Imperial College London

Penning Traps

Richard Thompson
Quantum Optics and Laser Science Group
Department of Physics
Imperial College London

Outline of lecture 1

Lecture 1: Penning Trap Basics

1. History of Penning Traps
2. Principles of Operation
3. Advantages and Disadvantages
4. Practical Aspects

Lecture 2: Review of Experiments

Lecture 3: Rotating Frame and Axialisation

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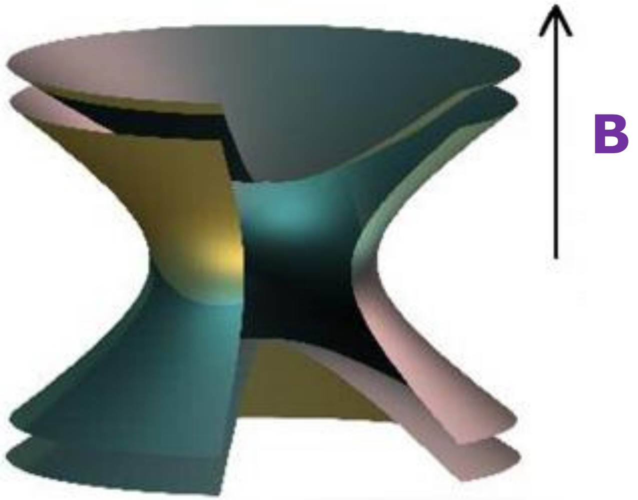
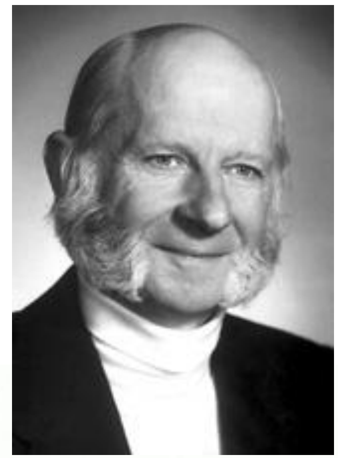
1. History of the Penning trap

- Penning's original experiments
- Development of the conventional Penning trap geometry
- Early experiments with Penning traps
- First demonstration of laser cooling



History of Penning Traps

- The name “Penning trap” was coined by Dehmelt
  - Penning (1936) didn’t actually trap charged particles
  - Electrons confined in discharges with B-field present



2.2. PENNING TRAP

Even though this scheme does not strictly belong to those admissible under our title, we will begin with a discussion of the Penning arrangement which we are using for the containment of electrons as well as cyclotron resonance and thermalization studies of these particles (Dehmelt, 1961). The confinement mechanism is that of the Penning discharge widely used in cold cathode ionization gages and ion-getter pumps. In our embodiment of this principle (Penning, 1936; Pierce, 1949) hyperbolic electrodes (see Fig. 1) create a dc electric potential of the form

$$\phi(x, y, z) = A(x^2 + y^2 - 2z^2), \qquad A = A_0 = \text{const}, \qquad (2.3)$$

Dehmelt, H. G. (1961). Unpublished.

History of Penning Traps

- Pierce (1949) treated the 3-electrode trapping geometry:

Thus, the condition under which electrons cannot get indefinitely far away from the axis is that

$$\omega_c > \sqrt{2}\,\omega_0 \qquad (4{\cdot}33)$$

We see that it is possible to obtain a pure sinusoidal motion of electrons trapped in this combination of electric and magnetic fields.

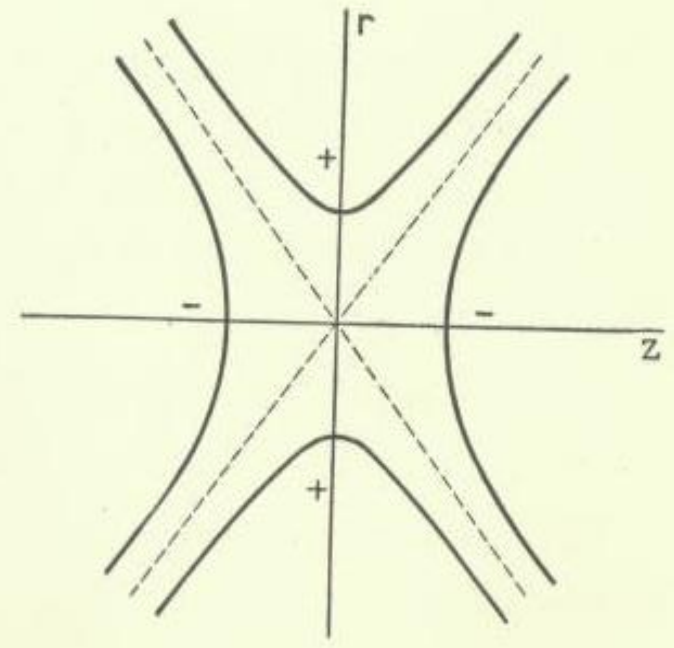


Fig. 4.7—Electron motion between hyperbolic electrodes may be limited to a certain region by use of an axial magnetic field.

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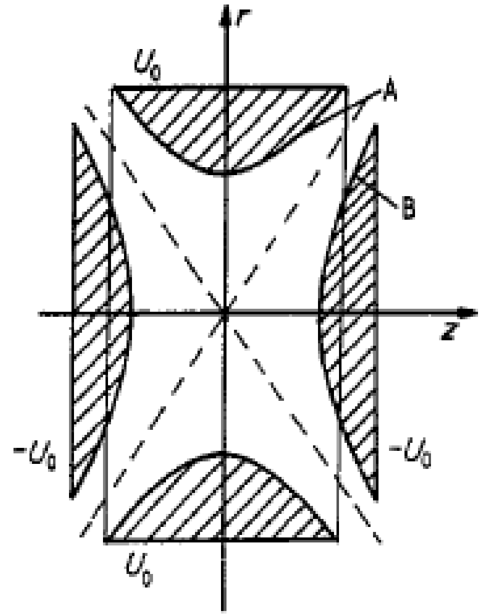
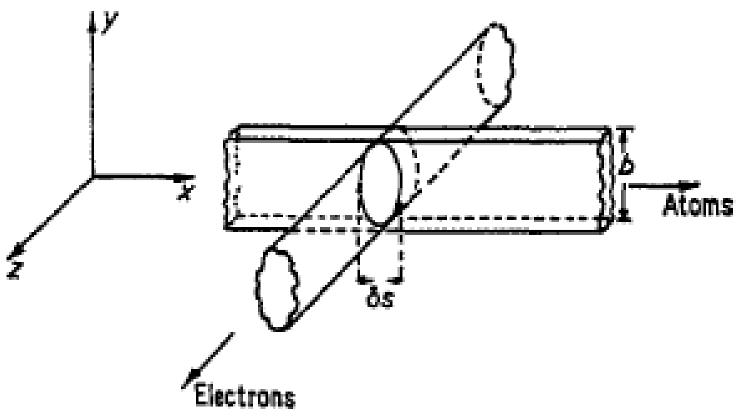
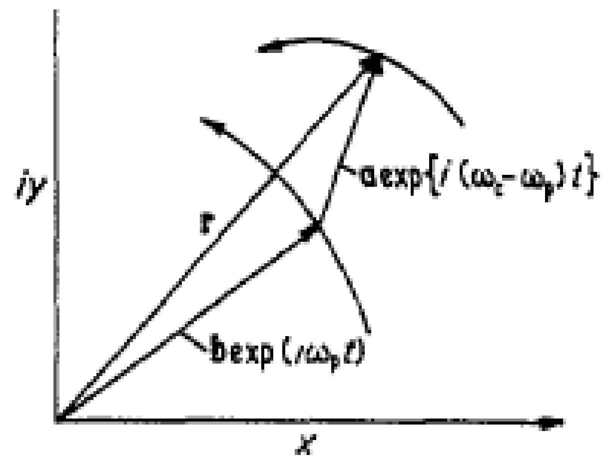
A trap for polarising electrons

