Imperial College London

Penning Traps Lecture 3

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Outline of lecture 3

Lecture 1: Penning Trap Basics

Lecture 2: Review of Experiments

Lecture 3: Rotating Frame and Axialisation

7. The rotating frame
8. Cooling in the rotating frame
9. Axialisation
10. Conclusion

Instabilities in the Penning trap

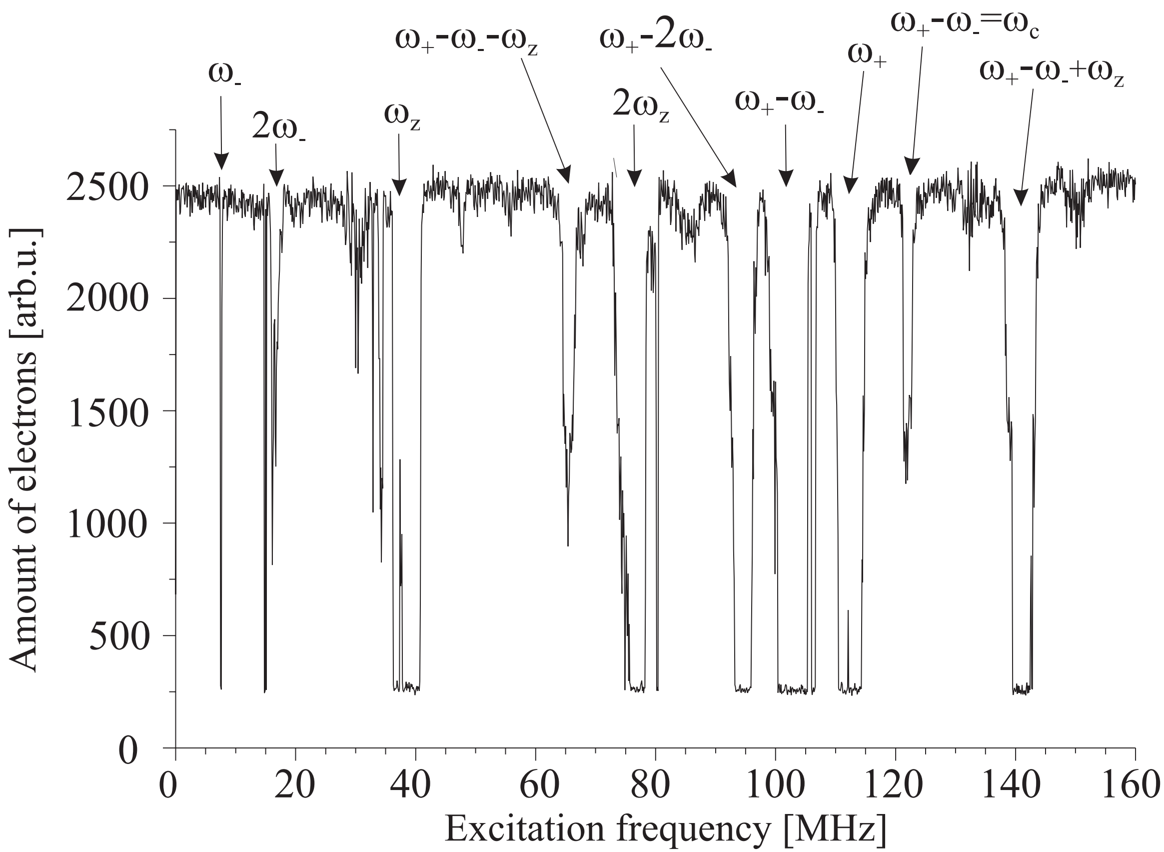

Fig. 4. Excitation spectrum of motional frequencies of the electron cloud.

You can see resonances with an external drive at integer combinations of trap oscillation frequencies in the Penning trap

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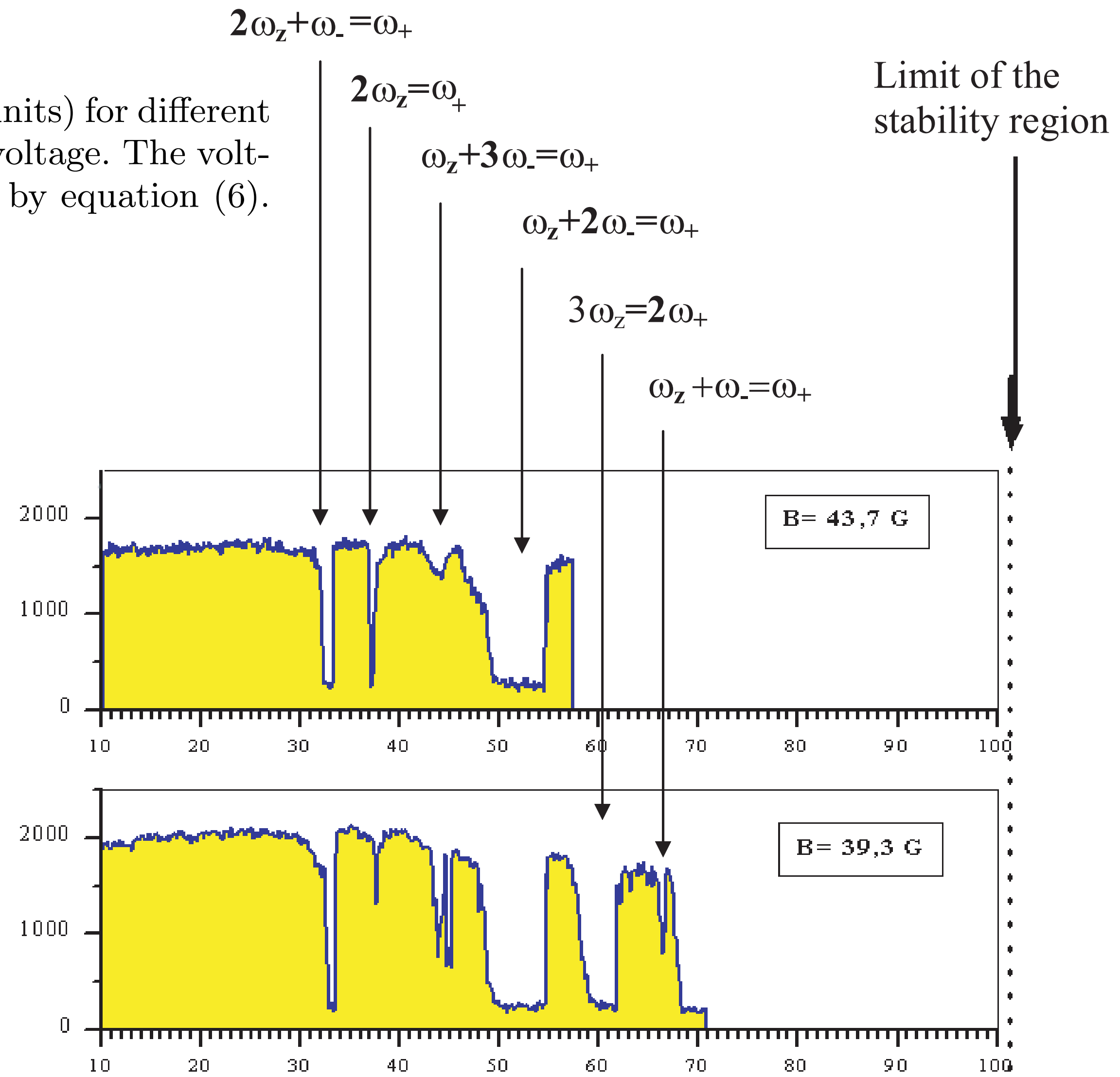
Instabilities in the Penning trap

Fig. 6. Stored electron number (in arbitrary units) for different magnetic fields as a function of the trapping voltage. The voltage is normalized to the stability limit given by equation (6). The storage time is fixed to 600 ms.



