2.8%) -4.7(2.8%)
LHC 8 TeV	239.1	+9.2(3.9%) -14.8(6.2%)	+6.1(2.5%) -6.2(2.6%)
LHC 14 TeV	933.0	+31.8(3.4%) -51.0(5.5%)	+16.1(1.7%) -17.6(1.9%)

NNLO

Collider	$\sigma_{\text{tot}}$ [pb]	scales [pb]	pdf [pb]
Tevatron	7.164	+0.110(1.5%) -0.200(2.8%)	+0.169(2.4%) -0.122(1.7%)
LHC 7 TeV	172.0	+4.4(2.6%) -5.8(3.4%)	+4.7(2.7%) -4.8(2.8%)
LHC 8 TeV	245.8	+6.2(2.5%) -8.4(3.4%)	+6.2(2.5%) -6.4(2.6%)
LHC 14 TeV	953.6	+22.7(2.4%) -33.9(3.6%)	+16.2(1.7%) -17.8(1.9%)

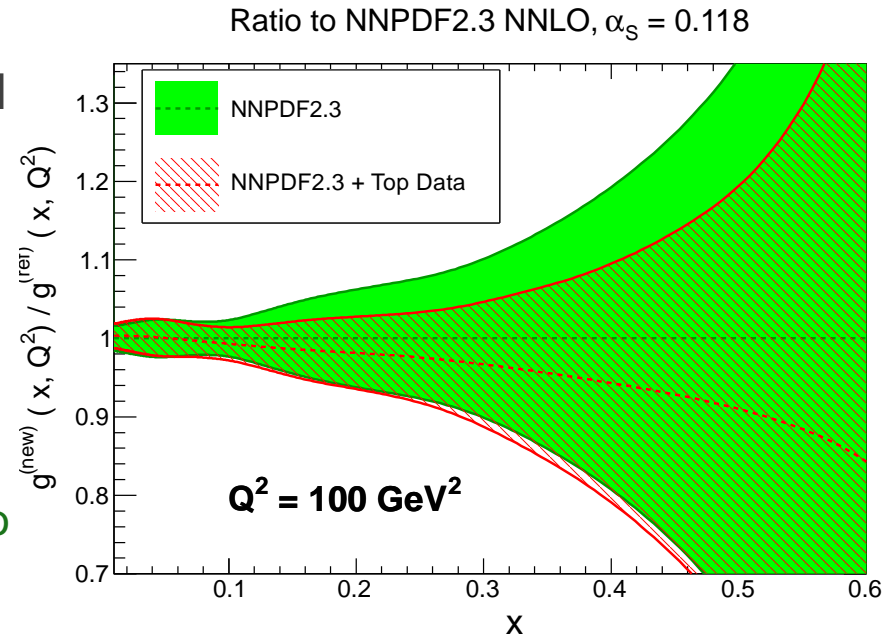
NNLO+NNLL



# Impact on gluon distribution

- ✓ Top production at the LHC dominated by  $qg$  and  $gg$  channels
- ✓ Total cross section sensitive to gluon PDF
- ✓ Impacts the NNLO global parton distribution fit

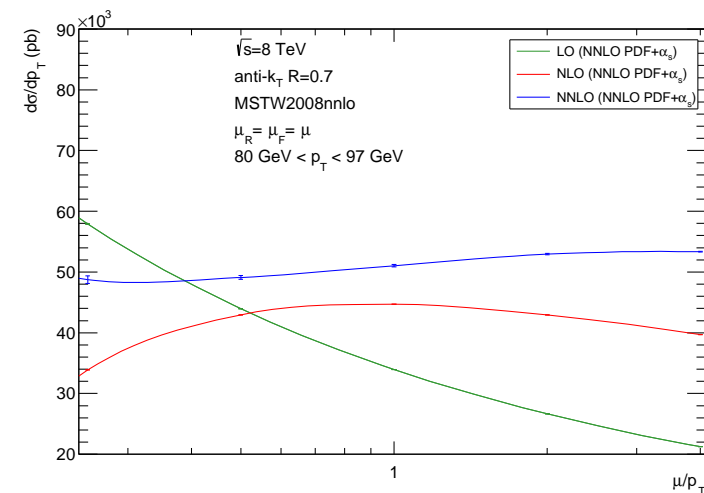
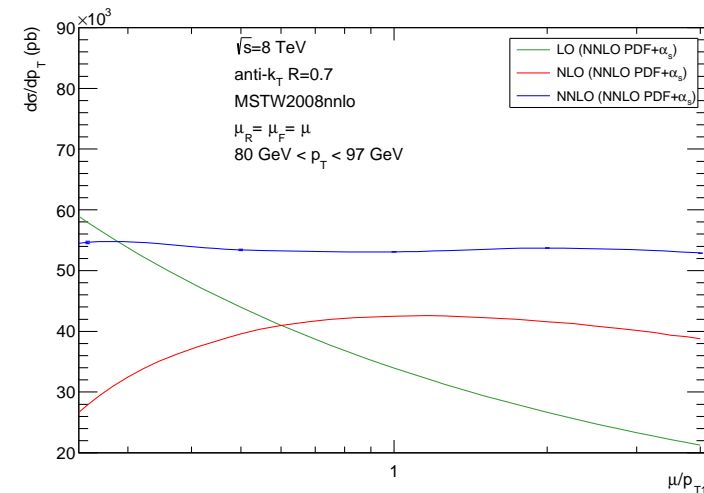
Czakon, Mangano, Mitov, Rojo