- Byrne and Farago (1965) studied the 3-electrode arrangement with B-field for a source of polarised electrons







Early experiments with Penning traps

- Gräff et al (1968) measured cyclotron and spin resonances of trapped electrons
- Gräff et al obtained electron $g-2$ value to 3 10^{-8} in 1969

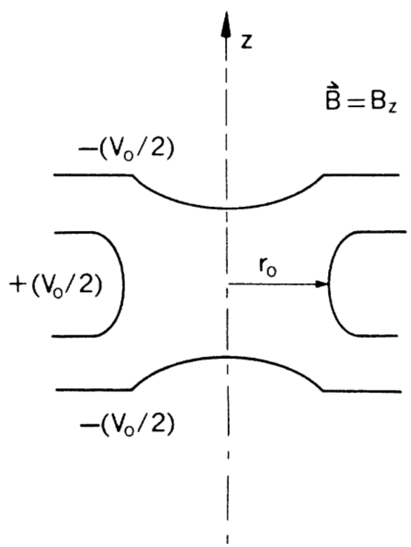


FIG. 1. The quadrupole electron trap used to contain free electrons in ultrahigh vacuum in order to observe their cyclotron- and spin-resonance frequencies.

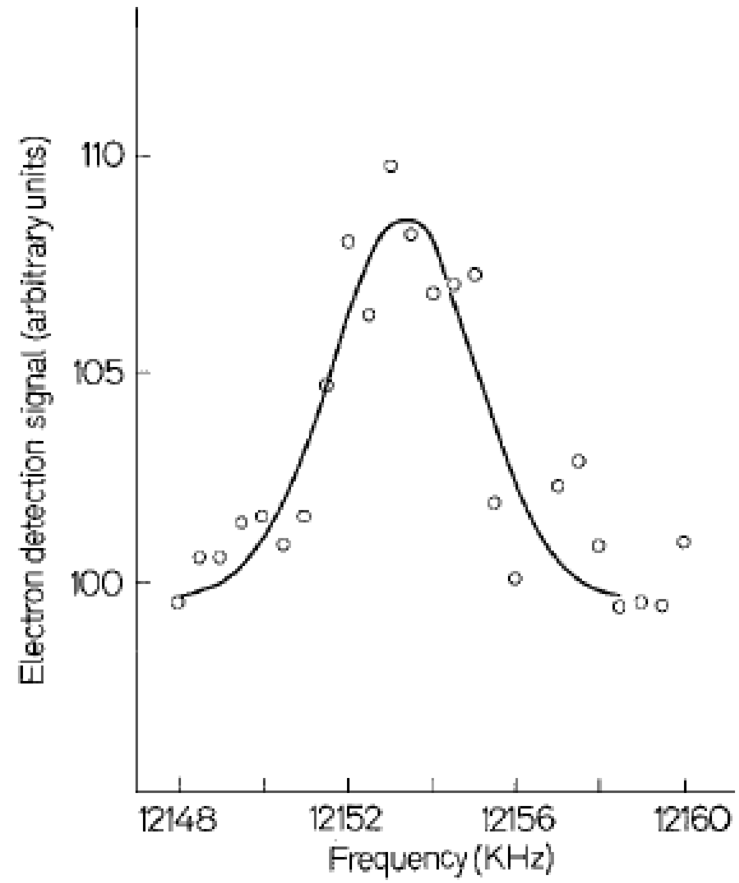
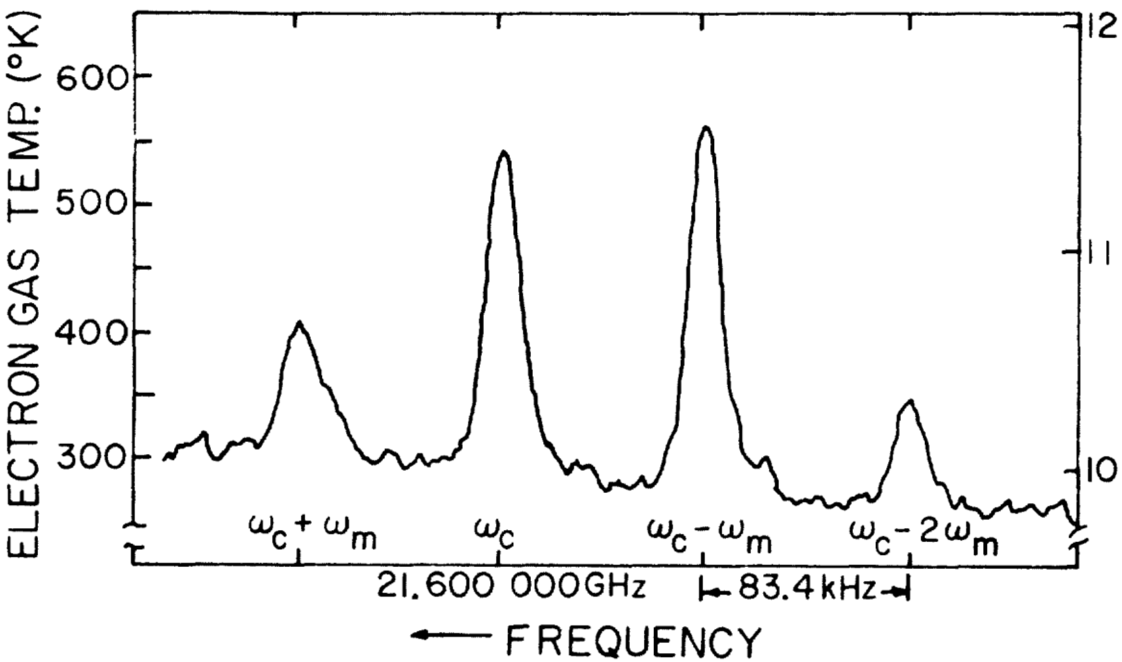


Fig. 2. The $(\omega_L' - \omega_B')$ line

RF spectroscopy in the Penning trap

- Dehmelt and Walls (1968) used currents induced in an external circuit to monitor the temperature of electrons in a Penning trap – the "Bolometric Technique"



Electron temperature as the drive is swept through the cyclotron resonance at 21.6 GHz

Sidebands from magnetron motion at 83 kHz

First laser cooling demonstration

- First laser cooling demonstration (Wineland et al 1978) also used the Bolometric technique

