CERN Summer School 2014
Introduction to Accelerator Physics

Part I

by
Verena Kain CERN BE-OP

Acknowledgement

These lectures are based on lectures given by

- Bernhard Holzer at the CAS (CERN Accelerator School)
- Oliver Bruning at the CAS
- Frank Tecker at the CAS
- Peter Forck at the JUAS (Joint Universities Accelerator School)
- Philipp Bryant at the JUAS

and the textbooks:

- S.Y. Lee, “Accelerator Physics”, World Scientific
- K. Wille, “Physics of Particle Accelerators and Synchrotron Radiation Facilities”, Teubner

and LHC talks by:

- M. Lamont
- J. Wenninger

Contents of these Lectures

Goal: provide basic understanding of accelerator physics

5 Lectures

Lecture 1: Introduction & Motivation, History

Lecture 2: Transverse Dynamics

Lecture 3: Longitudinal Dynamics

Lecture 4: Imperfections, Measurement, Correction, Injection, Extraction

Lecture 5: LHC, LHC performance and LHC Challenges

Goal of Accelerator

Goal of an accelerator: increase energy of CHARGED particles

- Increase energy how

$$\Delta E = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\, d\vec{r} = q \int_{\vec{r}_1}^{\vec{r}_2} (\vec{E} + \vec{v} \times \vec{B})\, d\vec{r}$$

- The particle trajectory direction d**r** parallel to **v**

$$\Delta E = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\, d\vec{r} = q \int_{\vec{r}_1}^{\vec{r}_2} \vec{E}\, d\vec{r} = qU$$

- **…**increase of energy with electric fields

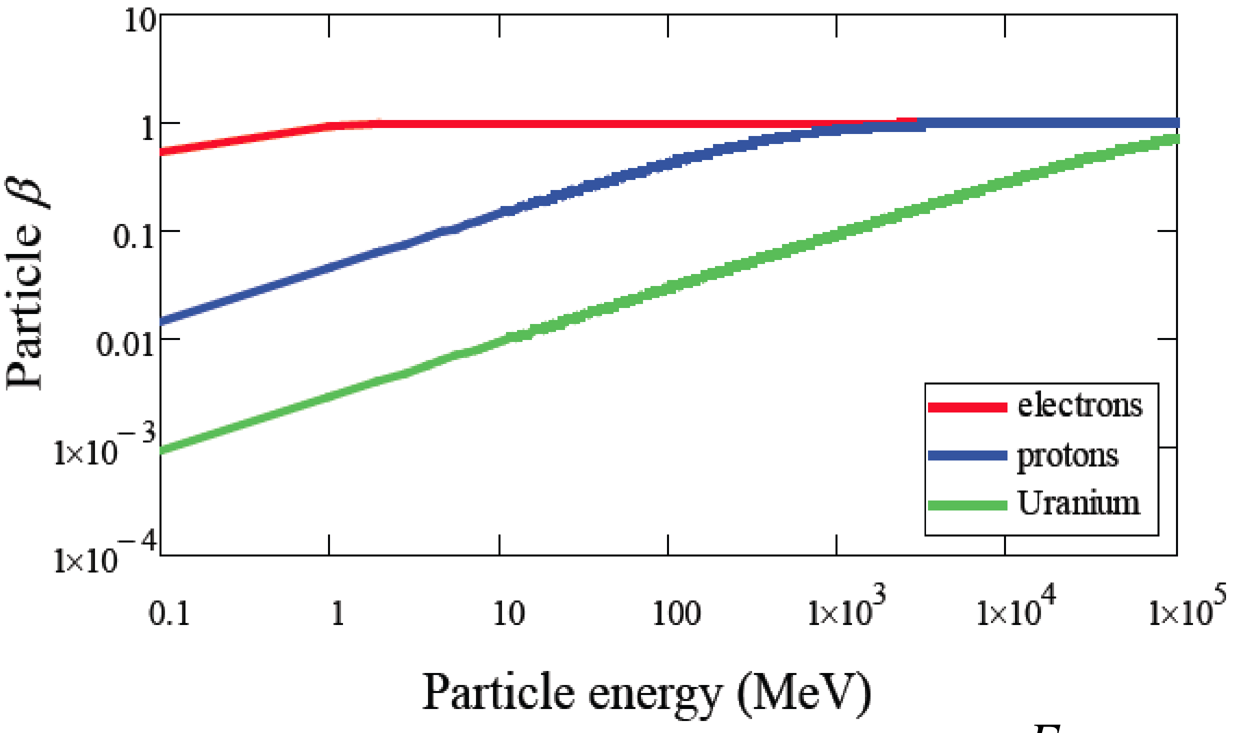
- Magnetic fields are needed for control of trajectories.

Goal of Accelerator

- Call them accelerator, but depending on the particle the velocity reaches maximum value often already at low energy
- Velocity does not change there after, momentum (mass) does

Particle rest mass:

electron 0.511 MeV
proton 938 MeV
^{239}U ~220000 MeV



$$g = \frac{E}{E_0} = \frac{m}{m_0}$$

The units we are using...

Energy: in units of eV:

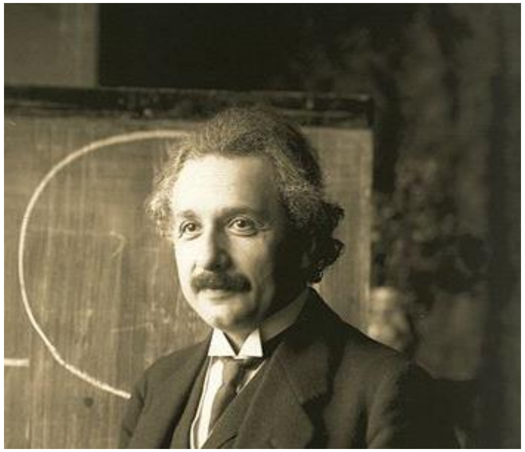
corresponds to the energy gained by charge of a single electron moved across a potential difference of one volt.

1 eV = 1.602176565(35) × 10^{-19} × 1 J

This comes from electrostatic particle accelerators.

Unit of mass m: we use $E = mc^2$

➔ Unit of mass is eV/c^2



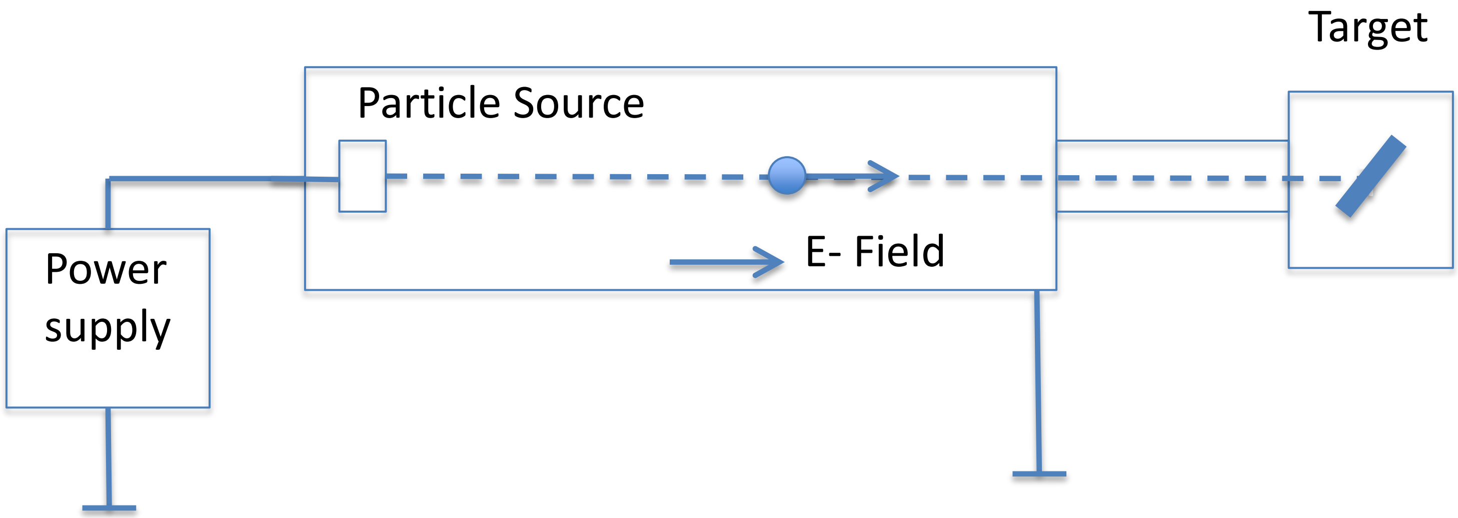
Unit of momentum p:

with $E^2 = (mc^2)^2 + p^2c^2$

➔ Unit of momentum is eV/c

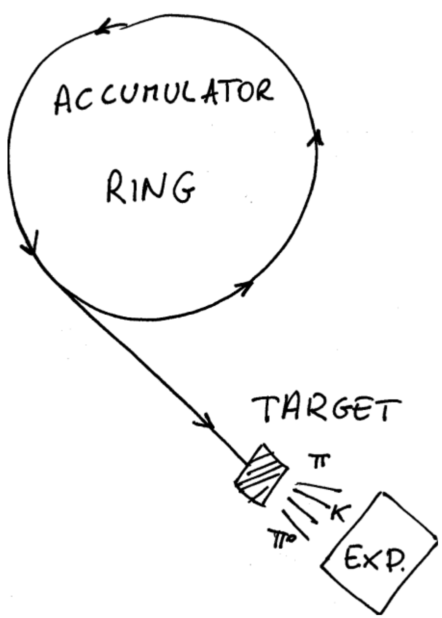
Units from Electrostatic Accelerator

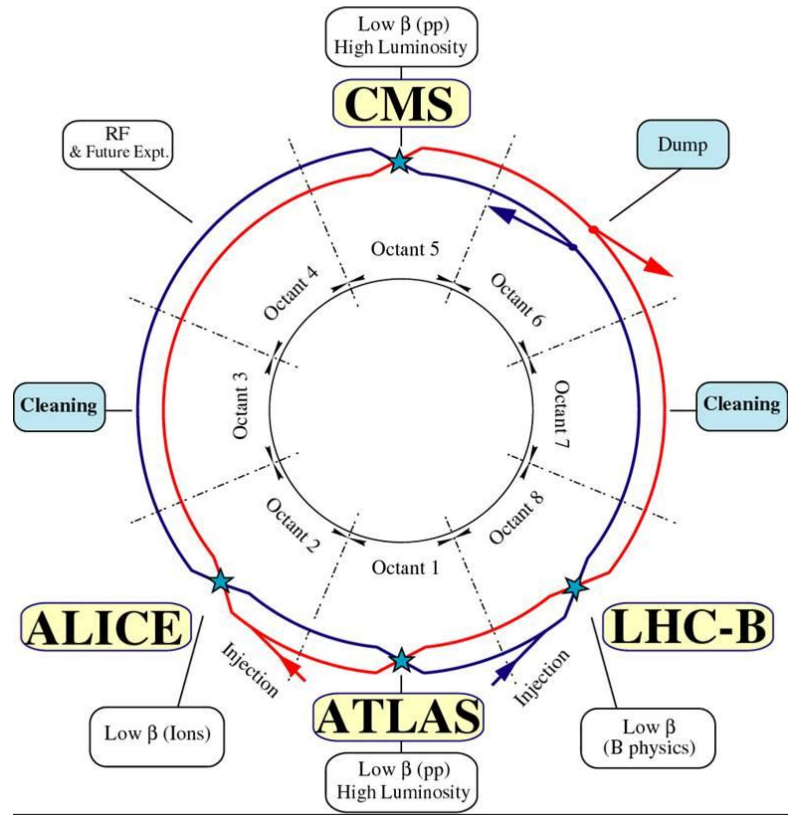
The basic principles of such an electrostatic accelerator:

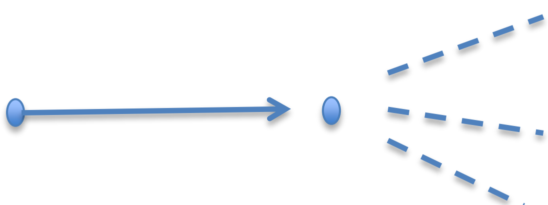


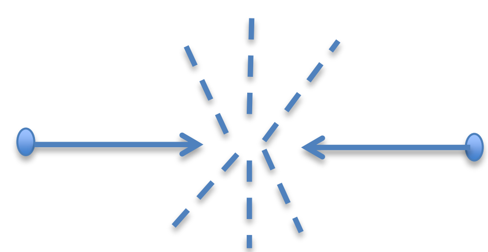
Goal of accelerators in particle physics

1) Provide particles with high energy
2) Provide collisions

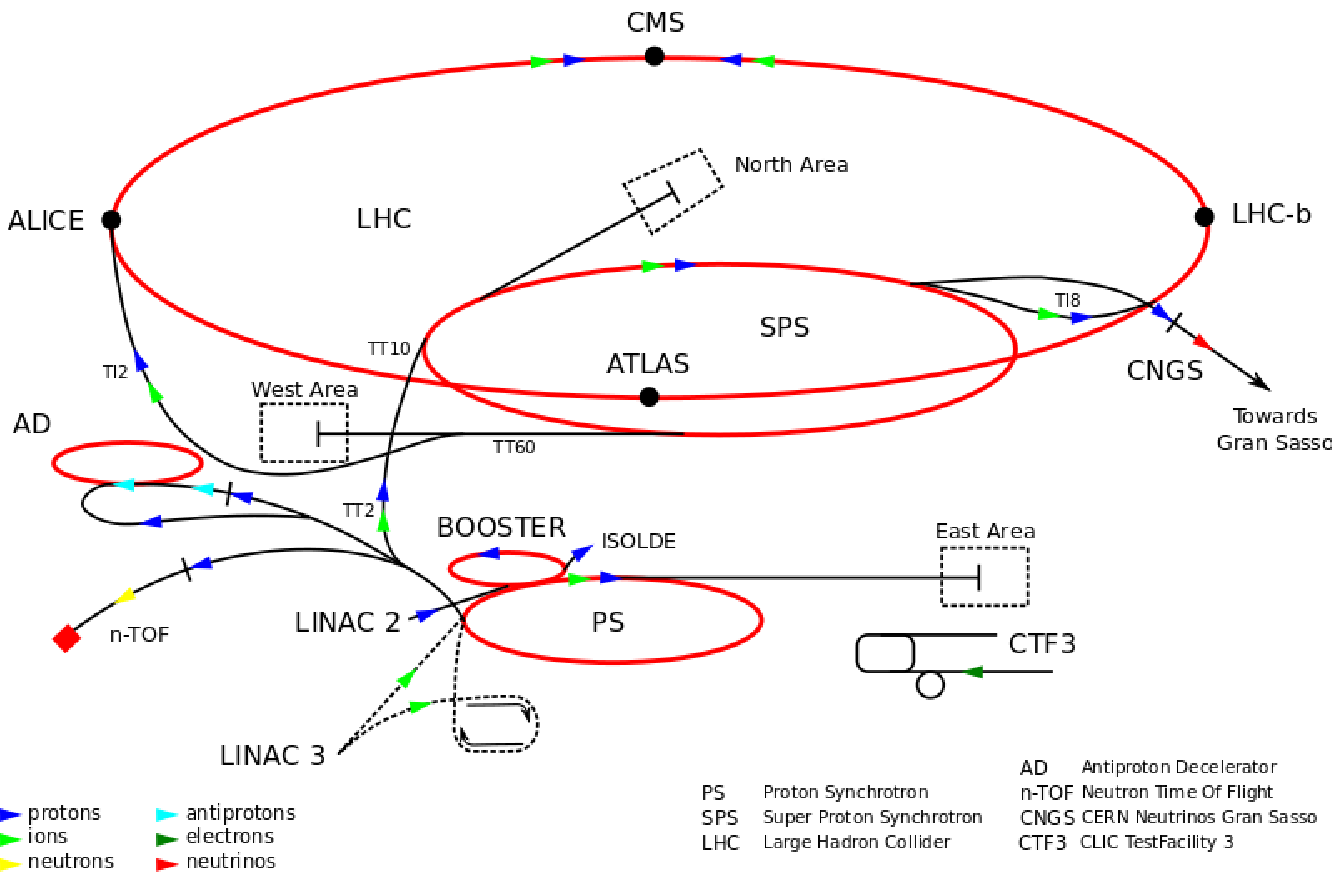








The CERN Accelerator Complex



PS (Proton Synchrotron): 1959
LHC (Large Hadron Collider): 2008

Circumference:
PS ~ 628 m
LHC ~ 27 km

Energy:
PS 26 × 10^9 eV (GeV)
LHC 7 × 10^{12} eV (TeV)

Particles produced through:
PS: fixed target collisions
LHC: beam-beam collisions

Why high energies? Why so large? Why collisions?