Motion instabilities occur in the Penning trap in the same way as RF traps

They occur when there are integer relations between oscillation frequencies

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7. The Rotating Frame

- It turns out that much of the physics of the Penning trap is simplified if we move to a frame rotating at half the cyclotron frequency.
  - The *centripetal force* ($-m\omega^2\mathbf{r}$) gives rise to a force that gives a confining potential in this frame
  - This overcomes the negative radial potential in the lab frame leading to a net confining potential
  - The *Coriolis force* ($-2\boldsymbol{\omega}\wedge\boldsymbol{v}$) gives rise to a force perpendicular to the velocity of a particle that cancels out the force due to the magnetic field
  - Therefore in this frame there is effectively no magnetic field
- **As a result, the radial motion in this frame reduces to standard two-dimensional SHM**

Equations of motion

- The original radial equations of motion were

$$\ddot{x} + \omega_c \dot{y} - (\omega_z^2 / 2)x = 0$$

$$\ddot{y} - \omega_c \dot{x} - (\omega_z^2 / 2)y = 0$$

or $$\ddot{u} - i\omega_c \dot{u} - (\omega_z^2 / 2)u = 0$$ with $u=x+iy$

- The transformation to a frame rotating at $\omega_c/2$ gives

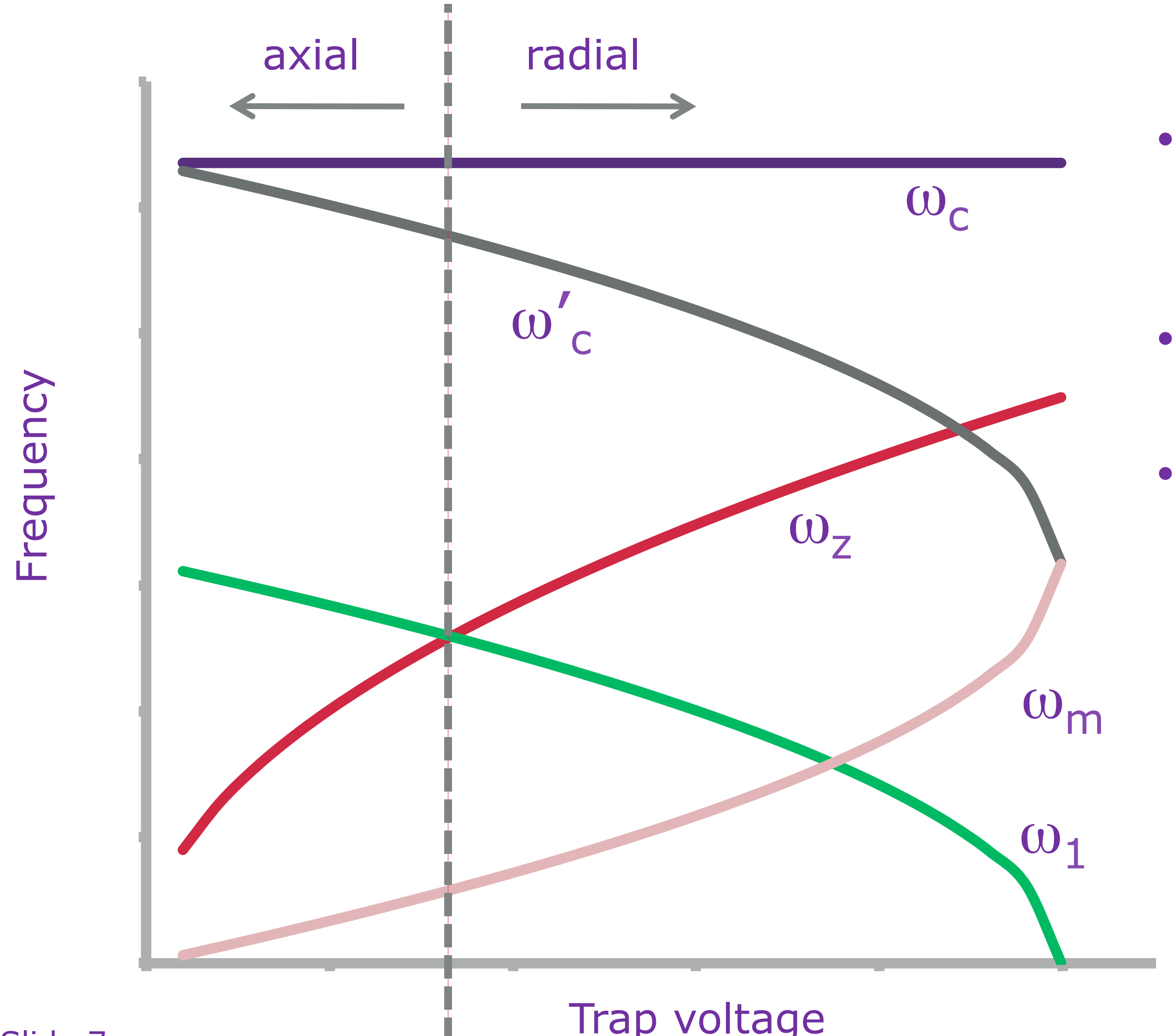
$$\ddot{u} + (\omega_c^2 / 4 - \omega_z^2 / 2)u = 0$$

or $$\ddot{u} + \omega_1^2 u = 0$$

with $$\omega_1^2 = \omega_c^2 / 4 - \omega_z^2 / 2$$

- This is just SHM in a potential well at frequency ω_1

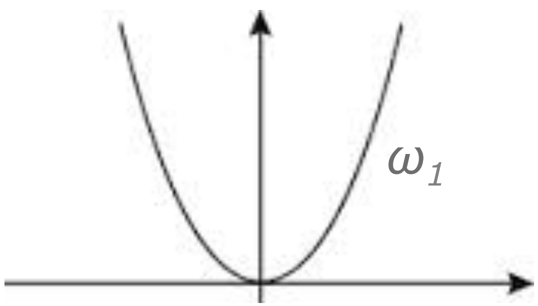
Oscillation frequencies for small crystals

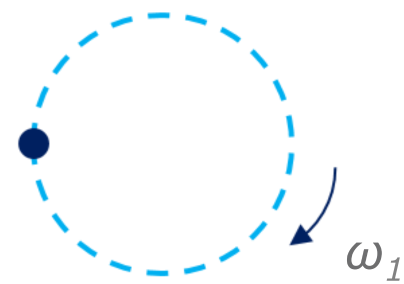


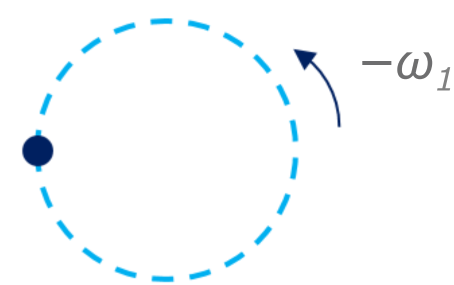
- ω_1 is the oscillation frequency in the rotating frame
- It reduces as the trap voltage is increased
- It becomes zero at the point where the trap becomes unstable

