CERN Summer School 2014
Introduction to Accelerator Physics

Part IV

by
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What's next?

- Imperfections, measurement and correction
  - E.g. how can we measure dispersion

- Injection and Extraction

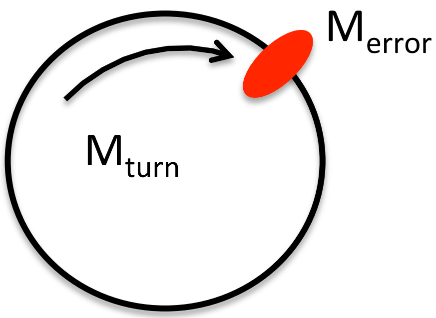
Gradient Error

What happens if there is an error in the quadrupole field?

Assume at one location in the ring quadrupole error of Δk over a distance l.

The effect on the focusing properties: the distorted one-turn matrix

$$M_{dist} = M_{error} \cdot M_{turn}$$



Remember: can write M from s_0 to s as function of β, α and ψ.

$$M = \begin{pmatrix} \sqrt{\frac{\beta}{\beta_0}}(\cos\psi + \alpha_0 \sin\psi) & \sqrt{\beta\beta_0}\sin\psi \\ \frac{(\alpha_0-\alpha)\cos\psi-(1+\alpha\alpha_0)\sin\psi}{\sqrt{\beta\beta_0}} & \sqrt{\frac{\beta_0}{\beta}}(\cos\psi - \alpha\sin\psi)) \end{pmatrix}$$

Gradient Error

$$M = \begin{pmatrix} \sqrt{\frac{\beta}{\beta_0}}(\cos\psi + \alpha_0 \sin\psi) & \sqrt{\beta\beta_0}\sin\psi \\ \frac{(\alpha_0-\alpha)\cos\psi-(1+\alpha\alpha_0)\sin\psi}{\sqrt{\beta\beta_0}} & \sqrt{\frac{\beta_0}{\beta}}(\cos\psi - \alpha\sin\psi)) \end{pmatrix}$$

One-turn matrix: $\psi_{turn} = 2\pi Q$

$$M_{turn} = \begin{pmatrix} \cos 2\pi Q + \alpha \sin 2\pi Q & \beta \sin 2\pi Q \\ -\gamma \sin 2\pi Q & \cos 2\pi Q - \alpha \sin 2\pi Q \end{pmatrix}$$

$$M_{dist} = M_{error} \cdot M_{turn}$$

Assuming a small error over a short length:

$$M_{error} = \begin{pmatrix} \cos(\sqrt{k}s) & \frac{1}{\sqrt{k}}\sin(\sqrt{k}s) \\ -\sqrt{k}\sin(\sqrt{k}s) & \cos(\sqrt{k}s) \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 \\ -\Delta kl & 1 \end{pmatrix}$$

Gradient Error

The new one-turn matrix:

$$M_{turn_{dist}} = \begin{pmatrix} \cos 2\pi Q + \alpha \sin 2\pi Q & \beta \sin 2\pi Q \\ -\gamma \sin 2\pi Q & \cos 2\pi Q - \alpha \sin 2\pi Q \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 0 \\ -\Delta kl & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos 2\pi Q_0 + \alpha \sin 2\pi Q_0 & \beta \sin 2\pi Q_0 \\ -\gamma \sin 2\pi Q_0 & \cos 2\pi Q_0 - \alpha \sin 2\pi Q_0 \end{pmatrix}$$

With $Q = Q_0 + \Delta Q$, ΔQ small and $\mathrm{Trace}(M_{dist}) = \mathrm{Trace}(M_{error}.M_{turn})$:

$$\Delta Q = \frac{1}{4\pi} \beta \Delta k \cdot l$$

β at the error location

The quadrupole error leads to a tune change. The higher the β, the higher the effect.

And also a change of the beta functions.

Chromaticity

The normalized quadrupole gradient is defined as

$$k = \frac{g}{p/e} \qquad\qquad p = p_0 + \Delta p$$

$$k = \frac{eg}{p_0 + \Delta p} \approx \frac{e}{p_0}(1 - \frac{\Delta p}{p_0})g = k_0 + \Delta k$$

$$\Delta k = -\frac{\Delta p}{p_0} k_0$$

...a gradient error. Particles with different $\Delta p/p$ will have different tunes.

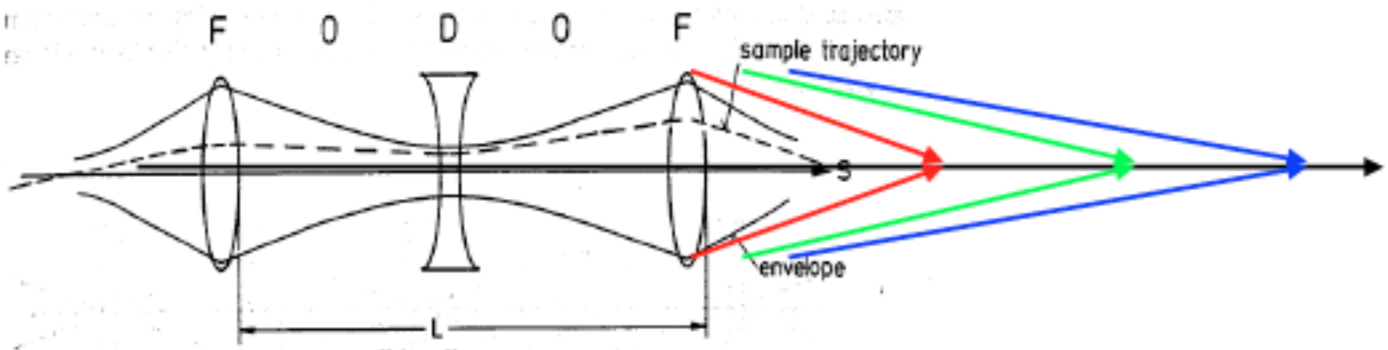


Figure 29: FODO cell

Chromaticity: Q'

The tune change for different Δp/p:

$$\Delta Q = \frac{1}{4\pi}\beta\Delta k \cdot l \quad \rightarrow \quad \Delta Q = -\frac{1}{4\pi}\frac{\Delta p}{p_0}k_0\beta l$$

Definition of chromaticity:

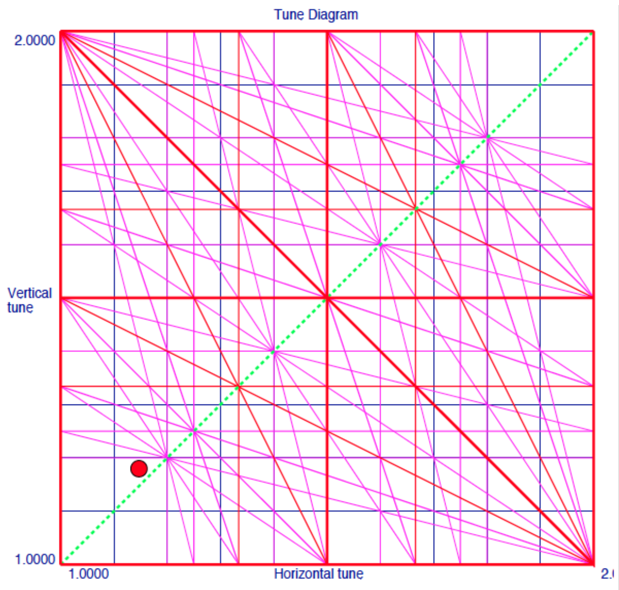
$$\Delta Q = Q'\frac{\Delta p}{p}$$

$$Q' = -\frac{1}{4\pi}\oint k(s)\beta(s)ds$$

With the beam momentum spread indicates the size of the tune spot in the tune diagram.

Chromaticity is created by quadrupole fields in the horizontal and vertical plane.

Chromaticity: Q'



We cannot leave chromaticity uncorrected:

Example LHC

Q'= 250 [no units]

$\Delta p/p = +/- 0.2 \times 10^{-3}$

$\Delta Q = 0.256 \ldots 0.36$!!!! ➔ Particles will go across resonance lines and will be lost.

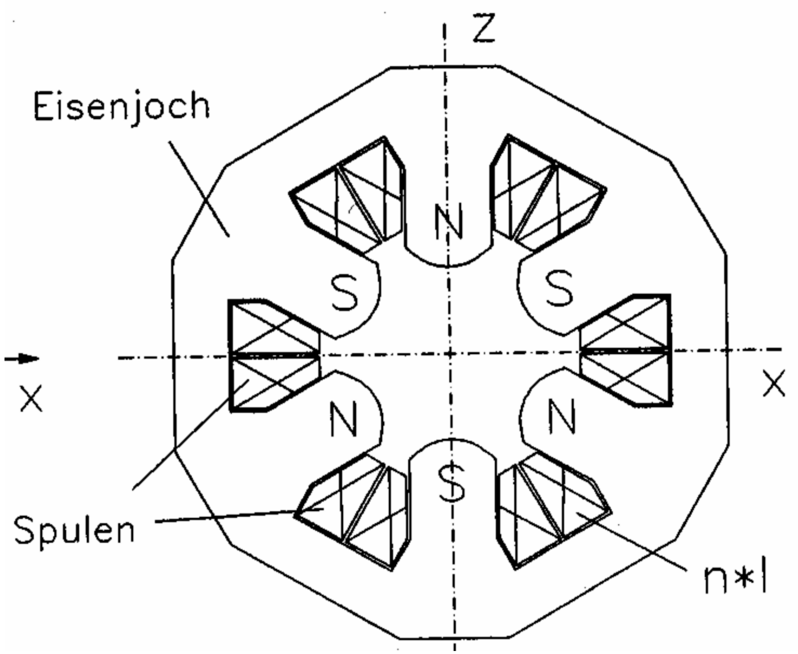
How to correct chromaticity?

Sextupole fields at locations of dispersion:

1) Sort the particles according to momentum: $x_D(s) = D(s)\frac{\Delta p}{p}$
2) Magnetic field with linear rising "gradient" ➔ prop. to x^2

Correcting chromaticity

Sextupole magnets:



$$B_x = \tilde{g}xy$$

$$B_y = \frac{1}{2}\tilde{g}(x^2 - y^2)$$

↓

$$\frac{\partial B_x}{\partial y} = \frac{\partial B_y}{\partial x} = \tilde{g}x$$

Linear rising "gradient"

Sextupoles give a normalized quadrupole strength of:

$$k_{sext} = \frac{\tilde{g}x}{p/e} = m_{sext}x \rightarrow k_{sext} = m_{sext}D\frac{\Delta p}{p}$$

Correcting chromaticity

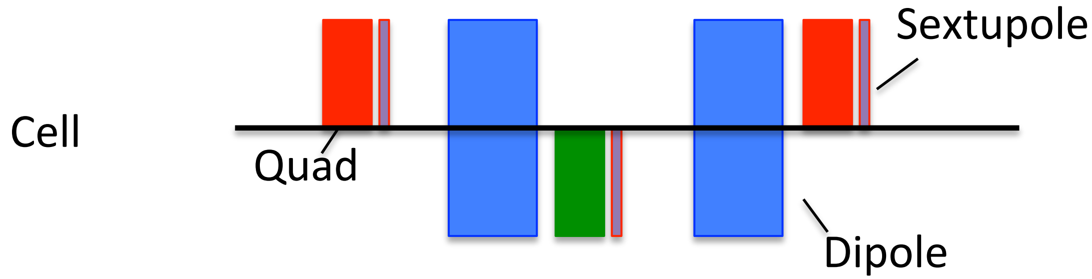
Only need dispersion in horizontal plane to correct chromaticity in horizontal and vertical plane.

$$Q' = -\frac{1}{4\pi} \oint \{k(s) - m(s)D(s)\}\beta(s)ds$$

Calculate m such that chromaticity vanishes.

(...we are neglecting collective effects.)

Add two families of sextupoles in your regular FODO lattice at the location with maximum dispersion: next to the quadrupoles



Effect of sextupoles on phase-space

Sextupoles create non-linear fields.

Depending on the tune the phase-space becomes more and more distorted. Motion becomes unstable close to the third order resonance.

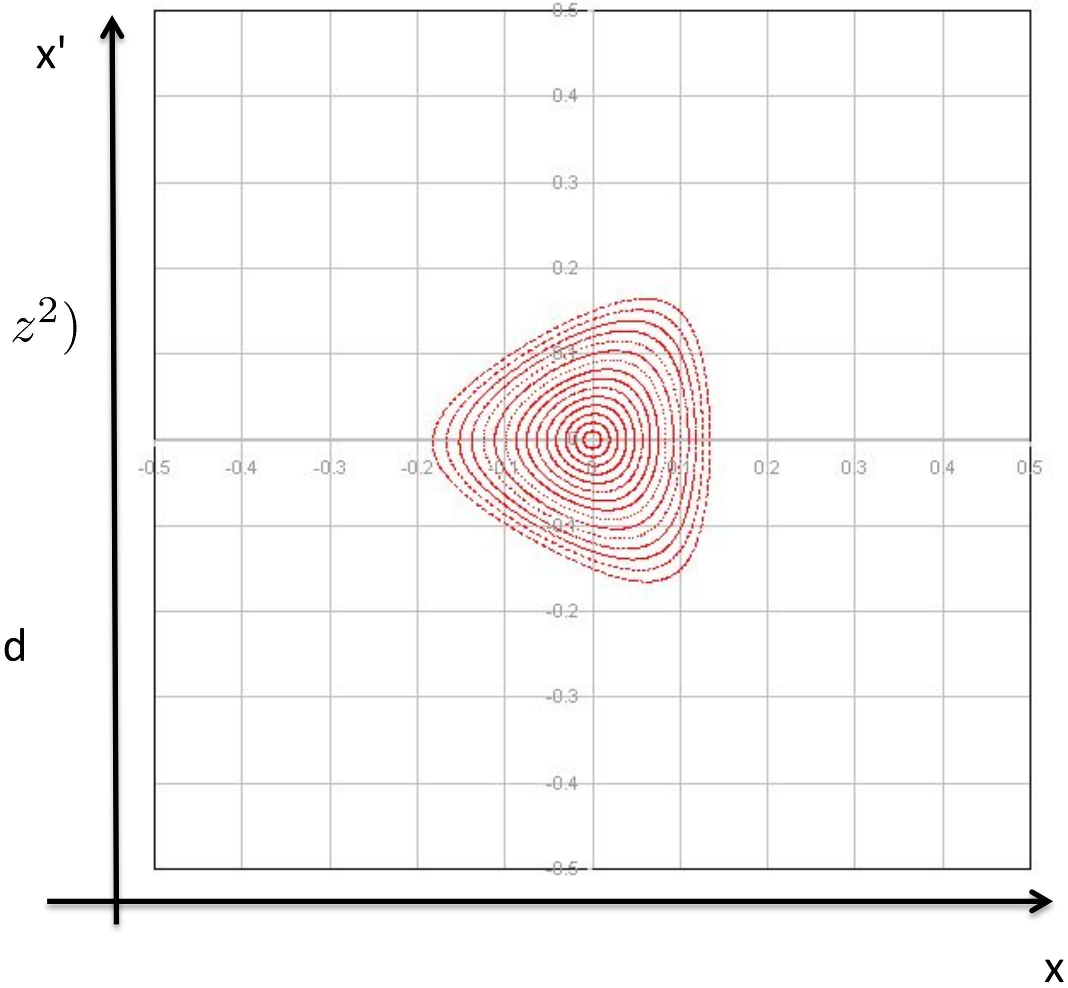
The perturbed Hill's equations with sextupoles:

$$x'' + K_x(s) = -\frac{1}{2} m_{sext}(s)(x^2 - z^2)$$

$$z'' + K_z(s) = m_{sext}(s) xz$$

Amplitude of separatrix (unstable fixed points):

$$\propto \frac{Q - \frac{p}{3}}{m_{sext}}$$



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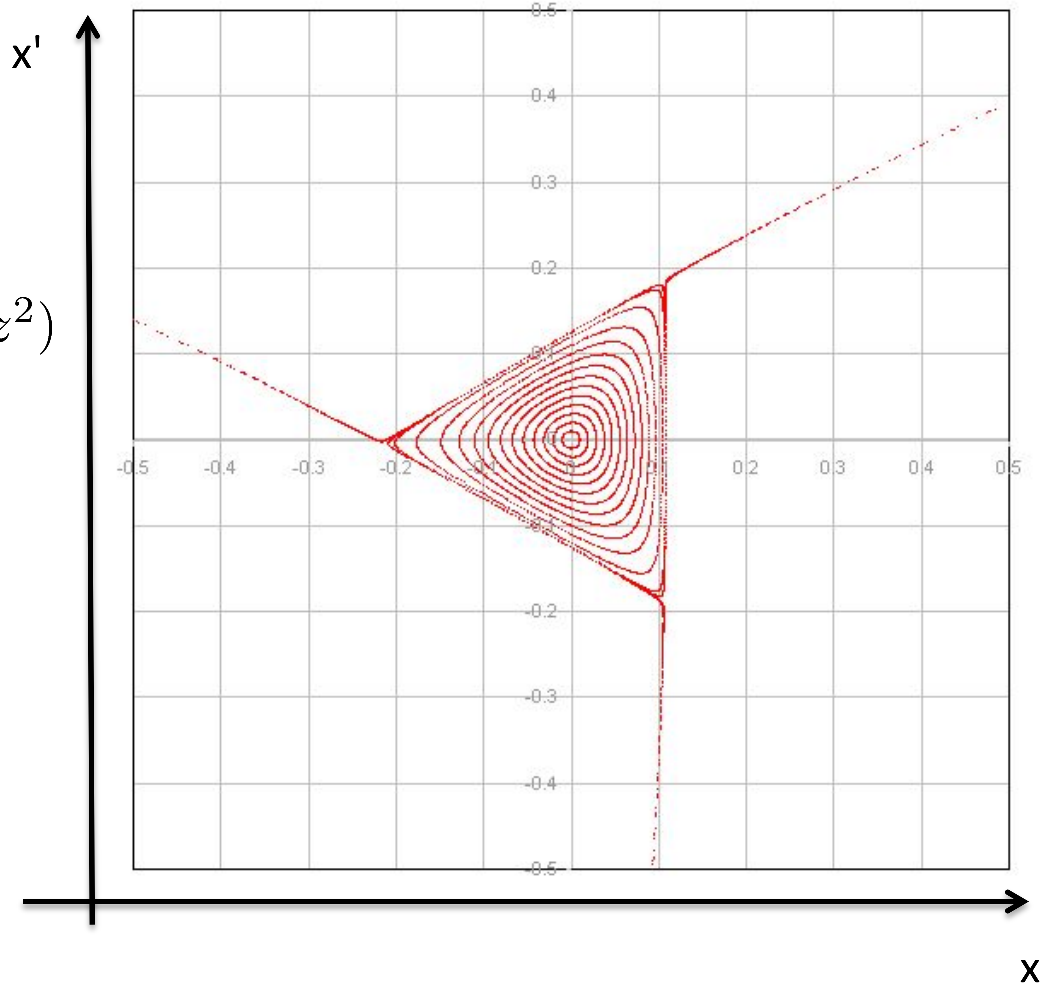
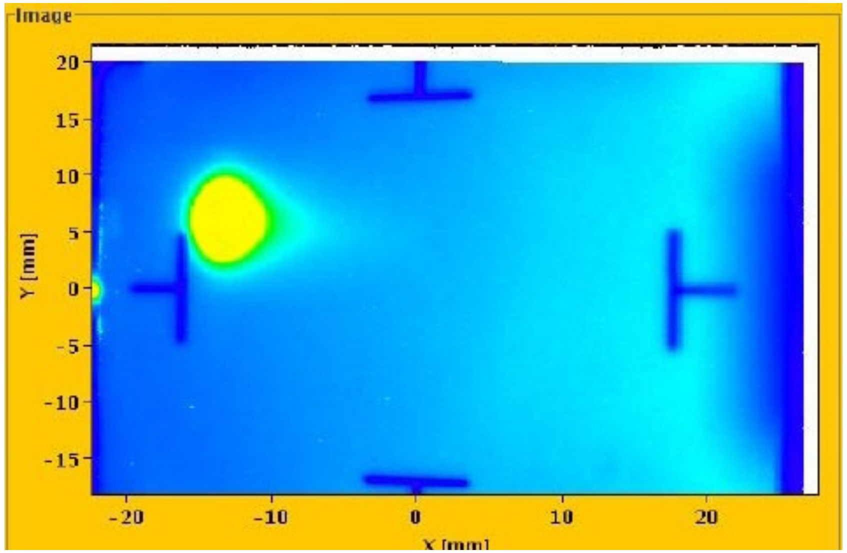


Diagnostics

Instruments to measure different beam parameters and optimize accelerator. The eyes of the accelerator…

Beam intensity
Beam position
Beam loss
Beam profile for beam size
…

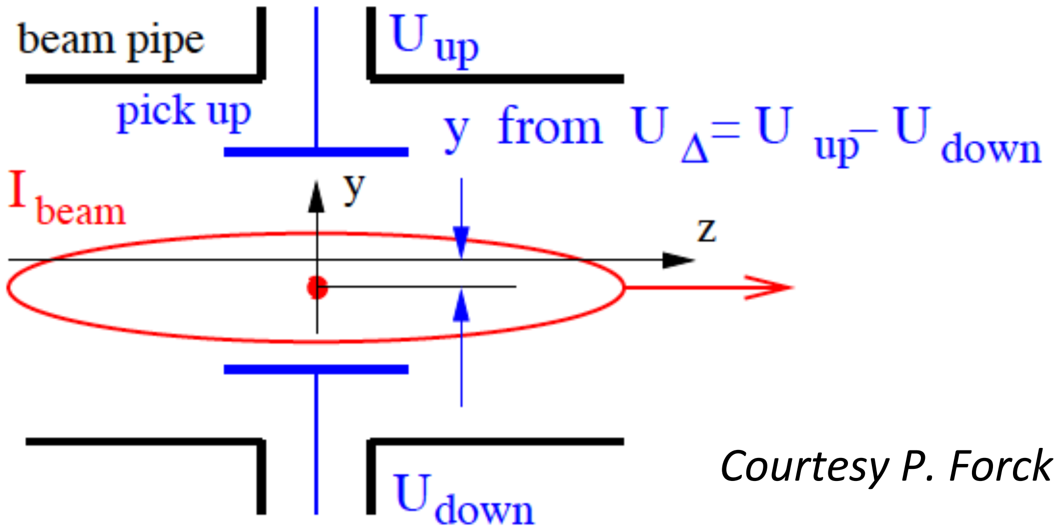


Beam profile from first injection into LHC 2008

Let's have a look at beam position monitors.

Position measurement with capacitive pick-up

The principle: isolated plates or buttons to measure difference of beam induced voltage



Courtesy P. Forck

'Proximity effect': the closer the beam to plate, the higher the induced voltage.

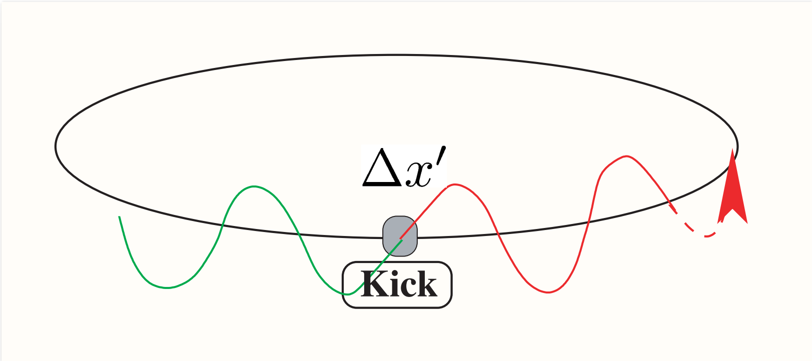
The intensity independent position is: $x = \frac{1}{S_x} \cdot \frac{U_{up} - U_{down}}{U_{up} + U_{down}} = \frac{1}{S_x} \cdot \frac{\Delta U}{\Sigma U}$

S_x = position sensitivity: position reading can depend on position itself

Is there an optimum position in the ring for the BPMs?

Let's have a look at the effect of dipole field errors
(e.g. from quadrupole misalignment)

Assume an error at one location (s_0) in the ring:



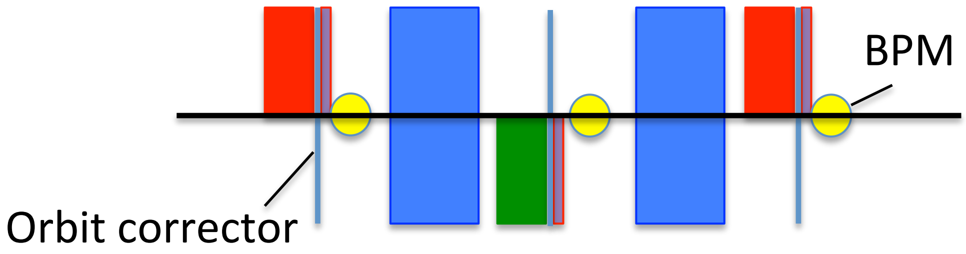
$$x(s) = \frac{\Delta x'}{2} \cdot \sqrt{\beta_0 \beta} \frac{\cos(\pi Q - \psi_{s_0 \to s})}{\sin(\pi Q)}$$

Note:

1) The orbit amplitude will be larger at locations in the ring with larger β ➔ this is where we will put the BPMs
2) The orbit amplitude will be larger if β_0 at the location of the kick is large ➔ this is where we will put the orbit correctors

Location of BPMs and orbit correctors

Our cell:



Why does the orbit have to be corrected. Typical alignment errors ~0.5 mm

Example SPS @ 450 GeV.: $\Delta x' = 0.5\,mm \cdot k \cdot l = 22\mu rad$

Maximum amplitude orbit: $x \approx 1\,mm$

How is the orbit corrected?

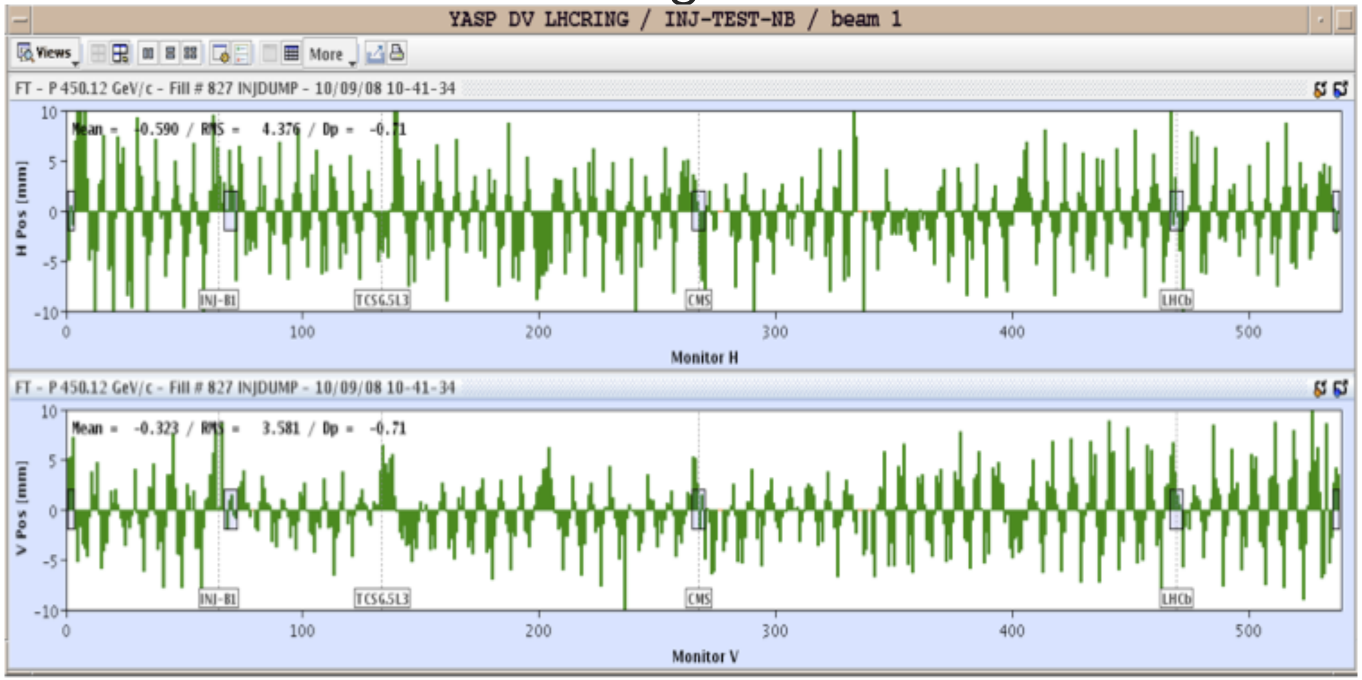
Different algorithms MICADO, SVD...

Let's have a look at SVD.

Orbit correction with Singular Value Decomposition

The steps:

- Measure the orbit around the ring



- Build “response matrix” R:

$$\begin{pmatrix} \Delta x_1 \\ \vdots \\ \Delta x_M \end{pmatrix} = R \cdot \begin{pmatrix} \Delta x'_1 \\ \vdots \\ \Delta x'_N \end{pmatrix}$$

Positions at all the BPMs: M BPMs

Angles from correctors: N correctors

SVD for orbit correction

The response matrix R has the form:

Remember the effect of a dipole kick at one location in the ring:

$$x(s) = \frac{\Delta x'}{2} \cdot \sqrt{\beta_0 \beta} \frac{\cos(\pi Q - \psi_{s_0 \to s})}{\sin(\pi Q)}$$

$$\to R_{ij} = \frac{\sqrt{\beta_i \beta_j}}{2 \sin \pi Q} \cos(|\psi_i - \psi_j| - \pi Q)$$

For the i-th BPM and the j-th corrector

Ideally we want: $\vec{\Delta x} = R \cdot \vec{\Delta x'} \to \vec{\Delta x'} = R^{-1} \cdot \vec{\Delta x}$

But the solution to this might not necessarily exist.
So we at least want to minimize $|R \cdot \Delta \vec{x'} - \Delta \vec{x}|$

And SVD provides such a solution with

$$R^{-1} = V \cdot W^{-1} U^T$$

SVD for orbit correction

Remember from linear algebra: can write a matrix R as a product of three matrices:

$$R = U \cdot W \cdot V^T$$

where U is a unitary M x M matrix, W is M x N diagonal matrix with positive or zero elements and V is an N x N unitary matrix.

$$U^T \cdot U = U \cdot U^T = V^T \cdot V = V \cdot V^T = I$$

The elements of W, w_n, are the eigenvalues.

$$W_{ij} = w_{min(i,j)}\delta_{ij}$$

The orthogonal basis vectors are related through

$$R \cdot \vec{v_n} = w_n \vec{u_n}$$

The SVD algorithm is implemented in Python, JAVA, matlab, ...

