Electromagnetic probes of the QGP

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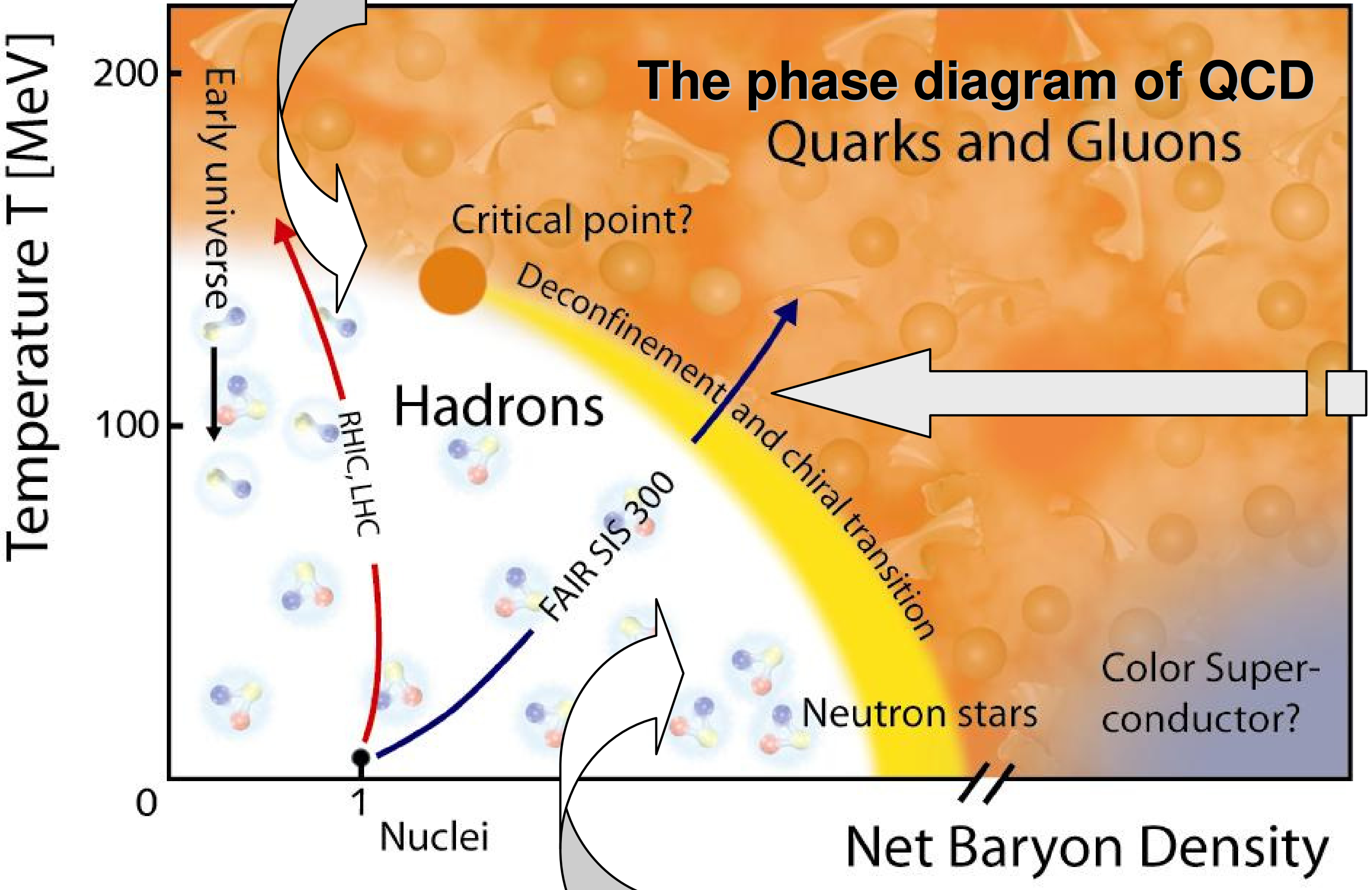
Ultimate goals of heavy-ion research

- Search for the critical point



- Study of the phase transition from hadronic to partonic matter (Quark Gluon Plasma)

photons & dileptons

- Study of the in-medium properties of hadrons at high baryon density and temperature

Electromagnetic probes: photons and dileptons

■ **Advantages:**

✓ dileptons and real photons are emitted from different stages of the reaction and not effected by final-state interactions

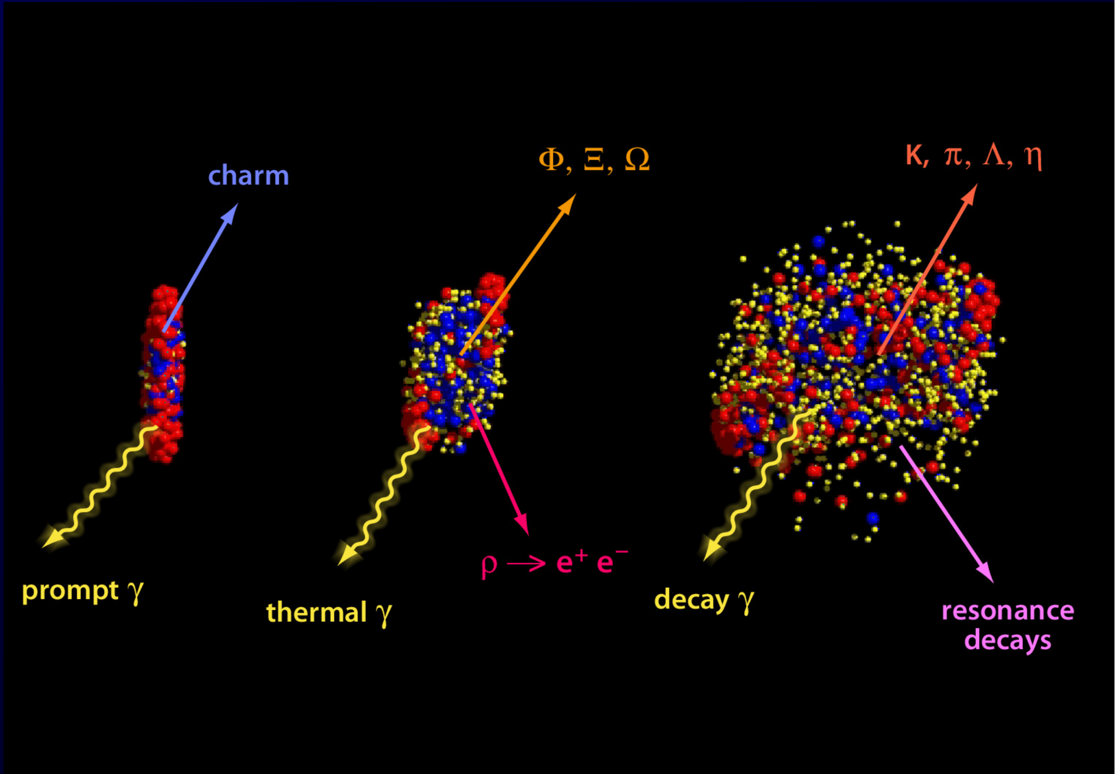
✓ provide undistorted information about their production channels

✓ promising signal of QGP – ‚thermal' photons and dileptons

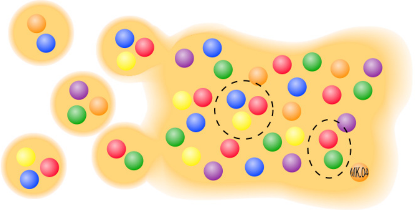
➔ Requires theoretical models which describe the dynamics of heavy-ion collisions during the whole time evolution!

❑ **Disadvantages:**

- low emission rate
- production from hadronic corona
- many production sources which cannot be individually disentangled by experimental data



From hadrons to partons



In order to study the dynamics of the phase transition from hadronic to partonic matter – Quark-Gluon-Plasma – we need a consistent non-equilibrium transport model with

☐ explicit parton-parton interactions (i.e. between quarks and gluons) beyond strings!

☐ explicit phase transition from hadronic to partonic degrees of freedom

☐ lQCD EoS for partonic phase

☐ Non-equilibrium transport theory: follows from the off-shell Kadanoff-Baym equations for the Green-functions $S^{<}_{h}(x,p)$ in phase-space representation for the partonic and hadronic phase

⇒ **Parton-Hadron-String-Dynamics (PHSD)**

W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919;
NPA831 (2009) 215;
W. Cassing, EPJ ST 168 (2009) 3

QGP phase described by

Dynamical QuasiParticle Model (DQPM)

A. Peshier, W. Cassing, PRL 94 (2005) 172301;
Cassing, NPA 791 (2007) 365: NPA 793 (2007)

Dynamical QuasiParticle Model (DQPM) - Basic ideas:

DQPM describes QCD properties in terms of ‚resummed' single-particle Green's functions – in the sense of a two-particle irreducible (2PI) approach:

Gluon propagator: $\Delta^{-1} = P^2 - \Pi$ gluon self-energy: $\Pi = M_g^2 - i2\Gamma_g\omega$

Quark propagator: $S_q^{-1} = P^2 - \Sigma_q$ quark self-energy: $\Sigma_q = M_q^2 - i2\Gamma_q\omega$

- the resummed properties are specified by complex self-energies which depend on temperature:
  - -- the real part of self-energies (Σ_q, Π) describes a dynamically generated mass (M_q,M_g);
  - -- the imaginary part describes the interaction width of partons (Γ_q, Γ_g)

- space-like part of energy-momentum tensor $T_{\mu\nu}$ defines the potential energy density and the mean-field potential (1PI) for quarks and gluons (U_q, U_g)

- 2PI framework guaranties a consistent description of the system in- and out-off equilibrium on the basis of Kadanoff-Baym equations

A. Peshier, W. Cassing, PRL 94 (2005) 172301;
Cassing, NPA 791 (2007) 365: NPA 793 (2007)

The Dynamical QuasiParticle Model (DQPM)

Properties of interacting quasi-particles: massive quarks and gluons (g, q, q_{bar}) with Lorentzian spectral functions :

$$(i = q, \bar{q}, g) \qquad \rho_i(\omega, T) = \frac{4\omega\Gamma_i(T)}{\left(\omega^2 - \vec{p}^2 - M_i^2(T)\right)^2 + 4\omega^2\Gamma_i^2(T)}$$

- **Modeling of the quark/gluon masses and widths → HTL limit at high T**
  - **quarks:**

mass: $$M^2_{q(\bar{q})}(T) = \frac{N_c^2 - 1}{8N_c} g^2 \left(T^2 + \frac{\mu_q^2}{\pi^2}\right)$$

width: $$\Gamma_{q(\bar{q})}(T) = \frac{1}{3}\frac{N_c^2 - 1}{2N_c}\frac{g^2 T}{8\pi}\ln\left(\frac{2c}{g^2} + 1\right)$$

  - **gluons:**

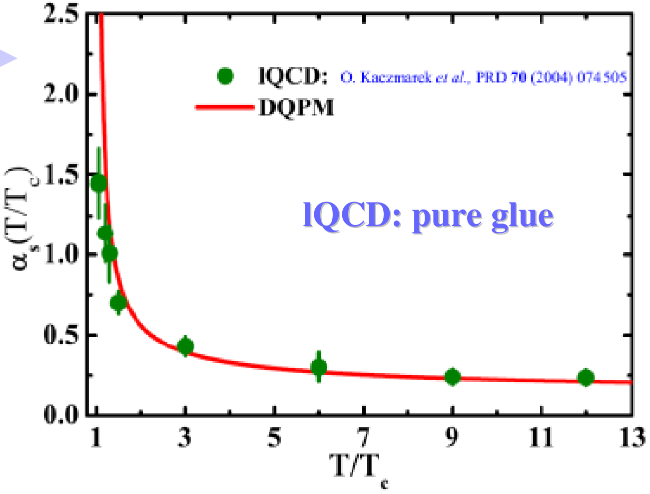
$$M_g^2(T) = \frac{g^2}{6}\left(\left(N_c + \frac{N_f}{2}\right)T^2 + \frac{N_c}{2}\sum_q \frac{\mu_q^2}{\pi^2}\right)$$

$$\Gamma_g(T) = \frac{1}{3} N_c \frac{g^2 T}{8\pi}\ln\left(\frac{2c}{g^2} + 1\right)$$

$N_c = 3$, $N_f = 3$

- **running coupling (pure glue):**

$$\alpha_s(T) = \frac{g^2(T)}{4\pi} = \frac{12\pi}{(11N_c - 2N_f)\ln[\lambda^2(T/T_c - T_s/T_c)^2]}$$



☐ **fit to lattice (lQCD) results (e.g. entropy density)**

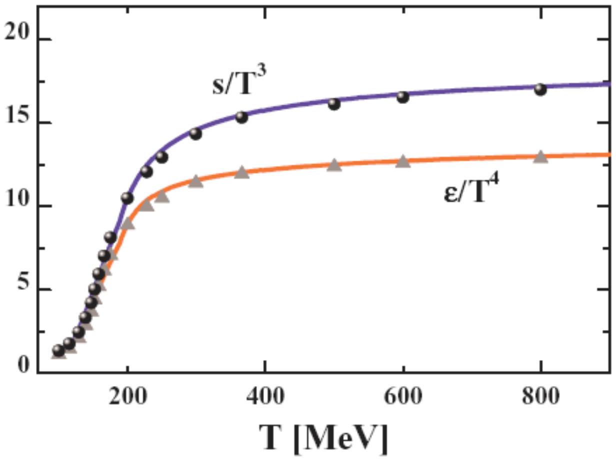
with 3 parameters: T_s/T_c=0.46; c=28.8; λ=2.42
(for pure glue N_f=0)

DQPM: Peshier, Cassing, PRL 94 (2005) 172301; Cassing, NPA 791 (2007) 365: NPA 793 (2007)

The Dynamical QuasiParticle Model (DQPM)

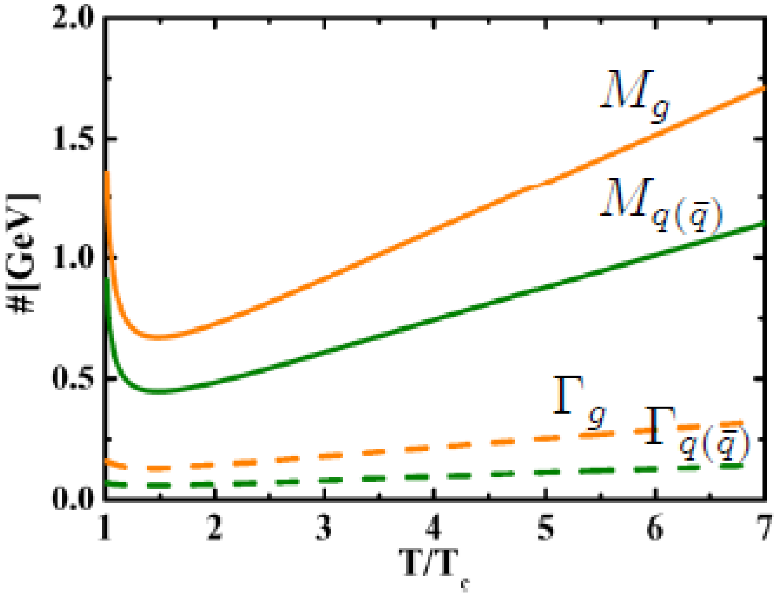
- fit to lattice (lQCD) results (e.g. entropy density)

* BMW lQCD data S. Borsanyi et al., JHEP 1009 (2010) 073



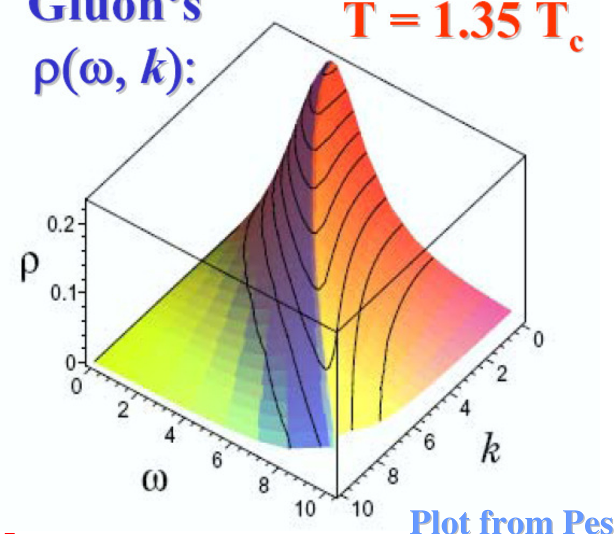
➔ Quasiparticle properties:

- large width and mass for gluons and quarks



T_C=158 MeV
ε_C=0.5 GeV/fm³

Gluon's $\rho(\omega, k)$:

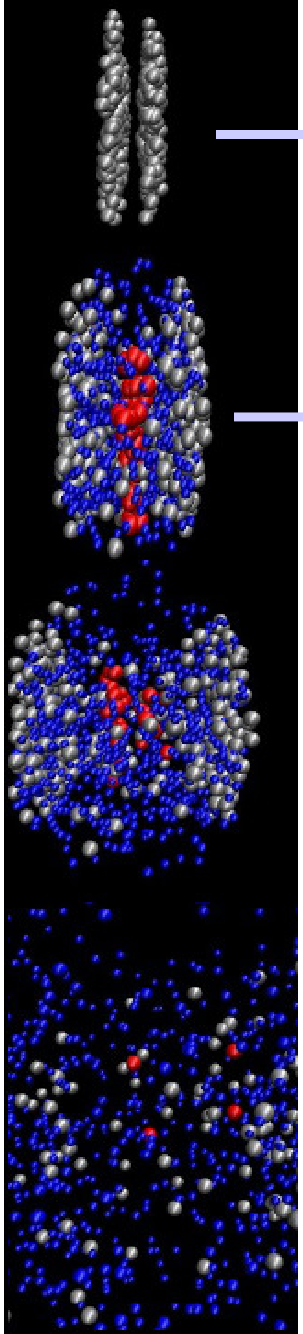


Plot from Peshier, PRD 70 (2004) 034016

- DQPM matches well lattice QCD
- DQPM provides mean-fields (1PI) for gluons and quarks as well as effective 2-body interactions (2PI)
- DQPM gives transition rates for the formation of hadrons → PHSD

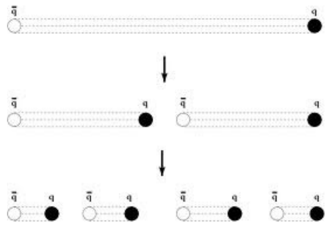
Peshier, Cassing, PRL 94 (2005) 172301; Cassing, NPA 791 (2007) 365: NPA 793 (2007)

I. PHSD - basic concept



I. From hadrons to QGP:

LUND string model



- ☐ **Initial A+A collisions – as in HSD:**
  - **- string formation in primary NN collisions**
  - **- string decay to pre-hadrons (*B* - baryons, *m* - mesons)**

- ☐ **Formation of QGP stage by dissolution of pre-hadrons**
  (all new produced secondary hadrons)
  into massive colored quarks + mean-field energy

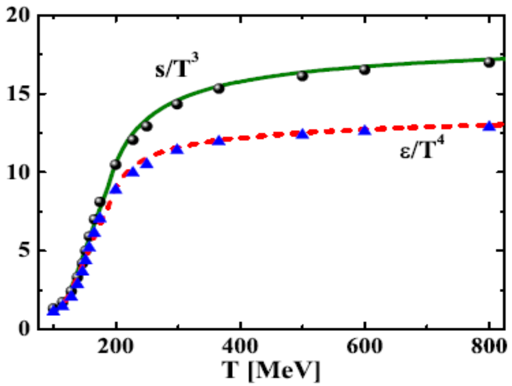
QGP phase:
$\varepsilon > \varepsilon_{critical}$

$$B \to qqq,\ \ m \to q\bar{q} \quad \forall \quad U_q$$

based on the Dynamical Quasi-Particle Model (DQPM) which defines quark spectral functions, i.e. masses $M_q(\varepsilon)$ and widths $\Gamma_q(\varepsilon)$

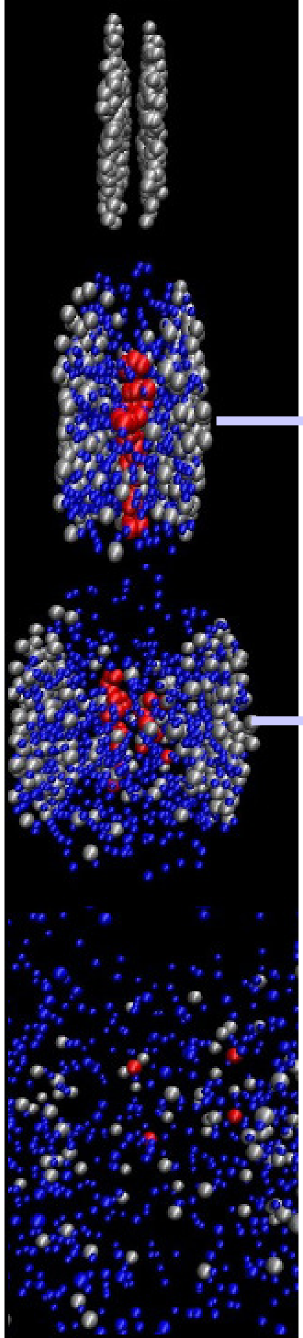
+ mean-field potential U_q at given ε – local energy density

(ε related by lQCD EoS to *T* - temperature in the local cell)



W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919; NPA831 (2009) 215; EPJ ST 168 (2009) 3; NPA856 (2011) 162.