Oscillation frequencies in the two frames

- In the frame rotating at $\omega_c/2$ the potential is a 2D harmonic potential well of frequency ω_1.
- We take the two normal modes to be
  - clockwise rotation (ω_1)
  - anti-clockwise rotation ($-\omega_1$)
- When these are transformed back to the lab frame they become
  - Cyclotron motion ($\omega_c/2+\omega_1$)
  - Magnetron motion ($\omega_c/2-\omega_1$)
  - Now **both clockwise**



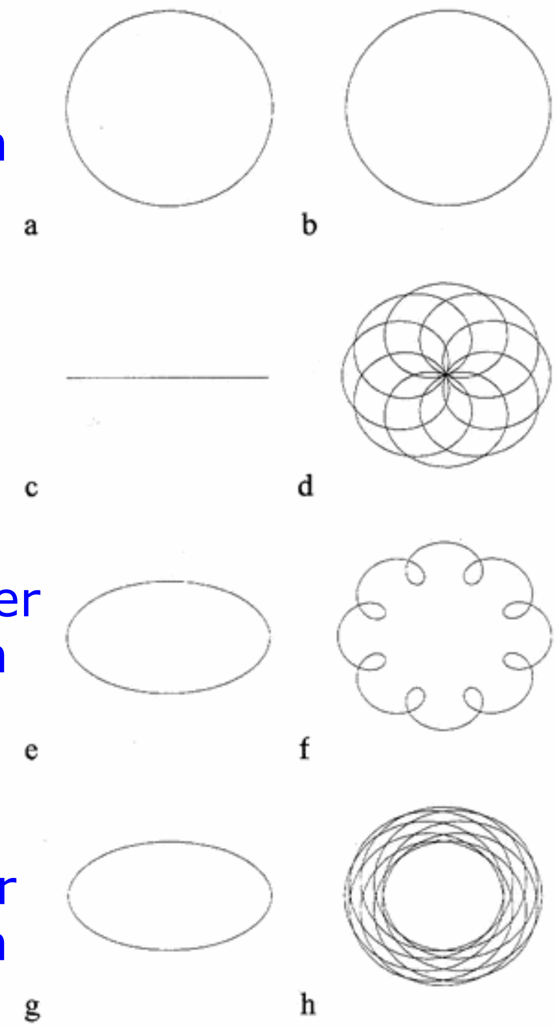




How does the motion transform?

- The motion is much simpler to describe in the rotating frame (RF)
- Why do we usually end up with $r_m > r_c$?
  - A particle at rest in the lab frame will be moving at $-\omega_c/2$ in the rotating frame so las a larger magnetron component

	RF	LAB
Pure cyclotron Pure magnetron		
Equal cyclotron and magnetron		
Cyclotron smaller than magnetron		
Cyclotron bigger than magnetron		



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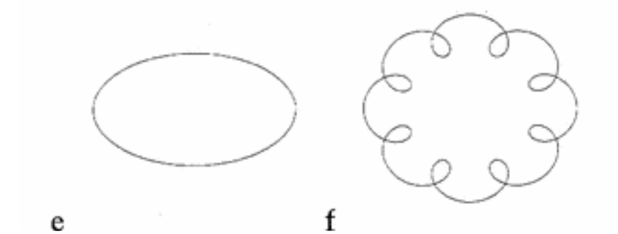
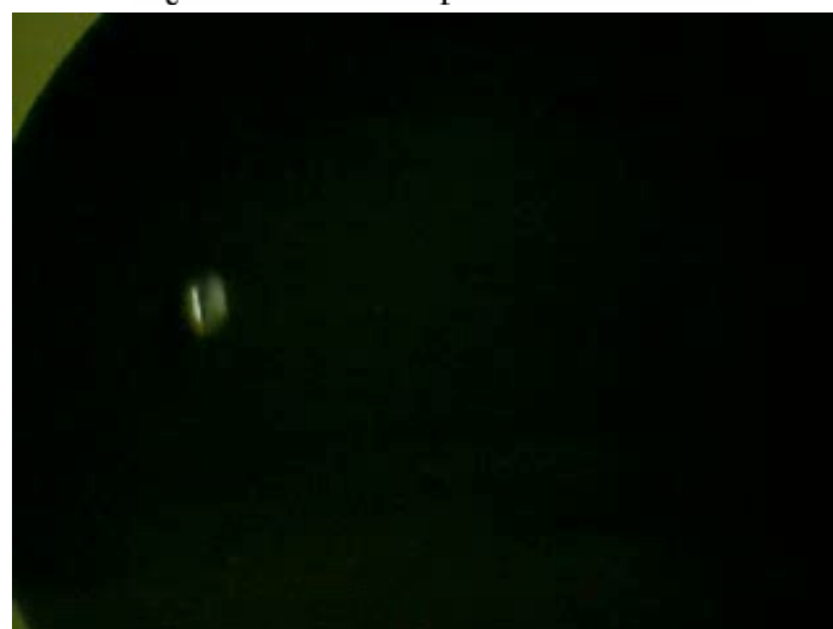
Can we visualise the motion?

- Yes we can!
- But in order to do so we need a 2D simple harmonic potential
  - A wok makes an excellent potential well
  - A ball bearing is an excellent model of an ion
  - But the motion of a ball bearing in a wok is not very interesting
- We need to simulate the effect of the magnetic field

Can we visualise the motion?

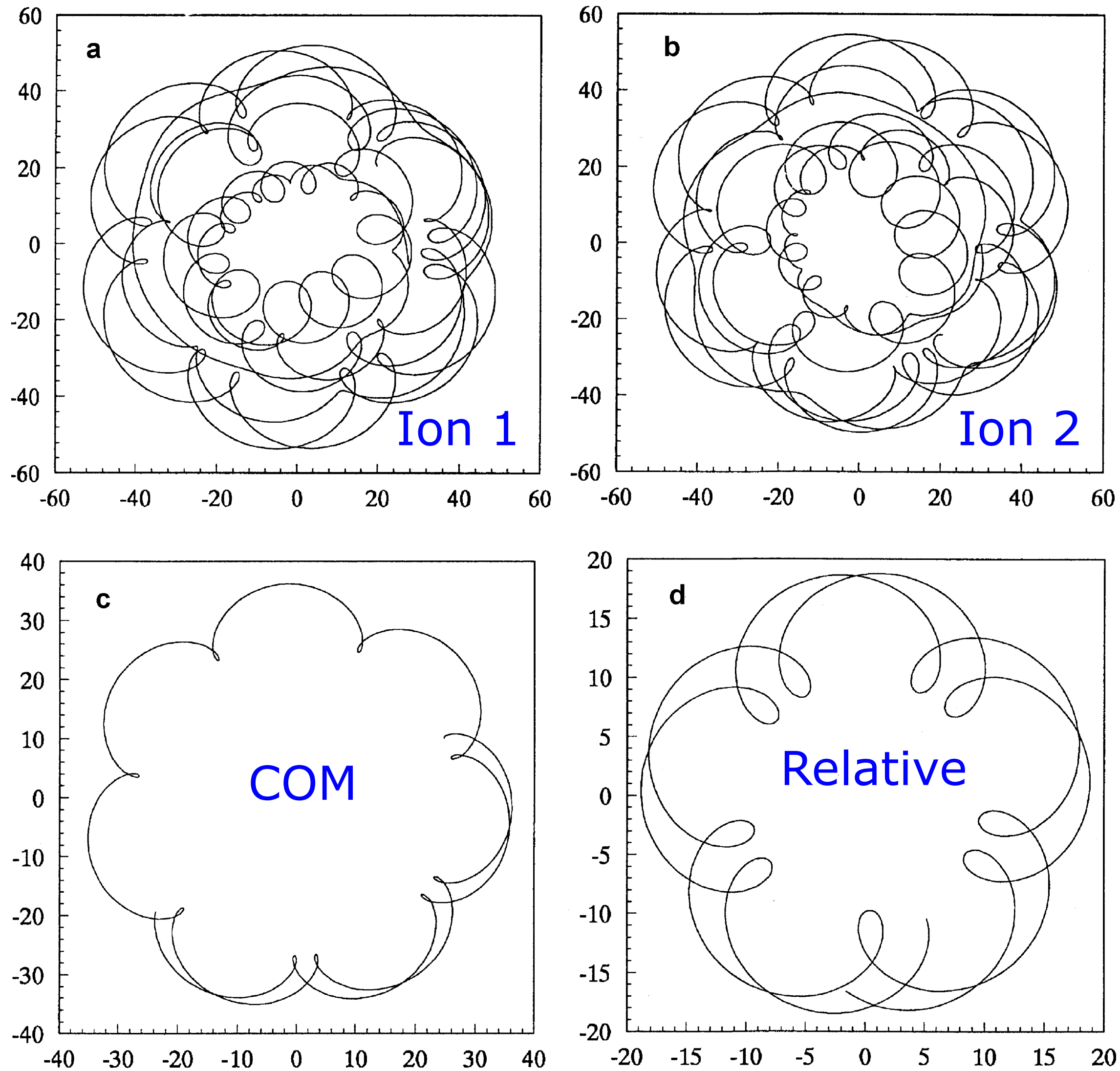
- In order to simulate the effect of the magnetic field we need to view the 2D simple harmonic potential **from a rotating frame**
  - Solution: use a rotating camera