SVD for orbit correction

We need

$$R^{-1} = V \cdot W^{-1} U^T$$

and (1 ≤ m ≤ min(M,N))

$$W_{ij}^{-1} = q_{min(i,j)} \delta_{ij} \qquad q_n = \begin{cases} 0 & w_n \leq \varepsilon \cdot w_{max} \\ 1/w_n & otherwise \end{cases}$$

ε: not necessarily all eigenvalues are used for correction. ε in range [0,1]

One calculates then

$$\vec{\Delta x'} = V \cdot W^{-1} \cdot U^T \vec{\Delta x}$$

...and applies $(-1) \cdot \vec{\Delta x'}$ to correct the orbit.

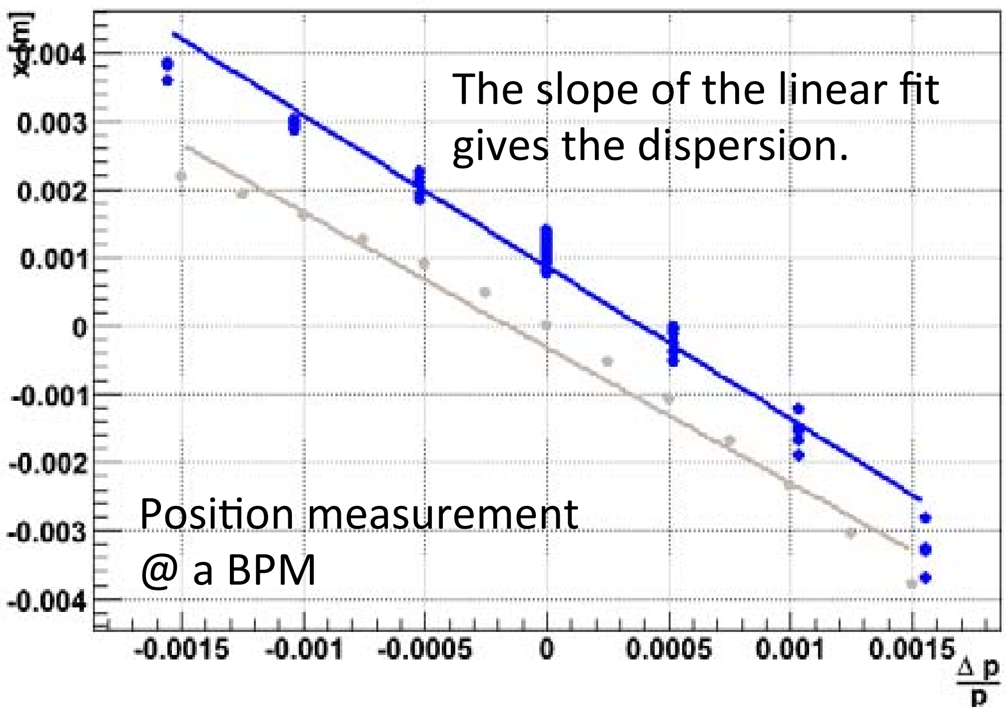
HOW TO MEASURE DISPERSION?

How to measure the dispersion?

Can measure dispersion at BPMs.

Remember:

$$x_{BPM}(s) = x_\beta(s) + D(s) \cdot \frac{\Delta p}{p}$$



How can we vary Δp/p?
Vary RF frequency

$$\frac{\Delta f_{RF}}{f_{RF}} = \left(\frac{1}{\gamma^2} - \alpha\right)\frac{\Delta p}{p}$$

HOW TO MEASURE CHROMATICITY?

How to measure chromaticity?

Chromaticity: $$\Delta Q = Q' \frac{\Delta p}{p}$$

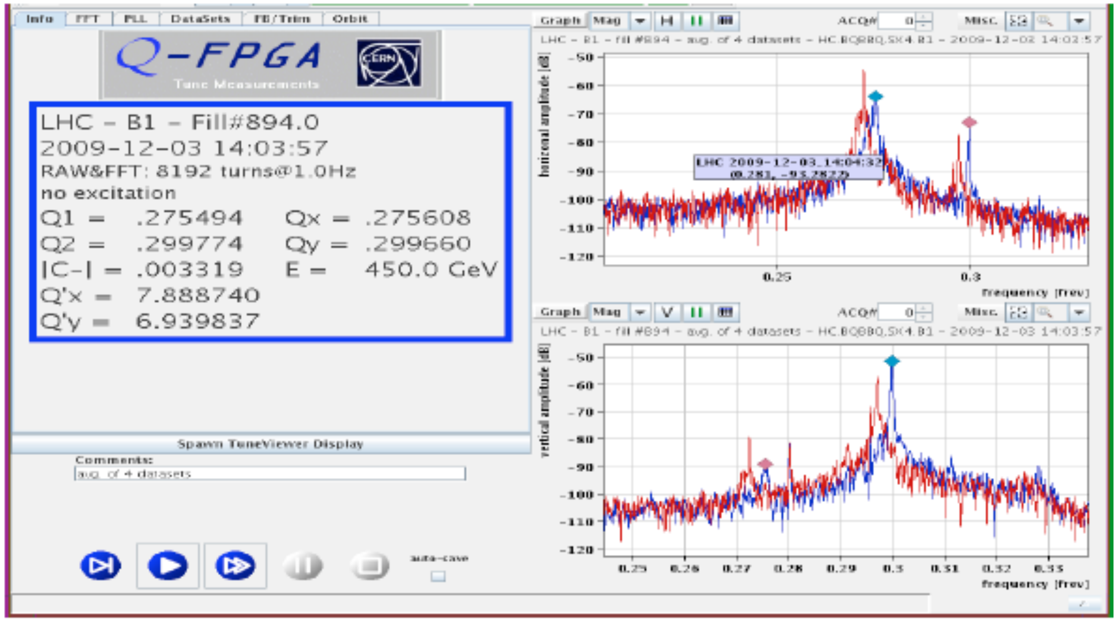
Need to measure tune as function of Δp\p.

Take turn-by-turn measurement of a sensitive beam position measurement.

Might have to create a big enough oscillation.

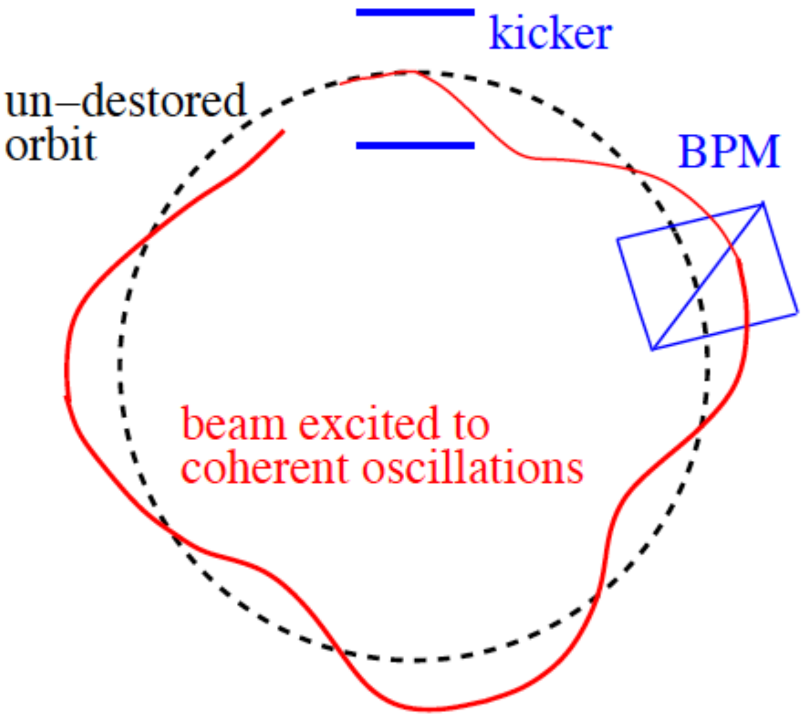
E.g. with tune kicker
Tune kicker gives a deflection only once with a duration of less than one turn.

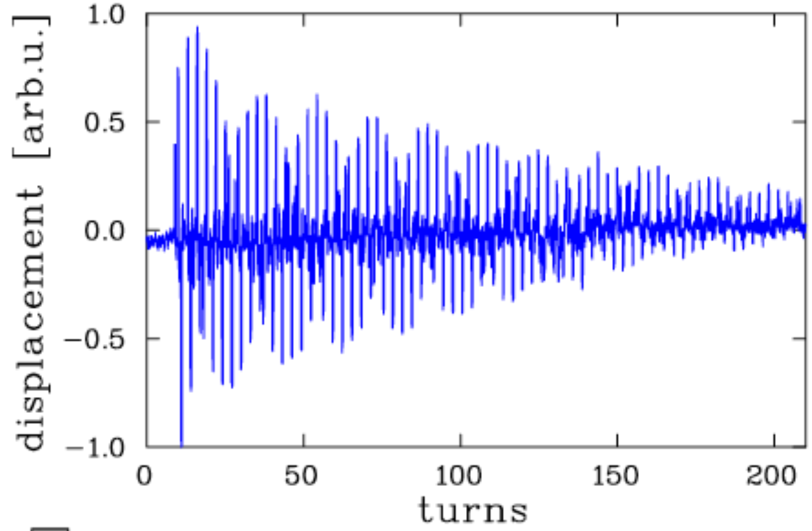
Free oscillation.

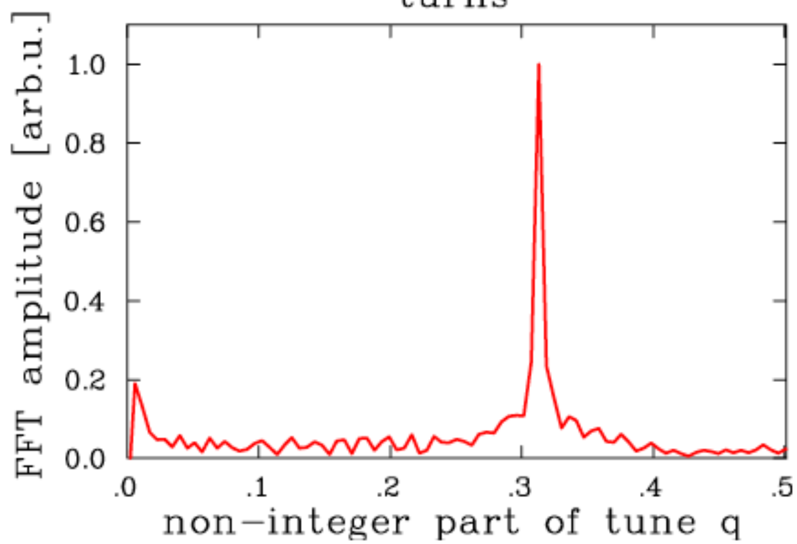


How to measure tune?

Record turn-by-turn data and perform FFT. The frequency of the peak amplitude in the spectrum corresponds to the tune (...to first order).







How to measure β at a quadrupole?

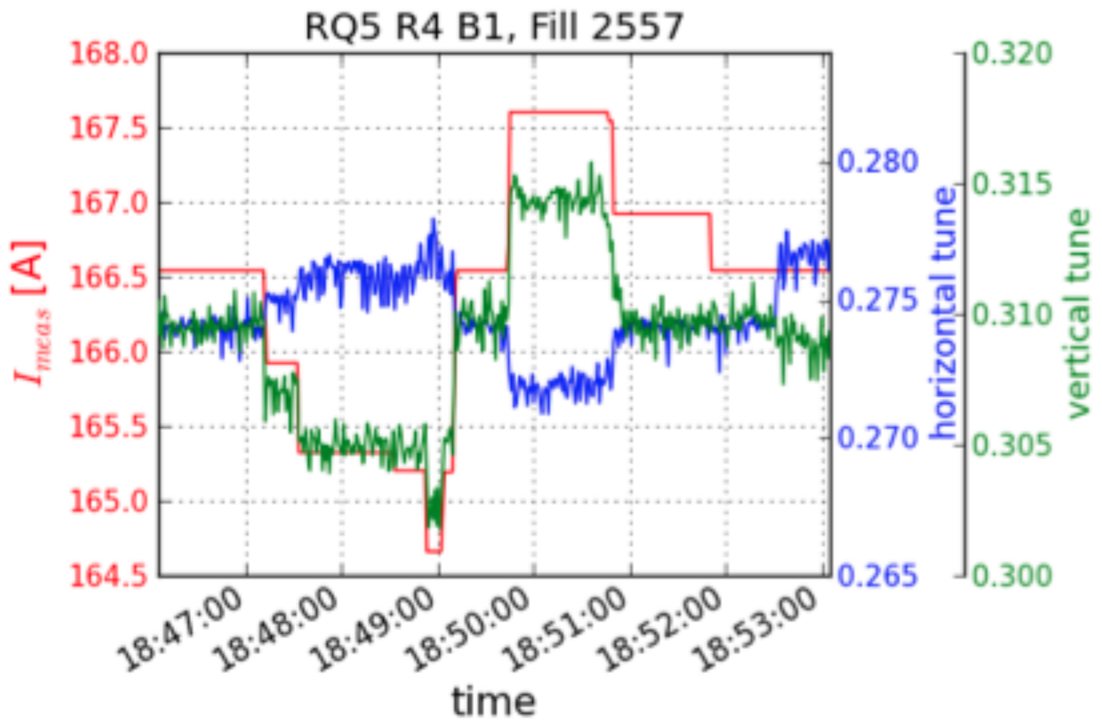
Remember formula from gradient error:

$$\Delta Q = \frac{1}{4\pi}\beta_{Quad}\Delta k \cdot l$$

Thus can measure β at quadrupole if we can change its strength and measure the tune change at the same time.

Obtain b from linear fit of tune change versus strength change.

This method is called **k-modulation.**

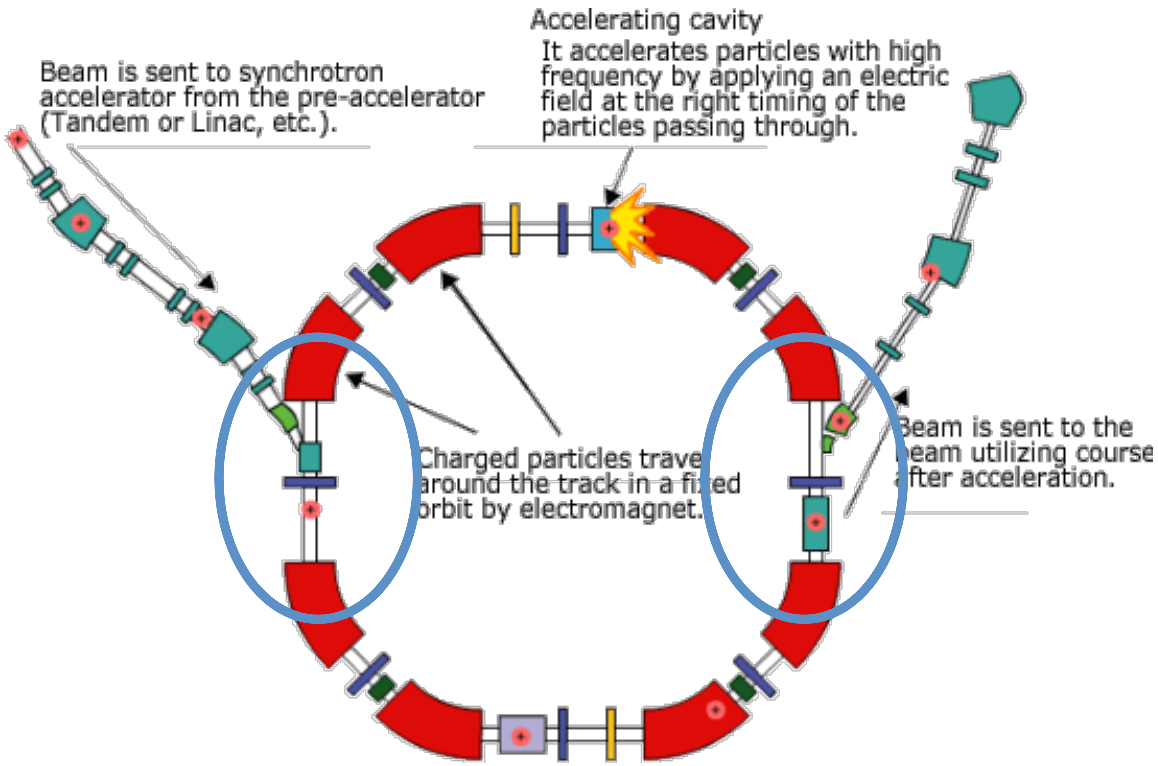


TRANSFER, INJECTION & EXTRACTION

Transfer, Injection & Extraction

We have looked at transverse motion and longitudinal motion of the beam in a synchrotron.

