Why study phase space cooling?

In accelerators/storage rings:

- Improve beam energy resolution for experiments
- Obtain denser beams (reduce transverse size) for beam – beam collision experiments
- Manipulate secondary (produced in a target) beams of exotic stuff like antiprotons
- Improve storage lifetime (Nature wants your beam to hit the wall)

In ion/atom traps:

- Fundamental studies of QM, QED, gravitation, symmetries, quantum logic, etc.

Why study phase space cooling – you want the truth?

Stochastic Cooling: discovery of W,Z particles; Top quark, Nobel Prize for Van der Meer (1984)

Laser cooling: Nobel Prize for Chu, Phillips, Cohen Tannoudji – 1997

Evaporative cooling: Nobel Prize (for BEC) for Ketterle, Cornell, Wieiman

Electron cooling: G.I. Budker – No Nobel prize but has an institute named after him (Novosibirsk, Russia)

The “are you paying attention slide” #1

WRONG!

The “are you paying attention slide” #2

Hardly!

Phase Space and the Liouville theorem

- We have a system of N particles
- We have generalised coordinates and momenta derived from a Lagrangian

$$p_i = \frac{\partial L}{\partial \dot{q}_i}, \qquad L(q, \dot{q}, t) = T - U$$

- The state of the system is described by a *single point* in 6N dimensional *phase space*
- Liouville's theorem addresses the time evolution of an *ensemble* of Hamiltonian systems having an initial *distribution* in *6N-dimensional* phase space

Phase Space and the Liouville theorem 2

- We define a probability density for the system to be found at a particular point in 6N-dimensional space

$$\varrho(q_1, \ldots, q_{3N}, p_1, \ldots, p_{3N}, t)\, d^{3N}p d^{3N}q$$

- The Liouville theorem states that under the application of Hamiltonian (derivable from a potential) forces:

$$\frac{\mathrm{d}\varrho}{\mathrm{d}t} = 0$$

- A very simple, elegant and powerful result about the classical dynamics of multi-particle systems!

Hardly!

Phase Space and the Liouville theorem 3

- This is one of the most misunderstood and useless theorems I have ever encountered in physics

- How good are your 6N-dimensional calculational and visualisation skills?

- We clearly need a more user-friendly phase space…

Phase Space and the Liouville theorem 4

- We can start by considering a 6-dimensional space where each particle is represented by a point

- We can define a density in this space

$$f(p, q, t)$$

- For *non-interacting particles,* it can be shown that

$$\frac{df}{dt} = 0$$

Phase Space and the Liouville theorem 5

- Wait, how useful is it to ignore interactions?
- After all, we are dealing with charged particles: the Coulomb interaction is derivable from a potential and was therefore included in the 6N-dimensional version of Liouville
- You just threw it out!
- It turns out that this formalism works out well in many accelerator and some plasma applications

Phase Space and the Liouville theorem 6

- In many accelerator applications, the interactions are weak enough that the six-dimensional phase space density is *approximately conserved on some time scale.*

- We can separate the interactions, and consider them as a perturbation

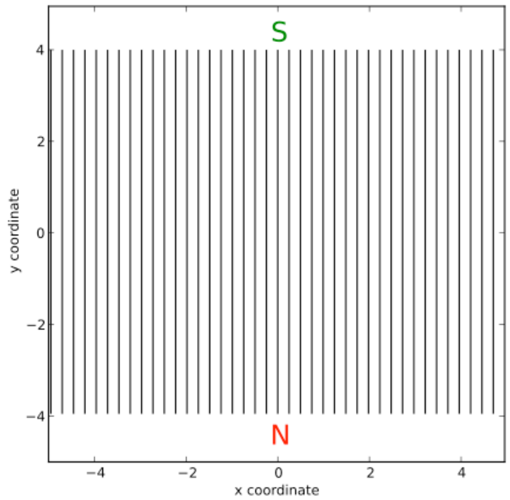
$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \nabla_r f + \frac{\boldsymbol{F}}{m} \cdot \nabla_v f = \left(\frac{\partial f}{\partial t} \right)_{\text{collisions}}$$

Here, **F** represents "bulk" forces, including external forces and the average force of the beam's own electromagnetic fields acting on itself. The collision force includes a lot of hidden evil… Note that the Coulomb interaction is just one thing, but here it has been arbitrarily divided up.

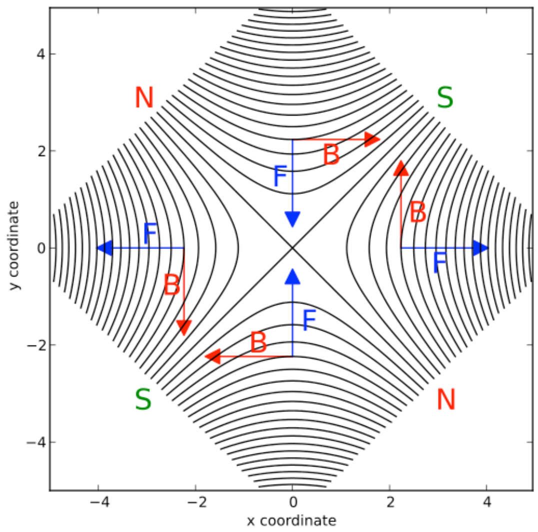
Phase Space and the Liouville theorem 7

- The accelerator guys use 2-D projections of 6-D phase space and refer to them as *emittances*
- We'll look at these in a minute, but we have finally arrived a at point where we can define what is meant by phase space cooling – at least to beam physics people
- Phase space cooling is any technique which increases the 6-D phase space density of a collection of particles, *without throwing any particles away intentionally.*

A very brief introduction to accelerator physics

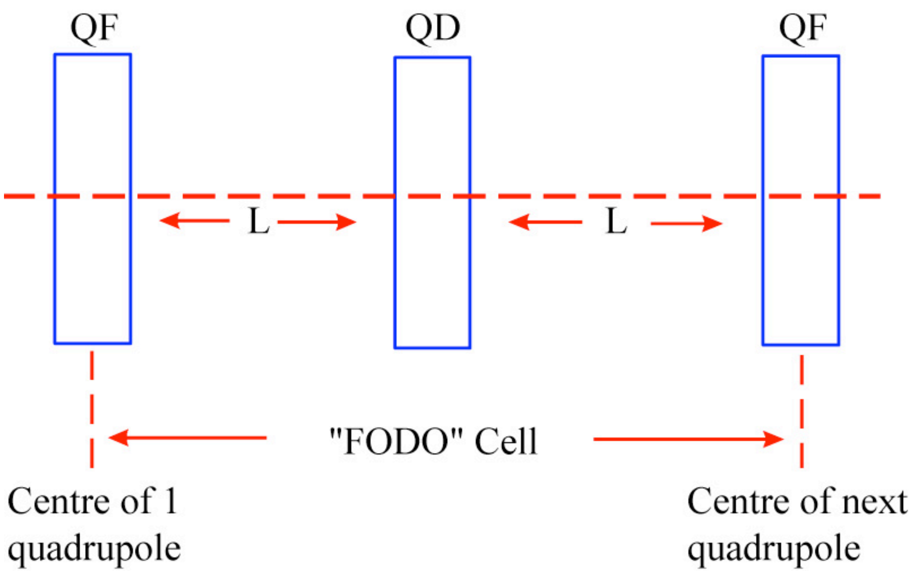


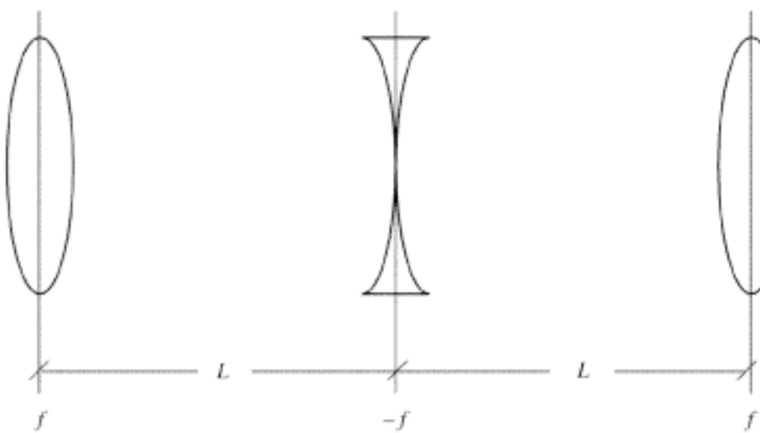
You need dipole magnets to get the charged particles to go around in a closed path. Constant field, vertically oriented.



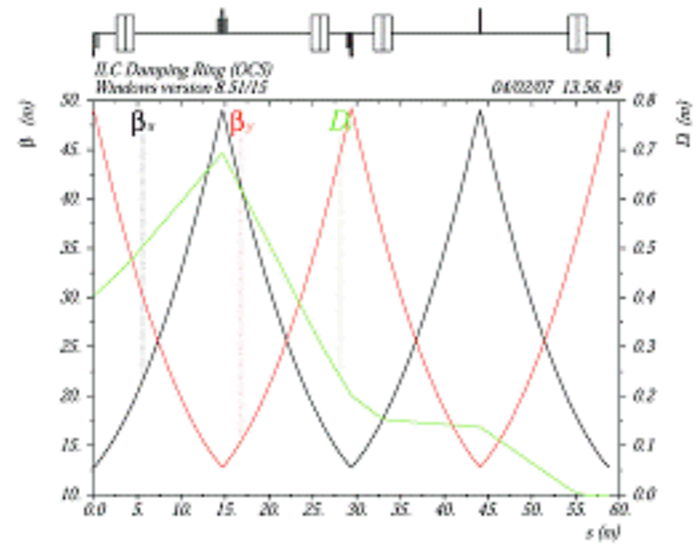
You need quadupole magnets to focus the beam about the ideal orbit. These *focus* in one plane and *defocus* in the other.

