Quantum information processing (QIP) with trapped ions

- Geometric phase gates
- Quantum operations with >2 ions
- Quantum simulation

Basic principles and first experiments

Les Houches, January 29, 2015

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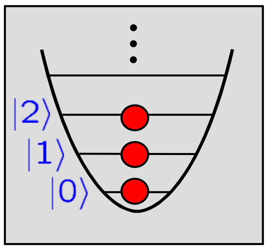
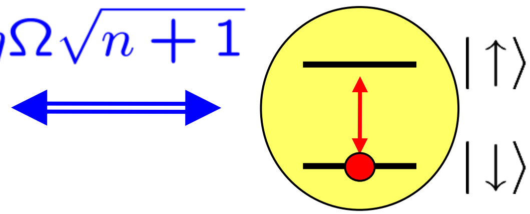
1. Addendum to this morning's lecture: motional state analysis by sideband excitation

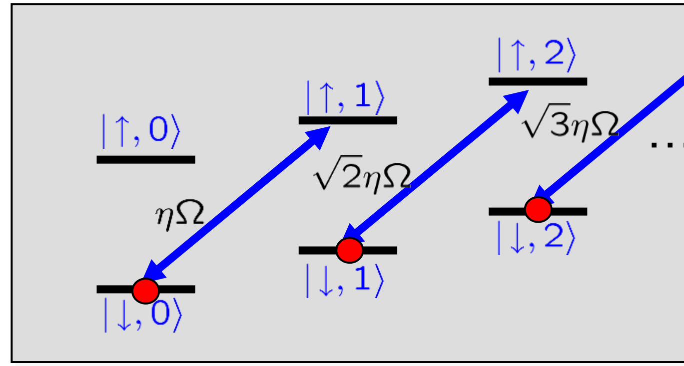
„Blue sideband“ pulses:

$$|\downarrow\rangle|n\rangle \longleftrightarrow |\uparrow\rangle|n+1\rangle$$



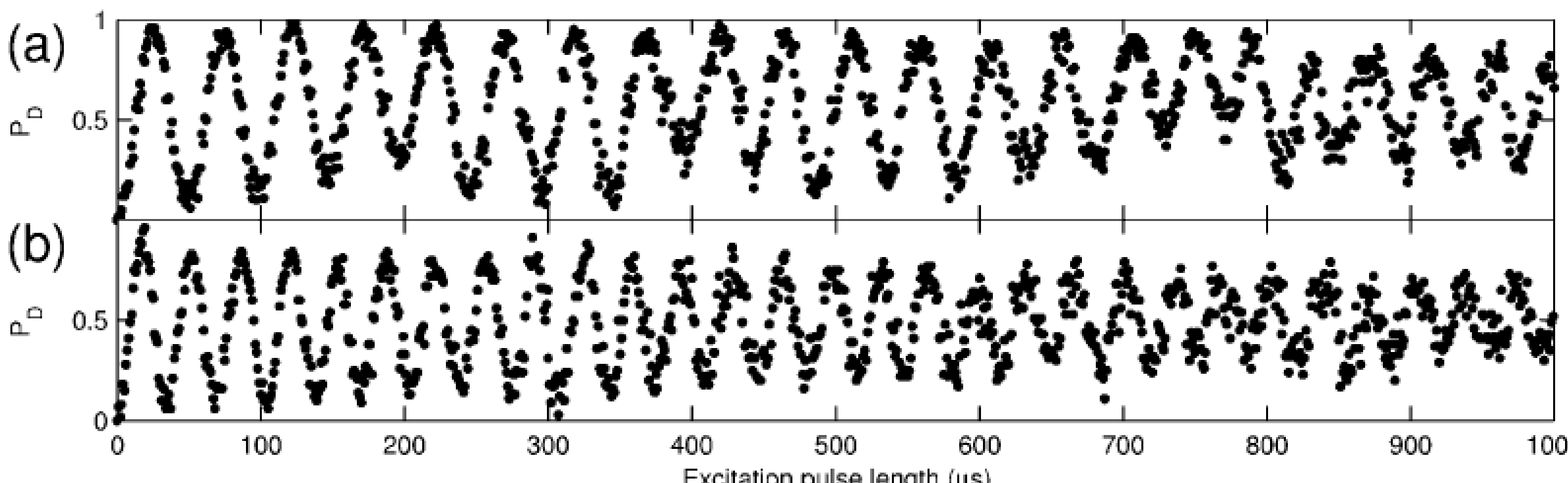
$$\eta\Omega\sqrt{n+1}$$





n=0

n=1

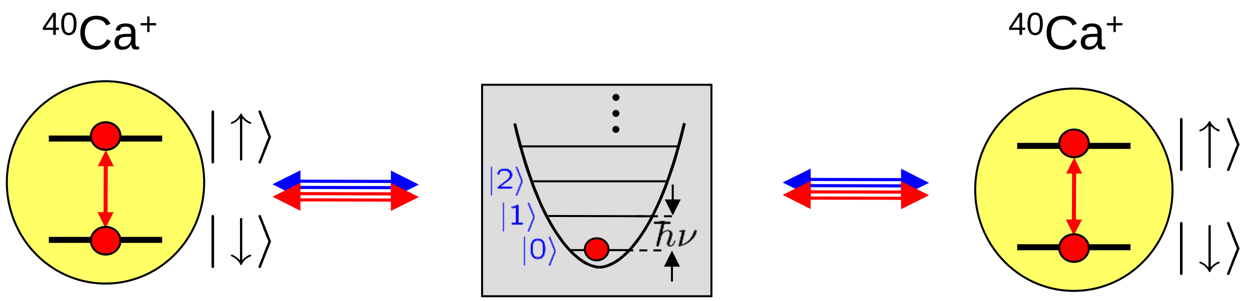


$$p_{\uparrow}(t) = \sum_n p_n \sin^2(0.5\eta\Omega\sqrt{n+1})$$ ⟶ see tutorial question Nr. 2

C. Roos *et al.*, PRL **83**, 4713(1999)

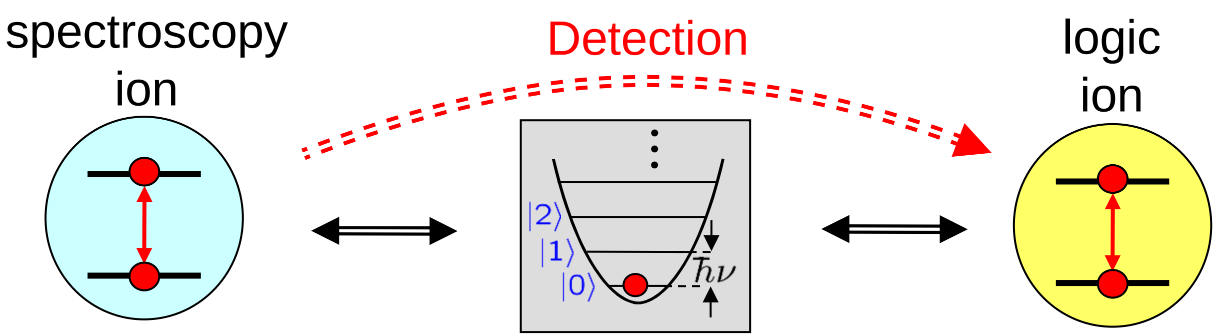
Quantum logic spectroscopy

Entangling gate mediated by harmonic oscillator:



Quantum logic spectroscopy:

Piet Schmidt's lecture



- Spectroscopy ion : object to be investigated
- Logic ion : cooling + state manipulation + detection

Geometric phases:
Another way to understand the Mølmer-Sørensen and other gate

1. Driven harmonic oscillator: phase space picture

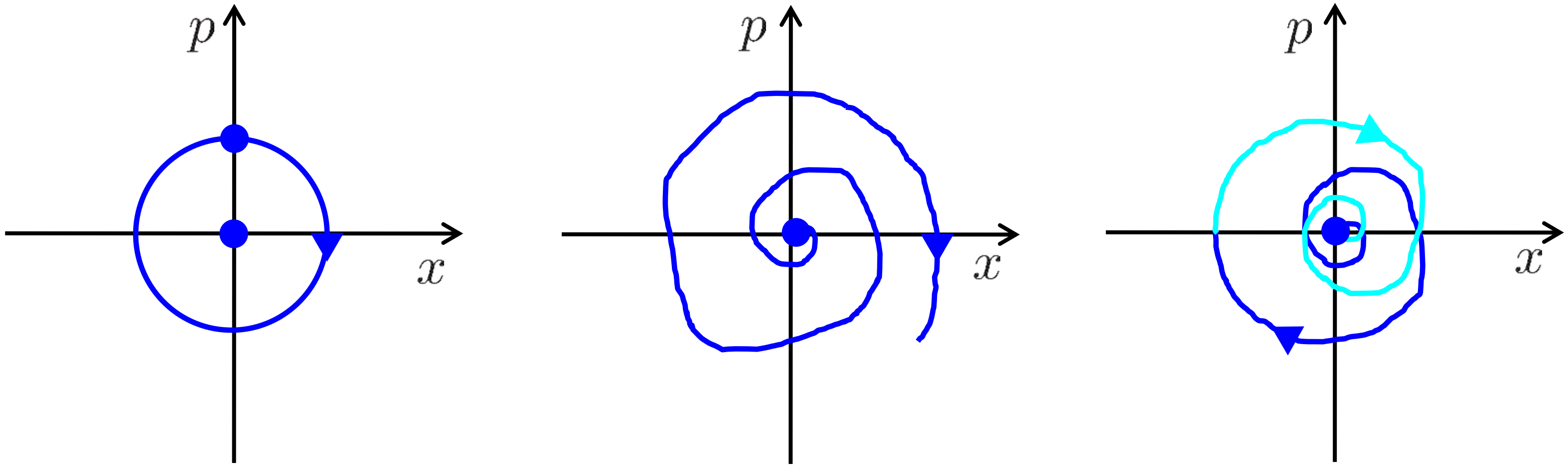
2. Driven harmonic oscillator with qubit state-dependent coupling

Classical harmonic oscillator in phase space

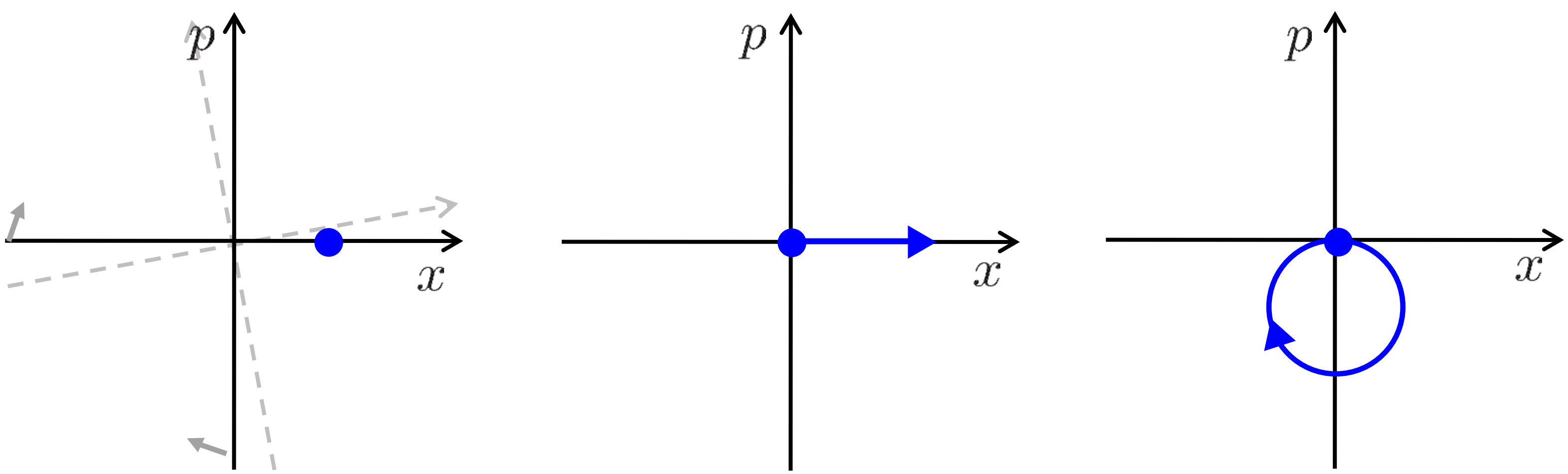
Phase space trajectory of harmonic oscillator

Resonantly driven harmonic oscillator

Off-resonantly driven harmonic oscillator

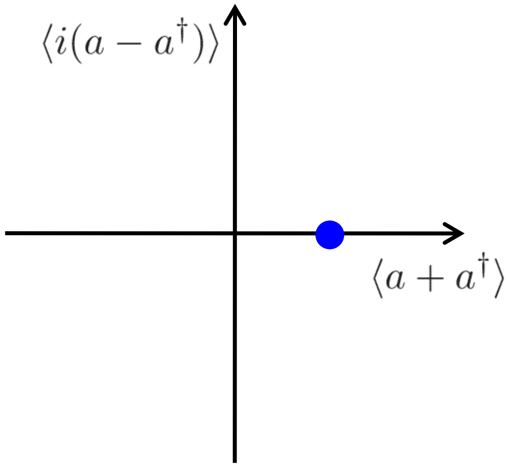


Rotating frame: get rid of the boring oscillation

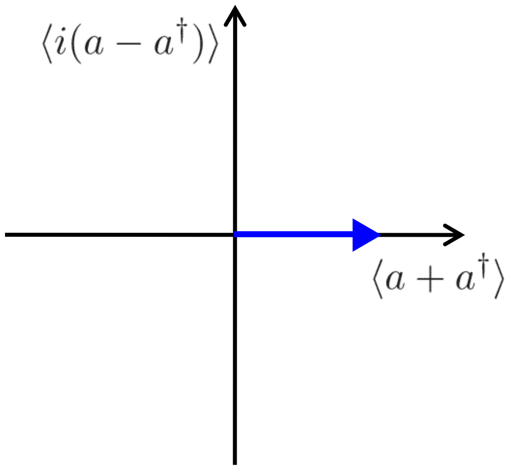


Quantum harmonic oscillator in phase space

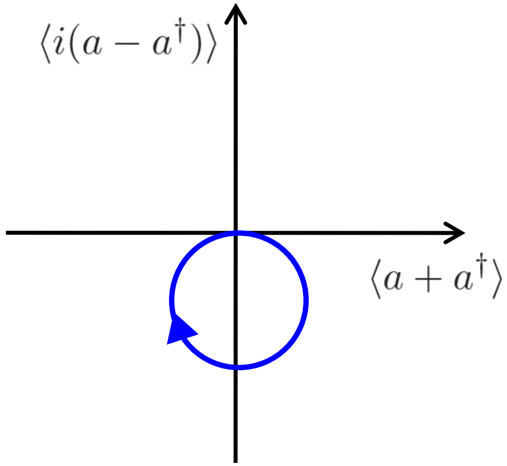
Free harmonic oscillator



Resonantly driven harmonic oscillator



Off-resonantly driven harmonic oscillator



Hamiltonian driving the motion:

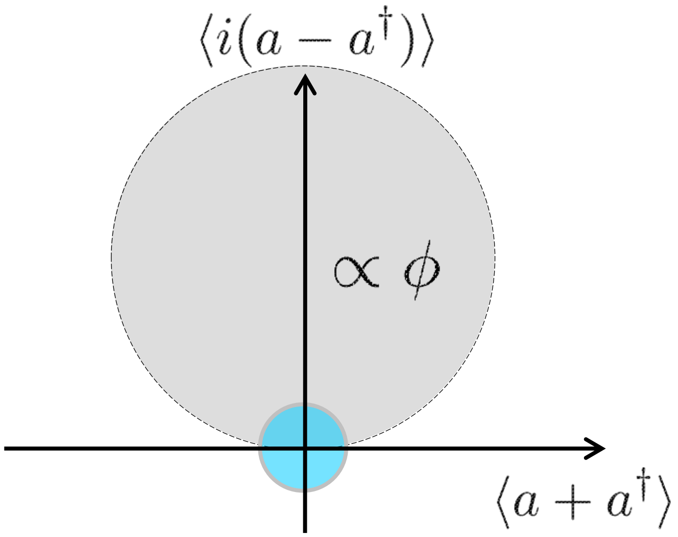
$$H = 0$$

$$H = \hbar\Omega i(a - a^\dagger)$$

$$H = \hbar\Omega i(ae^{i\delta t} - a^\dagger e^{-i\delta t})$$

Geometric phases in the harmonic oscillator

$$H = \hbar\Omega i(ae^{i\delta t} - a^\dagger e^{-i\delta t})$$



Geometric phase by cyclic quantum evolution:

$$|\psi\rangle \longrightarrow e^{i\phi}|\psi\rangle$$

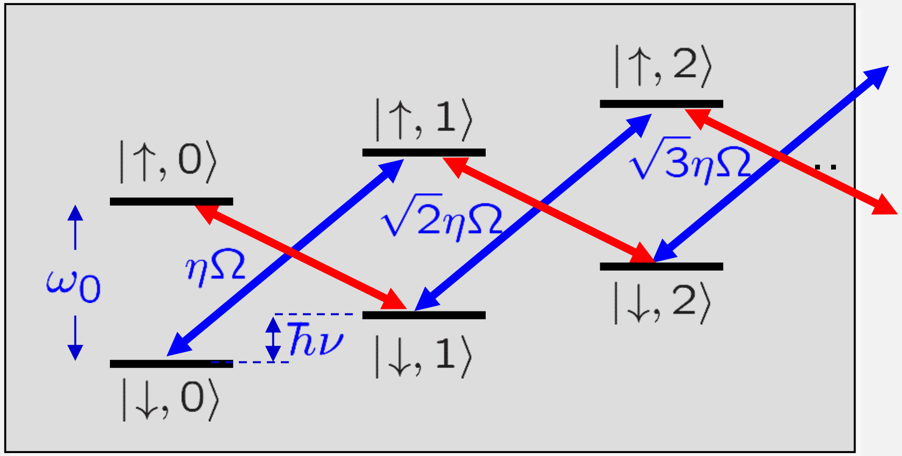
irrelevant global phase

If the phase could be made dependent on the quantum state, the phase would matter:

$$|\psi_1\rangle + |\psi_2\rangle \longrightarrow e^{i\phi_1}|\psi_1\rangle + e^{i\phi_2}|\psi_1\rangle$$

$\phi \propto$ enclosed area $\propto \Omega^2$ $\longrightarrow$ we need a state-dependent force