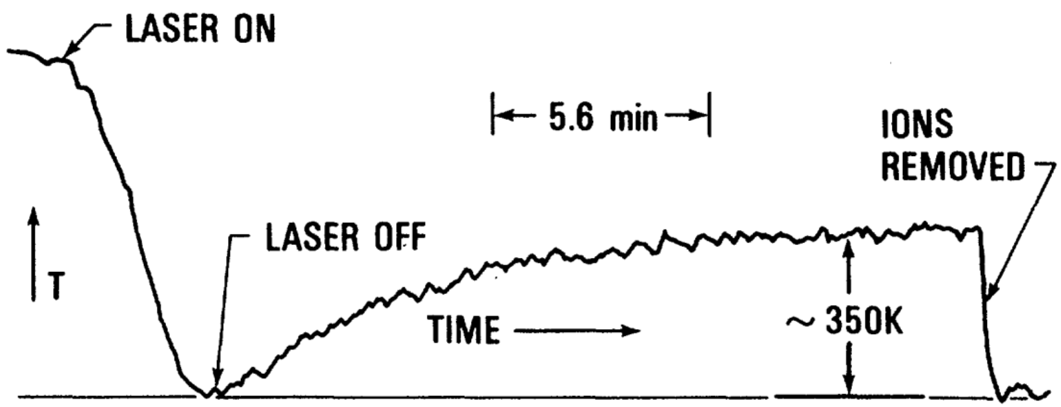
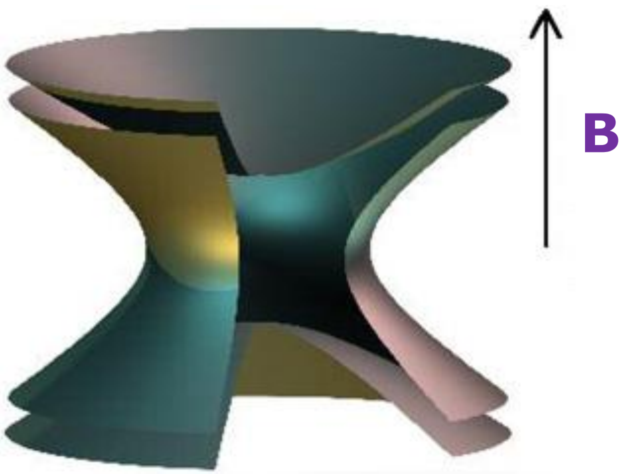
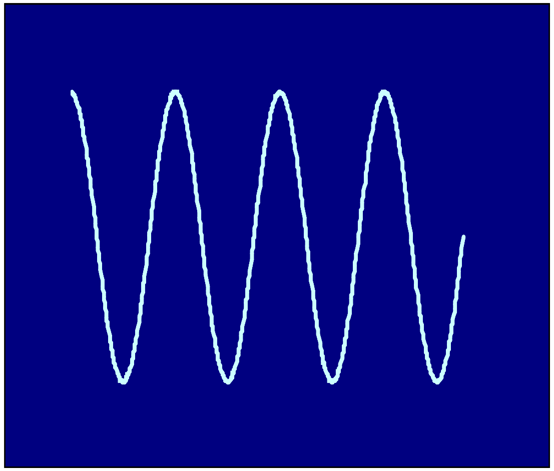


FIG. 2. Ion temperature vs time when laser cooling is applied for fixed $\nu_L - \nu_0$. The ions were initially heated above equilibrium temperature with the laser. Laser cooling was then applied on the $-\frac{1}{2} \leftrightarrow -\frac{3}{2}$ transition for a fixed time until a temperature approaching 0 K (< 40 K) was achieved. After the laser is turned off, the ions rethermalize to the ambient temperature.

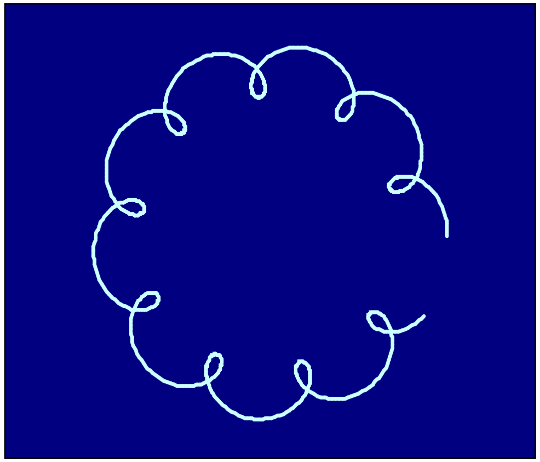
2. Principles of operation of Penning traps

- Electrode configuration
- Equations of motion
- Modes of oscillation
- Potential and kinetic energy
- Space charge effects





Axial motion



Radial motion

Earnshaw's theorem

- In order to trap a charged particle we would need a quadrupole potential with a three-dimensional minimum:

$$f(x,y,z) = Ax^2 + By^2 + Cz^2$$

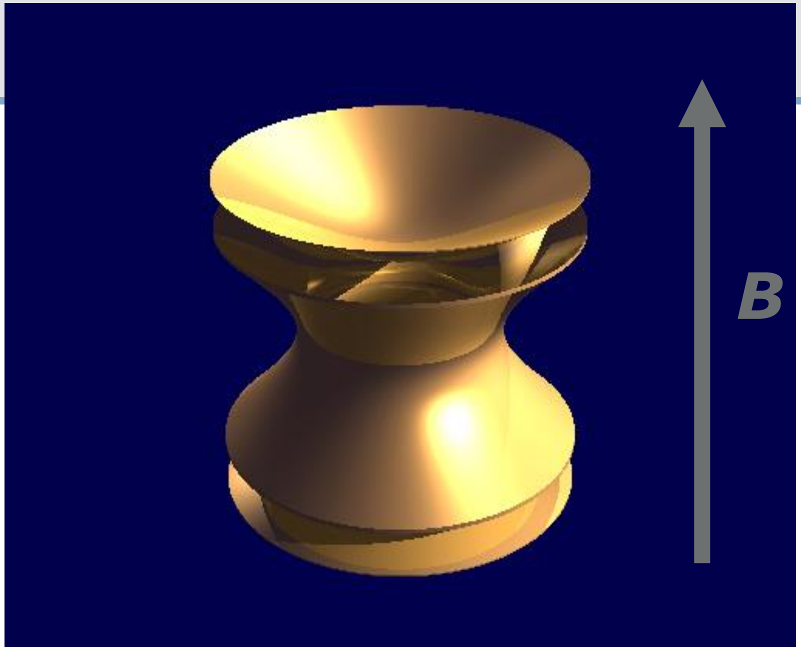
with *A*, *B* and *C* all positive

- However, Laplace's equation requires that

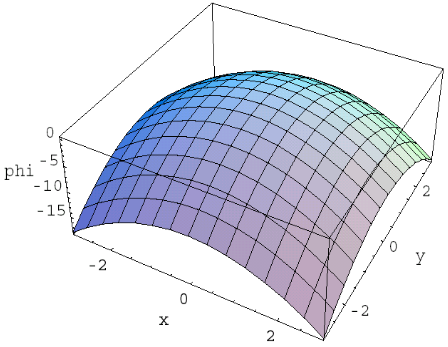
$$A + B + C = 0$$

- So three-dimensional trapping is not possible with static fields
- Two ways around this:
  - Add a magnetic field – the *Penning trap*
    - » these lectures
  - Use oscillating fields – the *Radiofrequency (Paul) trap*
    - » Martina Knoop's lectures

