Introduction to the SM (3)

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Last lecture

- Quantum SHO and PT
- $x \sim a + a^\dagger$
- We prefer to work with EV of H_0
- The form of the perturbation tells us what state can be created and annihilated
- a and $a^\dagger$ are important for PT of the SHO

Today: Feynman diagrams and formal symmetries

PT for 2 SHOs again

$$V(x,y) = \frac{kx^2}{2} + \frac{4ky^2}{2} + \alpha x^2 y \qquad i = |0,1\rangle \qquad x \sim a_x + a_x^\dagger$$

- Since $V' \sim x^2 y$ we see that $\Delta n_y = \pm 1$ and $\Delta n_x = 0, \pm 2$

$$f = |2,0\rangle \quad f = |2,2\rangle \quad f = |0,0\rangle \quad f = |0,2\rangle$$

- Energy cons. for the final state (not for int. states)
- Since $\omega_y = 2\omega_x$ only $f = |2,0\rangle$ is allowed

$$\mathcal{A} \sim \langle 2,0|\alpha x^2 y|0,1\rangle \sim \alpha\langle 2,0|(a_x+a_x^\dagger)(a_x+a_x^\dagger)(a_y+a_y^\dagger)|0,1\rangle$$

- Decay $y \to 2x$ with lifetime of $\tau_y \propto \alpha^2$

Even More PT

$$V' = \alpha x^2 z + \beta xyz \qquad \omega_z = 10,\ \omega_y = 3,\ \omega_x = 1$$

- Calculate $y \to 3x$ using 2nd order PT

$$\mathcal{A} \sim \langle 3,0,0|\mathcal{O}|0,1,0\rangle \qquad \mathcal{O} \sim \sum \frac{\langle 3,0,0|V'|n\rangle\langle n|V'|0,1,0\rangle}{E_n - E_{0,1,0}}$$

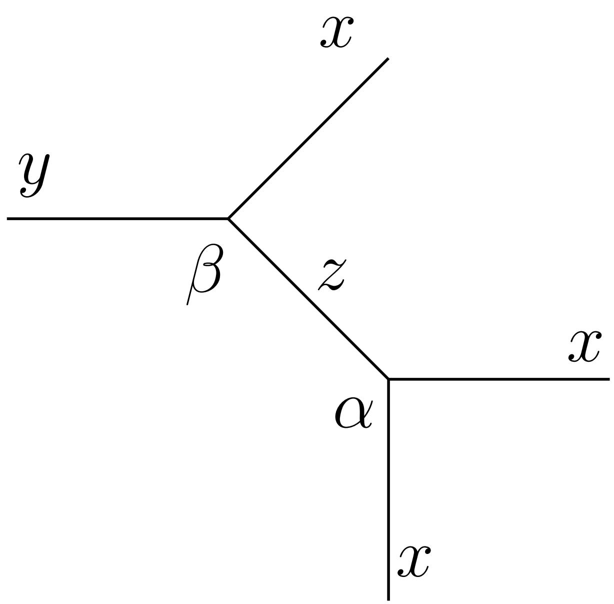
- Which intermediate states? $|1,0,1\rangle$ and $|2,1,1\rangle$
  - $\mathcal{A}_1 = |0,1,0\rangle \xrightarrow{\beta} |1,0,1\rangle \xrightarrow{\alpha} |3,0,0\rangle$
  - $\mathcal{A}_2 = |0,1,0\rangle \xrightarrow{\alpha} |2,1,1\rangle \xrightarrow{\beta} |3,0,0\rangle$
- The total amplitude is then

$$\mathcal{A} \propto \alpha\beta\left(\frac{\#}{\Delta E_1} + \frac{\#}{\Delta E_2}\right) \propto \alpha\beta\left(\frac{\#}{8} + \frac{\#}{12}\right)$$

Closer look

$$V' = \alpha x^2 z + \beta xyz \qquad \omega_z = 10,\ \omega_y = 3,\ \omega_x = 1$$