But we do not know yet how to fill the synchrotron with beam and how to get rid of it at the end of the operational cycle?

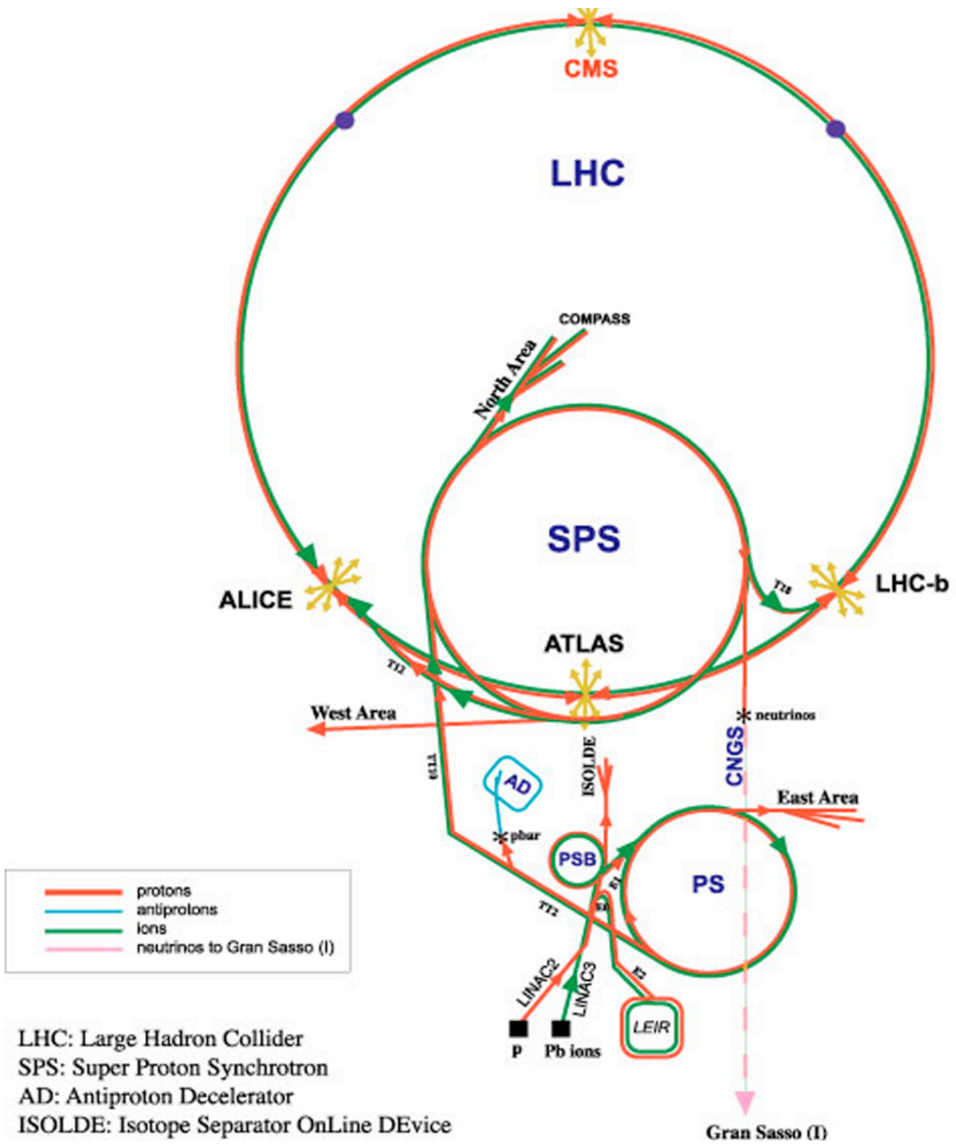


Injection, extraction and transfer

An accelerator has limited dynamic range.

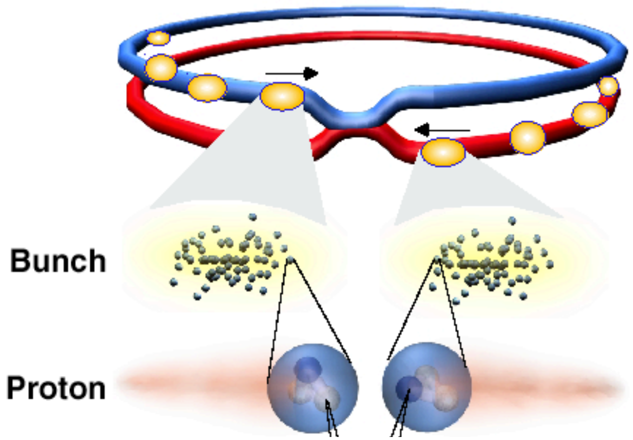
Chain of stages needed to reach high energy.

Transfer lines transport the beam between accelerators, and onto targets, dumps, instruments etc.



Injection and Emittance Preservation

Small emittances is key for collider performance.
What counts is the collision rate.



Collision rate = Luminosity x cross-section

$$\mathcal{L} \propto \frac{N_1 N_2 n_b}{\sigma^2}$$

$$\sigma^2 = \frac{\varepsilon_n \cdot \beta^*}{\gamma}$$

And injection errors can easily lead to emittance blow-up.

Injection with offset

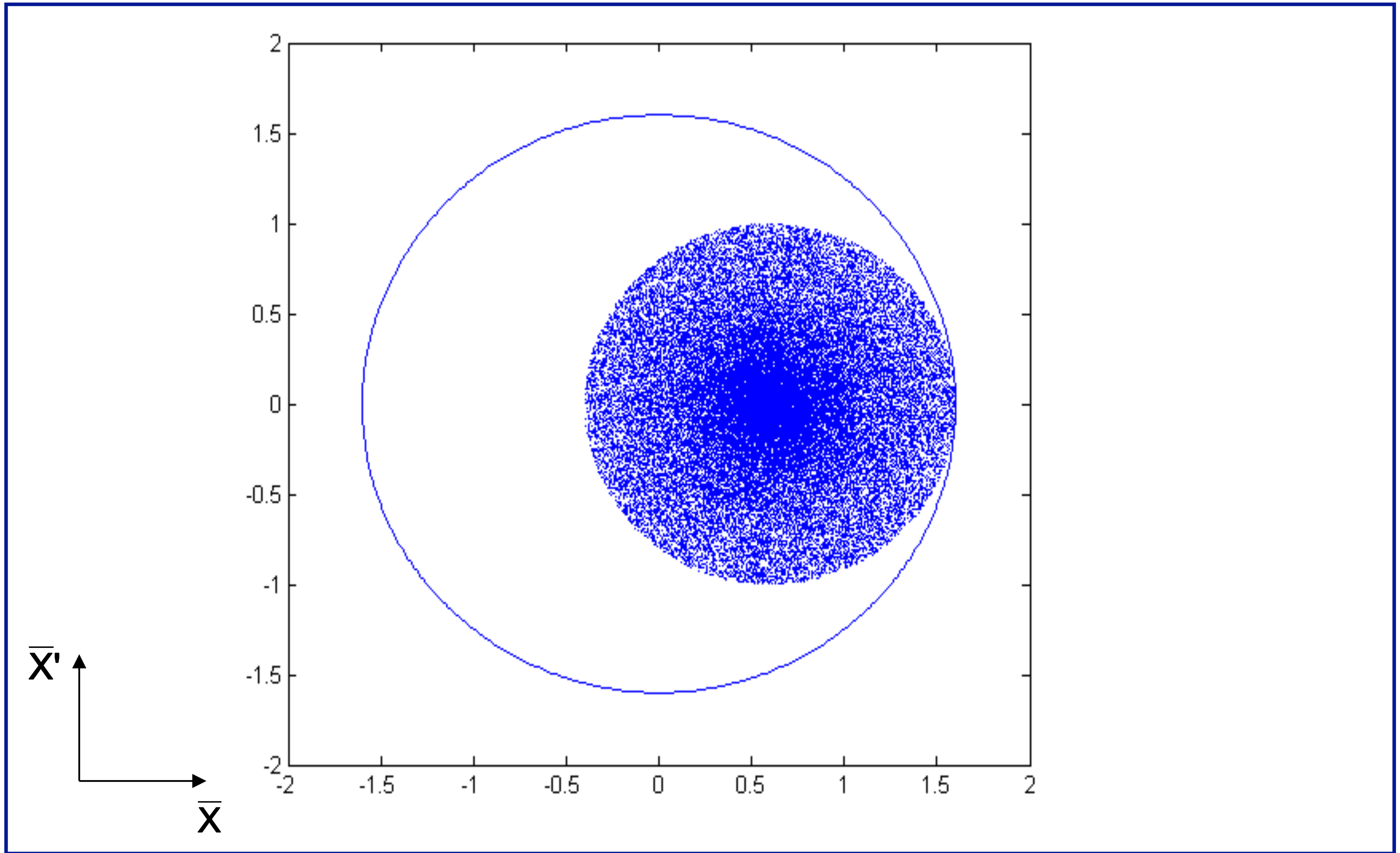
The beam has to be injected exactly on the orbit of the circular machine.

$$x_{inj}(s_0) = x_{CO}(s_0)$$
$$x'_{inj}(s_0) = x'_{CO}(s_0)$$
$$y_{inj}(s_0) = y_{CO}(s_0)$$
$$y'_{inj}(s_0) = y'_{CO}(s_0)$$

Otherwise: Filamentation and emittance blow-up due to non-linear fields in the circular machine.

➢ Transverse feedback to counteract.

Filamentation



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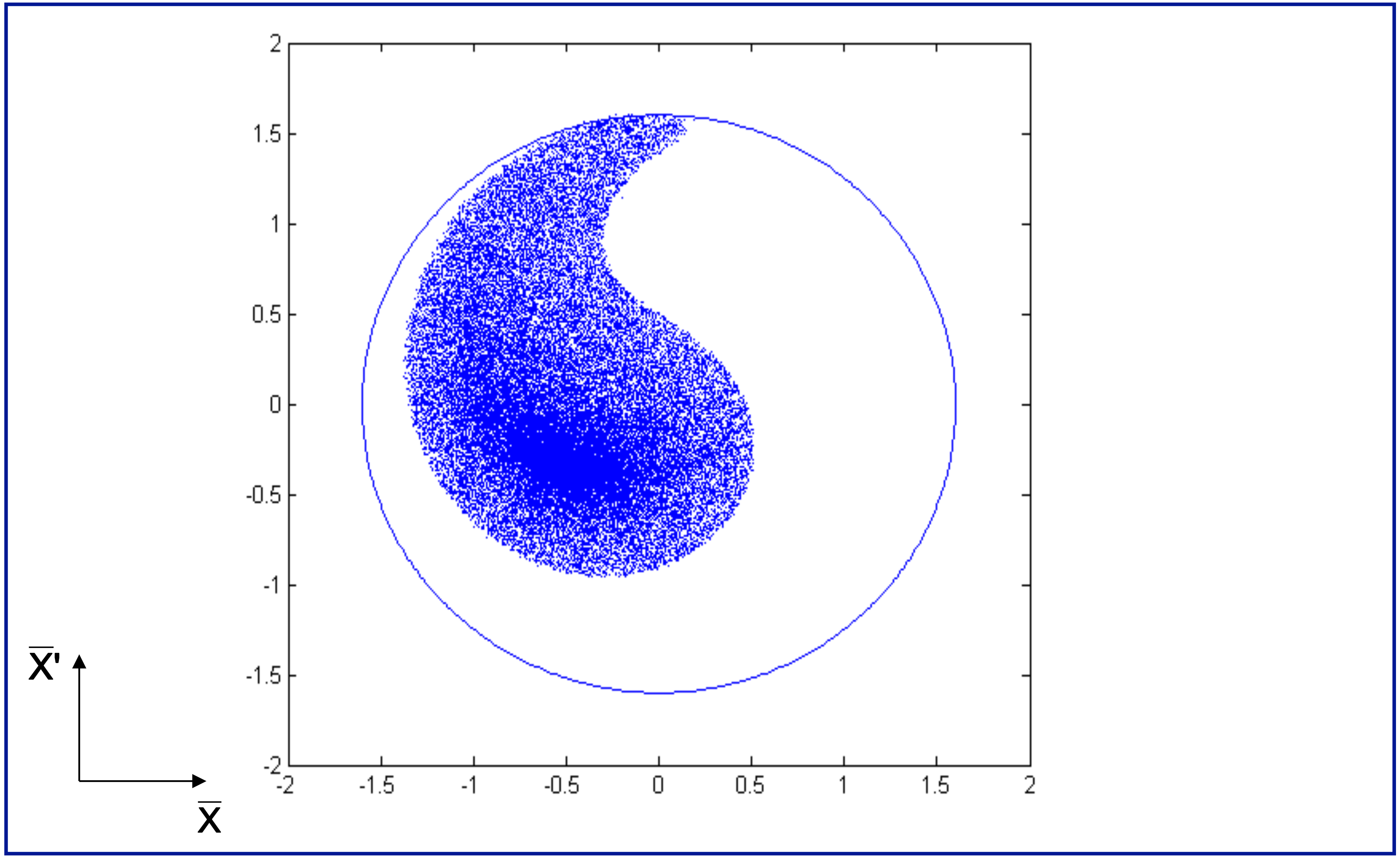