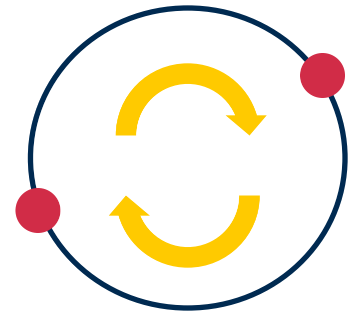
Centre of mass and relative motion of two ions

- The motion of two ions appears very complicated
- But actually it's very simple if you separate it into centre of mass and relative motions
- In the same way, things simplify in the rotating frame!

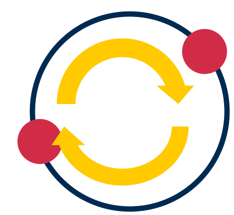


2 ions in the rotating frame

- Large separation:
  - $\omega_R \sim \omega_1$ in RF
  - $\omega_R \sim \omega_m$ in LAB



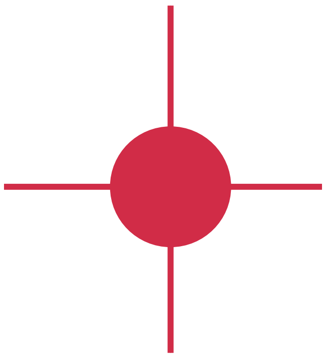
- Medium separation
  - $\omega_R < \omega_1$ in RF
  - $\omega_R > \omega_m$ in LAB

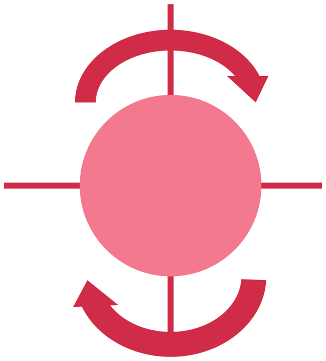


- Small separation:
  - No rotation
  - $\omega_R = 0$ in RF
  - $\omega_R \;\; \omega_c/2$ in RF
  - Brillouin Flow

Many ions in the rotating frame

- A plasma is easy to picture in a potential well and in the absence of the magnetic field
- If the plasma is not rotating in the rotating frame, it will sit at the bottom of the potential and have the maximum density
  - Brillouin flow!
- If it rotates in the rotating frame, centrifugal force will force the particles out and the density will drop





Many ions in the rotating frame

- A plasma is easy to picture in a potential well and in the absence of the magnetic field
- If the plasma is not rotating in the rotating frame
  - Frequency of the confining potential is ω_1 so density is

$$n = m\omega_c^2 \varepsilon_0 / 2e^2$$

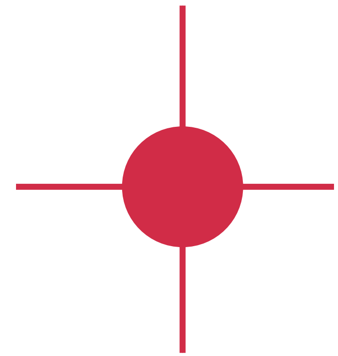
- If it rotates at ω_R‘ in the rotating frame:

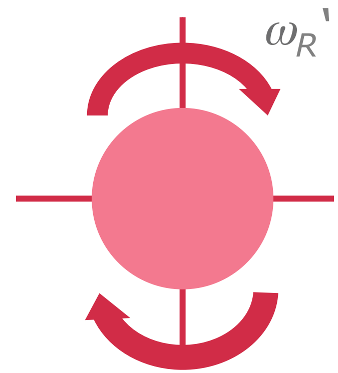
$$\omega_1^2 \rightarrow \omega_1^2 - \omega_R'^2$$

- Rearranging:

$$n = 2\varepsilon_0 m \omega_R (\omega_c - \omega_R) / e^2$$

as we had before





8. Cooling in the rotating frame

- We will now look again at cooling in the Penning trap as viewed from the rotating frame
- We will find this gives us new insights into how cooling works

Effect of a buffer gas in the rotating frame

- A buffer gas gives a uniform damping force in the lab frame
- In the rotating frame this damping force is rotating at $-\omega_c/2$
  - This looks like a whirlpool
- It creates a torque that rotates a single ion in the negative sense at $-\omega_1$ and accelerates it outwards
  - This corresponds to increasing magnetron radius
- This is why buffer gas damping increases the magnetron radius

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Effect of laser cooling in the rotating frame

- A **uniform cooling laser** beam also gives a uniform damping force in the lab frame
- In the rotating frame this damping force is rotating at $-\omega_c/2$
  - This looks like a whirlpool
- It creates a torque that rotates a single ion in the negative sense at $-\omega_1$ and accelerates it outwards
  - This corresponds to increasing magnetron radius
- This is why a uniform laser beam increases the magnetron radius

Effect of laser cooling in the rotating frame

- A laser beam **offset from the centre of the trap** gives a rotating damping force in the lab frame
  - The rotation speed of the damping (say ω_L) is proportional to the slope of laser intensity across the trap centre
- In the rotating frame this damping force is rotating at $-\omega_c/2+\omega_L$
  - This still looks like a whirlpool but if it rotates at a speed between $-\omega_1$ and $+\omega_1$ it will cool the motion
- This is why an offset laser beam cools the magnetron radius