Why high energies?

Accelerators are instrument to study smaller and smaller structures and heavy short-lived objects with

$$E = m \cdot c^2$$

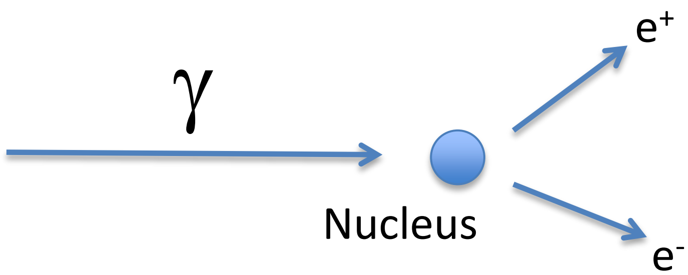
Wavelength of probe radiation needs to be smaller than object to be resolved

$$\lambda = \frac{h}{p} = \frac{h \times c}{E}$$

Object	size	Radiation energy
Atom	10^{-10} m	0.00001 GeV
Nucleus	10^{-14} m	0.01 GeV
Nucleon	10^{-15} m	0.1 GeV
Quarks	-	> 1 GeV

Why collisions?

- Conservation laws: e.g. momentum and energy conservation



Photon into e^+,e^- only in proximity of nucleus. Nucleus takes part of momentum (and part of available energy…)

- Center-of-mass Frame and Center-of-mass Energy (E_{CM})
  - Center-of-mass frame defined where: $\sum \vec{p}_i = \vec{0}$
  - The energy available for creation of particles corresponds to E_{CM}

Center-of-Mass Energy

Transformation to center-of-mass frame: Lorentz transformation

$$p^\mu = (E/c, \vec{p})$$ 4-momentum

$$p^\mu p_\mu = \frac{E^2}{c^2} - \vec{p}^2$$

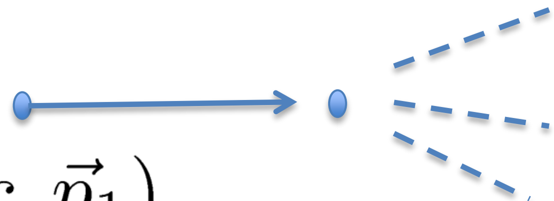
$$p'^\mu = L^\nu_\mu p^\nu$$ Lorentz Transformation

$$p^\mu p_\mu = p'^\mu p'_\mu$$ The norm: is Lorentz invariant

$$\frac{E^2_{CM}}{c^2} - \vec{0}^2 = \frac{E^2_{tot}}{c^2} - \vec{p}^2_{tot}$$

$$\frac{E^2_{CM}}{c^2} = \frac{E^2_{tot}}{c^2} - \vec{p}^2_{tot}$$

E_{CM} in Fixed Target Experiment



$$p_2 = (m_2 c, \vec{0})$$

$$p_1 = (E_1/c, \vec{p}_1)$$

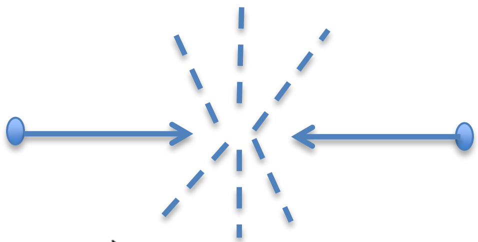
$$p_{tot} = (E_1/c + m_2 c, \vec{p}_1)$$

$$E_{CM}^2 = (m_1^2 + m_2^2)c^4 + 2E_1 m_2 c^2$$

$$E_{CM} \propto \sqrt{E_1}$$

E_{CM} in Collider Experiment

Laboratory Frame = CM Frame



$$p_1 = (E_1/c, \vec{p}_1) \qquad p_2 = (E_2/c, -\vec{p}_1)$$

$$E_{CM} = E_1 + E_2$$

➔ Collider more energy efficient;
But also more complex: two beams to be accelerated and to be brought into collision

Why do accelerators become larger and larger?

This is due to technological limitations.

- Magnetic field, accelerating gradient,…
- Circular machine:
  - the higher the particle momentum, the higher the magnetic field to keep beam on trajectory
  - the higher bending angle (the smaller R), the higher the magnetic field

Which particle to use?

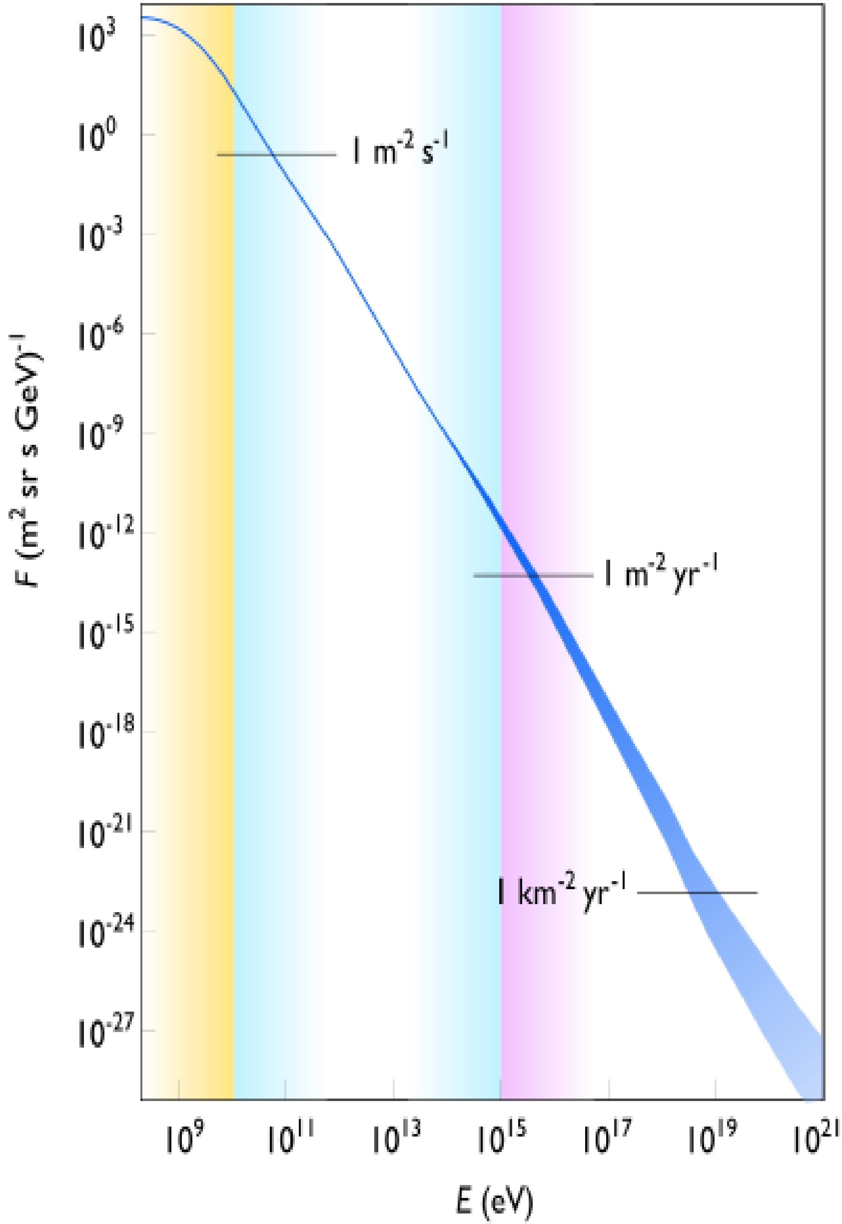
e^- , e^+	p^+
Elementary particles with no internal structure	Consist of quarks held together by gluons
The total energy of the collider is transferred into the collision	The constituents of the protons collide. The energy available for the collision less than the collider energy
Precision measurements: beam energy can be exactly tuned to optimize the analysis	Discovery machine: with a single chosen beam energy different processes at different energies can be scanned
Disadvantage for circular colliders: low mass of these leptons. High power loss due to synchrotron radiation $P_{loss} \propto \frac{E^4}{m^4} \times \frac{current}{R}$ Solution: linear accelerators - long	