Alternating gradient focusing

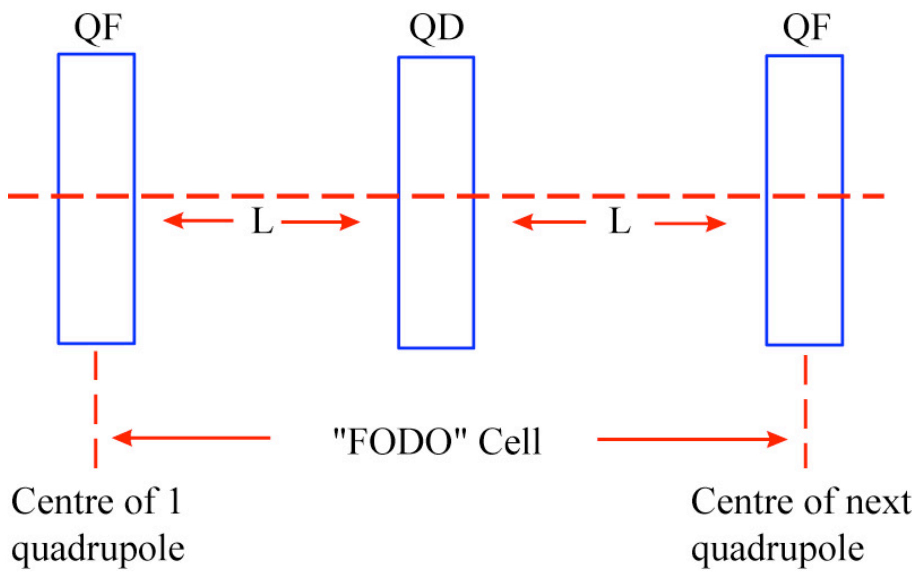


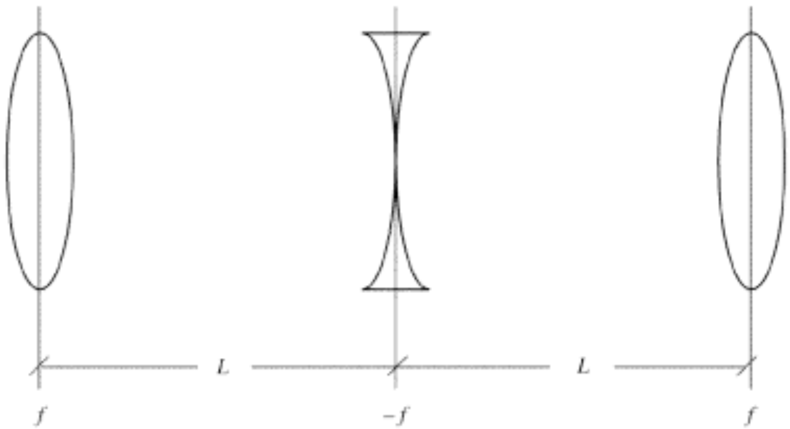


Note: we can use lens-matrix formalism and do ray tracing in complete analogy to optics



Alternating gradient focusing 2





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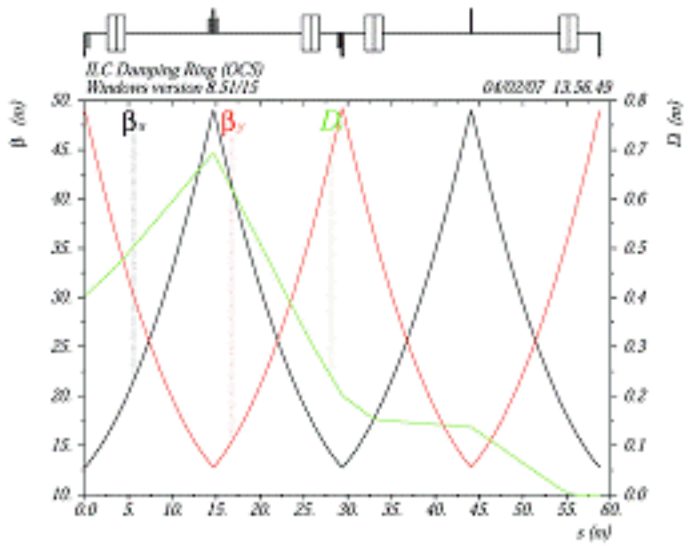
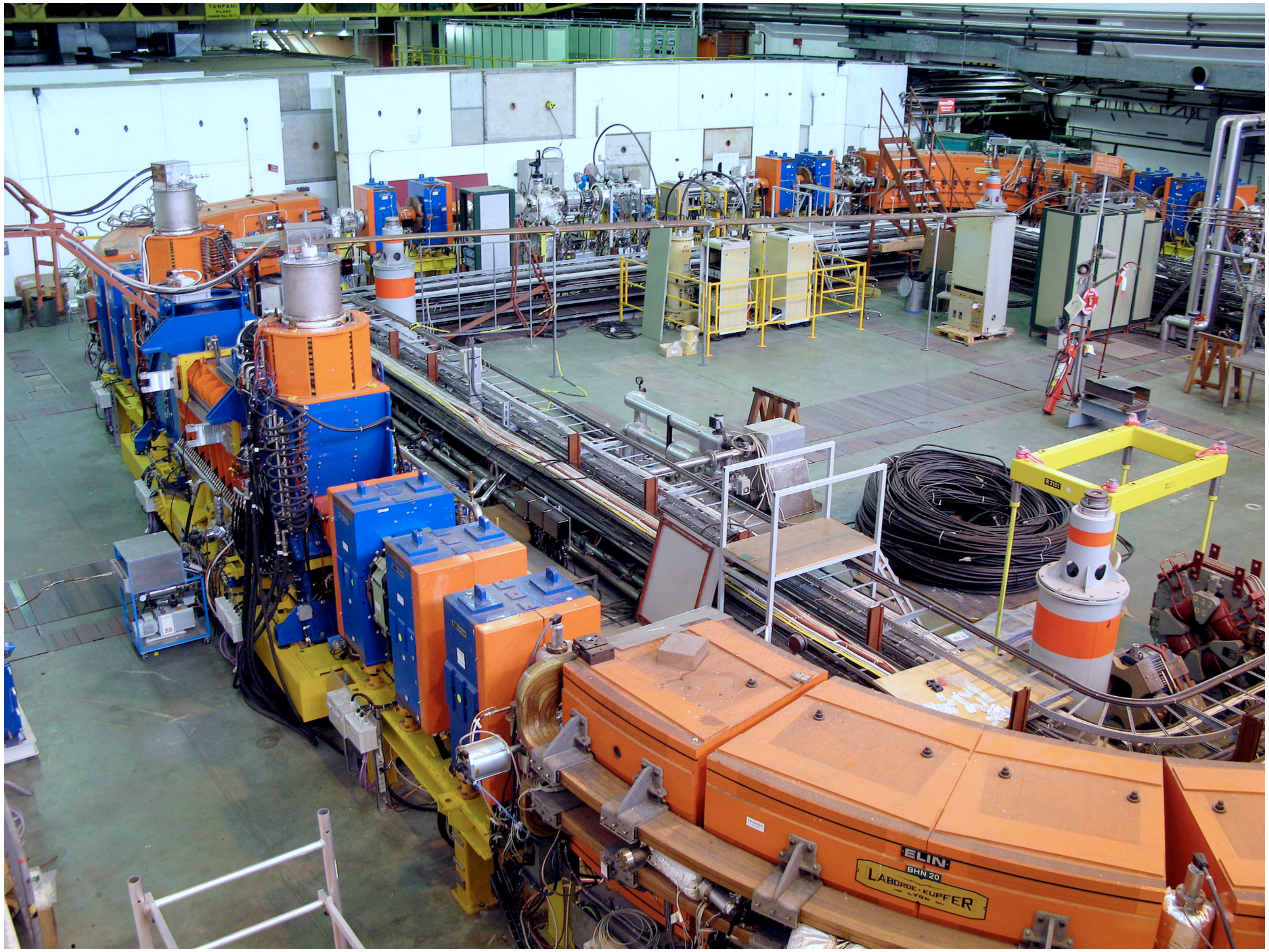


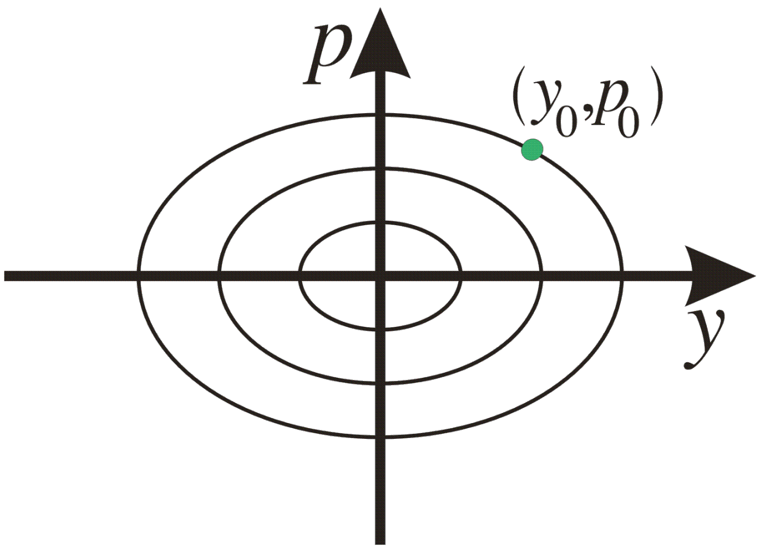
LEIR at CERN



Note: the principle of operation of these machines is the same from keV to TeV

Transverse, 2-D Phase space

- In a storage ring or accelerator, the charged particles oscillate transversely about an ideal orbit that goes through the center of the magnets. This is known as *betatron* motion and can be thought of as quasi-linear
- The number of oscillations made during each revolution is called the betatron tune Q_x, Q_y
- Machine design avoids resonances $nQ_x+mQ_y=p$; *n,m,p* integers

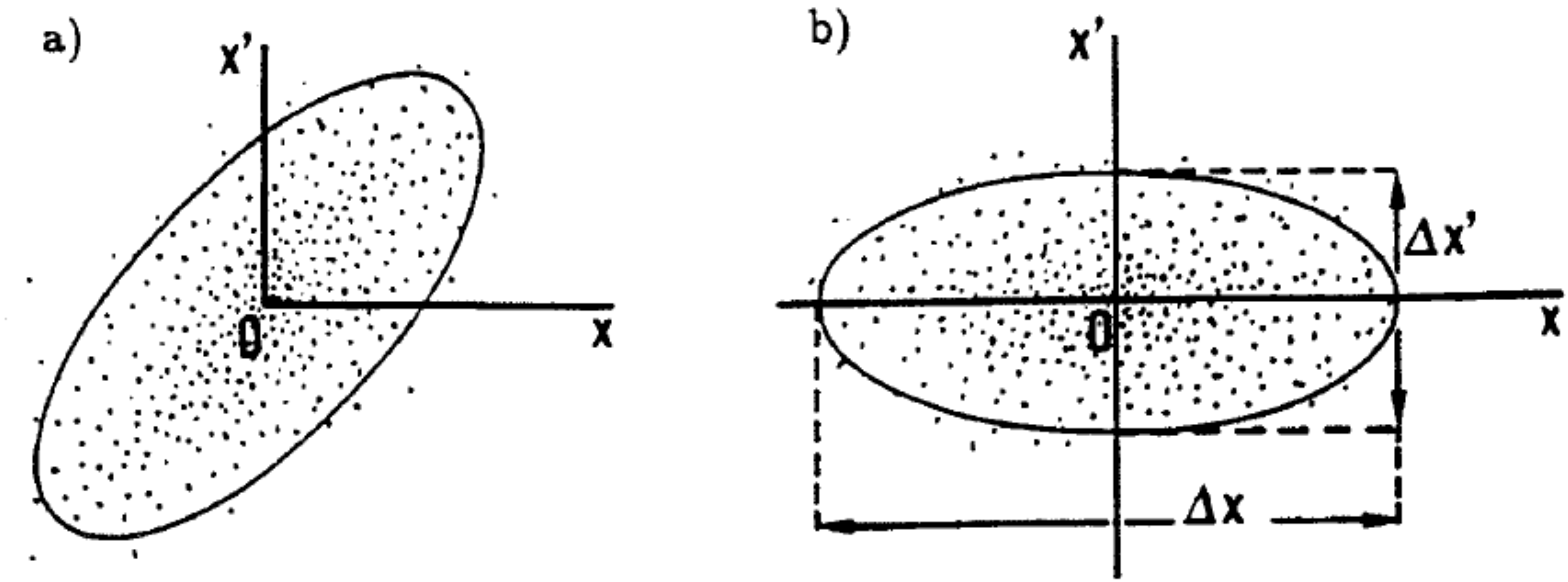


A typical harmonic oscillator phase space diagram

In accelerator physics, the phase space coordinates employed are usually the transverse position and the transvers angle in each plane:

X, X’, Y, Y’

Transverse emittance



The emittance is the area of an ellipse in this phase space plane that includes a specified fraction of all particles (typically 95% or 1-sigma or something else)

Our watered-down Liouville theorem for accelerators is that this area is conserved.