- ✓ leads to reduced gluon uncertainty at large  $x$

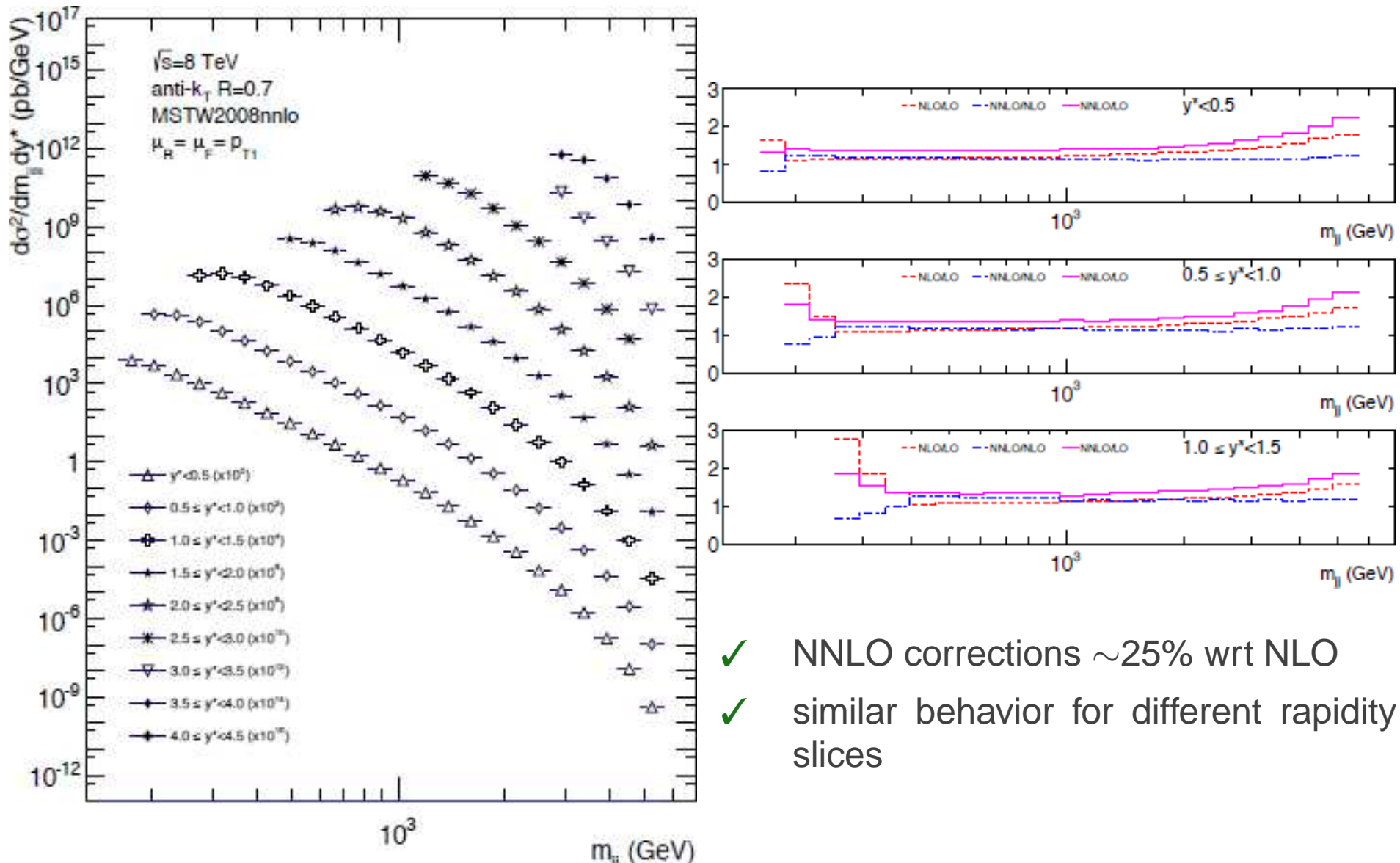
# pp $\rightarrow$ 2 jets at NNLO

- ✓ One of key processes for perturbative QCD
- ✓ Current experimental precision  $\mathcal{O}(5-10\%)$  for jets from 200 GeV/c-1 TeV/c
- Need NNLO QCD and NLO EW
- ✗ Only process currently included in global PDF fits that is not known at NNLO
- ✓ gg channel