Effect of laser cooling in the rotating frame

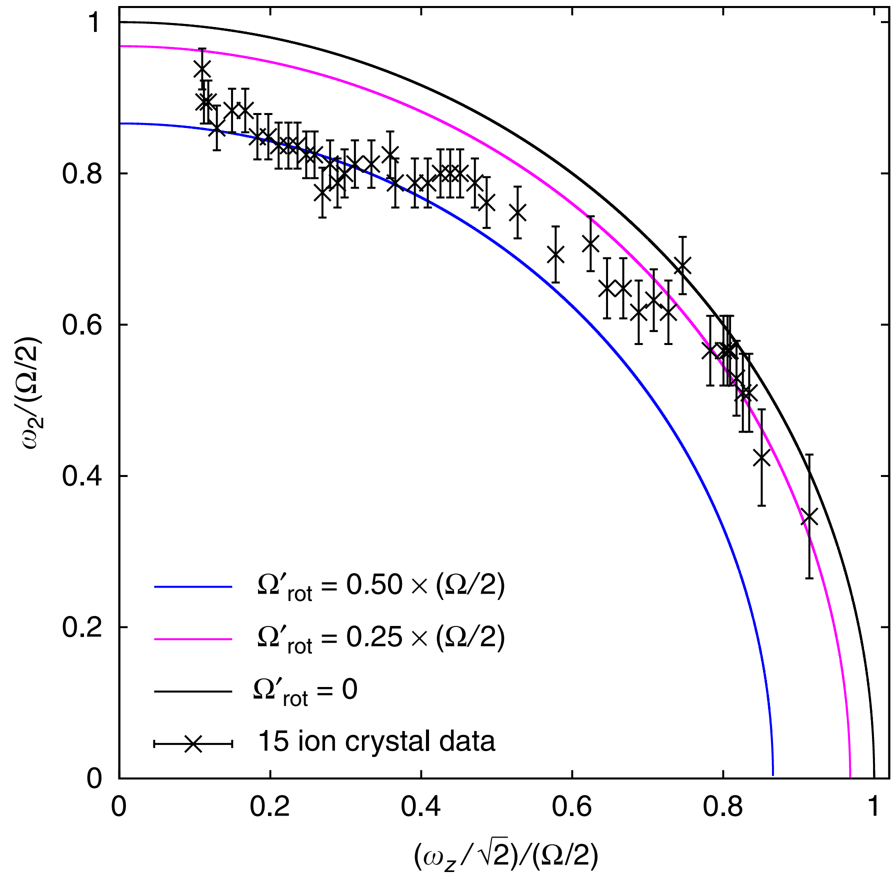
- For a Coulomb crystal, the rotating damping force in the rotating frame will drag the crystal till they both rotate at the same frequency
- We can therefore control the density and shape of the crystal by changing the laser parameters

Images of 15-ion crystals

- We match our simulations to the observed crystal configuration, and that determines the rotation frequency

Rotation of the crystal

- Comparison of experimental images with simulations yields the effective radial potential seen by the crystal
- From this we can determine the rotation frequency in the lab frame (ω)
- We find that ω is determined only by the parameters of the laser cooling beam
  - Independent of trap parameters, the crystal conformation and the number of ions (for small crystals)
- Measured values of ω are consistent with known laser parameters



Plot of the effective radial potential seen by the crystal as a function of axial frequency

9. Axialisation

- Axialisation is a technique for cooling the magnetron motion
- It requires two things:
  - Coupling of the magnetron motion to another motion in the trap
  - A damping mechanism for the second motion
- When set up properly, it results in cooling of both motions at the same time
- BEWARE: It also goes by other names:
  - Sideband cooling [not to be confused with optical sideband cooling]
  - Magnetron centering

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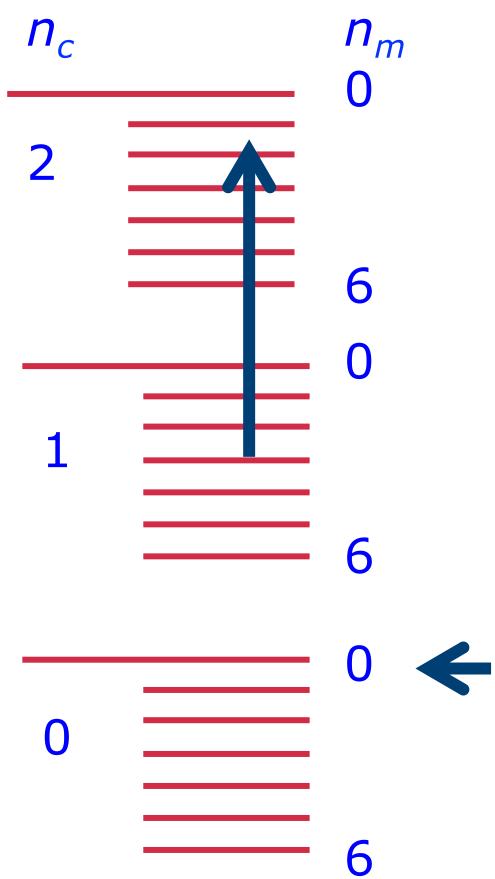
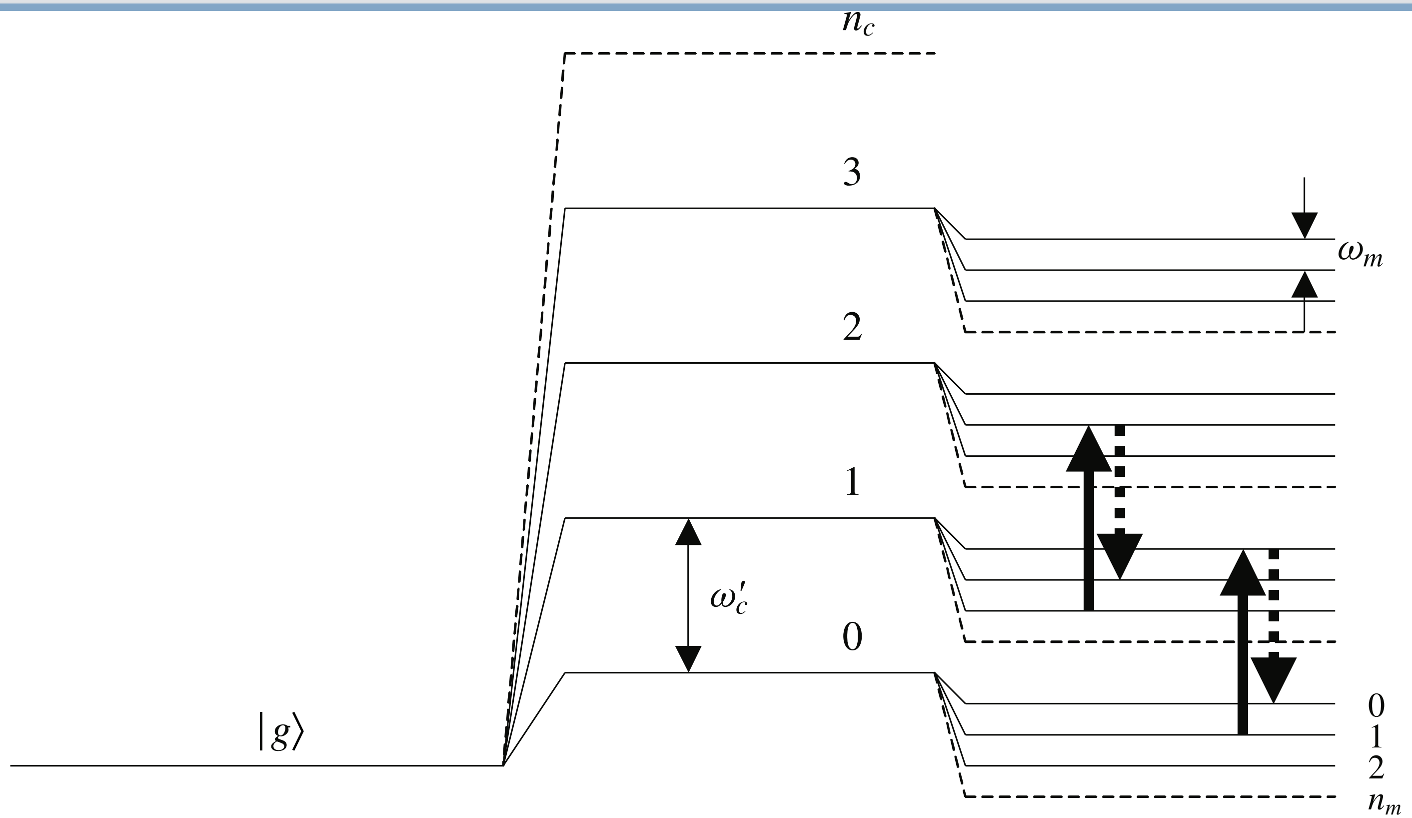
Coupling of motions