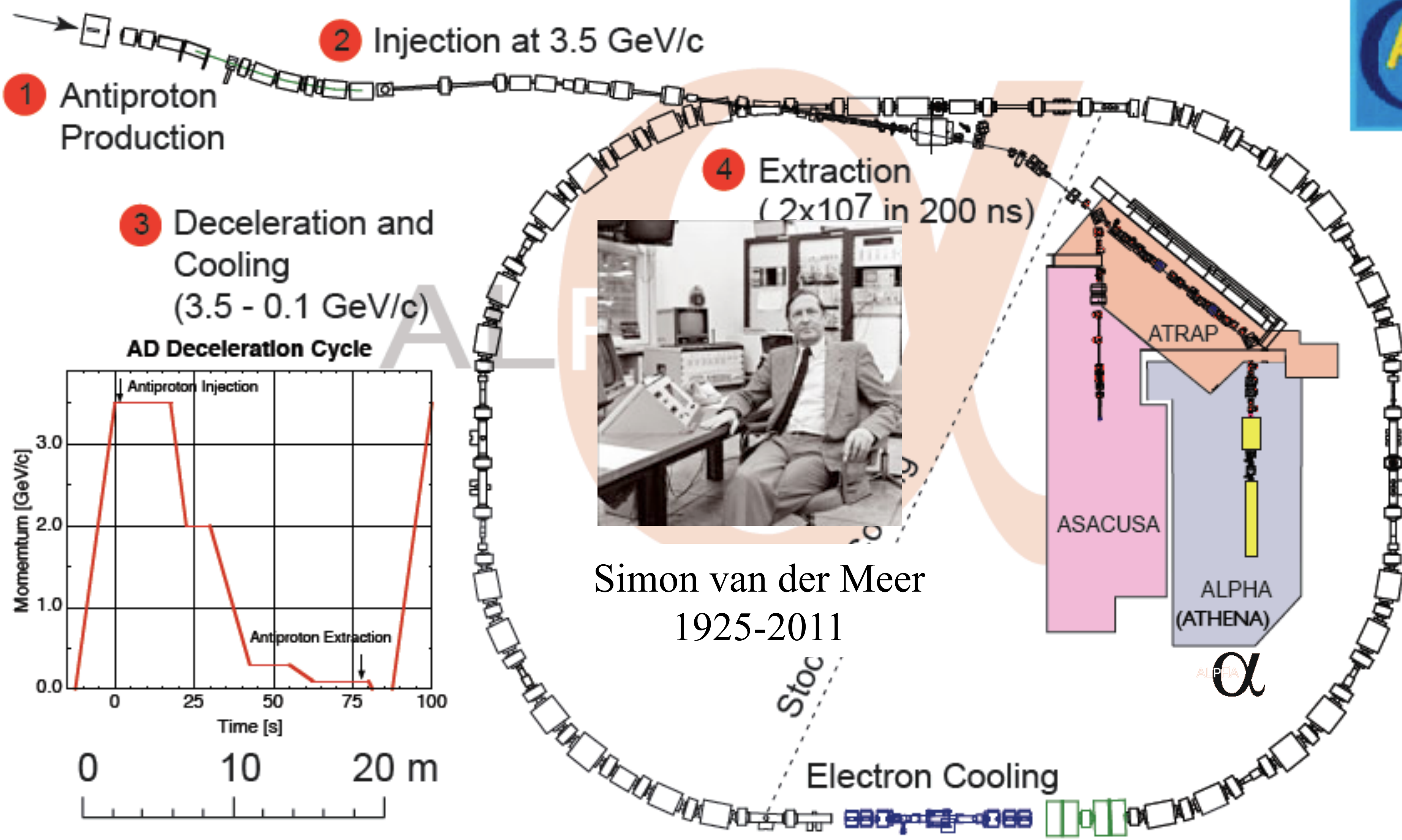
It never is.

A list I found – emittance non-conservation

- coupling between degrees of freedom (horizontal-vertical coupling, chromaticity...)
- intrabeam scattering
- beam-beam scattering
- scattering on residual gas
- multiple scattering through a thin foil
- electron cooling
- stochastic cooling
- laser cooling
- synchrotron radiation emission
- filamentation due to non-linearities
- wake fields
- space charge effects.

Beam cooling concerns itself with reducing or maintaining the emittance of a stored particle beam. We will start with stochastic cooling of transverse motion

The CERN AD



Simon van der Meer
1925-2011

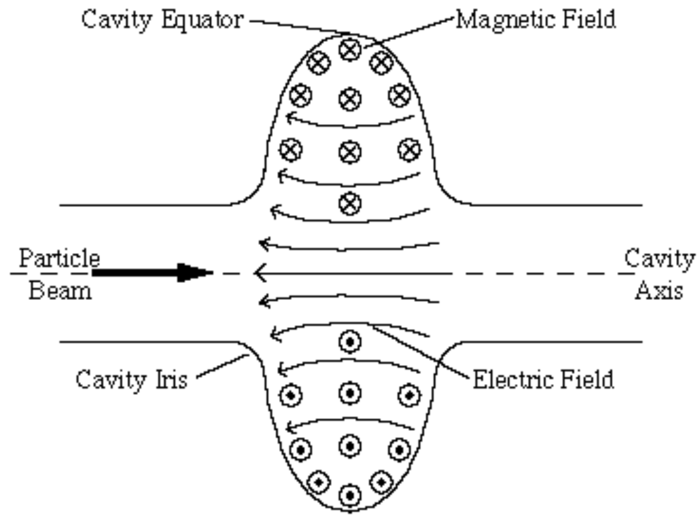
Longitudinal Phase Space 1

- DC or 'coasting beam': the figure of merit is the momentum spread dp/p of the beam

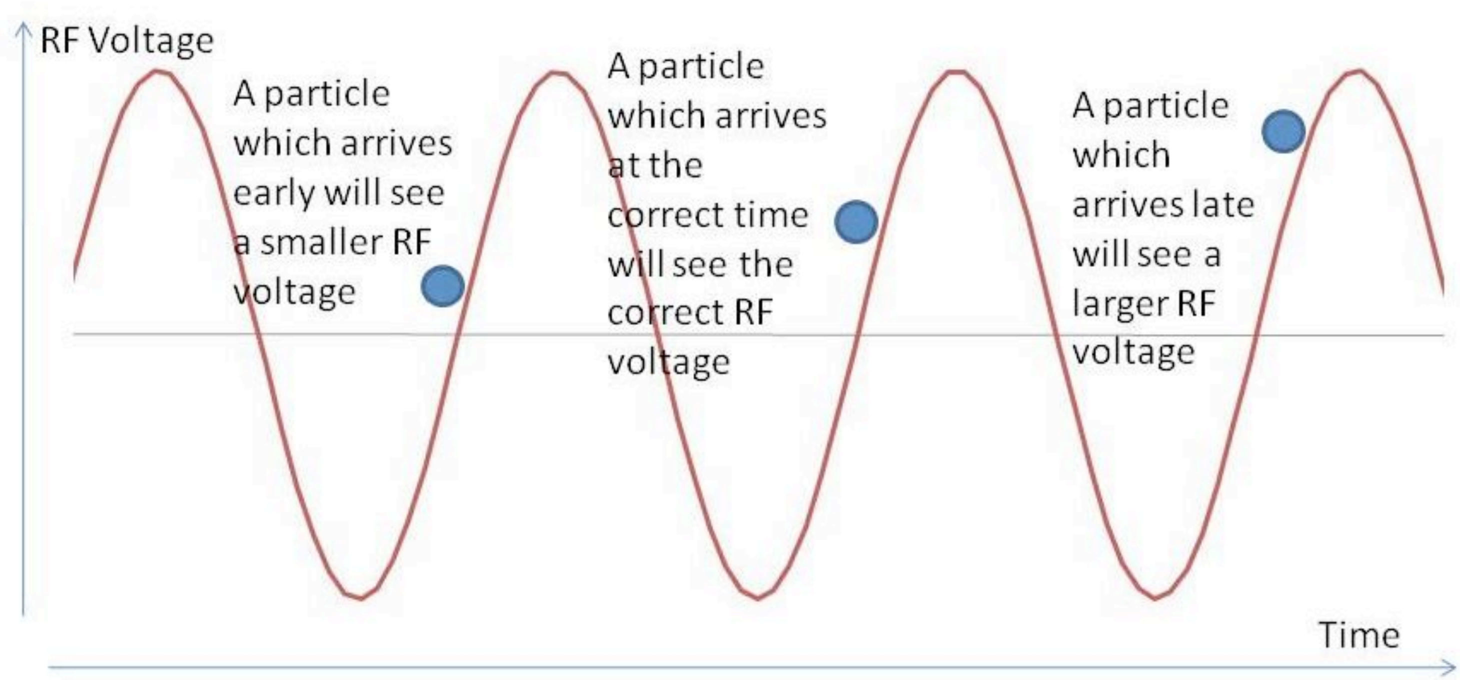
- Bunched beam: the beam can be spatially separated into longitudinal bunches through the use of an RF cavity

- This can be simply thought of as a gap electrode that can apply a longitudinal electric field that applies a 'kick' to particles as they pass

- Located at one point in the circumference, the cavity operates at a harmonic *h* of the revolution frequency

