Correlations in Trapped Plasma

Part 1

Effect of correlations on equilibrium properties

Correlations change the structure of confined ion plasma

Part 2

Effect of correlations on a dynamical property

Correlations increase the perpendicular to parallel collision rate
Correlations increase collision rate in non-magnetized plasma

François Anderegg

UCSD

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Strongly correlated plasmas

Thermodynamic state of an One Component Plasma determined by:

- Size
- Shape
- Coupling parameter

$$\Gamma = \frac{q^2}{a_{WS}\ k_B T}$$

with $\frac{4}{3}\pi\ a_{WS}^3\ n = 1$

$$a_{WS} = \left(\frac{3}{4\pi\ n}\right)^{\frac{1}{3}}$$

$$\Gamma \approx \frac{\text{potential energy between neighboring ions}}{\text{ion thermal energy}}$$

$\Gamma > 1 \Rightarrow$ strongly coupled OCP

$$\Gamma = \frac{q^2}{a_{WS} T}$$

$$a_{WS} = \left(\frac{3}{4\pi n} \right)^{\frac{1}{3}}$$

Non-neutral plasma

$T \sim 10^{-5}$ eV
$n \sim 2 \times 10^{7} \mathrm{cm}^{-3}$
$\Gamma \sim 10$

Giant planet interiors

$T \sim 1$ eV
$n \sim 10^{24} \mathrm{cm}^{-3}$
$\Gamma \sim 10$

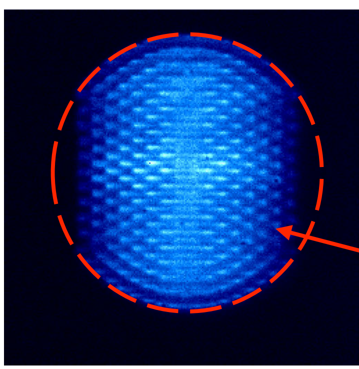
White dwarf stars

$T \sim 100$ eV
$n \sim 10^{30} \mathrm{cm}^{-3}$
$\Gamma \sim 10$

Dusty plasma

$T \sim 1$ eV
$n \sim 1 \mathrm{cm}^{-3}$
$q \sim 10^{4}$ e
$\Gamma \sim 10$

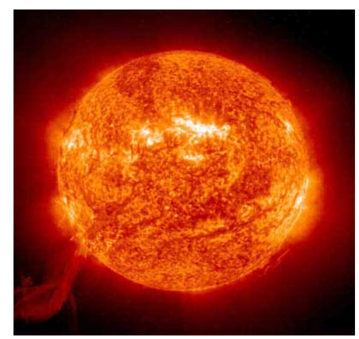
Our Sun : $\Gamma \sim 0.05$



John Bollinger, NIST

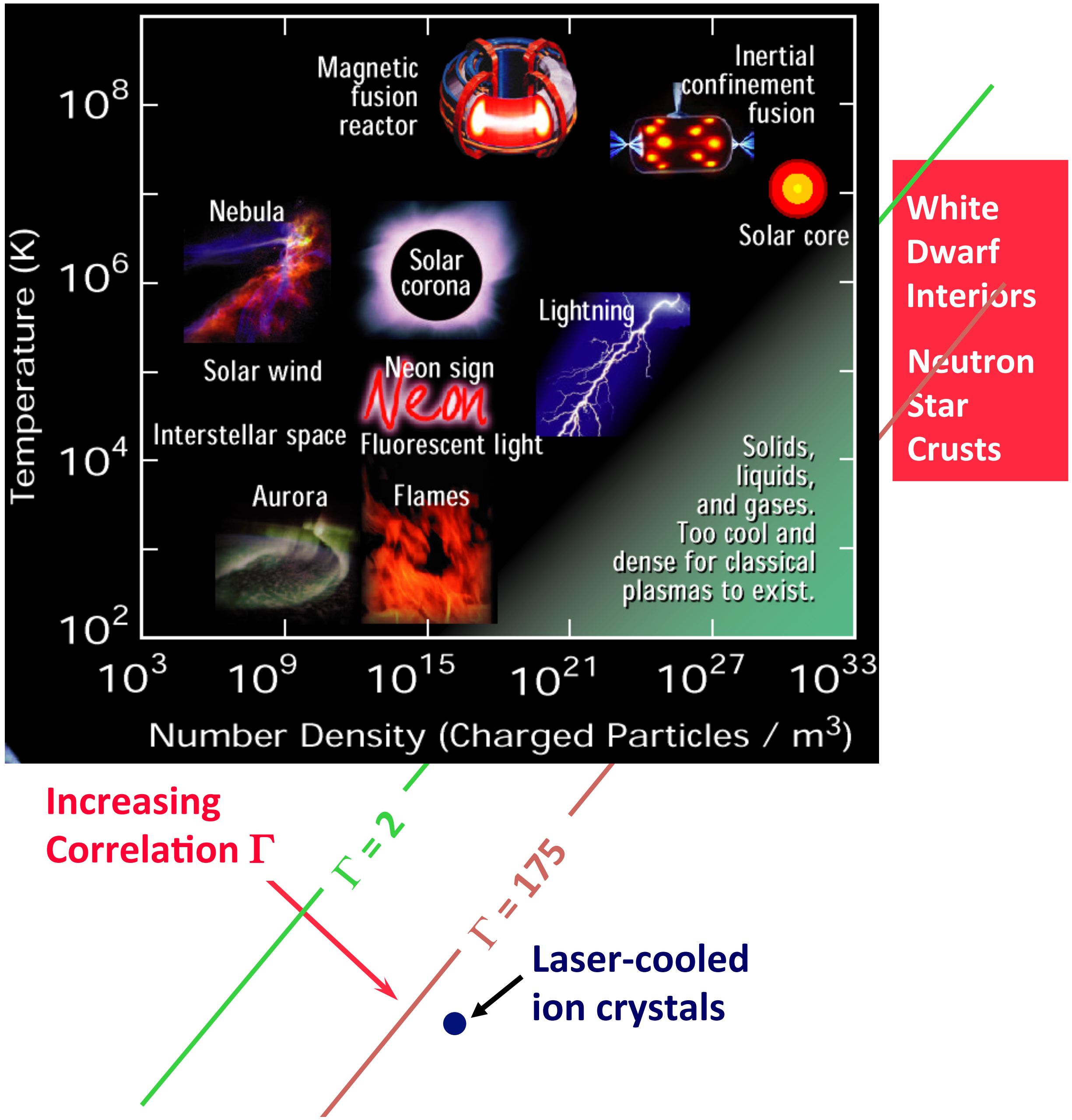
$\Gamma << 1$: plasma state
$\Gamma > 1$: liquid state

$\Gamma > 172$: bcc crystal



Frontiers in High Energy Density Physics, NRC, (2003)

Plasmas vs strongly coupled plasmas



Even when Γ is large, the mean field $e\phi >> \frac{e^2}{a}$ (for N >>1)

The shape of plasma remain ~ unchanged by correlations

In the absence of correlations

Boltzman distribution *"one particle distribution"*

$$f(r,\mathrm{v}) = C\ \exp\left(-\frac{1}{k_b T}\ \left[H + \omega P_\theta\right]\right)$$

$$H = \frac{m\mathrm{v}^2}{2} + \mathrm{e}\ (\phi_T + \phi_P) \qquad \nabla^2 \phi_P = -4\pi e n_p$$

$$P_\theta = m\mathrm{v}_\theta r + \frac{eB}{2c}\ r^2$$

In the presence of correlations

Gibbs distribution *"N particles distribution"*

$$f(r_1,\mathrm{v}_1,r_2,\mathrm{v}_2,\ldots,r_N,\mathrm{v}_N) = C\ \exp\left(-\frac{1}{k_b T}\ \left[H^{(N)} + \omega P_\theta^{(N)}\right]\right)$$

$$H^{(N)} = \sum_{i=1}^{N} \frac{m\mathrm{v}_i^2}{2} + \mathrm{e}\ \left(\phi_T(r_i) + \sum_{j>i} \phi_{ij}\right)$$

$$\phi_{ij} = \frac{e^2}{|r_i - r_j|} + \text{"image charge"}$$

$$P_\theta^{(N)} = \sum_{i=1}^{N}\ m\mathrm{v}_{\theta i} r_i + \frac{eB}{2c}\ r_i^2$$

$$= C\ \exp\left(-\sum_{i=1}^{N} \frac{m}{2k_b T}\ \left[\mathrm{v}_i + \omega r_i\ \hat{\theta}_i\right]^2\right)\ \tilde{C}\ \exp\left(-\sum_{j=1}^{N}\ \frac{1}{k_b T}\ \left\{\ \frac{1}{2}\ \sum_{i\neq j}\ \frac{e^2}{|r_i - r_j|} + e\phi_T\ \right\}\right)$$

Reduced distributions

Spatial distribution $$\rho^{(M)}(r_1,r_2,\ldots,r_M) = \int d^3r_{M+1}\ldots\, d^3r_N \;\; f(r_1,\ldots,r_N)$$

Density $\quad n(r) = N\rho^{(1)}(r) \quad$ *First reduced distribution*

$$\rho^{(2)}(r_1,r_2) = \rho^{(1)}(r_1)\;\; \rho^{(2)}(r_2)\; [\; 1+\; g(r_1,r_2)\;]$$

Two body spatial correlation

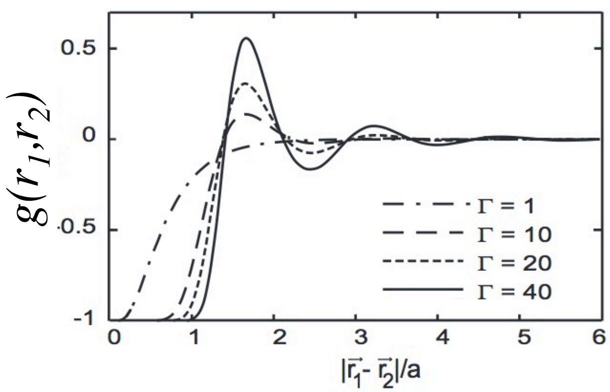


Figure 2.3: Correlation function for one component plasma.

