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SU(8) FAMILY UNIFICATION WITH BOSON-FERMION BALANCE

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MOTIVATION:

SM HAS BOSONS AND FERMIONS → SUPERSYMMETRY?

BUT:
- NO DEFINITIVE MECHANISM FOR SUSY BREAKING
- SO FAR, SUSY NOT FOUND

TRY WEAKER PRINCIPLE THAN EXACT SUSY:

BOSON-FERMION BALANCE

THAT IS, EQUAL NUMBERS OF MASSLESS BOSON AND FERMION HELICITY STATES

② STARTING POINT: SO(8) N=8 SUPERGRAVITY

	PARTICLE	HELICITIES
1	GRAVITON	2
8	GRAVITINOS	16
28	VECTORS	56
56	MAJORANA FERMIONS	112
70	SCALARS	70

16+112 = 128

2+56+70 = 128

28 ≡ ADJOINT OF SO(8) TOO SMALL TO CONTAIN SM

REDISTRIBUTE 70 SCALAR HELICITIES TO VECTORS

$\frac{70}{2} = 35$ $35 + 28 = 63$ CAN BE ADJOINT OF SU(8) BIG ENOUGH!

SPIN →	2	WEYL 3/2	1	WEYL 1/2	COMPLEX 0
"SU(8) GRAVITON MULTIPLET"	1	8_L	63	56_L	← SU(8) REPS
"SU(8) MATTER MULTIPLET"	SU(8) REPS →			$\overline{28}_L$ $\overline{28}_L$	56

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SU(8) ANOMALY CANCELLATION

CHIRAL ANOMALY OF SPIN 3/2

= 3 TIMES CHIRAL ANOMALY OF SPIN 1/2

$$3 \times \text{ANOMALY}(8_L) = 3 \times 1 = 3$$

$$2 \times \text{ANOMALY}(\overline{28}_L) = 2 \times (-4) = -8$$

$$\text{ANOMALY}(56_L) = 5$$

$$\text{TOTAL ANOMALY} = 3 - 8 + 5 = 0$$

[THIS COUNTING FIRST NOTED BY MARCUS (1985)]

[SPIN 3/2 ANOMALY DUFF (1982)
NIELSEN + RÖMER (1985)]

SO THE MODEL IS VIABLE

WILL NOW SHOW:

- MODEL CONTAINS FIELDS OF SM
- MODEL HAS A SYMMETRY BREAKING PATH TO SM

$$SU(8) \to SU(3)_{\text{FAMILY}} \times \underbrace{SU(5) \times U(1)/\mathbb{Z}_5}_{\text{"FLIPPED" } SU(5)}$$

$$\underbrace{SU(3)_{\text{FAMILY}} \times \text{"FLIPPED" } SU(5)}_{\text{FROM 56 SCALARS}} \;\; \underbrace{\text{"FLIPPED" } SU(5) \to SM \to \text{ELECTROMAGNETIC } U(1)}_{\text{FROM DYNAMICAL SYMMETRY BREAKING}}$$

UNDER $SU(8) \to SU(3) \times SU(5) \times U(1)$

$$56 \to (1,1)(-15) + (1,\overline{10})(9) + (\bar{3},5)(-7) + (3,10)(1)$$

$U(1)$ GENERATOR $\neq 0$

⑤

MODULAR GROUND STATE

$|\Omega\rangle$ = GROUND STATE BEFORE DYNAMICAL SYMMETRY BREAKING

NEED $\langle\Omega|\,(1,1)(-15)\,|\Omega\rangle \neq 0$

$\rightarrow$ $|\Omega\rangle$ MUST BE A SUPERPOSITION OF $U(1)$ EIGENSTATES DIFFERING BY 15 UNITS