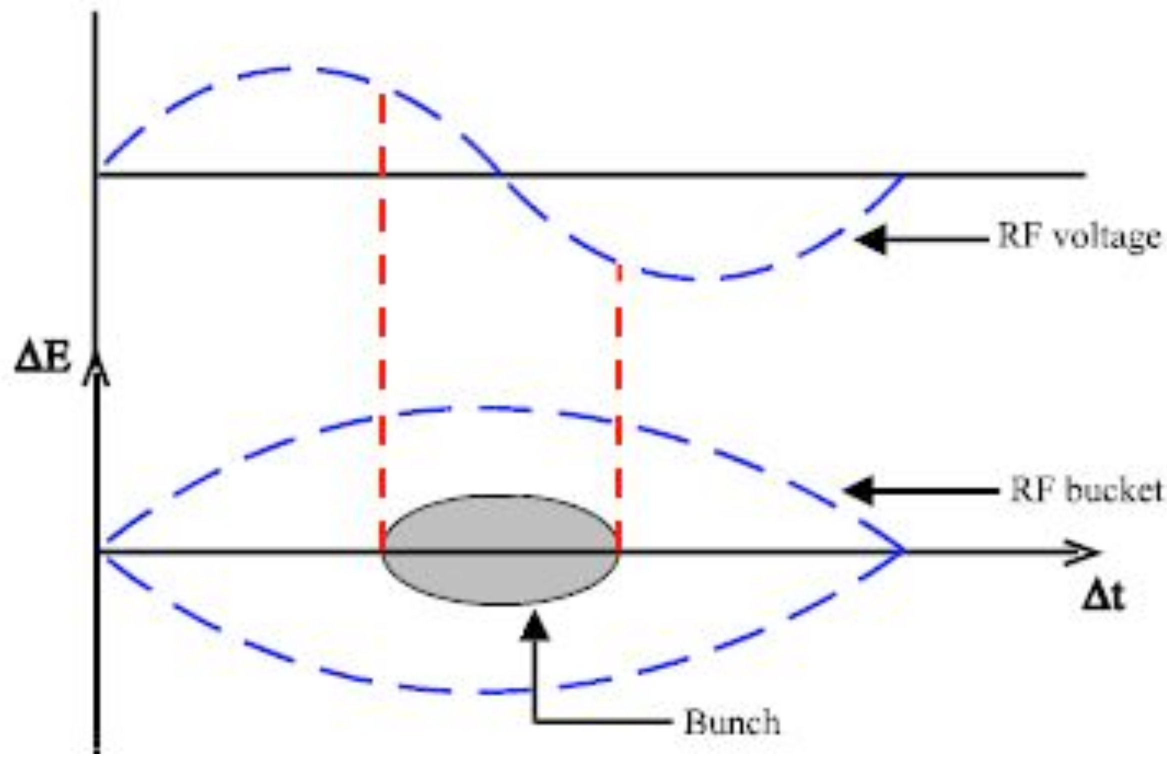


Longitudinal Phase Space 2



- The beam separates into *h* separate bunches around the stable points
- For an accelerator, the picture is as above, the particles are kicked by a given amount on each pass as the magnets are ramped up to higher energy
- For a storage ring, the frequency is fixed and the stable points are around zero field

Longitudinal Phase Space 3



- Non-ideal particles oscillate around the *synchronous particle* as they travel around
- This so-called *synchrotron oscillation* is – guess what – quasi harmonic.
- The synchrotron frequency is a fraction of the revolution frequency
- Accelerator guys don't normally talk about temperature – the area in this phase plane is normally quoted in eV-seconds.

Accelerators and Ion Traps 1

- So far we have:

  - Transverse, quasi-harmonic confinement by quadrupole magnets
  - Longitudinal, quasi-harmonic confinement by an RF cavity

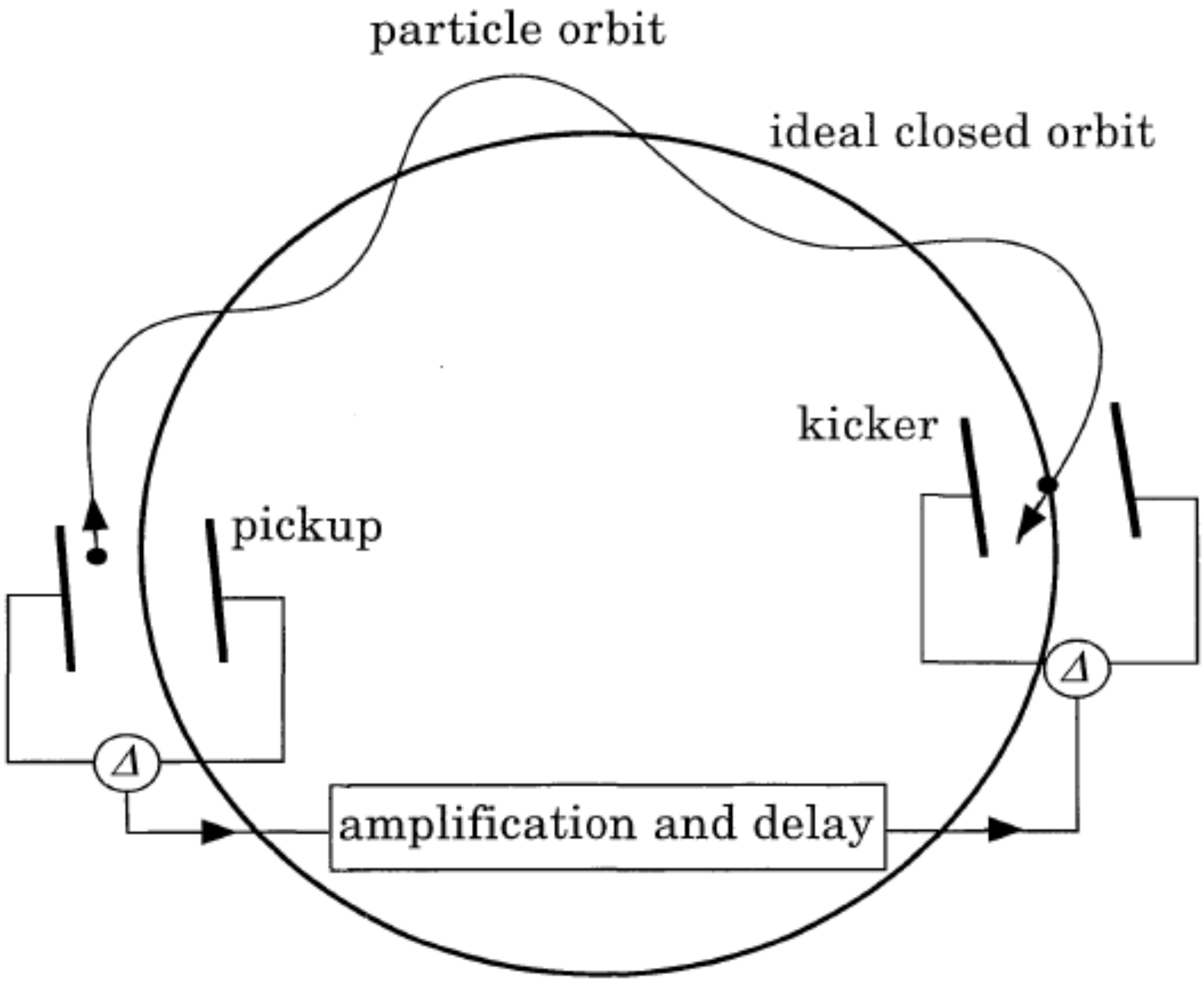
- A storage ring is like having *h* ion traps circulating, perhaps at relativistic velocities.

Note that W. Paul was inspired by the AG snchrotron when he invented his famous trap

So we have a lot in common between the two fields

Unfortunately, we are rarely able to communicate between the fields…

Stochastic Cooling 1



A very simple and straightforward method of increasing the transverse phase space density of a stored beam!

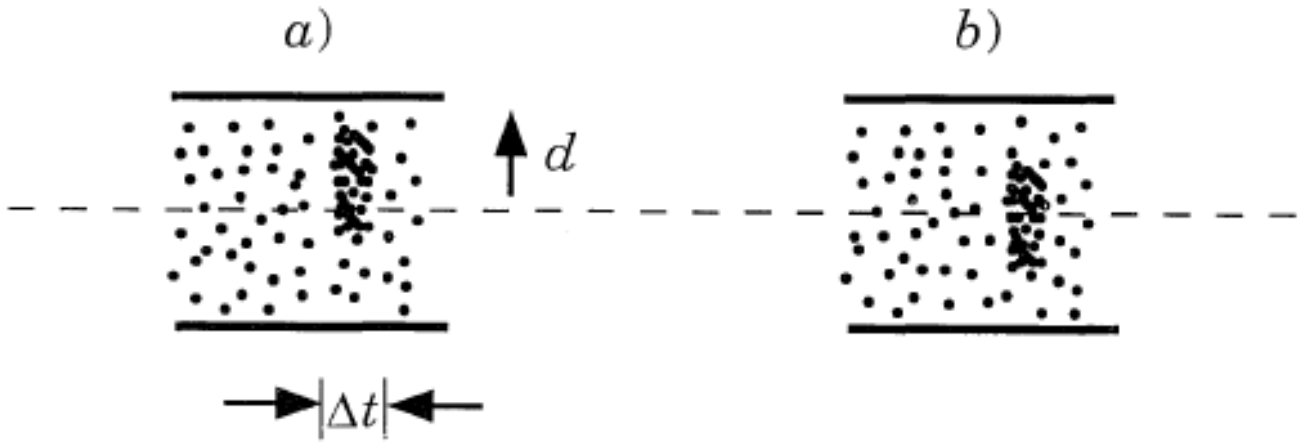
WRONG!