$g(r_1,r_2)$ *: correlation function measures the extra probability beyond what would be expected of a completely random distribution of finding particles at r_1 and r_2*

Coulomb interaction is a binary interaction, all thermodynamics quantities can be evaluated from $\quad n(r) \qquad g(r_1,r_2) \qquad T$

Correlations with small plasmas

Dubin and O'Neil, Computer Simulation of Ion Clouds in a Penning Trap, PRL **60**, 511 (1988)

$$d\mathbf{x}_i/dt = (c/B)\mathbf{E}_i \times \hat{\mathbf{z}} + U_i\hat{\mathbf{z}},$$

$$dU_i/dt = (e/m)\mathbf{E}_i \cdot \hat{\mathbf{z}},$$

$$\mathbf{E}_i = -\partial\Phi/\partial\mathbf{x}_i$$

$$e\Phi(\mathbf{x}_1,\ldots,\mathbf{x}_N) = \underbrace{\sum_{i>j} \frac{e^2}{|\mathbf{x}_i - \mathbf{x}_j|}}_{\text{Ion}} + \underbrace{\sum_i \tfrac{1}{2} m\omega_z^2 (z_i^2 - \rho_i^2/2)}_{\text{Trap}}$$

Ion   Trap

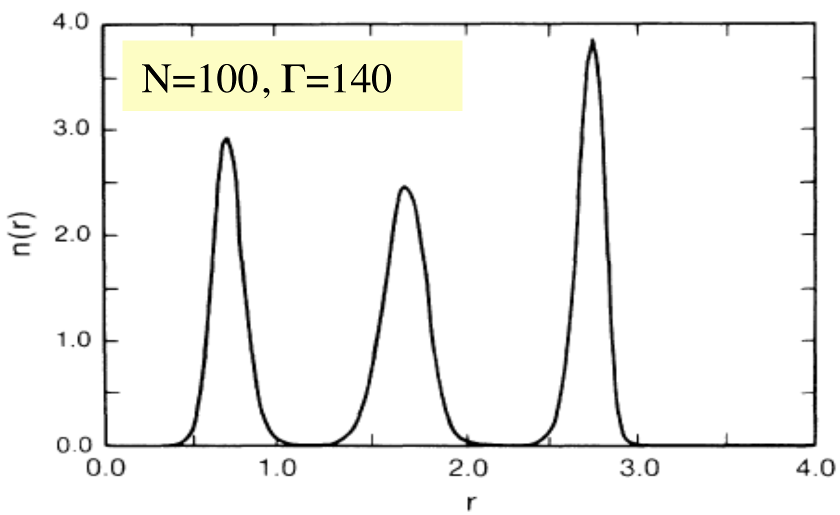
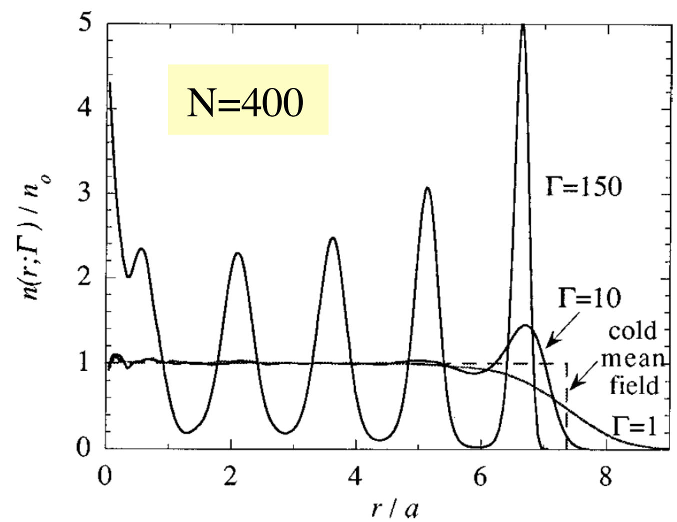


FIG. 1. Density as a function of spherical radius for $N = 100$, $\Gamma = 140$.

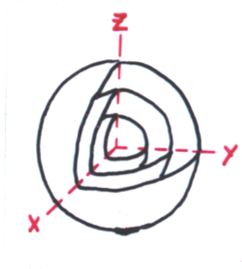


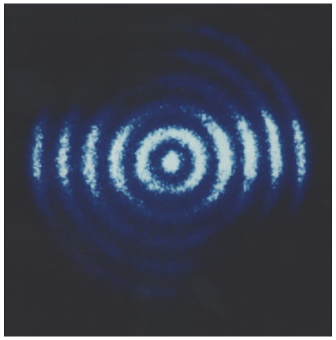
Boundary effects dominate => Shell structure is observed

Observations of shell structure

- **1988 – shell structures in Penning traps**
  NIST group

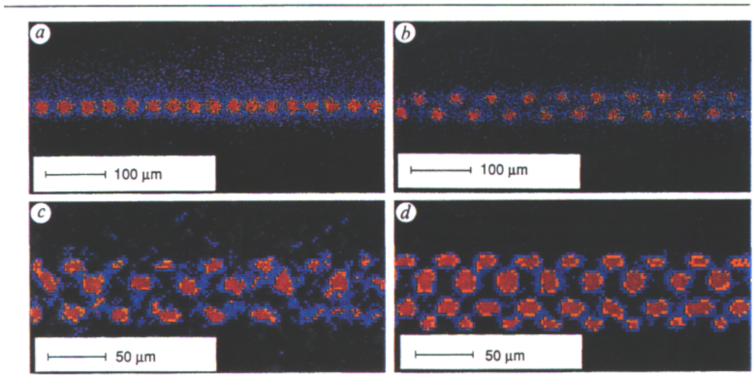
  PRL <u>60</u>, 2022 (1988)





- **1992 – 1-D periodic crystals in linear Paul traps**
  MPI Garching

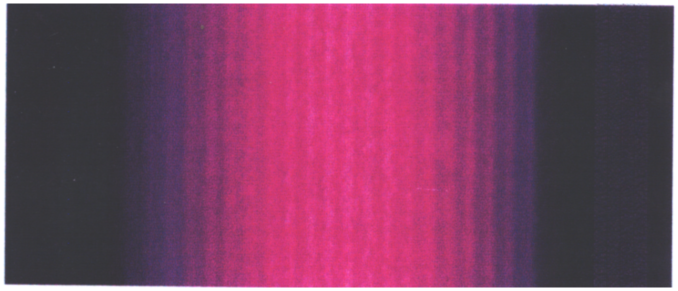
  Nature <u>357</u>, 310 (92)



- **1998 – 1-D periodic crystals with plasma diameter > 30 a_{WS}**
  Aarhus group

  PRL <u>81</u>, 2878 (98)

  See Drewsen presentation next week

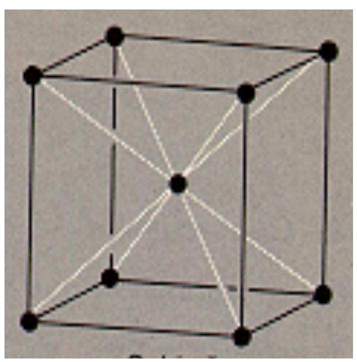


Large plasma

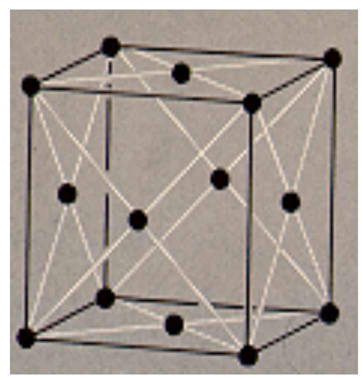
Influence of surface is limited

Interior comparable to infinite size crystal

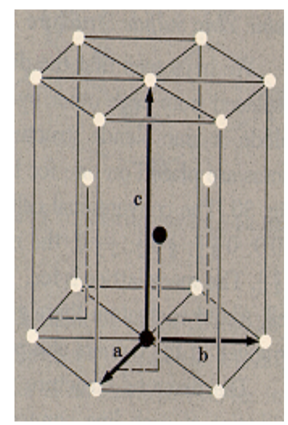
body centered cubic



face centered cubic



hexagonal close packed



Coulomb energies/ion of bulk bcc, fcc, and hcp lattices differ by < 10^{-4}

How large must a plasma be to exhibit a bcc lattice?

1989 - Dubin, planar model PRA 40, 1140 (89)

result: plasma dimensions ≥ 60 interparticle spacings required for bulk behavior

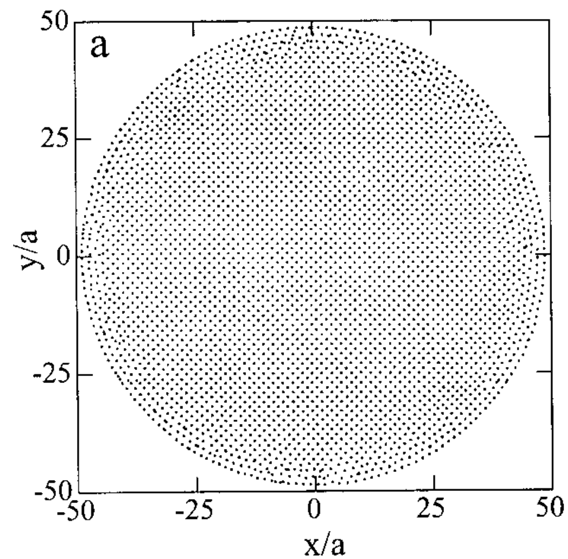
N > 10^5 in a spherical plasma ⇒ bcc lattice

2001 – Totsji, simulations, spherical plasmas, N≤120 k

PRL 88, 125002 (2002)