HISTORY OF ACCELERATORS

Natural Accelerators

- Radioactive Accelerators
  - Rutherford experiment 1911
  - Used α particles tunneling through the Coulomb barrier of Ra and Th to investigate the inner structure of atoms
  - Existence of positively charged nucleus, d ~ 10^{-13} m
  - α particle kinetic energy ~ 6 MeV

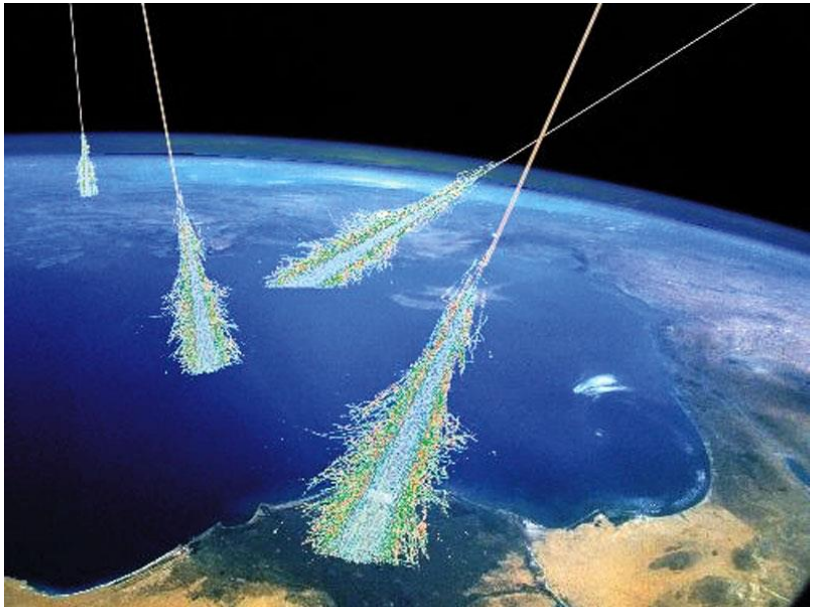
- Cosmic rays
  - Energies up to 3 x 10^{20} eV for heavy elements have been measured. ~ 40 x 10^{6} times what the LHC can do.
  - "Ultra high energy" cosmic rays are rare…



Why accelerators then....?

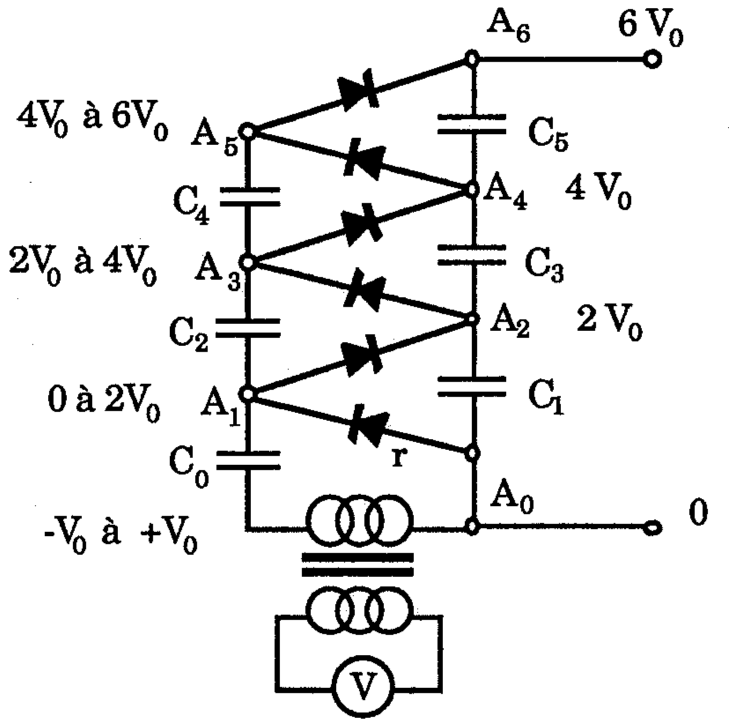
Accelerators have the advantage:

High energies, high fluxes of a given particle species, controlled energies at a specific location where a detector can be installed.

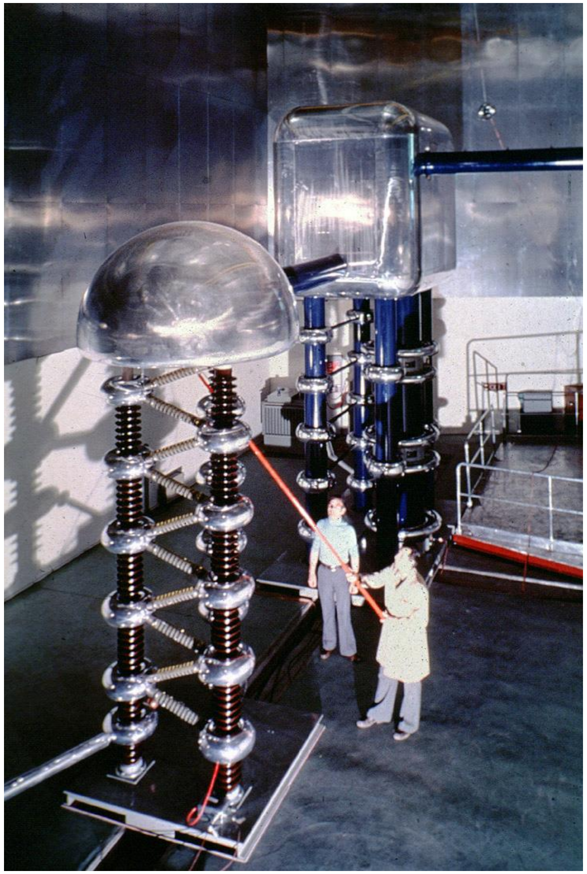


Electrostatic Accelerators – 1930s

- Cockcroft-Walton electrostatic accelerator
  - High voltage source by using high voltage rectifier units
  - High voltage limited due to sparking in air. Limit ~ 1 MV



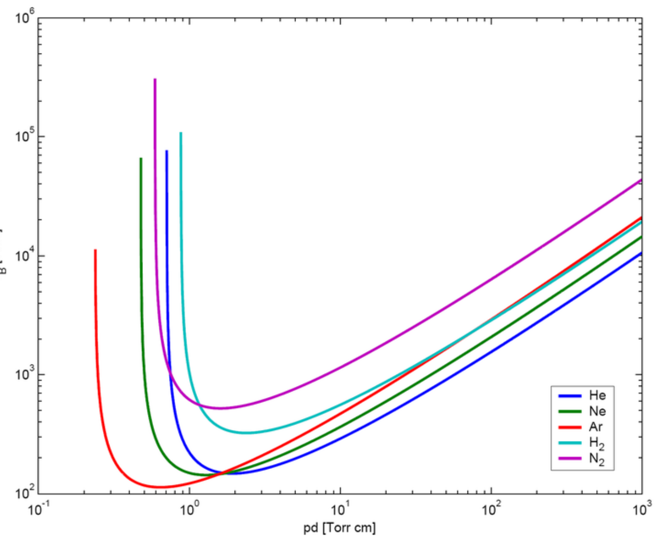
CERN used until 1993 as ion-source: 750 kV