MODULAR GROUND STATE MODULO DIVISOR OF 15

ASSUME MOD 5 IS NATURE'S CHOICE

$$|\Omega\rangle = \sum_{n=-\infty}^{\infty} e^{in\omega}\,|5n\rangle = |0\rangle + e^{i\omega}|5\rangle + e^{2i\omega}|10\rangle + e^{3i\omega}|15\rangle + e^{-i\omega}|-5\rangle + \dots$$

$\{\ \}$ = MODULO 5 EQUIVALENT TO $(\ )$

$$56 \rightarrow (1,1)(-15)\{0\} + (1,\overline{10})(9)\{-1\} + (\overline{3},5)(-7)\{-2\} + (3,10)(1)\{1\}$$

WILL SHOW MODULO 5 REPRESENTATIVES THAT CORRESPONDS TO PATH TO SM: ASSUME UNIQUE MOD 5 REPRESENTATIVE IS PICKED BY DYNAMICAL SYMMETRY BREAKING

⑥

FIELD CONTENT OF MODEL

FIELD	SPIN	SU(8) REP	HELICITIES	BRANCHING → $SU(3) \times SU(5) \times U(1)$
$h_{\mu\nu}$	2	1	2	1
ψ_μ^α	WEYL $\frac{3}{2}$	8_L	16	$(3,1)(-5)\{0\} + (1,5)(3)\{-2\}$
A_μ^A	1	63	126	$(1,1)(0)\{0\} + (8,1)(0)\{0\} + (1,24)(0)\{0\}$ $+ (3,\bar{5})(-8)\{2\} + (\bar{3},5)(8)\{-2\}$
$\chi^{[\alpha\beta\gamma]}$	WEYL $\frac{1}{2}$	56_L	112	$(1,1)(-15)\{0\} + (1,\overline{10})(9)\{-1\} + (\bar{3},5)(-7)\{3\} + \underline{(3,10)(1)\{1\}}$
$\lambda_{1[\alpha\beta]}$	WEYL $\frac{1}{2}$	$\overline{28}_L$	56	$(3,1)(10)\{5\} + (1,\overline{10})(-6)\{-1\} + (\bar{3},\bar{5})(2)\{-3\}$
$\lambda_{2[\alpha\beta]}$	WEYL $\frac{1}{2}$	$\overline{28}_L$	56	$\underline{(3,1)(10)\{5\}} + (1,\overline{10})(-6)\{-1\} + \underline{(\bar{3},5)(2)\{-3\}}$
$\phi^{[\alpha\beta\gamma]}$	COMPLEX 0	56	112	$(1,1)(-15)\{0\} + \underline{(1,\overline{10})(9)\{-1\}} + \underline{(3,10)(1)\{1\}}$ $+ (\bar{3},5)(-7)\{-2\}$

↔ STATES WITH FLIPPED SU(5) REPRESENTATIONS

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SYMMETRY BREAKING $SU(8) \rightarrow SU(3) \times SU(5) \times U(1)$

BREAK USING SCALAR $\phi^{(\alpha\beta\gamma)}$

$$A_\mu^A \qquad \underset{U(1)}{(1,1)(0)\{0\}} + \underset{SU(3)}{(8,1)(0)\{0\}} + \underset{SU(5)}{(1,24)(0)\{0\}}$$

$$+ \underbrace{(3,\bar{5})(-8)\{2\} + (\bar{3},5)(8)\{-2\}}_{\text{ABSORB 30 REAL COMPONENTS OF } \phi^{(\alpha\beta\gamma)} \text{ TO BECOME MASSIVE}}$$

$$\phi^{(\alpha\beta\gamma)} \qquad \underset{\text{GETS VACUUM EXPECTATION}}{(1,1)(-15)\{0\}} + \underbrace{(1,\overline{10})(9)\{-1\} + (3,10)(1)\{1\}}_{\text{RESIDUAL SCALARS}}$$

$$+ \underbrace{(\bar{3},5)(-7)\{-2\}}$$

30 REAL COMPONENTS THAT ARE ABSORBED BY VECTOR $(\bar{3},5)(8)\{-2\}$ AND CONJUGATE

NOTE: $8 \equiv -7 \quad MOD(15)$

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"STANDARD" SU(5)

