Tutorial questions: Trapped-ion quantum information

Les Houches Winter School, January 2015

1. **Measuring observables**

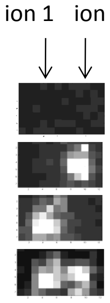
   In your experiment, you have created 1000 copies of a two-qubit quantum state and decided to analyze it by fluorescence measurements which project the state onto the $|\uparrow\rangle = \binom{1}{0}$ and $|\downarrow\rangle = \binom{0}{1}$ basis.

   (a) Use the measurement results shown in Fig.1 to calculate the expectation values of the observables $\sigma_z^{(1)}$, $\sigma_z^{(2)}$, and $\sigma_z^{(1)}\sigma_z^{(2)}$.

   (b) Write down two quantum states that could have produced these measurement results.

   (c) Using a focused laser beam resonant with the atomic transition, we apply the unitary $U = e^{i\frac{\pi}{4}\sigma_y^{(1)}} = \frac{1}{\sqrt{2}}(I + i\sigma_y^{(1)})$ before carrying out the fluorescence measurement. Which observables A could be measured in this experiment?

   Reminder: $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$



Detected state	Number of observations
$\|\uparrow\uparrow\rangle$	500
$\|\uparrow\downarrow\rangle$	100
$\|\downarrow\uparrow\rangle$	0
$\|\downarrow\downarrow\rangle$	400

FIGURE 1: Measurement results of a two-qubit state. The four different states shown to the left can be distiguished by spatially resolved fluorescence measurement. Table shows the number of observations found in a set of 1000 experiments repeatedly preparing and measuring the state.

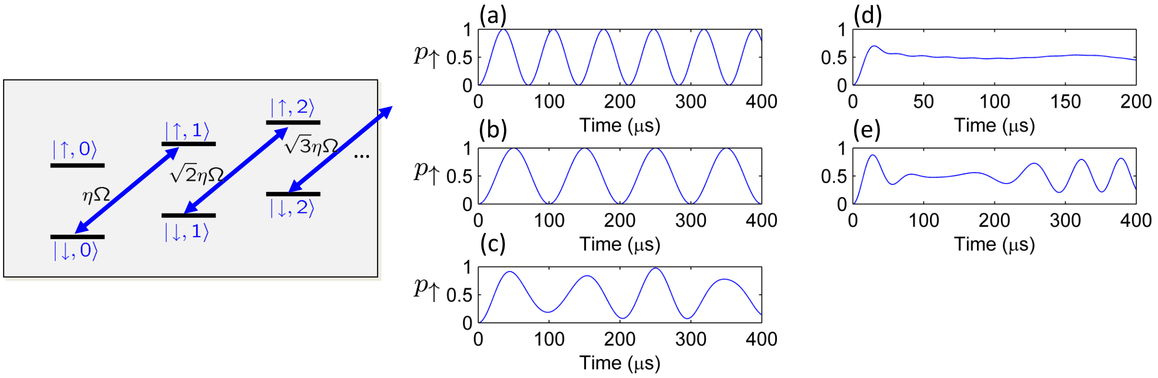


FIGURE 2: Excitation of an ion on the blue motional sidebands. Panels (a)-(e) show the excitation probability for various motional states.

2. **Analysis of the vibrational state of a trapped ion**

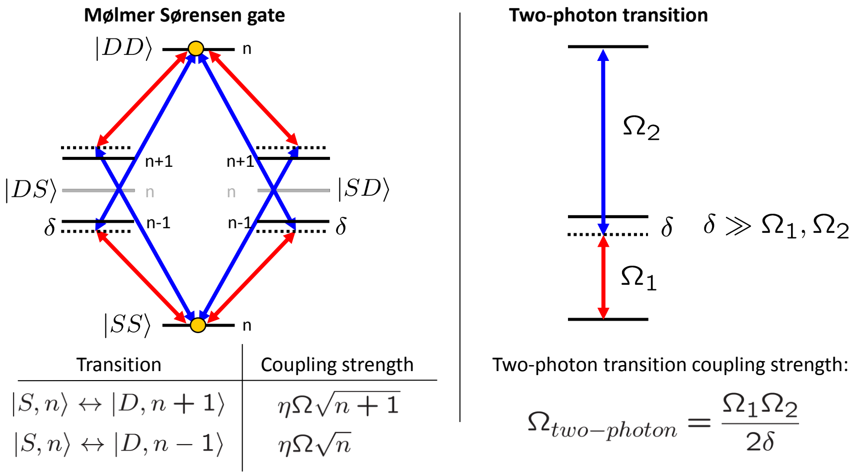
We are interested in analyzing the motional state of a trapped ion. Starting with a $^{40}\mathrm{Ca}^+$ ion initially prepared in the $S_{1/2}$ state, we excite the ion on the blue (upper) vibrational sideband of the $S_{1/2} \longleftrightarrow D_{5/2}$ transition with laser pulses of variable length τ and measure the probability $p(\tau)$ of finding in the ion in the metastable state. For different initial states, we find the measurement results shown in Fig.2 (a)-(e).

