Cooling atoms with lasers

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Outline

1. The concept of radiation pressure force
2. Beyond the concept : Motion induced effect on the atom-laser interaction
3. The dark side of the force

the mechanical effect of light

To understand the interaction of light with atoms, one must consider atoms as system with quantised energy level.

- the lowest energy level is the ground state g and is stable (if the nucleus is stable!!)
- the higher energy levels are associated to excited states e_i, with a finite lifetime τ_{e_i}.
- when the energy difference between these levels is in the optical domain :

$$E_e - E_g = h\nu \qquad \lambda = \frac{c}{\nu} \simeq 400 \text{ nm} - 1\mu\text{m}$$

most excited states have a lifetime of the order of 10 ns

the mechanical effect of light

Absorption and emission of light by atoms :

- if the photon energy is close enough to the energy gap between levels g and e, there is absorption of the photon by the atom, initially in its ground state
- the atom remains in this excited state e, for a duration τ_e on average. It **emits a photon spontaneously** to give back the energy and end in the ground state.
- if the driving by the laser is strong enough, it can also force the atom to emit one photon back to the laser wave : this is **stimulated emission**
- the two emission processes result in different wave vector direction

the mechanical effect of light

A photon carries energy **and** momentum $\hbar\mathbf{k}_L$

- there is a recoil induced by one photon absorption, $v_{rec} = \hbar k_L/m$.
  o.m : $v_{rec} = 3$ m/s for hydrogen and 3.5 mm/s for cesium, both excited on their resonance line.
- if there is stimulated emission, the net gain in recoil is null
- if spontaneous emission is most probable, the net gain over N cycles abs/em is Nv_{rec}.

How to reduce the atomic velocity ?

Make sure that the atom absorbs a counter-propagating photon!!

- The Doppler effect shifts the laser frequency *seen* by the atoms :

$$\omega_L^{at} = \omega_L - \mathbf{k}_L.\mathbf{v}$$

- Transition occurs if energy and momentum are conserved!!

$$\mathbf{p}' = \mathbf{p} + \hbar \mathbf{k}_L \quad (1)$$

$$E_g + \hbar\omega_L + \mathbf{p}^2/2m = E_e + (\mathbf{p} + \hbar\mathbf{k}_L)^2/2m \quad (2)$$

  it induces

$$\omega_L - \mathbf{k}.\mathbf{v} = \omega_L^{at} = \omega_0 + \hbar \mathbf{k}_L^2/2m$$

- If $\omega_L < \omega_0$, the laser reaches the atomic resonance for atom moving against the laser beam and then $||\mathbf{p}'|| < ||\mathbf{p}||$.

orders of magnitude

- $\hbar^2\mathbf{k}_L^2/2m = \hbar\omega_{rec}$ is the recoil energy, of the order of few 10 kHz
- for Doppler laser cooling, strong dipole transition are used (natural linewidth $\simeq$ 10 MHz)
- The recoil energy is often not taken into account in Doppler laser cooling and its condition reduces to

$$\omega_L - \mathbf{k}.\mathbf{v} = \omega_L^{at} = \omega_0$$

- To optimise the efficiency and control the limit temperature, one must go back to the fundamental of atom-laser interaction.

Looking inside the radiation pressure

- the force on a atom is (number of photons scattered per input time)$\times\hbar\mathbf{k}$
- number of photons scattered per input time=(number of photons scattered per input time when in the excited state)$\times$(probability to be in the excited state)
- $\mathbf{F} = \Gamma \times P_e \times \hbar\mathbf{k}$: one needs to know how to control P_e

the two-level atom in a laser electric field

- $|g\rangle, |e\rangle$ are the two involved atomic states, with energy defined by $E_e - E_g = \hbar\omega_0$ and $\omega_L \simeq \omega_0$.
- without any laser field, the hamiltonian of an atom at rest is

$$\hat{H}_0 = \frac{\hbar\omega_0}{2}\left(|e\rangle\langle e| - |g\rangle\langle g|\right)$$

- within the **dipolar approximation**, the laser electric field couples to the transition dipole $d_{e.g.}$ by

$$V_{AL}(\mathbf{r}, t) = -\mathbf{d}_{eg}.\mathbf{E}_L(\mathbf{r}, t)$$

where **r** is the position of the atom center of mass.
- the transition is driven by the dipole operator polarised along the local electric field polarisation ϵ_L

$$\mathbf{d}_{eg} = d_{eg}\epsilon_L\left(|e\rangle\langle g| - |g\rangle\langle e|\right)$$

the two-level atom in a laser electric field

- in a single active electron atom

$$\mathbf{d}_{eg} = q_e \langle e | \mathbf{r}_e | g \rangle$$

- if the laser wave is a plane wave

$$\mathbf{E}_L(\mathbf{r}, t) = E_L \epsilon_L \cos(\omega t - \Phi(\mathbf{r}))$$