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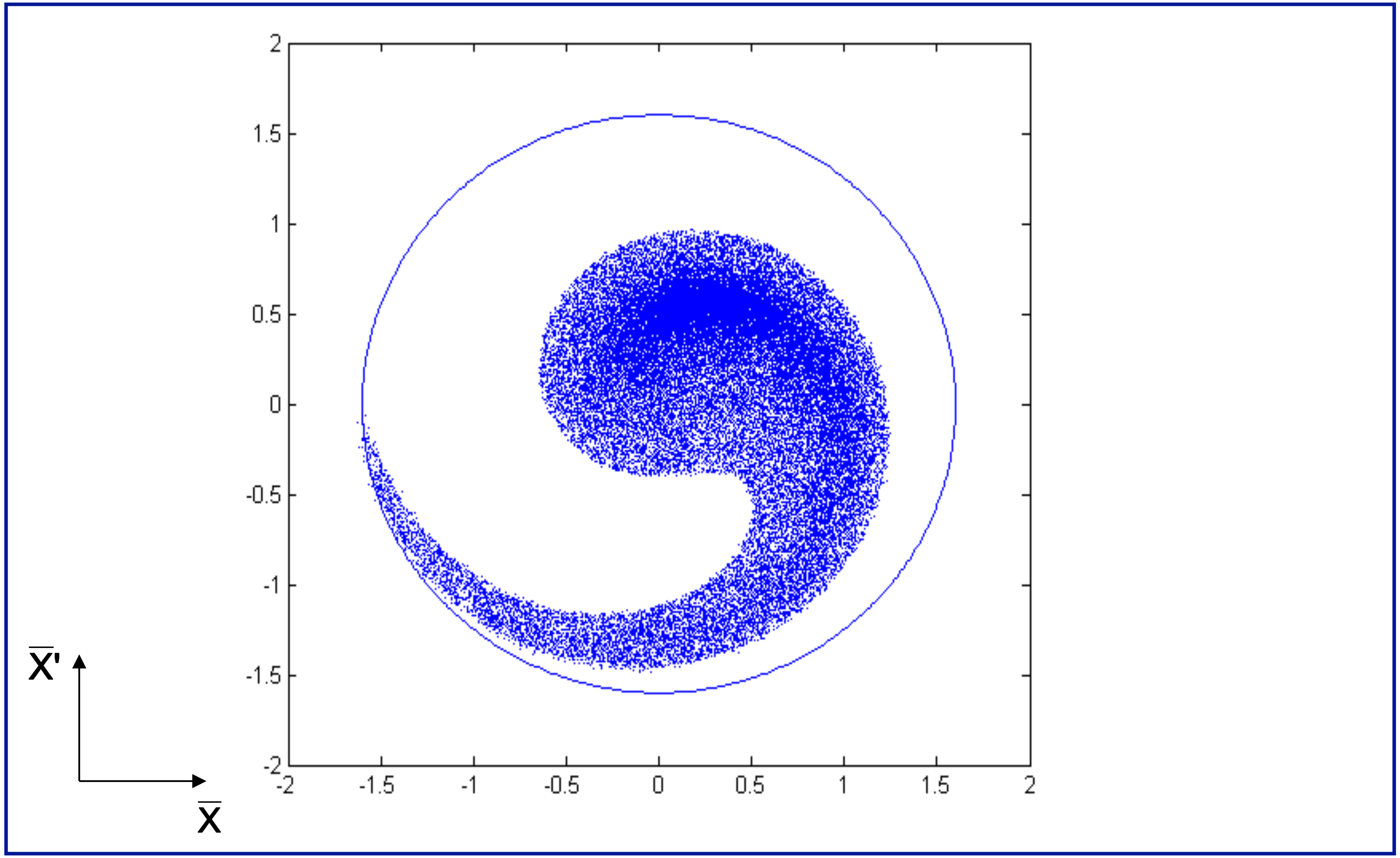

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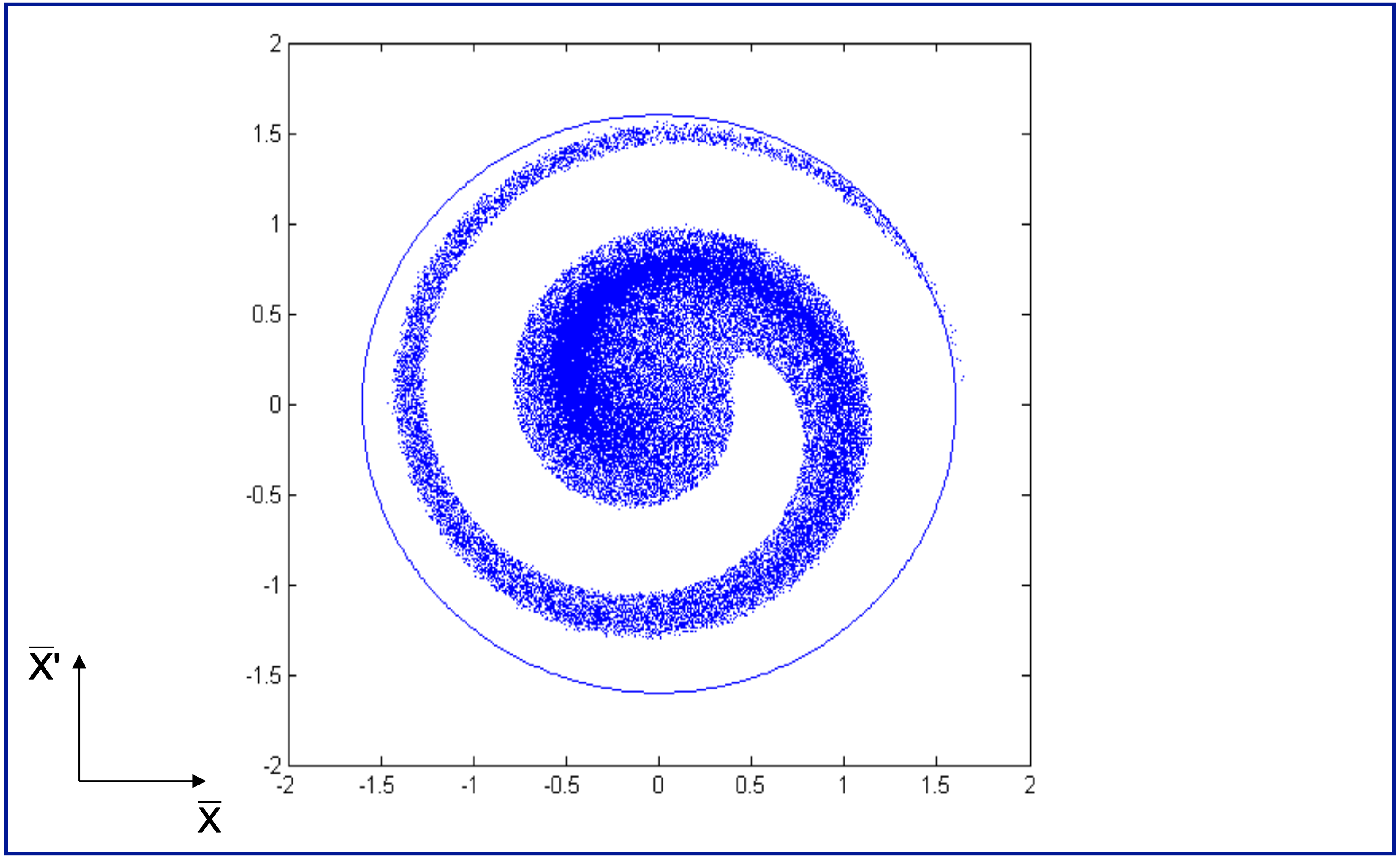

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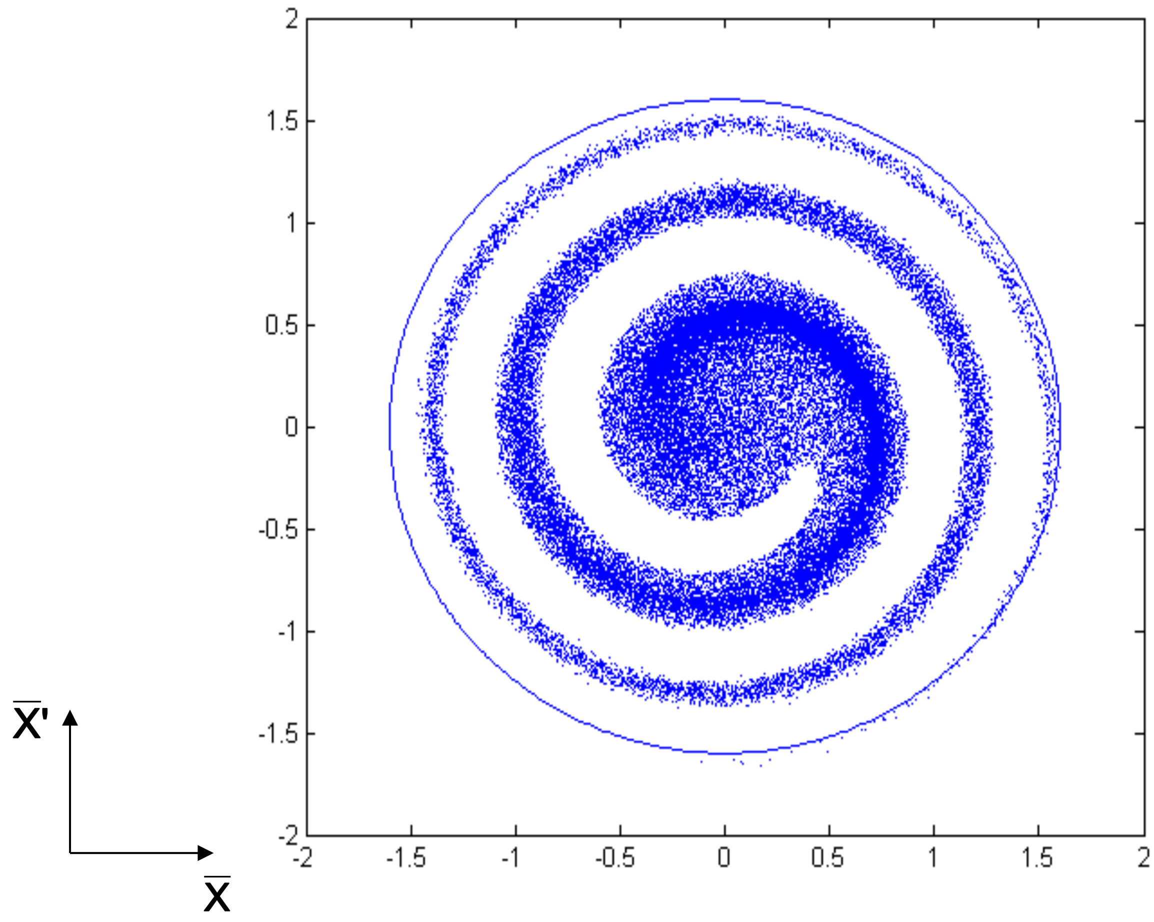

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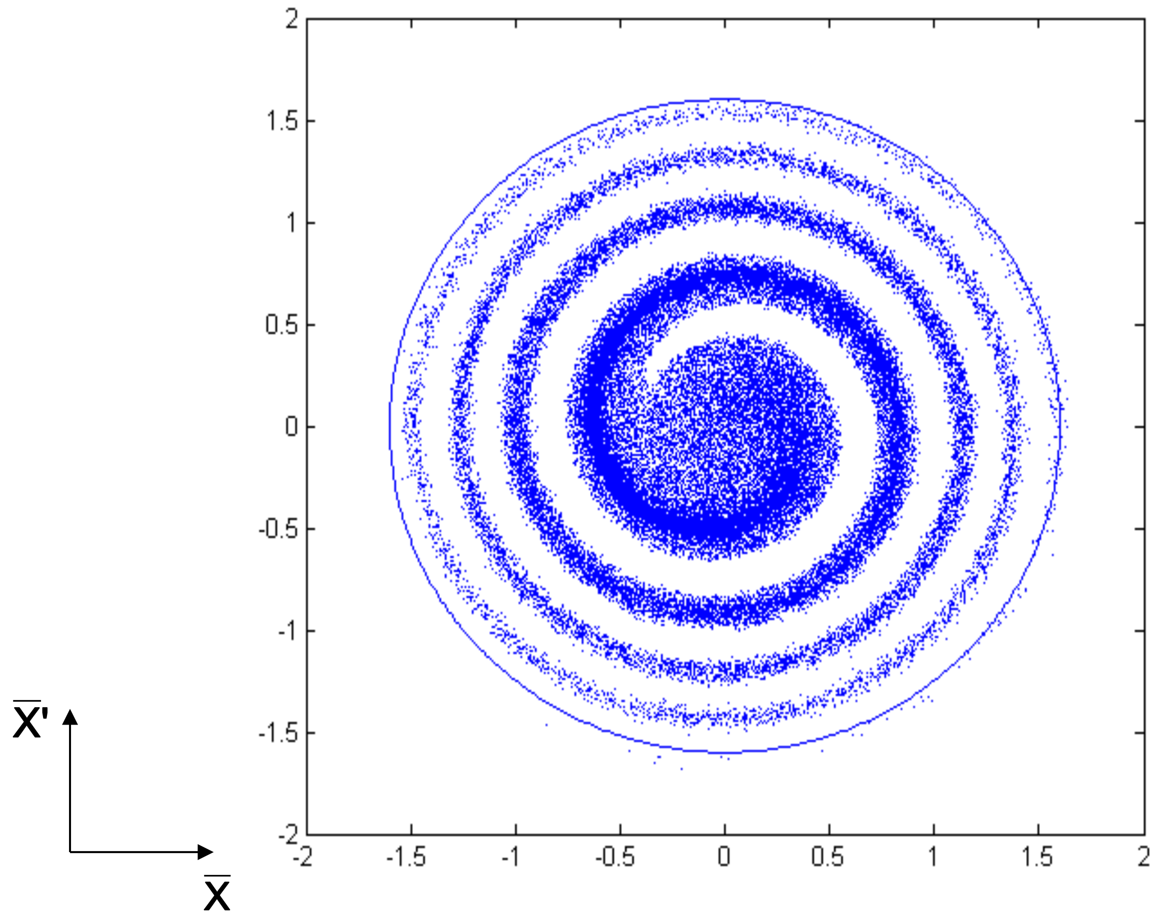

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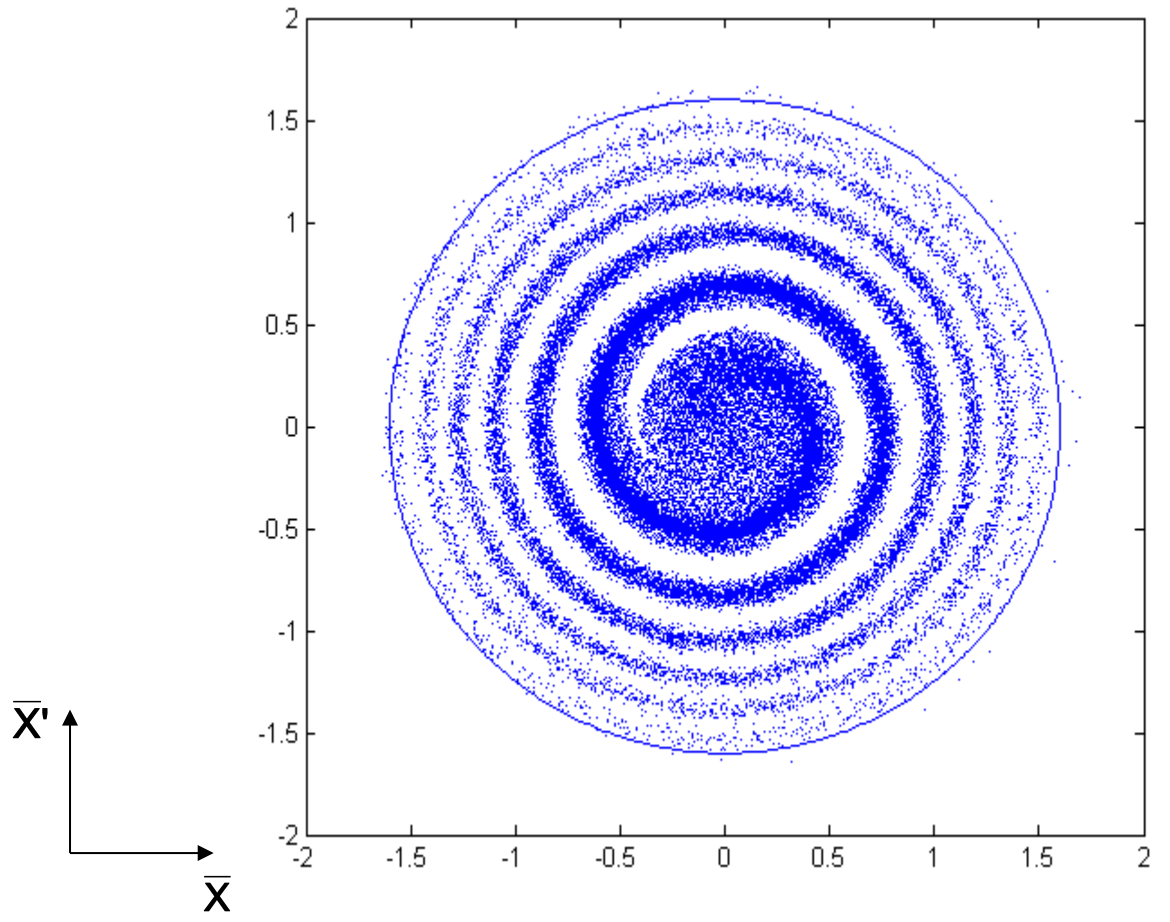