(single particle arguments are almost never right)

Stochastic Cooling 2

What can my pickup really see?:

Dipole moment current fluctuations



Note that if nothing else happens, this process just stops after one correction…

Note also that bad things can happen to the particles that aren't a part of the measured Fluctuation – thus the name "stochastic"

Stochastic Cooling 3

A figure of merit for a stochastic cooling system is thus how effective it is in dividing the beam up into small time intervals for measurement of individual fluctuations

The cooling time can indeed be roughly estimated by

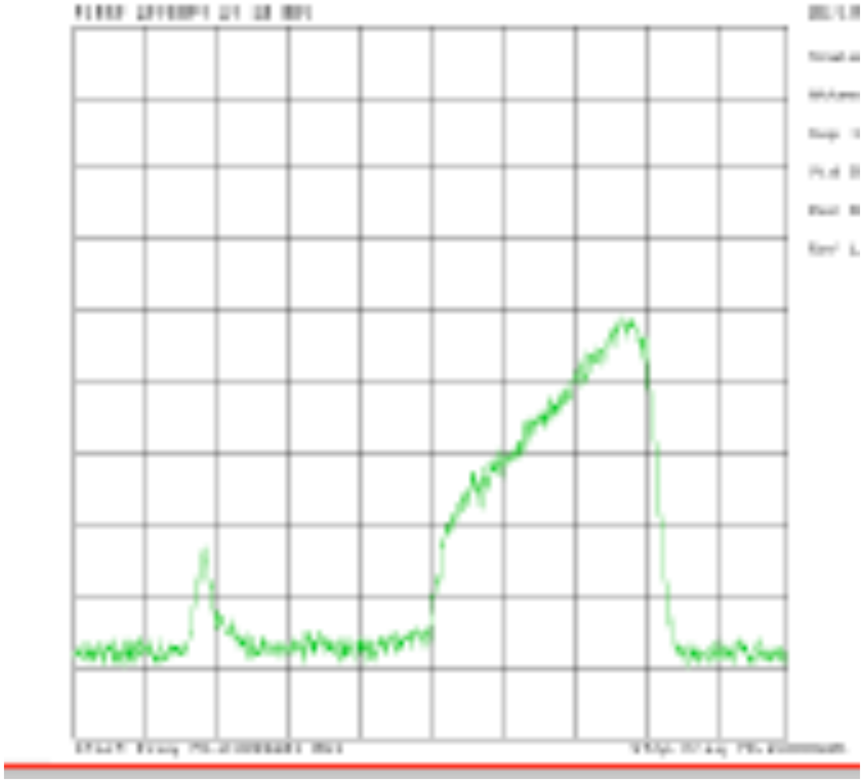
$$\frac{1}{\tau} = \frac{2W}{N}.$$

Where W is the system bandwidth and N is the number of particles. Typical bandwidths are Several GHz.

Stochastic cooling also works longitudinally, but we won't get into that.

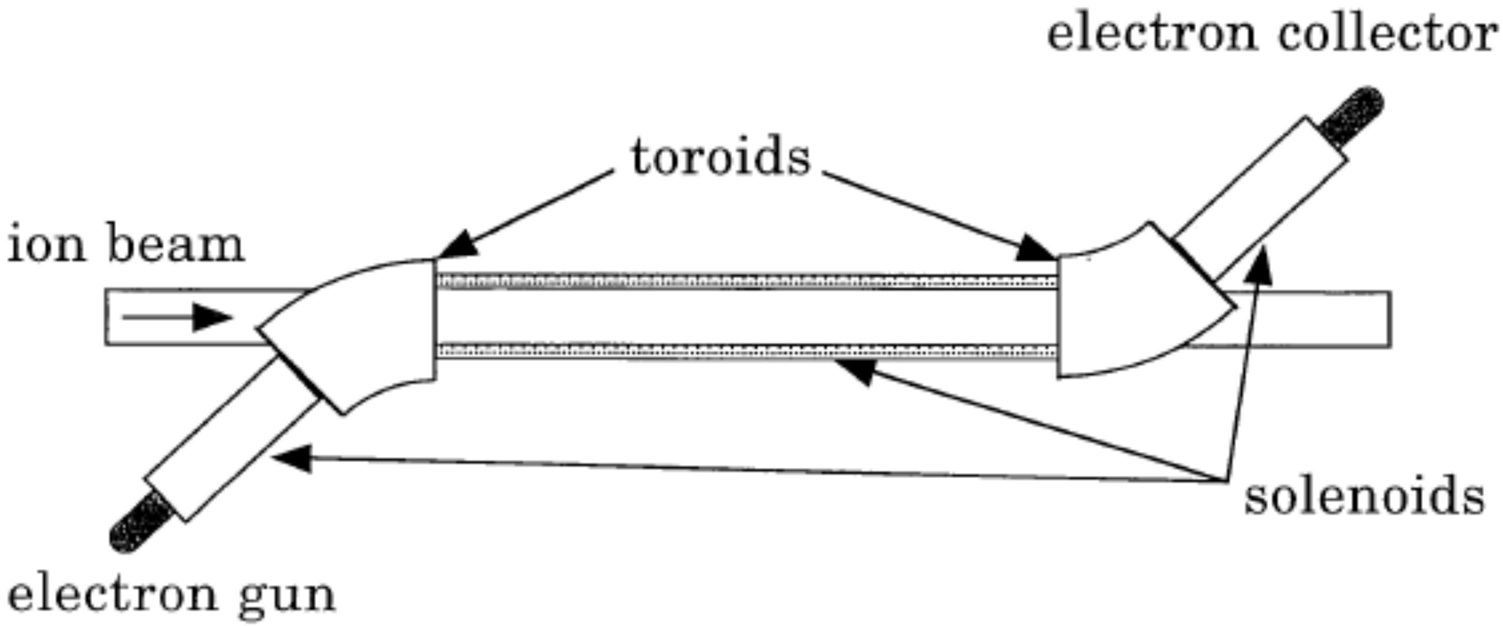
Stochastic *stacking* is an important process for antiproton accumulation.

Stochastic Stacking - antiprotons



- Note: Since stochastic cooling works by sensing fluctuation signals (Schottky “noise”), It doesn’t work very well with bunched beams – despite a lot of effort to try this

Electron Cooling (Budker, 1968)

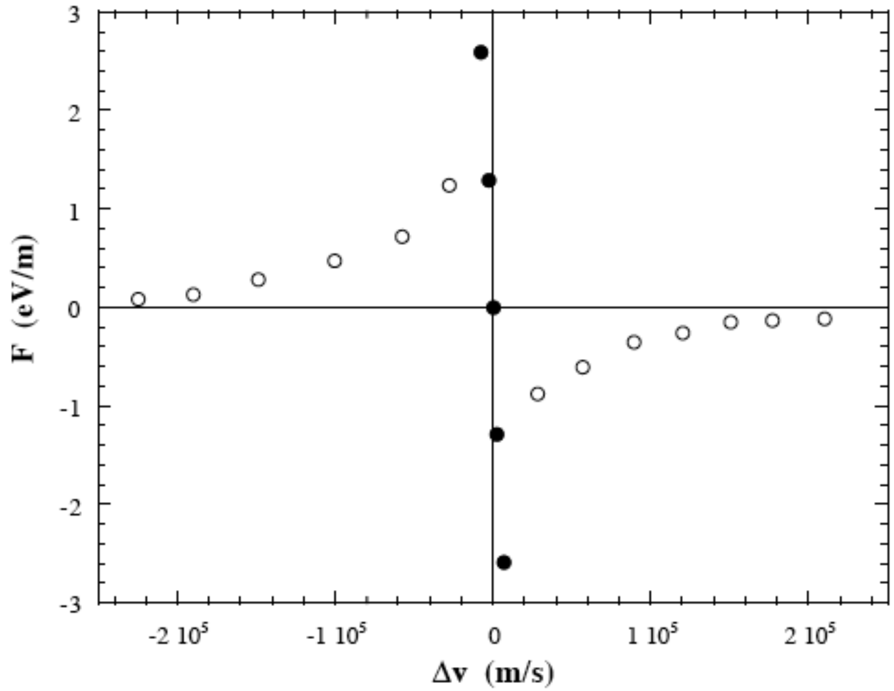


- A continuous(DC) electron beam is overlapped with the circulating ion beam at one point in the circumference of the storage ring
- The electron beam cools the circulating ion beam via Coulomb collisions

Electron Cooling 2

- The electron beam must have the same velocity as the ions – not really applicable at very high energies
- The electron beam can be much "colder" – smaller dp/p because it originated from a cold source and was electrostatically accelerated to a much higher velocity
- No problem with Liouville here as we are coupled to an external system
- Some typical numbers: 30 keV electron beam with longitudinal energy spread of 10^{-4} eV and transverse energy spread of 0.5 eV
- Electron cooling works with ANY species of charged particle
- Electron cooling works with bunched beams

Electron Cooling 2

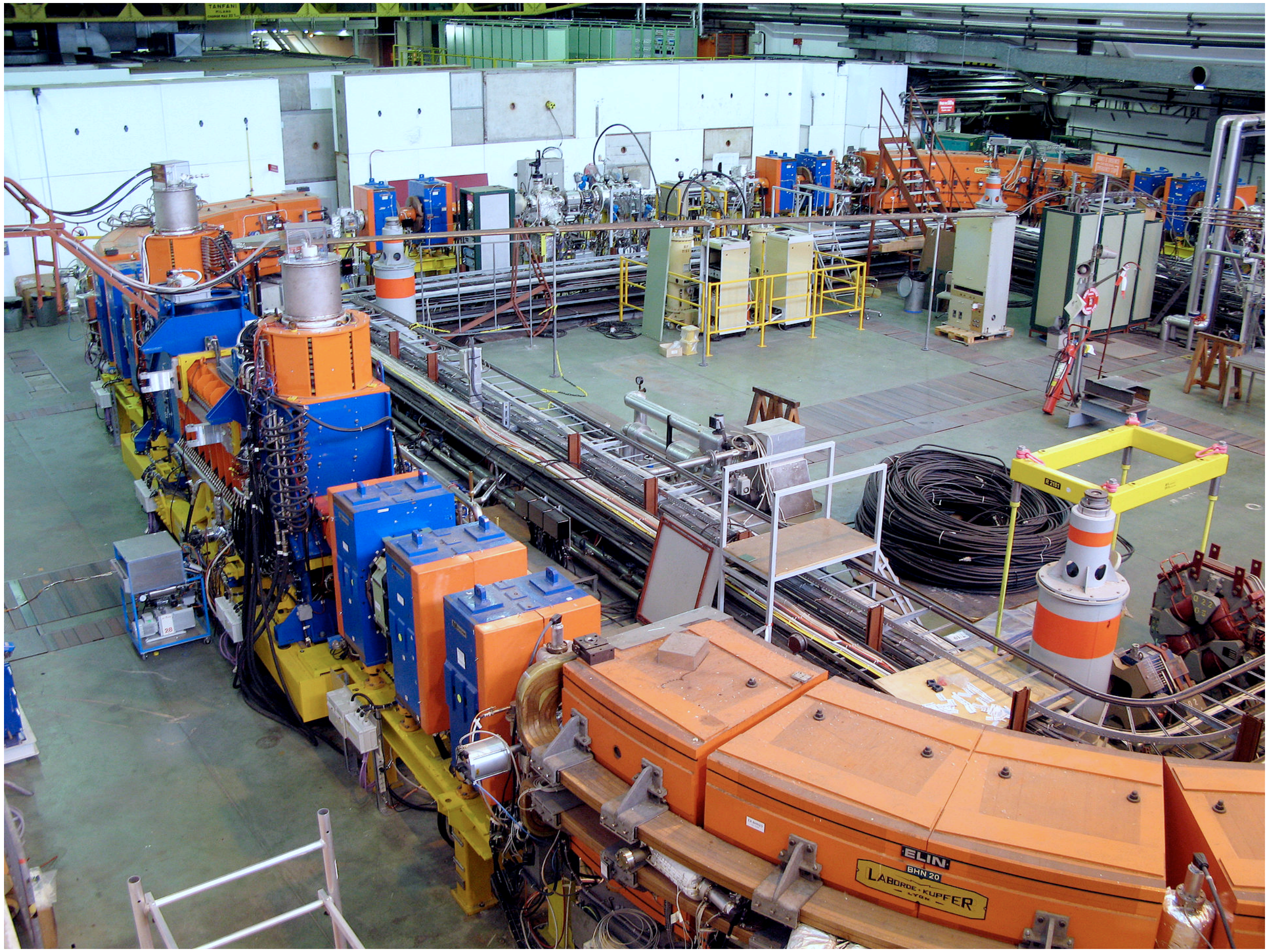


- Electron cooling of 40 MeV alpha particles at TARN2 (Japan)
- Note that, about 0 velocity, the force is roughly:

$$F = -\beta v$$

- Morale: if you don't like the rules, change them – Liouville has nothing to say about the ultimate, non-conservative, damping force like this