- In general two oscillators are coupled by excitation at their *difference frequency* to exchange energy
  - e.g. a laser driving an atomic transition
- We can couple the magnetron and cyclotron motions by excitation at their *sum frequency*
  - This is because of the negative energy of the magnetron motion
  - The sum frequency is just the cyclotron frequency $\omega_c = \omega'_c + \omega_m$
- We can also think of it in terms of quantum mechanical levels
  - ω_c drives n_c to n_c+1 and n_m to n_m−1



Quantum mechanical picture



Excitation at $\omega_c = \omega_c' + \omega_m$ by axialization drive

Cooling of cyclotron motion via laser cooling

Damping

Damping can be provided by a number of means:

- Buffer gas – used in mass spectrometry experiments
  - Especially Fourier Transform ICR (ion cyclotron resonance)
  - Gives a well controlled damping force on all particles
- Resistive cooling
  - Also used widely
- Laser cooling
  - Damping force is only applied to ions located in the laser beam and this is not ideal
  - Ions are much better localised when axialisation applied
  - Laser beam can be directed through trap centre as offset is no longer required
- Note that the magnetron can also be coupled to the *axial motion* using excitation at $\omega_z - \omega_m$ (needs different field symmetry)

Effect of the coupling

- The coupling causes energy exchange at a rate δ between the two modes of motion
- If the modes are damped at rates γ_c and γ_m then

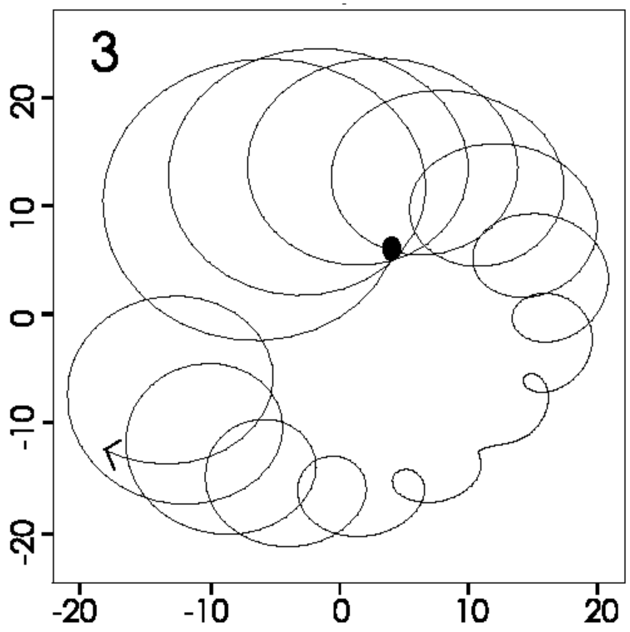
$$\dot{r}_c = \delta r_m - \gamma_c r_c, \qquad \dot{r}_m = -\delta r_c - \gamma_m r_m,$$

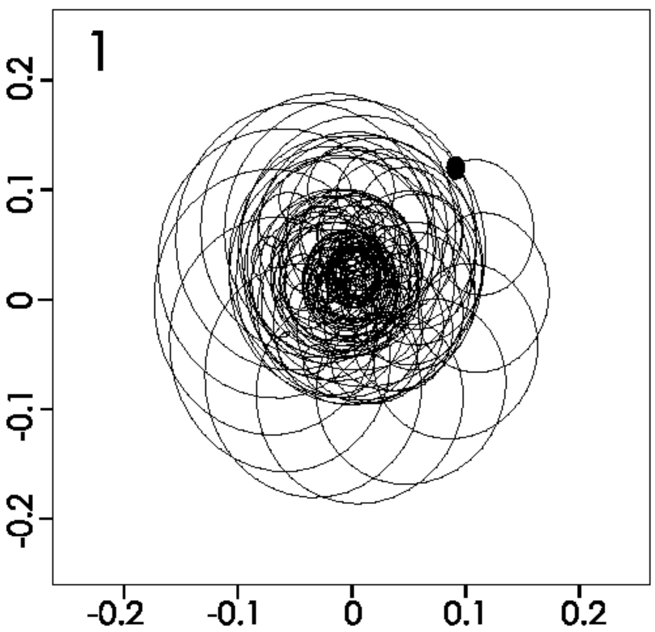
- The condition for axialisation to work is

$$\delta^2 > -\gamma_c \gamma_m,$$

Simulation of axialisation

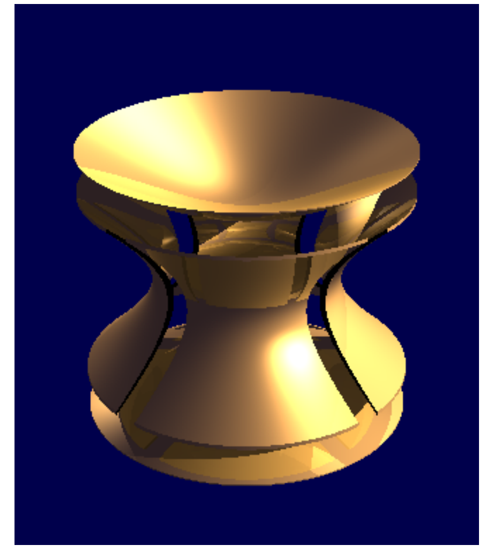
- With coupling alone, the orbital energy exchanges between magnetron and cyclotron motion
- With damping as well, the amplitude of both motions decreases

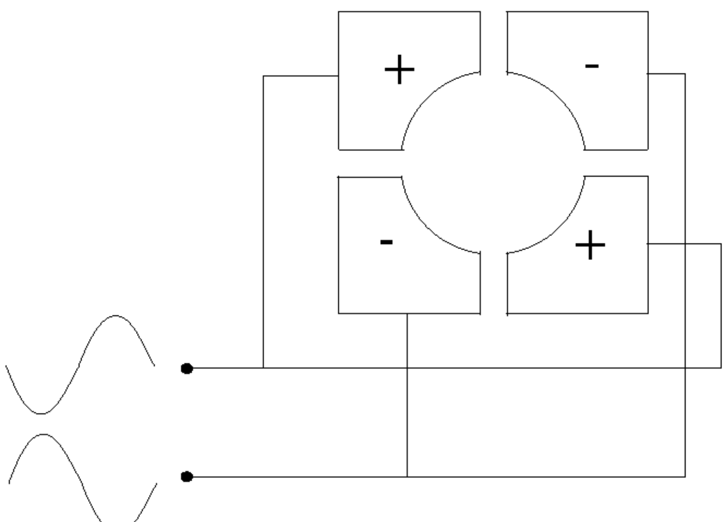


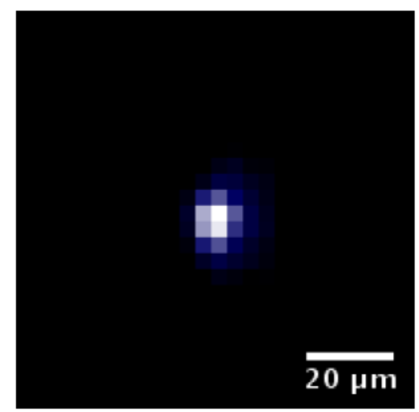


How do we apply the field?