II. PHSD - basic concept



II. Partonic phase - QGP:

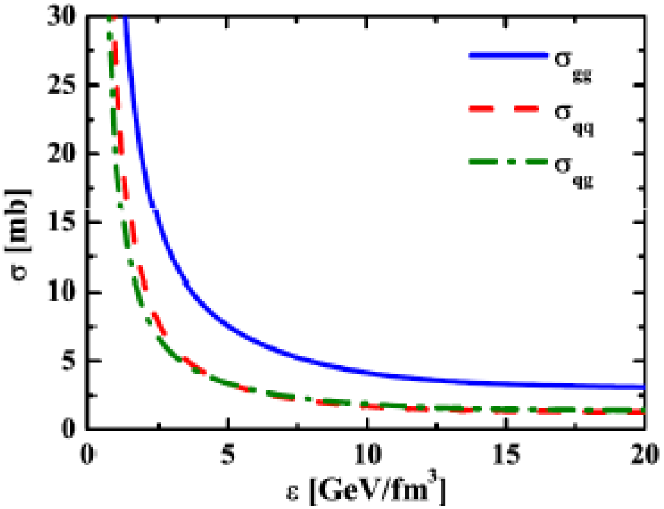
quarks and gluons (= ‚dynamical quasiparticles')
with off-shell spectral functions (width, mass) defined by the DQPM

- ☐ **in self-generated mean-field potential for quarks and gluons U_q, U_g from the DQPM**
- ☐ **EoS of partonic phase: ‚crossover' from lattice QCD (fitted by DQPM)**
- ☐ **(quasi-) elastic and inelastic parton-parton interactions:**

using the effective cross sections from the DQPM

- **(quasi-) elastic collisions:**

$$q+q \to q+q \qquad g+q \to g+q$$
$$q+\bar{q} \to q+\bar{q} \qquad g+\bar{q} \to g+\bar{q}$$
$$\bar{q}+\bar{q} \to \bar{q}+\bar{q} \qquad g+g \to g+g$$



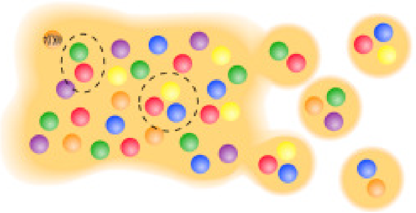
- **inelastic collisions:**

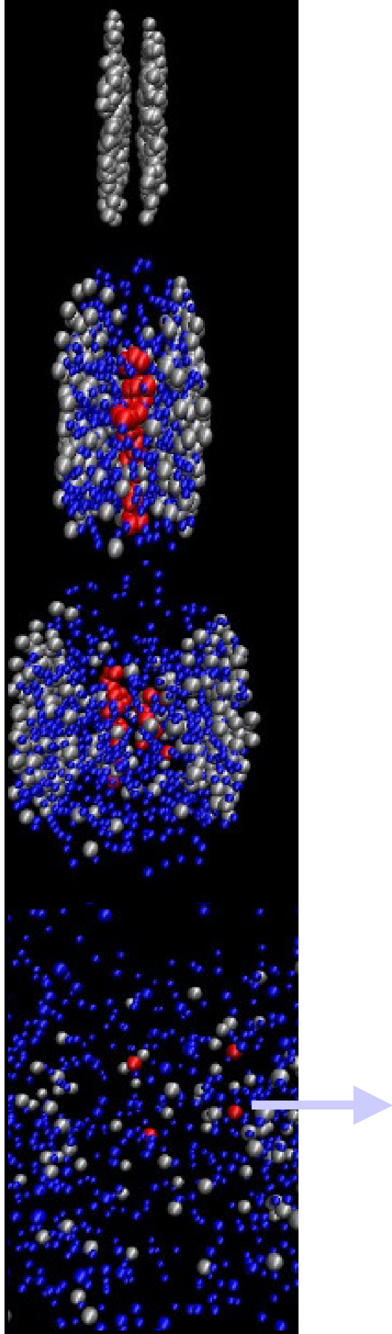
(Breight-Wigner cross sections)

$$\left\{ \begin{array}{l} q+\bar{q} \to g \\ g \to q+\bar{q} \end{array} \right.$$

$$\left. \begin{array}{l} \cancel{q+\bar{q} \to g+g} \\ \cancel{g \to g+g} \end{array} \right\}$$ **suppressed (<1%) due to the large mass of gluons**

III. PHSD - basic concept





III. Hadronization:

❑ **Hadronization:** based on DQPM

- massive, off-shell (anti-)quarks with broad spectral functions hadronize to off-shell mesons and baryons or color neutral excited states - ‚strings‘ (strings act as ‚doorway states‘ for hadrons)

$$\boxed{\begin{aligned} g \to q+\bar q, \qquad q+\bar q &\leftrightarrow meson\ \ ('\,string\,'\,) \\ q+q+q &\leftrightarrow baryon\ \ ('\,string\,'\,) \end{aligned}}$$

- Local covariant off-shell transition rate for q+qbar fusion

➔ meson formation:

$$Tr_j = \sum_j \int d^4x_j d^4p_j/(2\pi)^4$$

$$\frac{dN^{q+\bar q\to m}}{d^4x\ d^4p} = Tr_q Tr_{\bar q}\,\delta^4(\,p-p_q-p_{\bar q}\,)\,\delta^4\!\left(\frac{x_q+x_{\bar q}}{2}-x\right)\delta(\,flavor, color\,)$$

$$\cdot N_q(\,x_q,p_q\,)N_{\bar q}(\,x_{\bar q},p_{\bar q}\,)\cdot \omega_q\underline{\rho_q(\,p_q\,)}\cdot \omega_{\bar q}\underline{\rho_{\bar q}(\,p_{\bar q}\,)}\cdot |\,M_{q\bar q}\,|^2\ \underline{W_m\left(x_q-x_{\bar q},p_q-p_{\bar q}\right)}$$

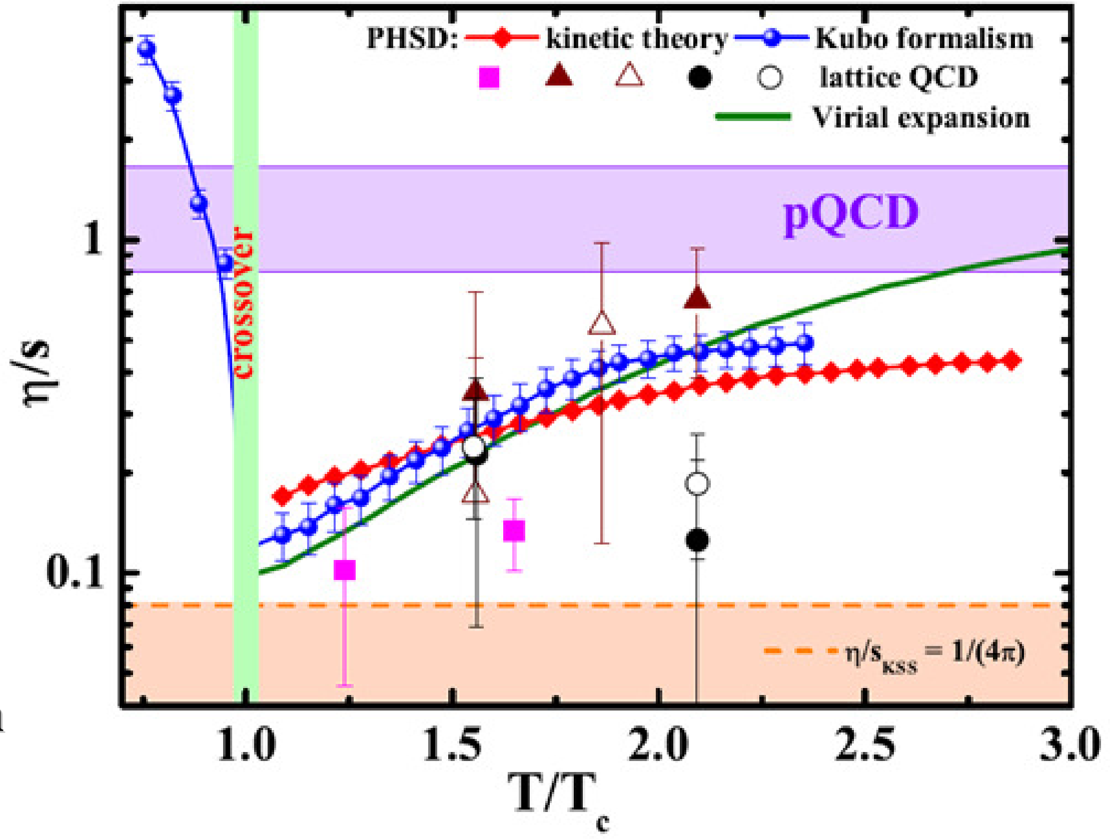
- $N_j(x,p)$ is the phase-space density of parton j at space-time position x and 4-momentum p
- W_m is the phase-space distribution of the formed ‚pre-hadrons‘ (Gaussian in phase space)
- $|M_{qq}|^2$ is the effective quark-antiquark interaction from the DQPM

IV. Hadronic phase: hadron-string interactions – off-shell HSD

Properties of parton-hadron matter – shear viscosity

η/s using **Kubo formalism** and the **relaxation time approximation** (‚kinetic theory')

- **$T=T_C$:** η/s shows **a minimum (~0.1)** close to the critical temperature
- **$T>T_C$:** QGP - **pQCD limit** at higher temperatures T >3 Tc
- **$T<T_C$:** fast increase of the ratio η/s for **hadronic matter** ➔
    - lower interaction rate of hadronic system
    - smaller number of degrees of freedom (or entropy density) for hadronic matter compared to the QGP

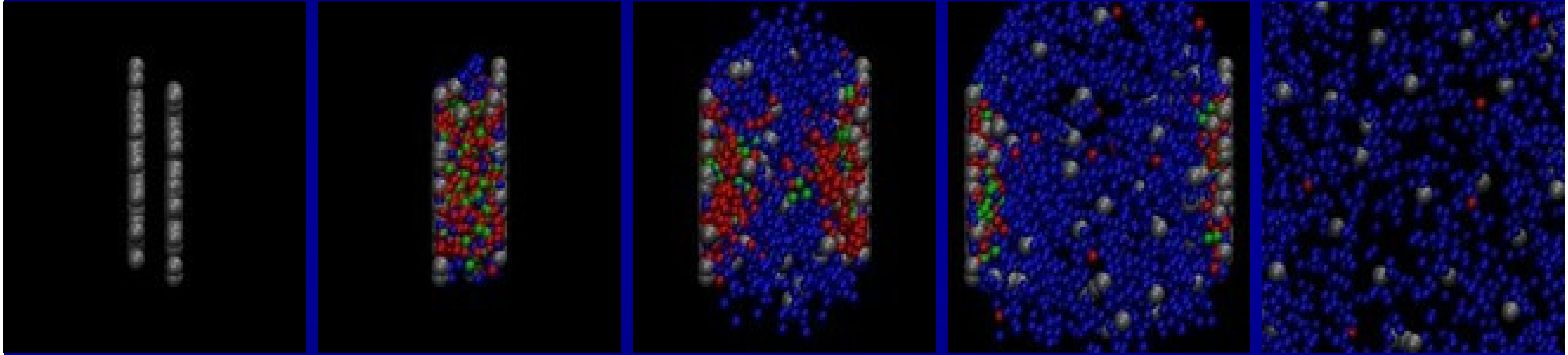


Virial expansion: S. Mattiello, W. Cassing, Eur. Phys. J. C 70, 243 (2010).

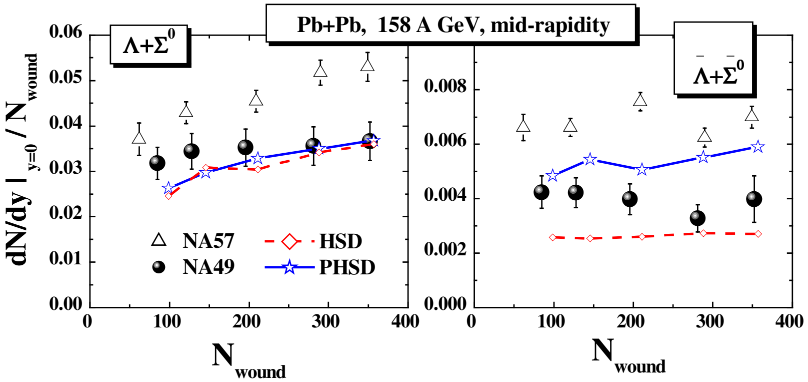
QGP in PHSD = strongly-interacting liquid

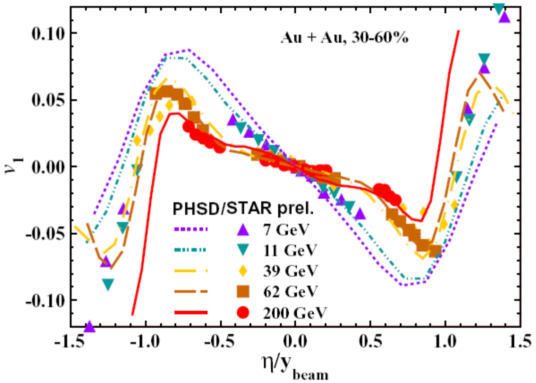
V. Ozvenchuk et al., PRC 87 (2013) 064903, arXiv:1212.5393

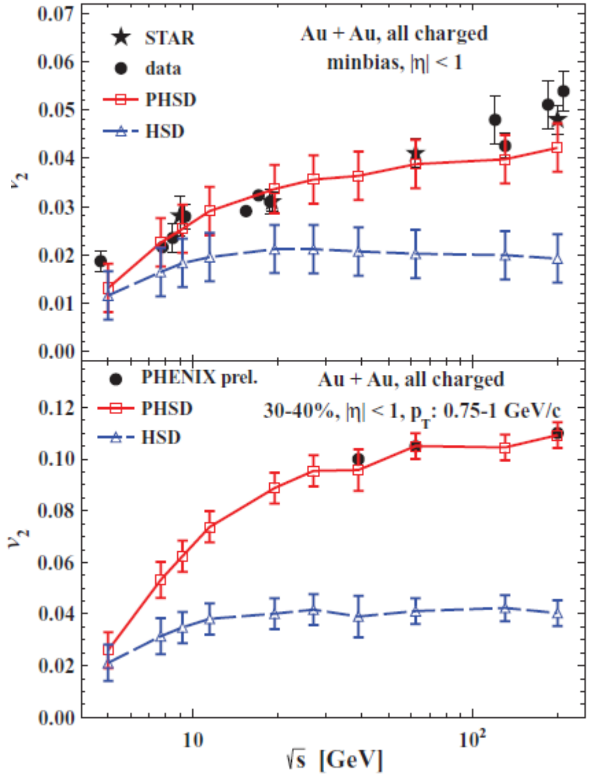
Au+Au, 21.3 TeV, central

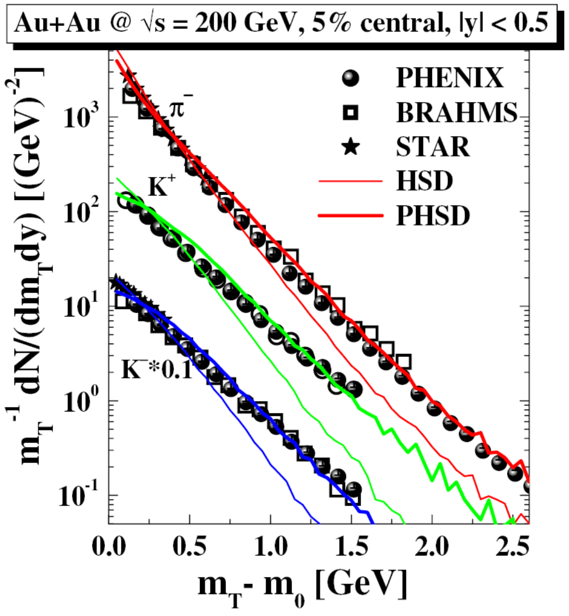


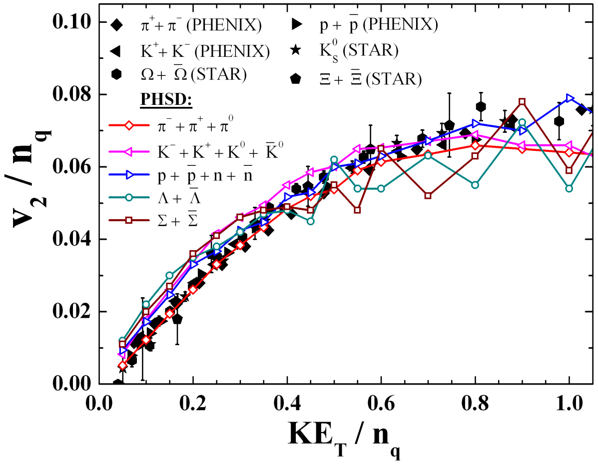
PHSD for HIC (highlights)