LEIR at CERN

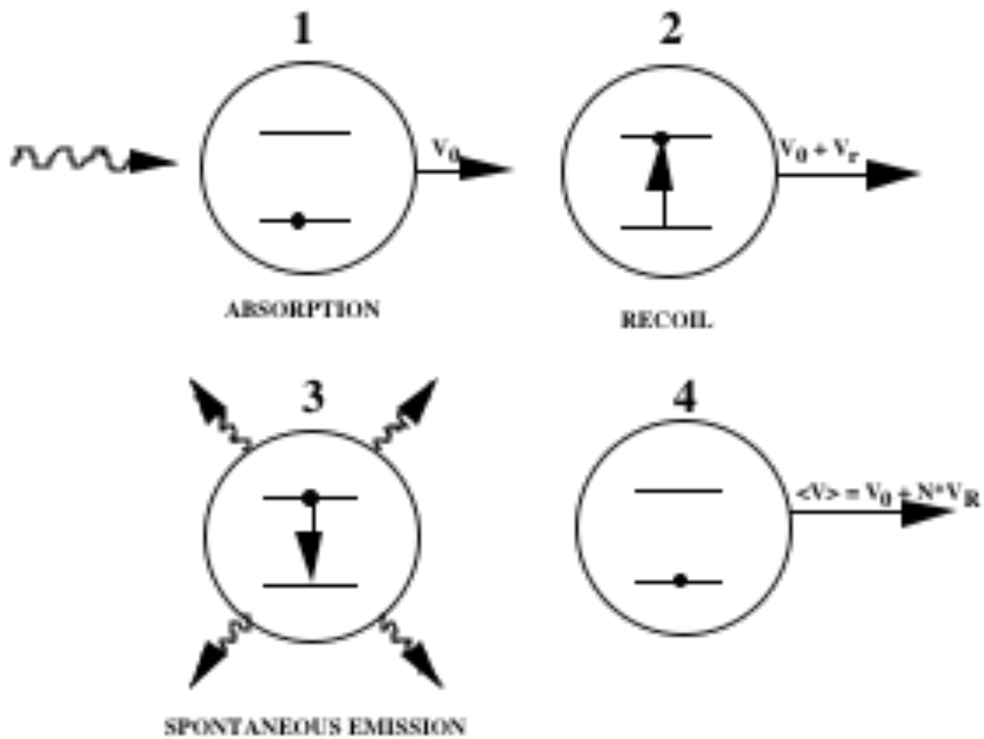


Things are bigger in America…

High Energy Electron Cooling – Fermilab Antiproton Recycler

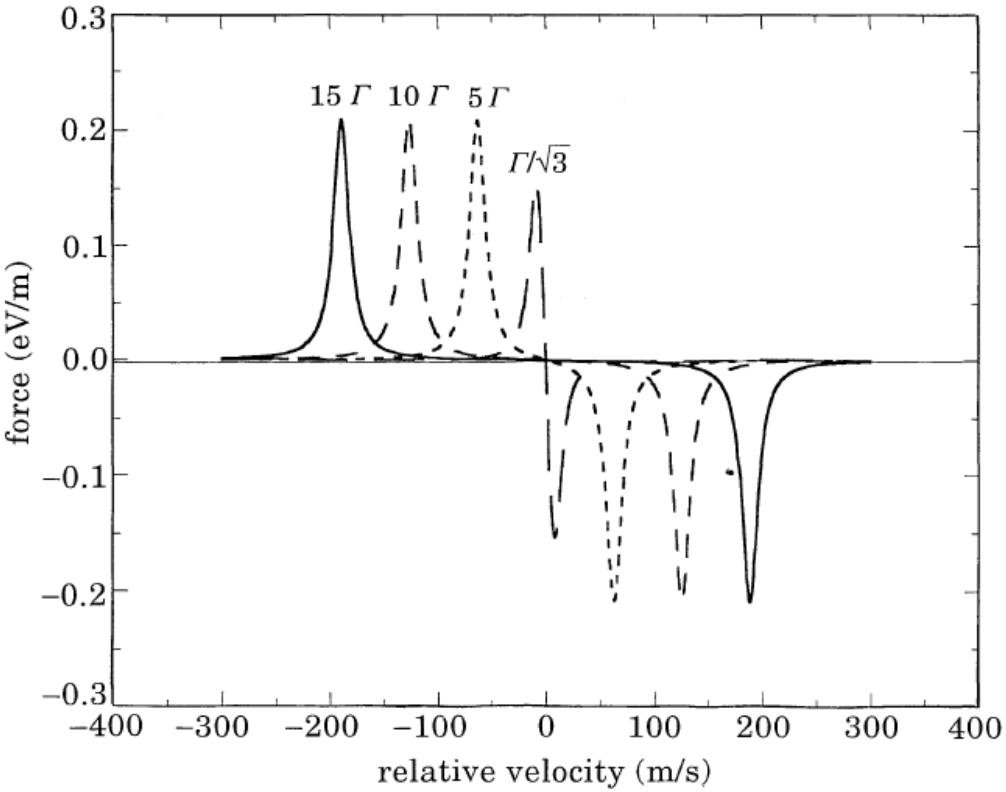
- 8 GeV antiprotons cooled by 4.3 MeV electrons
- 0.5 A electron beam - that's 2.15 MW of power
- 20 m of overlap
- Pelletron electron accelerator

Laser cooling 1

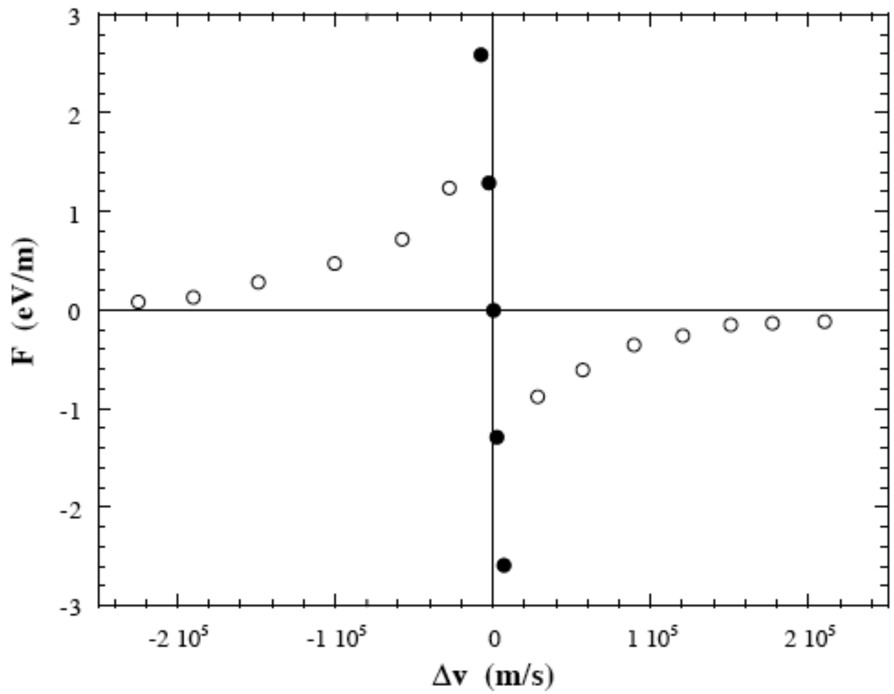


- Need a closed, two level system
- Typical photon energies of a few eV, recoil momentum of a few eV/c
- High scattering rates on resonance – transitions can be *ns*
- The Doppler shift introduces velocity dependence
- Level width determines ultimate temperature obtainable – Doppler cooling

Laser cooling 2



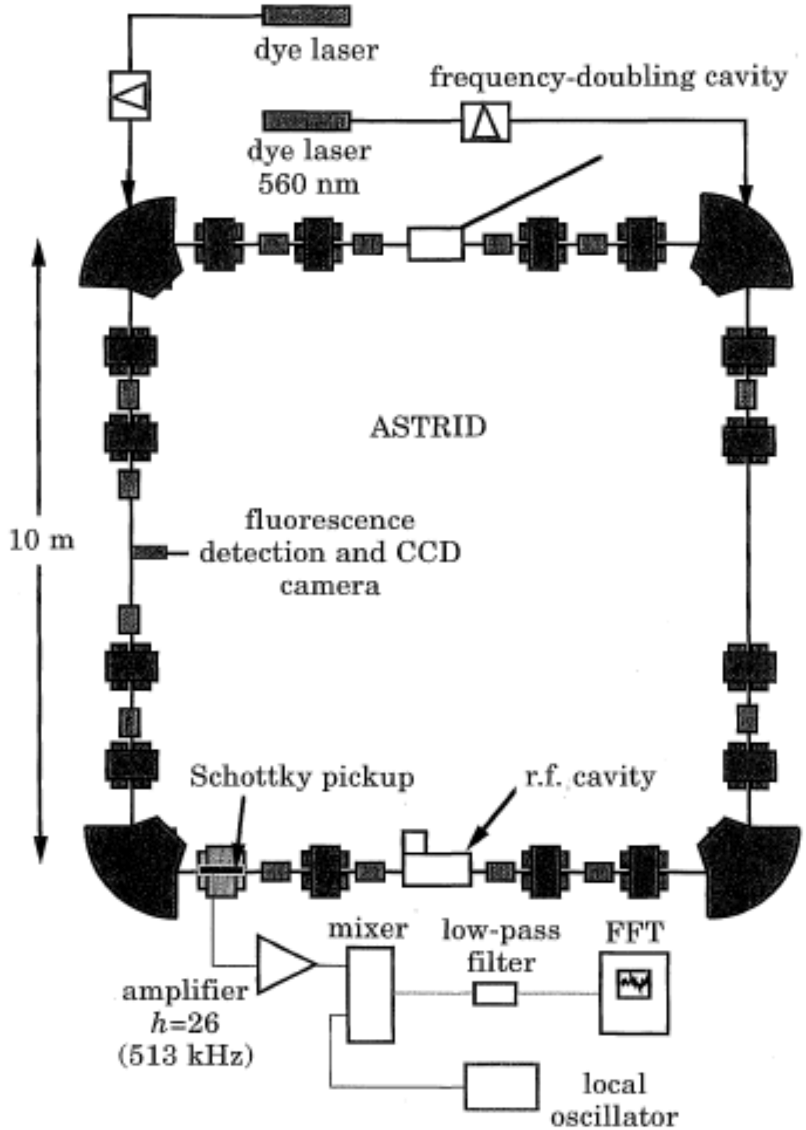
- Cooling force for two counterpropagating lasers
- $^{24}Mg^{+}$ ions; 280 nm
- Narrow reach in velocity
- mK ultimate temperature