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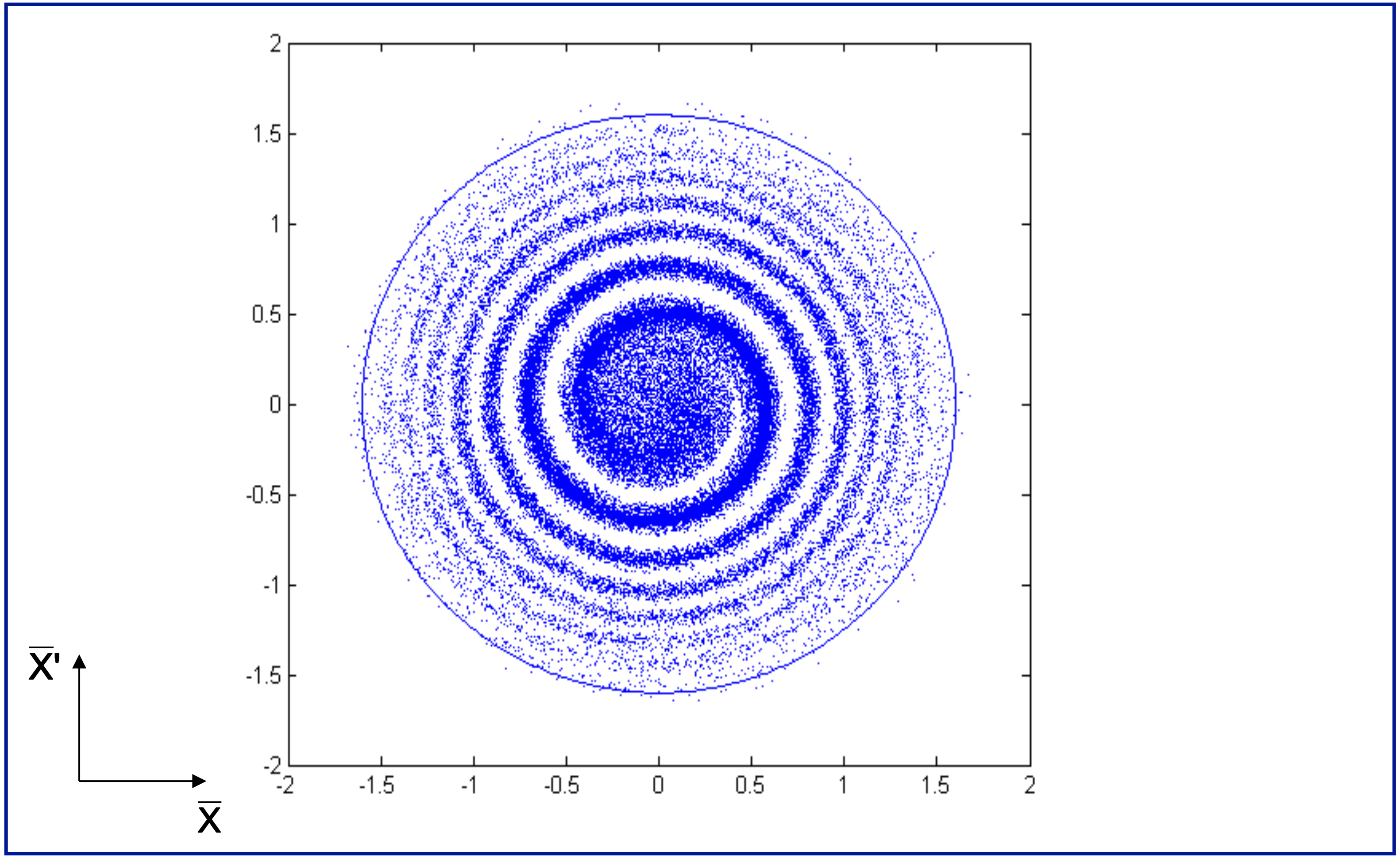

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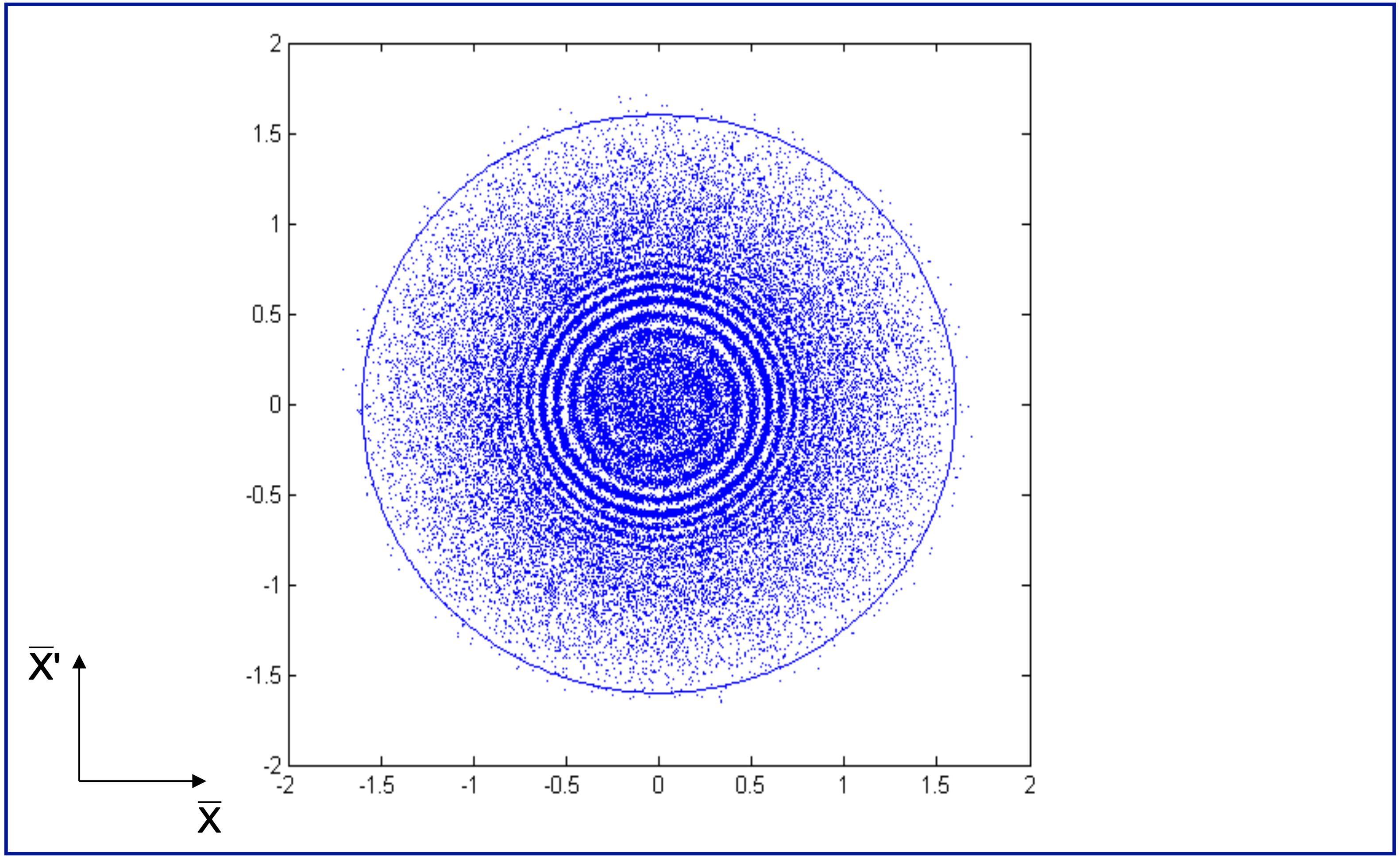

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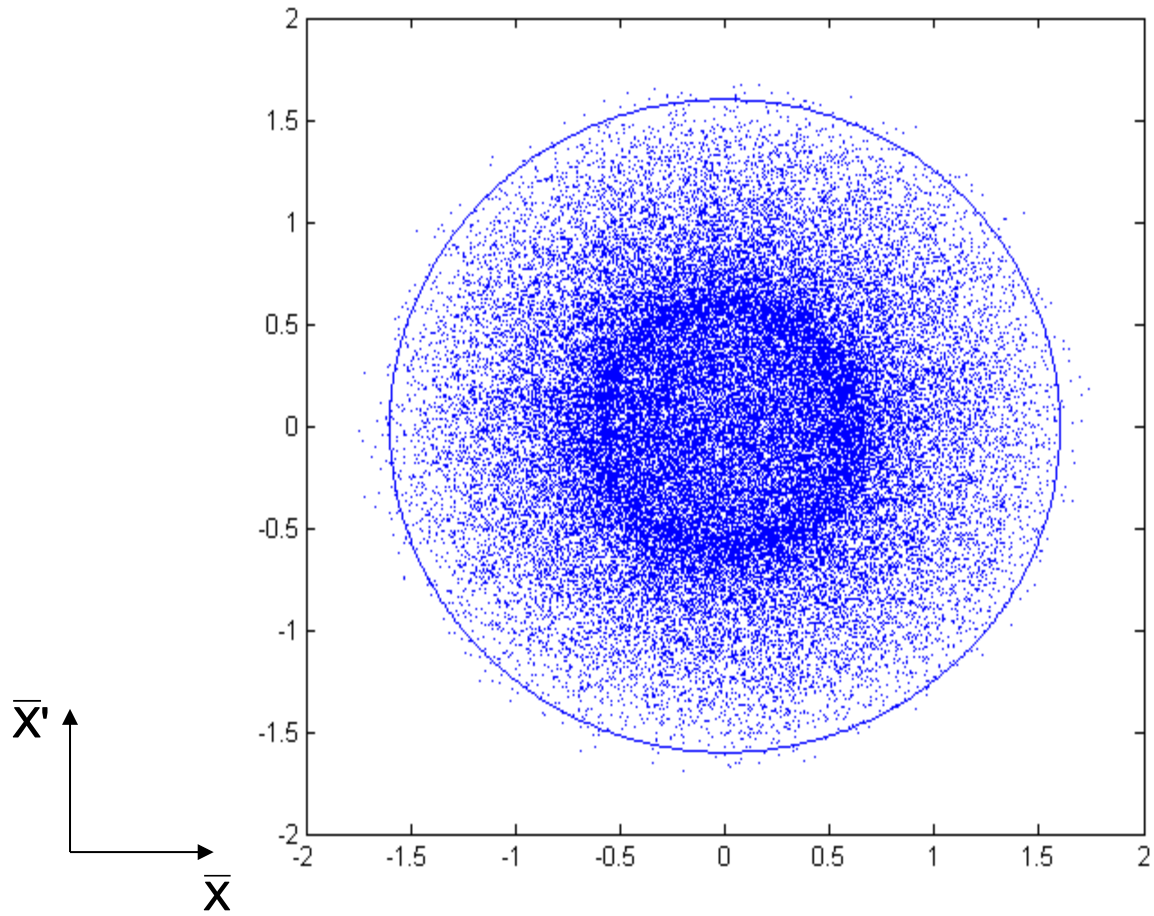
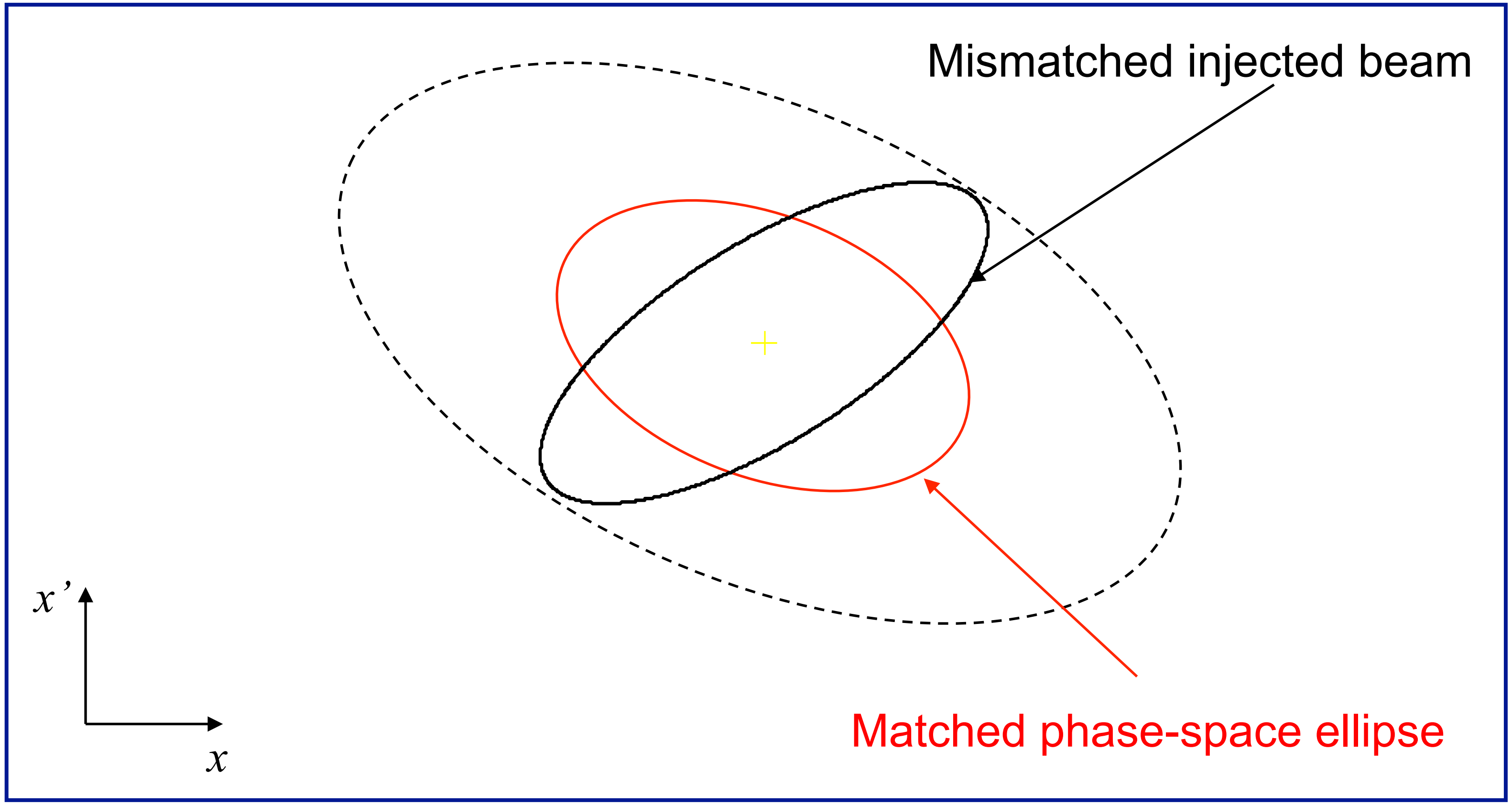