- With axialisation we apply a radial quadrupole field at ω_c
  - We need four segments (minimum) to apply a radial quadrupole field
  - e.g. by splitting the ring electrode into 4 segments
  - This allowed us to get our first well localised single ion images

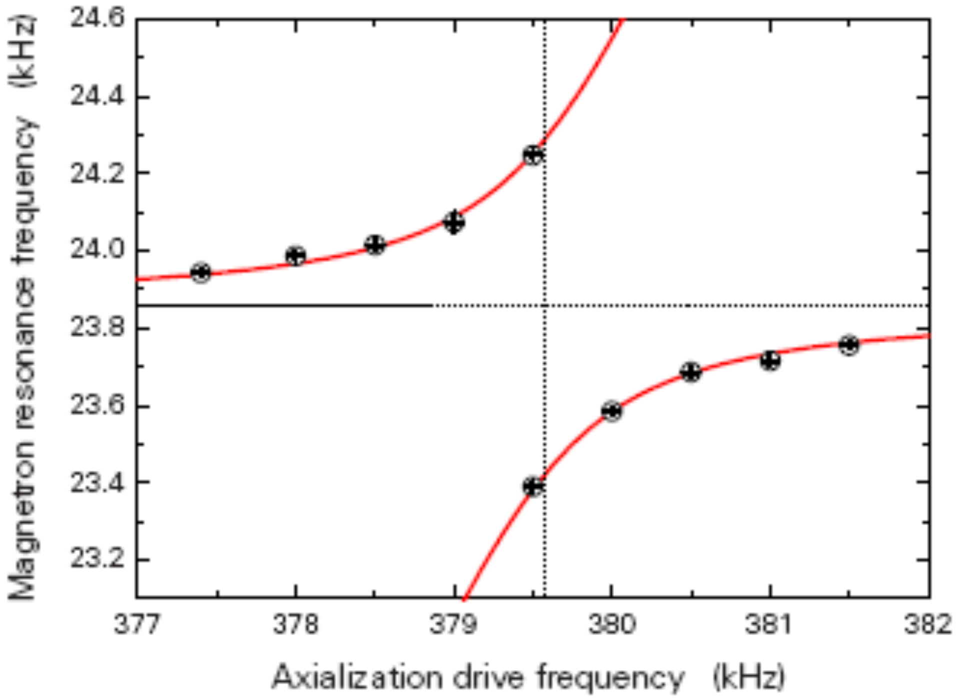


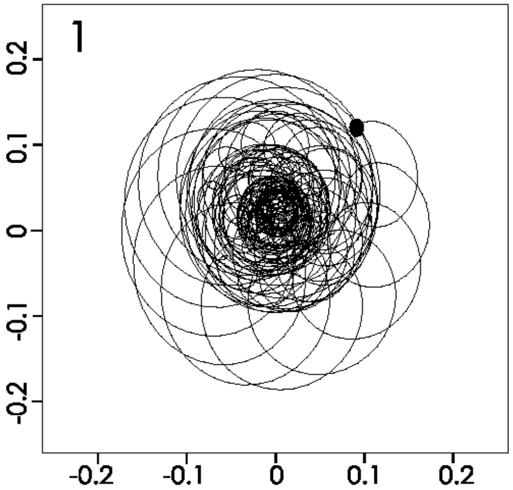




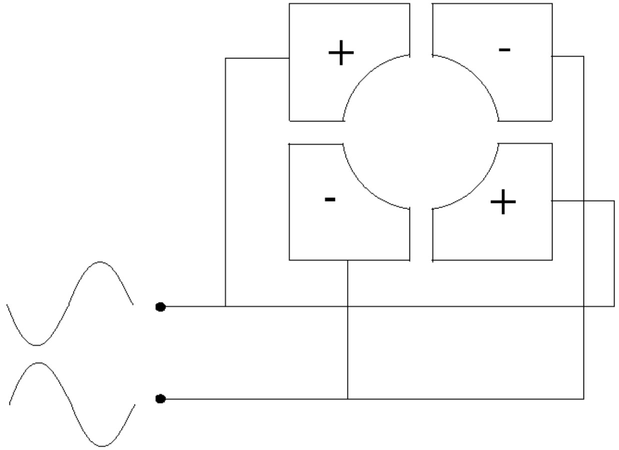
(d) Single ion

Results of axialisation experiments





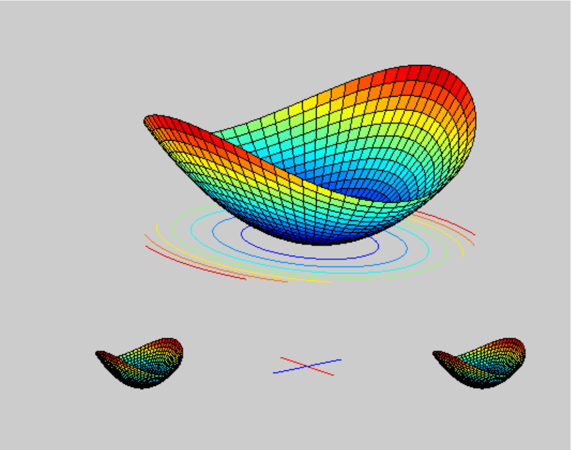
Effect of quadrupole drive with cyclotron damping



- Equivalent to two coupled and damped simple harmonic oscillators
- There is therefore an “avoided crossing” close to resonance

Rotating frame picture

- For axialisation we apply an oscillating quadrupole field at ω_c.
- This can be decomposed into two counter-rotating quadrupoles at frequency $\omega_c/2$
  - One of them is therefore stationary in the rotating frame
  - It "squeezes" the potential
  - The potential in this frame is no longer cylindrically symmetrical
  - The normal modes are now **linear oscillations** parallel and perpendicular to the axis of the squeeze
  - If the initial condition is circular motion in one direction this sets **both** normal modes in motion and this gives beats between them
  - The particle oscillates between clockwise (cyclotron) and counterclockwise (magnetron) rotation



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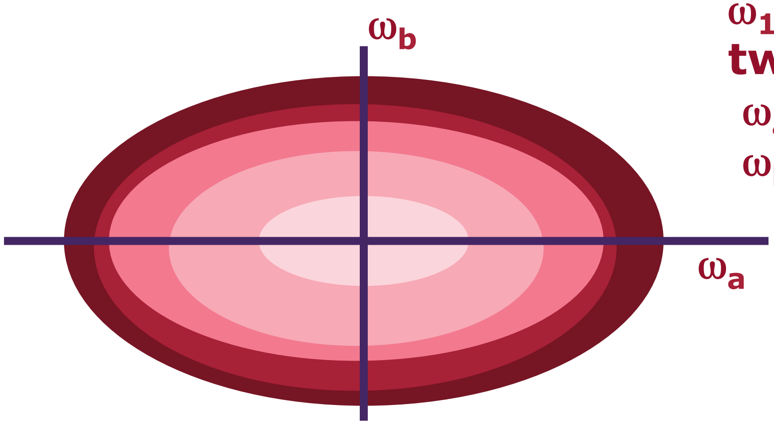
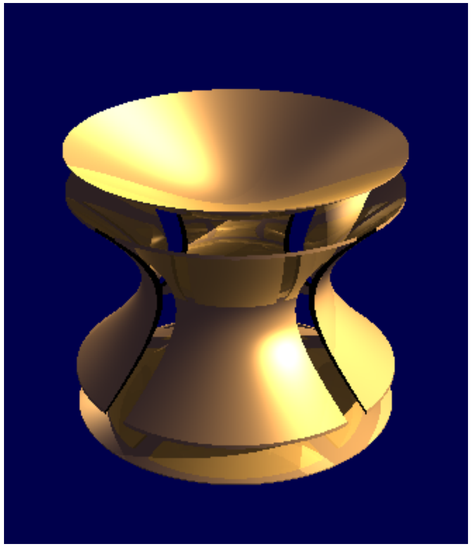
Use of oscillating field to force alignment