Principles of operation



- Three electrodes
  - Hyperboloids of revolution
  - Generate pure quadrupole potential
- DC potential applied between endcaps and ring
  - Traps ions in the axial direction
  - Radial motion unstable
- Large **B** field applied along the axis
  - provides radial confinement (ions forced into magnetron loops)
- Requires ultra-high vacuum
  - Avoids collisions leading to ion loss due to instability of radial motion



Radial potential

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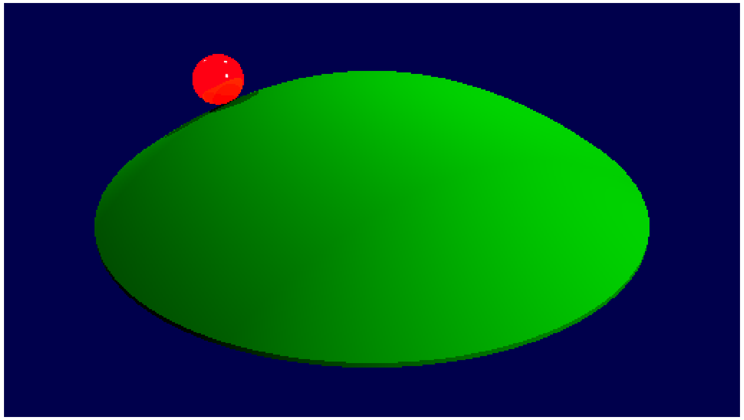
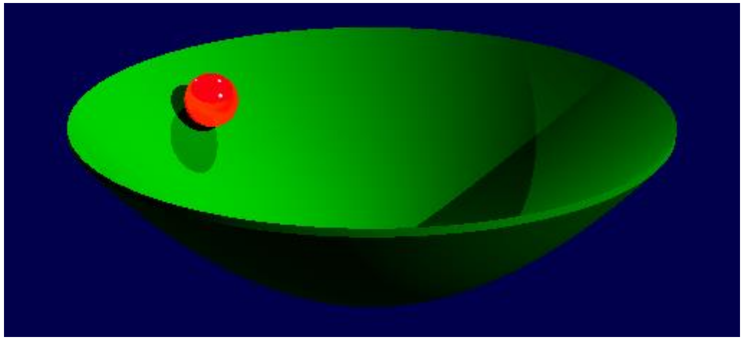
Radial motion in the Penning trap

- The axial motion is stable, as it is just simple harmonic motion in a quadratic potential
- In the plane of the ring the electric field pushes the ion out towards the ring
- The magnetic field stops it falling out by making it orbit round the centre of the trap in a new *effective* potential





Equations of motion

The equations of motion for a single particle are

$$\ddot{x} + \omega_c \dot{y} - (\omega_z^2 / 2)x = 0$$

$$\ddot{y} - \omega_c \dot{x} - (\omega_z^2 / 2)y = 0$$

$$\ddot{z} + \omega_z^2 z = 0$$

where $\omega_c = eB/m$ and $\omega_z^2 = 4eV/mR^2$ (R is related to the trap dimensions).

For the radial motion it is convenient to write this in complex form:

$$\ddot{u} - i\omega_c \dot{u} - (\omega_z^2 / 2)u = 0$$

Motion in the Penning trap

We end up with three types of motion:

- Axial motion
  - » oscillation between the endcaps (~200 kHz)

$$\omega_z = 4e(-V)/mR^2$$

- Modified cyclotron motion
  - » orbit around a magnetic field line (500 kHz)

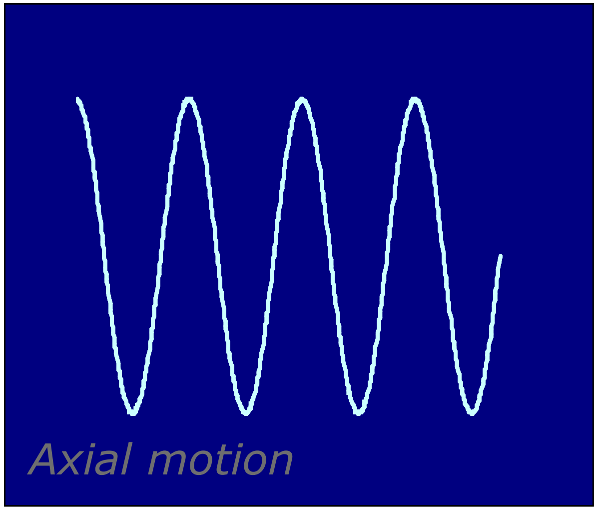
$$\omega_c' = \omega_c/2 + \sqrt{(\omega_c/2)^2 - \omega_z^2/2}$$

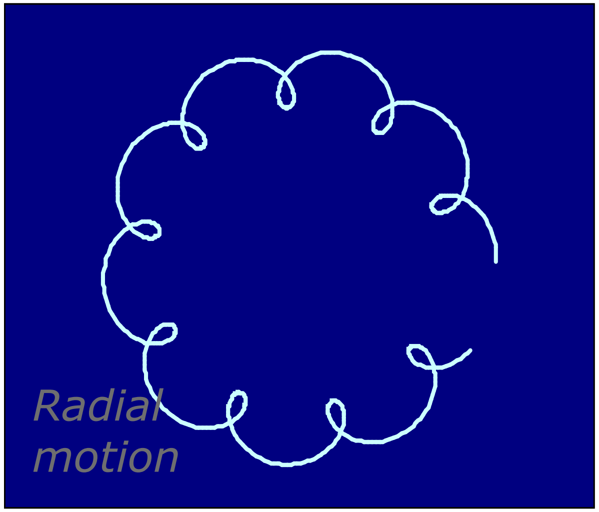
- Magnetron motion (**E**^B drift)
  - » slow orbit around trap centre (50 kHz)

$$w_m = w_c/2 - \sqrt{(w_c/2)^2 - w_z^2/2}$$

  - » This motion is unstable (negative total energy)