Again we have a force like:

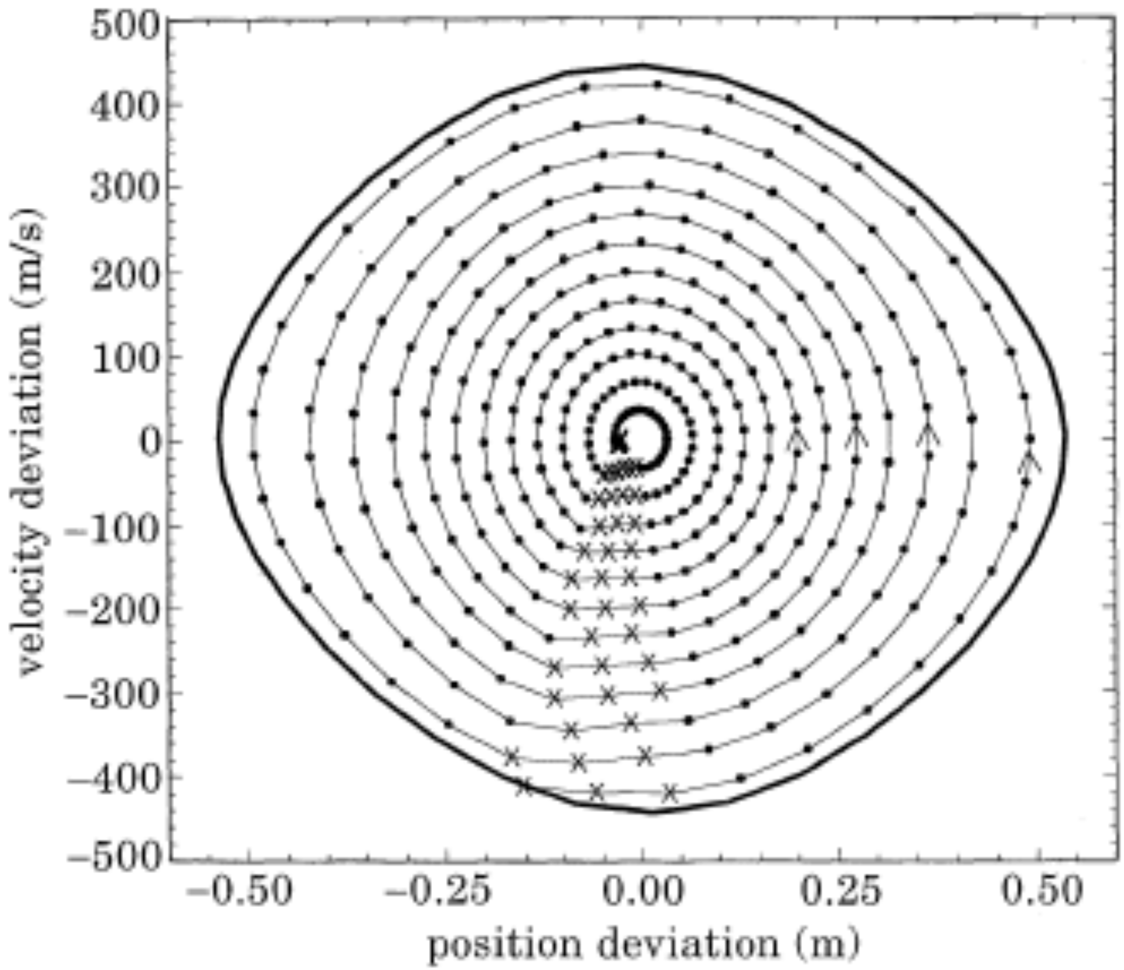
$$F = -\beta v$$

Laser cooling in a storage ring – ASTRID in Aarhus



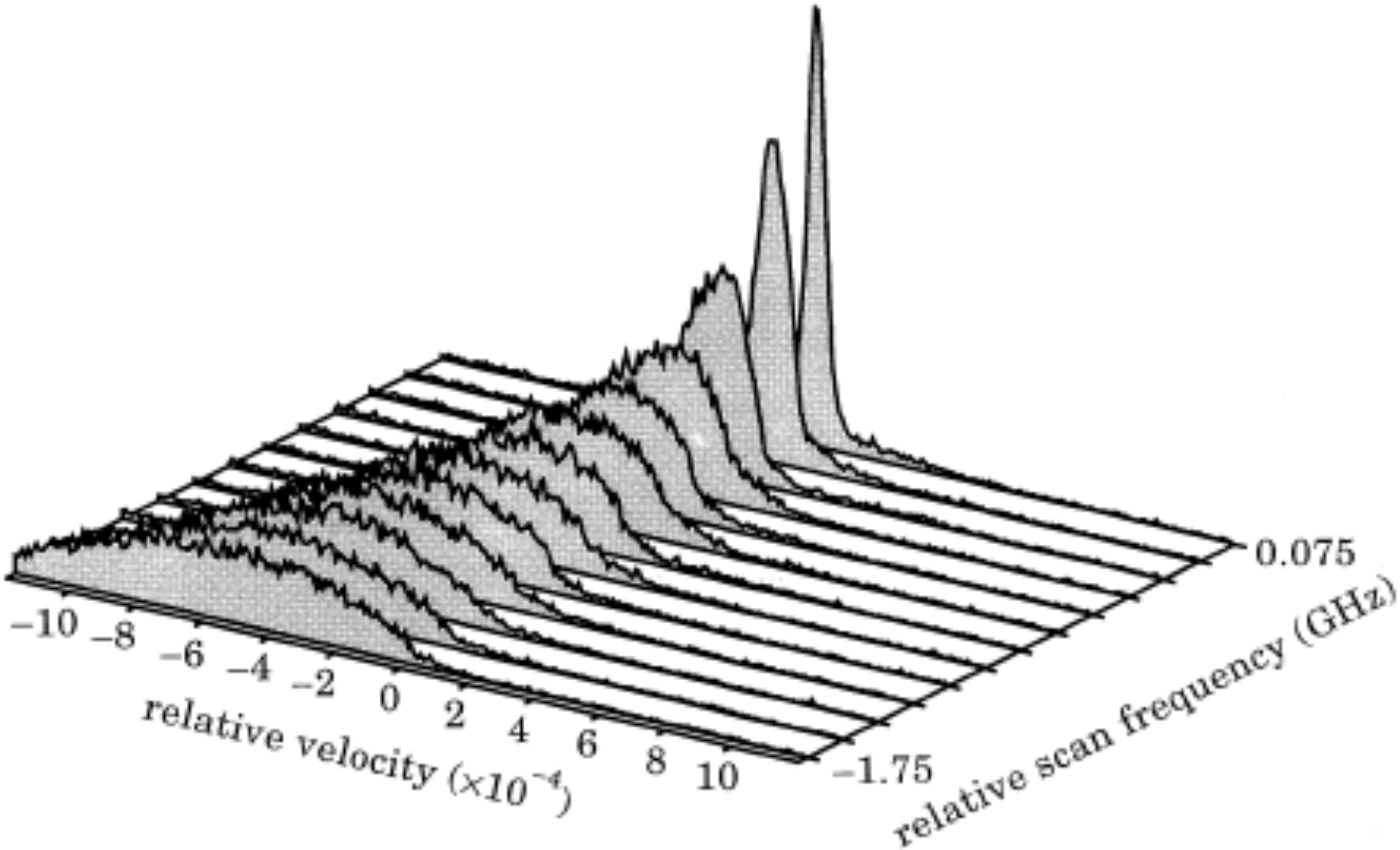
- 100 keV Mg ions
- About 10^6 m/s velocity
- $10^7 – 10^9$ stored ions
- Can detect fluorescence, make movies, etc.