- We look at $\mathcal{A} = |0,1,0\rangle \xrightarrow{\beta} |1,0,1\rangle \xrightarrow{\alpha} |3,0,0\rangle$



$$\mathcal{A} \propto \frac{\alpha\beta}{\Delta E_z}$$

Feynman diagrams

Using PT for fields

- For SHOs we have $x_i \sim a_i + a_i^\dagger$
- For fields we then have

$$\phi \sim \int \left[a(k) + a^\dagger(k)\right] \, dk$$

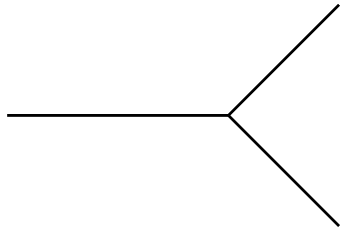
Quantum field $=$ creation and annihilation operators

Feynman diagrams

- A graphical way to do perturbation theory with fields
- Unlike SHOs before, a particle can have any energy as long as $E \geq m$
- Operators with 3 or more fields generate transitions between states. They give decays and scatterings
- Decay rates and scattering cross sections are calculated using the Golden Rule
- Amplitude are calculated from $\mathcal{L}$
- We generate graphs where lines are particles and vertices are interactions

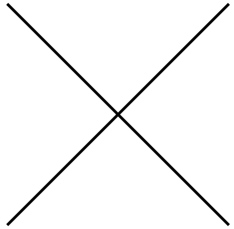
Examples of vertices

$\mathcal{L} = \lambda \phi_1^2 \phi_2$:



λ

$\mathcal{L} = \lambda \phi^4$:



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Calculations

- We usually care about $1 \to n$ or $2 \to n$ processes
- We need to make sure we have energy conservation
- External (Internal) particles are called on(off)–shell
- The amplitude is the product of all the vertices and internal lines
- Each internal line gives $1/(\Delta E)^2$ suppression
- There are many more rules to get all the factors right
- From the amplitude we get the rate

Examples of amplitudes

$$\mathcal{L} = \lambda_1 XYZ + \lambda_2 X^2 Z$$

$$\Gamma(Z \to XY)$$

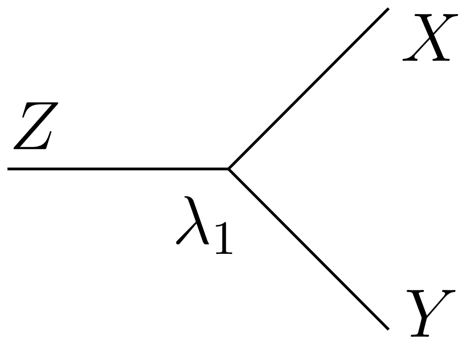
- Energy conservation condition
- Draw the diagram and estimate the amplitude

Examples of amplitudes

$$\mathcal{L} = \lambda_1 XYZ + \lambda_2 X^2 Z$$

$\Gamma(Z \to XY)$

- Energy conservation condition $m_Z > m_X + m_Y$
- Draw the diagram and estimate the amplitude



$$\mathcal{A} \propto \lambda_1$$

Examples of amplitudes (2)

$$\mathcal{L} = \lambda_1 XYZ + \lambda_2 X^2 Z$$

$\Gamma(Y \to 3X)$

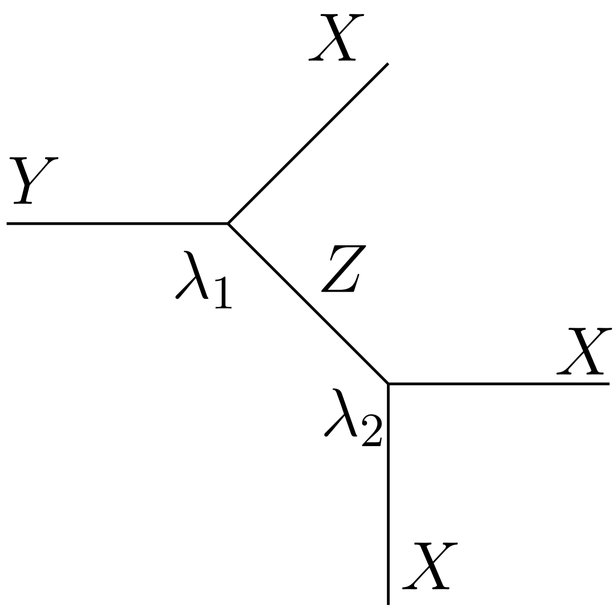
- Energy conservation condition
- Draw the diagram and estimate the amplitude