State-dependent forces for entangling gates

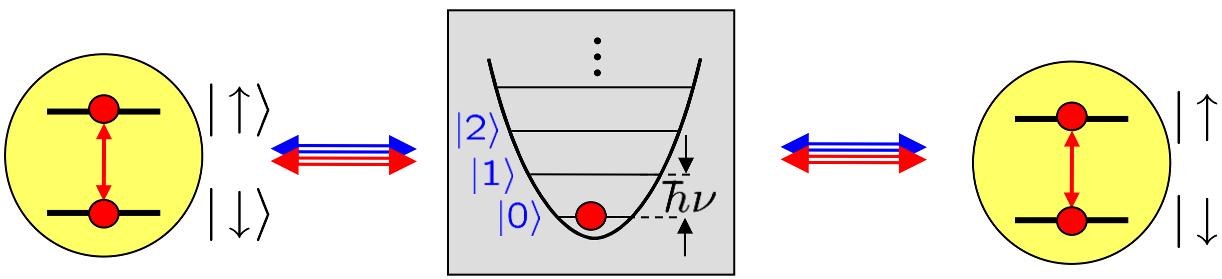


Bichromatic excitation

$$\omega_{laser} = \omega_0 \pm (\nu + \delta)$$

$$H \propto \ (ae^{i\delta t} + a^\dagger e^{-i\delta t})\sigma_x$$

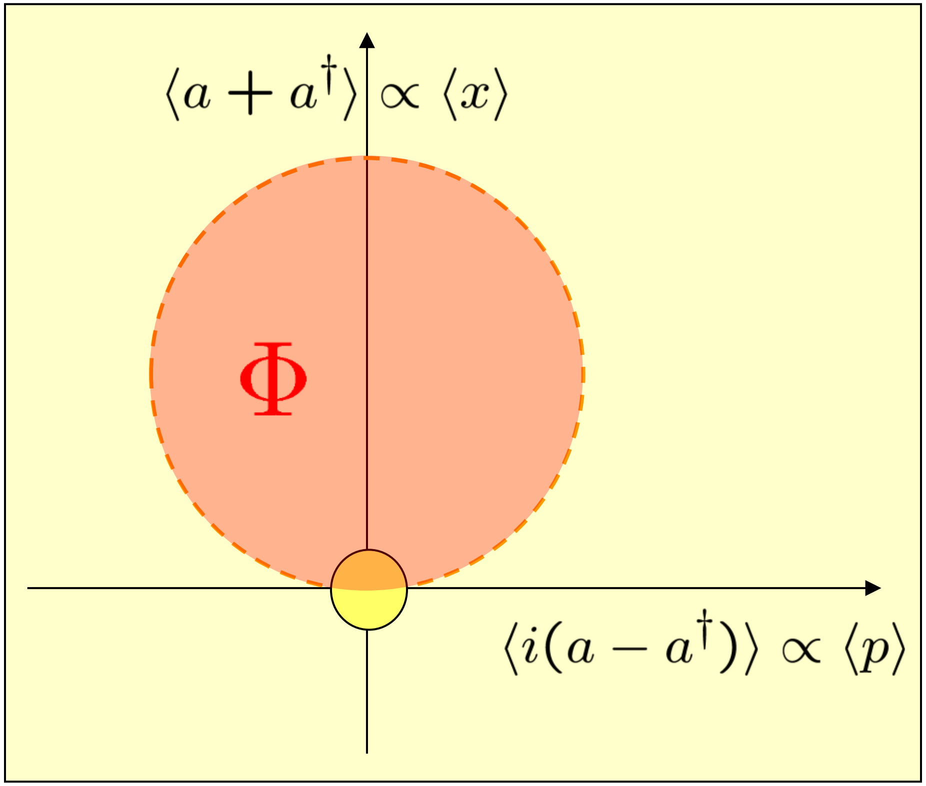
Two ions: $$H \propto \ (ae^{i\delta t} + a^\dagger e^{-i\delta t})(\sigma_x^{(1)} + \sigma_x^{(2)})$$

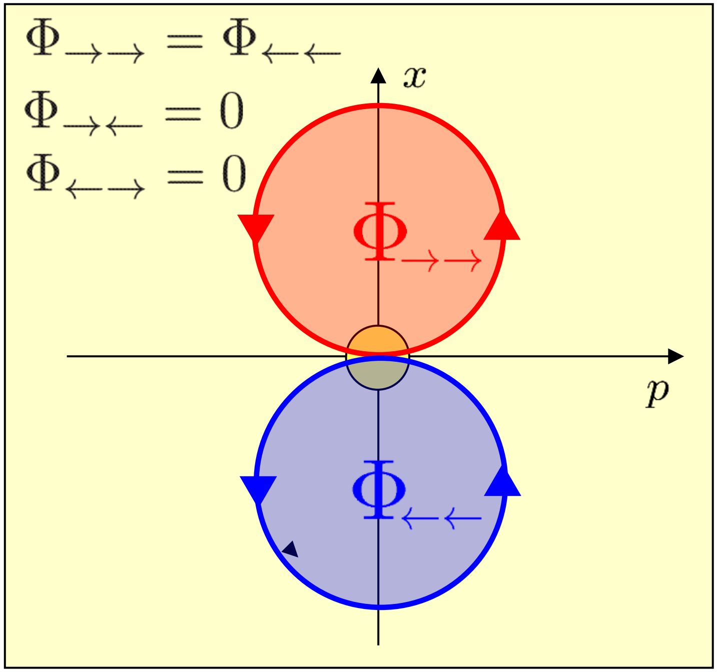


A. Sørensen, K. Mølmer,
Phys. Rev. Lett. **82**, 1971 (1999)

Geometric phase gate

$$H(t) \propto (ae^{i\delta t} + a^\dagger e^{-i\delta t})(\sigma_x^{(1)} + \sigma_x^{(2)})$$



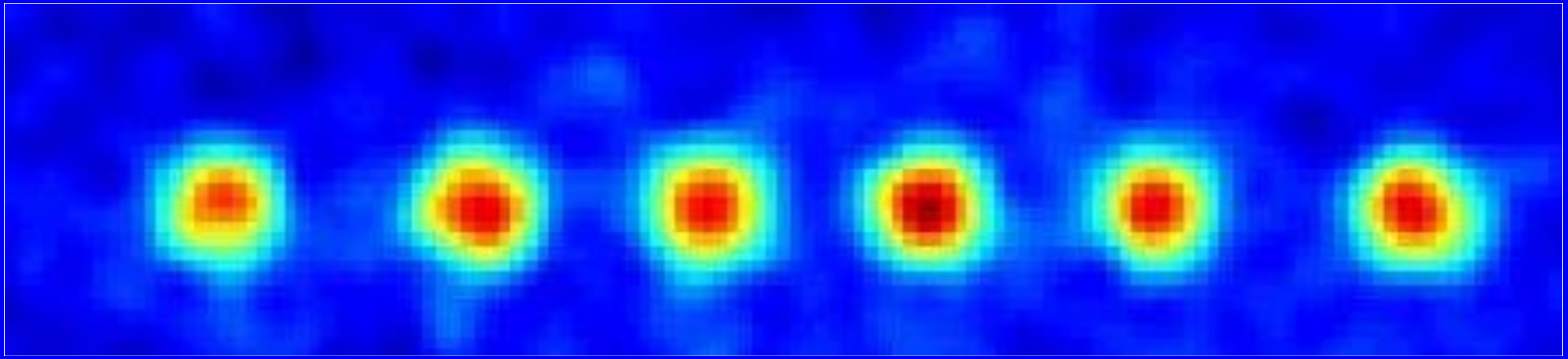


$$H_{eff} = \hbar\Omega\sigma_x^{(1)}\sigma_x^{(2)}$$

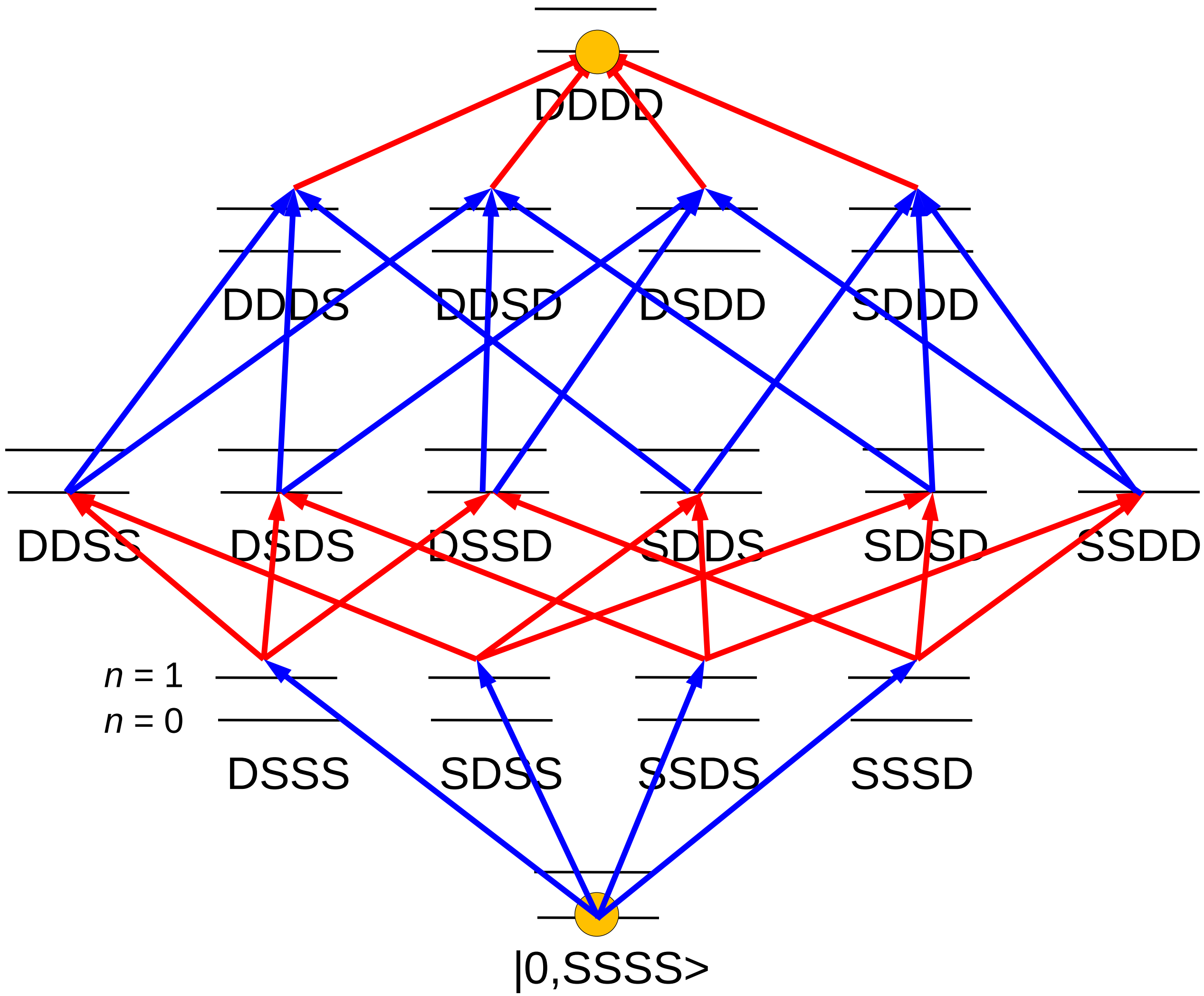
$\longrightarrow$ The phase Φ depends nonlinearly on the internal states of the ions

Quantum physics with more than 2 ions

GHZ-states
Scaling the system up

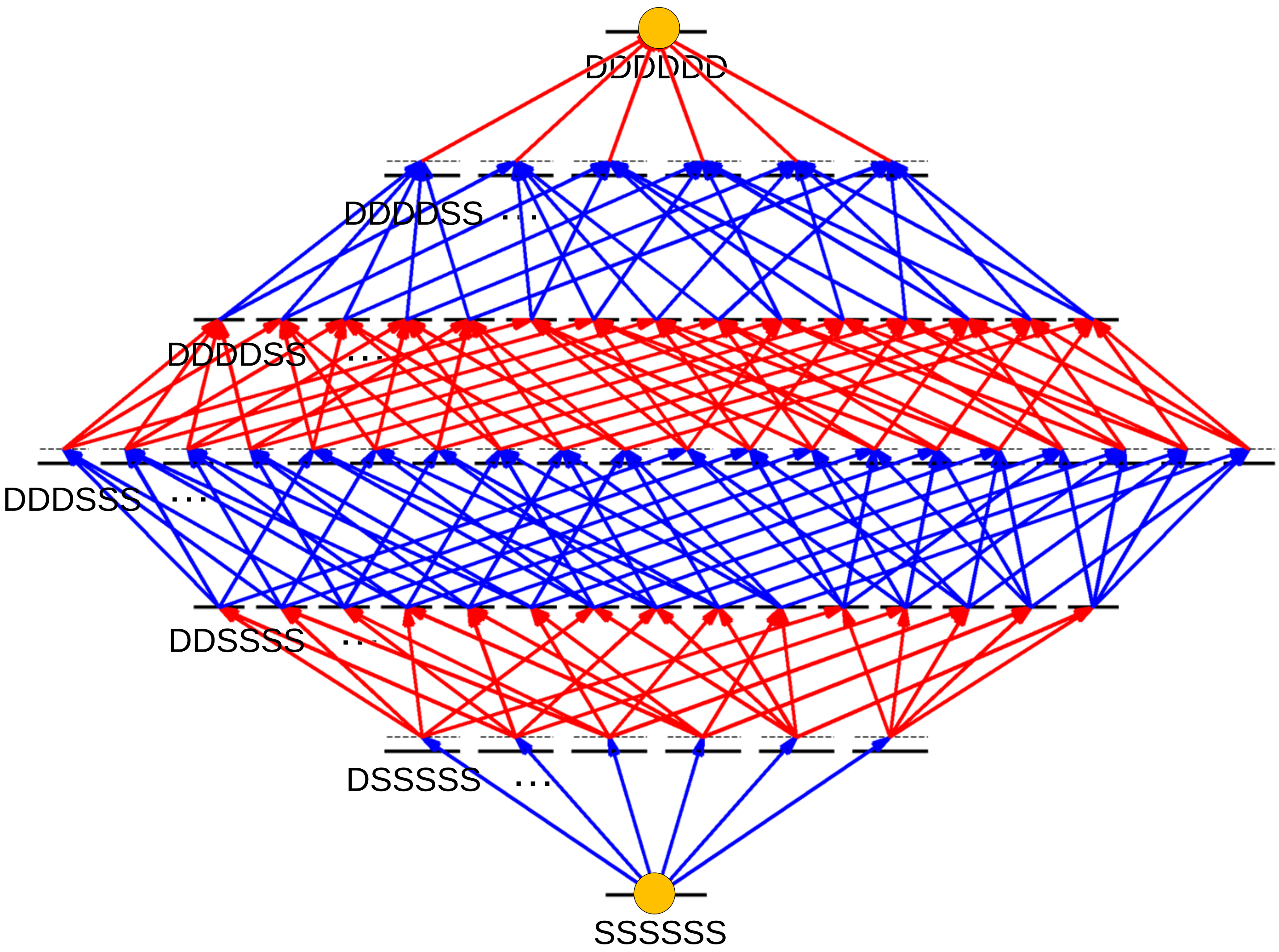


Creating GHZ-states with 4 ions



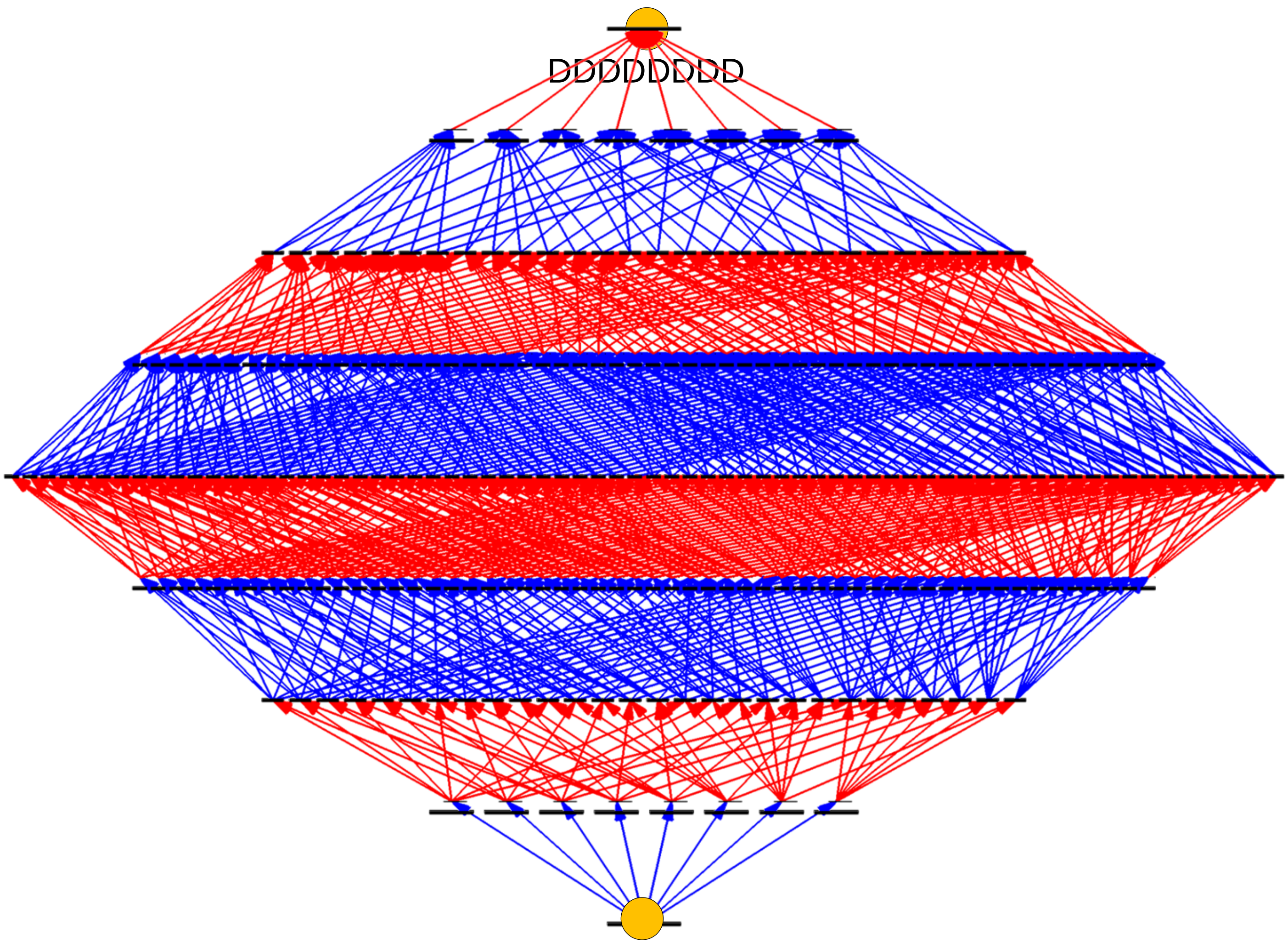
$$|SSSS\rangle \longrightarrow (|SSSS\rangle + |DDDD\rangle)/\sqrt{2}$$