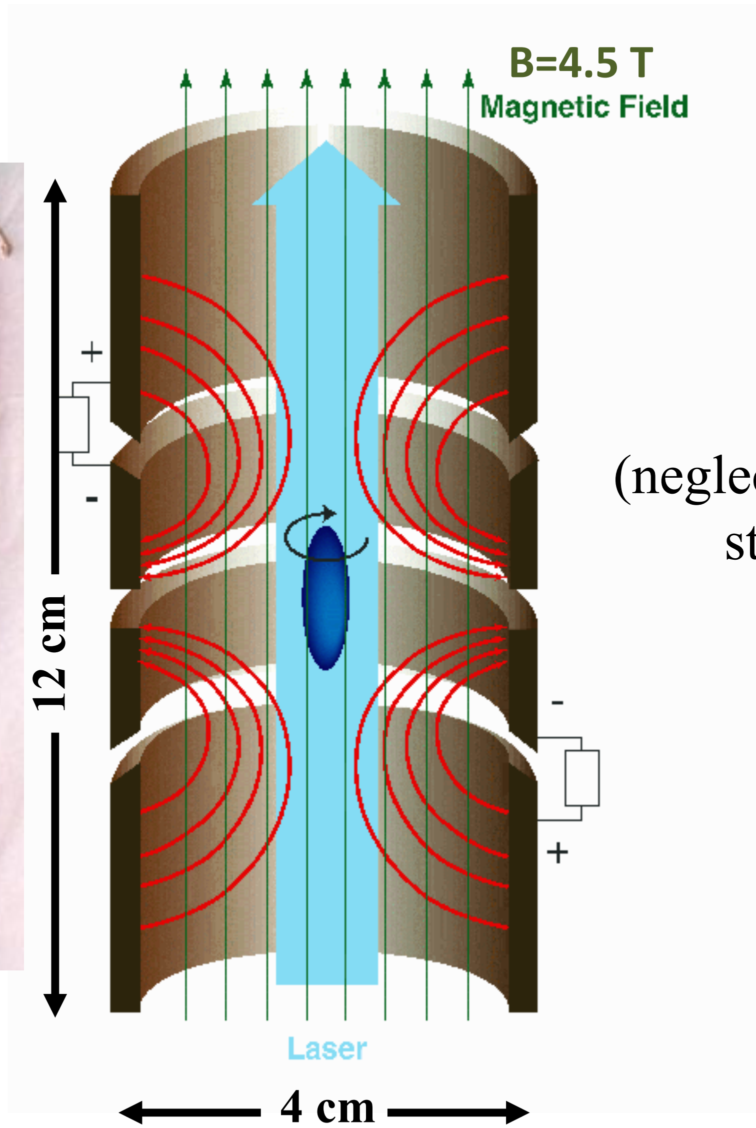
result: N>15 k in a spherical plasma

⇒ bcc lattice

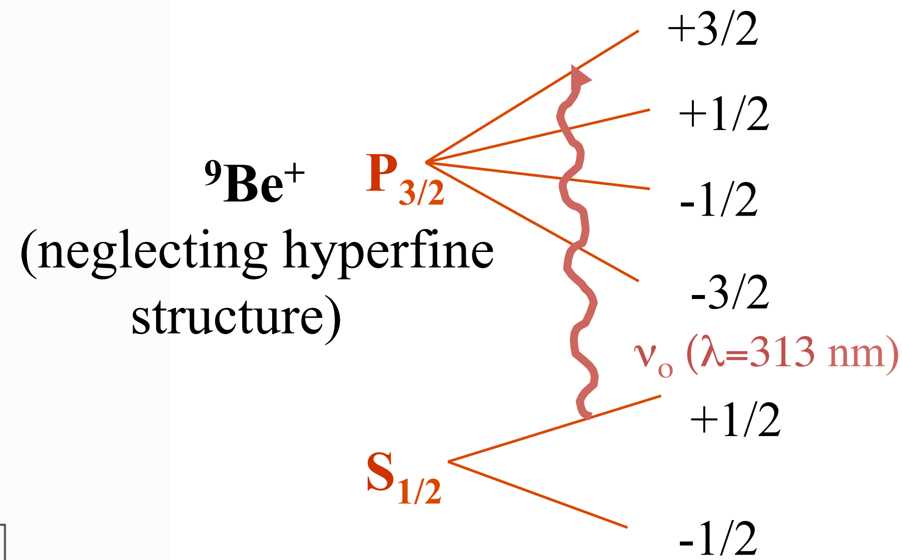


NIST Penning trap – designed to look for "large" bcc crystals

John Bollinger NIST



Doppler laser cooling

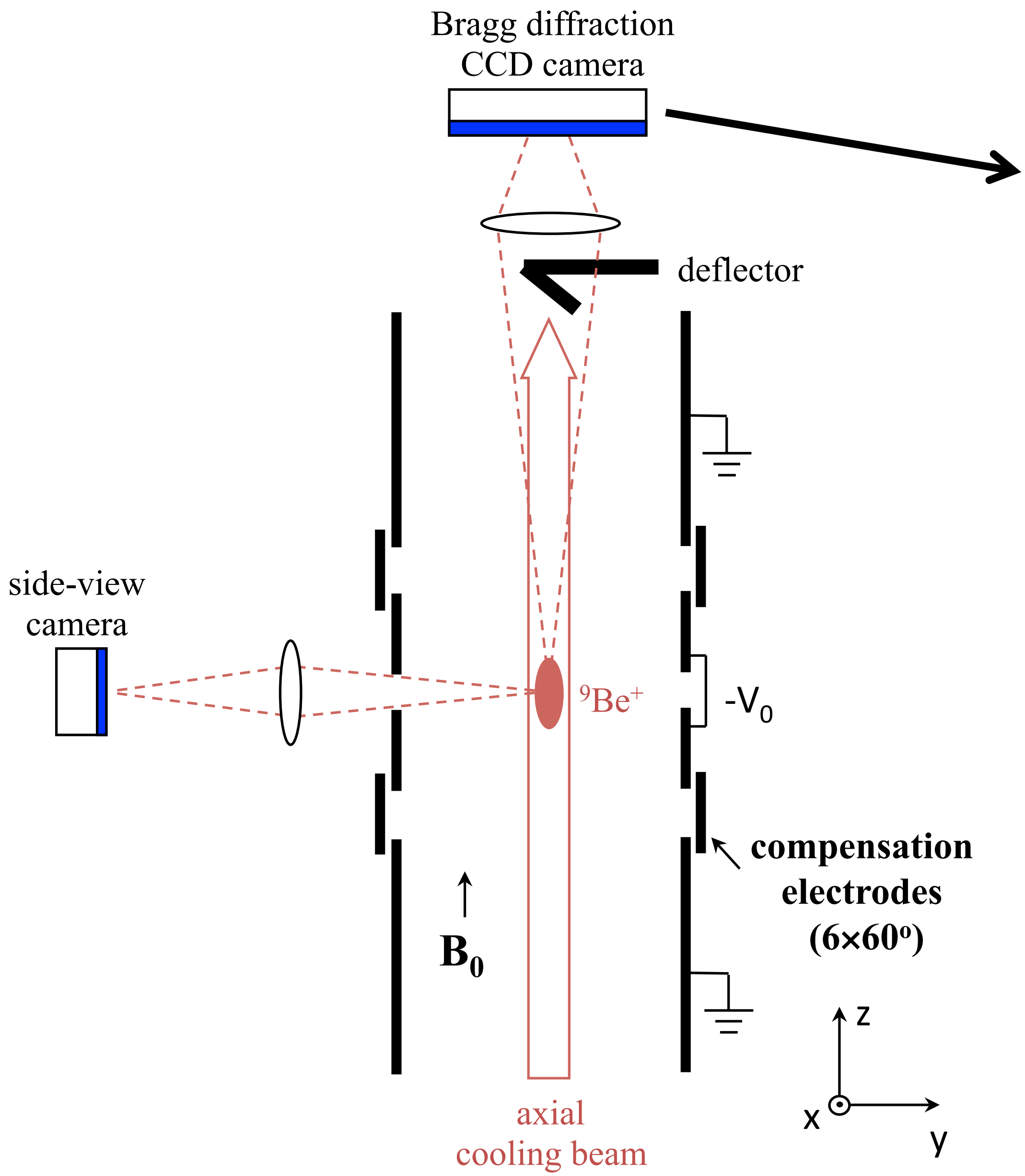


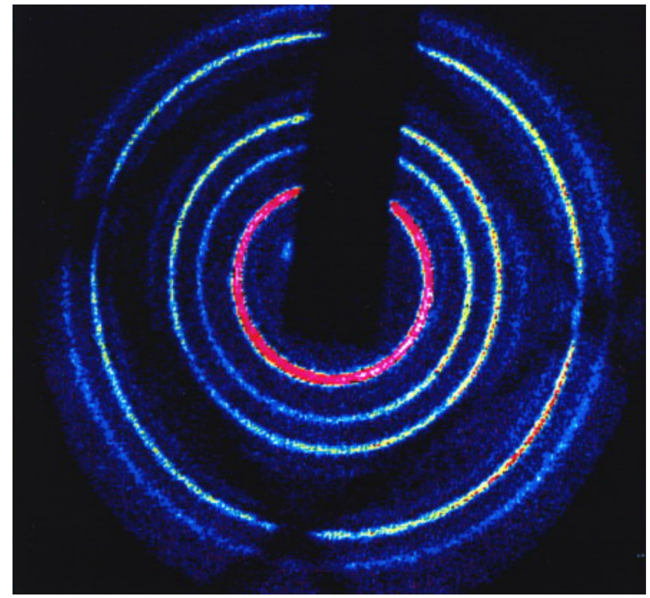
$T_{min}(^9Be^+) \sim 0.5$ mK

$T_{measured} < 1$ mK

John Bollinger NIST

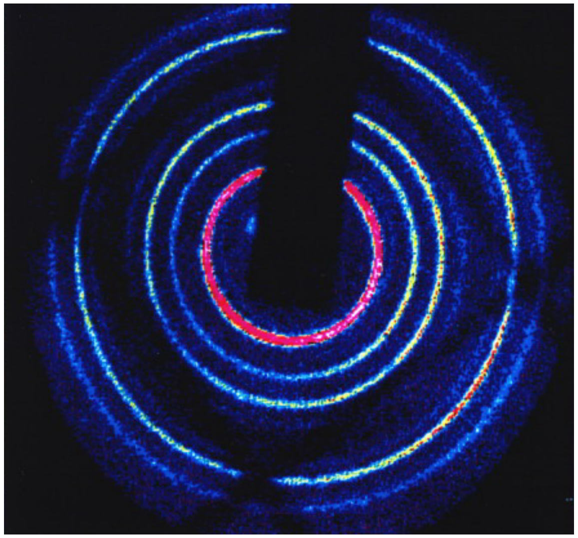
Bragg scattering

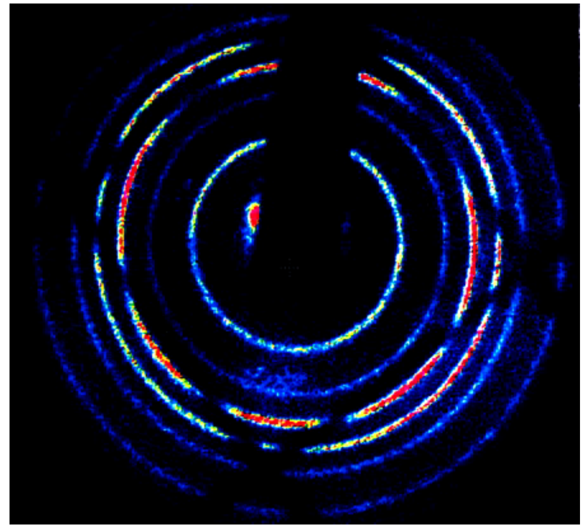




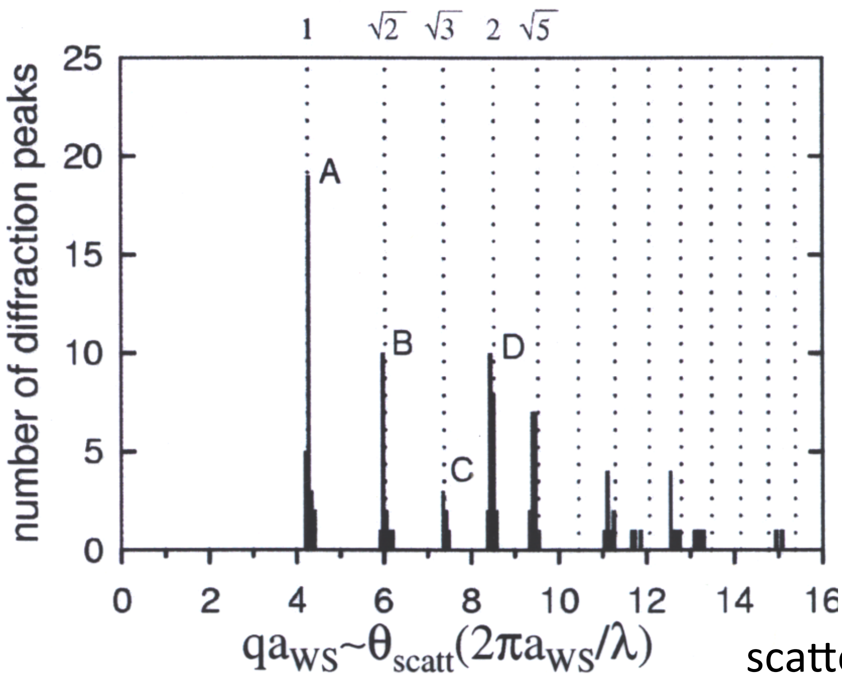
J.N. Tan, *et al.*, Phys. Rev. Lett. **72**, 4198 (1995)
W.M. Itano, *et al.*, Science **279**, 686 (1998)