Electrostatic Accelerators

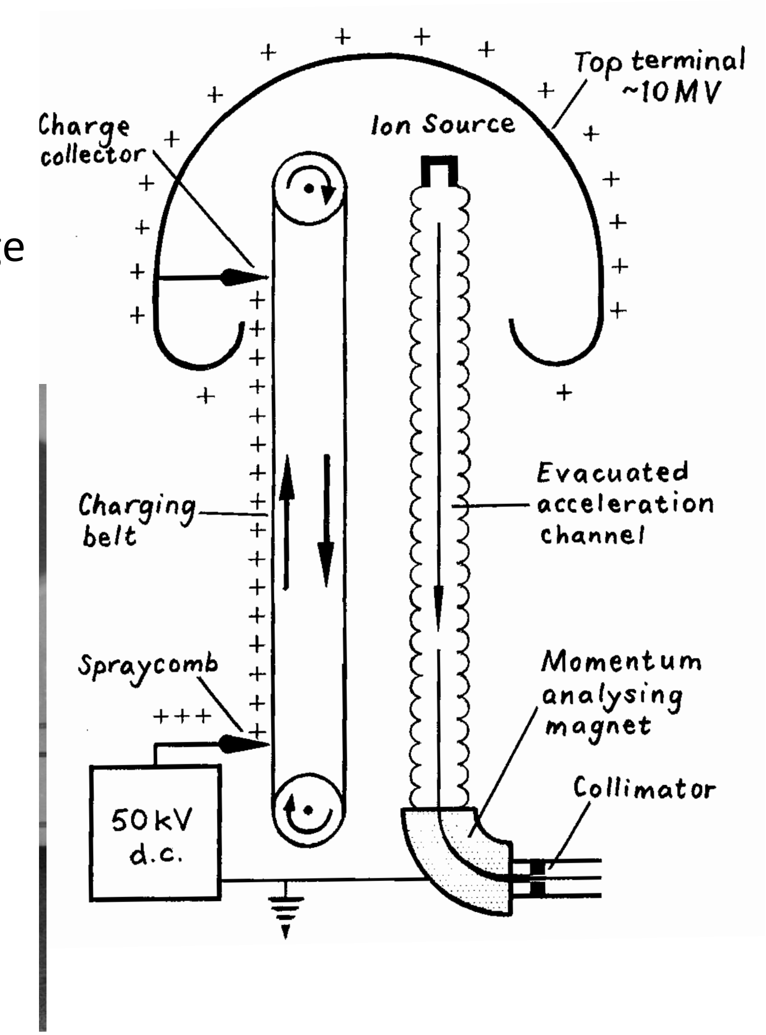
- Limit of 1 MV overcome: placing the electrodes under high pressure gas. Paschen's law

Break down voltage



Break down voltage depends on gas pressure and gap between electrodes.

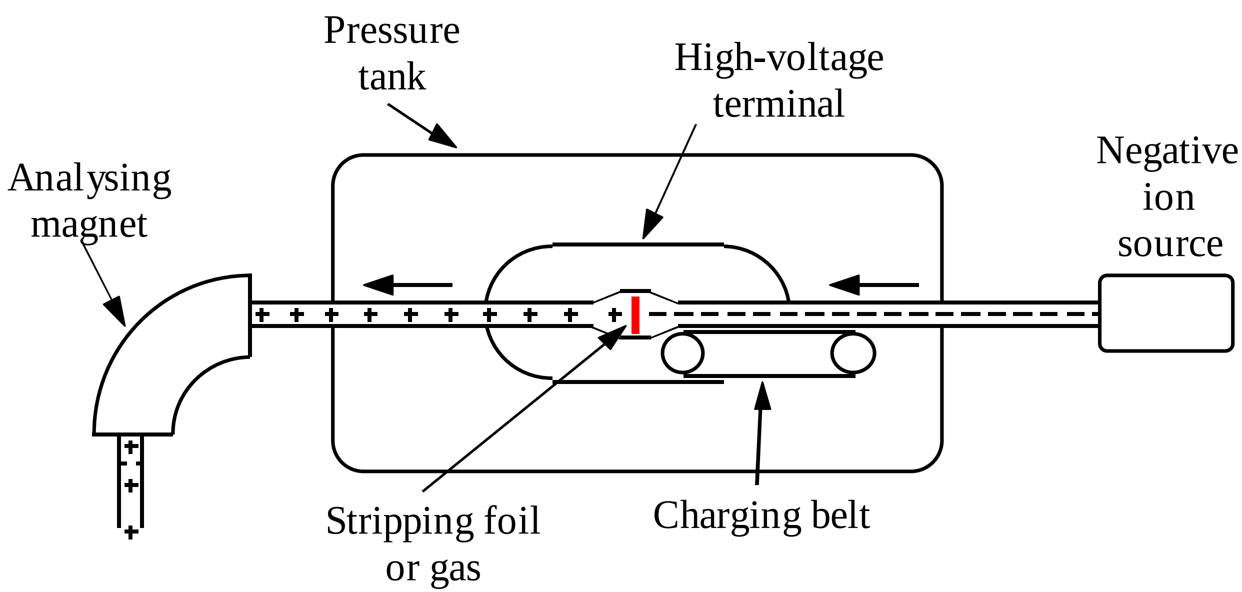
Product of pressure x gap



- ➔ Van De Graaf generator
  - 1 – 10 MV

Tandem Van de Graaf Generator

- …use the accelerating voltage twice

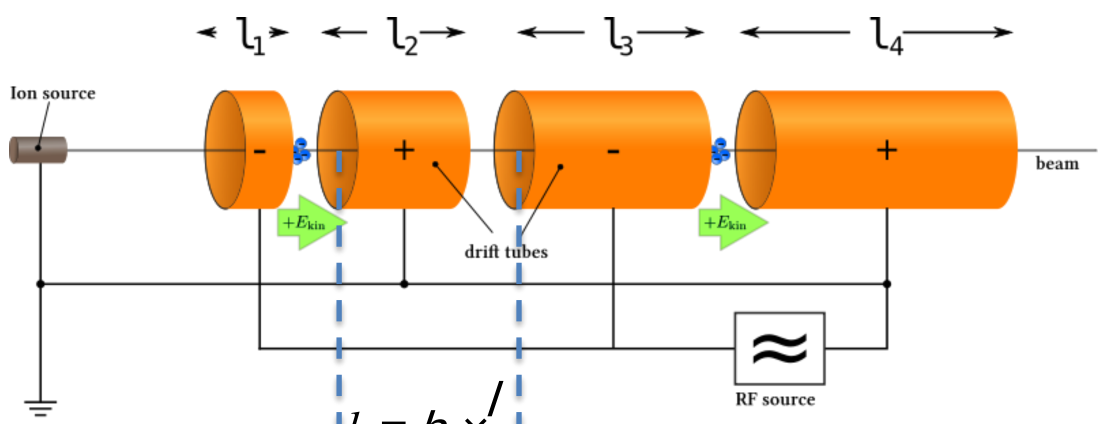


- Up to 25 MV
- Advantages of Van de Graaf:
  - Great variety of ion beams
  - Very good energy precision, small energy spread
- Applications in nuclear physics, accelerator mass spectroscopy,…

Alternating RF Field – the Revolution

Electrostatic accelerator limitation: maximum voltage before sparking for acceleration over single gap

- ➔ pass through acceleration gap many times (Ising)
- 1928 Wideroe: first working RF accelerator



$$l_i = \beta_i \times \frac{\lambda}{2}$$

Energy gain per gap:

E = q $V_{RF} \sin(\phi_s)$

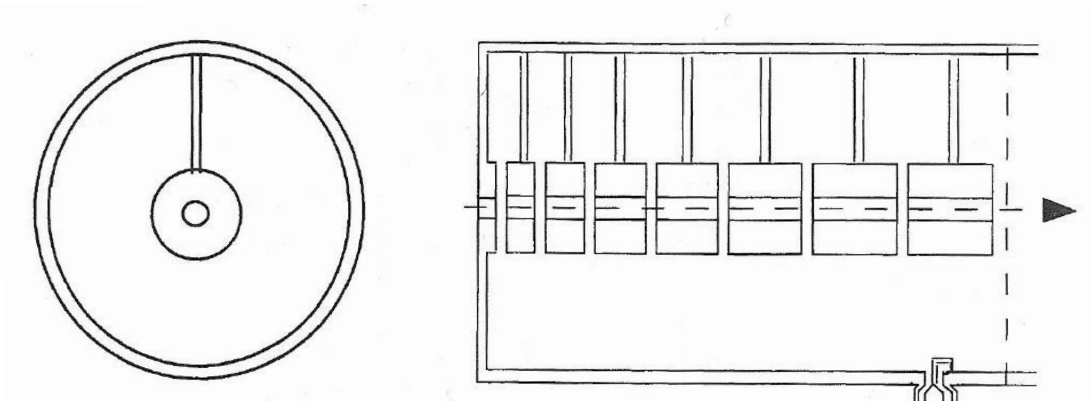
Φs…phase wrt to RF field

- Particle synchronous with field. In shielding tube when field has opposite sign. Voltage across each cell the same.

- Remark: tubes have to become longer and longer, as particles become faster and faster
- or higher frequency $\lambda = c/f_{RF}$
- But radiation power loss: $P = \omega_{RF} C V_{RF}^2$, C gap capacitance

Alvarez Linac or Drift Tube Linac

- Eliminate power loss: drift tube placed in cavity
  - Electromagnetic field oscillating in cavity. Standing wave, TM mode (longitudinal E-Field, transverse B-Field)
  - Resonant frequency of cavity = accelerating field frequency
  - Reduce power loss
  - Exploit Farraday's law: $\nabla \times \vec{E} = -\frac{\partial}{\partial t}\vec{B}$



Circular Accelerators

- Linear accelerators can in principle accelerate to arbitrarily high energies.
- ….but become longer and longer

- ➔ Particles on circular paths to pass accelerating gap over and over again

- ➔ Cyclotron proposed by E.O. Lawrence in 1929 and built by Livingston in 1931.

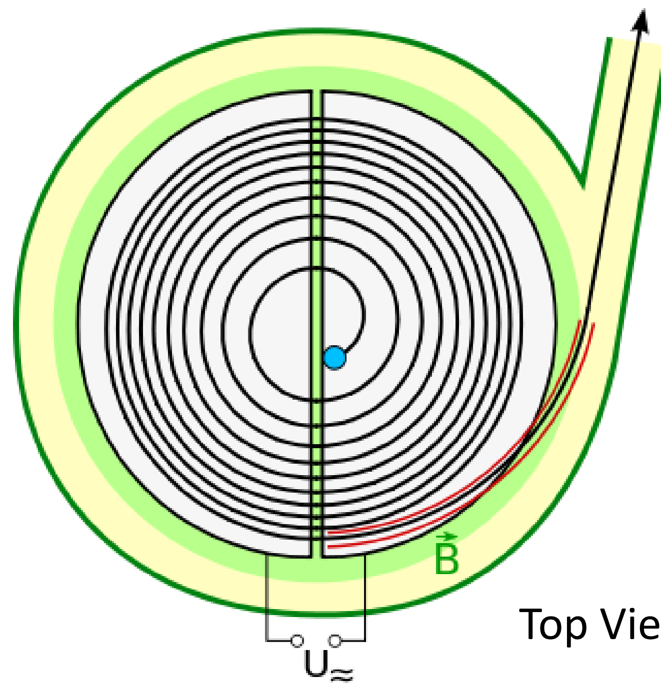
Cyclotron

Particle Source in the middle

Between the two "Dees" acceleration gap connected to RF source. $\omega_{RF} = \omega_{cyclotron}$

Vertical magnetic field to guide the particles in the horizontal plane. The radius of particle trajectory becomes larger and larger with larger energy

Particles extracted with a deflector magnet or an electrode.



Top View

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \longrightarrow F_L = q\, v\, B \longrightarrow$$ Vertical B
No E

$$F_c = m\frac{v^2}{r} \longrightarrow$$ centrifugal force

$$F_L = F_c \longrightarrow \omega = \frac{v}{r} = \frac{qB}{m} \longrightarrow$$ revolution period

Cyclotron Limitation

- Cyclotron frequency is constant for constant mass
- For relativistic particles mass is not constant

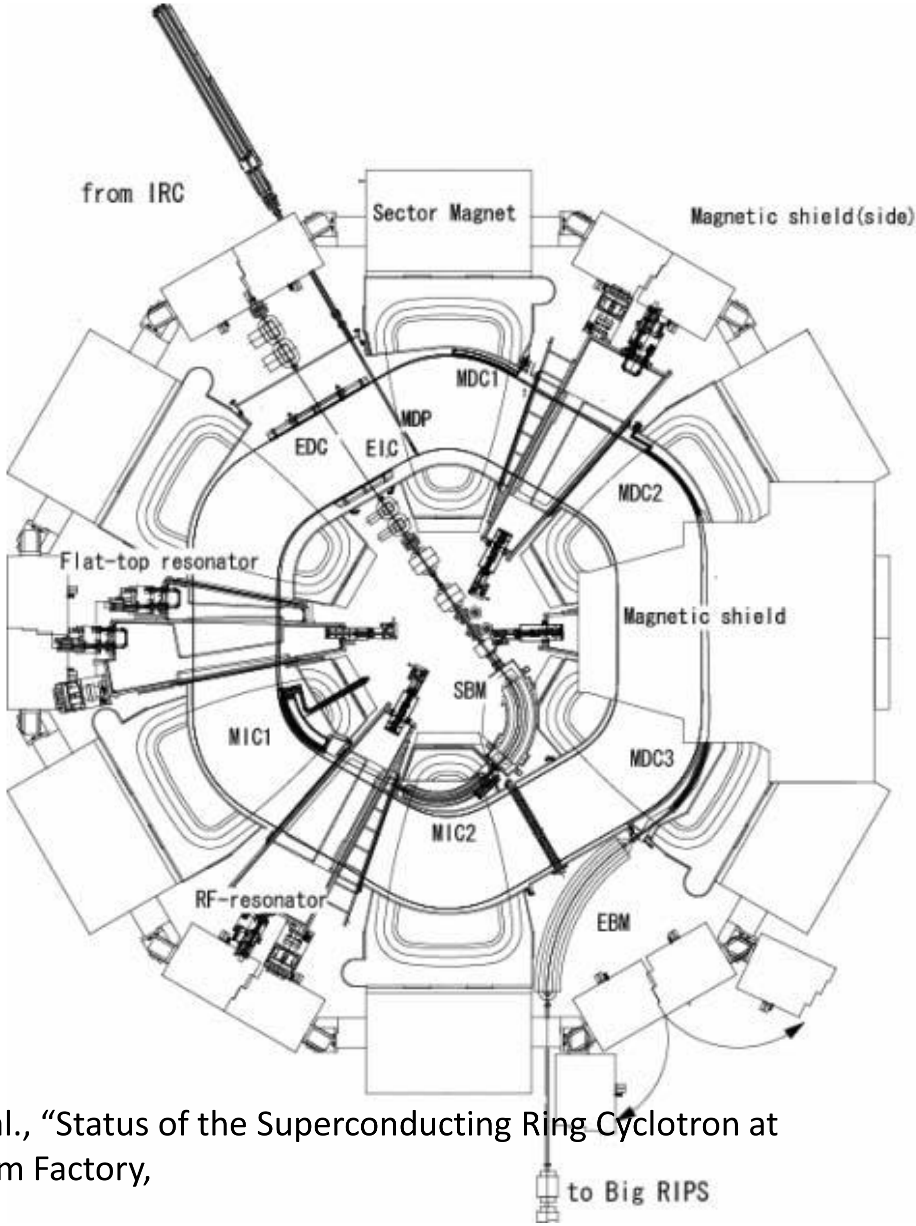
$$w = \frac{v}{r} = \frac{Bq}{m} = \frac{Bq}{m(E)}$$