Examples of amplitudes (2)

$$\mathcal{L} = \lambda_1 XYZ + \lambda_2 X^2 Z$$

$\Gamma(Y \to 3X)$

- Energy conservation condition $m_Y > 3m_X$
- Draw the diagram and estimate the amplitude



$$\mathcal{A} \propto \lambda_1 \lambda_2 \times \frac{1}{(\Delta E_Z)^2}$$

Examples of amplitudes (HW)

$$\mathcal{L} = \lambda_1 XYZ + \lambda_2 X^2 Z$$

$$\sigma(XX \to XY)$$

- Energy conservation condition
- Draw the diagram and estimate the amplitude

Some summary

- Quadratic terms describe free fields. Free particles cannot be created nor decay
- We use perturbation theory were terms with 3 or more fields in $\mathcal{L}$ are consider small
- These terms can generate and destroy particles and give dynamics
- Feynman diagrams are a tool to calculate transition amplitudes
- Many more details are needed to get calculation done
- Once calculations and experiments to check them are done, we can test our theory

Symmetries

How to “built” Lagrangians

- $\mathcal{L}$ is:
  - The most general one that is invariant under some symmetries
  - We work up to some order (usually 4)
- We need the following input:
  - What are the symmtires we impose
  - What DOFs we have and how they transform under the symmtry
- The output is
  - A Lagrangian with N parameters
  - We need to measure its parameters and test it

Symmetries and representations

Example: 3d real space in classical mechanics

- We require that $\mathcal{L}$ is invariant under rotation
- All our DOFs are assigned into vector representations $(\vec{r}_1,\ \vec{r}_2, ...)$
- We construct invariants from these DOFs. They are called singlets or scalars

$$C_{ij} \equiv \vec{r}_i \cdot \vec{r}_j$$

- We then require that V is a function of the C_{ij}s

Generalizations

- In mechanics, $\vec{r}$ lives in 3d real space and is a vector
- Fields do not live in real space. They live in some mathematical space
- They also do not have to be vectors, but can be scalars or tensors (representation)
- The idea is similar to what we did in mechanics
  - We require $\mathcal{L}$ to be invariant under rotation in that mathematical space
  - Thus $\mathcal{L}$ depends only on combinations of fields that form singlets
- All this is related to a subject called Lie groups
- We usually care about $SO(N)$, $SU(N)$ and $U(1)$

Combining representations

- It is all about generating singlets
- We all know that we can combine vectors in real space to generate singlets
- Some of you know how to make a spin zero from 2 spin half spinors (spin zero is a singlet!)
- There is a generalization of this procedure to any mathematical space
- As of now, all we need to know are $SU(3)$, $SU(2)$ and $U(1)$

$U(1)$

- $U(1)$ is rotation in 1d complex space, or 2d real space
- Complex numbers live in that space (or in our case, complex fields)
- When we rotate the space, each number can transform differently
- When we rotate by an angle θ we have

$$\phi \to e^{iq\theta}\phi \qquad \phi_1\phi_2 \to e^{i(q_1+q_2)\theta}\phi_1\phi_2$$

- Now we know how to build invariants: $\sum q_i = 0$
- Charge conservation $=$ symmetry under rotation in a mathematical $U(1)$ space
- So far we did not call it electric charge, just charge

$SU(2)$

- $U(2)$ is rotation in 2d complex space. We have $U(2) = SU(2) \times U(1)$
- $SU(2)$ is rotation in 2d complex space or 3d real space
- Rotations in this space are non-Abelian (non-commutative)
- It depend on the representation: scalar, spinor, vector
- A representation is labeled by the number of DOFs it has, like doublet or triplet
- For the SM all we care is that $2 \times 2 = 1 + 3$
- In the SM, the electron and the neutrino are in a doublet

$SU(3)$

- $U(3)$ is rotation in 3d complex space. We have $U(3) = SU(3) \times U(1)$
- The representations we care about are singlets, triplets and octets
- The three quarks form a triplet
- For the SM all we care is that $3 \times \overline{3} = 1 + 8$
- For non-Abelian group the charge is not just a number, but a representation