GEORGI + GLASHOW
PRL 32, 438 (1974)

$SU(5) \supset SM$

$Q = T_L^3 + Y$ ← (LANGACKER DEFINITION)

$Y = \frac{1}{6}$ SLANSKY $U(1)_{SM}$ GENERATOR

↑ PHYS. REP. 79, 1 (1981)

SU(5) REPS:
10
$\bar{5}$

"FLIPPED" SU(5)

BARR
PL B 112, 219 (1982)) FROM SO(10)

DERENDINGER, KIM, NANOPOULOS
PLB 139, 170 (1984)

ANTONIADIS, ELLIS, HAGELIN, NANOPOULOS
PLB 194, 231 (1987)

$SU(5) \times U(1)_X \supset SM$

$Q = T_L^3 + Y$

$Y = \frac{1}{5}(X - Z)$

$Z = \frac{1}{6}$ SLANSKY $U(1)_{SM}$ GENERATOR

SU(5) REPS:	X VALUES:
10	1
$\bar{5}$	−3
1	5

DESCENT FROM SO(10) 16 REP.

⇒ ALL $SU(5) \times U(1)_X$ ANOMALIES CANCEL

ALSO, ALL $SU(2)_{FAMILY} \times SU(5) \times U(1)_X$ " " "

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STATE ASSIGNMENTS IN STANDARD; FLIPPED SU(5)

$(SU(2)_L, SU(3)_{COLOR})_Y$	NAME	STD. SU(5)	FLIPPED SU(5)	
$(1, \bar{3})_{-2/3}$	u_L^c	10	$\bar{5}$	FLIP $10 \leftrightarrow \bar{5}$
$(2,1)_{-1/2}$	$\begin{pmatrix} e_L \\ \nu_L \end{pmatrix} = \ell_L$	$\bar{5}$	$\bar{5}$	
$(1,1)_0$	N_L	ABSENT	10	
$(1,\bar{3})_{1/3}$	d_L^c	$\bar{5}$	10	FLIP $\bar{5} \leftrightarrow 10$
$(2,3)_{1/6}$	$\begin{pmatrix} u_L \\ d_L \end{pmatrix} = q_L$	10	10	
$(1,1)_1$	e_L^c	10	1	

SYMMETRY BREAKING TO SM

$$SU(5) \to SU(2) \times SU(3) \times U(1)$$

$$\bar{5} = (2,1)(-3) + (1,\bar{3})(2)$$

$$10 = (1,1)(6) + (1,\bar{3})(-4) + (2,3)(1)$$

$$24 = (1,1)(0) + (3,1)(0) + (1,8)(0) + (2,3)(-5) + (2,\bar{3})(5)$$

STANDARD $SU(5)$: USES $(1,1)(0)$ OF REAL SCALAR 24

FLIPPED $SU(5)$: USES $(1,1)(6)$ OF COMPLEX SCALAR 10

$U(1)$ GENERATOR $\neq 0$

MODULAR GROUND STATE MOD (6)