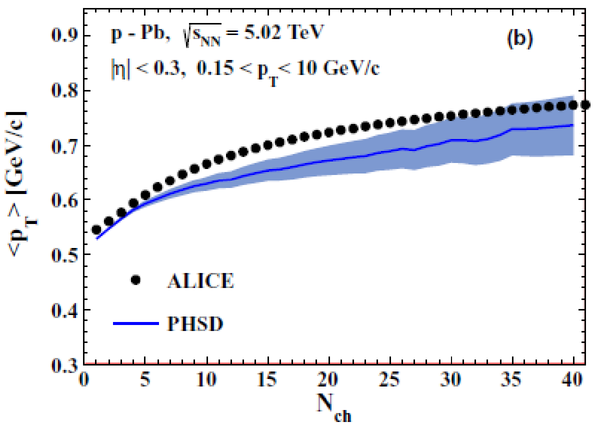


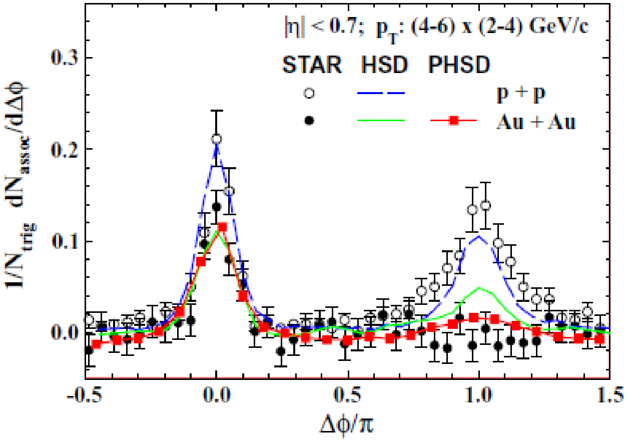








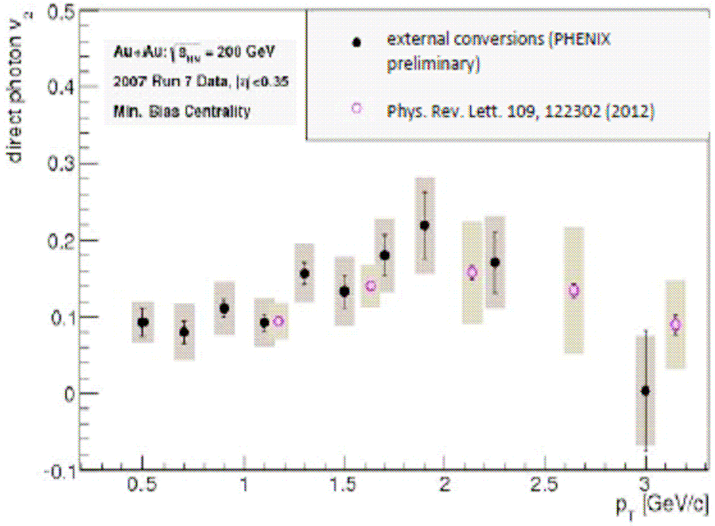




- PHSD provides a consistent description of HIC dynamics

Photons from SPS to LHC

I. Direct photon flow puzzle



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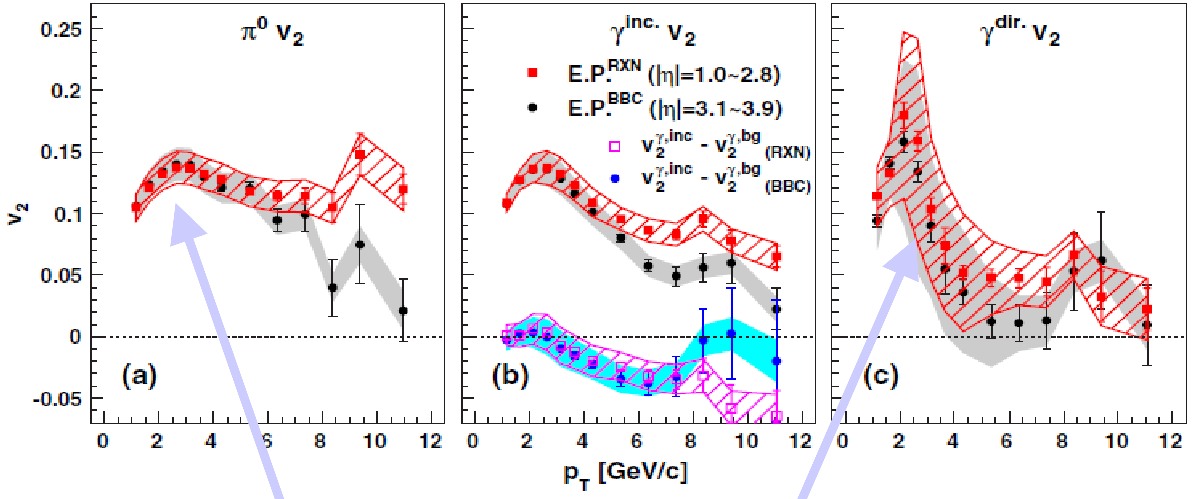
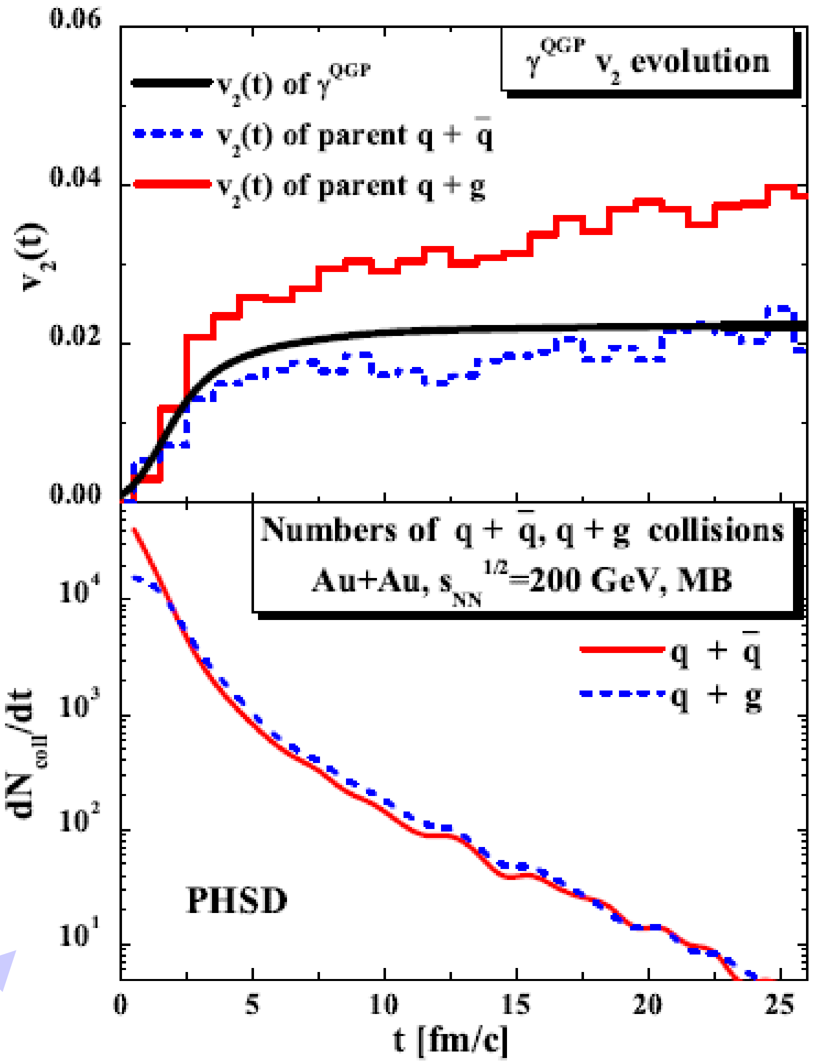
EMMI Rapid Reaction Task Force ‚Direct Photon Flow Puzzle‘,
24-28 February 2014, GSI Darmstadt,
Organizers: Klaus Reygers and Johanna Stachel

Photon v_2 puzzle



- **Strong elliptic flow of photons** ($v_2(\gamma^{dir})$~ $v_2(\pi)$) seen by **PHENIX** is surprising, if the origin would be the QGP !
- Variety of models: $v_2(\gamma^{dir}) << v_2(\pi)$



- QGP radiation occurs at early time when flow is not yet developed!

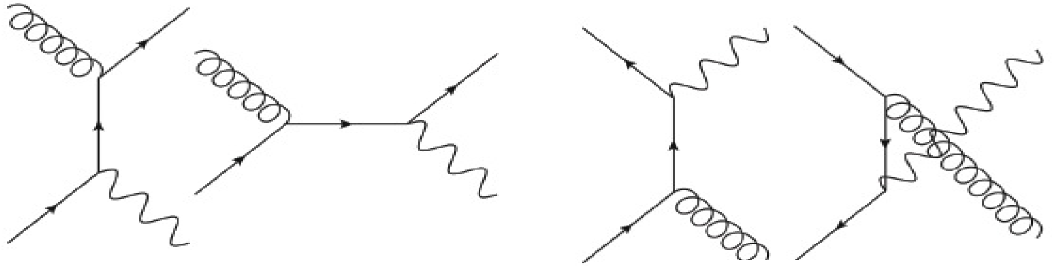
Olena Linnyk et al., PRC 88 (2013) 034904; arXiv:1304.7030

Photons from the hot and dense medium

Photon sources:

- ☐ from the QGP via partonic interactions:

Compton scattering q-qbar annihilation



$$q(\bar{q}) + g \rightarrow q(\bar{q}) + \gamma \qquad q + \bar{q} \rightarrow g + \gamma$$

- ☐ from hadronic sources:

- **decays of mesons:** $\pi \rightarrow \gamma + \gamma,\ \eta \rightarrow \gamma + \gamma,\ \omega \rightarrow \pi + \gamma$ $\eta' \rightarrow \rho + \gamma,\ \phi \rightarrow \eta + \gamma,\ a_1 \rightarrow \pi + \gamma$
- **secondary meson interactions:** $\pi + \pi \rightarrow \rho + \gamma,\ \rho + \pi \rightarrow \pi + \gamma$

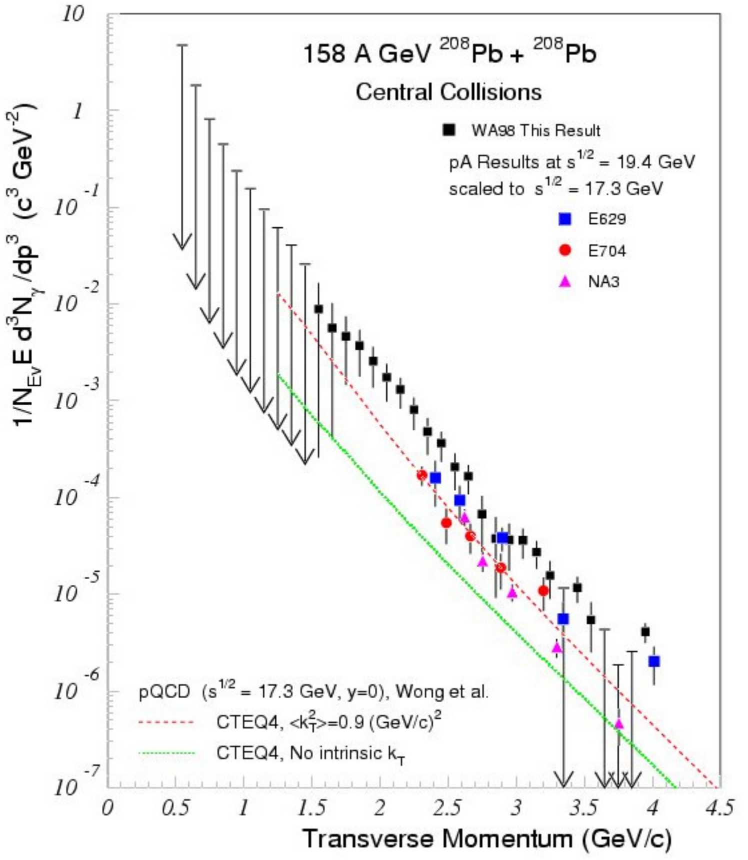
using the off-shell extension of Kapusta et al. in PRD44 (1991) 2774

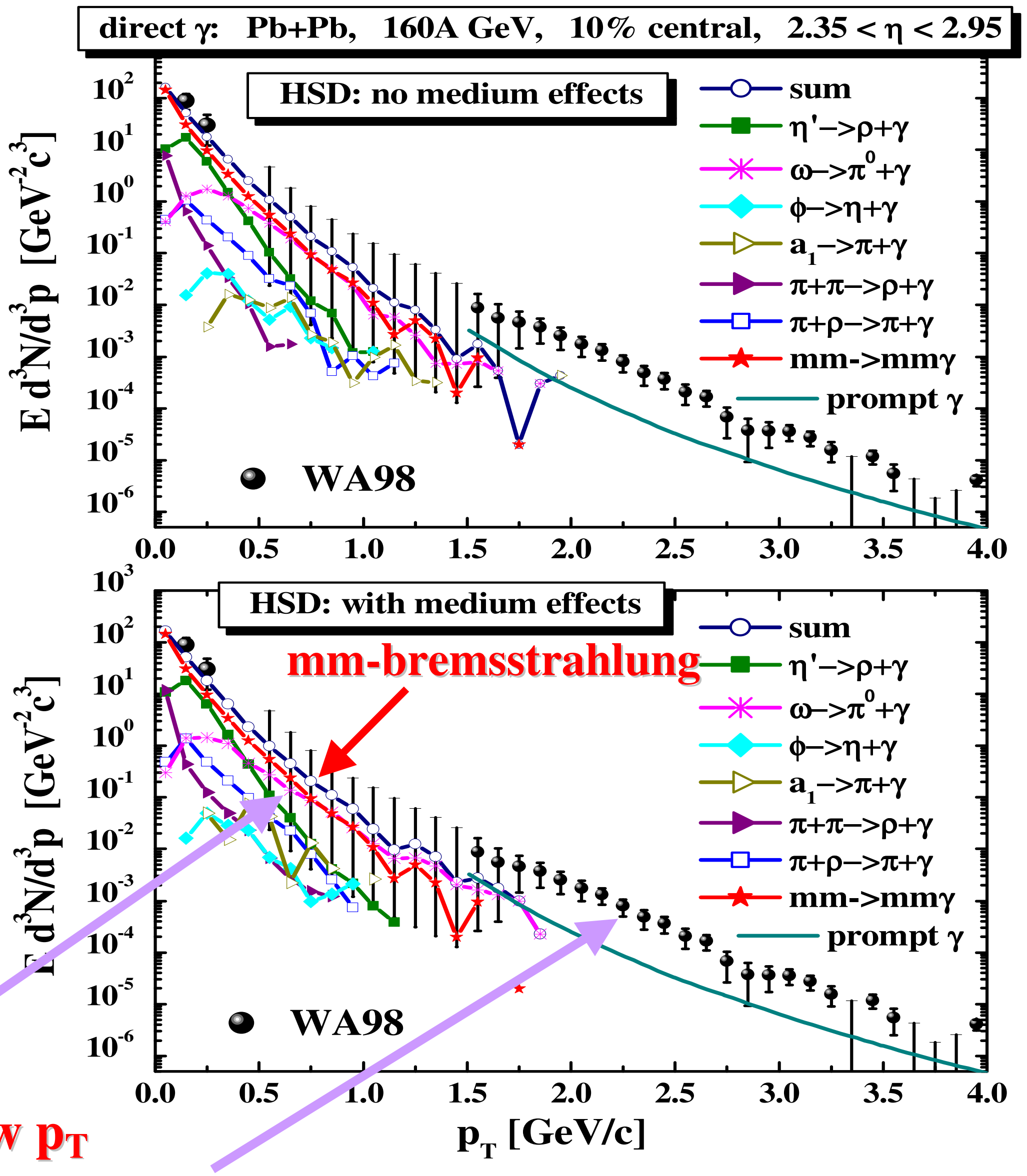
- **meson-meson and meson-baryon bremsstrahlung:**

$m+m \rightarrow m+m+\gamma,\quad m+B \rightarrow m+B+\gamma,\quad m=\pi,\eta,\rho,\omega,K,K^*,\ldots,\quad B=p,\Delta,\ldots$

using the soft-photon approximation

Direct photons at SPS: WA98





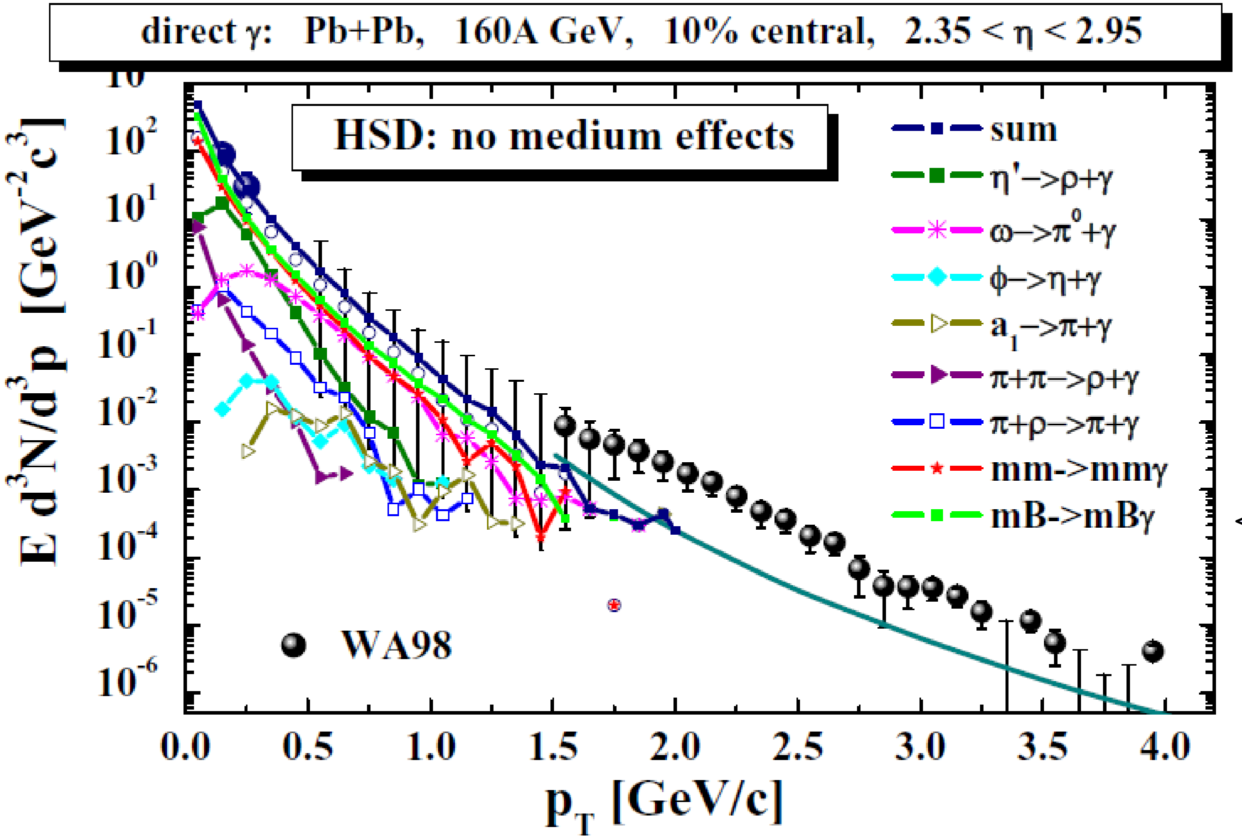
mm-bremsstrahlung

- **Hadronic sources** dominate at **low p_T**
- **High p_T: dominated by thermal photons from QGP**

E. B., S.M. Kiselev, and G.B. Sharkov, PR C78 (2008) 034905

Photon spectra at SPS

Updated HSD (2014) including meson-baryon bremsstrahlung



- **HSD: meson-meson and meson-baryon bremsstrahlung using SPA**

- **Bremsstrahlung rates are uncertain !!!**

Meson-meson Bremsstrahlung at SPS within SPA

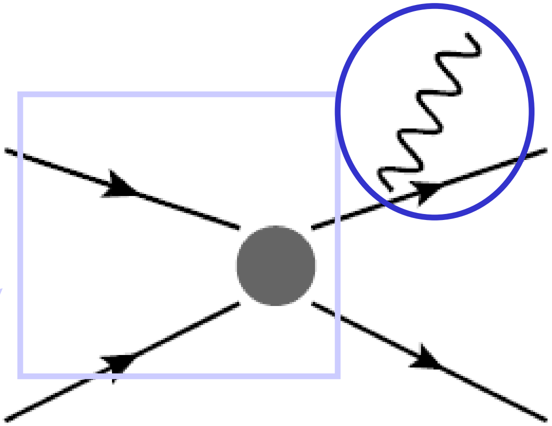
C. Gale, J. Kapusta, Phys. Rev. C 35 (1987) 2107

Soft Photon Approximation:

$m_1+m_2 \rightarrow m_1+m_2+\gamma$

$$q_0 \frac{d^3\sigma^\gamma}{d^3q} = \frac{\alpha}{4\pi}\frac{\bar{\sigma}(s)}{q_0^2}$$

$$\bar{\sigma}(s) = \frac{s-(M_1+M_2)^2}{2M_1^2}\sigma(s),$$

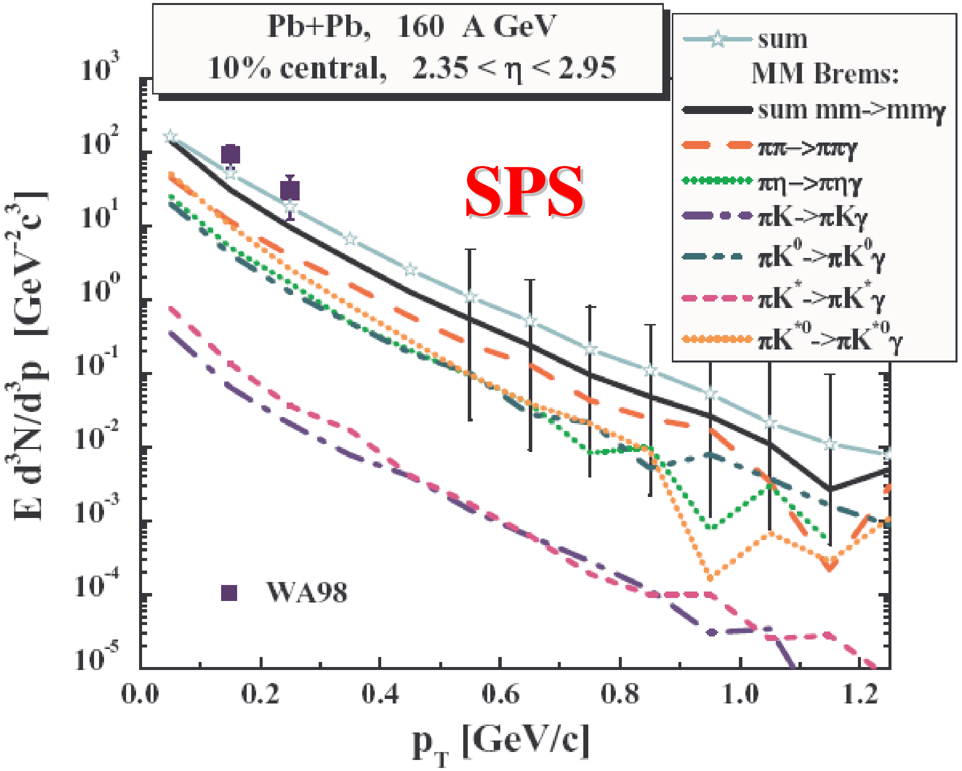


σ(s) – elastic meson-meson cross section

$m_1+m_2 \rightarrow m_1+m_2$ -???