Creating GHZ-states with 6 ions



$$|SSSSSS\rangle \longrightarrow (|SSSSSS\rangle + |DDDDDD\rangle)/\sqrt{2}$$

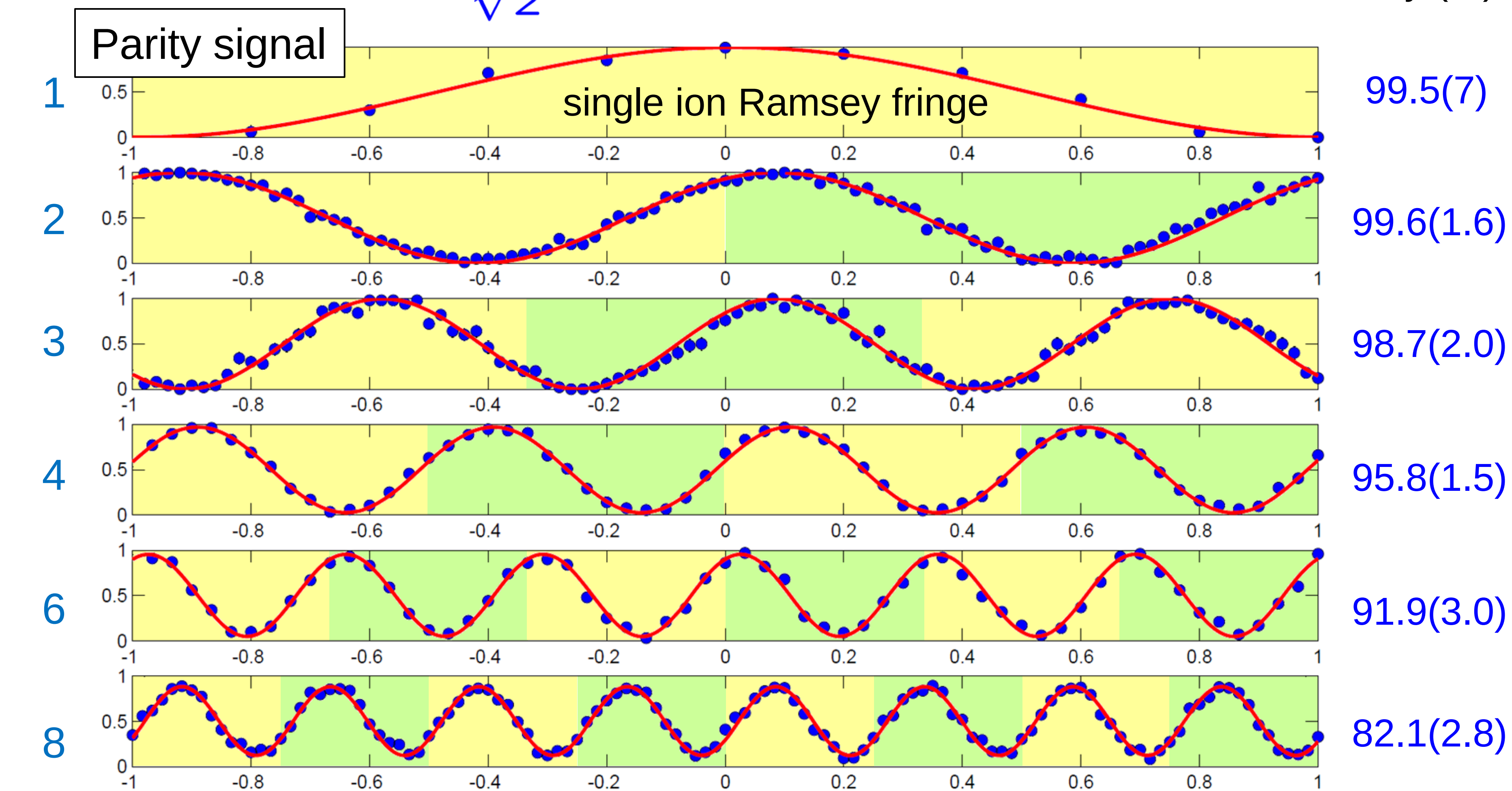
Creating GHZ-states with 8 ions



$$|SSSSSSSS\rangle \longrightarrow (|SSSSSSSS\rangle + |DDDDDDDD\rangle)/\sqrt{2}$$

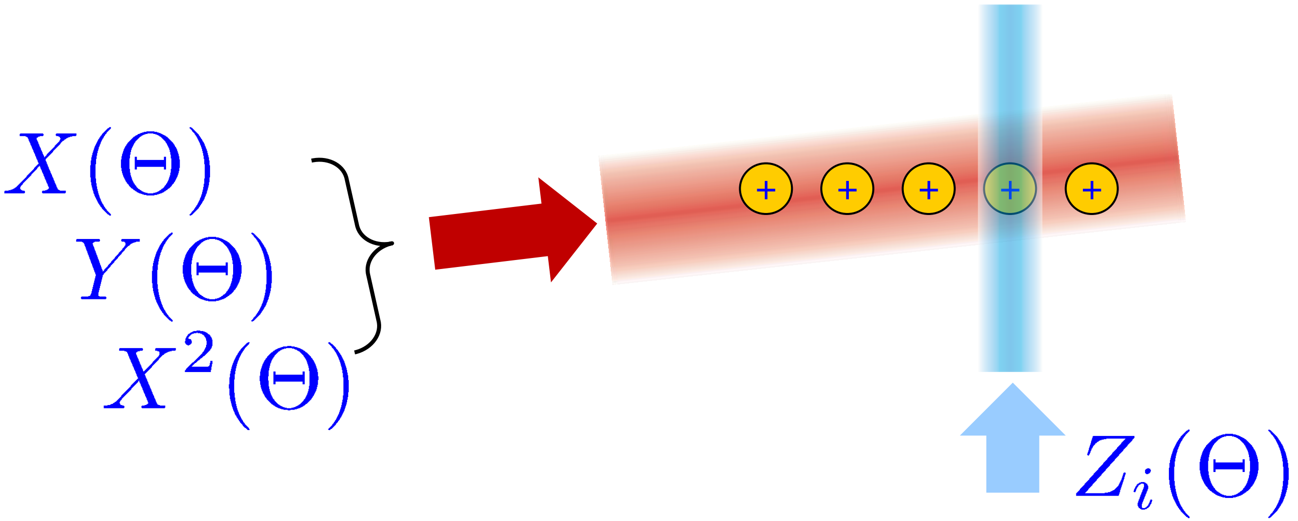
N - qubit GHZ state generation

$$\Psi = \frac{1}{\sqrt{2}}(|SS\ldots S\rangle + |DD\ldots D\rangle)$$



T. Monz et al., PRL 106, 130506 (2011)

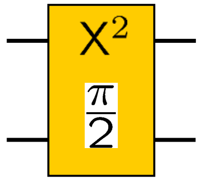
Quantum gate operations: universal toolbox



$X(\Theta)$, $Y(\Theta)$	collective local operations
$Z_i(\Theta)$	single-qubit z-rotations
$X^2(\Theta)$	Mølmer-Sørensen entangling operation

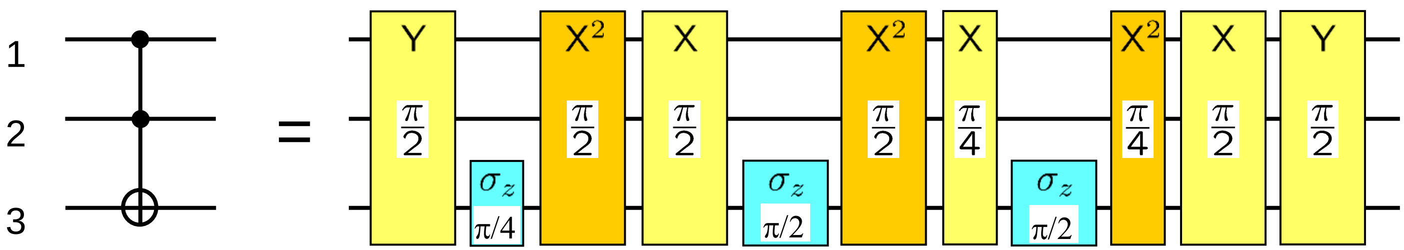
Entangling gates for quantum algorithms

Entangling gate



Building block for realizing quantum algorithms

Example: quantum Toffoli gate



V. Nebendahl *et al*., PRA **79**, 012312 (2009)

Current experiments: 2 to 7 qubits, > 100 gate operations

J. Barreiro *et al*., Nature **486**, 470 (2011), B. Lanyon *et al*., Science **334**, 57 (2011)

Scaling the ion trap quantum processor …

- more ions, larger traps, phonons carry quantum information

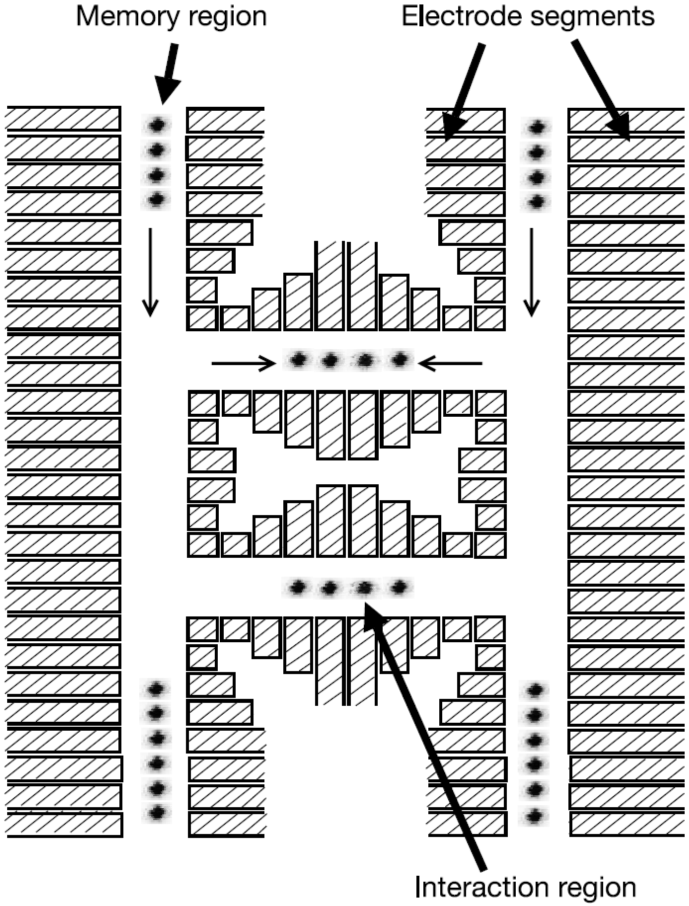
  Cirac-Zoller, slow for many ions (few 10 ions may be possible)

- move ions, carry quantum information around

  Kielpinski et al.,
  Nature **417**, 709 (2002)

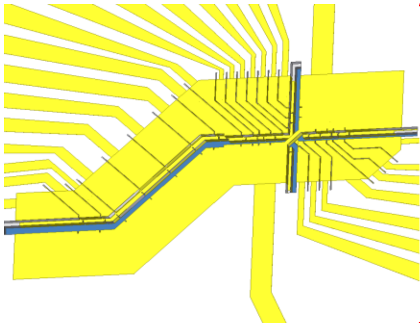
  requires small,
  integrated trap structures,

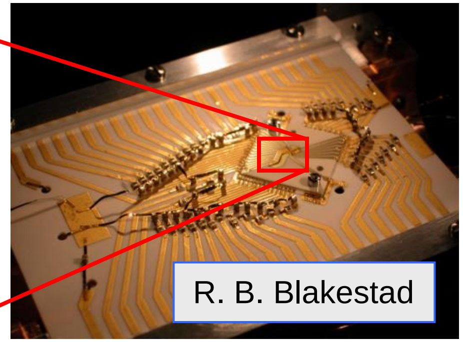
  miniaturized optics
  and electronics



Microchip traps at NIST Boulder

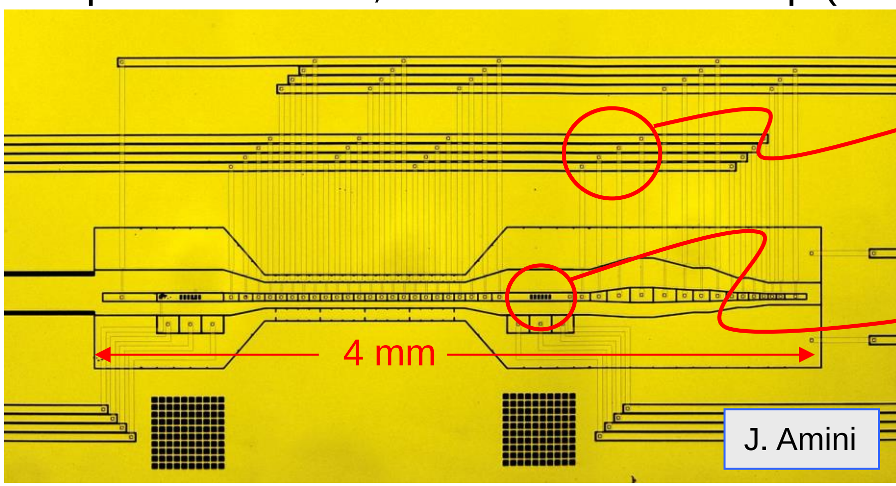
2-layer, 2-D, X-junction, 18 zones (Au on Al_2O_3)





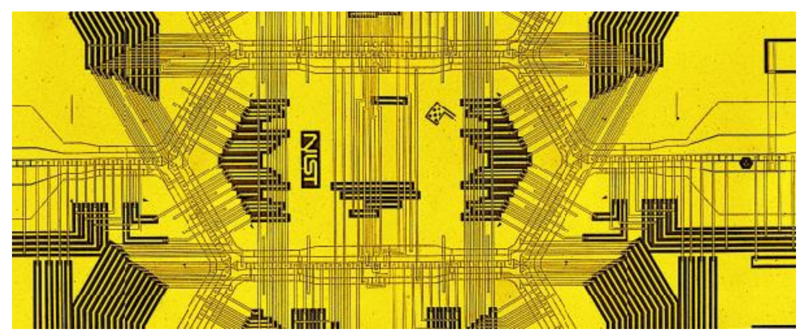
- Transport through junction ($^9Be^+$,$^{24}Mg^+$)
  - ◊ minimal heating ~ 20 quanta
  - ◊ transport error < 3 x 10^{-6}

transport in 40-zone, surface-electrode trap (Au on quartz)



- multi-layer structures
- back-side loading of ions (prevents electrode shorting)

200 – zone "racetrack"



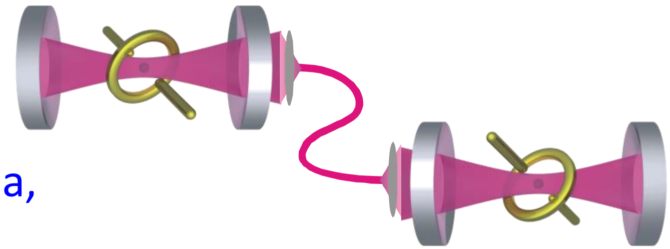
Scaling the ion trap quantum computer: other approaches

- cavity QED: atom – photon interface, use photons for networking

  J. I. Cirac et al., PRL **78**, 3221 (1997)

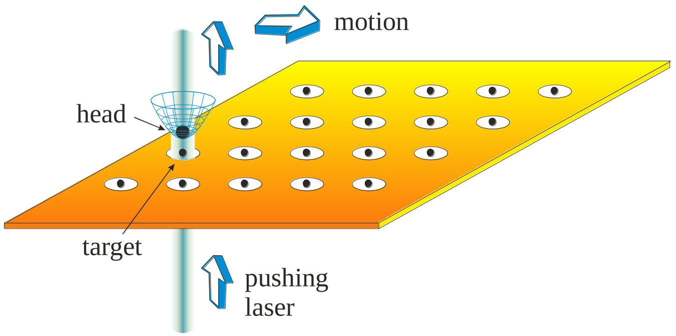
  T. Northup et al., Univ. Innsbruck

  further exp: JQI, Sussex, Bonn, Duke, Sandia, Saarbrücken,SK telekom…



- trap arrays, using single ion as moving head

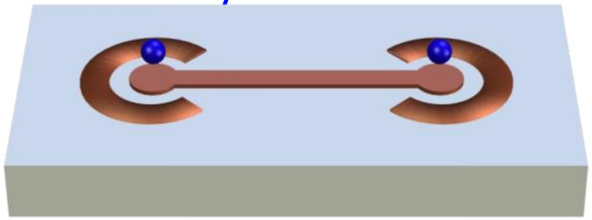
  I. Cirac und P. Zoller, Nature **404**, 579 (2000)

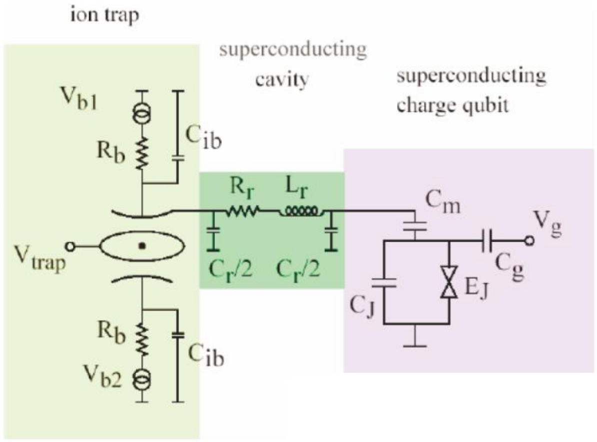


- ion – solid state qubits (e.g. charge qubit)

  L. Tian et al., PRL **92**, 247902 (2004)

  H. Häffner et al., UC Berkeley





- …more ideas …?