IN 24 VECTOR, $(2,3)(-5) + (2,\bar{3})(5)$ ABSORB 12 REAL

COMPONENTS OF COMPLEX 10 TO BECOME MASSIVE

THESE COME FROM $(2,3)(1)$ AND CONJUGATE

$$(2,3)(1) \Updownarrow (2,3)(-5) \qquad 1 \equiv -5 \mod (6)$$

SYMMETRY BREAKING TO SM - CONTINUED

$$SU(8) \rightarrow SU(3)_{FAMILY} \times SU(5) \times U(1)$$

$$\uparrow$$

$$\underbrace{\qquad\qquad U(1)_X}_{\text{FLIPPED } SU(5)}$$

USES SCALAR 56

$\hookrightarrow$ RESIDUAL $(1, \overline{10})\{-1\} + (3, 10)\{1\}$

CORRECT ASSIGNMENTS TO BREAK FLIPPED $SU(5) \rightarrow SM$

$$10 \supset (1,1)(6) \qquad Z = \frac{1}{6}(6) = 1$$

$$X = 1$$

$$\Rightarrow Y = \frac{1}{5}(X - Z) = 0$$

$$T_L^3 = 0 \text{ (SINGLET)} \Rightarrow Q = 0$$

$(3, 10)$ CAN ALSO BREAK FAMILY SYMMETRY $SU(3) \rightarrow SU(2) \times \frac{U(1)}{Z_2}$

AFTER $SU(5)$ BREAKING 10 DECOMPOSES INTO THREE COMPONENTS $10_{A,B,C}$

$(3, 10_{A,B,C})$ CAN COMPLETELY BREAK FAMILY $SU(3)$

FERMION STATE SUMMARY $\left(SU(3)_{FAMILY}, SU(5)\right)$

χ $\quad (1,1)\{0\} + (1,\overline{10})\{-1\} + (\bar{3},5)\{3 \equiv -2\} + (3,10)\{1\}$

λ_1 $\quad (3,1)\{5\} + (1,\overline{10})\{-1\} + (\bar{3},\bar{5})\{-3 \equiv 2\}$

λ_2 $\quad (3,1)\{5\} + (1,\overline{10})\{-1\} + (\bar{3},\bar{5})\{-3\}$

3 SETS WITH FLIPPED SU(5) ASSIGNMENTS

3 VECTOR-LIKE SETS BARR arXiv: 1307.5770 ROTATE AWAY PROTON DECAY ?

DARK MATTER CANDIDATE ?

POSSIBLE CONDENSATES:

- AFTER FAMILY BREAKING, THREE $(1,\overline{10})\{-1\}$ WITH $(3,10)\{1\}$
- $3 \times \bar{3} \supset 1$ PAIRINGS

$$\left.\begin{array}{l} \chi\,(\bar{3},5)\{3\} \quad \lambda(3,1)\{5\} \supset (1,5)\{3 \equiv -2\} \\ \chi\,(3,10)\{1\} \quad \lambda(\bar{3},\bar{5})\{-3\} \supset (1,5)\{-2\} \\ \lambda_1\,(3,1)\{5\} \quad \lambda_2(\bar{3},\bar{5})\{-3\} \supset (1,\bar{5})\{2\} \end{array}\right\}$$

MECHANISM TO GET SM HIGGS (MORE TO SAY ON THIS)

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ASYMPTOTIC FREEDOM AND GLOBAL SYMMETRIES

SU(8) IS ASYMPTOTICALLY FREE: C(S) = INDEX OF SO(8) REP. OF SPIN S

$$\frac{1}{3}\left[11\, C(1) - 26\, C(\text{WEYL } \tfrac{3}{2}) - 2\, C(\text{WEYL } \tfrac{1}{2}) - C(\text{COMPLEX } 0) \right]$$

$$= \frac{1}{3}\left[11 \times 16 - 26 \times 1 - 2 \times (15 + 2 \times 6) - 15 \right] = 27 > 0$$

GLOBAL CHIRAL SYMMETRIES

- OVERALL U(1) REPHASING OF ALL FERMIONS - BROKEN BY USUAL INSTANTON + ANOMALY MECHANISM
- RELATIVE U(1) REPHASING OF 56_L χ
- RELATIVE U(2) REPHASING AND MIXING OF TWO $\overline{28}_L$ $\lambda_{1,2}$

DYNAMICAL SYMMETRY BREAKING: GENERAL FORMALISM

- FROM LEFT CHIRAL SPINORS, CAN FORM TWO TYPES OF SCALARS

$$\bar{\Psi}_{L1}\Psi_{L2} \equiv 0$$

$$\left.\begin{aligned}\bar{\Psi}^c_{L1}\Psi_{L2} &= \Psi^T_{L1}\, i\gamma^0\, \Psi_{L2} \\ &= \Psi^T_{L2}\, i\gamma^0\, \Psi_{L1}\end{aligned}\right\} \neq 0$$