- ☐ **Taken σ(s) =10 mb for ALL m_1+m_2 channels !**
- ☐ **No isospin factors!**

→ Needs to be improved!



E. B., S.M. Kiselev, and G.B. Sharkov, PR C78 (2008) 034905

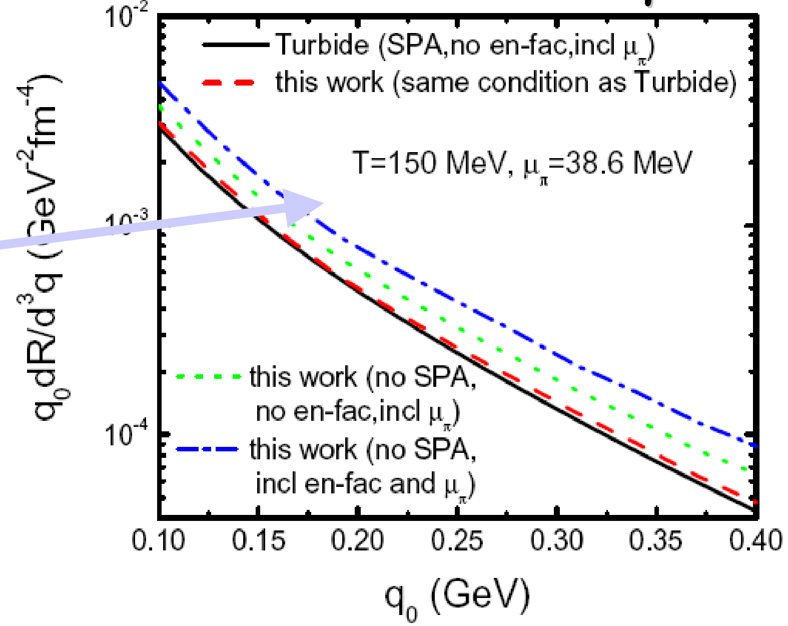
mm bremsstrahlung beyond SPA

W. Liu and R. Rapp, Nucl. Phys. A 96 (2007) 101

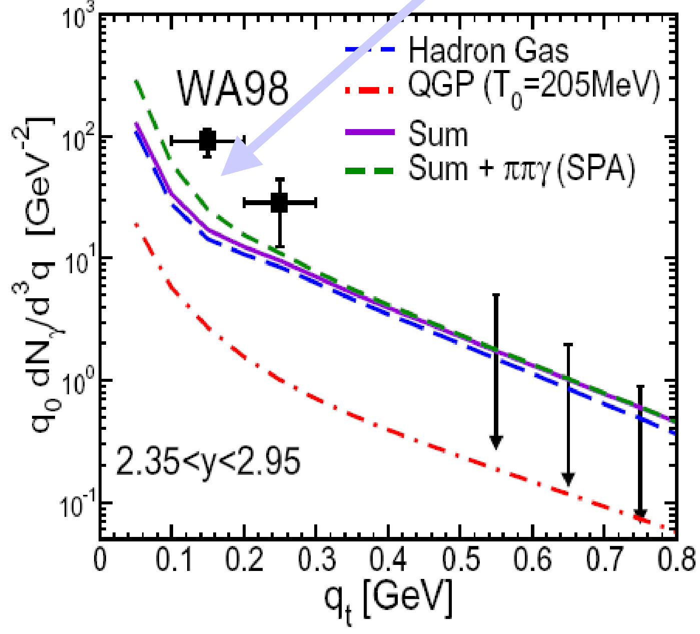
- **ππ→ππγ, πK→πKγ bremsstrahlung:**

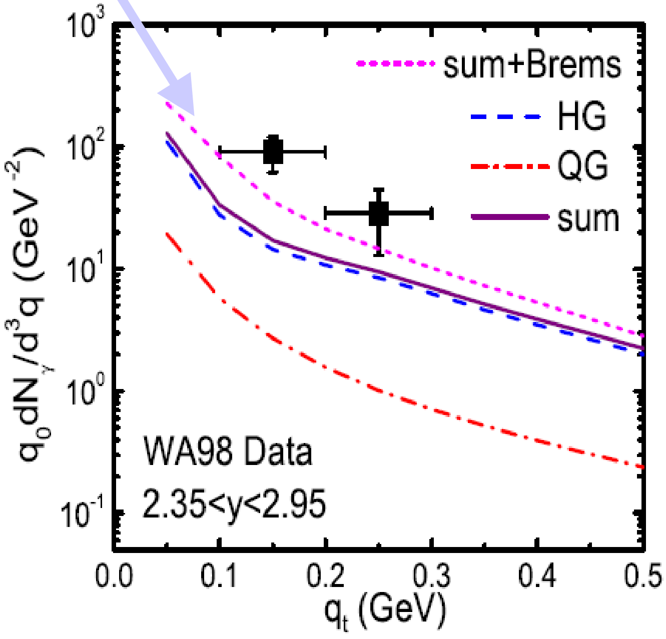
the photon yield within an effective chiral hadronic model including electromagnetic interaction via $U_{em}(1)$ gauge is larger than using SPA !

Rate for ππ→ππγ



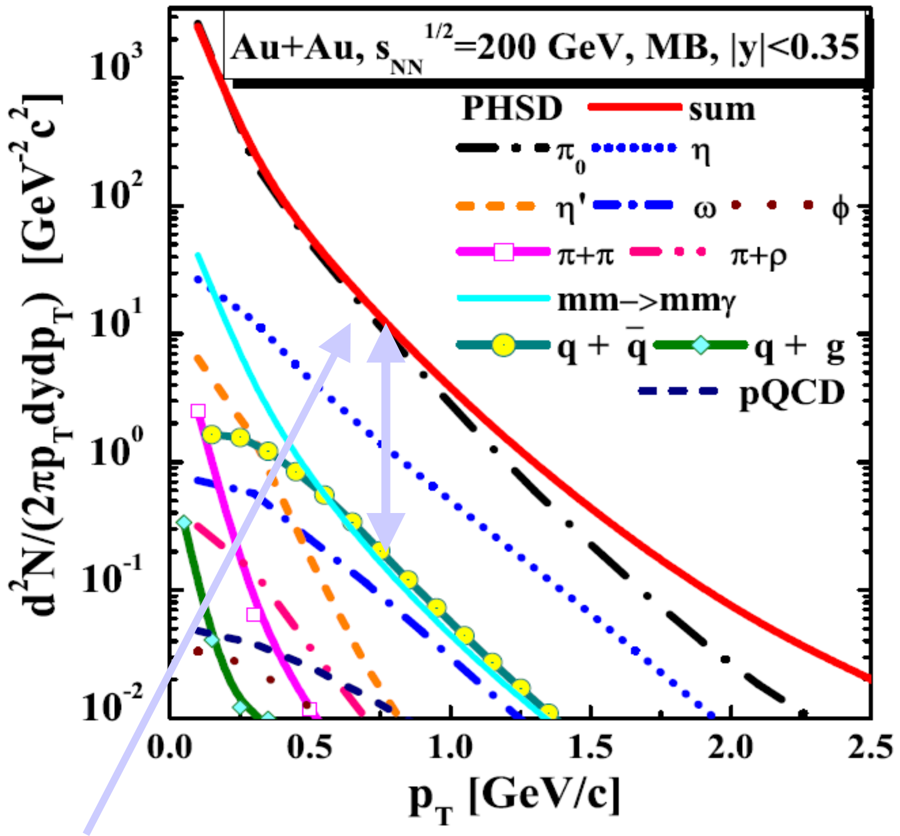
Central Pb-Pb $s^{1/2}$=17.3AGeV

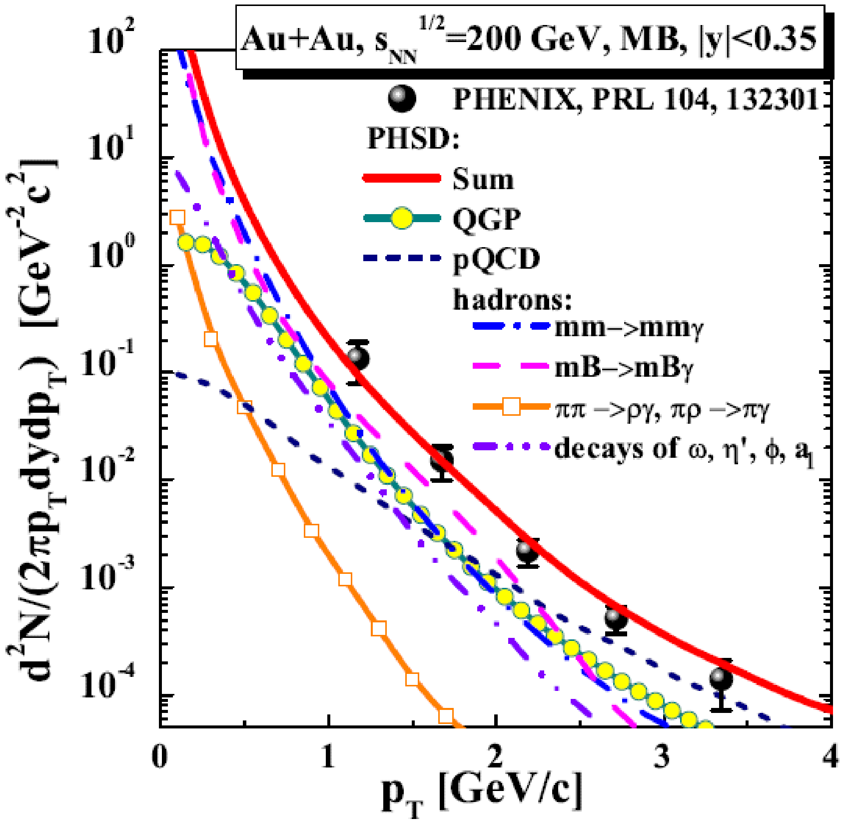




Photon spectra at RHIC

- Inclusive photon spectrum
- π^0 and η subtracted photon spectrum





- π^0 and η decays dominate the low p_T spectra

- QGP sources are mandatory to explain the spectrum (~50%), but hadronic sources are considerable, too !

- The 'effective temperature' T_{eff}:

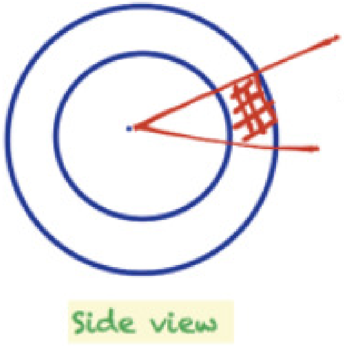
The slope parameter T_{eff} (in MeV)			
PHSD			PHENIX
QGP	hadrons	Total	[38]
260 ± 20	200 ± 20	220 ± 20	233 ± 14 ± 19

O. Linnyk et al., PRC 88 (2013) 034904; arXiv:1304.7030

Are thermal photons a QGP thermometer?

* Pictures from Charles Gale talk at
ITP Colloquium, Frankfurt, April 2014

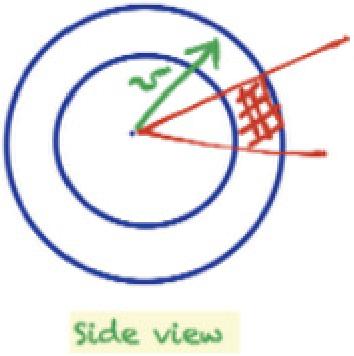
- **Static source:**



$$E\frac{d^3n}{d^3p} = E\,e^{-\beta E}$$

β=1/T, T is a ‚true‘ temperature

- **Moving source:**



$$E\frac{d^3n}{d^3p} \approx E\,e^{-\beta\gamma E+\beta\gamma v E}$$

➔Doppler shift:

effective T_{eff} deduced from the slopes is NOT a ‚true‘ temperature

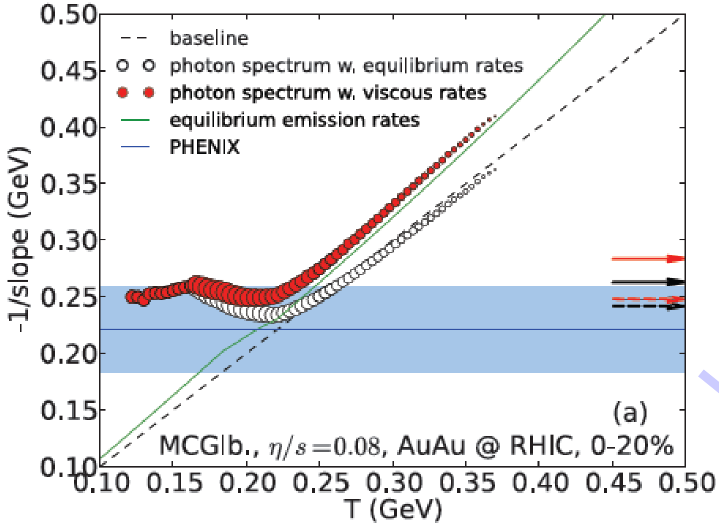
$$T_{eff} = \sqrt{\frac{1+v}{1-v}}\,T$$

Time evolution of the effective temperature

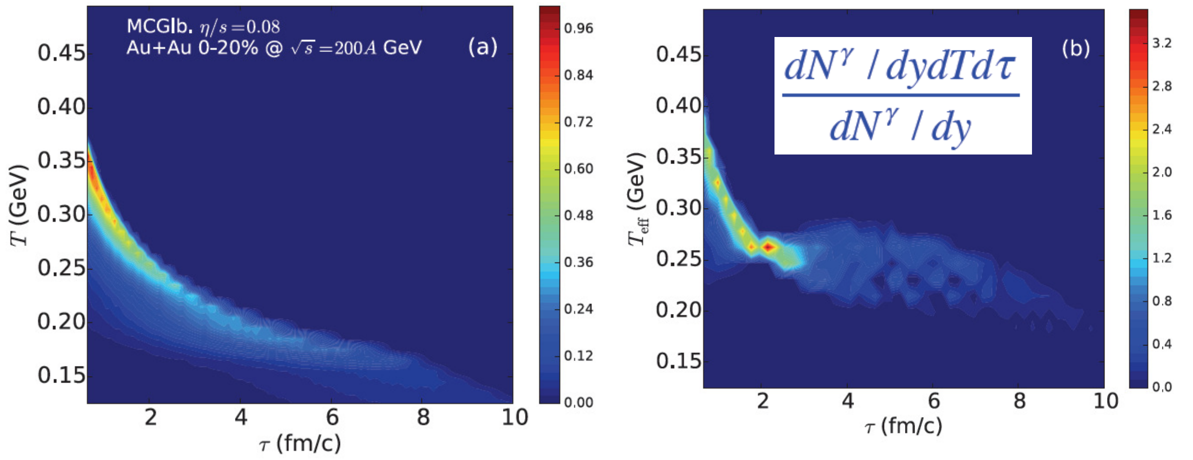
❑ (2+1)d viscous hydro (Ohio)

C. Shen et al., PRC89 (2014) 044910; arXiv:1308.2440

- T_{eff}= -1/slope vs. local fluid cell temperature T



- Contour plots of differential photon yield vs. time and T (upper) and Teff (lower)



Range of photon emission	Fraction of total photon yield	
	AuAu@RHIC 0–20% centr.	PbPb@LHC 0–40% centr.
$T = 120$–165 MeV	17%	15%
$T = 165$–250 MeV	62%	53%
$T > 250$ MeV	21%	32%
$\tau = 0.6$–2.0 fm/c	28.5%	26%
$\tau > 2.0$ fm/c	71.5%	74%

Exp. Data:

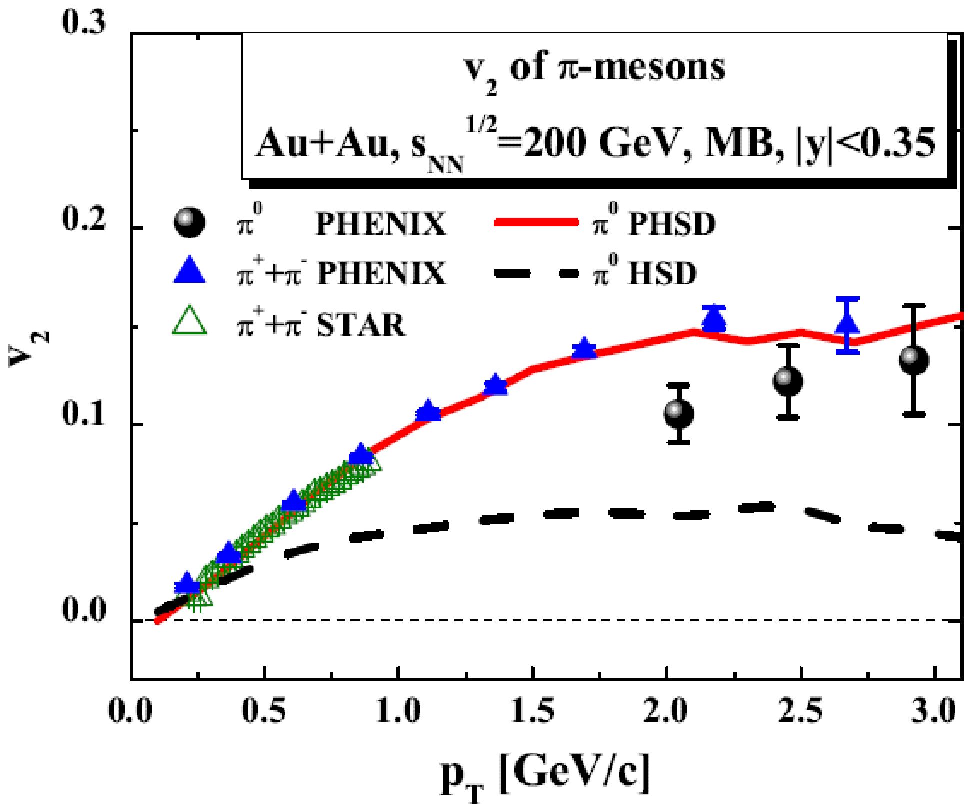
- RHIC: T_{eff}=221+19+19 MeV
- LHC: T_{eff}=304+51 MeV

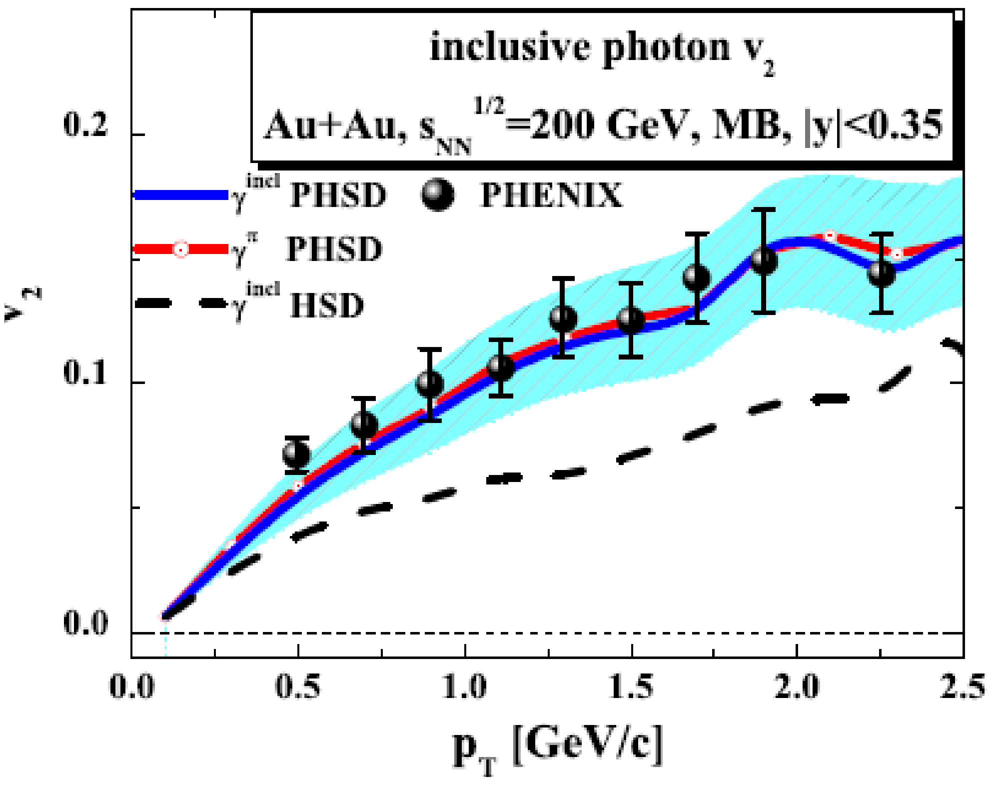
❑ Measured Teff > ‚true‘ T

❑ ‚blue shift‘ due to the radial flow!