These values are for an atomic ion in a field of ~1 T





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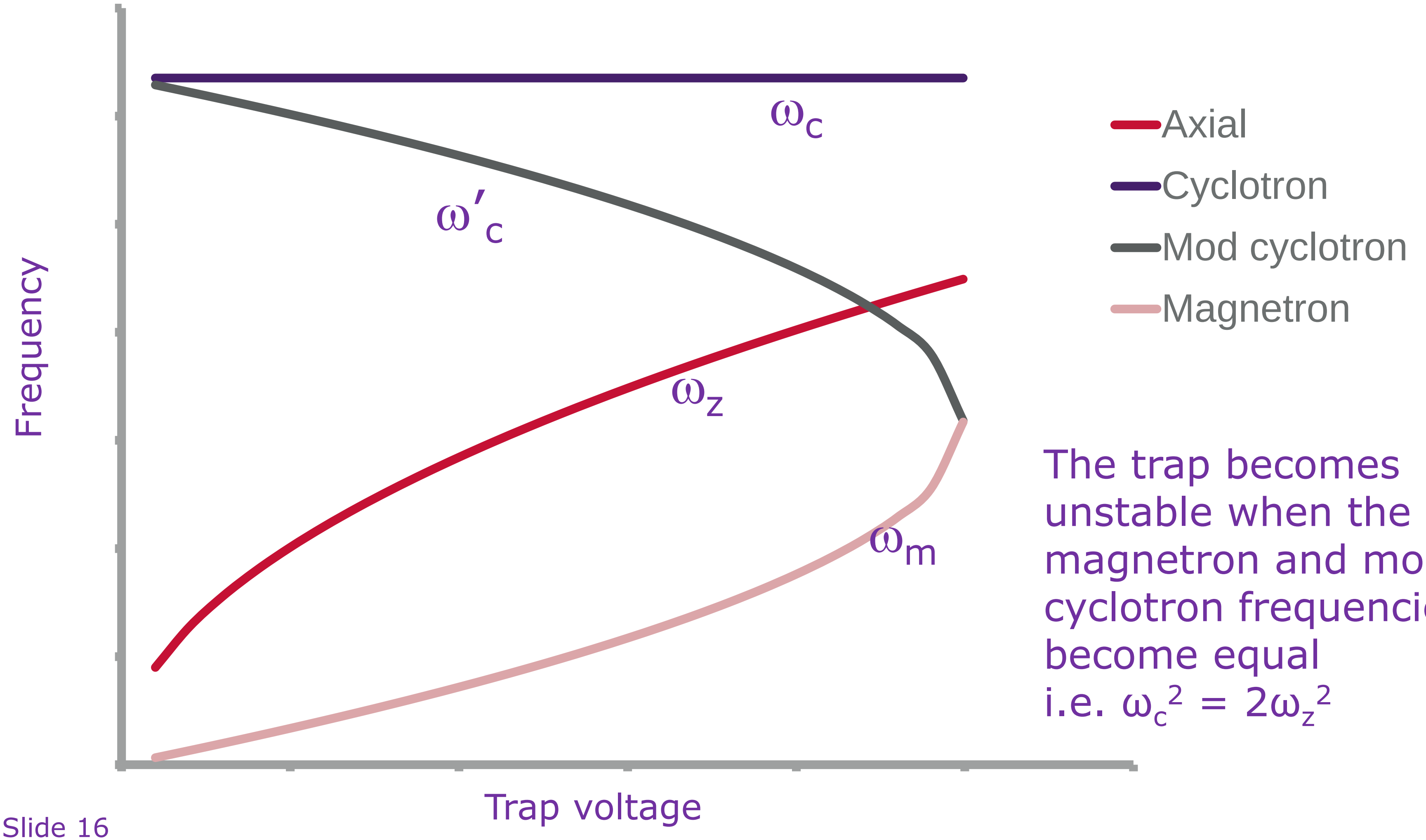
Oscillation frequencies



The trap becomes unstable when the magnetron and modified cyclotron frequencies become equal
i.e. $\omega_c^2 = 2\omega_z^2$

Invariance theorem

- From the expressions for the three oscillation frequencies, we find:

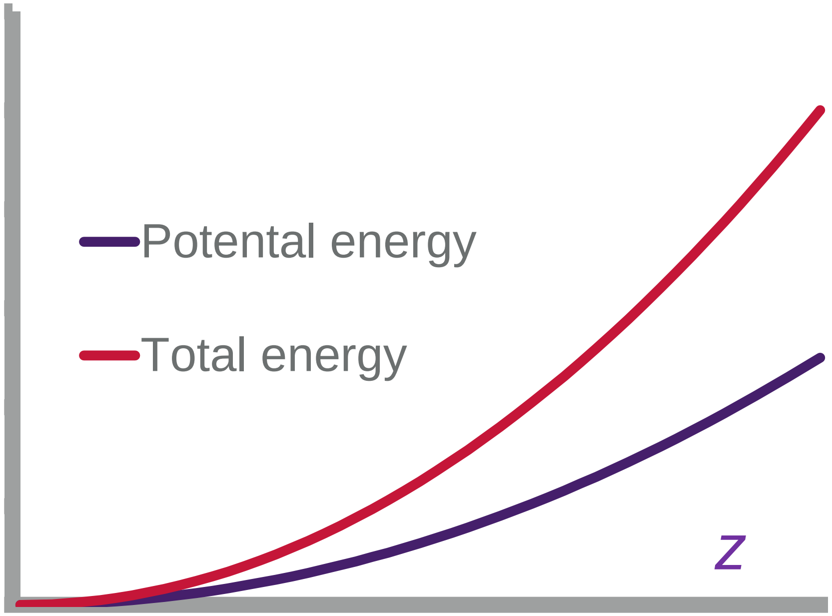
$$\omega_c = \omega'_c + \omega_m$$

- and

$$\omega_c^2 = \omega'^2_c + \omega_z^2 + \omega_m^2$$

- The last equation (named after Gabrielse) is still true for any small angle misalignment of the trap or quadratic imperfections of the field
  - It is therefore very important for experiments where precision measurements of the cyclotron frequency are necessary
  - This allows extremely precise measurements of the true cyclotron frequency to be made

Potential and kinetic energy

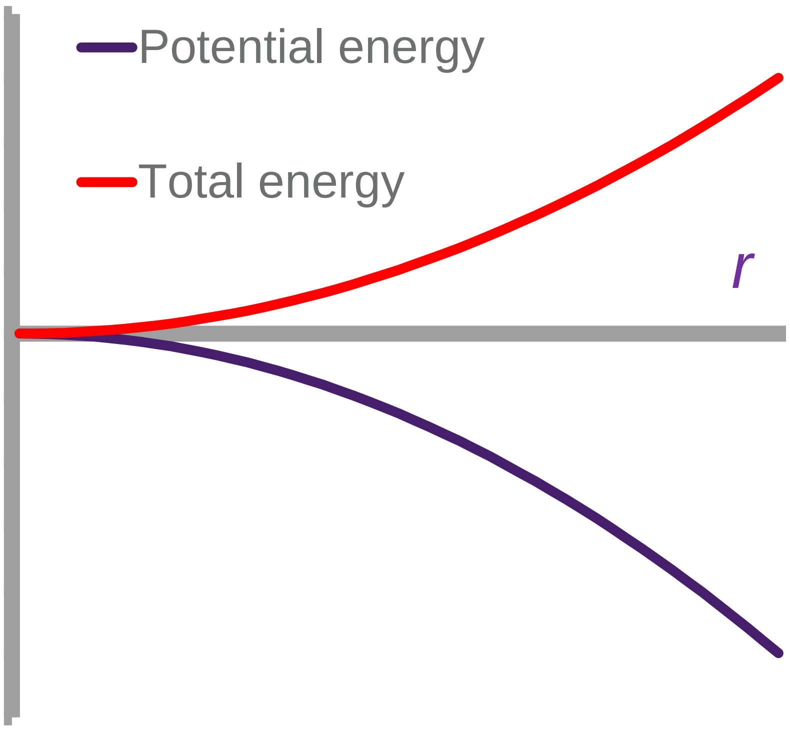


- For the **axial motion** the potential energy and the kinetic energy are equal

- $E = (1/2)\,[mz^2\omega_z^2 + mv_z^2]$
  $= (1/2)\,mz_0^2\omega_z^2$
  (z_0 is amplitude of motion)