ALLOWED FORM OF CONDENSATES

GROUP REP. CONTENT OF CONDENSATE IS DIRECT PRODUCT OF REP. CONTENT OF Ψ_1 AND Ψ_2

- VECTOR GLUON POTENTIAL FOR $A+B \to A+B$

$$V = \frac{g^2\, K(A+B;\, A, B)}{2r}$$

$$K(A+B;\, A,B) = C_2(A+B) - C_2(A) - C_2(B)$$

$C_2 =$ CASIMIR > 0

$C_2(1) = 0$

SPECIAL CASES:

$$\left.\begin{aligned} K(1;\, A, \bar{A}) &= C_2(1) - C_2(A) - C_2(\bar{A}) = -2\,C_2(A) < 0 \\ K(A;\, \bar{A}, B) &= C_2(A) - C_2(\bar{A}) - C_2(B) = -C_2(B) < 0 \end{aligned}\right\}$$

BOTH ATTRACTIVE

$$K(A;\, A, 1) = C_2(A) - C_2(A) - C_2(1) = 0$$

(15) DYNAMICAL SYMMETRY BREAKING TO GET SM HIGGS

$SU(3)$: $\quad 3 \times \bar{3} = \underline{1} + 8$

— = MOST ATTRACTIVE

$SU(5)$: $\quad 10 \times \bar{5} = \underline{5} + 45$

$1 \times 5 = 5 \quad , \quad 1 \times \bar{5} = \bar{5}$

ALL FAMILY $3 \times \bar{3} \supset 1$ PAIRINGS:

$$\chi(\bar{3},5)\{3\} \quad \lambda_a(3,1)\{5\} \supset (1,5)\{3 \equiv -2\}$$

$$\chi(3,10)\{1\} \quad \lambda_a(\bar{3},\bar{5})\{-3\} \supset (1,5)\{-2\}$$

$$\lambda_1(3,1)\{5\} \quad \lambda_2(\bar{3},\bar{5})\{-3\} \supset (1,\bar{5})\{2\}$$

SO GET CONDENSATE $(1,5)\{-2\}$ AND ITS CONJUGATE $(1,\bar{5})\{2\}$

GLOBAL $U(2)$ CHIRAL SYMMETRY OF λ_a BROKEN BY CONDENSATE $\Rightarrow$ GET 4 GOLDSTONES WITH REPRESENTATION $(1,5)\{-2\}$ THIS IS WHAT IS NEEDED IN FLIPPED $SU(5)$ TO BREAK SM ELECTROWEAK

ELIMATION OF HIGGS TRIPLET (ANTONIADIS ET AL)

$$10\{1\} \times 10\{1\} \times 5\{-2\} \supset (2,3)_{\frac{1}{6}} \times (2,3)_{\frac{1}{6}} \times (1,3)_{-\frac{1}{3}} \supset (1,1)_0$$

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ABSENCE OF SCALAR-FERMION YUKAWA COUPLINGS

CHIRALITY $\Rightarrow$ YUKAWA COUPLINGS OF SPIN $\frac{1}{2}$ FERMIONS HAVE GENERAL FORM

$$\Psi_{L1}^T \, i\gamma^0 \, \Psi_{L2} \, \Phi$$

$\Psi_{L1,2}$ ANY OF χ, λ

Φ EITHER ϕ OR ϕ^*

NO WAY TO CONTRACT SU(8) INDICES TO FORM A SINGLET:

$\chi \, \chi$	6 UPPER INDICES
$\lambda \, \lambda$	4 LOWER INDICES
$\chi \, \lambda$	3 UPPER, TWO LOWER INDICES
ϕ	THREE UPPER INDICES
ϕ^*	THREE LOWER INDICES