❑ only ~1/3 of total photon yield is from T>250 MeV

Inclusive photon elliptic flow





- **Pion elliptic flow is reproduced in PHSD and underestimated in HSD (i.e. without partonic interactions)**

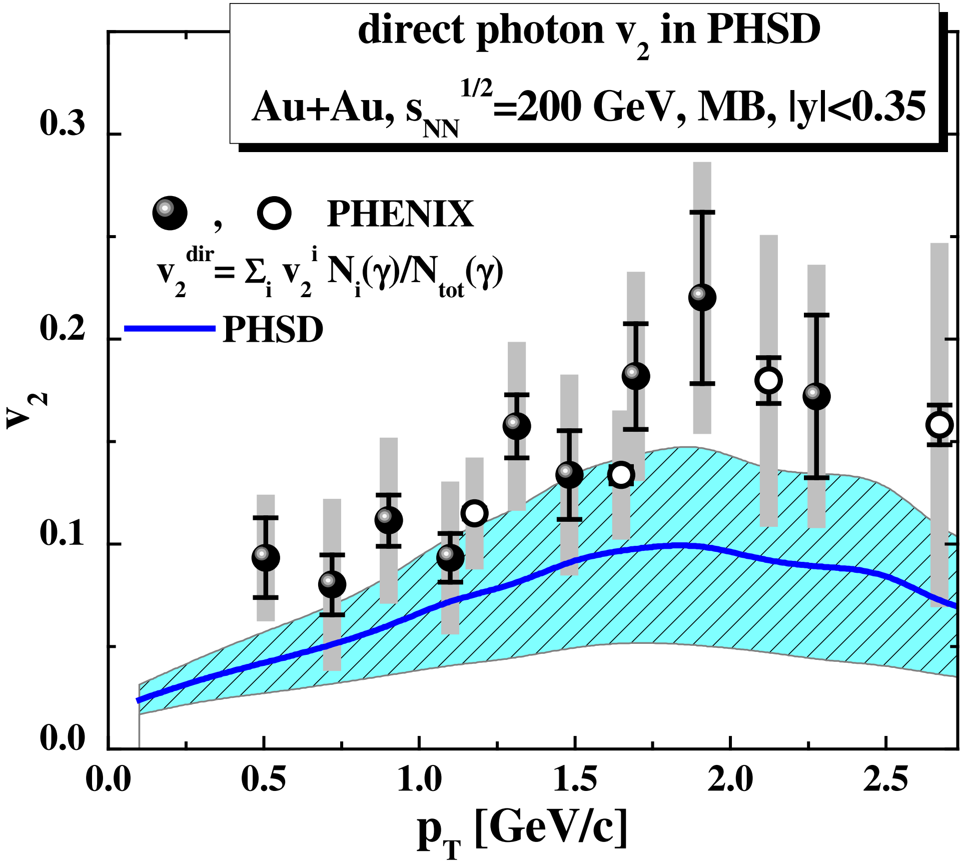
- **➔ large inclusive photon v_2 - comparable to that of hadrons - is reproduced in PHSD, too, because the inclusive photons are dominated by the photons from pion decay**

O. Linnyk et al., PRC 88 (2013) 034904; arXiv:1304.7030

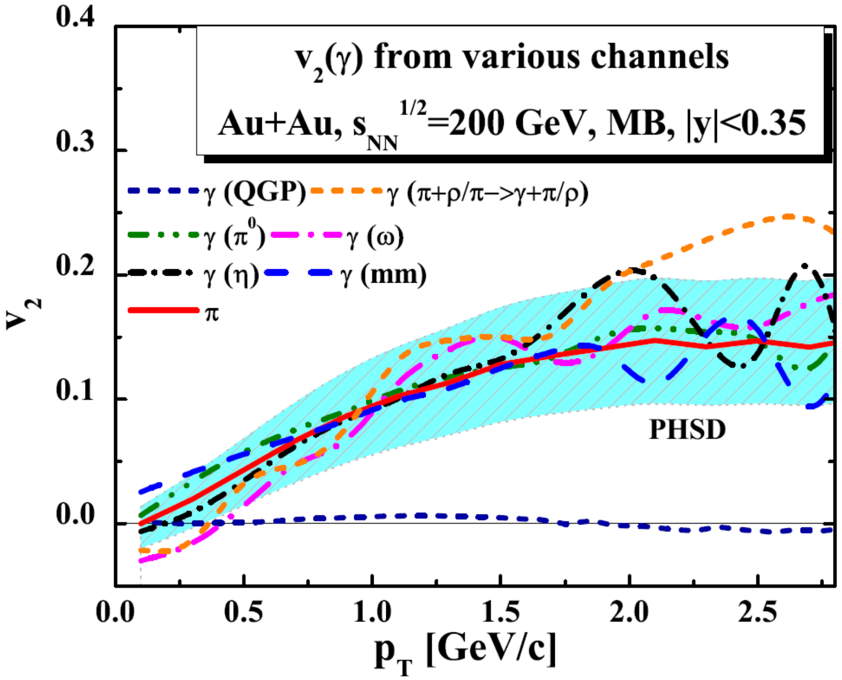
Elliptic flow of direct photons at RHIC

- Sum of v_2 of the individual channels, using their contribution to the spectra with the relative p_T -dependent weights $\omega_i(p_T)$: $\omega^i(p_T)=\frac{N^i(p_T)}{\sum_i N^i(p_T)}$

$$v_2(p_T)=\frac{\sum_i N^i(p_T)\cdot v_2^i(p_T)}{\sum_i N^i(p_T)}=\sum_i \omega_i(p_T)\cdot v_2^i(p_T)$$



- v_2^i of the individual channels „i“

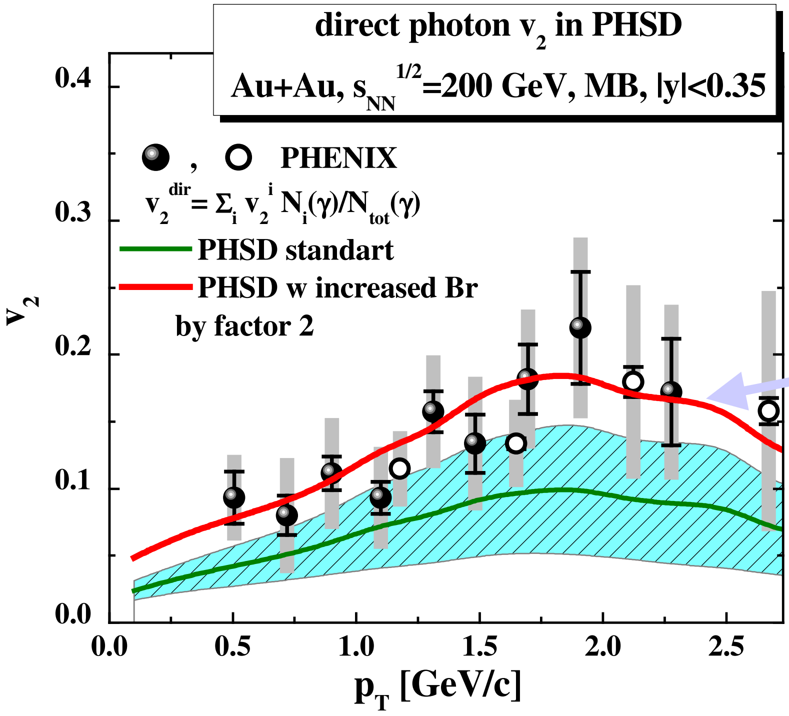


➔ v_2 of direct photons in PHSD - as evaluated by the weighted average of direct photon channels –underestimates the exp. data

O. Linnyk et al. Phys.Rev. C88 (2013) 034904; Phys. Rev. C 89 (2014) 034908

Towards the solution of the v_2 puzzle

- Is bremsstrahlung a solution?



?

Bremsstrahlung increased by a factor 2

(might be due to the uncertainties in SPA and mm and mB elastic cross sections)

Other ideas:

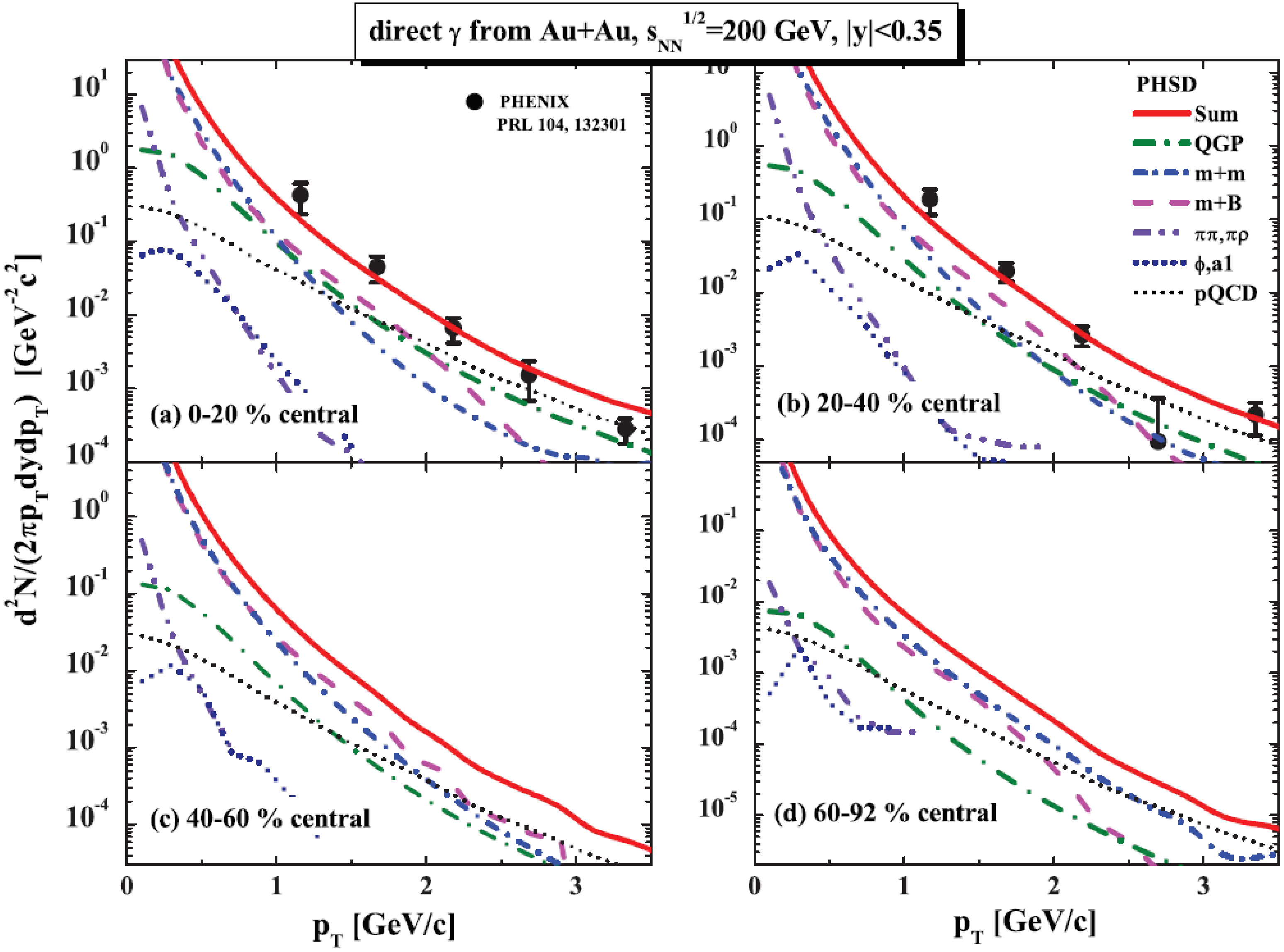
- **Early-time magnetic field effects ?**

(Basar, Kharzeev, Skokov, PRL (2012); Basar, Kharzeev, Shuryak, arXiv:1402.2286)

- **Glasma effects ?** (L. McLerran)
- **Primodial flow ?** (R. Rapp, H. van Hees)
- **???**

➢ **More experimental information is needed ➔ new PHENIX data on centrality dependence**

Centrality dependence of the direct photon yield



O. Linnyk et al, Phys. Rev. C 89 (2014) 034908

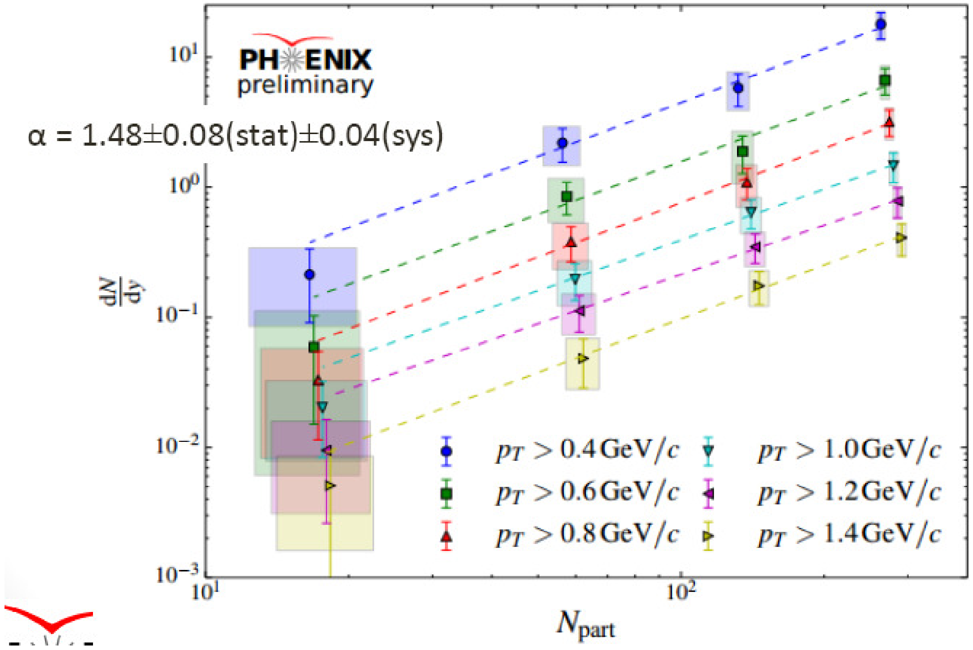
Centrality dependence of the ‚thermal‘ photon yield

PHENIX:

scaling of thermal photon yield vs centrality:

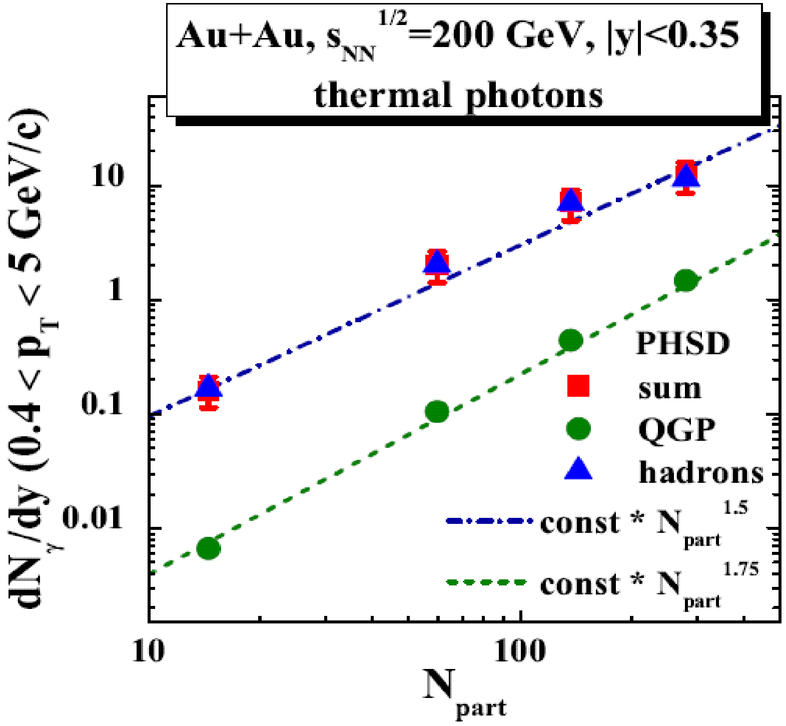
$dN/dy \sim N_{part}^{\alpha}$ with $\alpha \sim 1.48 \pm 0.08$

(‘Thermal’ photon yield = direct photons - pQGP
- hadronic decays)



PHSD predictions:

- ☐ Hadronic channels scale as ~ $N_{part}^{1.5}$
- ☐ Partonic channels scale as ~$N_{part}^{1.75}$

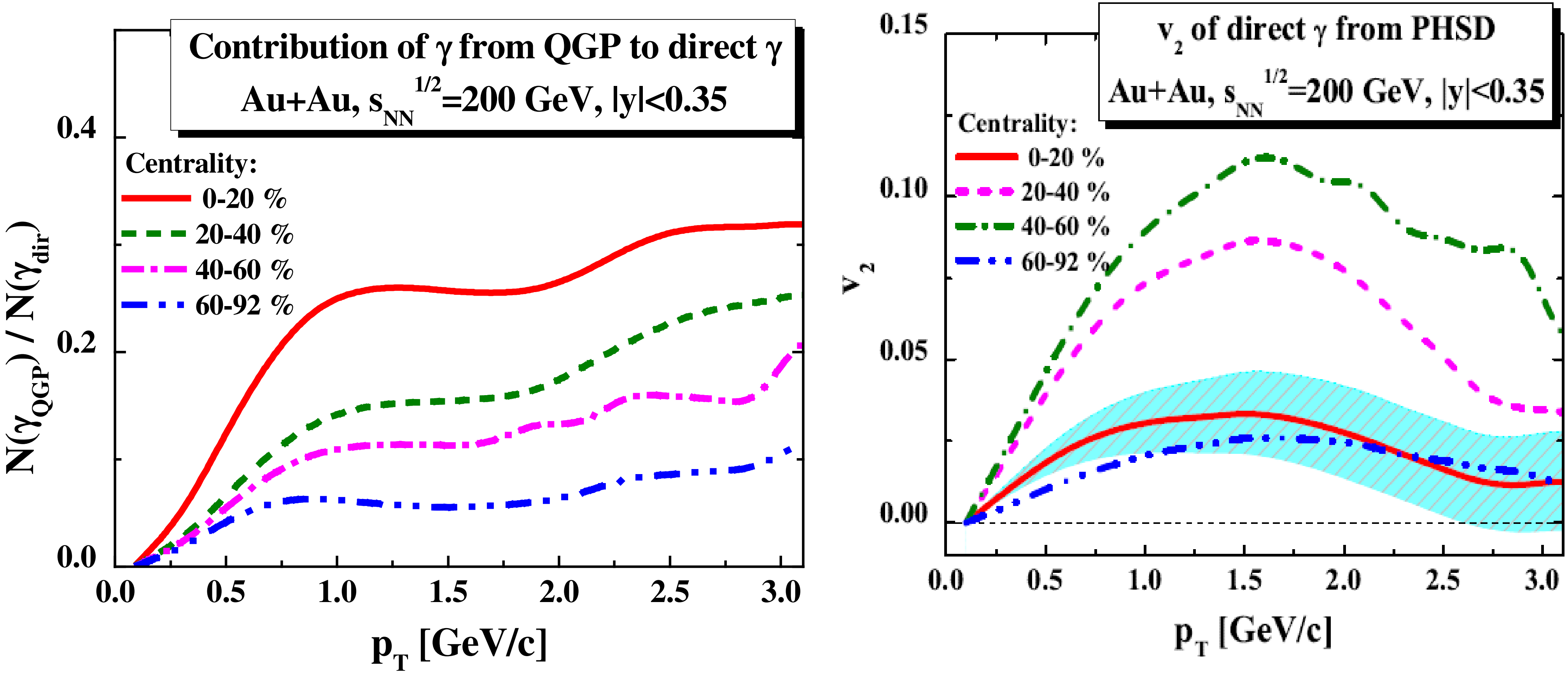


- PHSD: scaling of the direct photon yield with the number of participants to the power 1.5
- similar results from (2+1)d viscous hydro (Ohio): HG ~1.46, QGP ~2

➔ indication for a hadronic origin ?!