Injection with optical error

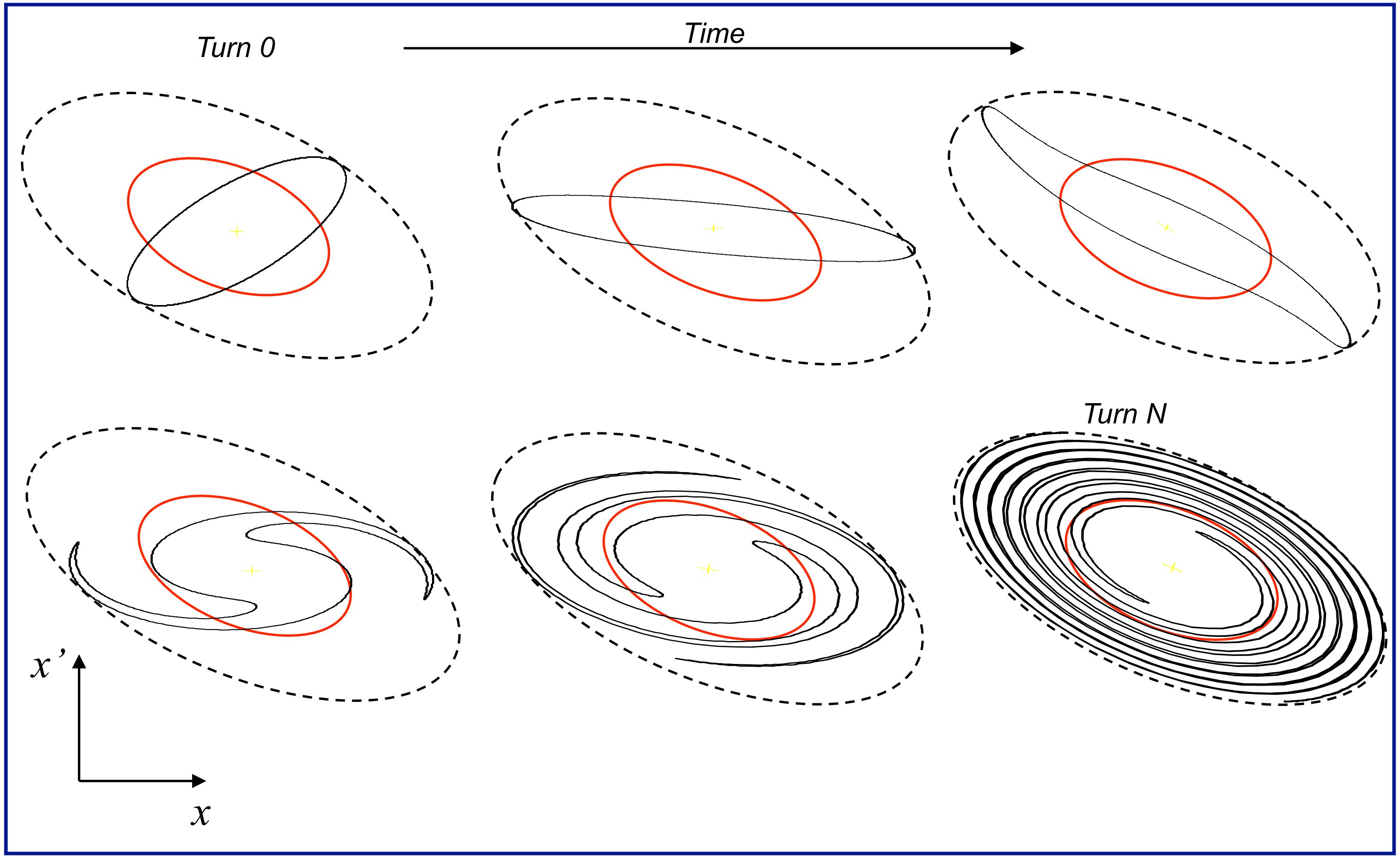
The orientation and shape of the ellipse is defined by the β, α.

- in a circular machine there is only one possible β, α per location.
- in a transfer line up to the injection point: "any" β, α possible.
    - E.g. for the same transfer line magnet settings, different initial β, α will give different β, α at the end



Injection with optical error

Injection with optical error will also lead to emittance blow-up.



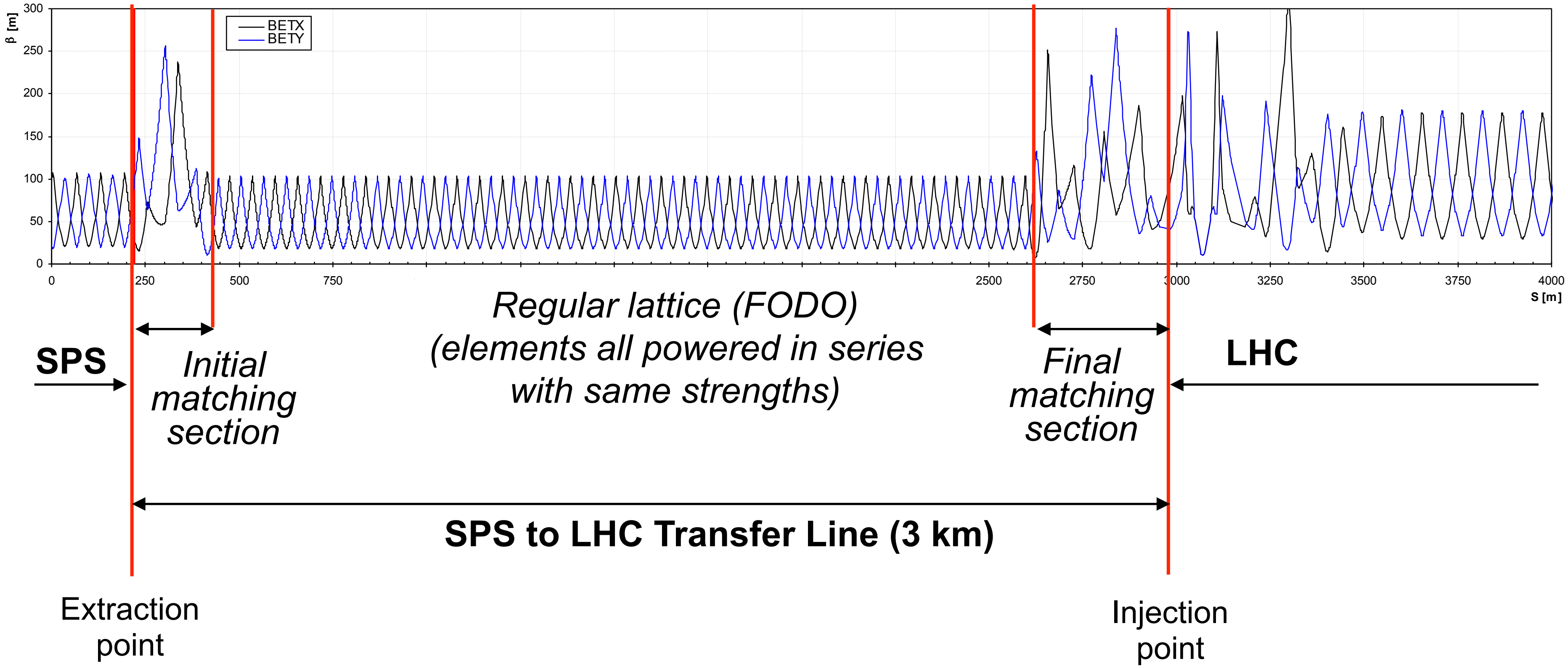
Many other possible errors....

Transfer lines – linking machines

Similar principles as in ring: FODO structure.

Powerful trajectory correction: sufficient number of correctors and BPMs – same rules as in ring.

Individually powered quadrupoles: matching quadrupoles to match to circular machine: β_x, α_x, D_x, D'_x, β_y, α_y, D_y, D'_y.



Injection

There are many different types of injections.

Depending on the beam and the requirements in the circular machine downstream different injection schemes can be implemented:

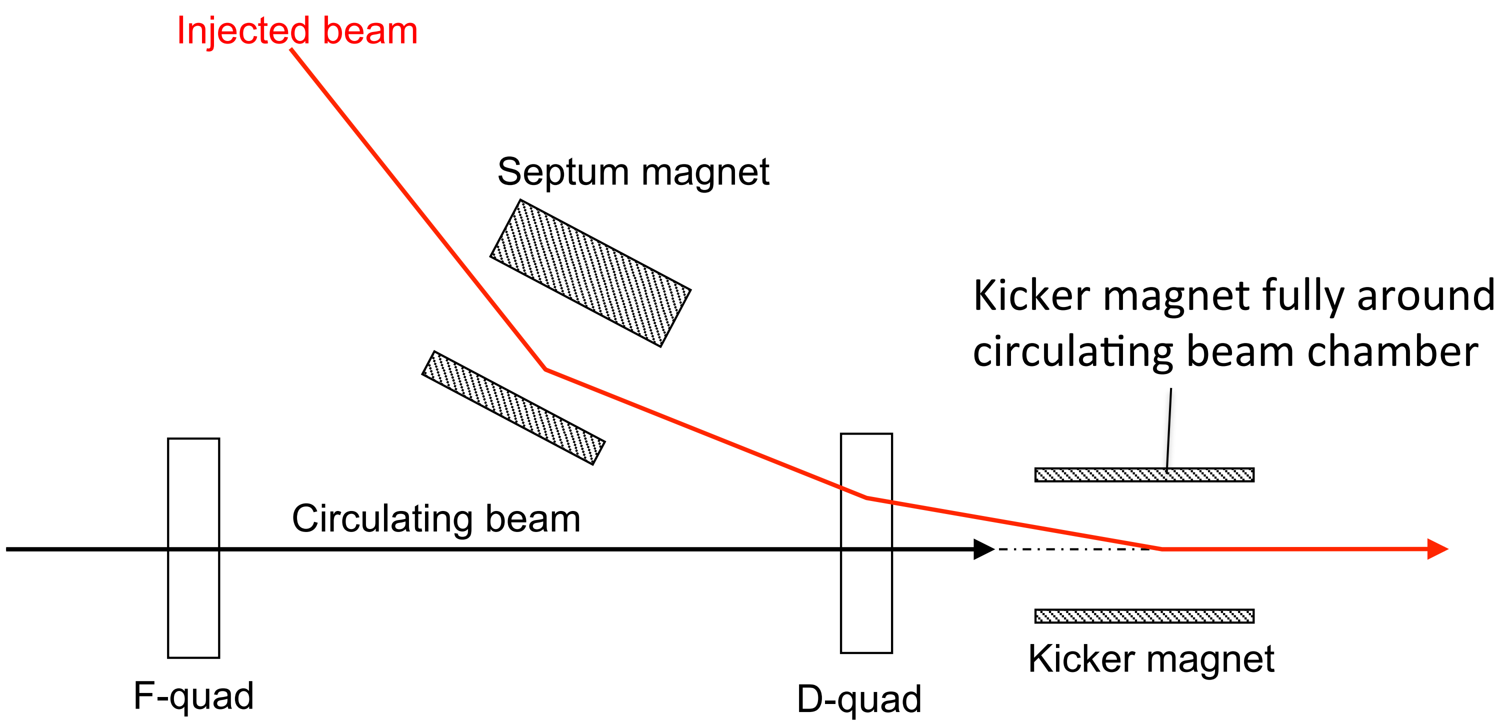
- Fast Injection
- Lepton synchrotron injection
- H^- injection
- Etc.

Injection (and also extraction) involves special equipment.

- Septum magnets
- Fast rising kicker magnets

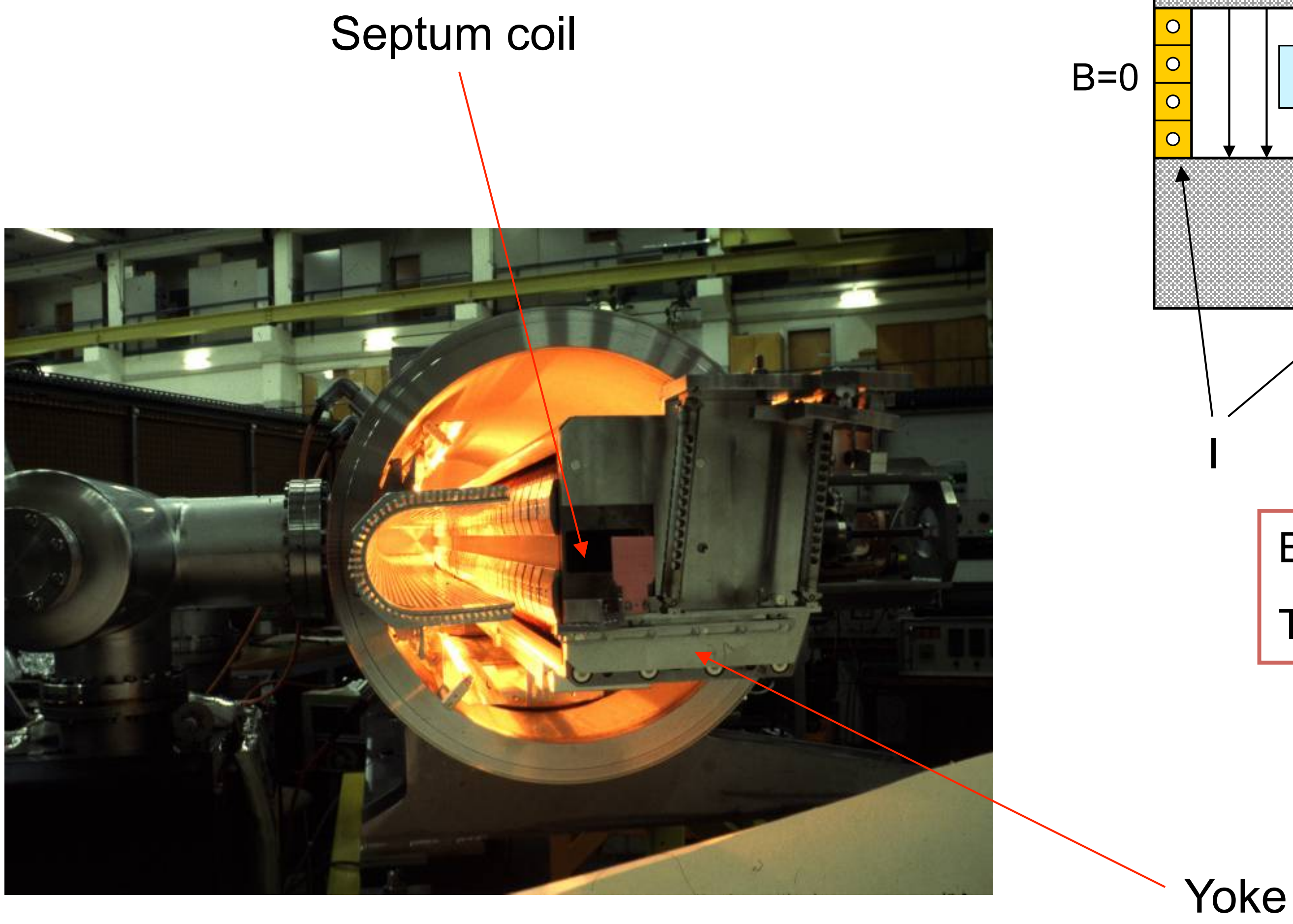
Single turn injection – same plane

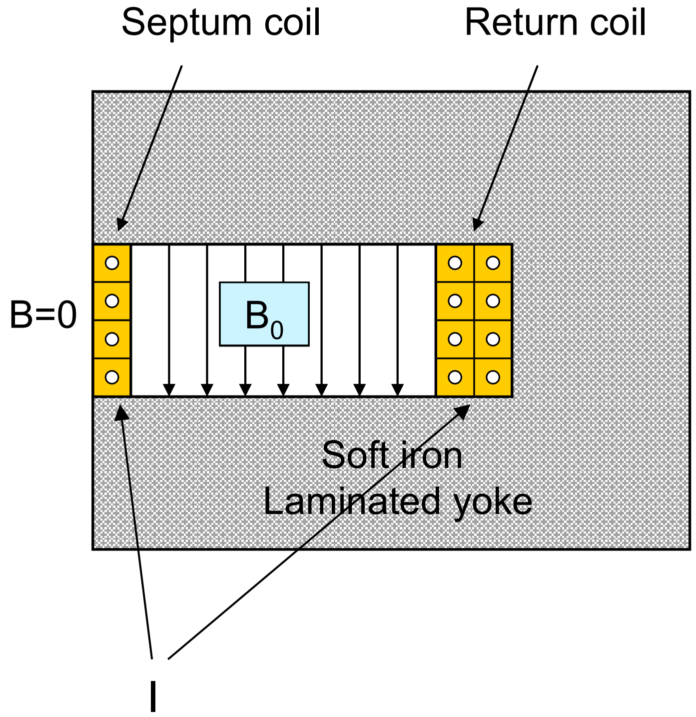
- Septum deflects the beam onto the closed orbit at the centre of the kicker
- Kicker compensates for the remaining angle
- Septum and kicker either side of D quad to minimise kicker strength