NO YUKAWA COUPLINGS INVOLVING SPIN $\frac{1}{2}$ FERMIONS MUST BE GENERATED BY RADIATIVE CORRECTIONS

SO AFTER SU(8) BREAKING, SPIN $\frac{1}{2}$ FERMIONS REMAIN MASSLESS

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SUPERSYMMETRIES IN LIMIT OF ZERO GAUGE COUPLING g

TWO CONSERVED REP. 8 SUPERCURRENTS

$$J_{a}^{\mu\alpha} = \gamma^{\nu}(\partial_{\nu}\phi^{[\alpha\beta\gamma]})\gamma^{\mu}\lambda_{a\,[\beta\gamma]} \qquad a=1,2$$

ONE CONSERVED SINGLET SUPERCURRENT

$$J^{\mu} = \gamma^{\nu}(\partial_{\nu}\phi^{*}_{[\alpha\beta\gamma]})\gamma^{\mu}\chi^{(\alpha\beta\gamma)}$$

$$\partial_{\mu} J_{a}^{\mu\alpha} = 0 \qquad \partial_{\mu} J^{\mu} = 0$$

NOT INVARIANCES OF SCALAR SECTOR SELF-COUPLINGS

$$S_{SELF}(\phi) = \phi^{*}_{[\rho\kappa\tau]}\phi^{*}_{(\alpha\beta\gamma)}\left(g_{1}\phi^{[\rho\kappa\tau]}\phi^{(\alpha\beta\gamma)} + g_{2}\phi^{(\alpha\kappa\tau]}\phi^{[\rho\beta\gamma)}\right)$$

$\Rightarrow$ $g_{1,2}$ ORDER g^{2} OR HIGHER

DISCUSSION

THREE NOVEL INGREDIENTS

- BOSON-FERMION BALANCE WITHOUT FULL SUSY
- CANCELING ANOMALIES BETWEEN SPIN $\frac{1}{2}$ AND SPIN $\frac{3}{2}$
- BREAKING GAUGE SYMMETRY WITH SCALAR REPRESENTATION WITH NONZERO $U(1)$ GENERATOR: MODULAR GROUND STATE

PROMISING FEATURES

- THREE FAMILIES
- ROUTE TO FLIPPED $SU(5)$
- $SU(8) \to SU(3)_{\text{FAMILY}} \times SU(5) \times U(1) \to SM$ USING SCALAR 56

  $SM \to$ ELECTROMAGNETIC $U(1)$ USING HIGGS GENERATED BY DYNAMICAL SYMMETRY BREAKING FROM MOST ATTRACTIVE CHANNELS
- VANISHING BARE YUKAWA COUPLINGS

  ZERO GAUGE COUPLING SUMS

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EXPERIMENTAL SIGNATURES

- STERILE NEUTRINOS (GENERIC)
- SINGLET HIGGS + SU(2) TRIPLET HIGGS SAME QUANTUM NUMBERS

OPEN ISSUES

- CHOICE OF MOD(5) REPRESENTATIVE – DISCRETE ANALOG OF VACUUM ALIGNMENT CONDITION?
- DYNAMICAL ISSUES – RUNNING COUPLINGS
  PROTON DECAY
  MASSES AND MIXINGS OF SM
  CP VIOLATION
  FLAVOR CHANGING NEUTRAL CURRENTS
  MONOPOLES

  EXTRA FERMION STATES BEYOND USUAL FAMILIES

- FURTHER UNIFICATION

  "SU(8) GRAVITY MULTIPLET" 128 BOSON AND FERMION HELICITIES

  "SU(8) MATTER MULTIPLET" 112 " " "

  MATCH NUMBERS OF HALF-INTEGER, INTEGER ROOTS OF E(8)

  IS THERE A CONNECTION TO THE E(8) ROOT LATTICE?