O. Linnyk et al, Phys. Rev. C 89 (2014) 034908

Centrality dependence of the ‚thermal‘ photon v_2

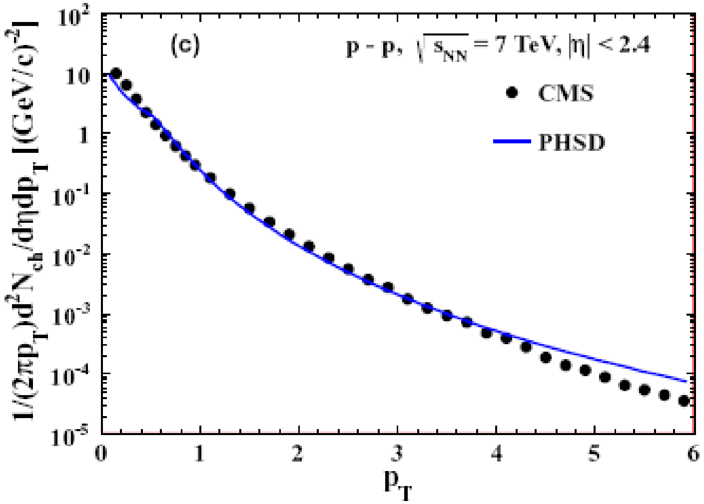


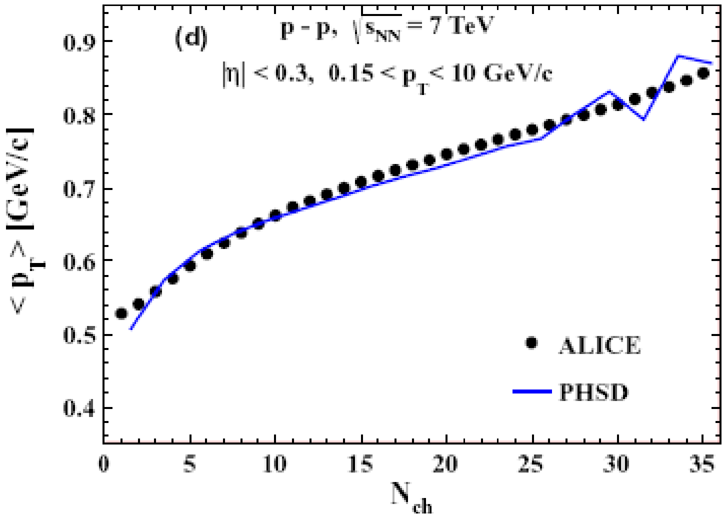
- ❑The contribution of the QGP photons decreases substantially for more peripheral collisions and the photon elliptic flow increases accordingly.

O. Linnyk et al, Phys. Rev. C 89 (2014) 034908

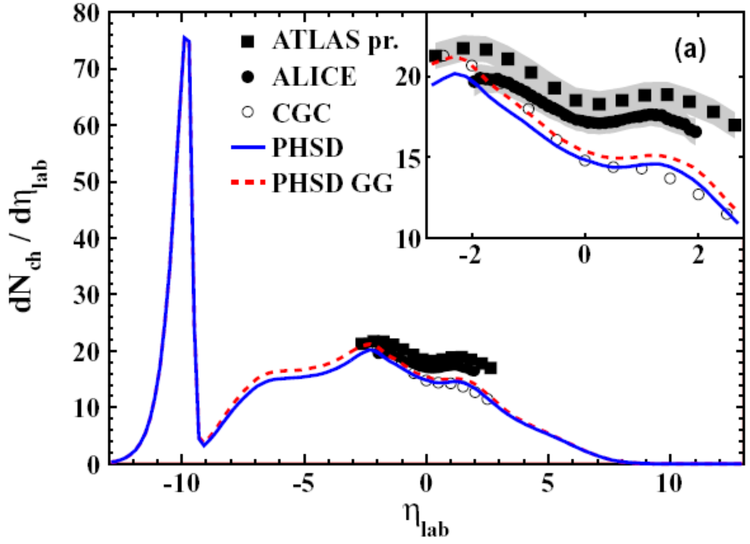
PHSD results for p+p and p+Pb at LHC

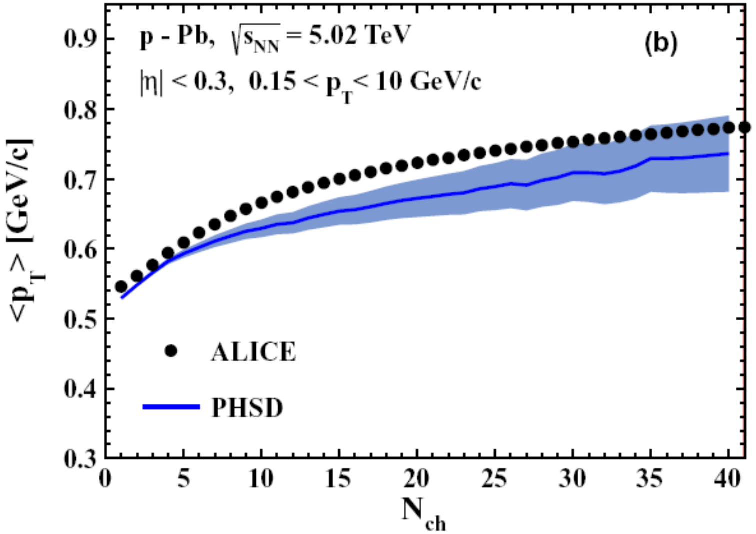
☐ pp at 7 TeV (charged particles)





☐ pPb at 5.02 TeV (charged particles)



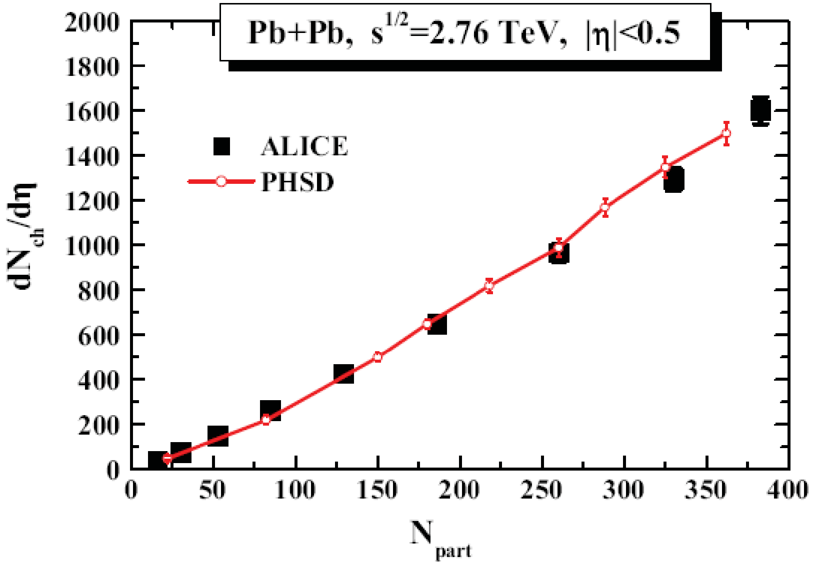


PHSD: V. Konchakovski et al., arXiv:1401.4409

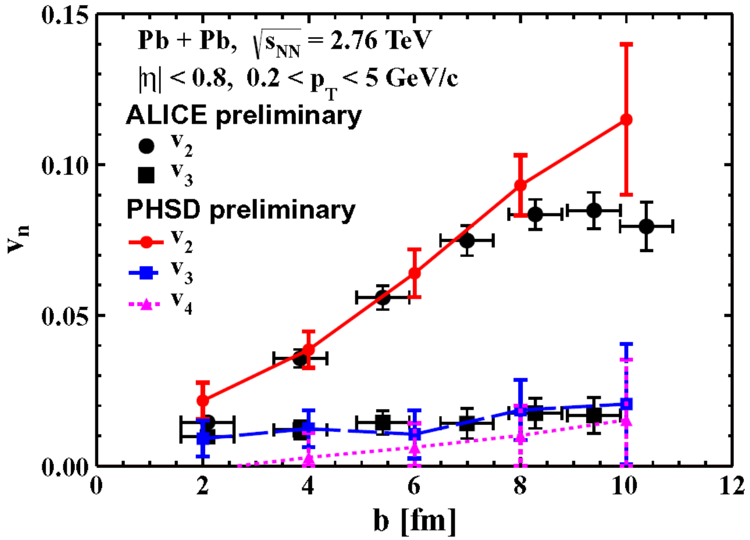
PHSD results for Pb+Pb at 2.76 TeV

❑ Charged particle multiplicity vs centrality

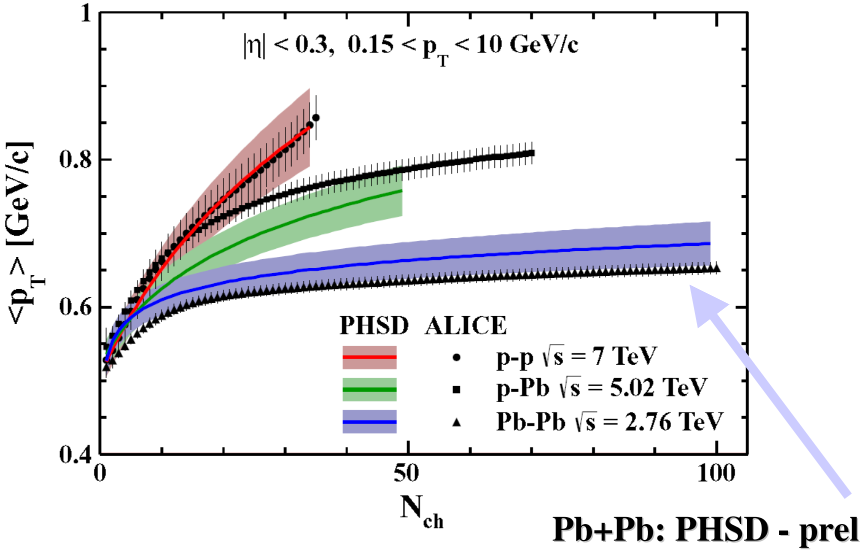
PHSD: Phys.Rev. C87 (2013) 014905; arXiv:1208.1279



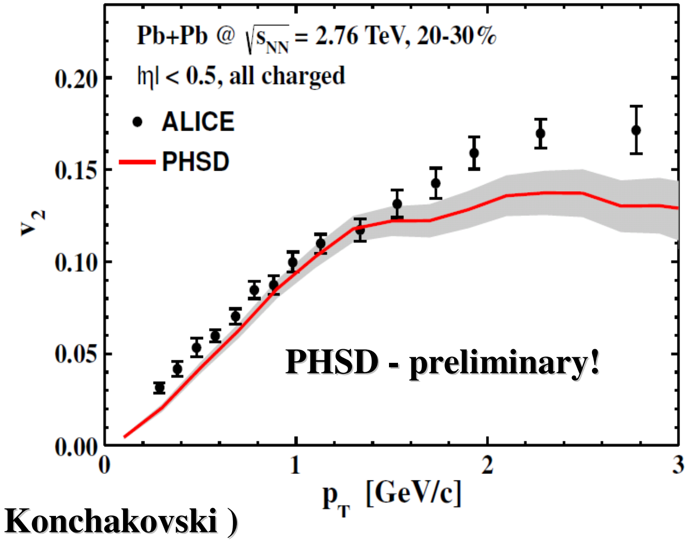
❑ centrality dependence of v_2, v_3, v_4 for charged particles



❑ $<p_T>$ vc N_{ch}

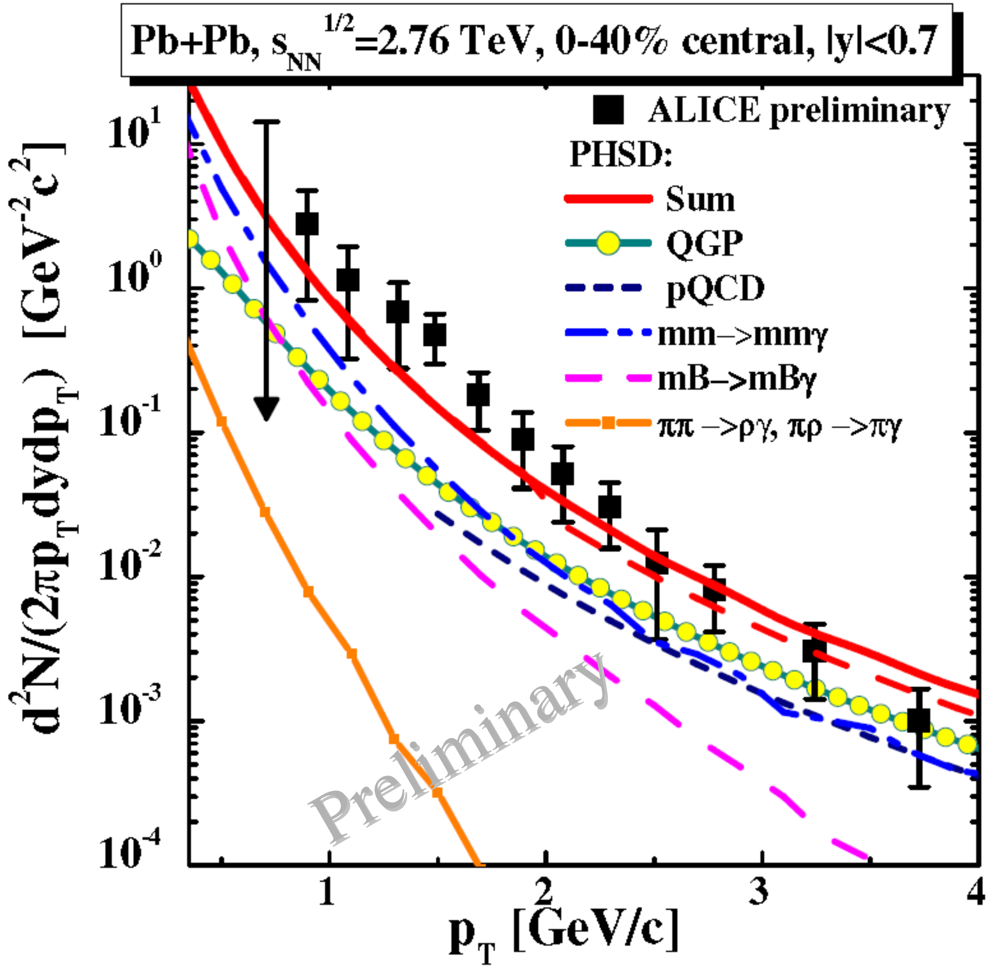


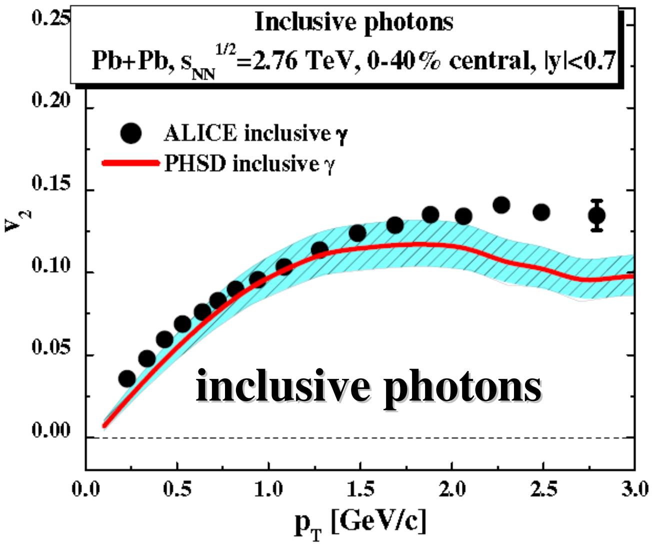
❑ $v_2(p_T)$ for charged particles

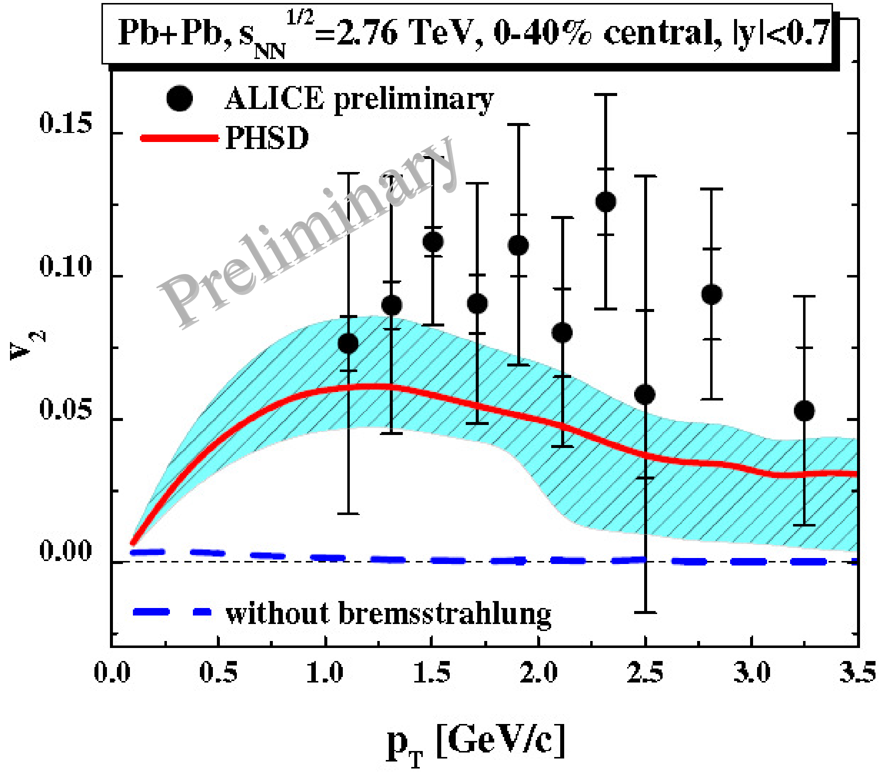


Pb+Pb: PHSD - preliminary! (V. Konchakovski)