- the Rabi frequency scales the interaction strength

$$\hbar\Omega_1(\mathbf{r}) = -\mathbf{d}_{eg}.\epsilon_L E_L(\mathbf{r})$$

the time evolution

- the evolution of the internal degrees of freedom is known through the density matrix of the internal state ρ. It obeys the master equation

$$\frac{\partial}{\partial t}\rho = -\frac{\mathrm{i}}{\hbar}[H,\rho] + \mathcal{L}\rho \tag{3}$$

- the hamiltonian is $H = H_0 + V_{AL}$ and $\mathcal{L}\rho$ rules the non hamiltonian evolution induced by relaxation (spontaneous emission, collisions,...). $\mathcal{L}$ is a Lindbald operator.
- the projection of the master equation on the basis $(|g\rangle, |e\rangle)$ forms the optical Bloch equations (OBE).
- the relative motion of the atom in the wave $\mathbf{r}(t)$ induces a modulation of the laser-atom interaction in

$$V_{AL}(\mathbf{r},t) = \hbar\Omega_1(\mathbf{r}(t))\,(|e\rangle\langle g| - |g\rangle\langle e|)\cos(\omega t - \Phi(\mathbf{r}(t)))$$

let's assume an atom at rest

- then $\mathbf{p} = \mathbf{0}$, $\Phi(\mathbf{r}) = \Phi_0$ and $\Omega_1(\mathbf{r}) = \Omega_1$.
- the OBE are

$$\frac{\mathrm{d}\rho_{ee}}{\mathrm{d}t} = i(\rho_{eg} - \rho_{ge})\Omega_1 \cos(\omega_L t - \Phi_0) - \gamma_p \rho_{ee} \quad (4)$$

$$\frac{\mathrm{d}\rho_{eg}}{\mathrm{d}t} = -i\omega_0 \rho_{eg} - i(\rho_{ee} - \rho_{gg})\Omega_1 \cos(\omega_L t - \Phi_0) - \gamma_d \rho_{eg} \quad (5)$$

  considering that $\rho_{gg} + \rho_{ee} = 1$ and that $\rho_{ge} = \rho_{eg}^*$, you know everything!
- if the only source of decoherence is spontaneous emission

$$\gamma_p = \Gamma \quad \text{and} \quad \gamma_d = \Gamma/2$$

let's assume an atom at rest

- we focus on the envelop evolution and get ride of the rapid evolution by

$$\rho_{eg} = \tilde{\rho}_{eg} e^{-i(\omega_L t - \Phi_0)}$$

- furthermore,we use the decomposition

$$\cos(\omega_L t - \Phi_0) = \frac{\exp(i(\omega_L t - \Phi_0) + \exp(-i(\omega_L t - \Phi_0)}{2}$$

- two kinds of terms for time evolution :

$$\frac{\mathrm{d}\rho_{ee}}{\mathrm{d}t} = i\Omega_1(\tilde{\rho}_{eg} e^{-i\omega_L t} - \tilde{\rho}_{ge} e^{i\omega_L t}) \frac{e^{(i(\omega_L t - \Phi_0)} + e^{(-i(\omega_L t - \Phi_0)}}{2} - \gamma_p \rho_{ee}$$

we neglect the fast oscillating terms $e^{\pm i 2\omega_L t}$ and keep the slow envelop effect : the "secular approximation"

also called the rotating wave approximation (RWA)

- in the RWA :

$$\frac{\mathrm{d}\rho_{ee}}{\mathrm{d}t} = i(\tilde{\rho}_{eg} - \tilde{\rho}_{ge})\Omega_1/2 - \gamma_p \rho_{ee} \quad (6)$$

$$\frac{\mathrm{d}\tilde{\rho}_{eg}}{\mathrm{d}t} = -i(\omega_0 - \omega_L)\tilde{\rho}_{eg} - i(\rho_{ee} - \rho_{gg})\Omega_1/2 - \gamma_d \tilde{\rho}_{eg} \quad (7)$$