For an ion oscillating at $\omega = (2\pi)940$ kHz, show that the Lamb-Dicke factor on the quadrupole transition ($\lambda = 729$ nm) is about $\eta = 0.1$ (for a laser beam parallel to the direction of the oscillation).

(a) For each subpanel of the figure, explain qualitatively the vibrational state distributions.

(b) For each subpanel of the figure, which process could have resulted in the vibrational state distributions?

The Rabi frequency on the carrier transition is assumed to be $\Omega = (2\pi)100$ kHz.



Transition	Coupling strength
$\vert S, n\rangle \leftrightarrow \vert D, n+1\rangle$	$\eta\Omega\sqrt{n+1}$
$\vert S, n\rangle \leftrightarrow \vert D, n-1\rangle$	$\eta\Omega\sqrt{n}$

Two-photon transition coupling strength:

$$\Omega_{two-photon} = \frac{\Omega_1\Omega_2}{2\delta}$$

FIGURE 3: Laser-ion interactions leading to a coupling between the two-qubit states $|SS\rangle$ and $|DD\rangle$ (left panel:). Coupling mechanism and strength of a two-photon transition (right panel).

3. Mølmer-Sørensen entangling gate

In the Mølmer-Sørensen gate, two ions are simultaneously illuminated by laser fields which off-resonantly couple to the red and blue sideband of the qubit transition (see Fig. 3). In this way, the joint vibrational state of the ions mediates couplings $|SS\rangle \longleftrightarrow |DD\rangle$ and $|SD\rangle \longleftrightarrow |DS\rangle$.

(a) Why does the coupling have to be off-resonant?

(b) In the limit of weak excitation, calculate the two-photon transition coupling strength $\Omega_{SS,DD}$ between levels $|SS\rangle$ and $|DD\rangle$ (see Fig 3). Show that the coupling strength $\Omega_{SS,DD} = -\frac{\eta^2\Omega^2}{\delta}$ does not depend on the vibrational state of the harmonic oscillator.

(c) For the ambitious ones: Show that the result of (b) also holds outside the regime of weak excitation.

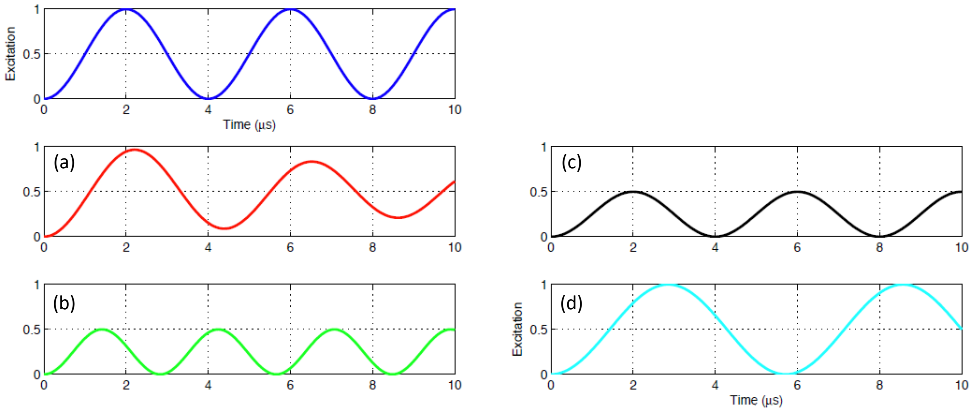


FIGURE 4: Rabi oscillations driven by resonant carrier excitation. (a)-(d) Rabi oscillations observed when repeating the experiment.

4. **I'd wish my experiment was more stable (coherent excitation of an ion)**

   In your experiment, you've managed to observe near-perfect Rabi oscillations that enable you to carry out a π-pulse in 2 μs (top plot of Fig.4). After a short break, you discover that something is going wrong when you repeat the experiment (plots (a)-(d)). For each of the cases, give a possible explanation of the cause.

5. **Entangled states**

   Which of the following two-qubit states are entangled?

   (a) $|\Psi_1\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

   (b) $|\Psi_2\rangle = \frac{1}{2}(|00\rangle + |01\rangle - |10\rangle + |11\rangle)$

   (c) $|\Psi_3\rangle = \frac{1}{2}(|00\rangle - |01\rangle - |10\rangle + |11\rangle)$

   (d)

$$\rho_1 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -i & 0 & 0 & 1 \end{pmatrix} \qquad \text{and} \qquad \rho_2 = \begin{pmatrix} 0.30 & 0.19 & 0.15 & 0.09 \\ 0.19 & 0.19 & 0.09 & 0.11 \\ 0.15 & 0.09 & 0.30 & 0.19 \\ 0.09 & 0.11 & 0.19 & 0.21 \end{pmatrix}$$