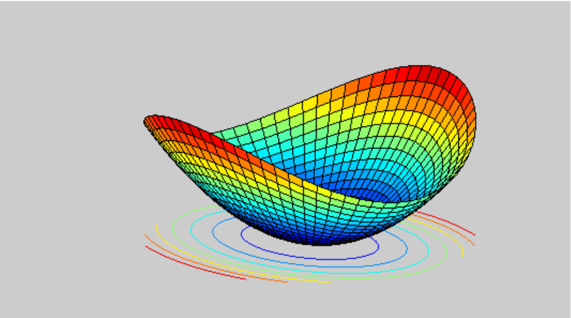


ω_1 splits into two modes:

$\omega_a < \omega_1$
$\omega_b > \omega_1$



- With a small number of cold ions this can be sued to force the particles to line up along the "soft" axis



Relation to the Rotating Wall

- Axialisation is the application of an *oscillating quadrupole at* ω_c
  - It can be decomposed into rotating quadrupoles at $\omega_c/2$
  - In general axialisation is used as a *resonant* process in a single (or few) particle system to couple the centre of mass frequencies
- The Rotating Wall is the application of a *rotating quadrupole* at some frequency ω_R
  - It's used to force a plasma of many particles to rotate at ω_R
  - (it is often also used with a rotating *dipole*)
- If $\omega_R = \omega_c/2$ then then we have Brillouin flow and the techniques are (nearly) equivalent

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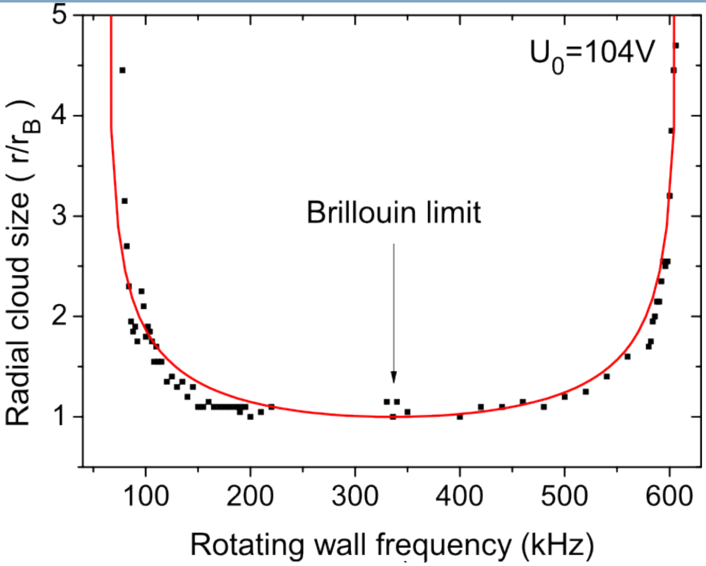
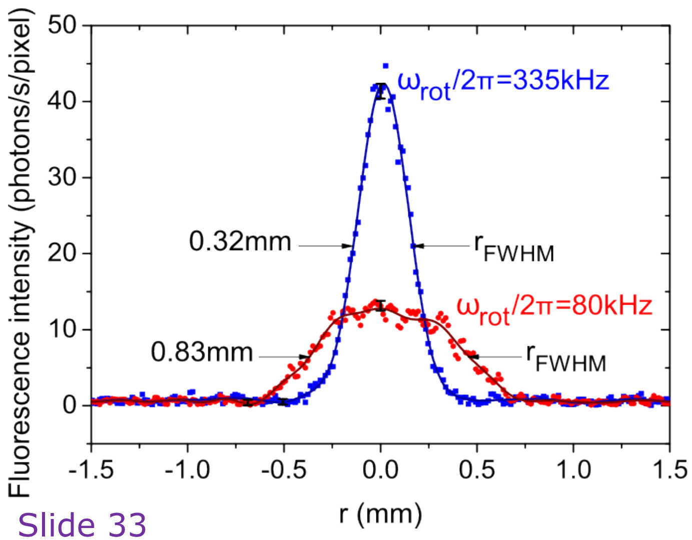
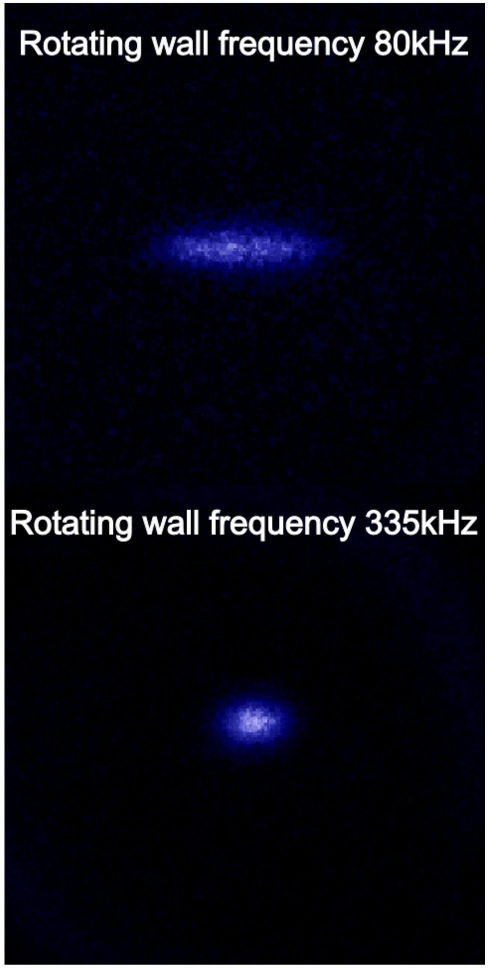
Rotating (dipole) wall results at Imperial





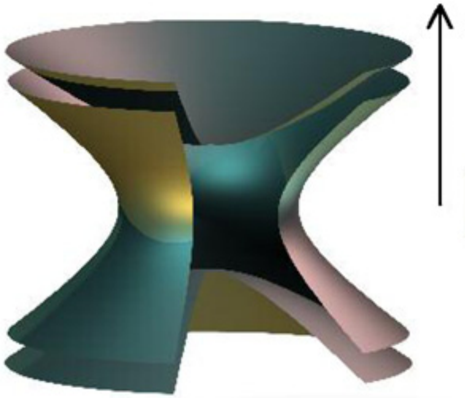


- Image of plasma shows compression is achieved
- Control over full range of plasma rotation frequency
- We can also drive plasma resonances
  - » Diagnostic for plasma parameters

Also extensive work at NIST

Conclusion

- Penning traps are really good for a wide variety of experiments in different fields of physics



- **Thanks for listening!**