Bunched beam laser cooling

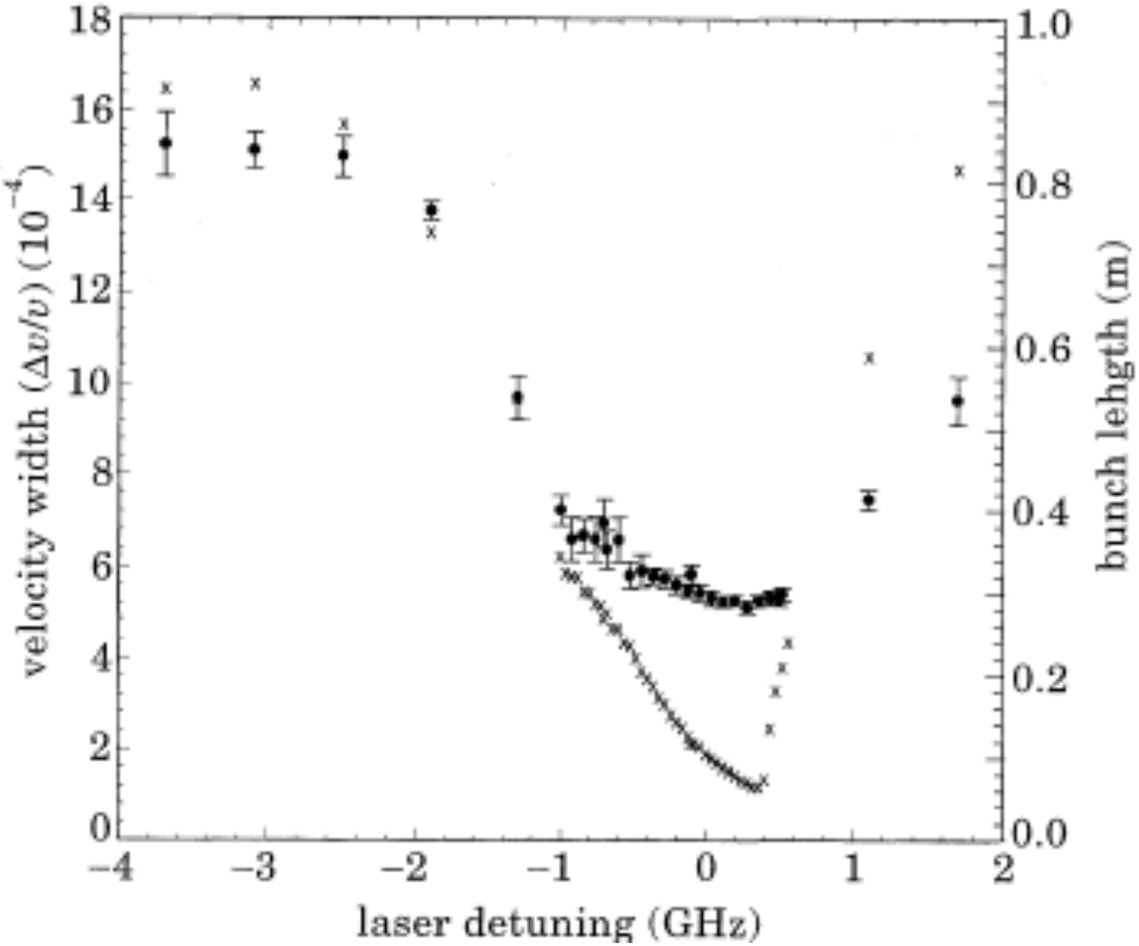


J.S. Hangst, Aarhus University

Bunched beam laser cooling 2



Bunched beam laser cooling 3



Plasma coupling parameter

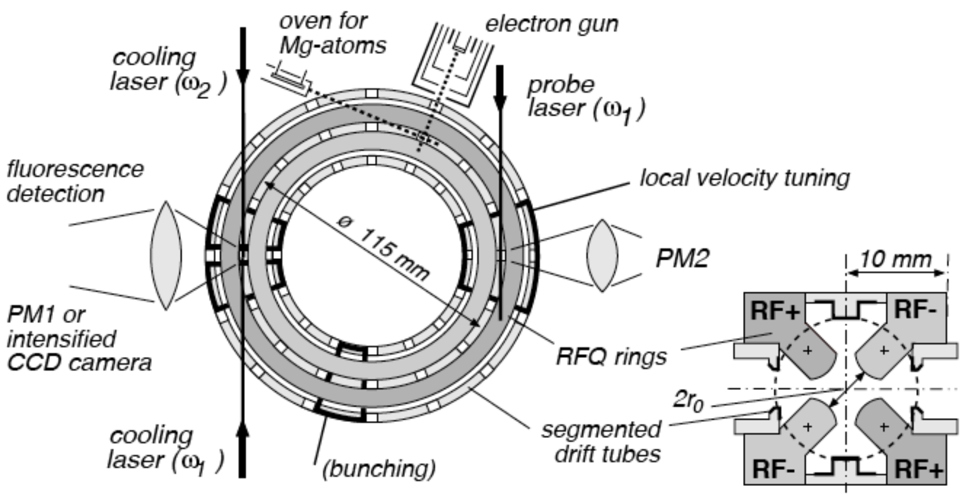
$$\Gamma_p = (e^2/4\pi\varepsilon_0 d) \cdot (1/kT)$$

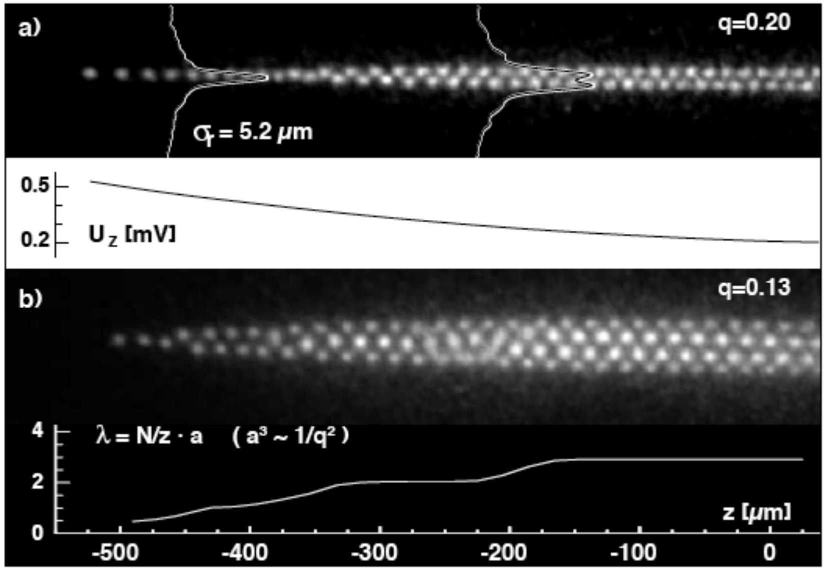
$\Gamma_p \sim 1$ liquid like behaviour

$\Gamma_p \sim 170$ crystalline structure

Goal was to produce strongly coupled or ‘crystalline’ ion beams

PALLAS RFQ storage ring – LMU Munich





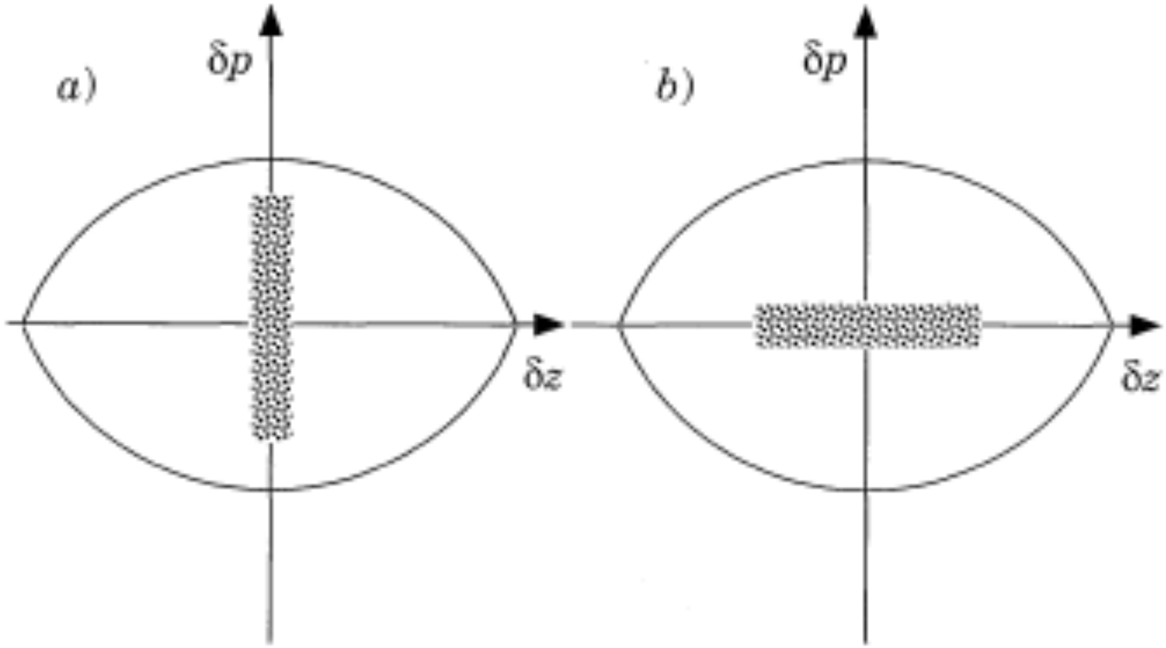
~30 cm in circumference

U. Schramm, T. Schätz, and D. Habs

Crystalline ion beams at

$$v = 2800\,\mathrm{m/s}$$

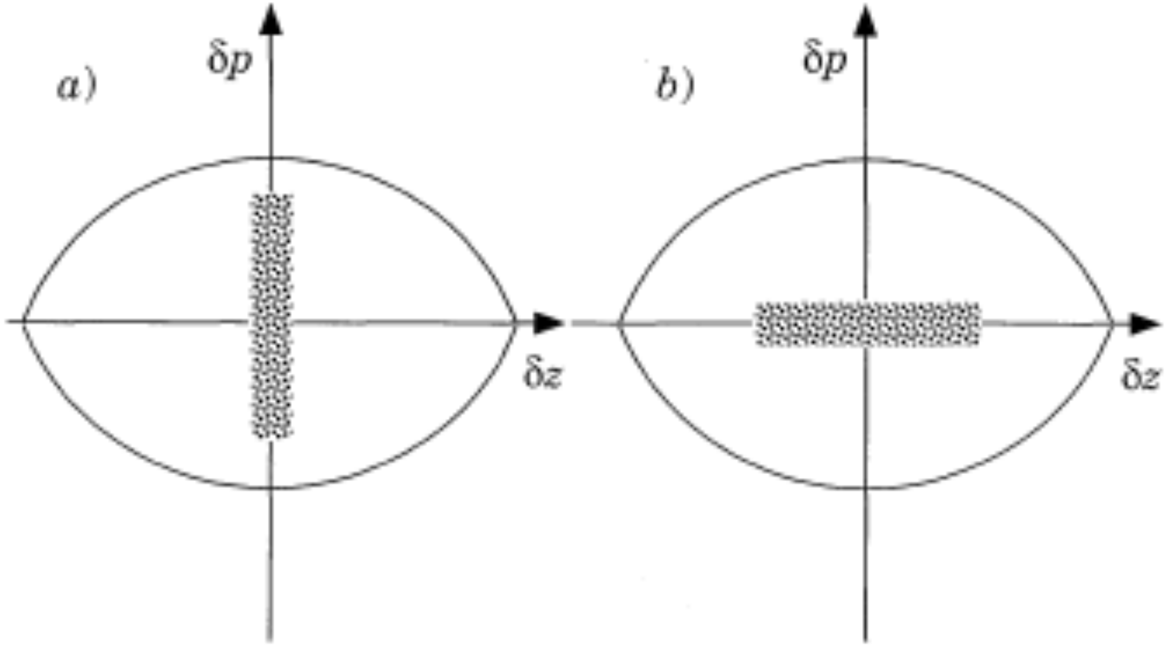
How to lie using temperature (kinetic energy spread)



- Start with a bunch of particles injected into a machine with a narrow time spread e.g. antiprotons produced from a proton beam hitting a target
- They are captured into an RF bucket with roughly the same phase in a)
- We let them do ¼ of a synchrotron oscillation, then we snap off the RF
- The momentum spread is greatly reduced, so we have cooled the beam

WRONG!

How to lie using temperature (kinetic energy spread)



- This technique, known as bunch rotation – while very useful – is NOT phase space cooling
- Plasma guys do something similar and call it 'adiabatic cooling'
- It is obvious to any trained physicist that this is not phase space cooling

WRONG!

I have seen lots of debates at scholarly conferences in which really smart guys confuse something like this for phase space cooling.

You might not care…

Summary

- Phase–space cooling can make you rich and famous
- Well, maybe famous…
- …among obscure groups of physicists
- It actually has much more potential to embarrass you..

Thanks for your attention!