- three relevant components: $Im(\tilde{\rho}_{eg})$, $Re(\tilde{\rho}_{eg})$, $(\rho_{ee} - \rho_{gg})$

$$\begin{aligned} u(t) &= Re(\tilde{\rho}_{ge}) \\ v(t) &= Im(\tilde{\rho}_{ge}) \\ w(t) &= (\rho_{ee} - \rho_{gg})/2 \\ &= \rho_{ee} - 1/2 \end{aligned}$$

$$\begin{aligned} \dot{u}(t) &= \Delta_L v(t) - \gamma_d u(t) \\ \dot{v}(t) &= -\Delta_L u(t) - \Omega_1 w(t) - \gamma_d v(t) \\ \dot{w}(t) &= -\Omega_1 v(t) - \gamma_p (w(t) + 1/2) \end{aligned}$$

$$\Delta_L = \omega_L - \omega_0$$

The stationary solutions when $\gamma_p = \Gamma$ and $\gamma_d = \Gamma/2$

- they depend on a parameter s called the **saturation parameter** :

$$s = \frac{\Omega_1^2/2}{\Delta_L^2 + \Gamma^2/4}$$

- on resonance

$$s = s_0 = 2\Omega_1^2/\Gamma^2 = I/I_s$$

  I_s is a characteristic of the transition strength and is of the order of few mW/cm^2 for alkali (sodium, rubidium, cesium...) and alkali-earth ion (Be^+, Ca^+, Sr^+...)

- assuming $\dot{u}(t) = \dot{v}(t) = \dot{w}(t) = 0$

$$u_{st} = \frac{\Delta_L}{\Omega_1}\frac{s}{1+s} \qquad v_{st} = \frac{\Gamma}{2\Omega_1}\frac{s}{1+s} \qquad \rho_{ee} = w_{st} + 1/2 = \frac{1}{2}\frac{s}{1+s}$$

Come back to the radiation pressure on **an atom at rest**

- the force behaves like ρ_{ee} :

$$\mathbf{F} = \Gamma \times \rho_{ee} \times \hbar \mathbf{k} \qquad \rho_{ee} = \frac{1}{2}\frac{s}{1+s} = \frac{1}{2}\frac{\Omega_1^2/2}{\Omega_1^2/2 + \Delta_L^2 + \Gamma^2/4}$$

- is maximum when $\Delta = 0$
- the spectral width (FWHM) is

$$\Delta\omega = \Gamma\sqrt{1 + \frac{2\Omega_1^2}{\Gamma^2}} = \Gamma\sqrt{1 + s_0}$$

For a moving atom, do we have to solve the full problem?

- if the dynamics of the internal state is a lot more rapid than the external dynamics, we can consider **r** and **v** as constant in the OBE.
- the internal time evolution is ruled by Ω_1, $\Gamma = 1/\tau_e$ and $\Delta_L = \omega_L - \omega_0$.
- one can show that whatever is the laser intensity, it takes $\tau_e = 1/\Gamma$ for the internal dynamics to reach its stationary state.
- the time T_{ext} it takes to drive the atom out of the resonance line is such that $k_L \Delta v(T_{ext}) = \Gamma/2$
- $\Delta v(T_{ext}) \simeq v_{rec} \times T_{ext} \times \Gamma/2$ so $T_{ext} = 1/2\omega_{rec}$
- if internal dynamics follows *instantaneously* the external dynamics if

$$T_{ext} \ll T_{int} \quad \text{which requires } \Gamma \gg 2\omega_{rec} = \frac{\hbar k_L^2}{m}$$

$\Rightarrow$ **the broad line condition**

For a moving atom, in the broad band limit

- $\Omega_1 = \Omega_1(\mathbf{r})$ and $\Delta_L \to \Delta_L - \mathbf{k}_L.\mathbf{v}$
- for an atomic gas in thermal equilibrium, it takes two laser beams for each direction
- two counter propagating laser beam with same intensity and detuning:

$$\mathbf{F}_\pm = \pm\hbar\mathbf{k}_L\Gamma\rho_{ee}^\pm(\mathbf{v})$$

- **within the low saturation limit** $\mathbf{F} = \mathbf{F}_+ + \mathbf{F}_-$ and $\rho_{ee}^\pm(\mathbf{v}) = s_\pm(\mathbf{v})/2$
- $$F = \frac{1}{2}\hbar k_L\Gamma(s_+(\mathbf{v}) - s_-(\mathbf{v}))$$
- if $k_L v \ll \Gamma$, the linear development of F for small v gives

$$\mathbf{F} = -m\alpha\mathbf{v} \quad \text{with} \quad \alpha = \omega_{rec}s_r\frac{-4\Gamma\Delta_L}{\Delta_L^2 + \Gamma^2/4} \quad s_r = \frac{\Omega_1^2/2}{\Delta_L^2 + \Gamma^2/4}$$