  - Currie, Gehrmann-De Ridder,
  - Gehrmann, Pires, NG
- ✓ Scale variation much reduced for  $0.5 < \mu/p_T < 2$ .
- ✓ Size of corrections, and uncertainty, still depends on scale choice  $p_{T1}$   $p_T$ .



# Di-jet mass distribution (gluons only) at NNLO

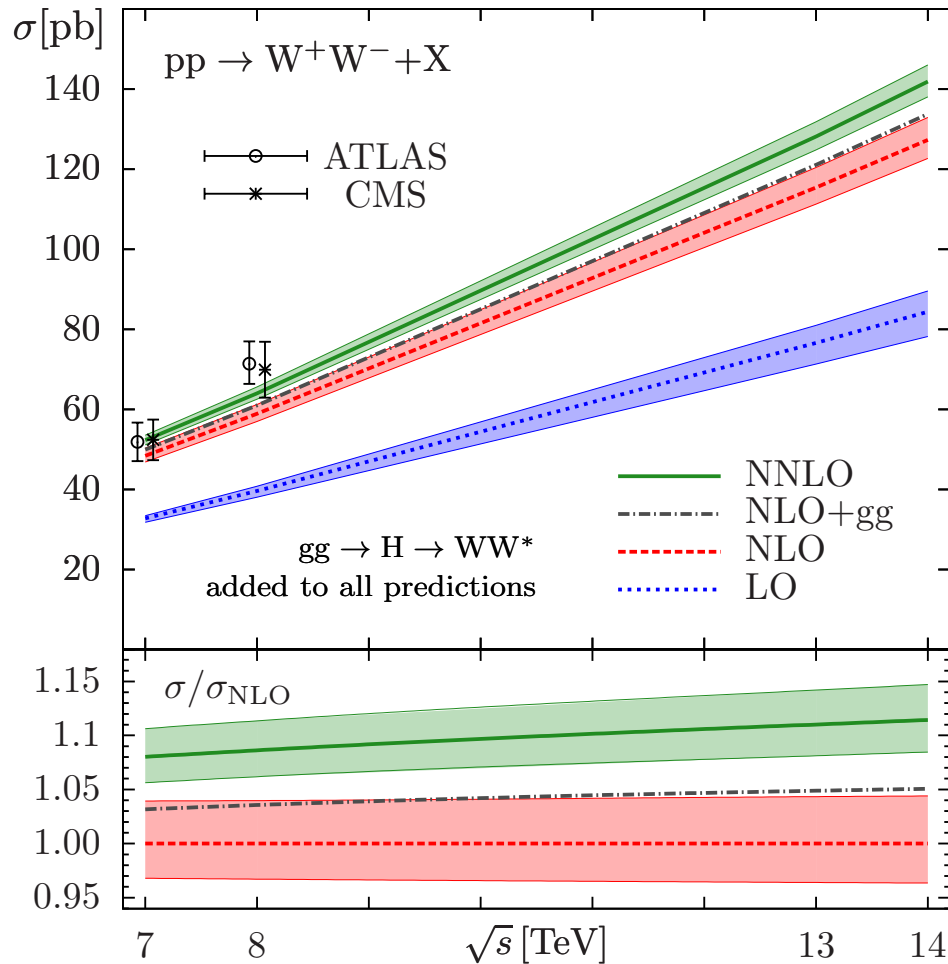
Gehrmann-De Ridder, Gehrmann, Pires, NG; Currie, Gehrmann-De Ridder, Pires, NG