PHSD results for Pb+Pb at 2.76 TeV: photons





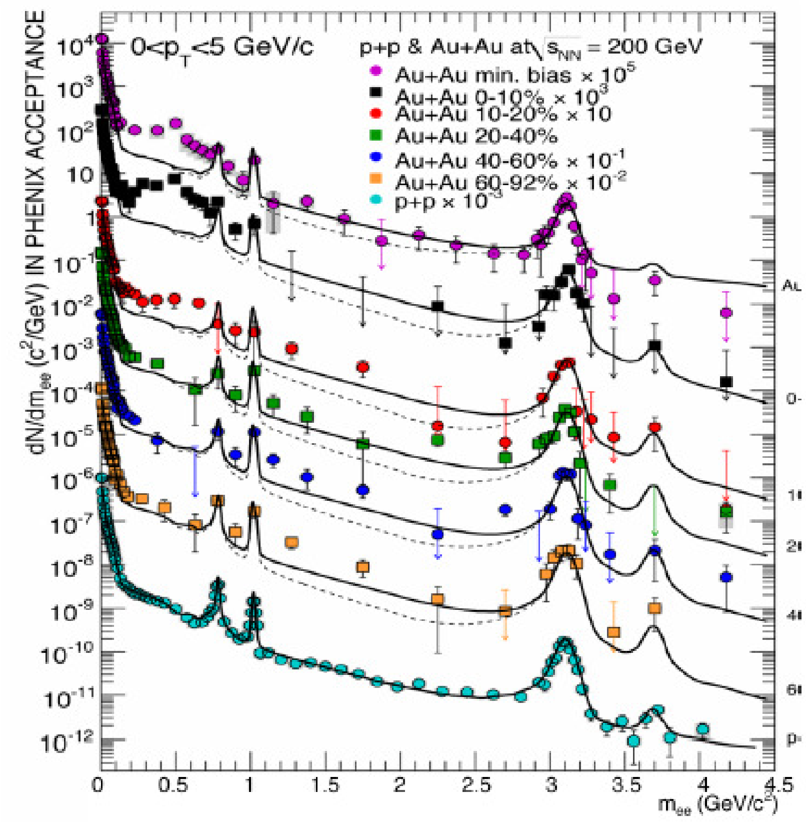


- **Is the considerable elliptic flow of direct photons at the LHC of hadronic origin ?!**

- **The photon elliptic flow at LHC is lower than at RHIC due to a larger/longer relative QGP contribution.**

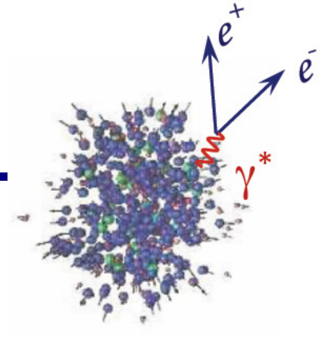
Dileptons: from SPS to LHC

II. PHENIX dilepton puzzle

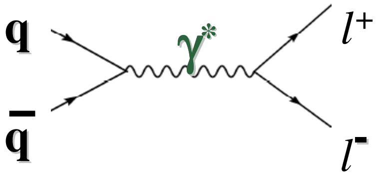


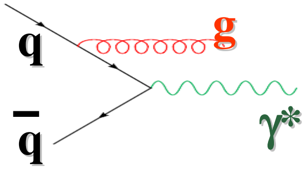
?

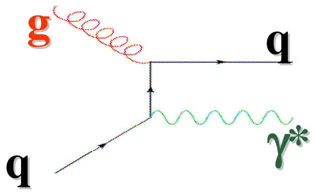
Dilepton sources

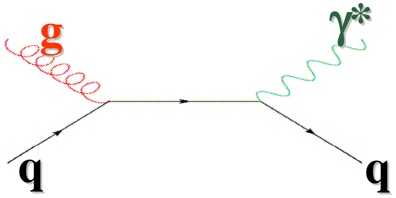


- **from the QGP via partonic (q,qbar, g) interactions:**

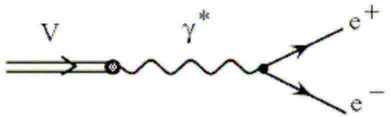




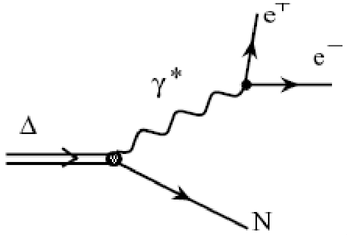




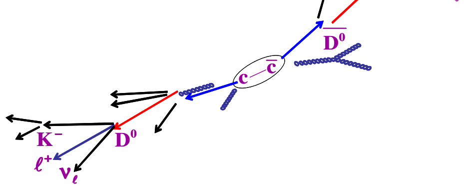
- **from hadronic sources:**



- **direct decay** of vector mesons (ρ,ω,φ,J/Ψ,Ψ‘)



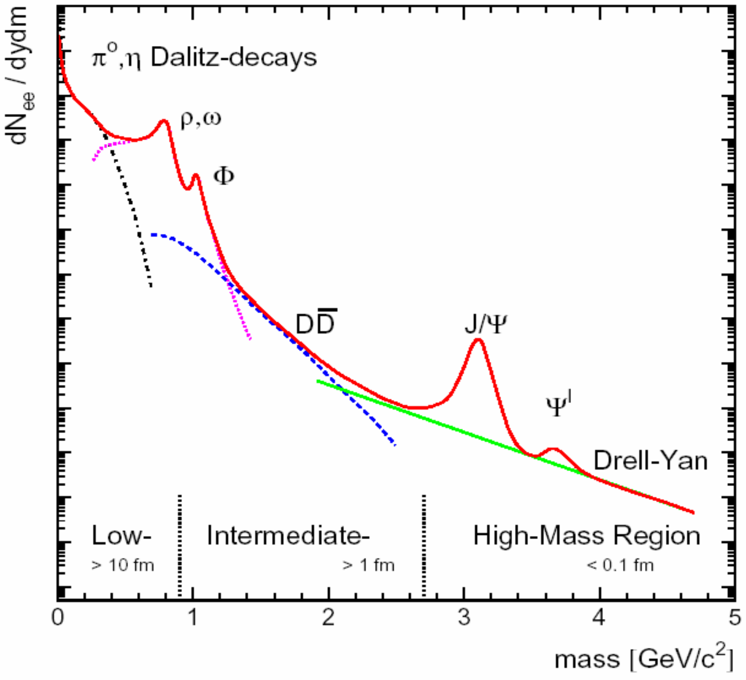
- **Dalitz decay** of mesons and baryons (π^0,η, Δ,…)



- **correlated D+Dbar pairs**

- radiation from **multi-meson reactions** (π+π, π+ρ, π+ω, ρ+ρ , π+a_1) - ‚4π‘

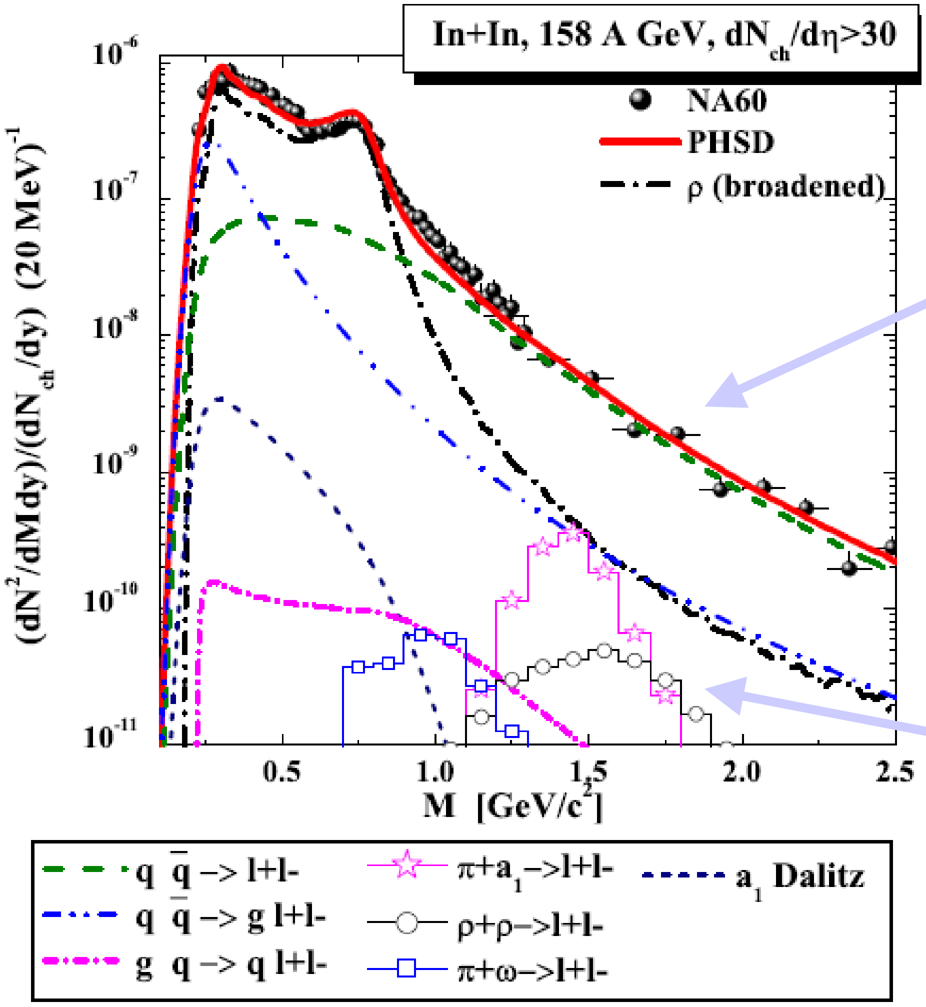
➔ **Dileptons are an ideal probe to study the properties of the hot and dense medium**

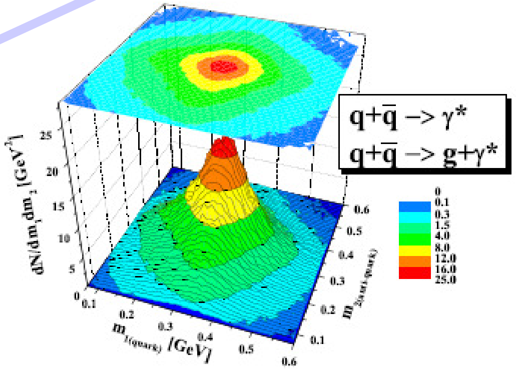


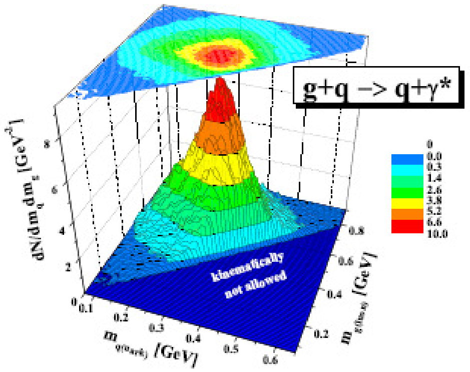
Dileptons at SPS: NA60

Acceptance corrected NA60 data

O. Linnyk, E.B., V. Ozvenchuk, W. Cassing and C.-M. Ko, PRC 84 (2011) 054917





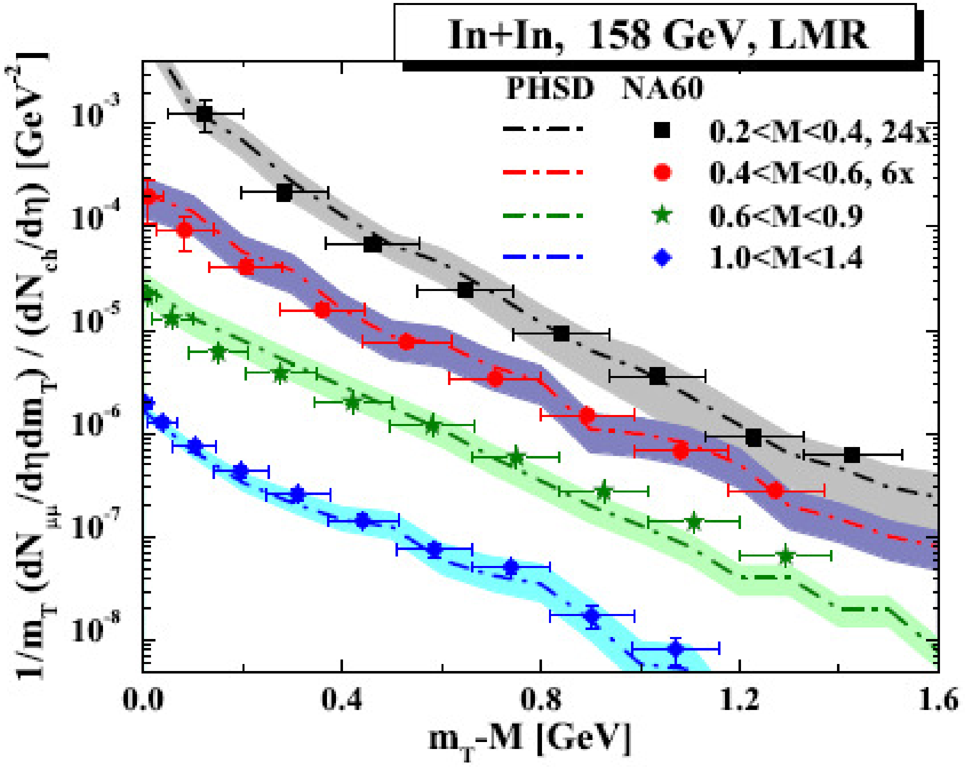


- Mass region above 1 GeV is dominated by partonic radiation !

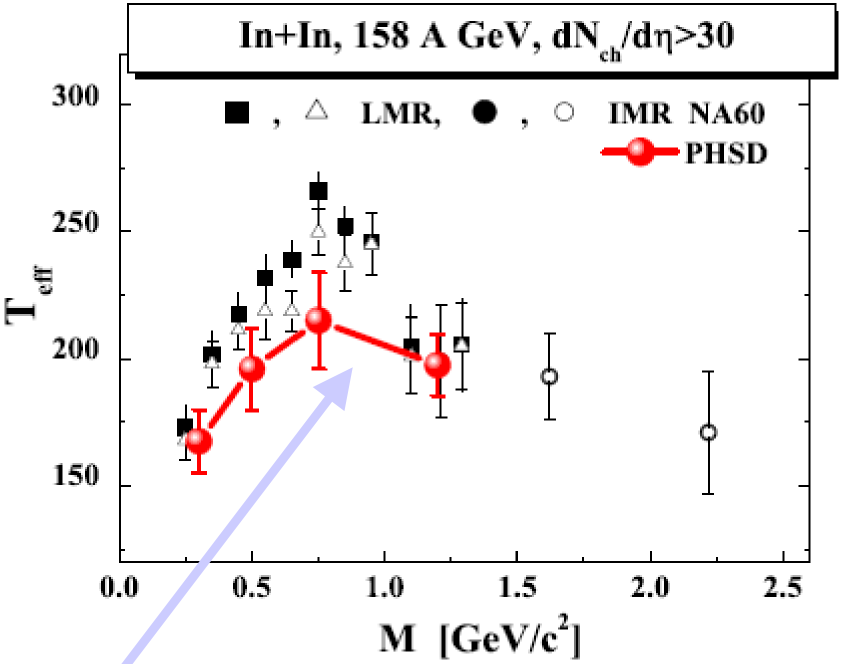
- Contributions of "4π" channels (radiation from multi-meson reactions) are small

* First discussion on "4π" : C. Song, C.M. Ko and C. Gale, PRD50 (1994) R1827

NA60: m_T spectra



- Inverse slope parameter T_{eff} for dilepton spectra vs NA60 data

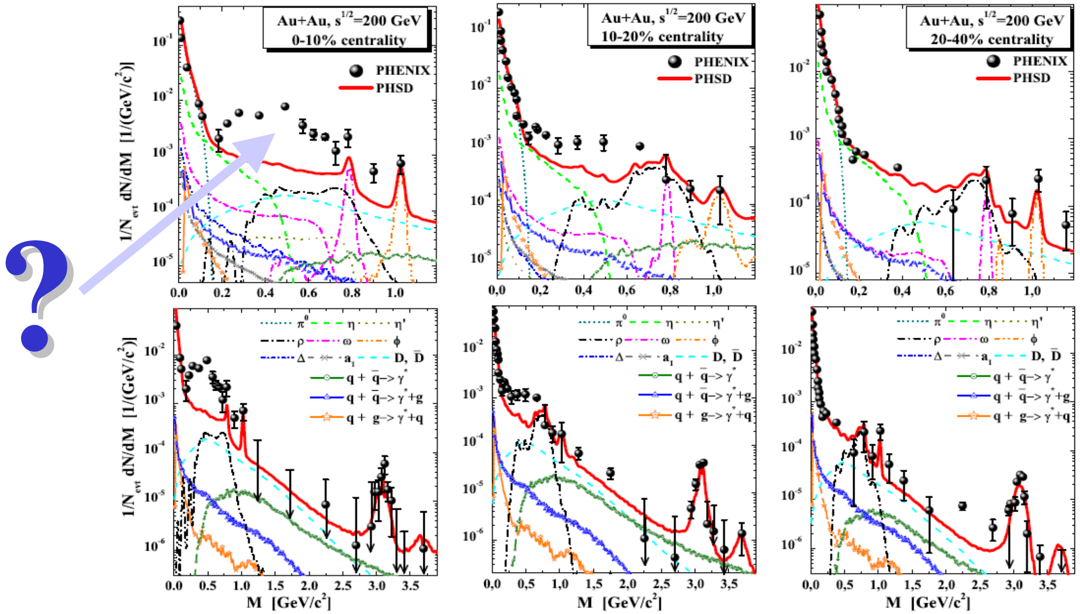


Conjecture:

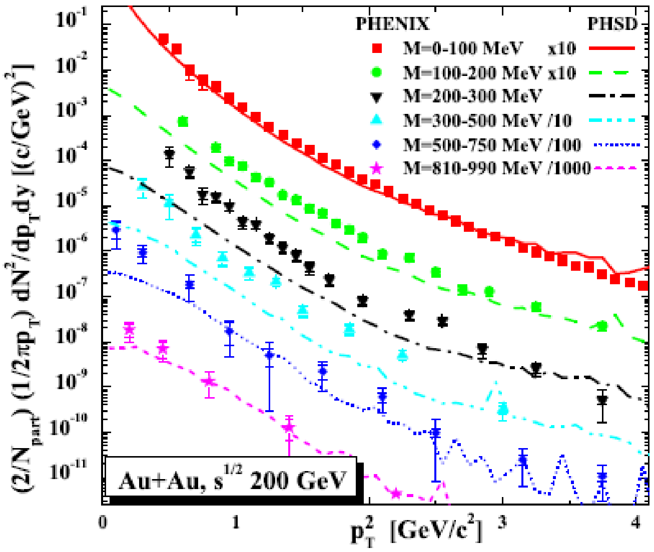
- spectrum from sQGP is softer than from hadronic phase since quark-antiquark annihilation occurs dominantly before the collective radial flow has developed (cf. NA60)