Doppler cooling in the broad line limit

$$\mathbf{F} = -m\alpha\mathbf{v} \quad \text{with} \quad \alpha = \omega_{rec} s_r \frac{-4\Gamma\Delta_L}{\Delta_L^2 + \Gamma^2/4} \quad s_r = \frac{\Omega_1^2/2}{\Delta_L^2 + \Gamma^2/4}$$

is a damping force if $\Delta_L < 0$

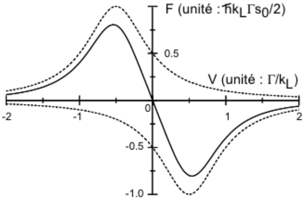


FIG. 2: Trait continu : somme des deux forces de pression de radiation agissant sur un atome en mouvement dans une mélasse optique, en fonction de la vitesse atomique. Traits pointillés : forces créées par chacune des deux onde planes progressives. Le désaccord est $\delta = -\Gamma/2$.

Doppler cooling in the broad line limit

- the typical timescale for velocity damping is

$$\tau_{DC} = \frac{1}{\omega_{rec} s_r} \frac{\Delta_L^2 + \Gamma^2/4}{4\Gamma|\Delta_L|}$$

- if Δ_L is too close to the resonance, this timescale tends to infinity.
- for $\Delta_L = -\Gamma/2$, $\tau_{DC} = 1/(4\omega_{rec} s_0)$
- for a typical low saturation parameter $s_0 = 1/10$, $\tau_{DC} = 2.5/\omega_{rec}$. For heavy alkalies, τ_{DC} is smaller than 1 ms (can be lower than 100 μs)
- in 2D, you can reduce the divergence of an atomic beam
- in 3D, you can cool atom in "optical molasses" with a capture range of the order of Γ/k_L

How cold can the atoms get?

- the norm $||\mathbf{v}^2||$ reaches a limit because of the fluctuation of the force, induced by spontaneous emission and absorption
- it looks like brownian motion : the fluctuation of the force around its mean value is responsible for an increase of the momentum variance, linear in time
- from spontaneous emission, the diffusion scales like

$$\Delta\mathbf{p}^2/\Delta t = (\hbar k_L)^2 \rho_{ee}\Gamma$$

- from the atom-laser interaction (for 1D, two beams)

$$\begin{aligned}\Delta\mathbf{p}^2/\Delta t &= (\hbar k_L)^2\Delta^2(N_+ - N_-)/\Delta t \\ &= (\hbar k_L)^2 < N_+ + N_- > /\Delta t = (\hbar k_L)^2 s_0\Gamma\end{aligned}$$

- in 3D, $\Delta\mathbf{p}^2/\Delta t = 6(\hbar k_L)^2 s_0\Gamma$ with s_0 defined for one beam.

How cold can the atoms get?

- by summing cooling and diffusion :

$$\Delta \mathbf{p}^2/\Delta t = 6(\hbar k_L)^2 s_0 \Gamma - 2\alpha \mathbf{p}^2$$

$$\left(\frac{\mathbf{p}^2}{2m}\right)_{eq} = \frac{3}{2} k_B T_{eq} = \frac{6(\hbar k_L)^2 s_0 \Gamma}{2m\alpha}$$

$$k_B T_{eq} = \frac{\hbar \Gamma}{4}\left(\frac{2|\Delta_L|}{\Gamma} + \frac{\Gamma}{2|\Delta_L|}\right)$$

- the smaller temperature is reached for $\Delta = -\Gamma/2$ and

$$k_B T_{min} = \hbar \Gamma/2$$

How cold can the atoms get?

$$k_B T_{min} = \hbar\Gamma/2$$

- ex: Rb ($\tau_e = 27$ ns), $T_{min} = 140\ \mu$K , for Na, $T_{min} = 240\ \mu$K
- the reached mean squared velocity v_D depends on the atomic mass :

$$v_D = \sqrt{\frac{\hbar\Gamma}{m}} = \sqrt{\frac{\hbar k_L}{m}\frac{\Gamma}{k_L}} = \sqrt{v_{rec} v_c}$$

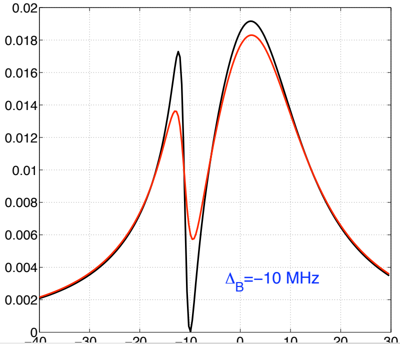
for sodium $v_{rec} = 0.03$ m/s and $v_c = 6$ m/s

- all this makes sens if $v_D \ll v_c$, which implies $\hbar k_L^2/m = 2\omega_{rec} \ll \Gamma$: **the broad line condition!**

Beware!!!