Bragg scattering from spherical plasmas with N~ 270 k ions





Evidence for bcc crystals



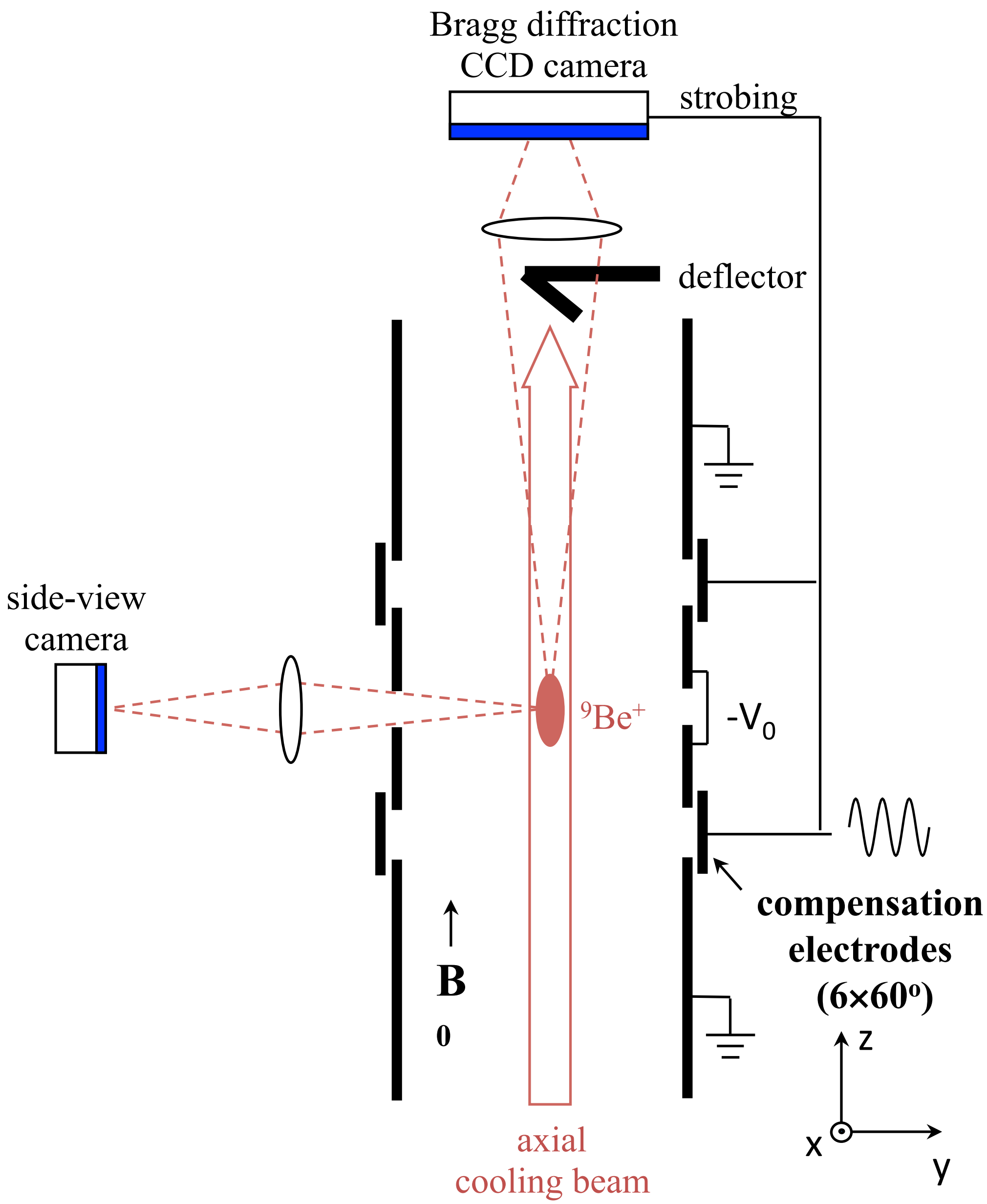
scattering angle →

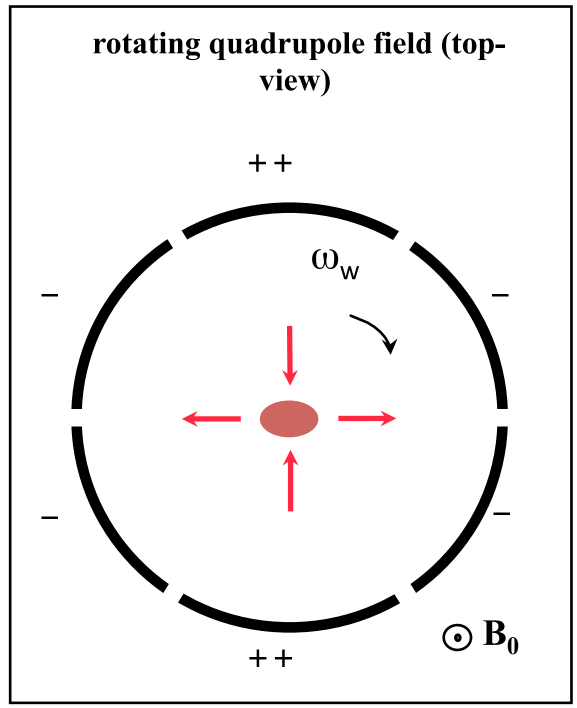
John Bollinger NIST

Rotating wall control of the plasma rotation frequency

John Bollinger NIST

Huang, *et al.* (UCSD), PRL 78, 875 (97)
Huang, *et al.* (NIST), PRL 80, 73 (98)



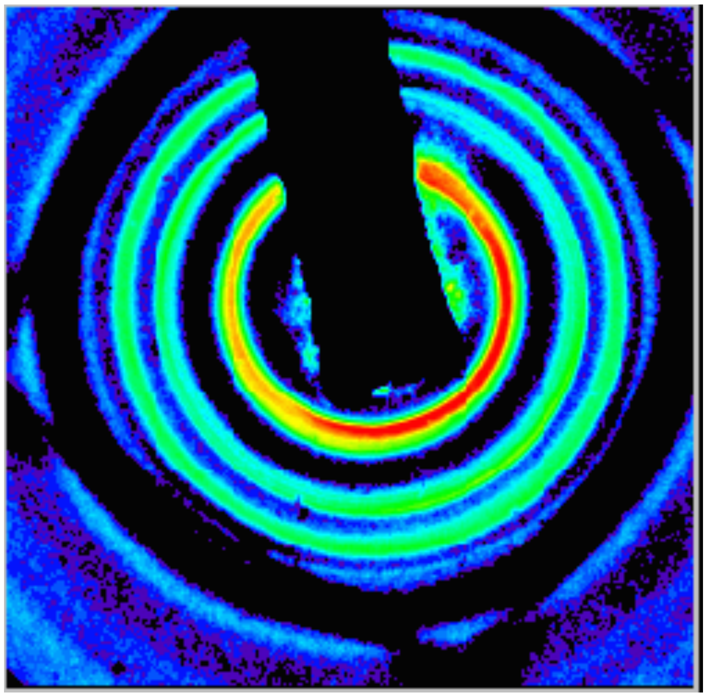


Phase-locked control of the plasma rotation frequency

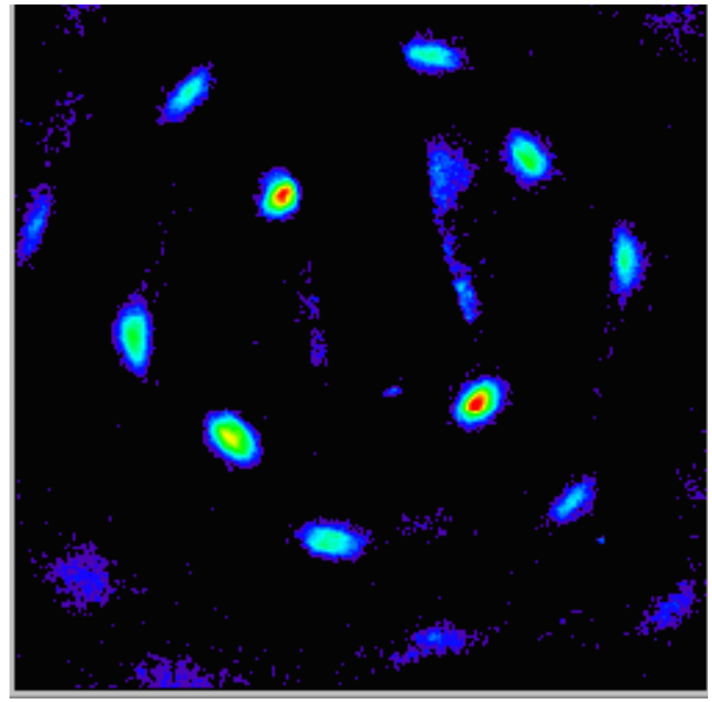
John Bollinger NIST

Huang, et al., Phys. Rev. Lett. 80, 73 (98)

time averaged Bragg scattering



camera strobed by the rotating wall

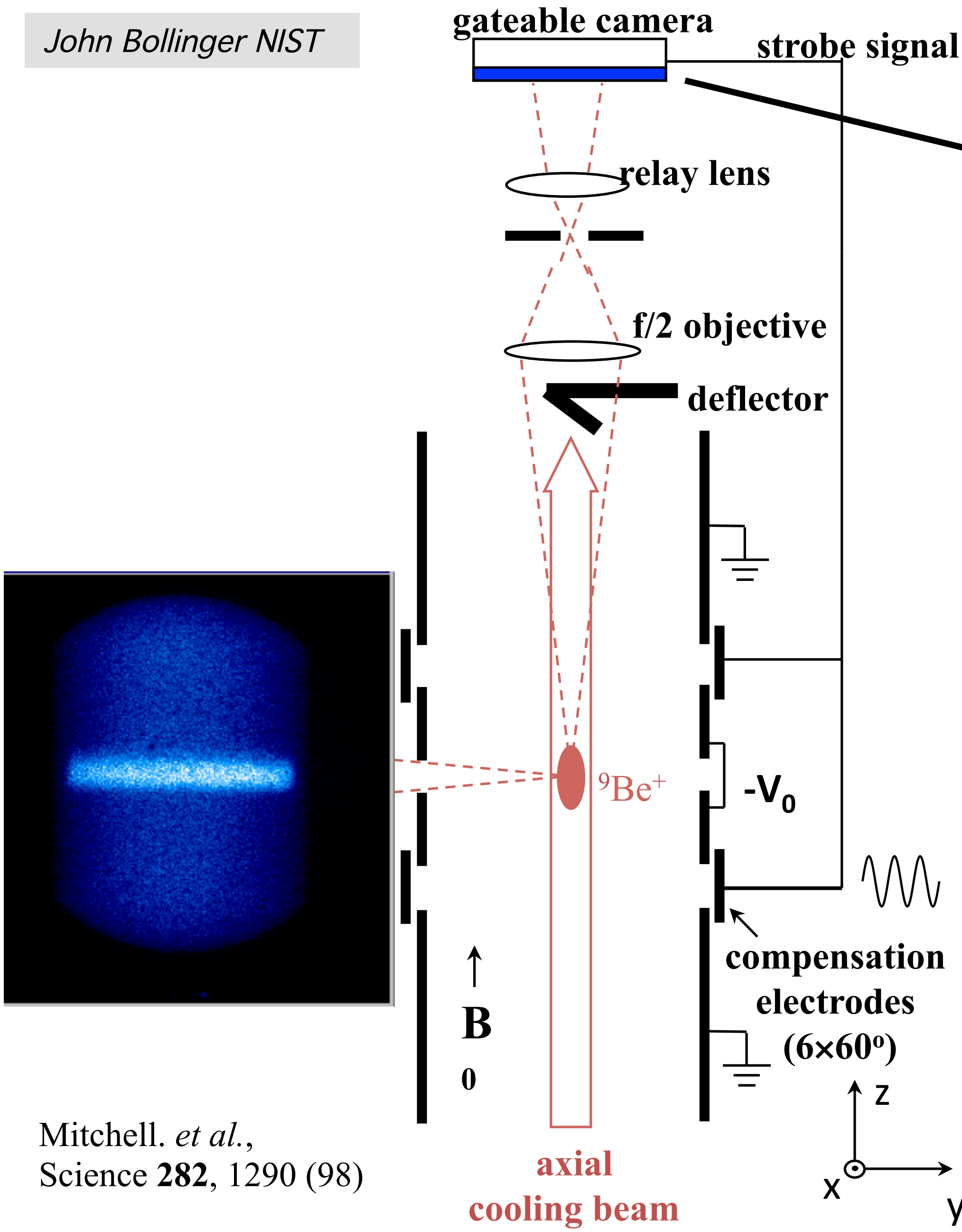


- **determine if crystal pattern due to 1 or multiple crystals**
- **enables real space imaging of ion crystals**