Magnetic septum

Pulsed or DC magnet with thin (2-20mm) septum between zero field and high field region





$B_o = \mu_0 I / g$

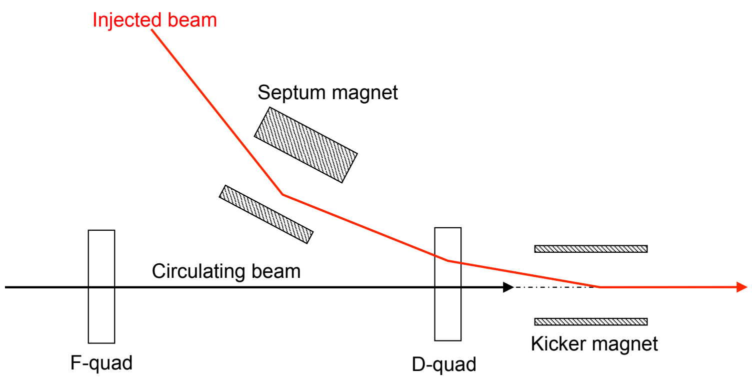
Typically I 5-25 kA

Kicker magnet

Example: LHC is filled from the SPS.

SPS circumference: 23 μs – LHC batch in SPS: ~ 8 μs

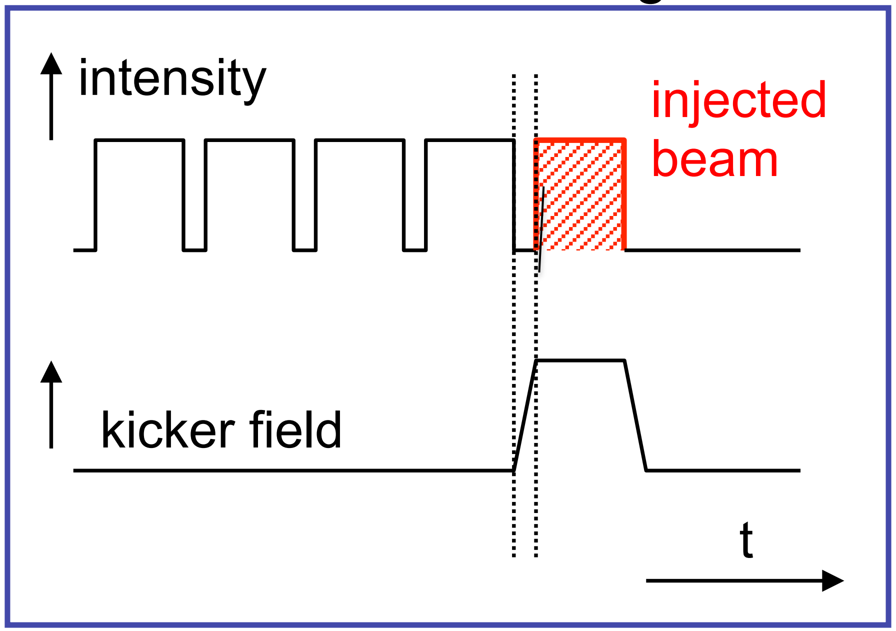
LHC circumference: 89 μs ➔ need several injections from SPS (12)



Pulsed magnet with very fast rise time (100ns – few μs)

LHC case: ~ 1 μs

'boxcar' stacking

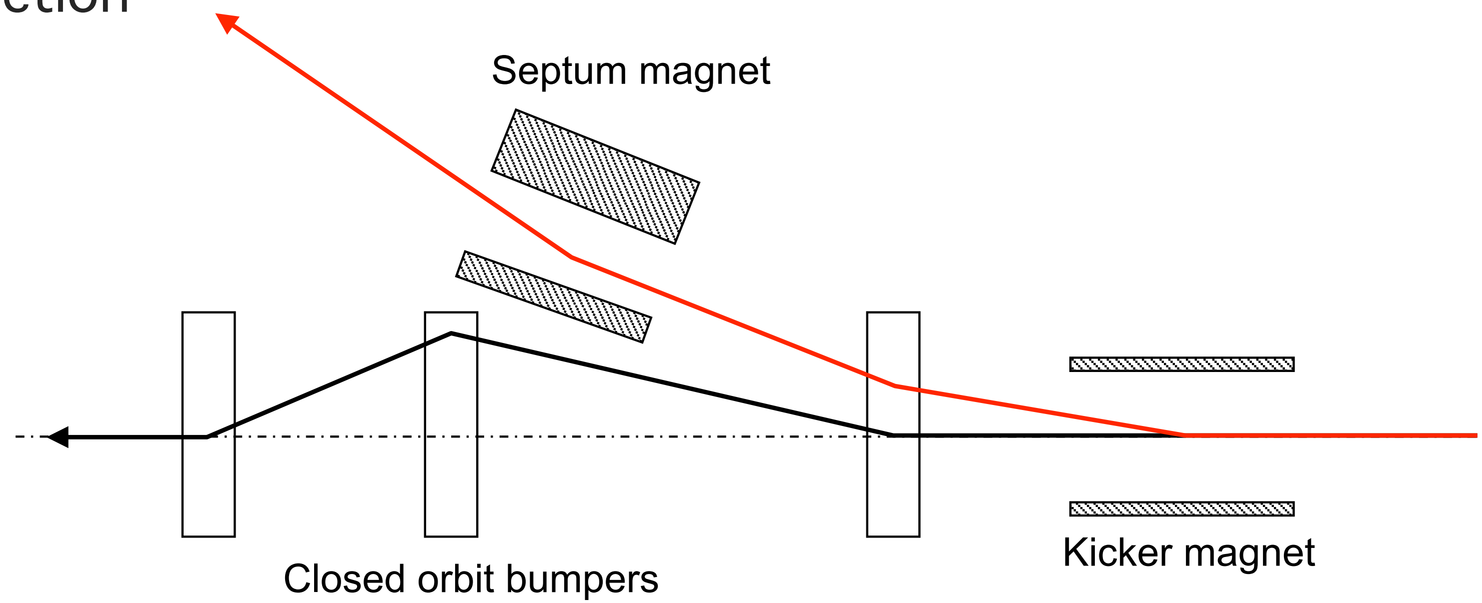


Extraction

Taking the beam out of a circular accelerator.

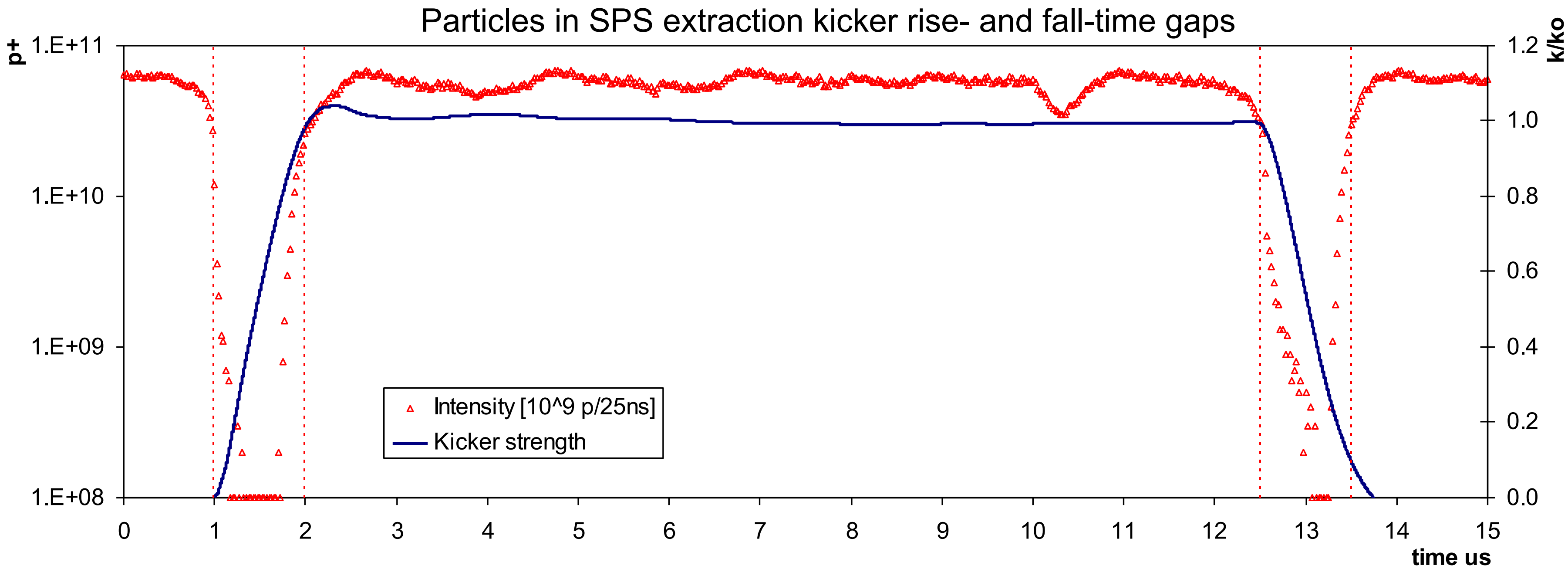
Again many different techniques depending on the requirements.
Some are virtually loss free, others intrinsically create significant particle losses.

For the extraction of the LHC beam from the SPS and the extraction from the LHC on the beam dump:
"Fast extraction"



Kicker magnets - synchronization

If kicker magnets are involved synchronization is very important.



Particles on the falling or rising edge are lost on the vacuum chamber.

In the LHC 3 μs long abort gap for the 3 μs long rise time: no beam is allowed to be injected into abort gap.

What's next?

LHC, LHC Performance, LHC Challenges

APPENDIX

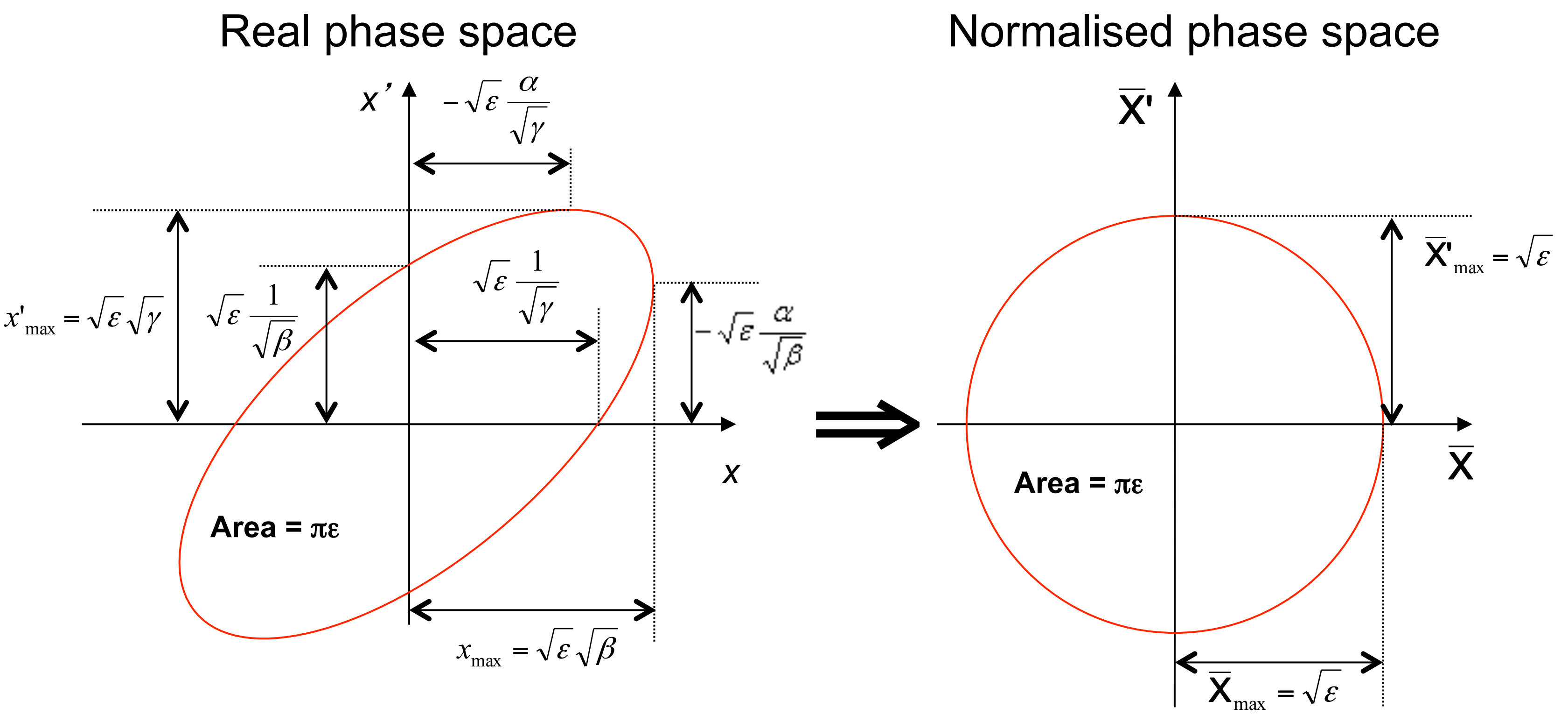
Normalised Phase Space

$$\begin{bmatrix} \bar{X} \\ \bar{X}' \end{bmatrix} = \mathbf{N} \cdot \begin{bmatrix} x \\ x' \end{bmatrix} = \sqrt{\frac{1}{\beta_S}} \cdot \begin{bmatrix} 1 & 0 \\ \alpha_S & \beta_S \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \end{bmatrix}$$

$$\bar{X} = \sqrt{\frac{1}{\beta_S}} \cdot x$$

$$\bar{X}' = \sqrt{\frac{1}{\beta_S}} \cdot \alpha_S x + \sqrt{\beta_S} x'$$

Normalised Phase Space



$$\varepsilon = \gamma \cdot x^2 + 2\alpha \cdot x \cdot x' + \beta \cdot x'^2$$

$$\varepsilon = \overline{X}^2 + \overline{X}'^2$$