# Les Houches wishlist for W/Z processes

Process	State of the Art	Desired
$V$	$d\sigma(\text{lept. } V \text{ decay}) @ \text{NNLO QCD}$ $d\sigma(\text{lept. } V \text{ decay}) @ \text{NLO EW}$	$d\sigma(\text{lept. } V \text{ decay}) @ \text{NNNLO QCD}$ and $@ \text{NNLO QCD+EW}$ NNLO+PS
$V + j(j)$	$d\sigma(\text{lept. } V \text{ decay}) @ \text{NLO QCD}$ $d\sigma(\text{lept. } V \text{ decay}) @ \text{NLO EW}$	$d\sigma(\text{lept. } V \text{ decay})$ $@ \text{NNLO QCD} + \text{NLO EW}$
$VV'$	$d\sigma(V \text{ decays}) @ \text{NLO QCD}$ $d\sigma(\text{on-shell } V \text{ decays}) @ \text{NLO EW}$	$d\sigma(\text{decaying off-shell } V)$ $@ \text{NNLO QCD} + \text{NLO EW}$
$gg \rightarrow VV$	$d\sigma(V \text{ decays}) @ \text{LO QCD}$	$d\sigma(V \text{ decays}) @ \text{NLO QCD}$
$V\gamma$	$d\sigma(V \text{ decay}) @ \text{NLO QCD}$ $d\sigma(\text{PA, } V \text{ decay}) @ \text{NLO EW}$	$d\sigma(V \text{ decay})$ $@ \text{NNLO QCD} + \text{NLO EW}$
$Vb\bar{b}$	$d\sigma(\text{lept. } V \text{ decay}) @ \text{NLO QCD}$ massive $b$	$d\sigma(\text{lept. } V \text{ decay}) @ \text{NNLO QCD}$ + NLO EW, massless $b$
$VV'\gamma$	$d\sigma(V \text{ decays}) @ \text{NLO QCD}$	$d\sigma(V \text{ decays}) @ \text{NLO QCD} + \text{NLO EW}$
$VV'V''$	$d\sigma(V \text{ decays}) @ \text{NLO QCD}$	$d\sigma(V \text{ decays}) @ \text{NLO QCD} + \text{NLO EW}$
$VV' + j$	$d\sigma(V \text{ decays}) @ \text{NLO QCD}$	$d\sigma(V \text{ decays}) @ \text{NLO QCD} + \text{NLO EW}$
$VV' + jj$	$d\sigma(V \text{ decays}) @ \text{NLO QCD}$	$d\sigma(V \text{ decays}) @ \text{NLO QCD} + \text{NLO EW}$
$\gamma\gamma$	$d\sigma @ \text{NNLO QCD} + \text{NLO EW}$	$q_T$ resummation at NNLL matched to NNLO

# pp $\rightarrow W^+W^-$ at NNLO



Gehrmann et al

- ✓ Provides a handle on the determination of triple gauge couplings, and possible new physics
- ✓ Severe contamination of the  $W^+W^-$  cross section due to top-quark resonances

$\frac{\sqrt{s}}{\text{TeV}}$	$\sigma_{LO}$	$\sigma_{NLO}$	$\sigma_{NNLO}$	$\sigma_{gg \rightarrow H \rightarrow WW^*}$
7	29.52 <sup>+1.6%</sup> <sub>-2.5%</sub>	45.16 <sup>+3.7%</sup> <sub>-2.9%</sub>	49.04 <sup>+2.1%</sup> <sub>-1.8%</sub>	3.25 <sup>+7.1%</sup> <sub>-7.8%</sub>
8	35.50 <sup>+2.4%</sup> <sub>-3.5%</sub>	54.77 <sup>+3.7%</sup> <sub>-2.9%</sub>	59.84 <sup>+2.2%</sup> <sub>-1.9%</sub>	4.14 <sup>+7.2%</sup> <sub>-7.8%</sub>
13	67.16 <sup>+5.5%</sup> <sub>-6.7%</sub>	106.0 <sup>+4.1%</sup> <sub>-3.2%</sub>	118.7 <sup>+2.5%</sup> <sub>-2.2%</sub>	9.44 <sup>+7.4%</sup> <sub>-7.9%</sub>
14	73.74 <sup>+5.9%</sup> <sub>-7.2%</sub>	116.7 <sup>+4.1%</sup> <sub>-3.3%</sub>	131.3 <sup>+2.6%</sup> <sub>-2.2%</sub>	10.64 <sup>+7.5%</sup> <sub>-8.0%</sub>

- ✓ The NNLO QCD corrections increase the NLO result by an amount varying from 9% to 12% as  $\sqrt{s}$  goes from 7 to 14 TeV.
- ✓ Scale uncertainties at the  $\pm 3\%$  level.