Quantum information processing

Quantum information

Investigating resources for information processing tasks, entanglement characterization,…

Quantum simulation

Investigating many-body Hamiltonians using well-controlled quantum systems

Quantum computing

Quantum algorithms for efficient computing

Quantum metrology

Entanglement-enhanced measurements

→ Piet Schmidt‘s lecture

Quantum communication

Quantum networks

→ Michael Drewsen‘s lecture

Simulating quantum physics

If there are quantum algorithms that run exponentially faster than their classical counterparts:

What stops us from simulating a quantum computer on a classical computer to find a solution in a much shorter time than with the classical algorithm?

Obstacle: There is no solution for simulating general quantum dynamics efficiently on a classical computer.

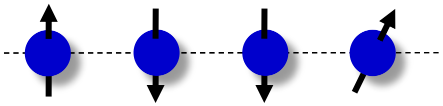
⇨ Maybe we should use a quantum processor to simulate the physics of quantum systems which is hard to simulate on classical computers

Quantum simulation

Quantum simulations with trapped ions

Simulating quantum-many body systems

How can we study the physics of quantum-many body systems?



Approaches:

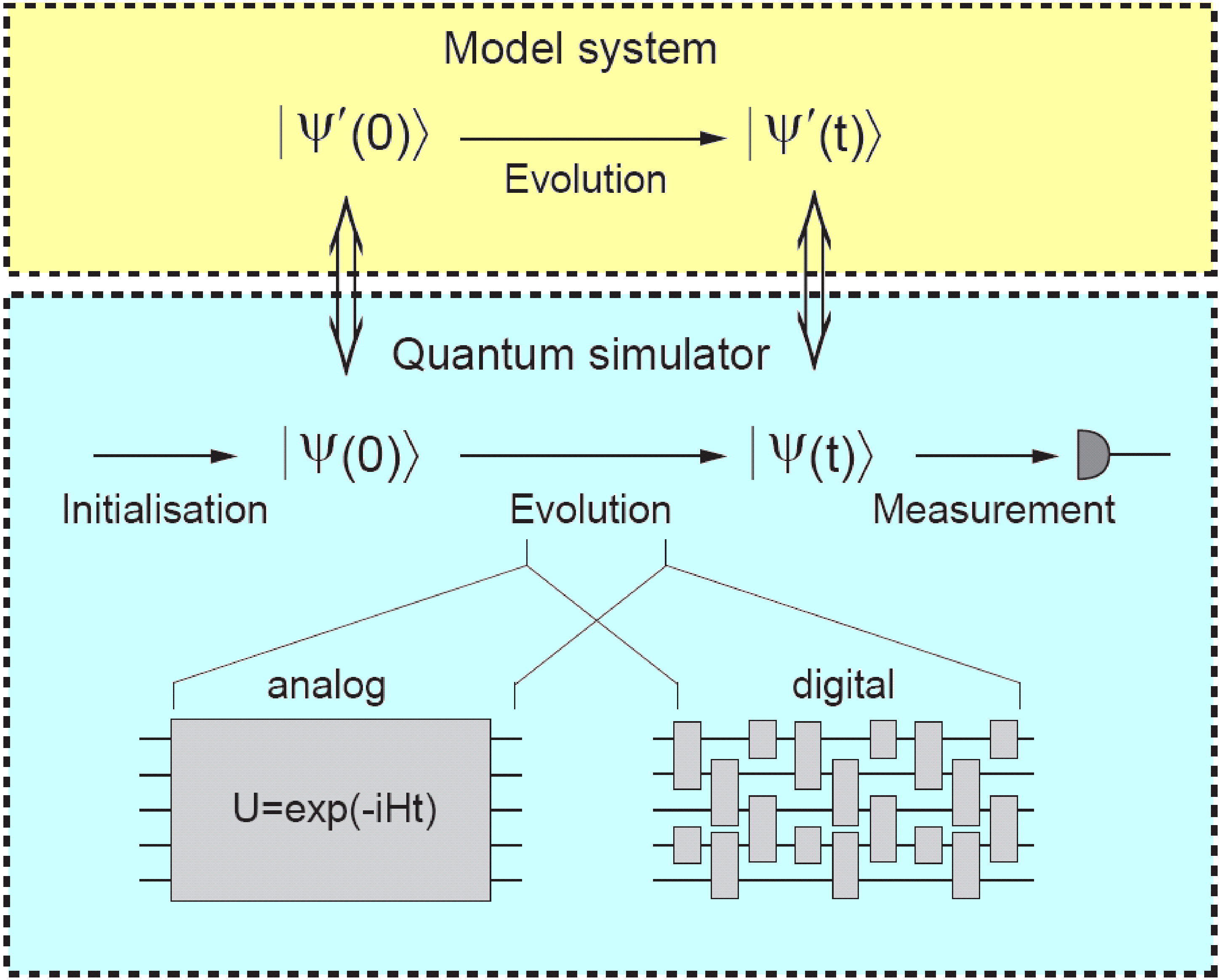
- In some cases: Analytical techniques
- Numerical simulation methods on a computer

But: Exponential scaling of resources with the system size severely restricts the number of particles that can be simulated.

Interacting spins: exact diagonalization techniques limited to N ~ 40 spins

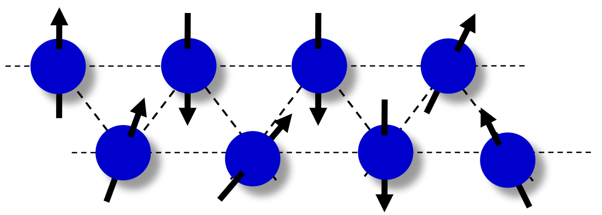
- Feynman (1982), Lloyd (1996): **Quantum simulators**
  Use a precisely controlled quantum system for simulating a model of interest

Quantum simulation principle



Simulating quantum spin systems

Hamiltonians:



- Ising model (with transverse field)

$$H = \frac{1}{2}\sum_{i,j} J_{ij}\sigma_i^x\sigma_j^x + B\sum_i \sigma_i^z$$

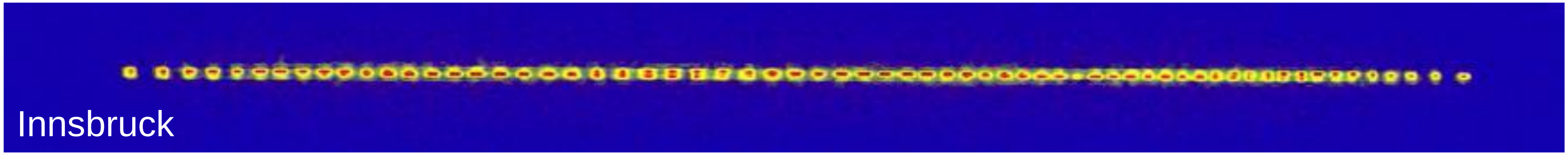
- XY model

$$H = \frac{1}{2}\sum_{i,j} J_{ij}^x\sigma_i^x\sigma_j^x + \frac{1}{2}\sum_{i,j} J_{ij}^y\sigma_i^y\sigma_j^y + B\sum_i \sigma_i^z$$

- Heisenberg model

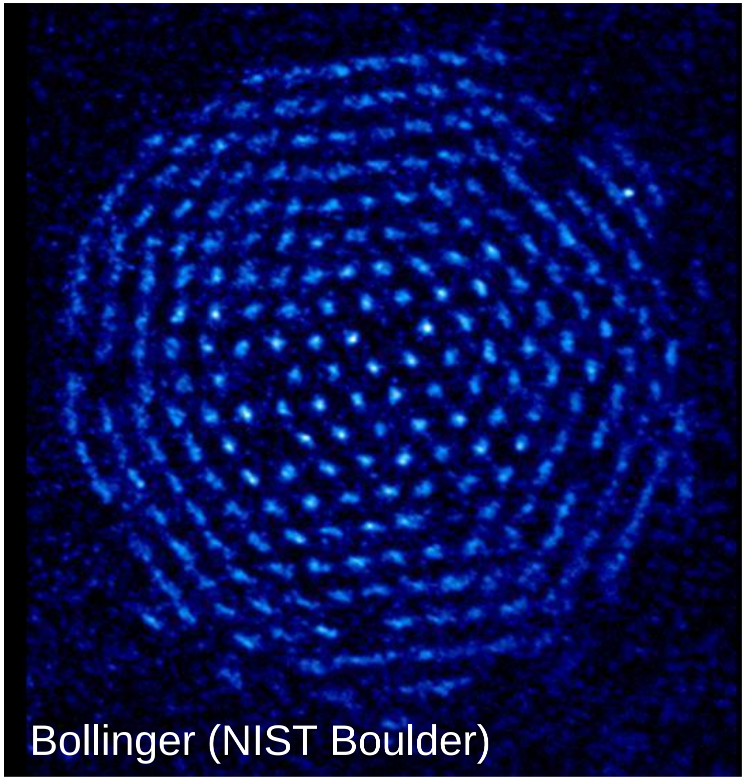
$$H = \frac{1}{2}\sum_{i,j} J_{ij}^x\sigma_i^x\sigma_j^x + \frac{1}{2}\sum_{i,j} J_{ij}^y\sigma_i^y\sigma_j^y + \frac{1}{2}\sum_{i,j} J_{ij}^z\sigma_i^z\sigma_j^z + B\sum_i \sigma_i^z$$

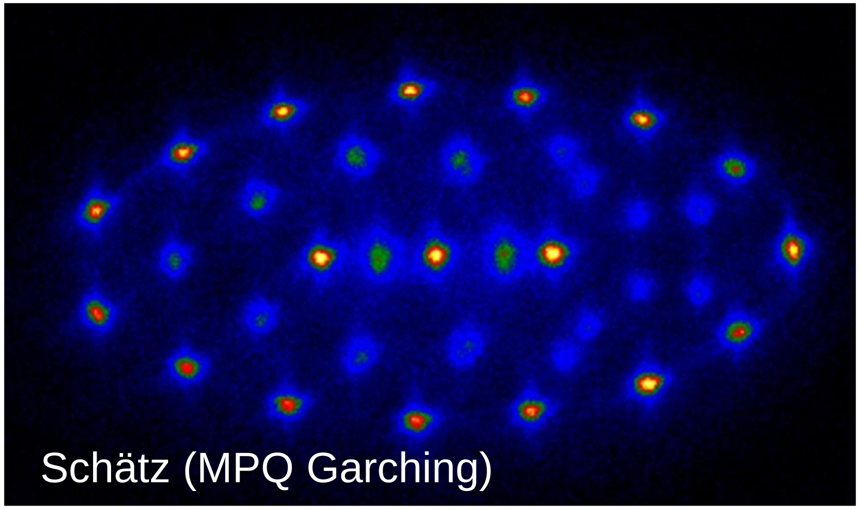
Trapped ions for simulating quantum magnetism



Challenges:

- Controlling the geometry
- Keeping decoherence low
- Engineering interactions





Trapping geometries: rf traps

Linear traps: Harmonic anisotropic potentials

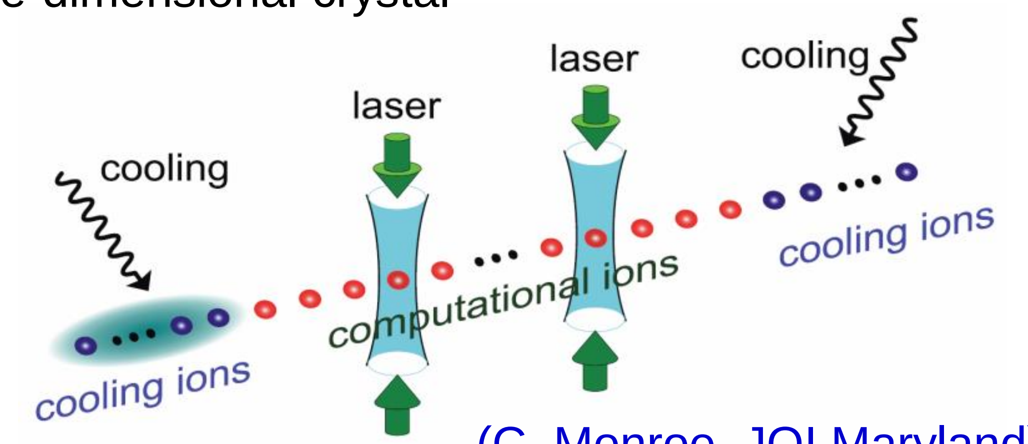
N = 2…40(?) ions in a one-dimensional crystal

$$\frac{\omega_r}{\omega_z} > 0.77\frac{N}{\log N}$$

longer crystals require very anisotropic potentials

Segmented microtraps: Anharmonic potentials for linear ion strings with equal spacing

N > 100(?) ions in a one-dimensional crystal

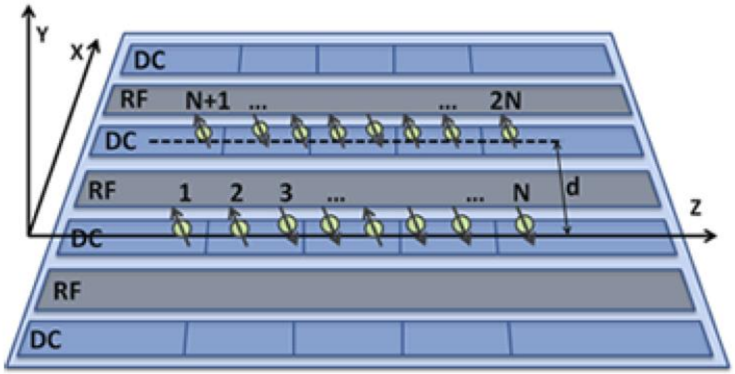


(C. Monroe, JQI Maryland)

G.-D. Lin, et al., Europhys. Lett. 86, 60004 (2009)

Trapping geometries: rf traps

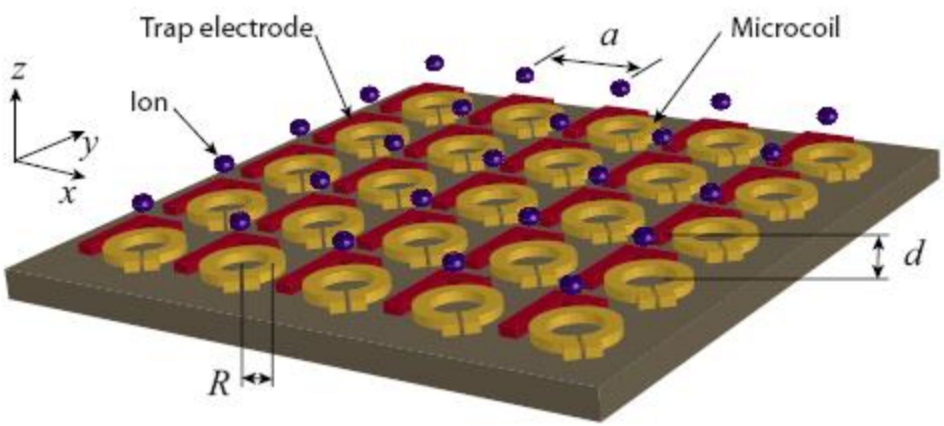
Segmented microtraps for Potentials with multipole trapping sites



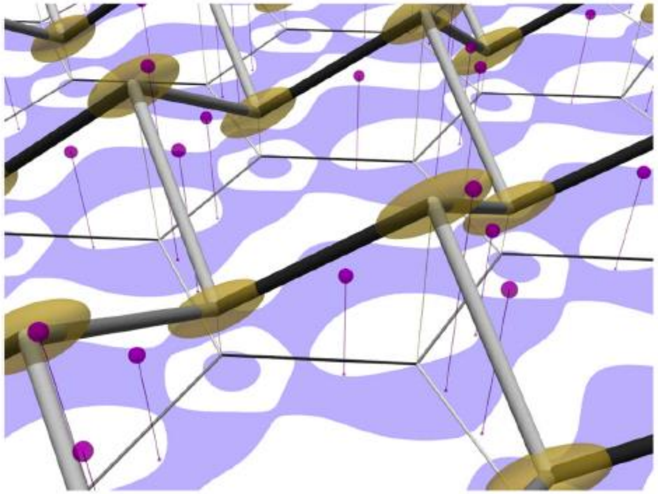
Multiple linear strings in close proximity

J. Welzel *et al*., EPJD **65**, 285 (2011)

2d-lattices of trapping sites



Chiaverini and Lybarger, Phys. Rev. A **77**, 022324 (2008)

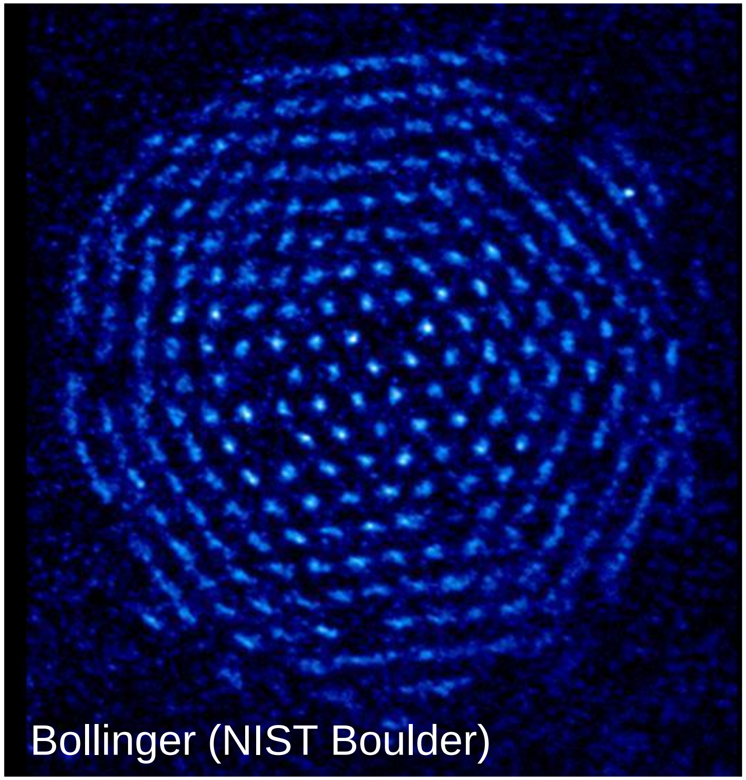


R Schmied *et al*, PRL **102**, 233002 (2009)

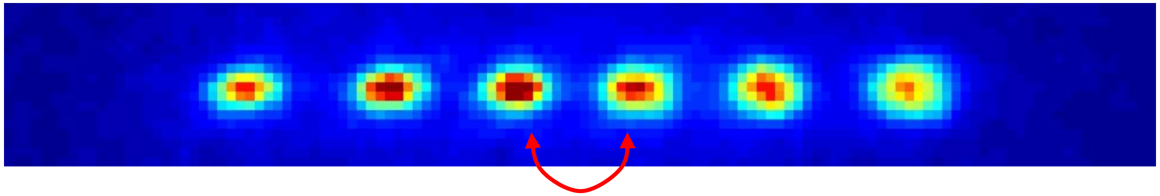
Trapping geometries: Penning traps

Penning trap: anisotropic potential for trapping 2d crystals

- N≈ 100 – 300 ions possible
- low internal state decoherence
- challenge: demonstrate same kind of quantum control as in rf-traps



How to engineer spin-spin interactions



$\sigma_z \otimes \sigma_z$

- Direct state-dependent forces between the ions (as in molecules) ?