- Cooling is not trapping!
  It takes a restoring force to trap.
- an atom is rarely a two level system.
  It is OK for $J = 0 \rightarrow J = 1$ transition like in Ca,
  It is OK for alkali in 1D without polarisation mixing that gives ride to polarisation gradient.

sometimes two transitions (3 levels) are implied:
ex : the heavy alkaline-earth ions (Ca^+, Sr^+, Ba^+)



There is another force...Luke!

- going back to the Heisenberg picture where **r** and **p** are operator with a time evolution, this evolution is ruled by

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = \frac{\mathbf{p}}{m}$$
$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = \mathbf{F} = -\langle \nabla V_{AL}(t) \rangle - \langle \nabla V_{AR} \rangle$$

- the average force induced by V_{AR} is null and, assuming the atomic wave packet is small compared to λ_L

$$\mathbf{F} = \langle \mathbf{d}_{eg}.\epsilon_L \rangle \nabla E_L(\mathbf{r}(t), t)$$

There is another force...Luke!

$$\mathbf{F} = \langle \mathbf{d}_{eg}.\epsilon_L \rangle \nabla E_L(\mathbf{r}(t), t)$$

- we now know the dipole through the time evolution of the internal degree of freedom

$$\begin{aligned} \langle \mathbf{d}_{eg}.\epsilon_L \rangle &= \mathbf{d}_{eg}.\epsilon_L(\rho_{eg} + \rho_{ge}) \\ &= 2\mathbf{d}_{eg}.\epsilon_L(u_{st}\cos(\omega_L t + \Phi(\mathbf{r})) - v_{st}\sin(\omega_L t - \Phi(\mathbf{r})) \end{aligned}$$

- u and v gives the dipole in phase and $\pi/2$ out of phase (quadrature???) with the electric field
- the force is then

$$\mathbf{F} = -2\hbar(u_{st}\cos(\omega_L t - \Phi(\mathbf{r})) - v_{st}\sin(\omega_L t - \Phi(\mathbf{r}))\nabla(\Omega_1(\mathbf{r})\cos(\omega_L t - \Phi(\mathbf{r}))$$

There is another force...Luke!

$$\mathbf{F} = -2\hbar(u_{st}\cos(\omega_L t - \Phi(\mathbf{r})) - v_{st}\sin(\omega_L t - \Phi(\mathbf{r}))\nabla(\Omega_1(\mathbf{r})\cos(\omega_L t - \Phi(\mathbf{r}))$$

- two contributions to the force, averaged over a time period of the electric field :

$$\mathbf{F} = -2\hbar\Omega_1(\mathbf{r})\left(u_{st}\frac{\nabla\Omega_1(\mathbf{r})}{\Omega_1(\mathbf{r})} - v_{st}\nabla\Phi(\mathbf{r})\right)$$

- in a travelling wave $\Phi(\mathbf{r}) = \mathbf{k}_L.\mathbf{r}$ and

$$\mathbf{F}_{RP} = \hbar\mathbf{k}_L\Omega_1 v_{st} = \hbar\mathbf{k}_L\Gamma\rho_{ee}$$

we have here the radiation pressure force deduced we know already

The dipolar force

$$\mathbf{F}_{dip} = -\frac{\hbar\Delta_L}{2}\frac{\nabla s(\mathbf{r})}{1+s(\mathbf{r})}$$

- this force is conservative (no dissipation) : $\mathbf{F}_{dip} = -\nabla U_{dip}(\mathbf{r})$

$$U_{dip}(\mathbf{r}) = \frac{\hbar\Delta_L}{2}\log(1+s(\mathbf{r}))$$

- depending on the sign of Δ_L, atoms are attracted or repealed by higher intensity.
- in practice, it is used with large detuning (to reduce radiation pressure), then for $\Delta_L \gg \Omega_1$

$$U_{dip}(\mathbf{r}) = \frac{\hbar\Omega_1^2(\mathbf{r})}{4\Delta_L}$$

- o.m : few mK