## 4. Improved precision for event simulation

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### Fixed order calculations

- ✓ Expansion in powers of the coupling constant
- ✓ Correctly describes hard radiation pattern
- ✓ Final states are described by single hard particles
- ✓ NLO: up to two particles in a jet, NNLO: up to three..
- ✓ Soft radiation poorly described

### Parton shower

- ✓ Exponentiates multiple soft radiation (leading logarithms)
- ✓ Describes multi-particle dynamics and jet substructure
- ✓ Allows generation of full events (interface to hadronization)
- ✓ Basis of multi-purpose generators (SHERPA, HERWIG, PYTHIA)
- ✓ Fails to account for hard emissions

**Ideally: combine virtues of both approaches**

# Matrix Element improved Parton Shower

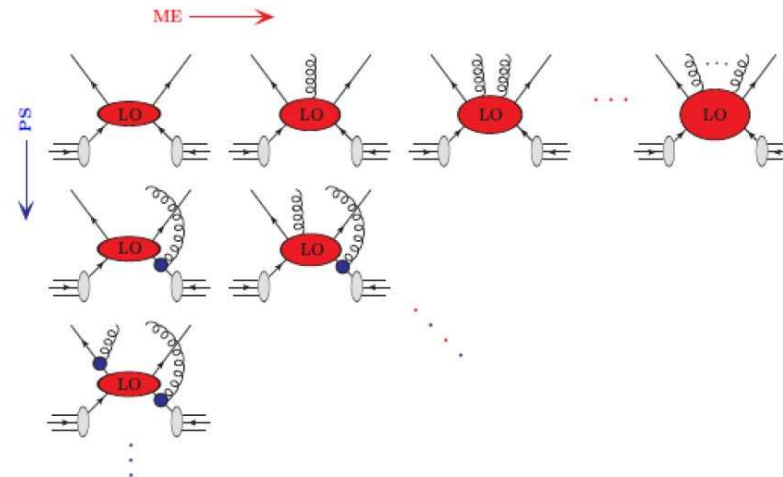
matrix elements and parton showers are approximations in different regions of phase space

## Merging

Several fixed order calculations of increasing multiplicity supplemented by PS

CKKW: Catani, Krauss, Kuhn, Webber

MLM: Mangano

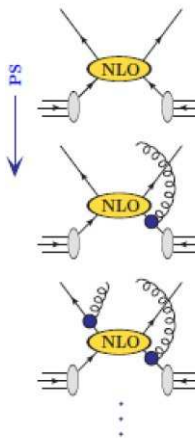


## Matching

One fixed order calculation supplemented by PS

MC@NLO: Frixione, Webber,

POWHEG: Nason, Oleari



## Now benefitting from automation of NLO

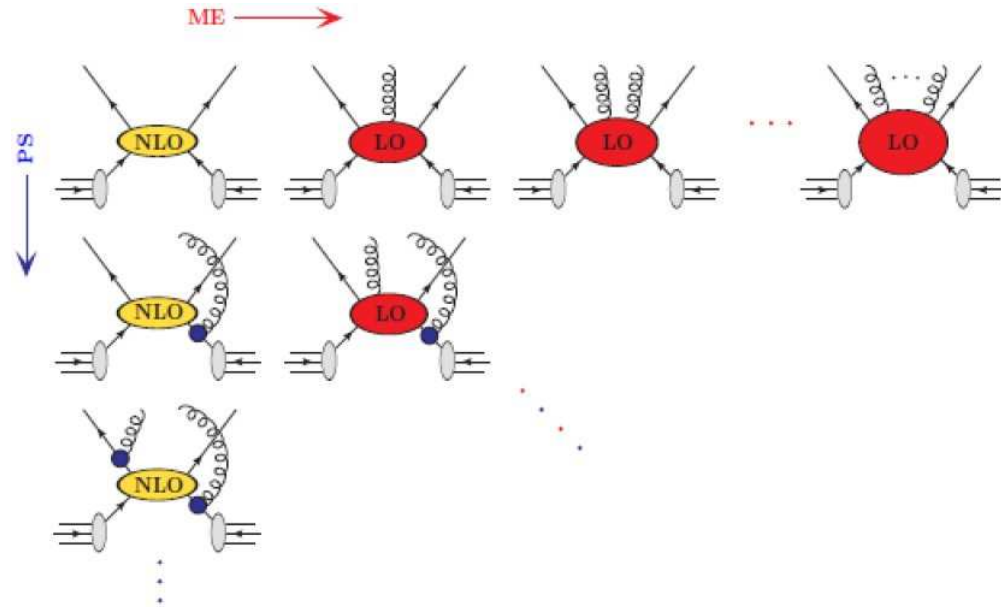
aMC@NLO: Frederix, Frixione, Hirschi, Maltoni, Pittau, Torrielli

