PROBLEMS FOR LL's PRACTICAL STATISTICS LECTURES

1) An experiment is searching for quarks of charge 2/3, which are expected to produce 4/9 the ionisation I_0 of unit charged particles. In an exposure in which 10^5 cosmic ray tracks are observed, 1 track has its ionisation measured as $0.44I_0$. The detector is such that ionisation measurements are Gaussian distributed about their true values with standard deviation σ. Calculate the probability that this could be a statistical fluctuation on the ionisation of a unit charged particle for the following different assumptions:

a) $\sigma = 0.07I_0$ for all 10^5 track, or

b) For 99% of the tracks $\sigma = 0.07I_0$, while for the remainder it is $0.14I_0$.

2) An experiment is determining the decay rate λ for a new particle X, whose probability density for decay at time t is proportional to exp(-λt). A total of nine decays are observed at decay times 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8 and 0.9 picoseconds. Calculate the likelihood function L(λ) at suitable values of λ (most easily done by a simple computer program), and draw a graph of the results. Find the best estimate of λ from the maximum of the likelihood curve, and a "±σ" range for λ by finding the values of λ where the logarithm to the base e of the likelihood function decreases by 0.5 units from its maximum value.

3) The coverage C(μ) is a property of a statistical technique for estimating a range for a parameter μ at a confidence level α (e.g. 68%, 90% or whatever). It is the fraction of times that, in repetitions of the procedure with different data, the estimated range contains the true value μ.
In a Poisson counting experiment with n observed events, one method of estimating a range for the Poisson parameter μ uses the estimate n +- sqrt(n) i.e. from n - sqrt(n) to n + sqrt(n). This is supposed to have 68% coverage. Determine the actual coverage C(μ) at μ = 3.41 and 3.42 as follows:
For each value of μ, determine for which measured values the nominal range from the "n+-sqrt(n)" procedure includes the specified true value, and then add up the Poisson probabilities for obtaining these measured values, again assuming the specified value of the Poisson parameter.
Explain why plots of the coverage C(μ) as a function of the Poisson parameter value μ have discontinuities.

4) An experiment is searching for New Physics. With no New Physics production, 100 events are expected; if there is New Physics, 110 events are expected. The experiment observes 130 events, which is 3σ above the 'No New Physics' prediction, so the p-value for the null hypothesis is 0.1%. The Lab Publicity Officer announces that this shows we now are 99.9% certain that New Physics has been discovered.

Comment.