  Reduce the ion-ion distance to a_0 ? Impossible!

  Make the ions bigger ? Difficult, but possible. Rydberg ions !

  (has been demonstrated for neutral atoms, but is challenging for ions because of the higher energies required for exciting into Rydberg state)

- Use external fields for engineering state-dependent forces

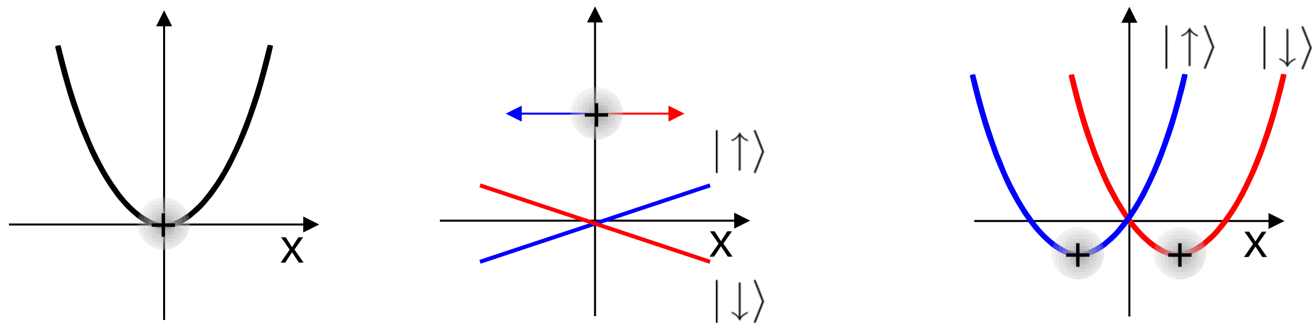
  Spin-spin interactions mediated by Coulomb interaction

  (a) Laser interactions: Absorption and stimulated emission transfer momentum!

  (b) Magnetic field gradients: Position-dependent Zeeman shifts

Spin-spin couplings by magnetic field gradients

Trapping potential   Zeeman energy



$$H = \hbar\nu a^\dagger a \; + \; \underbrace{\mu B' \hat{x} \sigma_z / 2}_{\propto\, (a + a^\dagger)\sigma_z}$$

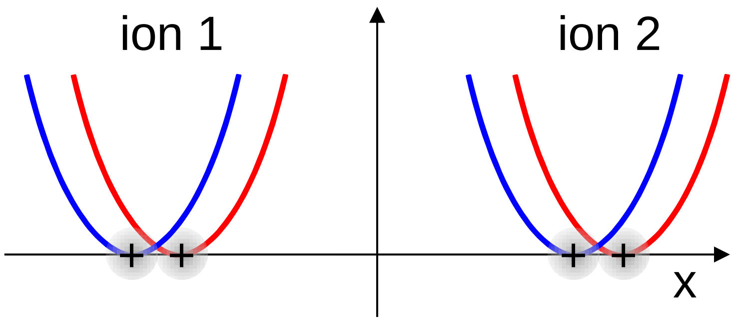
spin-dependent force

In a magnetic field gradient, any microwave-induced spin flip also couples to the vibrational state.

Mintert + Wunderlich, PRL **87**, 257904 (2001)

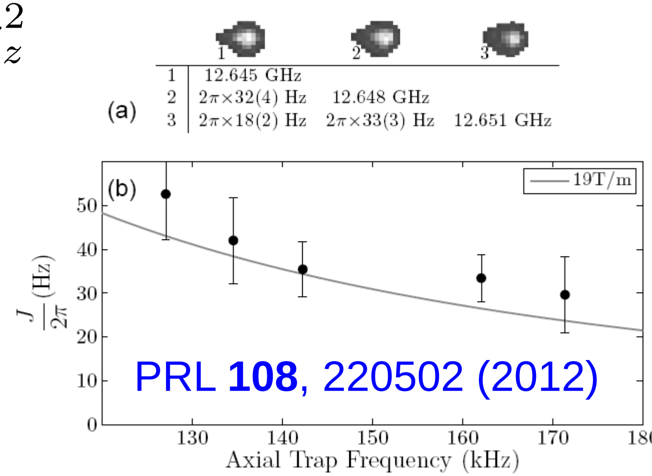
Spin-spin couplings by magnetic field gradients

(a) Ion crystals in a static field gradient



Potential energy (trap + Coulomb energy) depends on both internal states

$$H = J\sigma_z^1\sigma_z^2$$



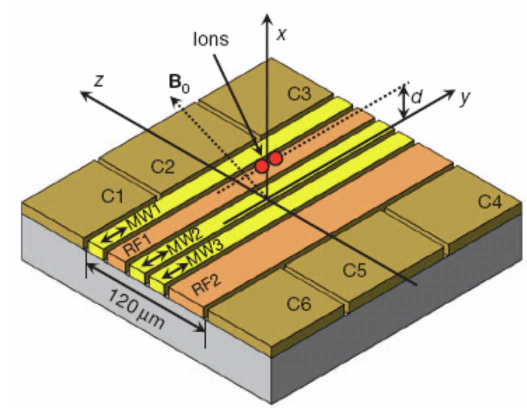
PRL **108**, 220502 (2012)

Wunderlich (Siegen)

(b) Ion crystals in a oscillating field gradient

$$H(t) \propto (ae^{-i\delta t} + a^\dagger e^{i\delta t})\sigma_z \longrightarrow H_{eff} = J\sigma_z^1\sigma_z^2$$

Off-resonant excitation of a vibrational mode with a state-dependent potential



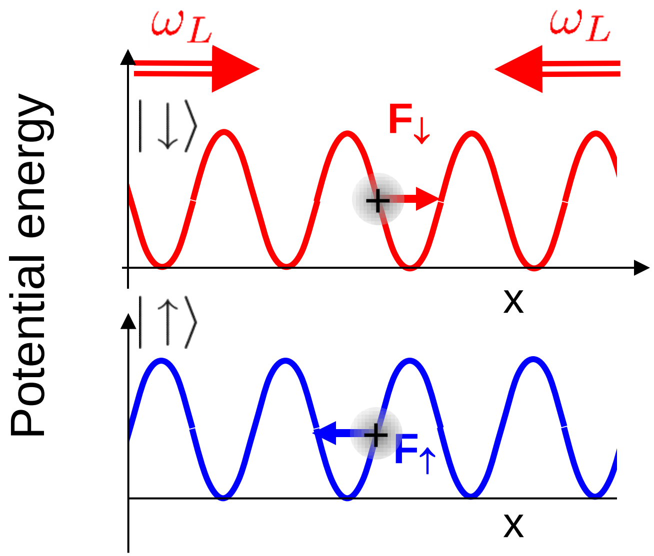
C. Ospelkaus *et al*., Nature **476**, 181 (2011)

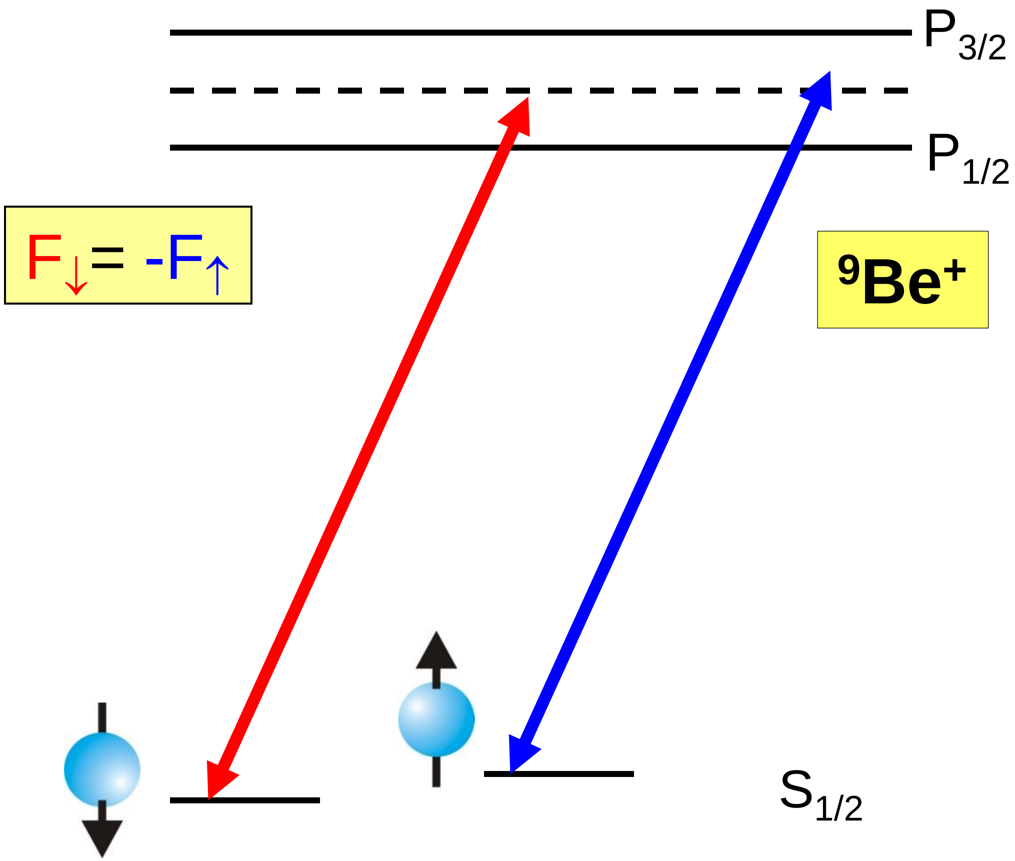
Wineland (NIST Boulder)

Spin-spin couplings by laser-induced potentials

1. Conditional phase shift gate

Spatial light shifts by off-resonant coupling to P-states in a standing wave





Spin-dependent force

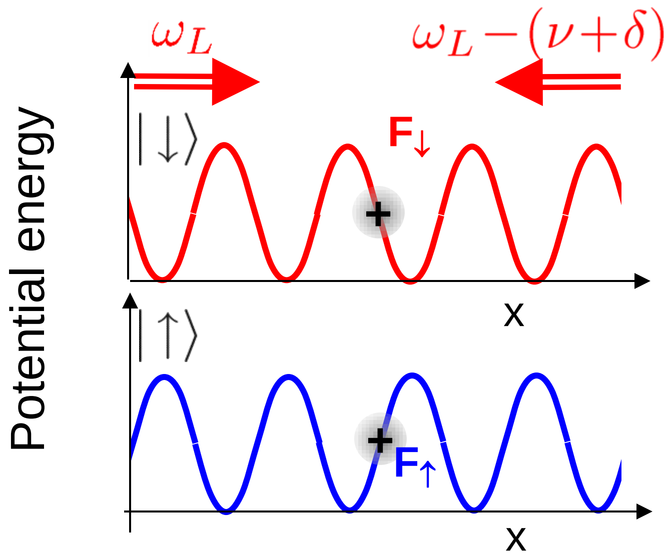
$$H \propto (a + a^\dagger)\sigma_z$$

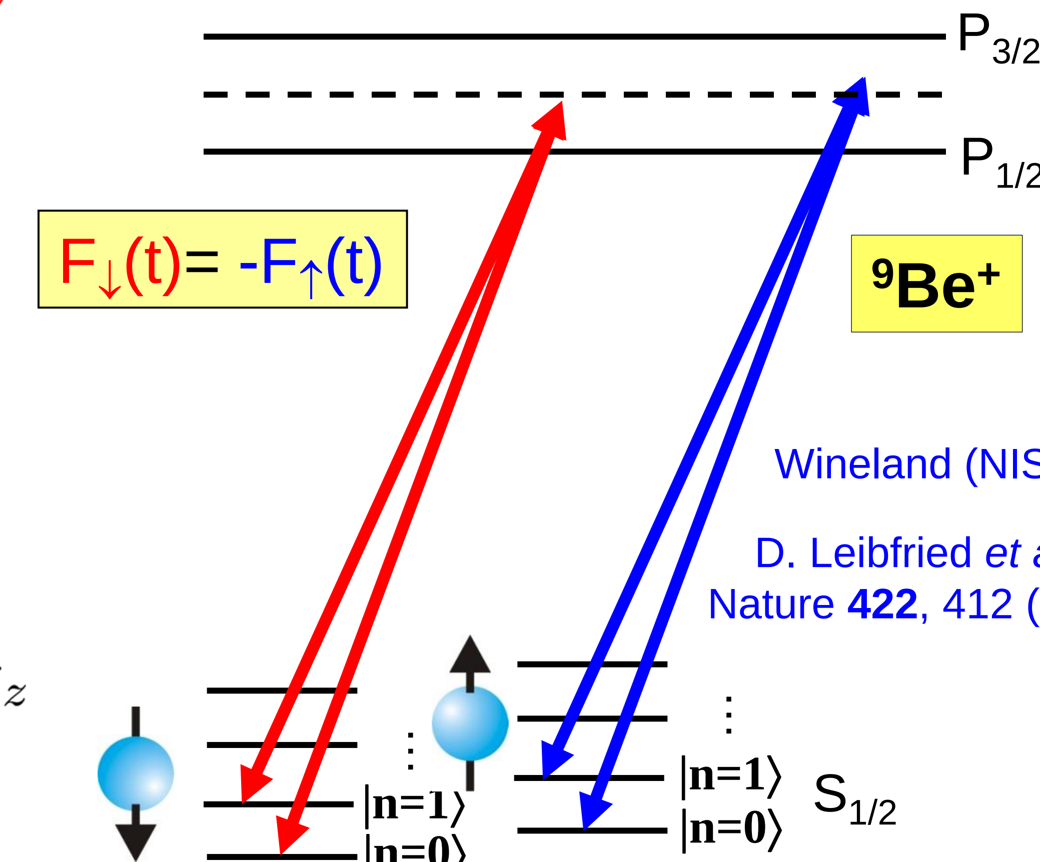
Spin-spin couplings by laser-induced potentials

1. <u>Conditional phase shift gate</u>

Spatial light shifts by off-resonant coupling to P-states in a standing wave

Frequency shifting one beam creates a moving standing wave.





Wineland (NIST)

D. Leibfried *et al*., Nature **422**, 412 (2003)

Spin-dependent force

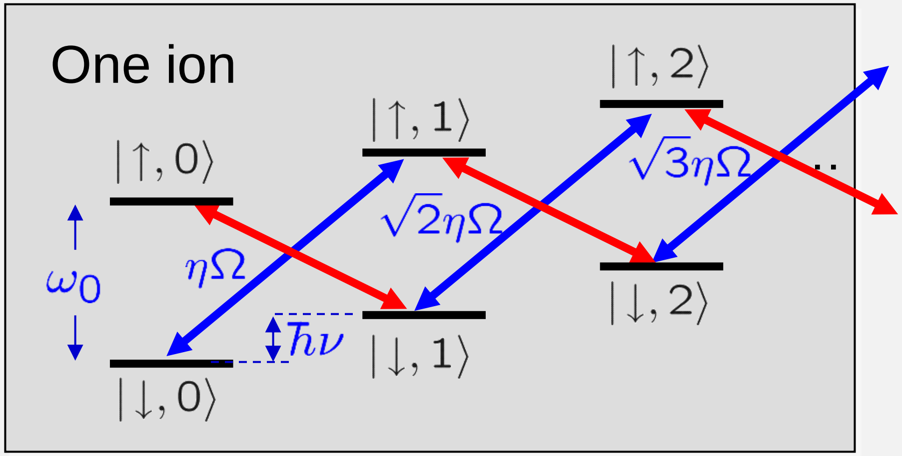
$$H(t) \propto (a e^{-i\delta t} + a^\dagger e^{i\delta t})\sigma_z$$

realizes Ising interaction

$$\xrightarrow{\text{with two ions}} H_{eff} = J\sigma_z^1\sigma_z^2$$

Spin-spin couplings by laser-induced potentials

2. Mølmer-Sørensen gate



qubit-motion coupling

$$\omega_{laser} = \omega_0 \pm \nu$$

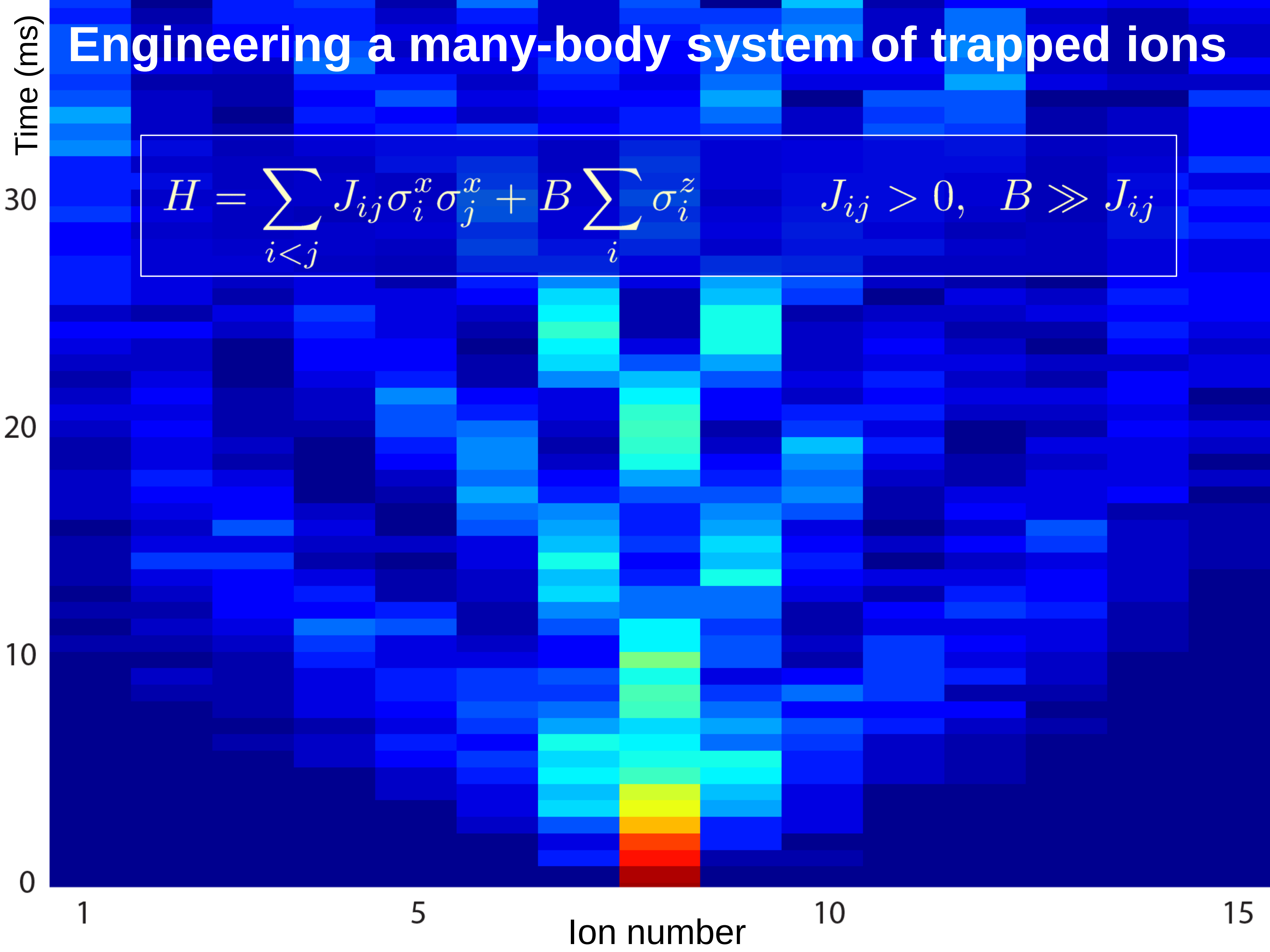
$$H \propto (a + a^\dagger)\sigma_x$$

realizes Ising interaction (for two ions)