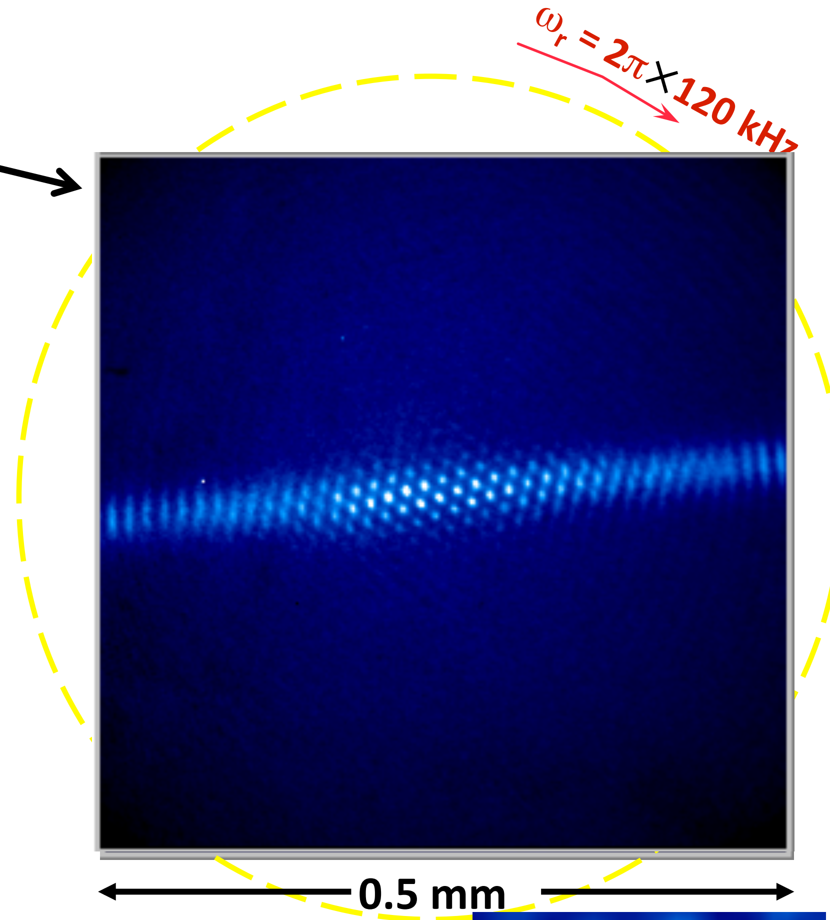
Real space imaging

John Bollinger NIST

Top-view images in a spherical plasma of 180,000 ions

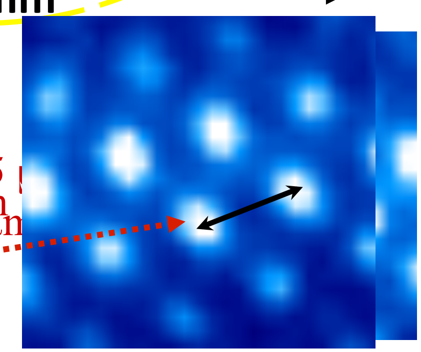


Mitchell. *et al.*,
Science **282**, 1290 (98)



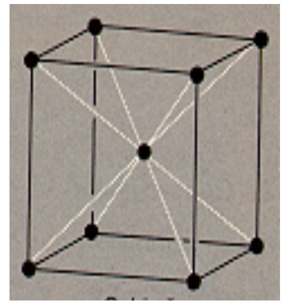
bcc (100) plane
predicted spacing: 12.5
measured: 12.8 ± 0.3 µm

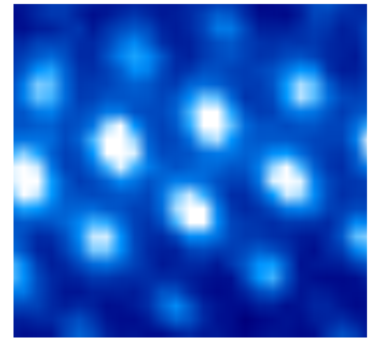
bcc (111) plane
predicted spacing: 14.4 µm
measured: 14.6 ± 0.3 µm



Summary of correlation observations in approx. spherical plasmas

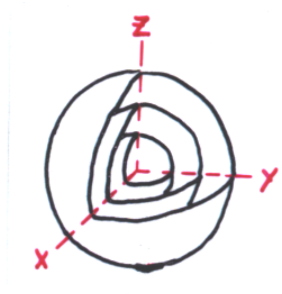
N ≥ 2 x 10^5
observe bcc crystal structure



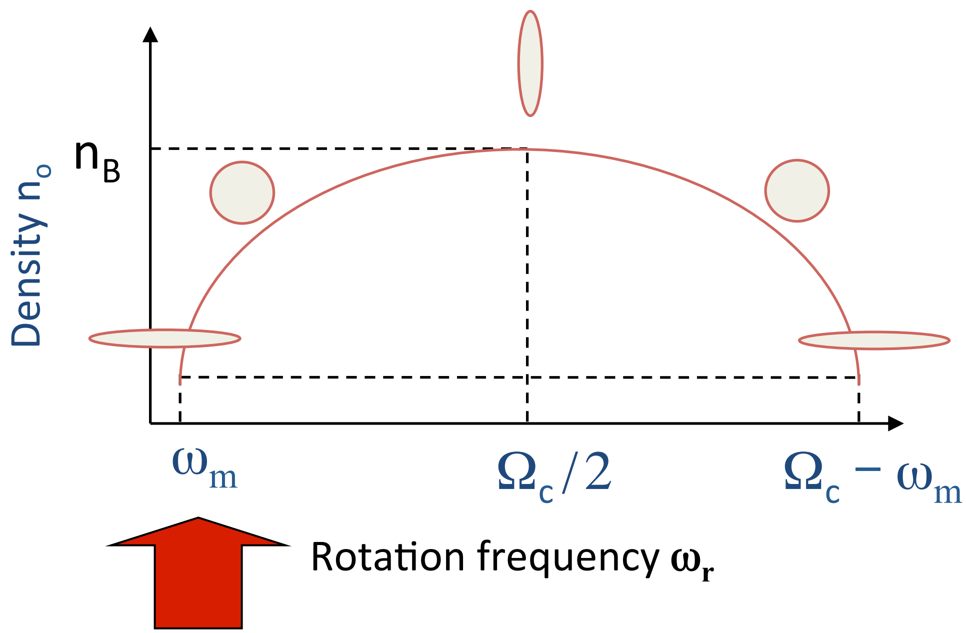


1 x 10^5 > N > 2 x 10^4
observe other crystal structures (fcc, hcp?, …) in addition to bcc structure