- The classical cyclotron only valid for particles up to few % of speed of light
  - Not useful for electrons…already relativistic at 500 keV
- Possibilities: synchrocyclotrons (change frequency (and magnetic field) with energy) or isochronous cyclotrons (increase magnetic field with r, frequency constant)
- Modern cyclotrons can reach > 500 MeV (PSI, TRIUMF, RIKEN)

Biggest Cyclotron in the world

- RIKEN, Japan
- 19 m diameter, 8 m high
- 6 superconducting sector magnets, 3.8 T
- Heavy ion acceleration

- Uranium ions accelerated up to 345 MeV/u



K. Yamda et al., “Status of the Superconducting Ring Cyclotron at RIKEN RI Beam Factory,
EPAC 2008

PSI cyclotron

High intensity P_{max} = 1300 kW

Betatron - 1940

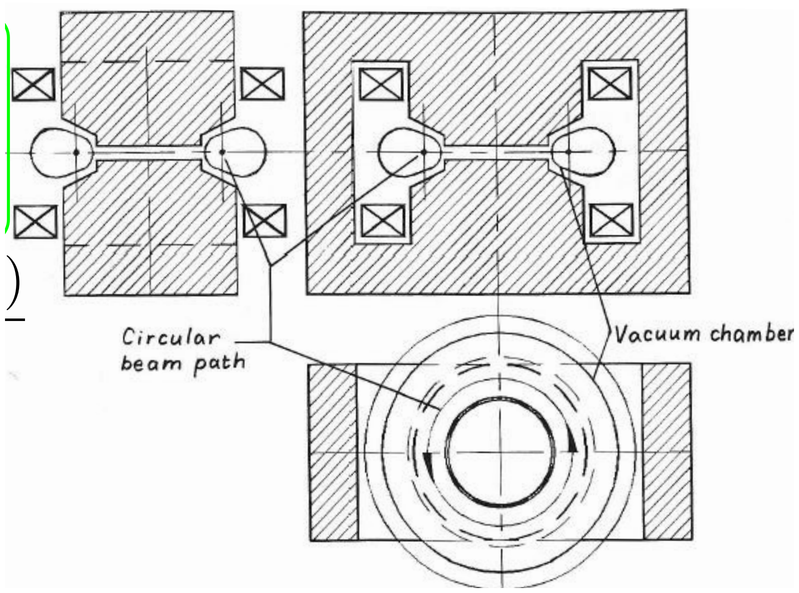
- Another early circular accelerator

- Idea by Wideroe in 1923. Kerst builds the first working betatron in 1940.

- Difference: constant radius and magnetic field changing with time to keep radius constant.
- Accelerating E-field generated through induction, no external E-field

- e^- accelerated to 2.3 MeV

- “Betatron” for beta rays (e^-)



Betatron – a few formulae

$$\oint \vec{E}\, d\vec{r} = -\iint_A \dot{\vec{B}}\, d\phi$$

Induction, Farraday's law

$$2\pi R\left|\vec{E}\right| = -\pi R^2 \frac{d}{dt}\left\langle\left|\vec{B}\right|\right\rangle$$

for d**E**/d$\Theta=0$

$$\left|\vec{F}\right| = \left|\dot{\vec{p}}\right| = -e\left|\vec{E}\right| = \frac{eR}{2}\left\langle\left|\dot{\vec{B}}\right|\right\rangle$$

The force on the particle

$$\left|\vec{p}\right| = eR\left|\vec{B}\right|$$

At the same time the magnetic field needs to compensate the centrifugal force

$$\left|\dot{\vec{B}}\right| = \frac{1}{2}\left\langle\left|\dot{\vec{B}}\right|\right\rangle \Rightarrow \left|\vec{B}(t)\right| = \frac{1}{2}\left\langle\left|\vec{B}(t)\right|\right\rangle + \left|\vec{B}_0\right|$$

Stability criterion

Synchrotron

- Higher and higher energies – larger and larger radii, limited B fields – cannot stay compact

- Fix trajectory ➔ R = constant; R can be large
- Dipole magnets with field only where the beam is
  - “small” magnets

- R= constant ➔B field increases **synchronously** with beam energy

- Synchrotron - all big modern machines are synchrotrons

Cosmotron – BNL – 1952 – 3 GeV

3 GeV p^+ synchrotron

Particle rigidity: $B\rho = \frac{p}{e}$

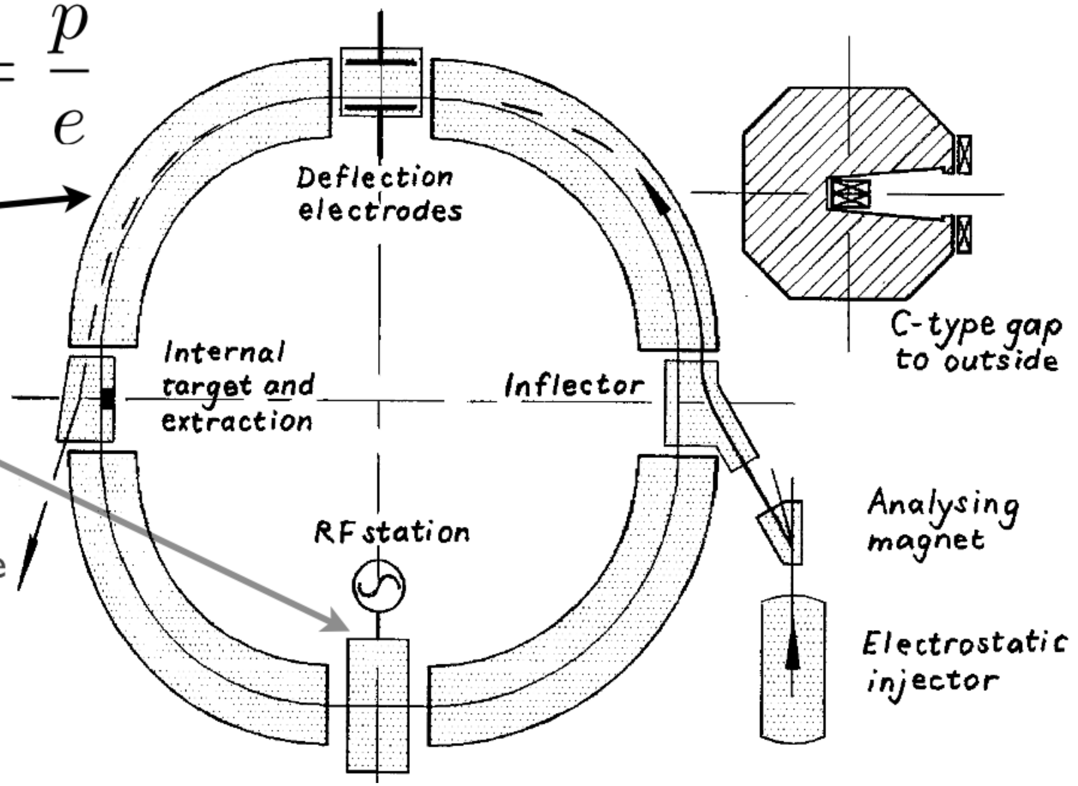
B = B(t) magnetic field from the bending magnets

p = p(t) particle momentum varies by the RF cavity

e electric charge

ρ constant radius of curvature

New magnetic elements for injection and extraction.