$$H_{eff} = J\sigma_x^1\sigma_x^2$$

Engineering a many-body system of trapped ions

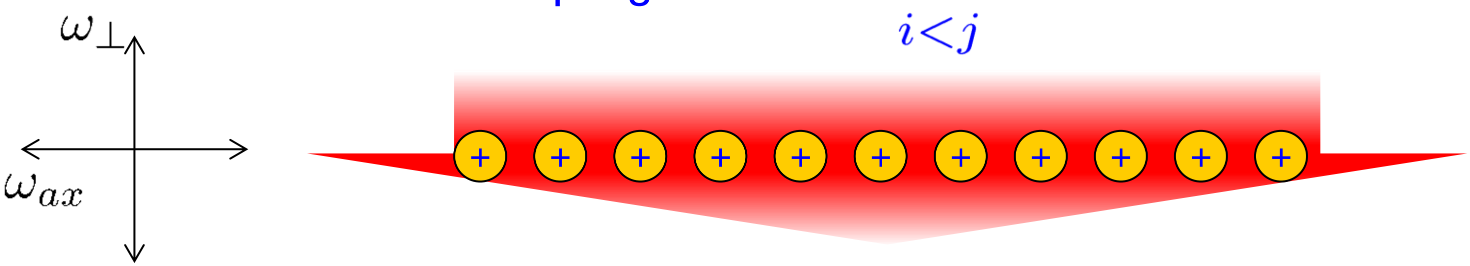
$$H = \sum_{i<j} J_{ij}\sigma_i^x\sigma_j^x + B\sum_i \sigma_i^z \qquad J_{ij} > 0, \quad B \gg J_{ij}$$



Geometry of laser-ion interaction

spin-spin coupling

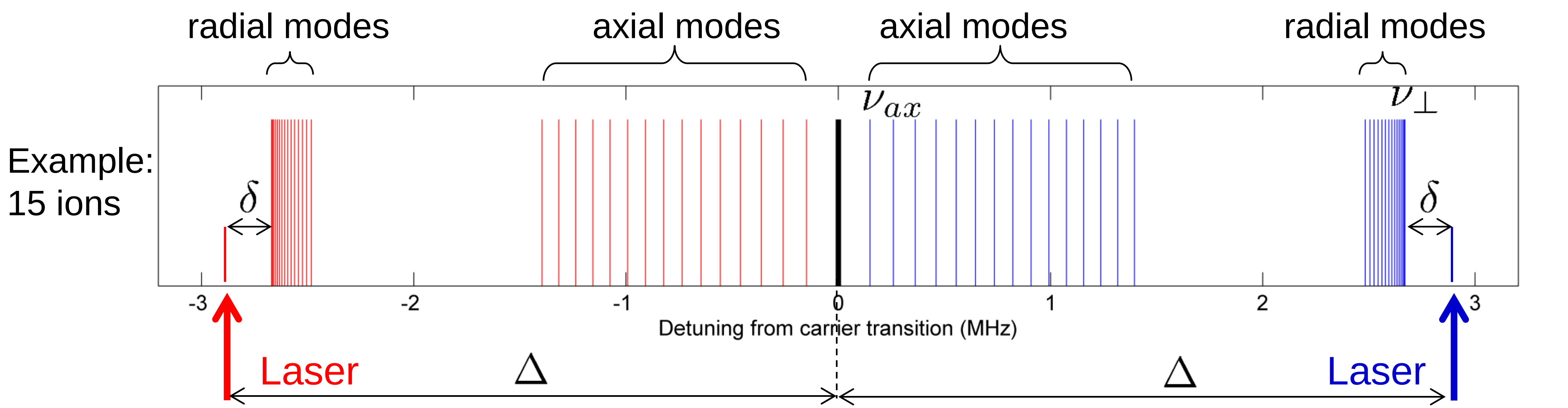
$$H = \sum_{i<j} J_{ij} \sigma_i^x \sigma_j^x$$

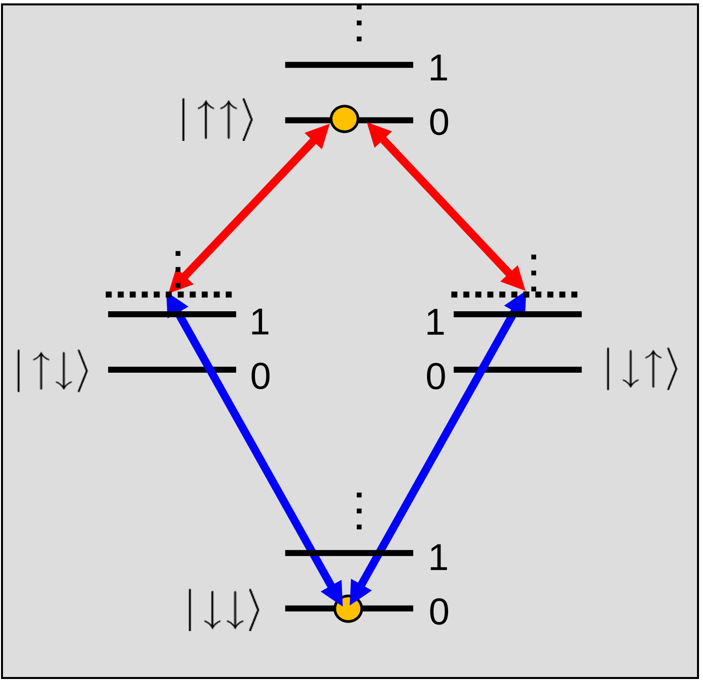


Features:

- Long strings $\Rightarrow$ strongly anistropic trapping potentials: $\omega_\perp / \omega_{ax} \approx 15 - 20$
- weak axial confinement $\Rightarrow$ 'hot' axial modes $\Rightarrow$ all laser beams $\perp$ to ion string

Variable-range interactions by coupling to transverse modes





$$H = \sum_{i<j} J_{ij} \sigma_i^x \sigma_j^x$$

with $$J_{ij} = \Omega^2 \frac{(\hbar k)^2}{2m} \sum_m \frac{b_{i,m} b_{j,m}}{\Delta^2 - \nu_m^2}$$

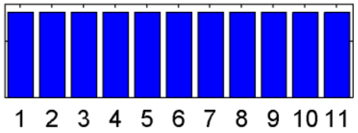
Variable-range interactions by coupling to transverse modes

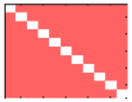
Example: 11 ions

$$J_{ij} = \Omega^2 \frac{(\hbar k)^2}{2m} \sum_m \frac{b_{i,m} b_{j,m}}{\Delta^2 - \nu_m^2}$$

vibrational mode

'COM'



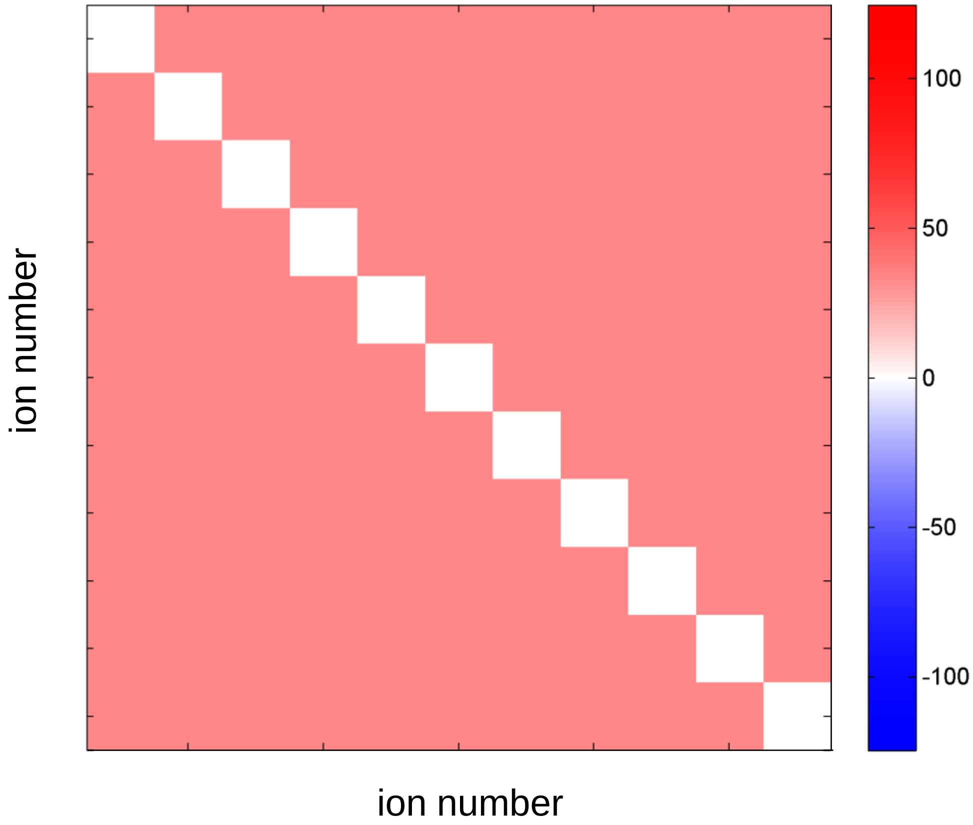


'Tilt'

⋮

⋮

Spin-spin coupling matrix J_{ij} (Hz)



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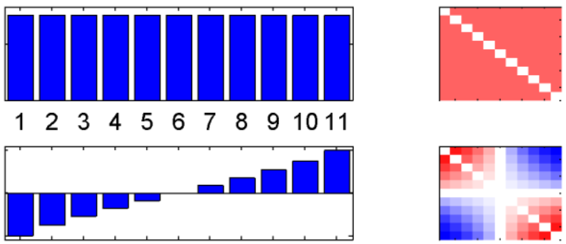

⋮

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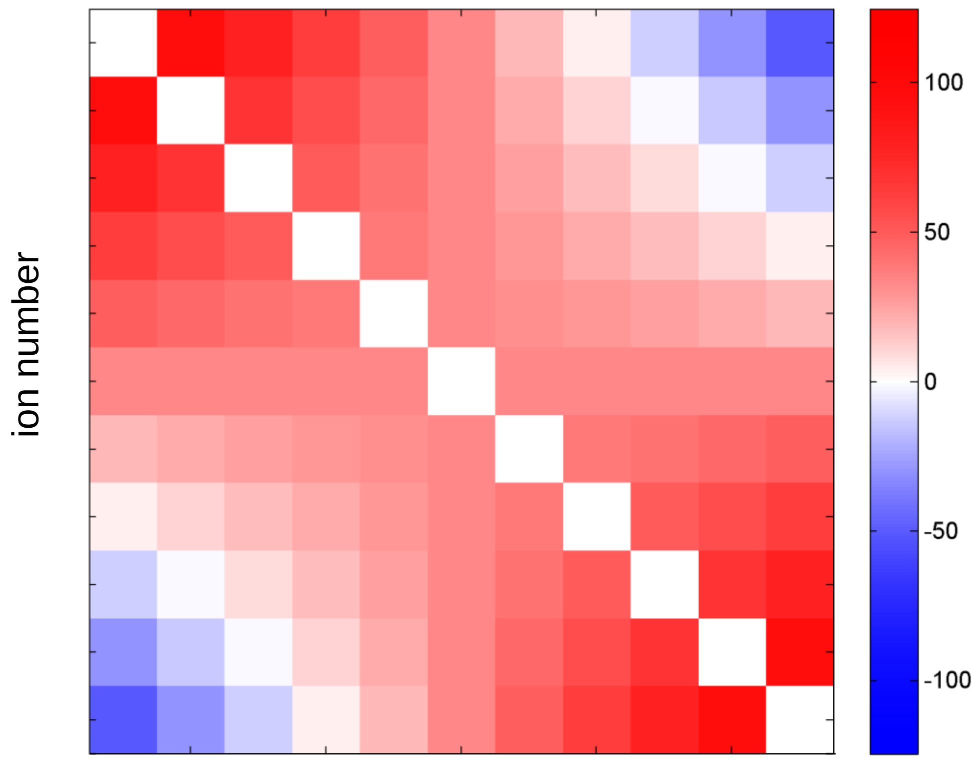

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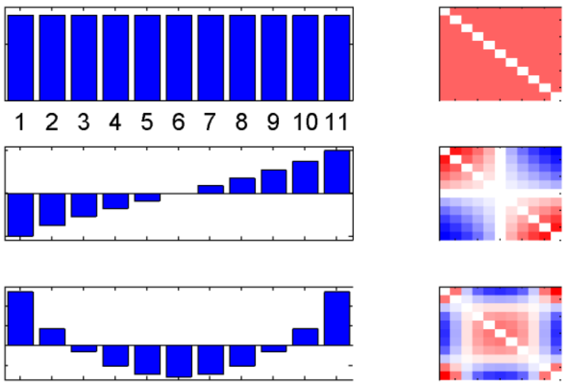

⋮

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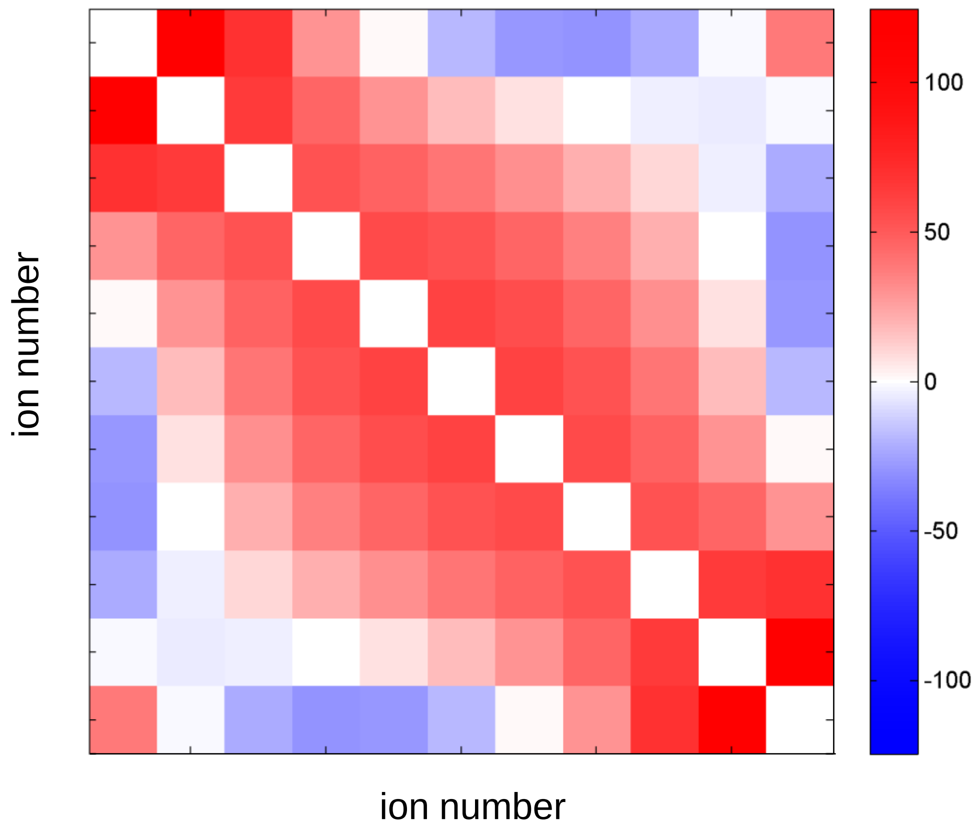