N < 2 x 10^4
Shell structure dominates



Planar Plasmas



with planar plasmas all the ions can reside within the depth of focus

Planar structural phases can be ‘tuned’ by changing ω_r

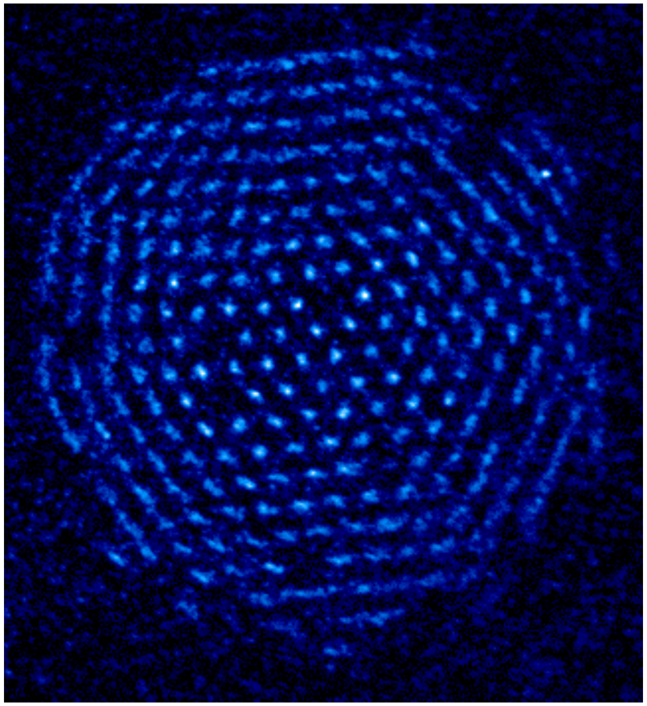
John Bollinger NIST

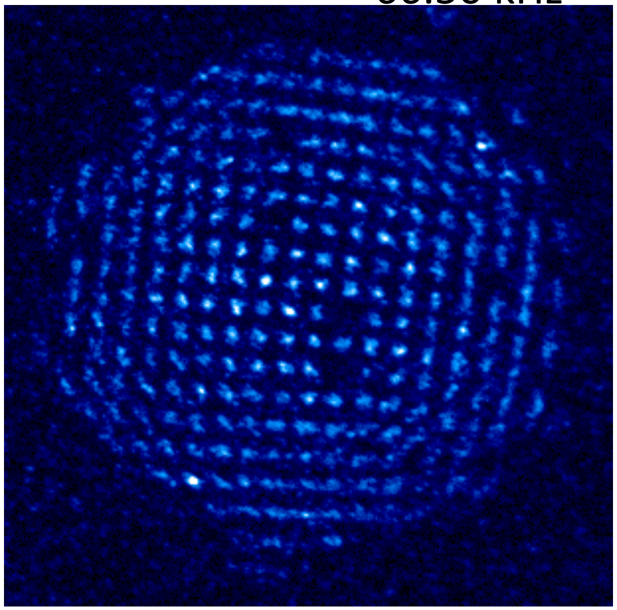
Real space images

65.70 kHz

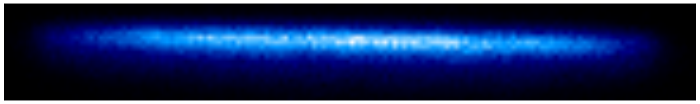
66.50 kHz

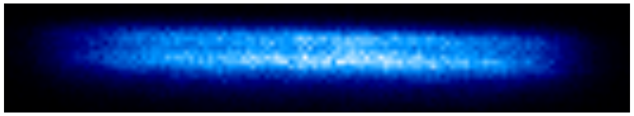
top-views





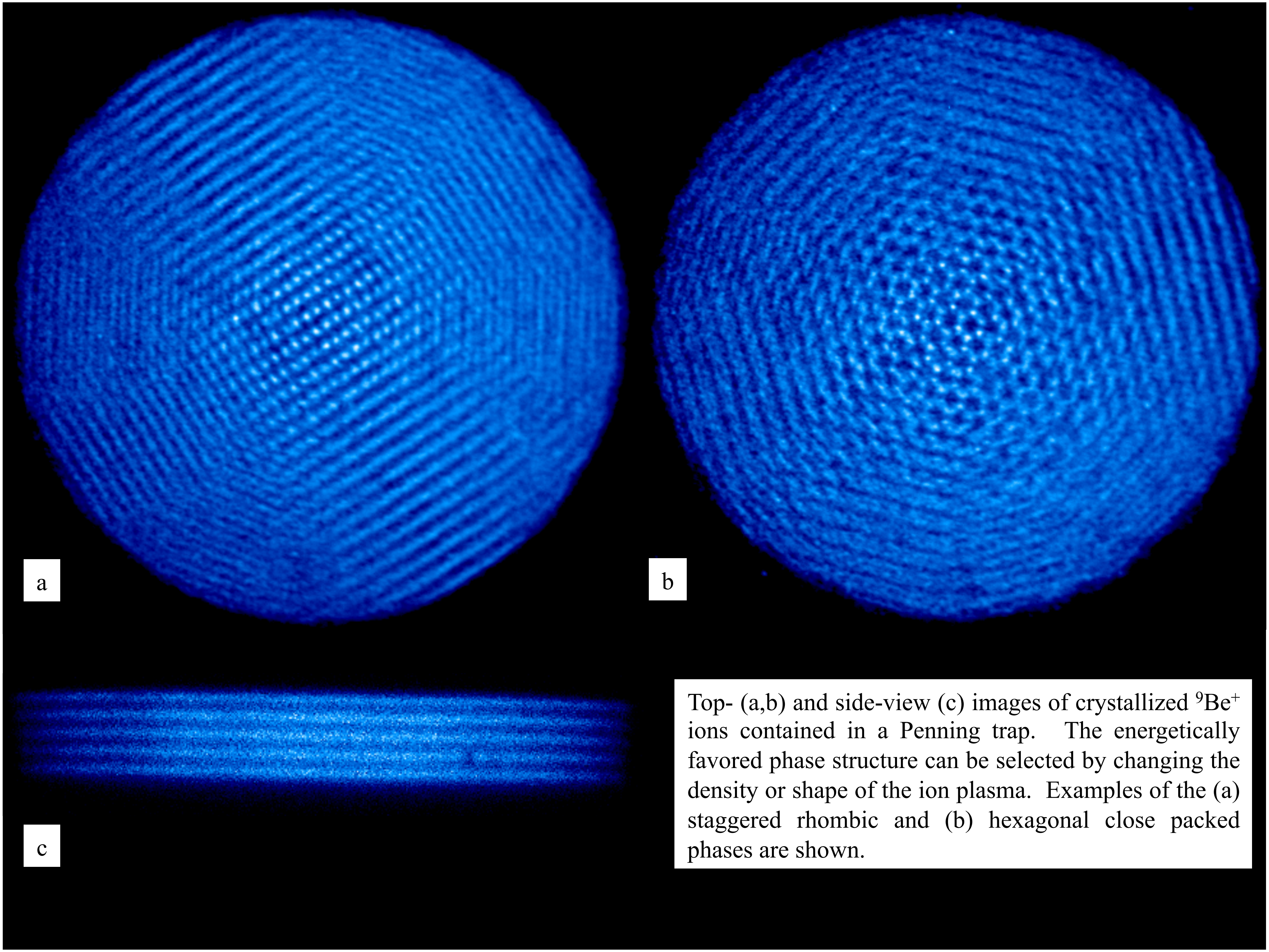
side-views





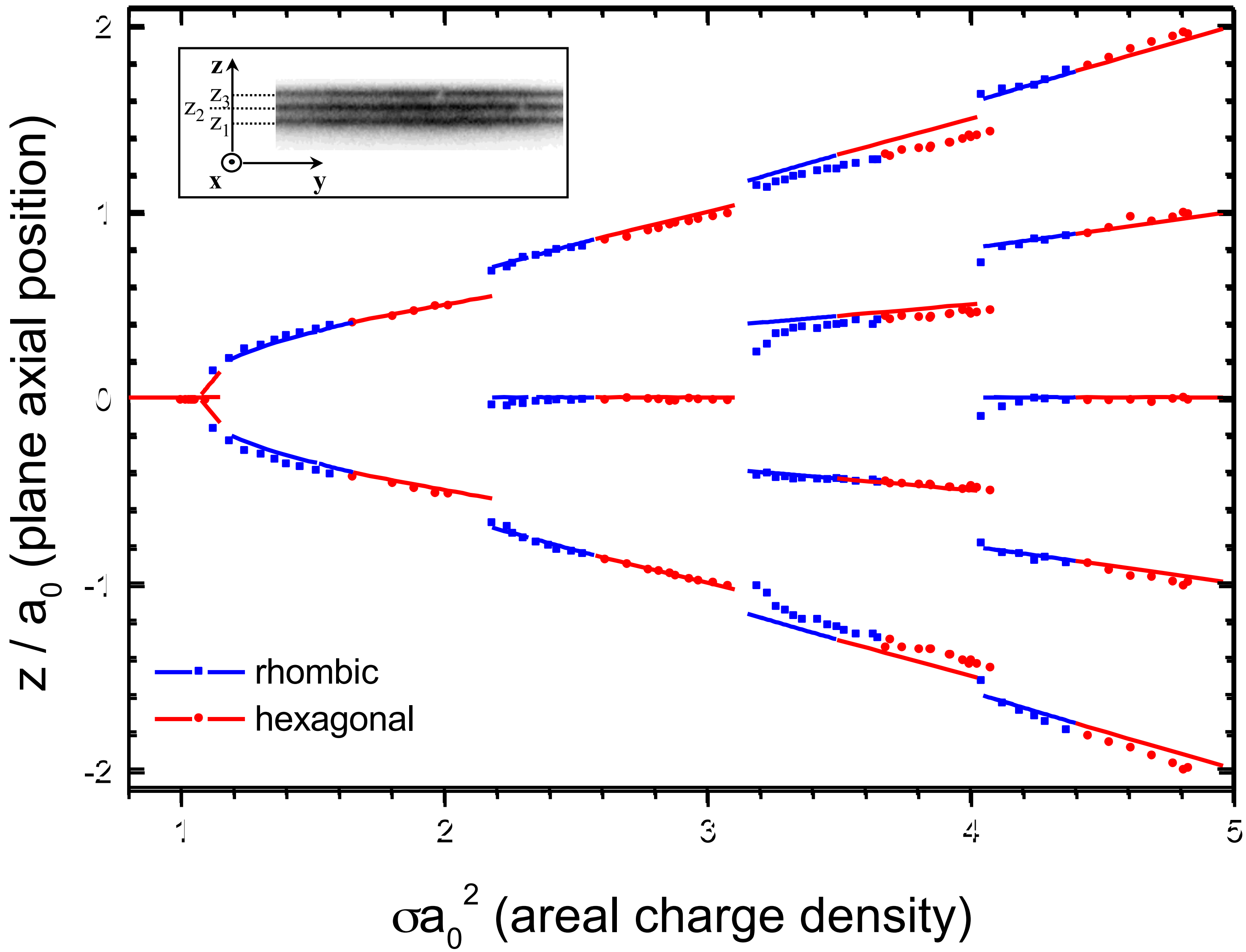
1 lattice plane, hexagonal order

2 planes, cubic order



Top- (a,b) and side-view (c) images of crystallized $^{9}Be^{+}$ ions contained in a Penning trap. The energetically favored phase structure can be selected by changing the density or shape of the ion plasma. Examples of the (a) staggered rhombic and (b) hexagonal close packed phases are shown.

Mitchell, *et al.,* Science **282**, 1290 (98)
Theoretical curve from Dan Dubin, UCSD



increasing rotation frequency →

John Bollinger NIST

Part 1

Effect of correlations on equilibrium properties

Correlations change the structure of confine ion plasma

Part 2

Effect of correlations on a dynamical property

Correlations increase the perpendicular to parallel collision rate

Correlations increase collision rate in non-magnetized plasma

Use Non Neutral Plasma to study properties of "stars"

-- Non-neutral plasmas are low density, low temperature
Nuclear reactions are NOT happening.

-- **But something analog to nuclear reaction IS happening.**

Dubin proved **nuclear fusion reaction in correlated plasma is isomorphic** to enhanced **perp-to-parallel collisions** in a pure ion plasma at low temperatures.

Phys. Plasmas 15, 055705 (2008)

Phys. Rev. Lett. 94, 025002 (2005)

-- The **correlation enhancement is directly analogous to correlation-enhanced fusion** in the Sun with $\Gamma \sim 0.05$, and in white dwarfs with $\Gamma >> 1$.

"Salpeter enhancement"

Perpendicular to Parallel Collisions:

$$\frac{d}{dt}T_{\perp} = \nu_{\perp //} \; (T_{//} - T_{\perp})$$

Distance of closest approach

$b = e^2 / T$

$$\nu_{\perp //} = n \, \bar{v} \, b^2 \; \underbrace{4\sqrt{2}\, I(\bar{\kappa})}_{\text{Magnetization suppression}} \; \underbrace{g(\Gamma)}_{\text{Correlation enhancement}}$$

Adiabaticity parameter

$\bar{\kappa} = \sqrt{2}\, b/r_c$

Correlation parameter

$$\Gamma = \frac{e^2}{a_{WS} T}$$

-- $I(\bar{\kappa})$ *Suppress* $\nu_{\perp //}$ collisions in the "highly magnetized" regime of $r_c < b$. In this regime, only rare, energetic collisions mix $E_{\perp}$ and $E_{//}$.

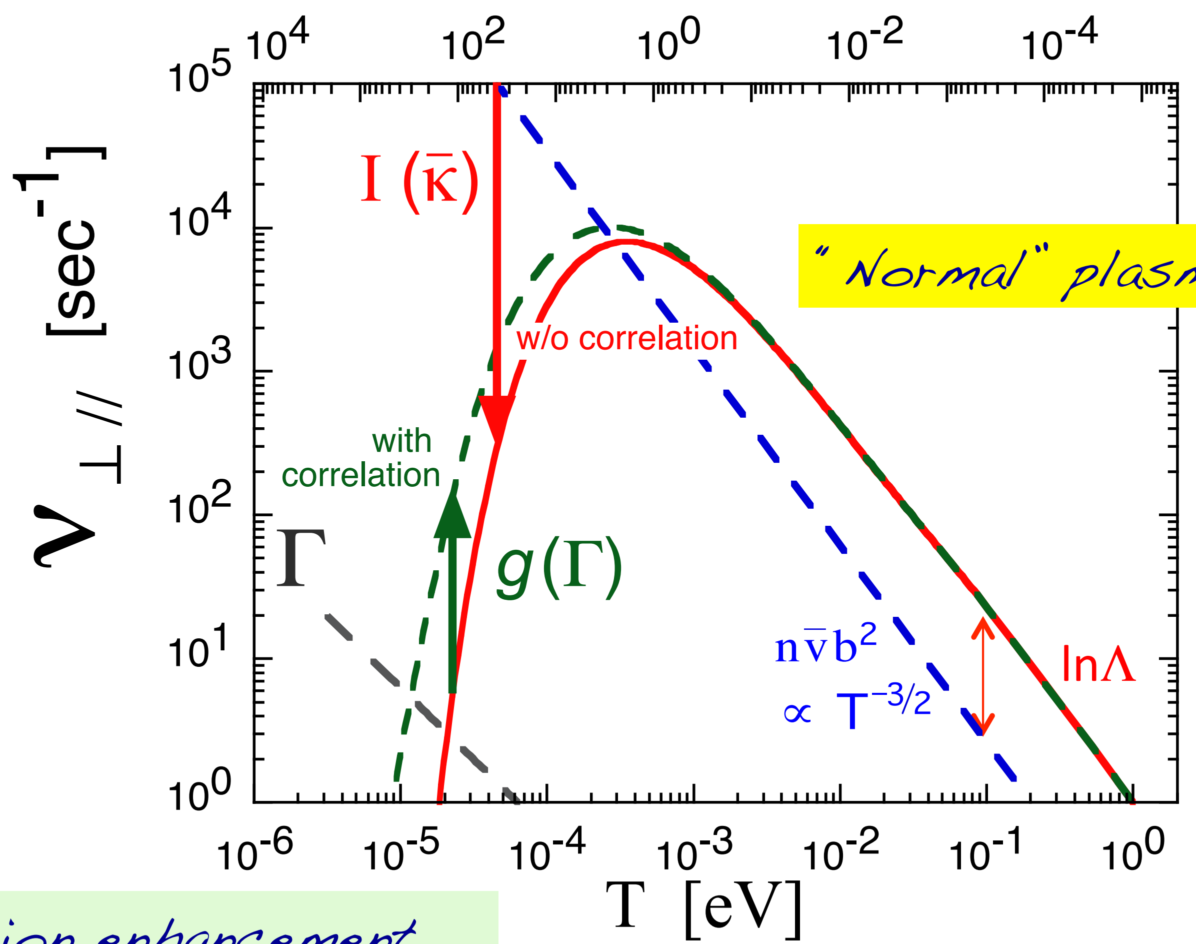
Dubin's lecture (yesterday)

-- $g(\Gamma)$ *Enhances* these rare collisions, due to particle correlations, in the cryogenic liquid regime of $\Gamma = 1 - 10$.