Particles do many 1000 turns – trajectories have an angular spread ➔ divergence

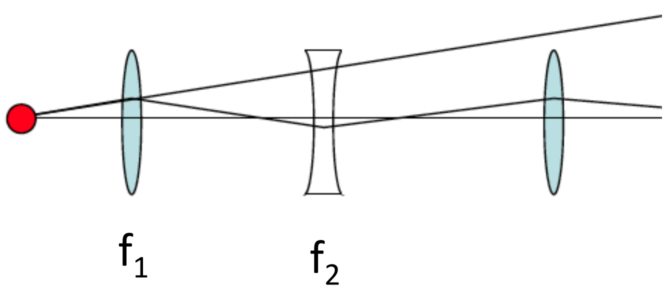
Need focusing elements. Cosmotron **weak focusing** machine

Strong Focusing

Idea by E. Courant, M. Livingston, H. Snyder in 1952 and earlier by Christofilos

Alternating gradient focusing

- Analogous to geometrical optics: a series of alternating focusing and defocusing lenses will focus.



$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

Consider $f_1 = f$, $f_2 = -f$ ➔ $F = d/f^2 > 0$

In our case the lenses will be magnets with alternating gradients

The first alternating gradient synchrotrons

Alternating gradient focusing was quickly adopted by synchrotrons and transfer lines.

- 1954: Cornell University, e^- accelerated to 1.5 GeV (Wilson et al.)

The following two machines are still in operation.

They use combined function magnets.

- 1959: CERN Proton Synchrotron (PS) accelerated protons to 28 GeV
- 1960: Brookhaven Alternating Gradient Synchrotron (AGS) accelerated protons to 33 GeV

The next step: storage ring colliders

Make use of all the particles' energy. 2-beam synchrotrons.

The first one: Ada (Frascatti), 1961-64, e+,e-, 250 MeV, 3m circumference

Many examples to come at DESY, SLAC, KEK, Fermilab with the Tevatron (980 GeV), BNL with RHIC

1971-1984: ISR (CERN), p+,p+, 31.5 GeV, 948 m circumference

1981-1984: SPS running p+, p-, 270 – 315 GeV, 6.9 km circumference; discovery of W and Z Bosons

1989-2000: LEP highest energy electron synchrotron, e+,e-, 104 GeV, 27 km circumference; three generations of quarks and gluons

2008 - : LHC highest energy proton synchrotron, p+,p+, heavy ions, 4 TeV (2.76 TeV per nucleon for $^{208}Pb^{82+}$); Discovery of Higgs

More tomorrow…

EXTRA SLIDES

Electron source

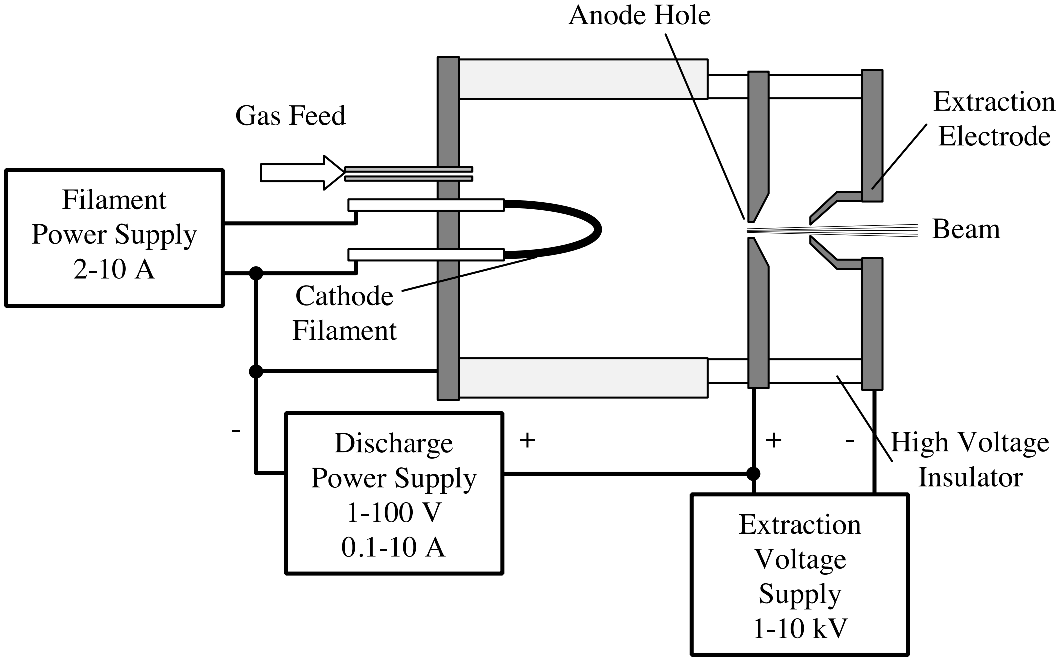
- Electrons: thermionic cathode:
  - Free electrons through heat. Heating with a filament to induce thermionic emission. Cathodes special metal layers with low work function to emit e- easily.

Positive Ion Sources

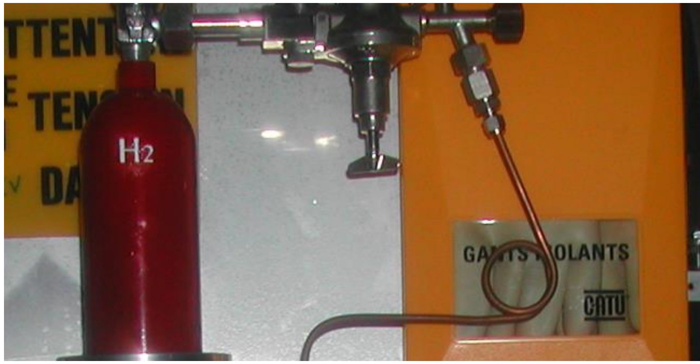
- The principle: Plasma, electron bombardement,…

- $H_2 + e^- \rightarrow H_2^+ + 2e^-$
- $H_2^+ + e^- \rightarrow H^+ + H + e^-$
- $H + e^- \rightarrow H^+ + 2e-$

- $H_2 \rightarrow 2\,H^+ + 2e^-$



- CERN for LHC protons: Duoplasmatron
  – Higher density plasma at anode. density plasmas hence the name.



The first cyclotron

- 30 cm across
- 0.5 T magnetic field
- p^+ accelerated to 1.2 MeV

Weak Focusing

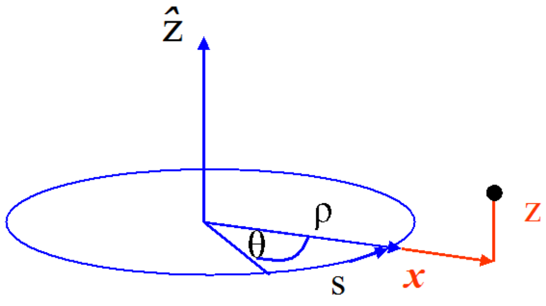
Particles with deviations from the design trajectory need to feel restoring forces otherwise the beam diverges.

For example for deviations in the vertical plane $F_z \propto -z$

Need either horizontal field component…

$$B_x = -const \cdot z \qquad \frac{\partial B_x}{\partial z} = -const$$

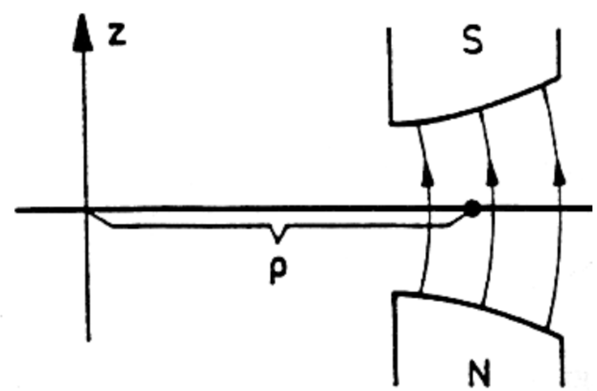
…or a radially decreasing guide field



circular coordinate system

Maxwells equation: $\vec{\nabla} \times \vec{B} = 0$

$$\rightarrow \quad \frac{\partial B_x}{\partial z} = \frac{\partial B_z}{\partial x} = \frac{\partial B_z}{\partial r} < 0$$



Similar considerations for horizontal plane

Pole shape in combined function magnet