O. Linnyk, E.B., V. Ozvenchuk, W. Cassing and C.-M. Ko, PRC 84 (2011) 054917

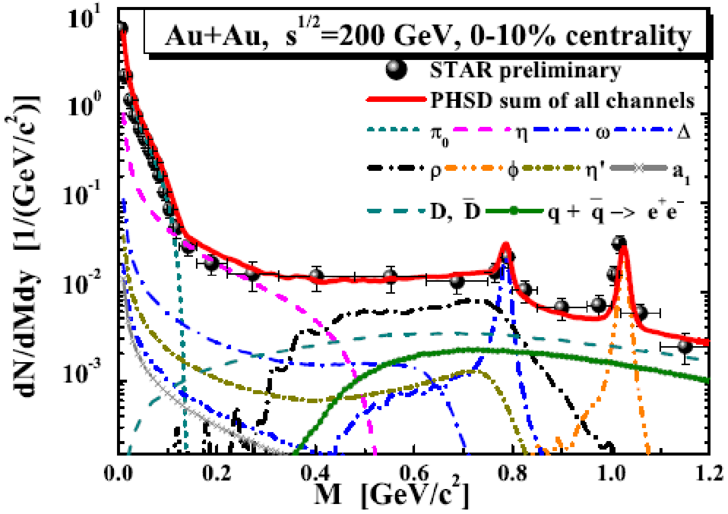
PHENIX vs. STAR dilepton spectra

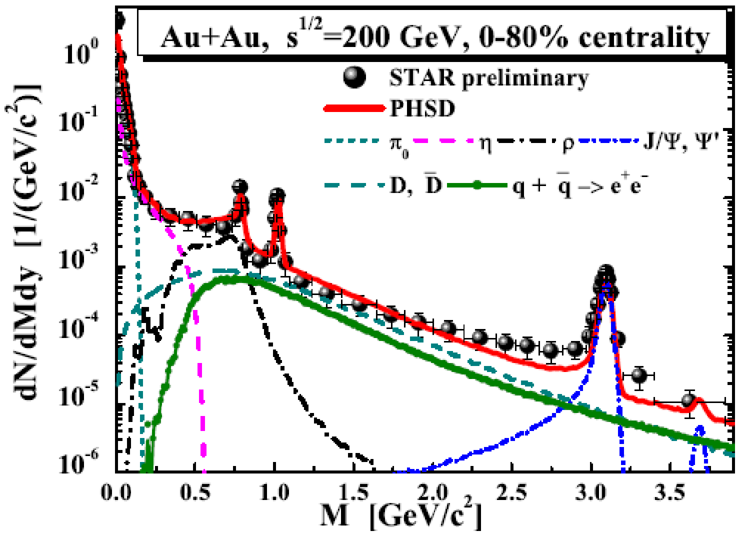


(a)0-10% centrality (b)10-20% centrality (c)20-40% centrality



- **PHENIX: Peripheral collisions (and pp) are well described, however, central fail!**





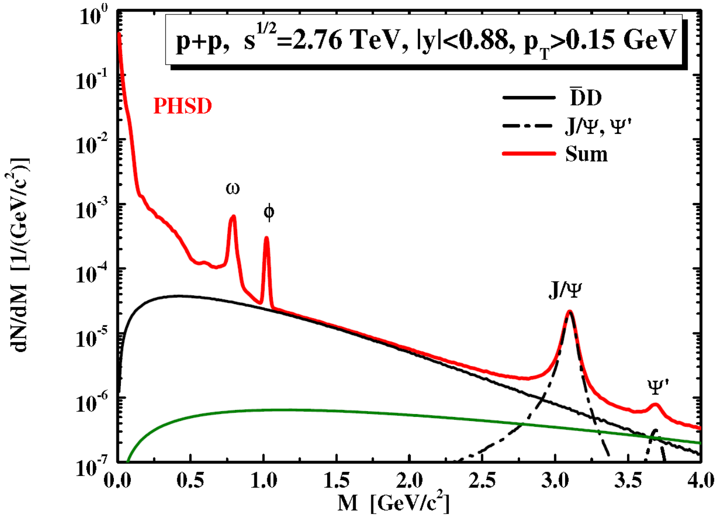
- **STAR data are well described!**

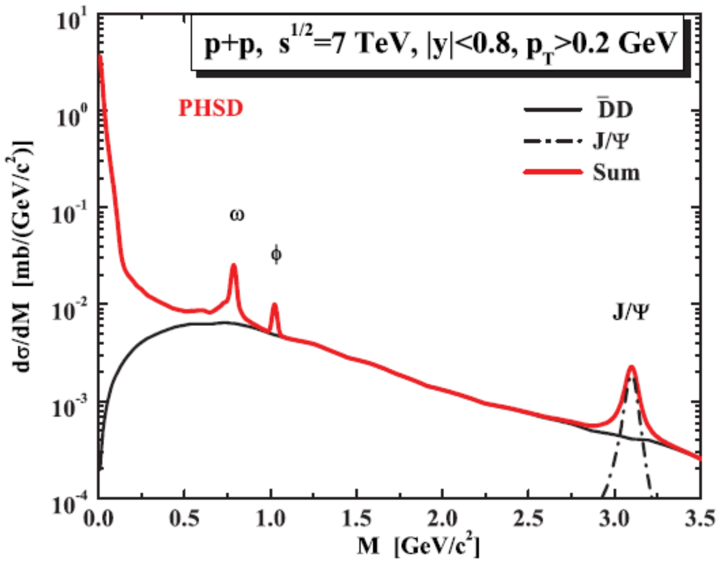
O. Linnyk et al., PRC 85 (2012) 024910

LHC: dileptons from pp and pPb

- ❑ dileptons from pp at $s^{1/2}$=2.76 and 7 TeV

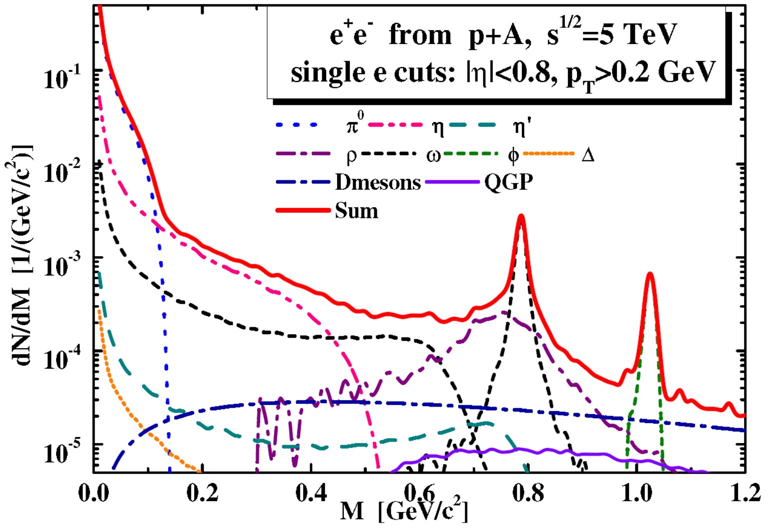
O. Linnyk et al., Phys.Rev. C87 (2013) 014905; arXiv:1208.1279

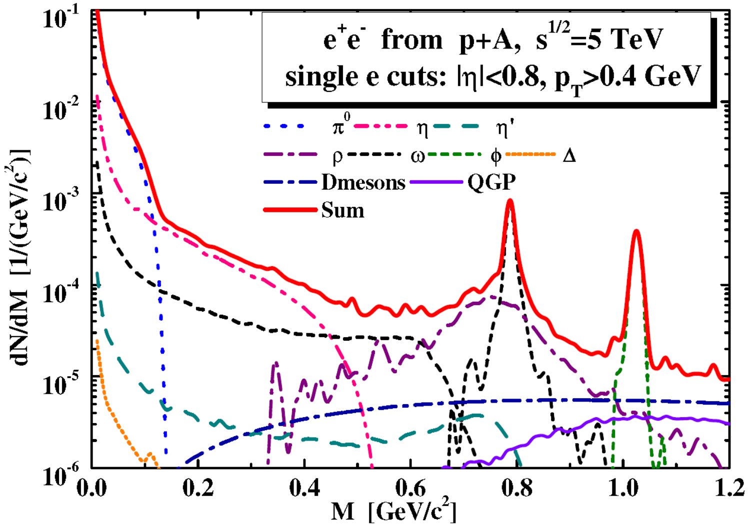




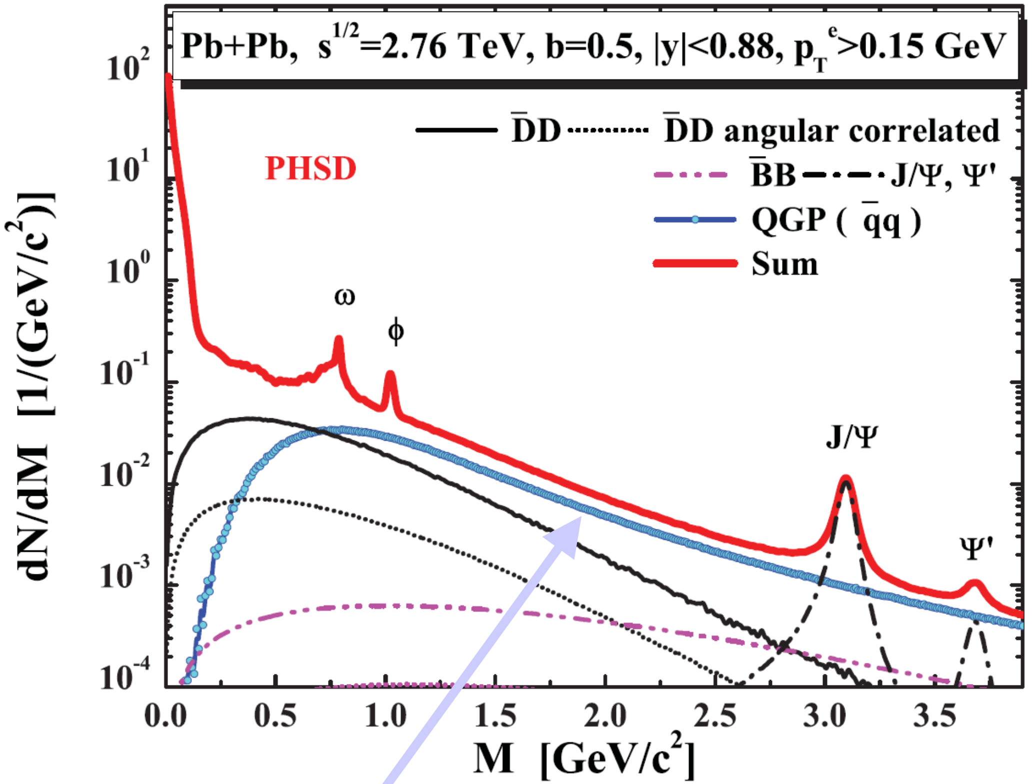
- ❑ PHSD predictions for the dilepton spectra from pPb at $s^{1/2}$=5 TeV

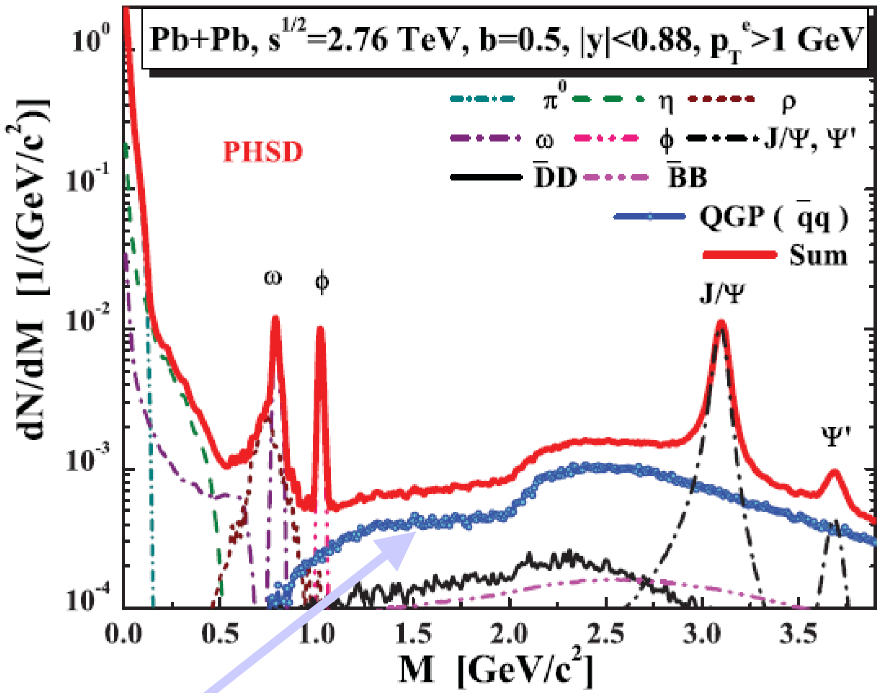
O. Linnyk, Oct. 2013



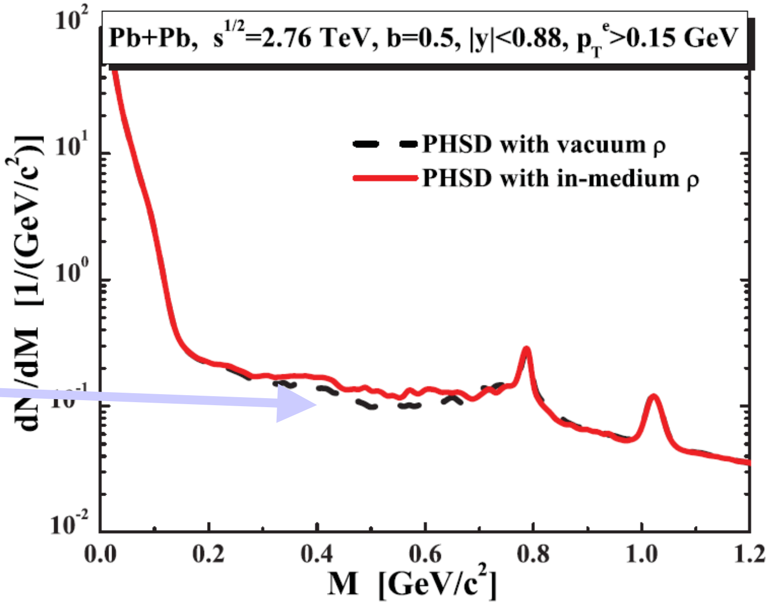


LHC: mass spectra with exp. cuts





- **QGP(qbar-q)** **dominates at M>1.2 GeV !**
- **p_T cut enhances the signal of QGP(qbar-q)**
- **in-medium effects for ρ mesons are small**



O. Linnyk, W. Cassing, J. Manninen, E.B., P.B. Gossiaux, J. Aichelin, T. Song, C.-M. Ko, Phys.Rev. C87 (2013) 014905; arXiv:1208.1279

Summary

I. Direct photons - the photons produced in the QGP contribute up to 50% to the observed spectrum, but have small v_2

- Large direct photon v_2 – comparable to that of hadrons – is attributed to the intermediate **hadronic bremsstrahlung and hadronic scattering channels** not subtracted from the data
- The **QGP** phase causes the strong elliptic flow of photons indirectly, by enhancing the v_2 of final hadrons due to the partonic interaction in terms of explicit parton collisions and the partonic mean-field potentials

II. Dilepton spectra - according to the PHSD predictions - show **sizeable changes due to the different in-medium scenarios** (as collisional broadening and dropping mass) which can be observed experimentally

- **In-medium effects** can be observed at all energies from SIS to LHC
- At SPS, RHIC and LHC the **QGP** (qbar-q) dominates at M>1.2 GeV

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Mark Gorenstein

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Laura Tolos
Angel Ramos

Thank you!

--Back up slides--

Dynamical description of strongly interacting systems

- ☐ Semi-classical BUU ➔ solution for weakly interacting systems of particles

How to describe strongly interacting systems?!

- ☐ Quantum field theory ➔

Kadanoff-Baym dynamics for resummed(!) single-particle Green functions $S^<$

$$\boxed{\hat{S}^{-1}_{0x}\, S^{<}_{xy} = \Sigma^{ret}_{xz} \odot S^{<}_{zy} + \Sigma^{<}_{xz} \odot S^{adv}_{zy}}$$

(1962)

Integration over the intermediate spacetime

Green functions $S^<$ /self-energies Σ:

$$\begin{cases} iS^{<}_{xy} = \eta\langle\{\Phi^{+}(y)\Phi(x)\}\rangle \\ iS^{>}_{xy} = \langle\{\Phi(y)\Phi^{+}(x)\}\rangle \\ iS^{c}_{xy} = \langle T^{c}\{\Phi(x)\Phi^{+}(y)\}\rangle \quad -causal \\ iS^{a}_{xy} = \langle T^{a}\{\Phi(x)\Phi^{+}(y)\}\rangle \quad -anticausal \end{cases}$$

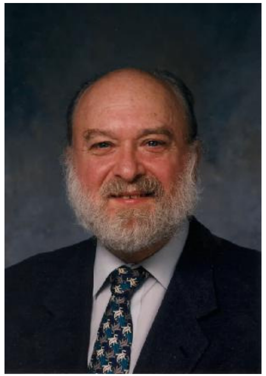
$$S^{ret}_{xy} = S^{c}_{xy} - S^{<}_{xy} = S^{>}_{xy} - S^{a}_{xy} \quad -retarded$$

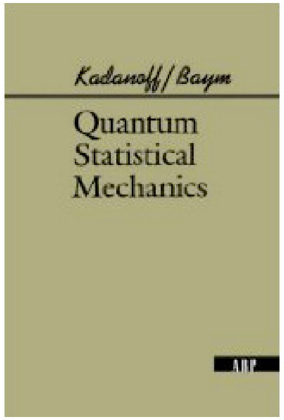
$$S^{adv}_{xy} = S^{c}_{xy} - S^{>}_{xy} = S^{<}_{xy} - S^{a}_{xy} \quad -advanced$$

$$\hat{S}^{-1}_{0x} \equiv -(\partial^{\mu}_{x}\partial^{x}_{\mu} + M_0^2)$$

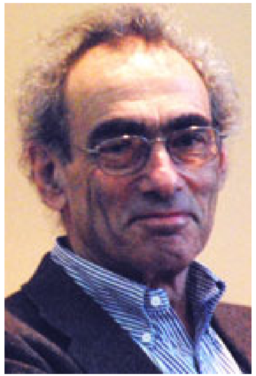
$\eta = \pm 1$ (bosons / fermions)

$T^a\,(T^c)$ − (anti−)time − ordering operator


Leo Kadanoff


Kadanoff/Baym

Quantum
Statistical
Mechanics


Gordon Baym

From Kadanoff-Baym equations to generalized transport equations

After the first order gradient expansion of the Wigner transformed Kadanoff-Baym equations and separation into the real and imaginary parts one gets:

Generalized transport equations (GTE):

drift term **Vlasov term** **backflow term** **collision term = ‚loss' term - ‚gain' term**

$$\diamond\{\underline{P^2 - M_0^2} - \underline{Re\Sigma^{ret}_{XP}}\}\{S^<_{XP}\} - \underline{\diamond\{\Sigma^<_{XP}\}\{ReS^{ret}_{XP}\}} = \frac{i}{2}\,[\,\underline{\Sigma^>_{XP}\, S^<_{XP}} - \underline{\Sigma^<_{XP}\, S^>_{XP}}\,]$$

Backflow term incorporates the off-shell behavior in the particle propagation

! vanishes in the quasiparticle limit $A_{XP} \rightarrow \delta(p^2-M^2)$

❑ **GTE: Propagation of the Green's function $iS^<_{XP}=A_{XP}N_{XP}$, which carries information not only on the number of particles (N_{XP}), but also on their properties, interactions and correlations (via A_{XP})**

Spectral function:

$$A_{XP} = \frac{\Gamma_{XP}}{(P^2 - M_0^2 - Re\Sigma^{ret}_{XP})^2 + \Gamma^2_{XP}/4}$$

$\Gamma_{XP} = -Im\,\Sigma^{ret}_{XP}$ – **width of spectral function**

= reaction rate of particle (at phase-space position XP)

4-dimentional generalizaton of the Poisson-bracket:

$$\diamond\{F_1\}\{F_2\} := \frac{1}{2}\left(\frac{\partial F_1}{\partial X_\mu}\frac{\partial F_2}{\partial P^\mu} - \frac{\partial F_1}{\partial P_\mu}\frac{\partial F_2}{\partial X^\mu}\right)$$

W. Cassing , S. Juchem, NPA 665 (2000) 377; 672 (2000) 417; 677 (2000) 445