# Matrix Element improved Parton Shower

## MENLOPS

Supplements core NLOPS with higher multiplicity LOPS

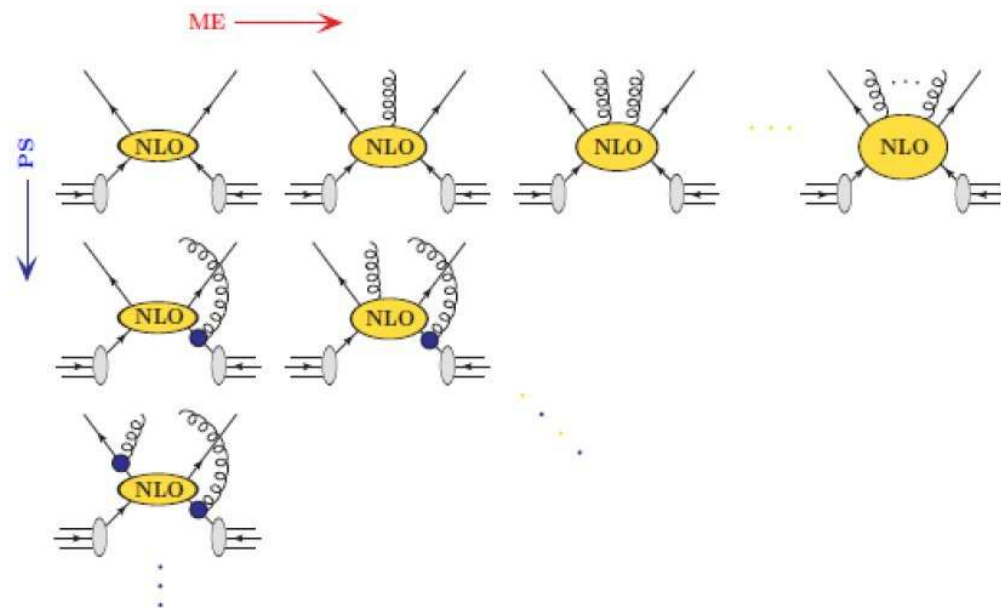
Hamilton, Nason; Hoeche, Krauss, Schonherr, Siegert; Lonnblad, Prestel