Overview

Magnetization suppression

$\overline{\kappa} = \sqrt{2}\, b/r_c$



"Normal" plasma regime

Correlation enhancement

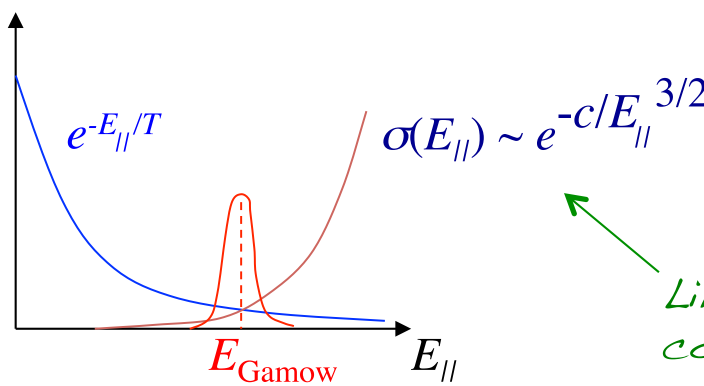
When $r_c < b$: $\nu_{\perp//}$ comes from rare energetic collisions

cross-section for $E_{//}$-$E_{\perp}$ sharing

$$\sigma(E_{//}) \propto e^{-\pi\left(\frac{b}{rc}\right)} \propto e^{-\pi\left(\frac{e^2\ \Omega_c}{T\ v_{//}}\right)} \approx e^{-\pi\left(\frac{C}{E_{//}}\right)^{\frac{3}{2}}}$$

κ *adiabaticity parameter*

$$\nu_{\perp//} = \int dE_{//}\ \frac{1}{T}\ e^{-E_{//}/T}\ \ \sigma(E_{//})$$



Like rare fusion collisions with $\sigma_{fusion}(E_{//})$

$$\nu_{\perp//} = n\bar{v}b^2\ 4\sqrt{2}\ I(\bar{\kappa})\ \ g(\Gamma)$$

$$E_{Gamow} \cong 1.23\ \bar{\kappa}^{\frac{2}{5}}\ T$$

$$I(\bar{\kappa}) \approx C\ \exp\left(-2.044\ \bar{\kappa}^{\ 2/5}\right)$$

Glinsky et al. Phys. Fluids B, 1992

$\bar{\kappa} = \sqrt{2}\ b / r_c$

E_{Gamow} *for* $\bar{\kappa} = 20$ *corresponds to particles at* $4\ \bar{v}$

Collision Enhancement from Shielding (correlation)

Debye shielding (correlation) reduces the energy barrier for close impact distances ρ

No shielding:

$$E_{//} = \frac{e^2}{\rho}$$

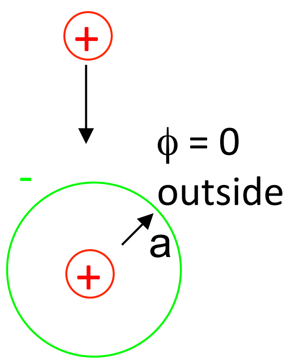
With Debye Shielding:

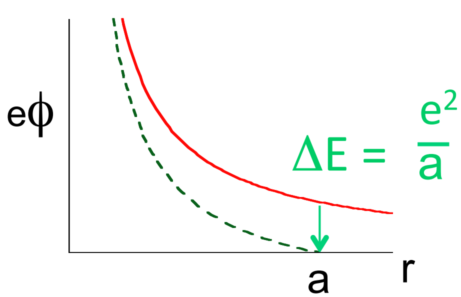
$$E_{//} = \frac{e^2}{\rho}\, e^{-\rho/\lambda_D}$$

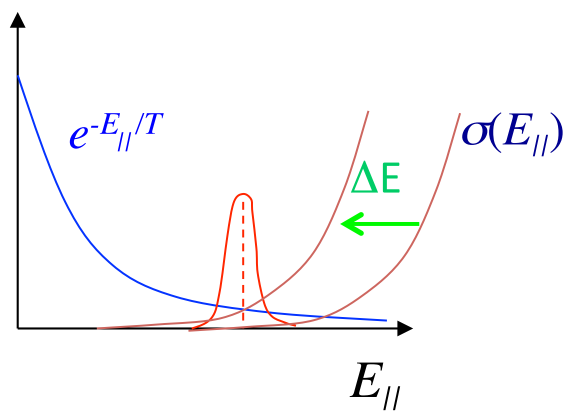
$$= \frac{e^2}{\rho}\left(1 - \frac{\rho}{\lambda_D}\right)$$

$$= \frac{e^2}{\rho} - \underbrace{\frac{e^2}{\lambda_D}}_{\Delta E}$$

$\Gamma > 1$ ($\lambda_D < a$) $\rightarrow \frac{e^2}{\rho} - \frac{e^2}{a}$







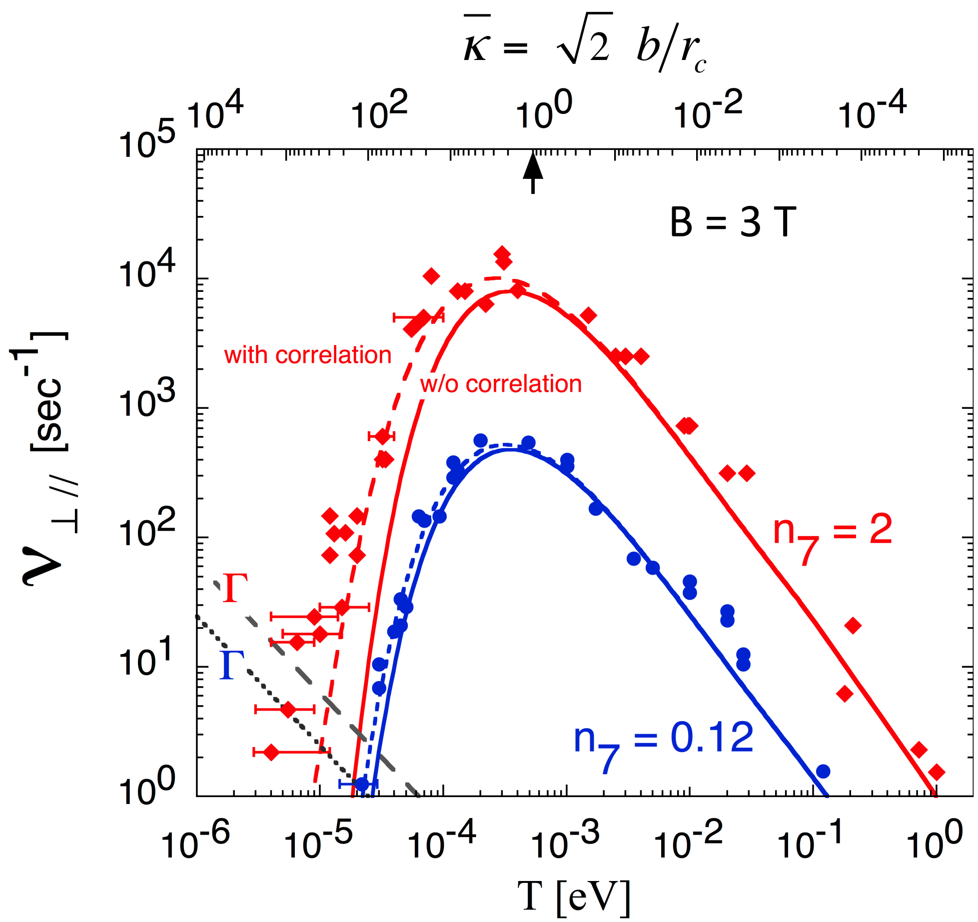
$$\nu_{\perp//} = \int \frac{dE_{//}}{T}\, e^{-(E_{//} - e^2/a)/T}\, \sigma(E_{//})$$

$$\nu_{\perp//} = e^{\frac{e^2}{aT}} \int \frac{dE_{//}}{T}\, e^{-E_{//}/T}\, \sigma(E_{//}) = e^{\Gamma}\, \nu_{\perp//}^{no\ corr}$$

Enhancement $g(\Gamma) = e^{\Gamma}$

Independent of σ(E); same for fusion

Measured Collision Rates at B = 3.Tesla



Low density:

Magnetization suppress $\nu_{\perp //}$

No correlation,

no $\nu_{\perp //}$ enhancement

High density:

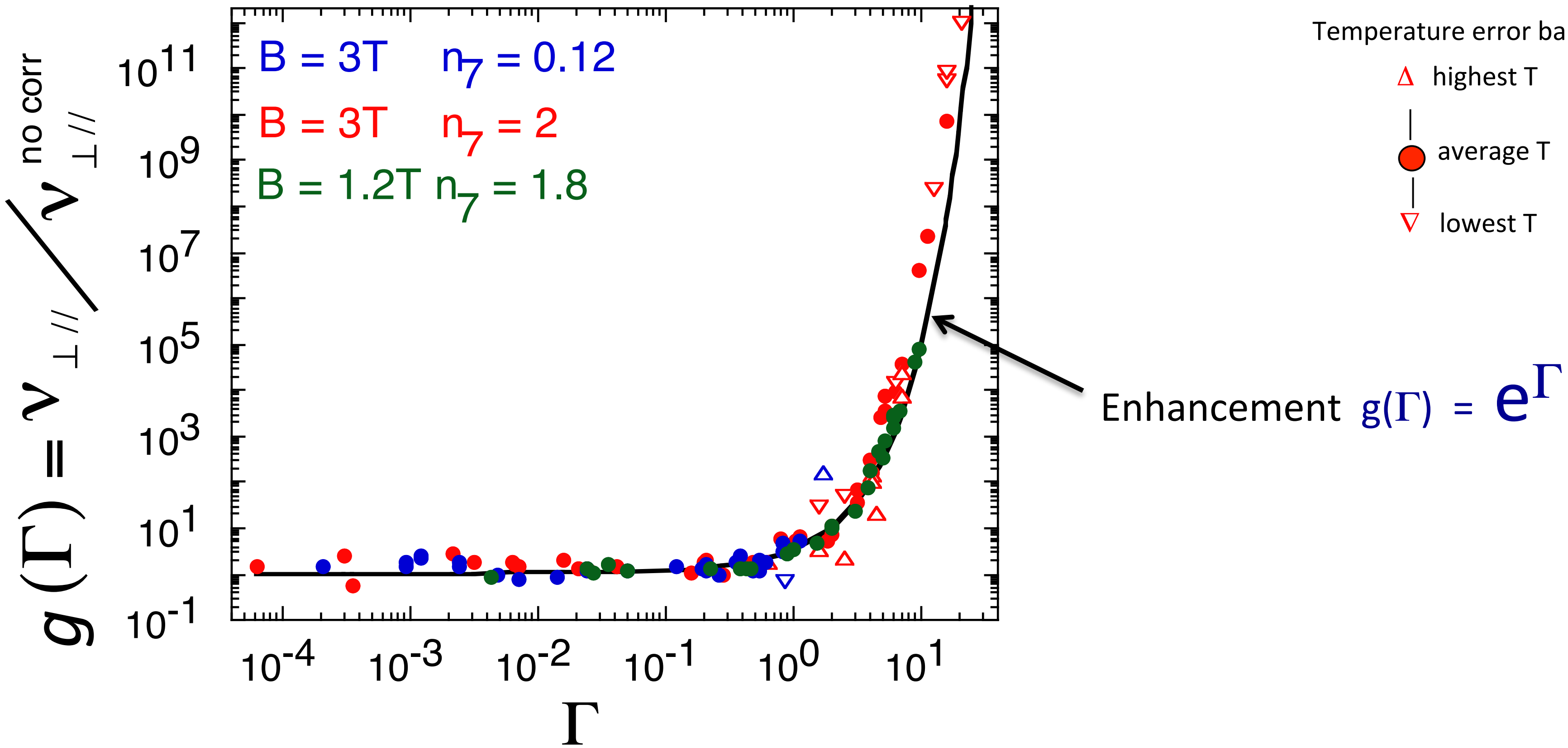
correlated at $T<10^{-4}$,

strong enhancement

Phys. Rev. Lett. ***102****, 185001 (2009)*

Phys. Plasmas 17, 055702 (2010)