- The usual principle of equipartition of energy applies

Potential and kinetic energy

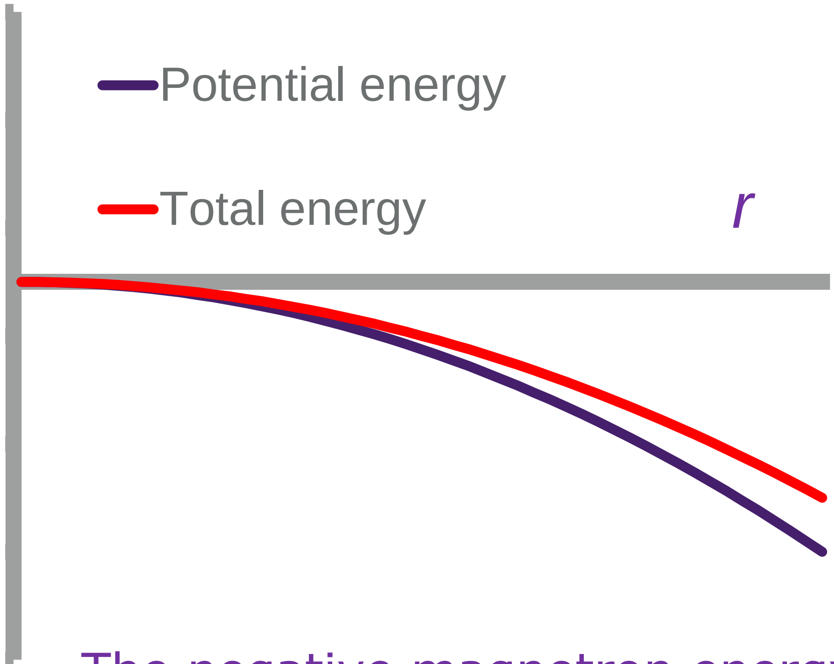


- For the **cyclotron motion** the potential energy and the kinetic energy are not equal

- $E = -(1/4)\, m r_c^2 \omega_z^2 + (1/2)\, m v_c^2$
  $= (1/2)\, m r_c^2 \omega'_c(\omega'_c - \omega_m)$

- Equipartition of energy does **not** apply here

Potential and kinetic energy



The negative magnetron energy has several consequences for the applications of the Penning trap

- For the **magnetron motion** the potential energy and the kinetic energy are not equal
- $E = -(1/4)\, m r_m^{\,2}\omega_z^{\,2} + (1/2)\, m v_m^{\,2}$
  $= -(1/2)\, m r_m^{\,2}\omega'_m(\omega'_c - \omega_m)$
- Equipartition of energy does **not** apply here and the total energy is negative

Interactions between ions

If two ions are in the trap, the equations of motion are modified:

$$\ddot{x}_1 + \omega_c \dot{y}_1 - (\omega_z^2/2)x_1 + (x_1 - x_2)e^2/4\pi\varepsilon_0 m r^3 = 0$$

$$\ddot{y}_1 - \omega_c \dot{x}_1 - (\omega_z^2/2)y_1 + (y_1 - y_2)e^2/4\pi\varepsilon_0 m r^3 = 0$$

$$\ddot{z}_1 + \omega_z^2 z_1 + (z_1 - z_2)e^2/4\pi\varepsilon_0 m r^3 = 0$$

$$r^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2$$

With a similar equation for particle 2.

These coupled equations can be separated:

$$\ddot{X} + \omega_c \ddot{Y} - (\omega_z^2/2)X = 0.....$$

$$\ddot{x} + \omega_c \dot{y} - (\omega_z^2/2)x + e^2 x/4\pi\varepsilon_0 m r^3 = 0......$$

Centre of mass motion

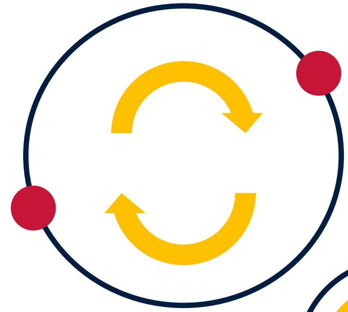
Unchanged oscillation freqs.

Relative motion

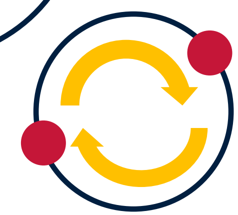
Modified oscillation freqs.

Motion of 2 ions

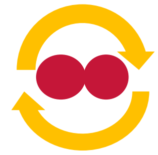
- The relative motion in the axial direction has an increased frequency.
  - If the ions are both on the *z*-axis their minimum energy is when $z^3 = e^2/16\pi\varepsilon_0 m\omega_z^2$. The relative oscillation frequency is then $\sqrt{3}\,\omega_z$
- If the ions are separated radially, they rotate about the z-axis at ω_R:
  - Large separation: $\omega_R \sim \omega_m$



  - Medium separation: $\omega_R > \omega_m$



  - Minimum separation: $\omega_R = \omega_c/2$



  - $r^3 = e^2/16\pi\varepsilon_0 m(\omega_c^2/4 - \omega_z^2/2)$

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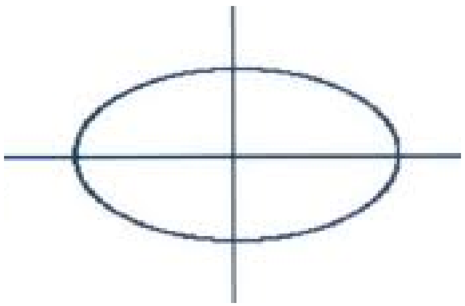
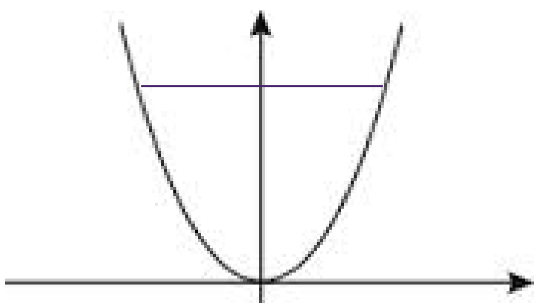
Space charge

- When large numbers of ions are present, the cloud of ions behaves like a plasma
  - Referred to as a *non-neutral plasma* or *single-component plasma*
- The main properties of this plasma are:
  - It has a uniform density n
  - It has sharp edges (at low temperature)
  - It has a spheroidal shape
- We can think of the plasma as "filling up" the quadratic potential well
- The whole plasma moves at the centre of mass oscillation frequencies