## MEPS@NLO

Combines multiple NLOPS

Lavesson, Lonnblad; Hoeche, Krauss, Schonherr, Siegert; Frederix, Frixione





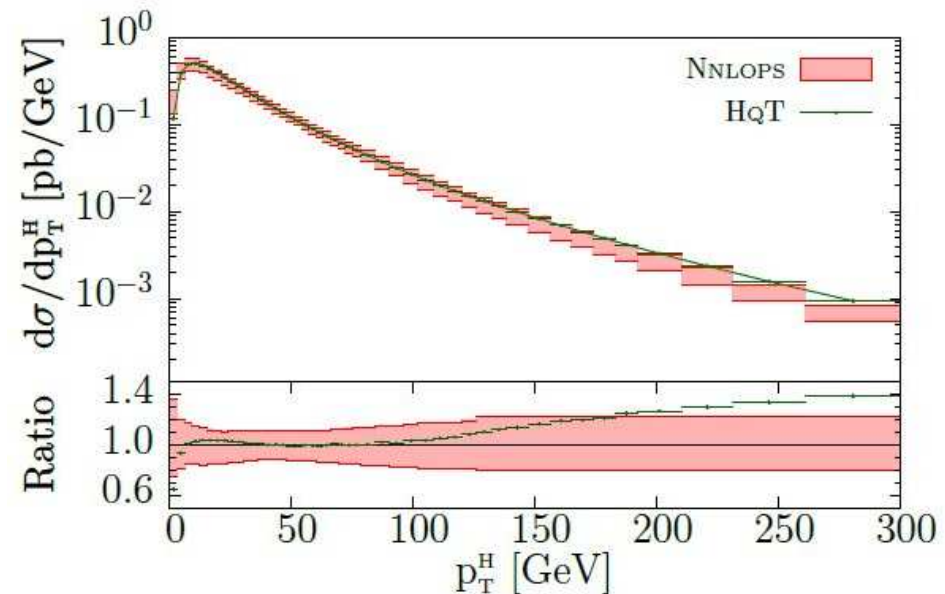
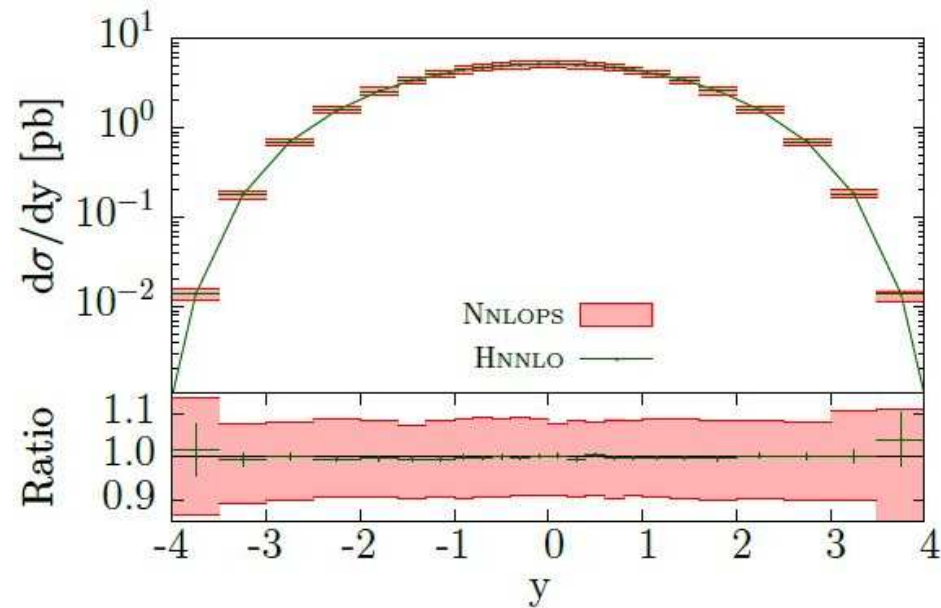
# Reaching NNLOPS accuracy

## MINLO

Multiscale improved NLO CKKW scale for Born pieces  
Sudakov form factors for Born functions in POWHEG

Hamilton, Nason, Zanderighi

Exciting idea! starting from HJ@NLO+PS generate H rapidity distribution at NNLO



Hamilton, Nason, Oleari, Re, Zanderighi

# Summary: Where are we now?

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- ✓ Witnessed a revolution that has established NLO as the new standard
  - previously impossible calculations now achieved
  - very high level of automation for numerical code
  - standardisation of interfaces - linkage of one-loop and real radiation providers
  - take up by experimental community
- ✓ Substantial progress in NNLO in past couple of years
  - several different approaches for isolating IR singularities
  - several new calculations available
  - codes typically require significant CPU resource

# Summary: Where are we going?

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## ✓ NNLO automation?

- as we gain analytical and numerical experience with NNLO calculations, can we benefit from (some of) the developments at NLO, and the improved understanding of amplitudes
- automation of two-loop contributions?
- automation of infrared subtraction terms?
- standardisation of interfaces - linkage to one-loop and real radiation providers?
- interface with experimental community

Next few years:

- ✓ Les Houches wishlist to focus theory attention
- ✓ New high precision calculations that will appear such as, e.g. N3LO  $\sigma_H$ , **could reduce Missing Higher Order uncertainty by a factor of two**
- ✓ NNLO will emerge as standard for benchmark processes such as dijet production leading to improved pdfs etc. **could reduce theory uncertainty due to inputs by a factor of two**