Enhancement versus Correlation Γ



Enhancement depends on correlation parameter Γ

But is independent of $\bar{\kappa}$

Phys. Rev. Lett. ***102***, *185001 (2009)*

Phys. Plasmas 17, 055702 (2010)

Summary of perpendicular to parallel colision

- **Perp-to-parallel collisions** are strongly ***suppressed*** in the "**strong magnetization**" regime of $r_c < b$.
  Only rare, energetic collisions cause $E_\perp$ to $E_{//}$ energy exchange.

- These **rare, energetic collisions** are strongly ***enhanced*** in the **correlated** liquid and crystal regimes.

- Enhancements up to 10^9 over uncorrelated theory are observed.

- **Same enhancement** applies to rare **energetic fusion collisions** in hot, dense, **correlated plasmas such as stars**.

Part 1

Effect of correlations on equilibrium properties

Correlations change the structure of confine ion plasma

Part 2

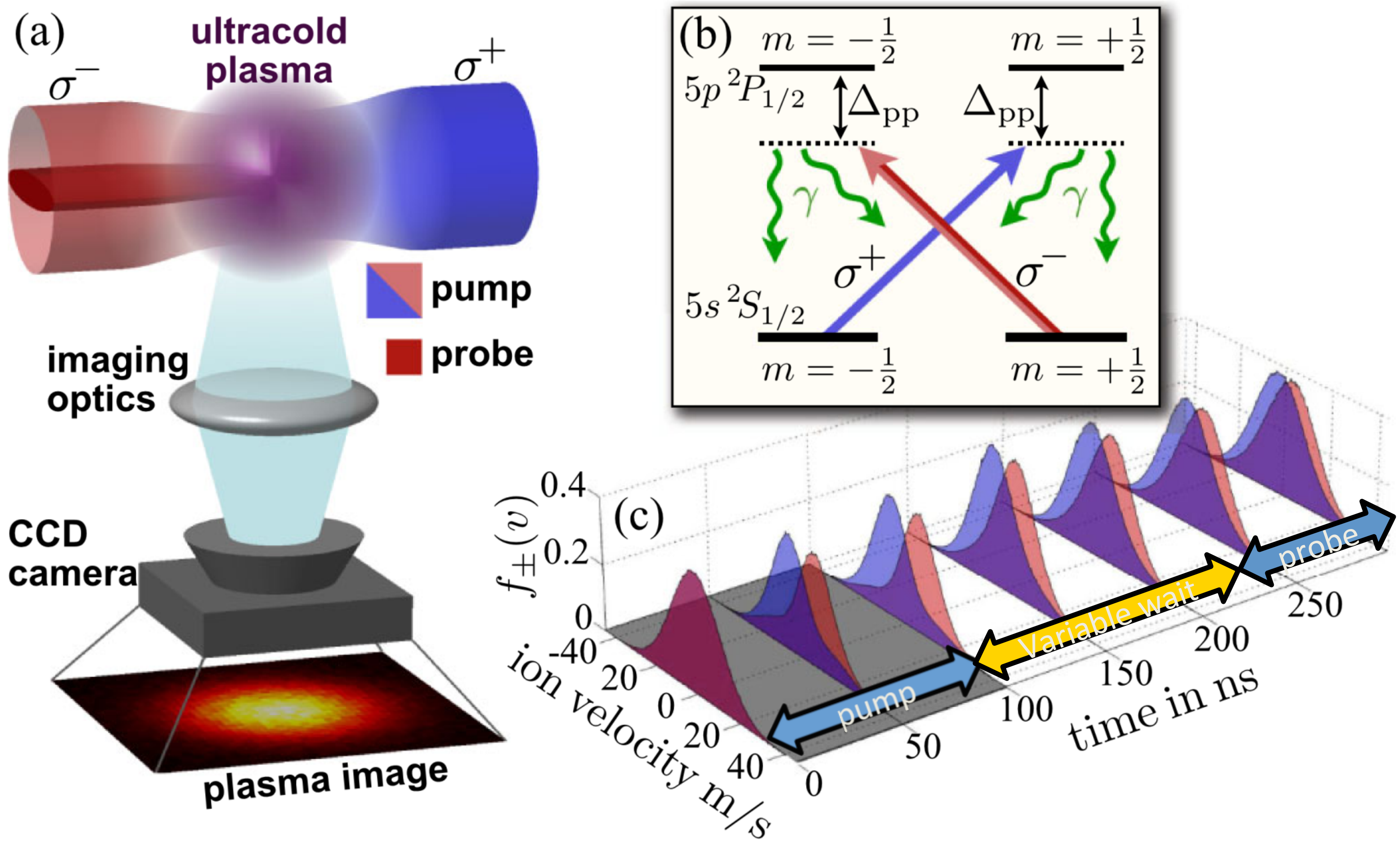
Effect of correlations on a dynamical property

Correlations increase the perpendicular to parallel collision rate

Correlations increase collision rate in non-magnetized plasma

Collisional Relaxation of Tagged Ions in Strongly Coupled Ultracold Neutral Plasmas

Thomas Killian RICE



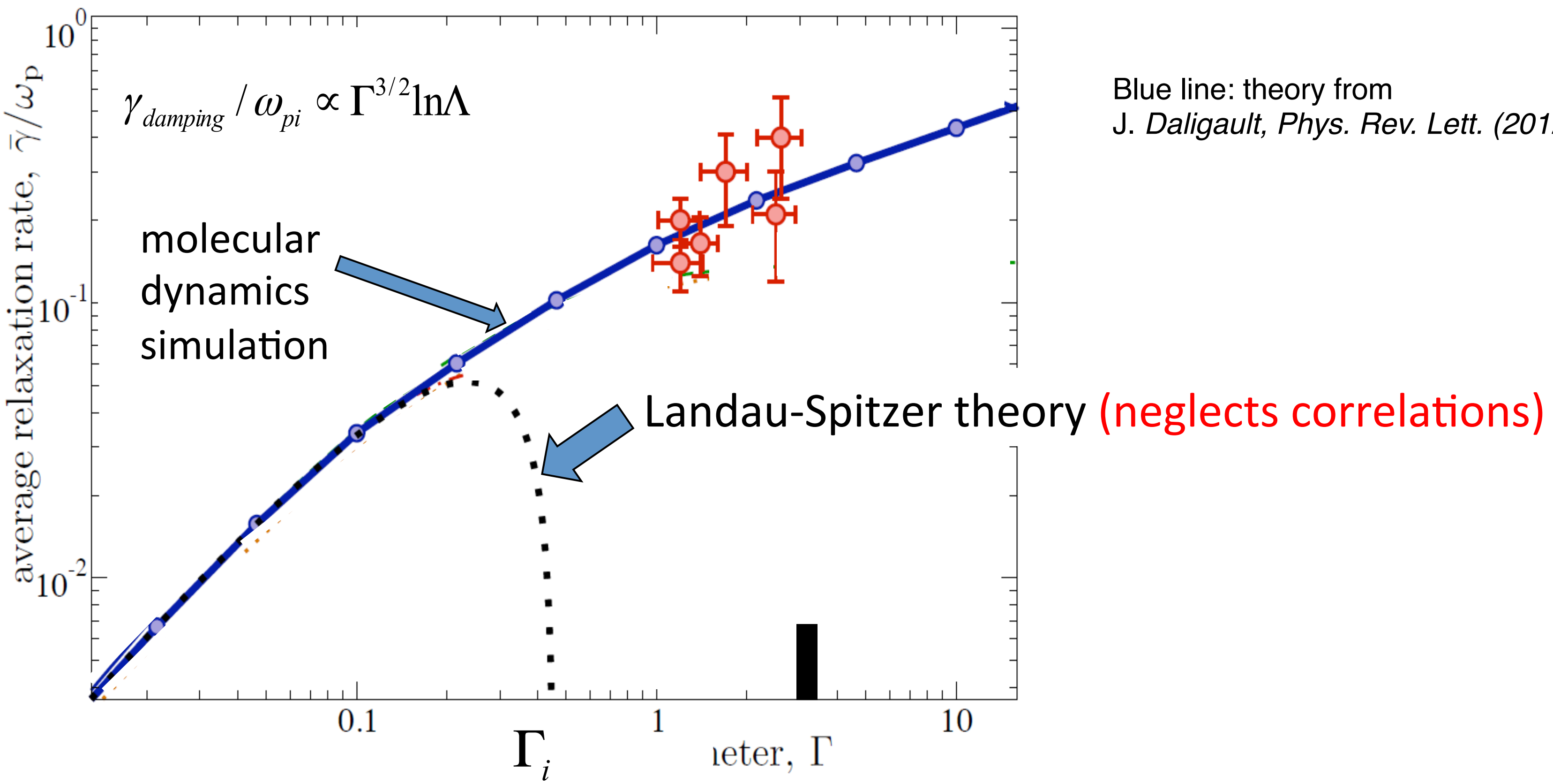
○Optical pumping on $\Delta m = \pm 1$ transitions perturbs ion velocity distribution of each spin state.

○ Perturbed velocity distributions relax towards a Maxwellian through collisions.

○We image only the $m = +1$ spin state

Bannasch et al.,, Phys. Rev. Lett. **109**, 185008 (2012)

Evolution of Average Velocity Gives Collision Rate and Diffusion Constant Beyond Landau-Spitzer



Blue line: theory from
J. *Daligault, Phys. Rev. Lett. (2012)*:

Observe dramatic increase of relaxation rate over the case neglecting correlations

Bannasch et al.,, Phys. Rev. Lett. **109**, 185008 (2012)

Thomas Killian RICE

Summary

- Correlations change the internal structure of plasma:
  - small plasma : Shell structure
  - large plasma $N > 10^5$ Crystals
  - planar plasma crystal (promising for quantum computing)

- Correlations increase perpendicular to parallel collision rate

$$\nu_{\perp//} = \nu_{\perp//}^{no\ \mathrm{cor}} \exp(\Gamma)$$

- Correlations increase the collision rate in non-magnetized plasma

Publications can be found at nnp.UCSD.edu