See other lectures in the school

Brillouin flow

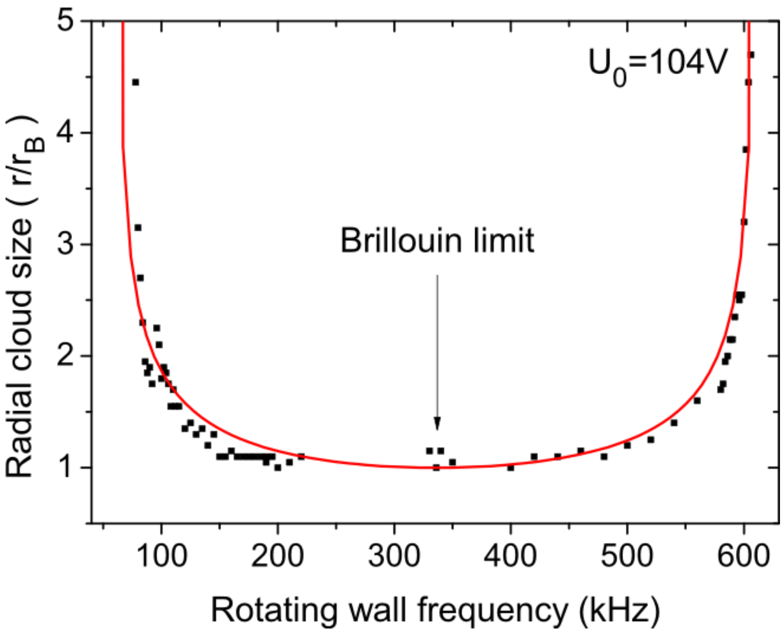
- When cold, the plasma rotates like a solid body at a frequency ω_R and this is linked to the number density of the plasma *n*:

$$n = 2\varepsilon_0 m \omega_R(\omega_c - \omega_R)/e^2$$

- This means that the maximum density is achieved when $\omega_R = \omega_c/2$ and then

$$n = \varepsilon_0 m \omega_c^2 / 2e^2 = \varepsilon_0 B^2 / 2m$$

- This is called *Brillouin flow.*



At Brillouin flow, the total relativistic energy density of the particles is equal to the energy density of the magnetic field - WHY?

3. Advantages and disadvantages

Penning traps offer a very unusual environment for experiments.

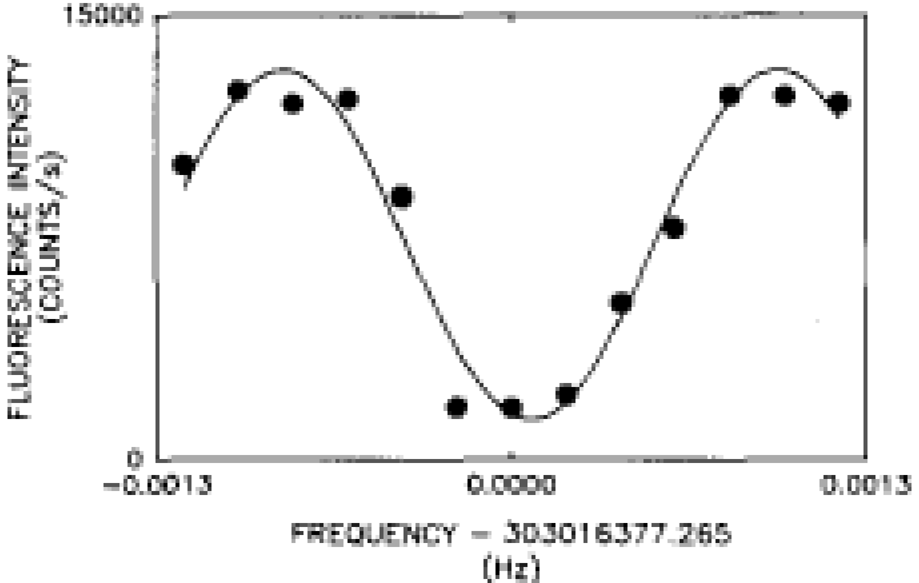
They allow some experiments to be performed that would be difficult or impossible in other systems.

What are the particular advantages and disadvantages that they offer?

Advantages of Penning traps (1)

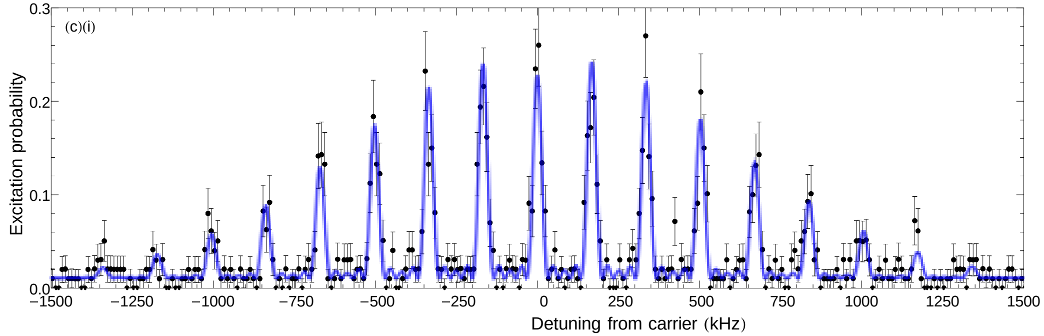
- An isolated and stable environment
  - Only static electromagnetic fields are used
  - The particles do not interact with the container walls or residual gas
  - Free of unwanted perturbations
- Tight localisation
  - Particles remain at the centre of the trap and cannot escape
  - Long interaction times

This Ramsey spectrum was taken with a 550 s interaction time giving a central fringe width of 1mHz (NIST)



Advantages of Penning traps (2)

- Magnetic field
  - Useful for many experiments (e.g. mass measurement)
  - Also good for guiding particles into the trap
- Cooling
  - There are lots of different cooling techniques available
  - See lectures by *Caroline Champenois* and *Laurent Hilico*
  - Eliminates Doppler effect: ultra-high resolution spectroscopy in optical domain



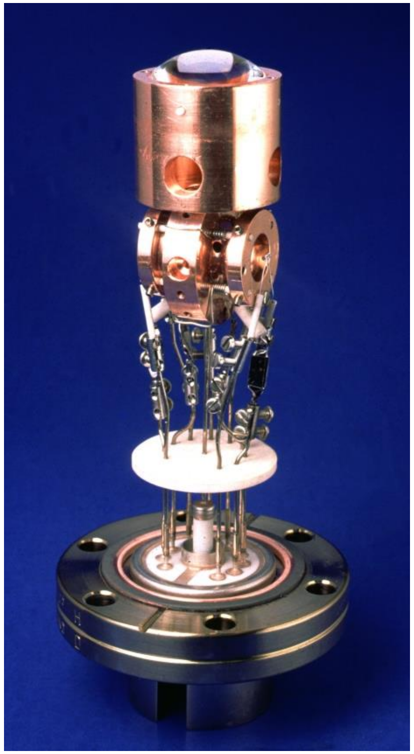
Doppler width after Doppler cooling ~1MHz

Disadvantages

- Magnetic field is expensive to provide
- Access to the ions is difficult
  - Superconducting magnet limits access severely
  - Trap electrodes make optical access difficult
  - All holes in electrodes distort the quadratic potential
- Vacuum system needs to be good for UHV ($<10^{-9}$mbar)
- Magnetron motion is unstable
  - The energy associated with this motion is negative
  - Collisions and perturbations all tend to make the radius increase
  - Hard to cool efficiently
  - Leads to a rotation of the whole plasma

