(A1)

APPENDIX: MODULAR GROUND STATE

(arXiv: 1909.1180)

FOR SU(8) BROKEN BY RANK THREE ANTISYMMETRIC TENSOR

$$\left.\begin{aligned} A_\mu^A \quad 63 &= (1,1)(0) + (8,1)(0) + (1,24)(0) + (3,\bar{5})(-8) + (\bar{3},5)(8) \\ \phi^{[\alpha\beta\gamma]} \quad 56 &= (1,1)(-15) + (1,\overline{10})(9) + (3,10)(1) + (\bar{3},5)(-7) \end{aligned}\right\}$$

(g) ARE $U(1)$ GENERATOR VALUES DEFINED BY

$$[G, (m,n)] = g\,(m,n)$$

$$G = \mathrm{Diag}(-5,-5,-5,3,3,3,3,3)$$

$(1,1)$ OF ϕ HAS $g = -15 \Rightarrow$ GROUND STATE AND STATE BASIS SHOULD HAVE MODULO 15 PERIODICITY IN $U(1)$ GENERATOR g

(A2)

- <u>MODULO 15 STATE BASIS</u>

$$|k\rangle = |k+15s\rangle \qquad s \text{ INTEGER}$$

- <u>CLUSTERING</u> $|k_A + k_B\rangle_{A+B} = |k_A\rangle_A \, |k_B\rangle_B$ WIDELY SEPARATED A, B

PERIODICITY ALLOWS $(\bar{3},5)(-7)$ OF ϕ TO BE ABSORBED AS LONGITUDINAL COMPONENT OF $(\bar{3},5)(8)$ OF A_μ^A

U(1) GAUGE BOSON MASS AFTER SYMMETRY BREAKING:
$(1,1)(0)$ HAS GENERATOR G

ϕ POTENTIAL MINIMUM AT $\bar{\phi}^{[123]} = d$
ALL OTHER COMPONENTS $= 0$

GIVES $SU(8) \supset SU(3) \times SU(5)$

WITHOUT MODULARITY, $U(1)$ BECOMES MASSIVE

(A3)

U(1) MASS CALCULATION

$$D_\nu \phi^{(\alpha\beta\gamma)} = \partial_\nu \phi^{(\alpha\beta\gamma)} + f A^A_\nu \left(t_A{}^\alpha{}_\delta \, \phi^{[\delta\beta\gamma]} + t_A{}^\beta{}_\delta \, \phi^{[\alpha\delta\gamma]} + t_A{}^\gamma{}_\delta \, \phi^{[\alpha\beta\delta]} \right)$$

FOR $\left\{ \begin{array}{l} \phi = \bar{\Phi}^{[123]}{}_a \\ t_A = G \end{array} \right.$ THIS IS

$$G^\alpha{}_\delta \, \bar{\phi}^{[\delta\beta\gamma]} + G^\beta{}_\delta \, \bar{\phi}^{[\alpha\delta\gamma]} + G^\gamma{}_\delta \, \bar{\phi}^{[\alpha\beta\delta]} = -15 \, \bar{\phi}^{(\alpha\beta\gamma)}$$

CONGRUENT TO 0
MODULO 15

SO IN MODULO 15 ARITHMETIC, U(1) MASS CAN BE ZERO, TO GIVE

$$SU(8) \supset SU(3) \times SU(5) \times U(1)$$

A4

SU(5) BREAKING BY RANK TWO ANTISYMMETRIC TENSOR

$$A_\mu^a \qquad 24 = (1,1)(0) + (3,1)(0) + (1,8)(0) + (2,\bar{3})(5) + (2,3)(-5)$$

$$\phi^{[\alpha\beta]} \qquad 10 = (1,1)(6) + (1,\bar{3})(-4) + (2,3)(1)$$

$$G = \mathrm{Diag}(3,3,-2,-2,-2)$$

(1,1) IN 10 HAS $g = 6$ $\Rightarrow$ NEED MODULO 6 BASIS

(2,3)(−5) CAN PAIR WITH (2,3)(1) MOD 6

$$\bar{G}^\alpha_\delta \, \bar{\phi}^{[\delta\beta]} + \bar{G}^\beta_\delta \, \bar{\phi}^{[\alpha\delta]} = 6\, \bar{\phi}^{[\alpha\beta]}$$

$\uparrow$ CONGRUENT TO 0 MODULO 6

SO CAN GET

$$SU(5) \supset SU(2) \times SU(3) \times U(1)$$

(A5) STATE BASIS MODULO 5 BUILT FROM BASIS MODULO 15

(arXiv PAPER DOES GENERAL CASE $5 \to p$ $15 \to N$)

$$|k\rangle_q = \frac{1}{3}\sum_{n=0}^{2} \zeta_q^{5n}\, |k+5n\rangle$$

$$\zeta_q = e^{2\pi i q/15} \qquad \zeta_q^{15} = 1 \qquad \left(\zeta_q^{5} = e^{2\pi i q/3}\right)$$

$$|k+5s\rangle = \frac{1}{3}\sum_{n=0}^{2} \zeta_q^{5n}\, |k+5n+5s\rangle = \zeta_q^{-5s}\,\frac{1}{3}\sum_{n=0}^{2} \zeta_q^{5(n+s)}\, |k+5(n+s)\rangle$$

OVER $n = 0,1,2$ $k+5(n+s)$ COVERS SAME VALUES AS $k+5n$ MODULO 15

EXAMPLE:

$k=1$, $s=0$	1	6	11
$k=1$, $s=1$	6	11	$16 \equiv 1$

SO $$|k+5s\rangle_q = \zeta_q^{-5s}\, |k\rangle_q$$

MODULO 5 INVARIANCE UP TO A PHASE

A6

CLUSTERING OF MOD 5 BASIS

$$|k\rangle_q = \zeta_q^{5s}\,|k+5s\rangle_q \;\Rightarrow\; |k\rangle_q = \frac{1}{3}\sum_{s=0}^{2} \zeta_q^{5s}\,|k+5s\rangle_q$$

$$\Rightarrow\; |k\rangle_q = \frac{1}{9}\sum_{s=0}^{2}\sum_{n=0}^{2} \zeta_q^{5s}\,\zeta_q^{5n}\,|k+5s+5n\rangle$$

WHEN $k = k_A + k_B$ $\quad | \;\rangle = | \;\rangle_{A+B}$ $\quad$ A, B SEPARATED

$$|k_A+k_B\rangle_{q;A+B} = \frac{1}{9}\sum_{s=0}^{2}\sum_{n=0}^{2} \zeta_q^{5s}\,\zeta_q^{5n}\,\underbrace{|k_A+k_B+5s+5n\rangle_{A+B}}_{|k_A+5s\rangle_A\;|k_B+5n\rangle_B}$$

$$= \frac{1}{3}\sum_{s=0}^{2}\zeta_q^{5s}\,|k_A+5s\rangle_A \;\; \frac{1}{3}\sum_{n=0}^{2}\zeta_q^{5n}\,|k_B+5n\rangle_B$$

$$= |k_A\rangle_{q;A}\;|k_B\rangle_{q;B}$$ FACTORIZES