Introduction to the SM (2)

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Happy 2nd birthday

Yesterday...

- What is mechanics: $x(t) \Rightarrow L \Rightarrow$ symmetries
  - Q: the implication of $x_1 \to -3x_2$ and $x_2 \to -x_1/3$?
  - A: $q_1 = x_1 - 3x_2,\ q_2 = x_2 + 3x_1$ and we have $V(q_1)$
- What is field theory: $\phi(x,t)$ are the coordinates, not x
- Harmonic oscillator
  - Leading term in the Taylor expansion
  - Normal modes and higher order couplings
  - We also expand around the minimum for fields

Today: QM, QFT, PT and Feynman diagrams

Quantum mechanics

What is QM?

- Many ways to formulate QM
- For example, we promote $x \to \hat{x}$
- We solve QM if we know the wave function $\psi(x,t)$
- Then we know things like $\langle x \rangle (t)$
- The wave function is mathematically a field

The quantum SHO

$$H = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2} \qquad E_n = (n + 1/2)\hbar\omega$$

- We also like to use

$$H = (a^\dagger a + 1/2)\hbar\omega \qquad a^{(\dagger)} \sim x \pm ip$$

- We call $a^\dagger$ and a creation and annihilation operators

$$a|n\rangle \propto |n-1\rangle \qquad a^\dagger|n\rangle \propto |n+1\rangle$$

- So far this is abstract. What can we do with it?

Fields

- With many DOFs, $a \to a_i \to a(k)$
- And the states

$$|n\rangle \to |n_i\rangle \to |n(k)\rangle$$

- And the energy

$$(n+1/2)\hbar\omega \to \sum (n_i+1/2)\hbar\omega_i \to \int [n(k)+1/2]\hbar\omega dk$$

- Just like in mechanics, we expand around the minimum of the fields, and to leading order we have SHOs
- In QFT fields are operators while x and t are not

SHO and photons

I have two questions:

- What is the energy that it takes to excite a SHO by one level?
- What is the energy of the photon?

SHO and photons

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- What is the energy that it takes to excite a SHO by one level?
- What is the energy of the photon?

Same answer

$$\hbar\omega$$

- Why the answer to both question is the same? Can we learn anything from it?

What is a particle?

Excitations of SHOs are particles

What about fermions?

- We see how photons are related to SHO
- We can construct a fermion SHO

$$[a, a^\dagger] = 1 \quad \to \quad \{b, b^\dagger\} = 1$$

- No classical analogue since $b^2 = 0$
- We can then think of fermionic fields. They can generate only one particle in a given state

What about mass and couplings?

- A SHO give a "free" Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2$$

- We can add "potential" terms (without derivatives)

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + m^2\phi^2$$

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- We can add terms. How do we choose what to add?
  - Must be invariant under the symmetries
  - We keep some leading terms (usually, up to ϕ^4)

A short summary

- Particles are excitations of fields
- The fundeumental Lagrangian is giving in term of fields
- Our aim is to find it

Perturbation theory

Perturbation theory

$$H = H_0 + H_1 \qquad H_1 \ll H_0$$

- In many cases pertubation theory is a mathematical tool
- There are cases, however, that PT is a better way to describe the physics
- Many times we prefer to work with EV of H (why?)
- Yet, at times it is better to work with EV of H_0 (why?)

PT for 2 SHOs

$$V(x,y) = \frac{kx^2}{2} + \frac{4ky^2}{2} + \alpha x^2 y$$

- Classically α moves energy between the two modes
- How it goes in QM?
- Recall the Fermi golden rule

$$P \propto |\mathcal{A}|^2 \times \text{P.S.}$$

- If the initial state is $|0,1\rangle$ what transitions are alowed?

PT for 2 SHOs again

$$V(x,y) = \frac{kx^2}{2} + \frac{4ky^2}{2} + \alpha x^2 y$$

- since it is $x^2 y$ all we have is

$$\mathcal{A} \sim \langle 2,0|\alpha x^2 y|0,1\rangle$$

- a_y annihilates the y "particle" and $(a_x^\dagger)^2$ creates two x "particles"
- We have

$$P \propto \alpha^2$$

- It is a "decay" of "particle" y into two x particles

Even More PT

$$V' = \alpha x^2 z + \beta xyz \qquad \omega_z = 10,\ \omega_y = 3,\ \omega_x = 1$$

- Calculate $y \to 3x$ using 2nd order PT

$$\mathcal{A} \sim \langle 3,0,0|\mathcal{O}|0,1,0\rangle \qquad \mathcal{O} \sim \sum \frac{\langle 3,0,0|V'|n\rangle\langle n|V'|0,1,0\rangle}{E_n - E_{0,1,0}}$$

- What intermediate states contribute? only $|1,0,1\rangle$

$$P \propto \frac{\alpha^2\beta^2}{64}$$

- What is the meaning of the $1/\Delta E$?

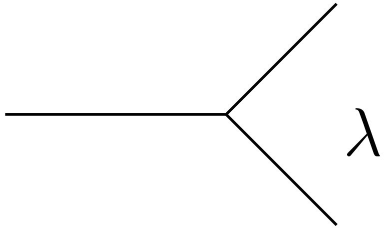
Feynman diagrams

Feynman diagrams

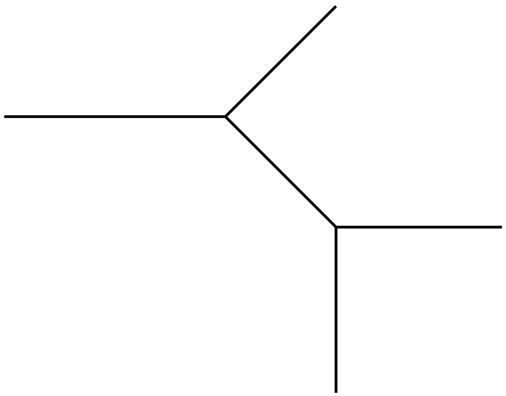
- A graphical way to do perturbation theory with fields
- Unlike SHOs before, a particle can have any energy as long as $E \geq m$
- Operators with 3 or more fields generate transitions between states. They give decay and scattering
- Decay rates and scattering cross sections are calculated using the Golden Rule
- Amplitude are calculated from $\mathcal{L}$
- Each coupling contributes to the amplitude linearly
- Each internal line corrspnod the the off-shellness of 2nd order perturbation theory and give $1/(\Delta E)^2$

Examples

$\mathcal{L} = \lambda\phi^3$



$\mathcal{L} = \lambda_1 XYZ + \lambda_2 X^2 Z$ and we look for $Y \to 3X$



Homework: with $\mathcal{L} = \lambda X^2 Z$ estimate $\sigma(2X \to 2X)$