4. Practical Aspects

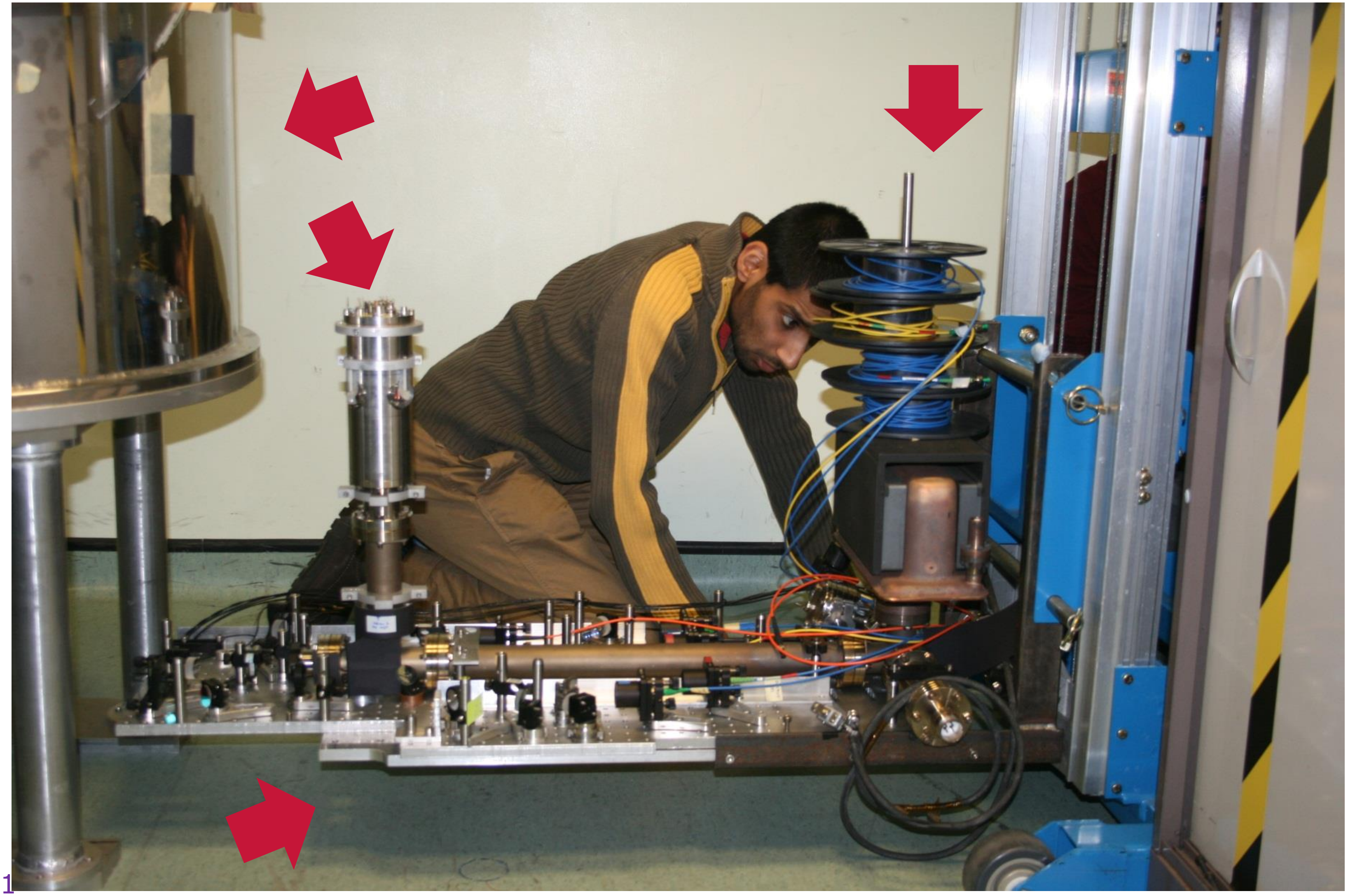
- Trap environment
- Loading ions in the trap
- Detection of ions
- Cooling of ions



Trap environment

- Magnetic field
  - We need a field of 1-10 T – generally requires a superconducting magnet
- Cryogenic environment
  - Some experiments require the whole trap to be at a few K
- Ultra-High Vacuum
  - Collisions will lead to heating up or loss of ions, so the trap has to be held under UHV conditions ($<10^{-9}$ mbar)
  - This also means that the trap components must be very clean to prevent degassing and contamination of the vacuum
  - Either create ions inside the trap or transport them in from outside

Trap practicalities



Loading of ions into the trap

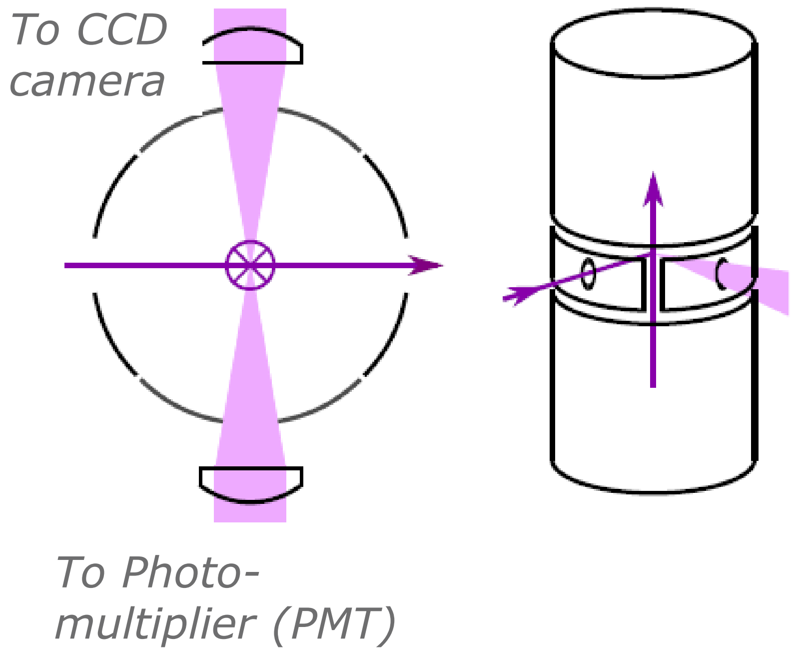
Ions can be loaded into the trap in two ways:

- Create the ions inside the trap:
  - Electron beam impact on a neutral vapour
  - Ablation from a surface using a pulsed laser
  - Photoionisation of neutral atoms
  - Ions are trapped instantly when they are created
- Transport the ions into the trap from elsewhere
  - Requires an external ion source (e.g. accelerator)
  - Trap potential needs to be changed rapidly to trap them

Detection of ions

Three main ways to detect ions in a trap:

- Detect the light that they emit when excited by a laser
  - Sensitive and non-invasive
  - But needs atom with suitable transition
- Detect induced currents in external circuit
  - Sensitive at cryogenic temperatures
- Detect particles by ejecting them from the trap to a detector (e.g. MCP)
  - Destructive but works for all particles



See lectures by Joel Fajans

Cooling

Several ways to reduce the energy of trapped particles:

- Buffer gas cooling
  - Allow the ions to interact with an inert buffer gas at room temp.
  - Works for cyclotron and axial but not magnetron motion
- Radiative cooling
  - For electrons/positrons at high energy for the cyclotron motion
- Resistive cooling
  - Extracts energy from the ions when they induce currents in an external circuit (mainly axial and cyclotron)
  - can cool to a few K if apparatus is at liquid helium temperature
- Laser cooling
  - Very effective if suitable transition available
  - Can cool to ~1mK or lower

See other lectures at the school

- Sympathetic cooling
  - Ions mixed with a laser-cooled species are cooled through collisions

Summary of Lecture 1

- The Penning trap is an elegant device with many excellent features for experiments in physics
  - Especially precision measurements due to the extreme isolation
- It makes possible experiments that would otherwise not be possible
  - Especially those involving delicate quantum effects
- In the next lecture we will look at some of the different areas of physics where they are used.