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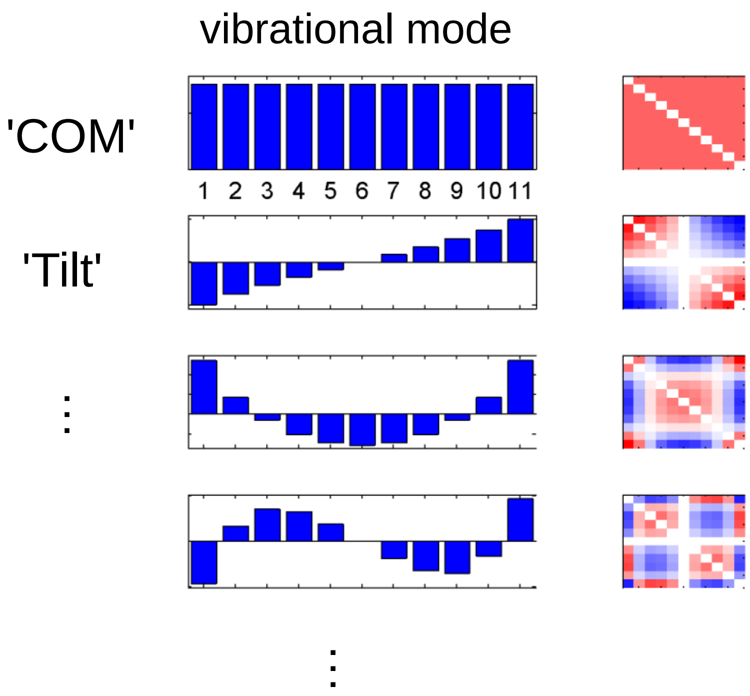
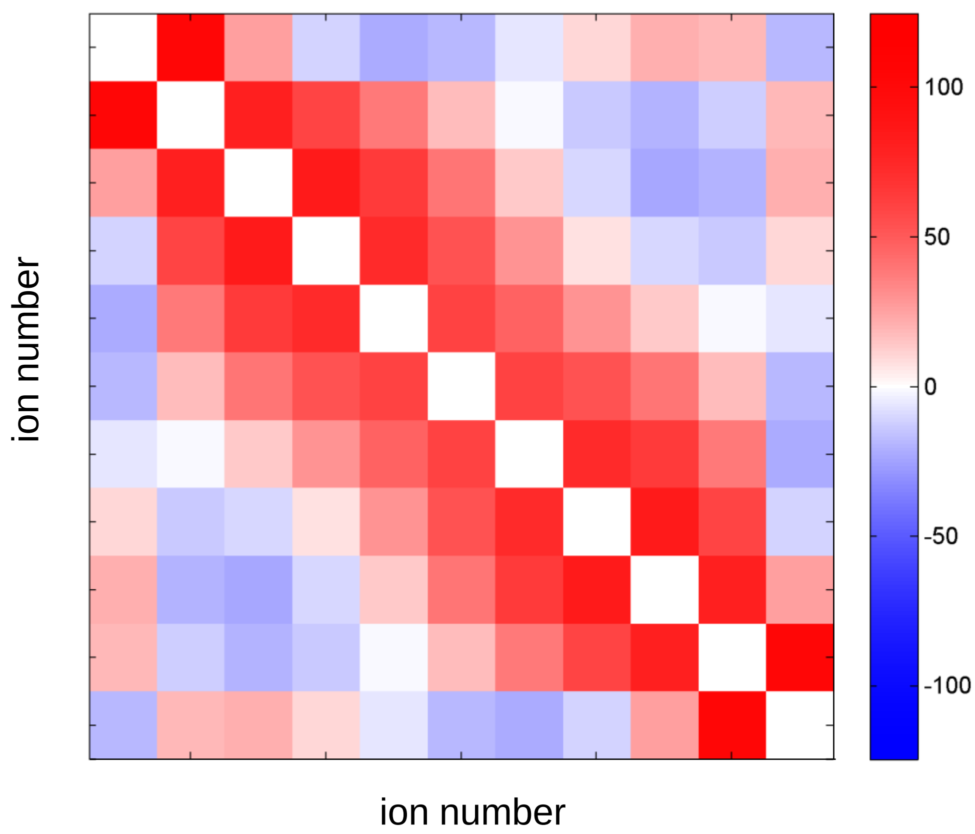


Spin-spin coupling matrix J_{ij} (Hz)

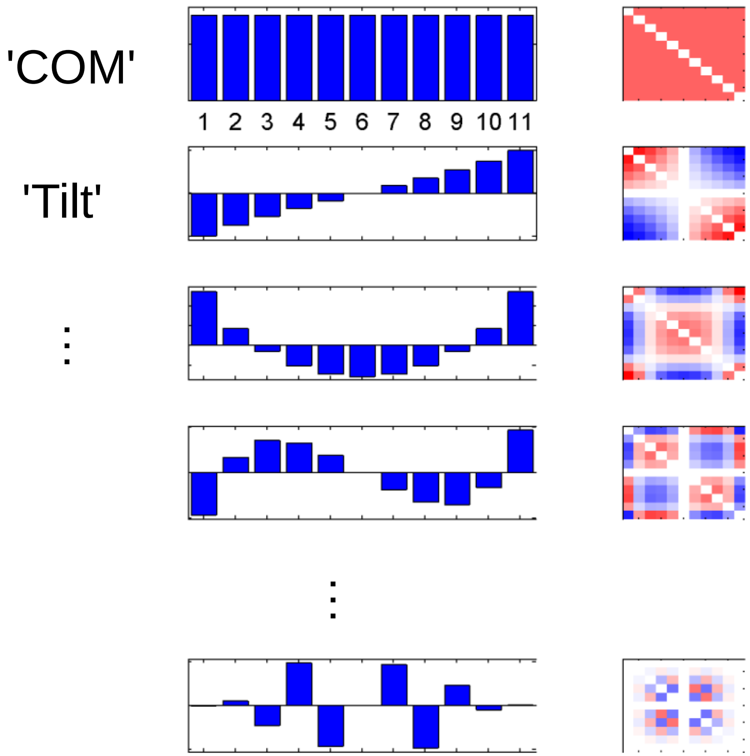


Variable-range interactions by coupling to transverse modes

Example: 11 ions

$$J_{ij} = \Omega^2 \frac{(\hbar k)^2}{2m} \sum_m \frac{b_{i,m} b_{j,m}}{\Delta^2 - \nu_m^2}$$

vibrational mode



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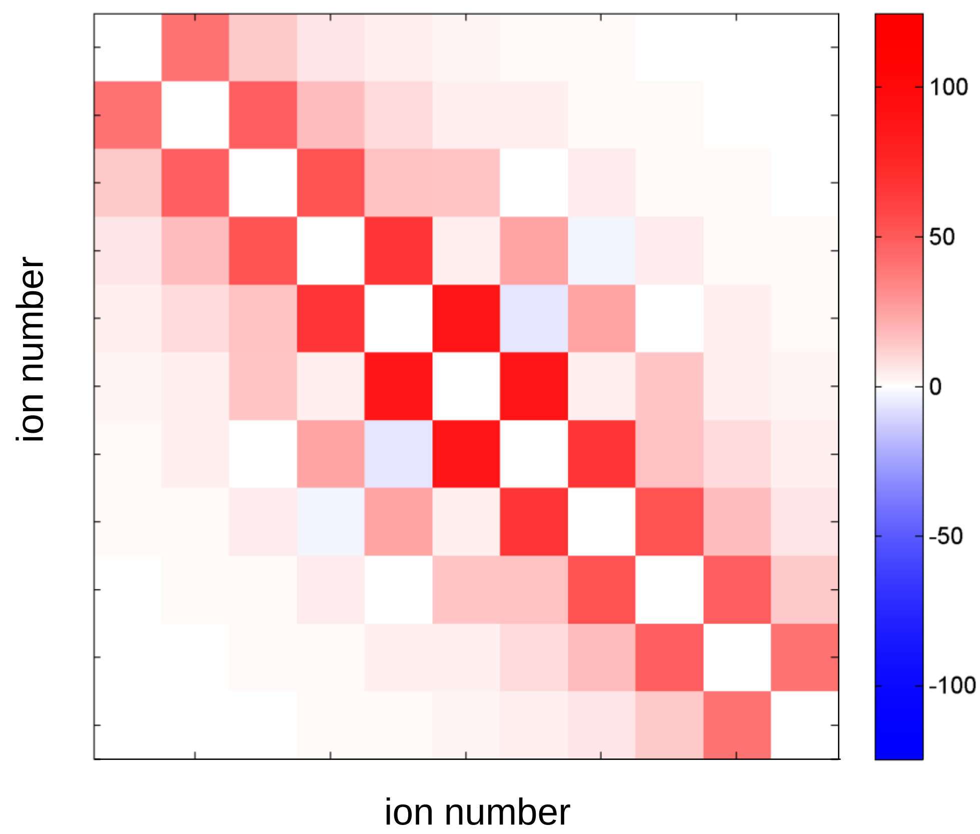


Variable-range interactions by coupling to transverse modes

Example: 11 ions

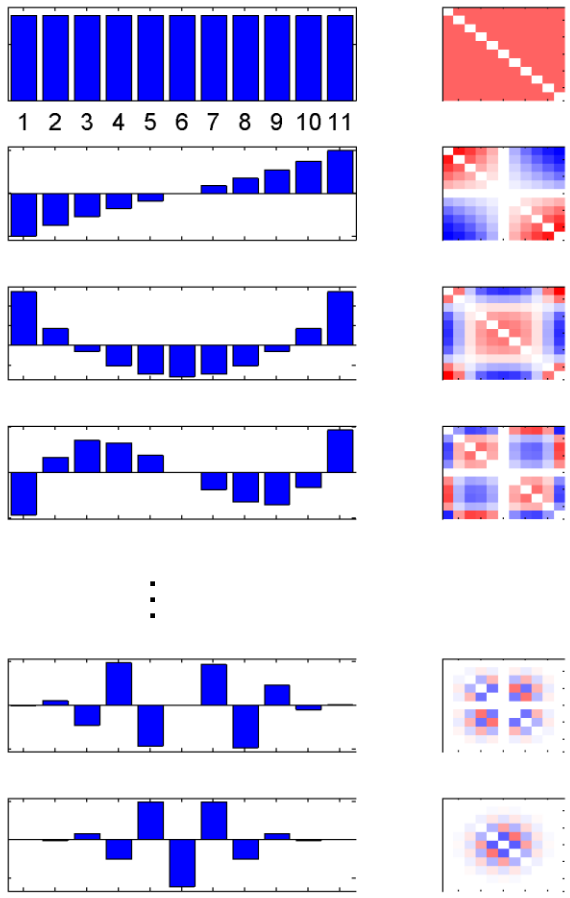
$$J_{ij} = \Omega^2 \frac{(\hbar k)^2}{2m} \sum_m \frac{b_{i,m} b_{j,m}}{\Delta^2 - \nu_m^2}$$

vibrational mode

'COM'

'Tilt'

⋮



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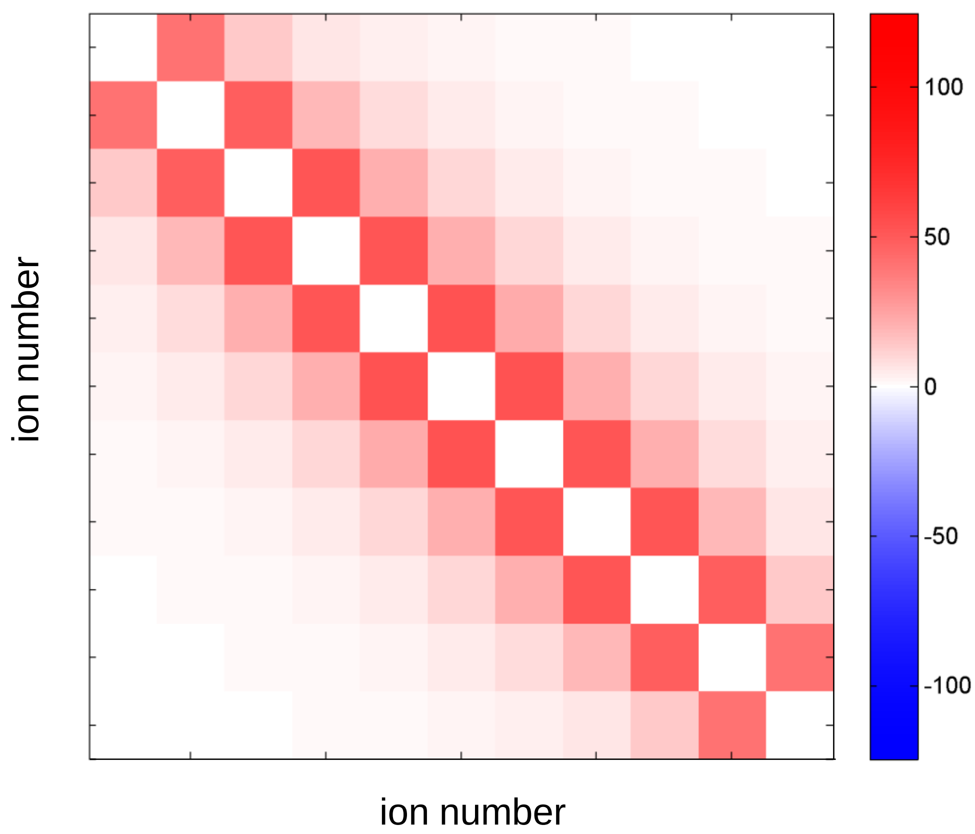
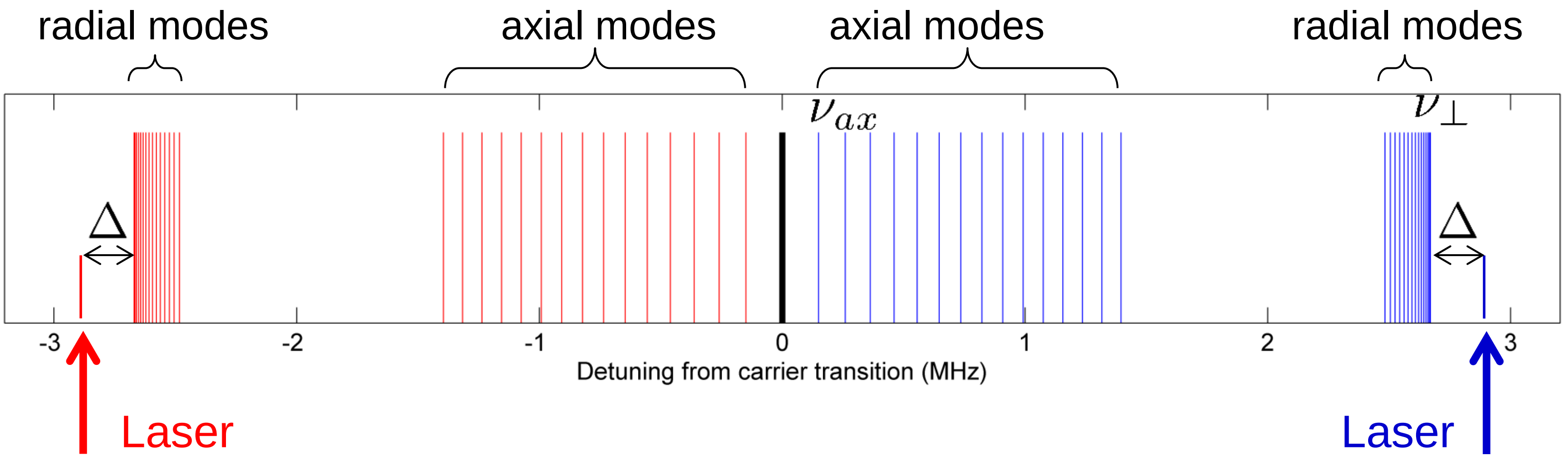


Variable-range interactions by coupling to transverse modes

Example:
15 ions



$$H = \sum_{i<j} J_{ij} \sigma_i^x \sigma_j^x \qquad \text{with} \qquad J_{ij} \approx \frac{J_0}{|i-j|^\alpha}$$

Interaction range: $0 < \alpha < 3$

couple only to center-of-mass

couple to all modes equally

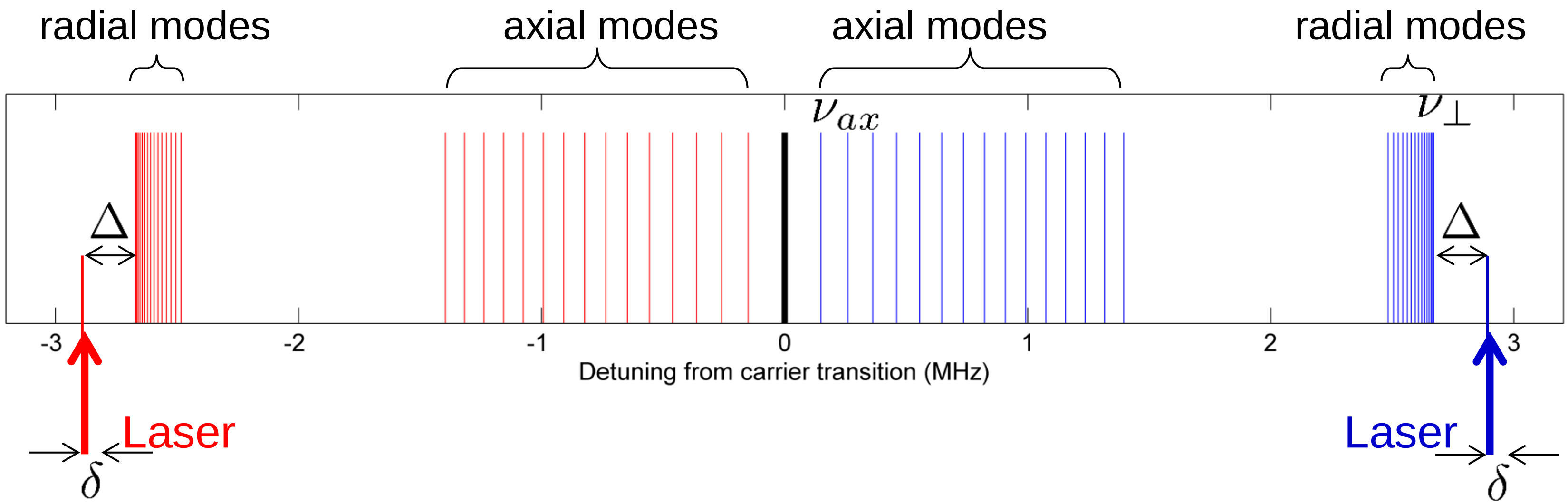
Knobs to turn:

- laser detuning Δ
- spread of radial modes

K. Kim et al, PRL **103**, 120502 (2009)

J. Britton et al, Nature **484**, 489 (2012)

Ising model with transverse field



$$H = \sum_{i<j} J_{ij}\sigma_i^x\sigma_j^x + B\sum_i \sigma_i^z \qquad B = \delta/2$$

$$\approx \sum_{i<j} J_{ij}(\sigma_i^+\sigma_j^- + h.c.) + B\sum_i \sigma_i^z \quad \text{for} \quad B \gg J$$

XY model of hopping hard core bosons

Measurement of the coupling matrix

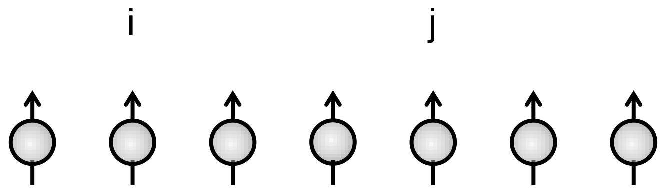
Protocol:

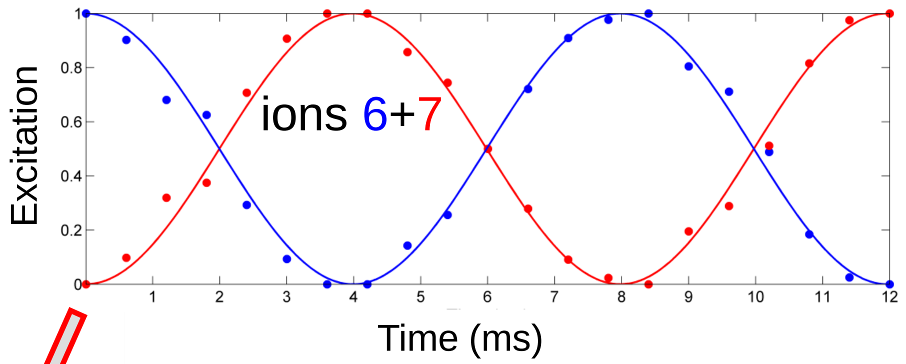
1. Initialize ions in state $|\uparrow\rangle_i|\downarrow\rangle_j$

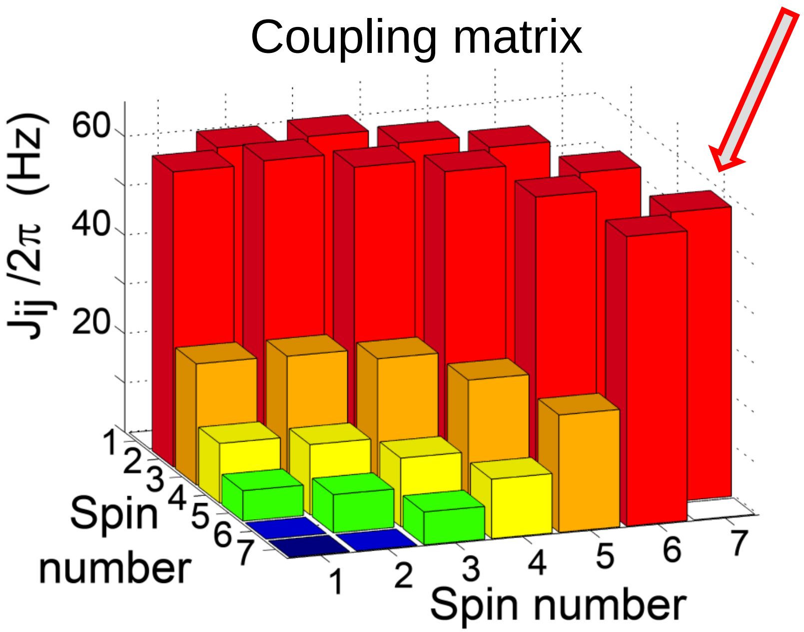
2. Switch on Ising Hamiltonian

$$|\uparrow\rangle_i|\downarrow\rangle_j \longleftrightarrow |\downarrow\rangle_i|\uparrow\rangle_j$$

3. Measure coherent hopping rate



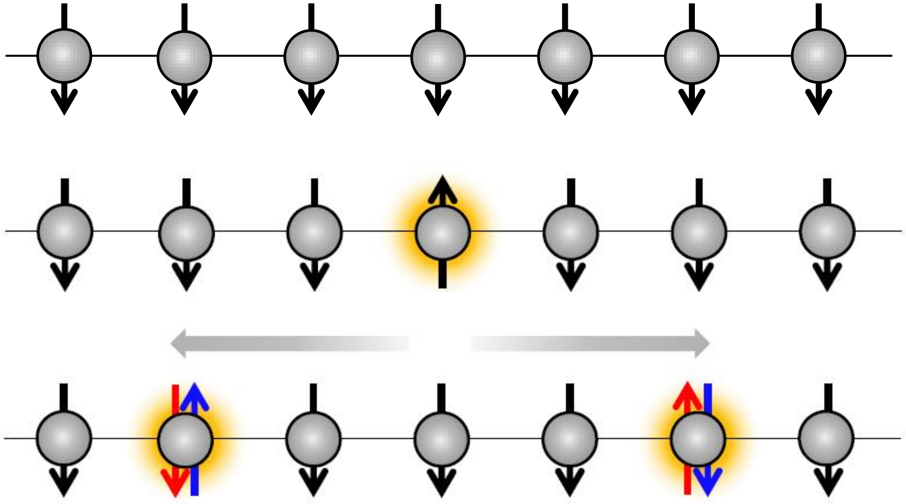




Spread of correlations after local quenches

$$H_{XY} = \sum_{i<j} J_{ij}(\sigma_i^+ \sigma_j^- + h.c.) + B \sum_i \sigma_i^z$$

Ground state: all spins aligned with transverse field



1. Local quench: flip one spin

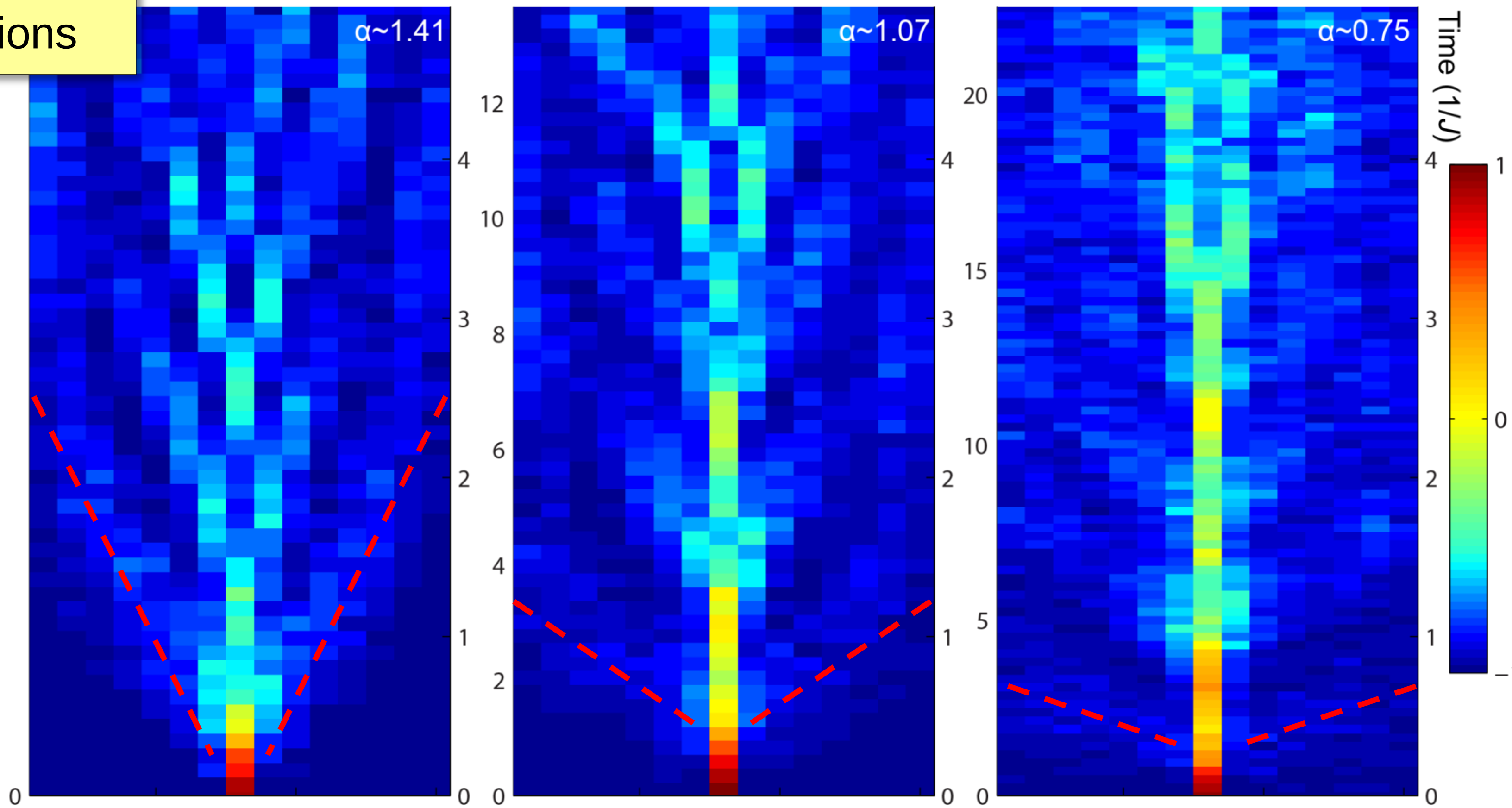
2. Spread of entanglement

3. Measure magnetization or spin-spin correlations

Magnetization dynamics after a local quench

15 ions

Time



increasing interaction range

P. Jurcevic et al., Nature **511**, 202 (2014)

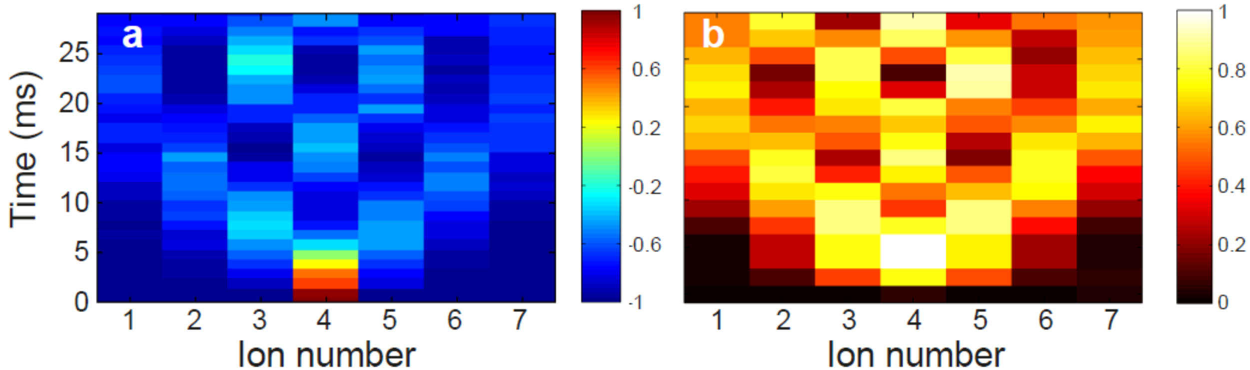
see also: P. Richerme et al., Nature **511**, 198 (2014)

$$J_{ij} \approx J_0 \frac{1}{|i-j|^\alpha}$$

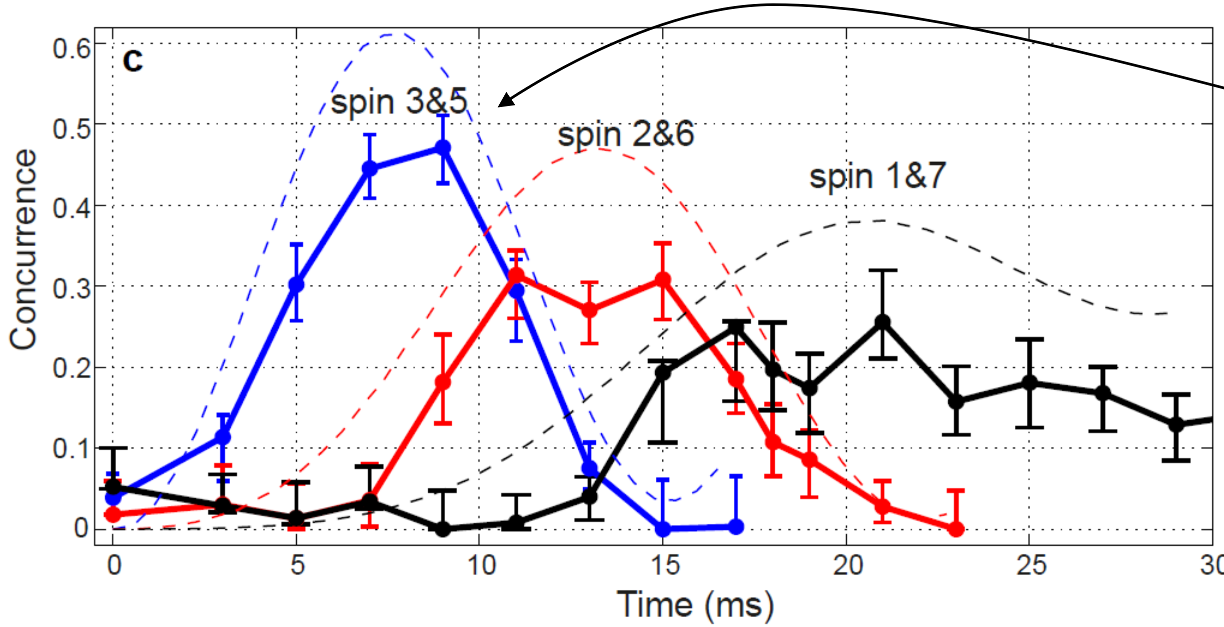
Spread of entanglement after a local quench

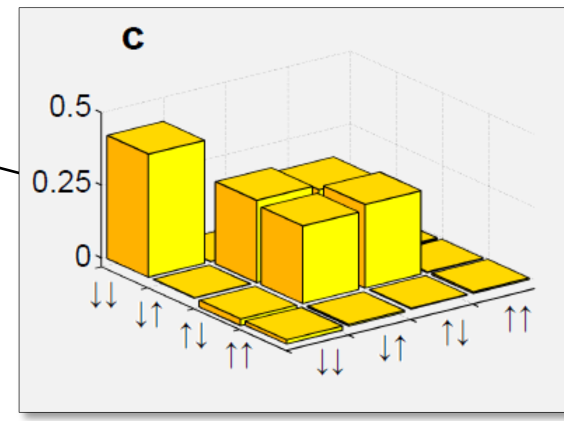
z-Magnetization

single-site entropy



7 ions
$a \approx 1.75$





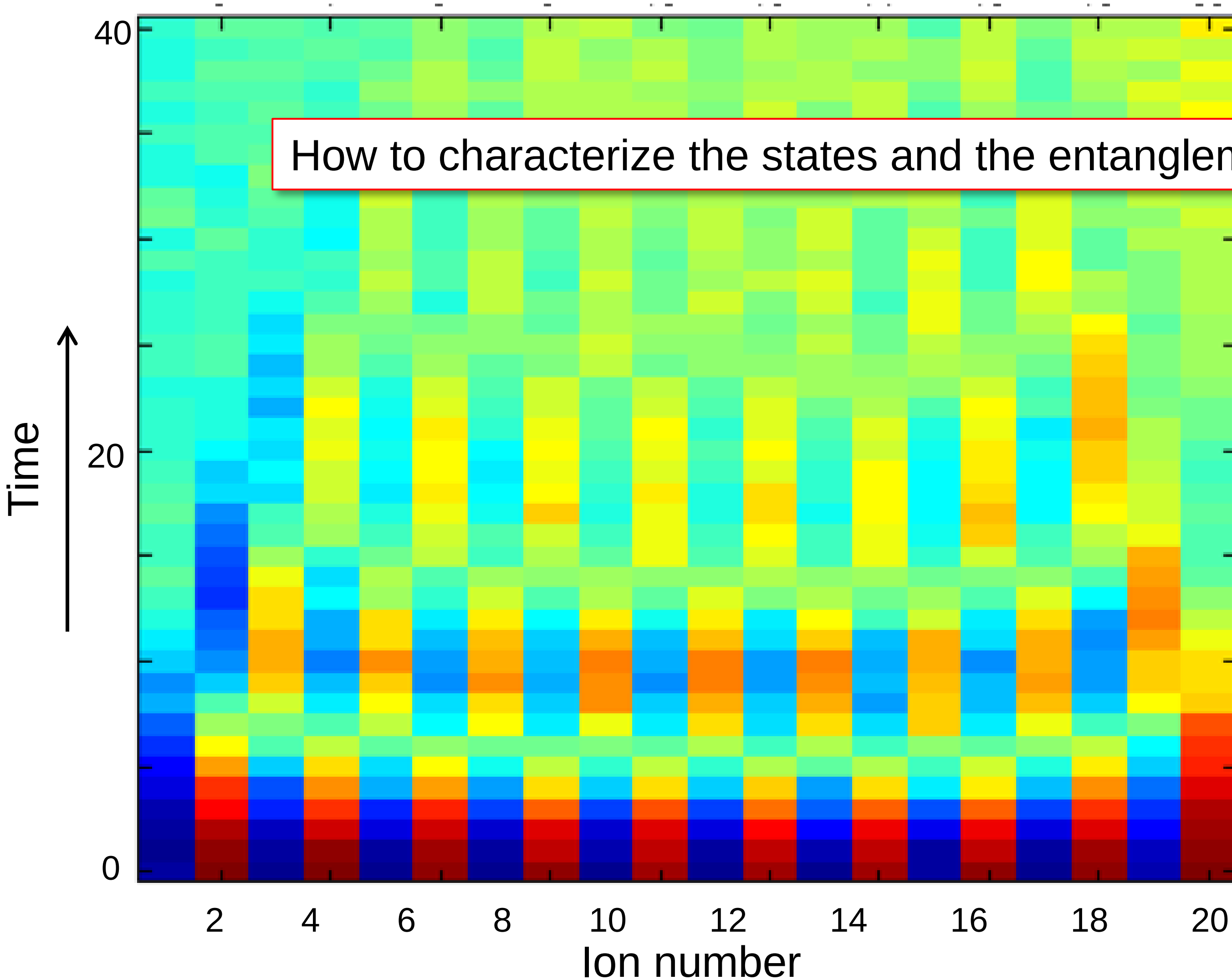
density matrix reconstruction
of spins 3 + 5
9 ms after the quench

$$\mathcal{F} = Tr(\rho|\psi\rangle\langle|\psi|) = 0.975$$

P. Jurcevic et al., Nature **511**, 202 (2014)

20-ion magnetization dynamics

N/2 excitation subspace: number of states growths exponentially with N



How to characterize the states and the entanglement created?

Summary

- A wide variety of two-ion quantum gates are geometric phase gates
  - the motional state carries out a closed loop in phase space
  - the area of the loop is proportional to the phase aquired by the quantum state
  - state-dependent forces make this phase depend on the joint two-qubit state.

- Scaling up a quantum processor to more ions necessitates
  - the ability to induce a wide variety of entangling interaction between the ions
  - building traps capable of holding tens of ions in flexible geometries

- Quantum simulation is the study of many-body physics using a well-controlled quantum system
  - Ions seem to be well-suited for simulating interacting spin systems
  - Ising interactions of variable range can be induced by coupling to many vibrational modes simultaneously

- In current state-of-the-art experiments up to 10-20 ions are brought into complex entangled states