Introduction to the SM (1)

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The goal

Maximize your knowledge

How do I do it?

The plan

1. What is particle physics
2. How we calculate
3. The basic of the SM
4. The Higgs mechanism and flavor
5. Neutrinos and open issues

What is HEP?

What is HEP

Find the basic laws of Nature

More formally

$$\mathcal{L} = ?$$

- We have quite a good answer
- It is very elegant, it is based on axioms and symmetries
- We use particles to answer this question

What is mechanics?

- Answer the question: what is $x(t)$?
- A system can have many DOFs, and then we seek to find $\vec{x}(t) \equiv x_1(t)$, $x_2(t)$,...
- Once we know $\vec{x}(t)$ we know any observable
- Solving for $q_1 \equiv x_1 + x_2$ and $q_2 \equiv x_1 - x_2$ is the same as solving for x_1 and x_2
- The idea of generalized coordinates is very important

How do we find $x(t)$?

$x(t)$ minimizes something

- This is an axiom
- The thing that $x(t)$ minimized is called "the action" and is denoted by S
- There is one action for the whole system
- Similar to a minimum of a function

$$\min[f(x)] \Rightarrow x_0, \qquad \min[S(x(t))] \Rightarrow x_0(t),$$

- The condition for a minimum of a function is $df(x)/dx = 0$. What is the equivalent one for the minimum of the action?

What is S?

$$S = \int_{t_1}^{t_2} L(x, \dot{x}) dt, \qquad \dot{x} \equiv dx/dt = v$$

- The solution of the requirement that S is minimal is given by the E-L equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{\partial L}{\partial x}$$

- Once we know L we can find $x(t)$ up to initial conditions
- Wow. Mechanics is reduced to the question “what is L?”

An example

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{\partial L}{\partial x} \qquad L = \frac{mv^2}{2} - V(x)$$

- The solution is $-V'(x) = m\dot{v}$, aka $F = ma$
- Here $F = ma$ is the output, not the starting point!
- So how do we find what is L?

What is L?

L is the most general one that is invariant under some symmetries

- We (again!) rephrase the question. Now we ask what are the symmetries of the system that lead to L
- Symmetires are related to conservation laws

Example of symmetries

What are the symmetries that give

$$L = \frac{mv^2}{2} - V(x)$$

- In 1d, if we require $x \to -x$ invariant what can we say about $V(x)$?
- In 3d, if we require rotation invariant?
- In 1d with two particles, if we require $x_1 \to x_1 + C$ and $x_2 \to x_2 + C$ invariant?
- What bout the kinetic term, $mv^2/2$?
- (homework) $x_1 \to -3x_2$ and $x_2 \to -x_1/3$?

What is field theory

What is a field?

- In math: something that has a value in each point. We can denote it as $\phi(x,t)$
  - Temperature (scalar field)
  - Wind (vector field)
  - Magnetic field (?)
  - Density of people (?)
- In physics a field used to be associated with a source
- We now know that fields are fundamental

How to deal with fields

- $\phi(x,t)$ have infinite number of DOFs
- It can be an approximation for many DOFs
- To solve mechanics of fields we need to find $\phi(x,t)$
- Here ϕ is the generalized coordinate and not x
- We still need to minimize S

$$S = \int \mathcal{L} dx dt \qquad \mathcal{L}[\phi(x,t), \phi'(x,t), \dot{\phi}(x,t)]$$

- We also have an E-L equation for field theories

An example

- We usually require Lorentz invariant
- Field theory is nice as x and t are treated the same
- The “kinetic term” is promoted

$$T \propto \left(\frac{dx}{dt}\right)^2 \Rightarrow T \propto \left(\frac{d\phi}{dt}\right)^2 - \left(\frac{d\phi}{dx}\right)^2 \equiv (\partial_\mu \phi)^2$$

- For example, a free field $\mathcal{L}$ gives a wave equation

$$(\partial_\mu \phi)^2 \Rightarrow \frac{\partial^2 \phi}{\partial x^2} = \frac{\partial^2 \phi}{\partial t^2}$$

- Again, we see that what is used to be the starting point, here is the final result

Harmonic oscillator

The harmonic oscillator

Why do we care so much about harmonic oscillators?

- Becuse we really care about springs?
- Because we really care about pendulums?

The harmonic oscillator

Why do we care so much about harmonic oscillators?

- Becuse we really care about springs?
- Because we really care about pendulums?

Because almost any function around its minima can be approximated as a harmonic function!

- Indeed, we usually expand the potential around one of its minima
- We identify a small parameter, and keep only few terms in a Taylor expansion

Classic harmonic oscillator

$$V = \frac{kx^2}{2}$$

We solve and get

$$x(t) = A\cos(\omega t) \qquad k = m\omega^2$$

- The period does not depend on the amplitude
- Energy is conserved

Which of the above two statements is a result of the approximation of keeping only the harmonic term in the expansion?

Coupled oscillators

Coupled oscillators

- There are normal modes
- The energy of each mode is conserved
- This is an approximation!
- Once we keep non-harmonic terms energy moves between modes

$$V(x,y) = \frac{k_1 x^2}{2} + \frac{k_2 y^2}{2} + \alpha x^2 y$$

- How fast energy is transferring?