Introduction to the SM (4)

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Yesterday

- Feynman diagrams: tool for PT

$$(\Delta E)^2 = E_1^2 - E_2^2 = (m^2 + p_1^2) - (m^2 + p_2^2) = \Delta(p^2) \xrightarrow{NR} \Delta E$$

- In principle we know now how to get from $\mathcal{L}$ to predictions
- Symmertires
  - Rotation in some mathematical spaces
  - We allways need to construct singlets

Today: $SU(N)$, local symmetries and SSB

HW

$$\mathcal{L} = \lambda_1 XYZ + \lambda_2 X^2 Z$$

$$\sigma(XX \to XY)$$

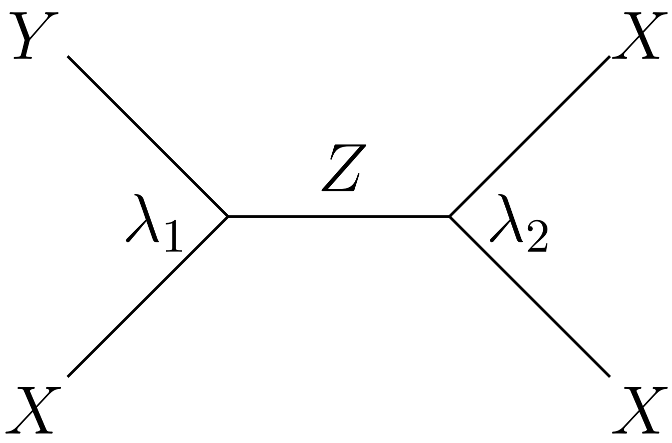
- Energy conservation
- Draw the diagram and estimate the amplitude

HW

$$\mathcal{L} = \lambda_1 XYZ + \lambda_2 X^2 Z$$

$$\sigma(XX \to XY)$$

- Energy conservation $2E_X > m_X + m_Y$ in CM frame
- Draw the diagram and estimate the amplitude



$$\mathcal{A} \propto \frac{\lambda_1 \lambda_2}{m_Z^2 - q^2}$$

$$q_\mu = p_\mu^X + p_\mu^Y$$

Symmetires

$U(1)$

- $U(1)$ is rotation in 1d complex space
- Each field comes with a q that tells us how much it rotates
- When we rotate by an angle θ we have

$$X \to e^{iq\theta} X$$

- Consider $q_X = 1,\ q_Y = 2,\ q_Z = 3$ and write 3rd and 4th order invariants

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$$XX^*YY^* \qquad X^2Y^* \qquad XYZ^* \qquad X^3Z^* \qquad Y^2X^*Z^*$$

$SU(2)$

- $U(2)$ is rotation in 2d complex space. We have $U(2) = SU(2) \times U(1)$
- $SU(2)$ is rotation in 2d complex space or 3d real space
- Rotations in this space are non-Abelian (non-commutative)
- It depends on the representation: scalar, spinor, vector
- Spin in QM is described by SU(2) rotations, so we use the same language to describe it
- A representation is labeled by the number of DOFs it has, like singlet, doublet or triplet
- For the SM all we care is that $2 \times 2 = 1 + 3$ so we know how to generate singlets

$SU(3)$

- $U(3)$ is rotation in 3d complex space. We have $U(3) = SU(3) \times U(1)$
- The representations we care about are singlets, triplets and octets
- Unlike $SU(2)$, in $SU(3)$ we have complex representations, 3 and $\bar{3}$
- The three quarks form a triplet (the three colors)
- To form a singlet we need to know that

$$3 \times \bar{3} = 1 + 8 \qquad 3 \times 3 = \bar{3} + 6$$

- For non-Abelian group the charge is not just a number, but a representation

A game

A game calls “building invariants”

- Symmetry is $SU(3) \times SU(2) \times U(1)$
  - $U(1)$: Add the numbers ($\bar{X}$ had charge $-q$)
  - $SU(2)$: $2 \times 2 = 1 + 3$ and recall that 1 is a singlet
  - $SU(3)$: we need $3 \times 3 = \bar{3} + 6$ and $3 \times \bar{3} = 1 + 8$
- Fields are

$$Q(3,2)_1 \qquad U(3,1)_4 \qquad D(3,1)_{-2} \qquad H(1,2)_3$$

- What 3rd and 4th order invariants can we built?

$$(HH^*)^2 \qquad H^3 \qquad UDD \qquad QUD \qquad HQU^*$$

- HW: Find more invariants

Local symmetires

Local symmetry

Basic idea: rotations depend on x and t

$$\phi(x_\mu) \to e^{iq\theta}\phi(x_\mu) \quad \xrightarrow{local} \quad \phi(x_\mu) \to e^{iq\theta(x_\mu)}\phi(x_\mu)$$

- It is kind of logical and we think that all imposed symmetries in Nature are local
- The kinetic term $|\partial_\mu \phi|^2$ in not invariant
- We want a kinetic term (why?)
- We can save the kinetic term if we add a field that is
  - Massless
  - Spin 1
  - Adjoint representation: $q = 0$ for U(1), triplet for $SU(2)$, and octet for $SU(3)$

Gauge symmetry

- Fermions are called matter fields. What they are and their representation is an input
- Gauge fields are known as force fields

Local symmetries $\Rightarrow$ force fields

Gauge symmetry

- The coupling of the new field is via the kinetic term. Recall classical electromagnetism

$$H = \frac{p^2}{2m} \Rightarrow H = \frac{(p - qA_i)^2}{2m}$$

- In QFT, for a local $U(1)$ symmetry and a field with charge q

$$\partial_\mu \to D_\mu \qquad D_\mu = \partial_\mu + iqA_\mu$$

- We get interaction from the kinetic term

$$|D_\mu \phi|^2 = |\partial_\mu \phi + iqA_\mu \phi|^2 \ni qA\phi^2 + q^2 A^2 \phi^2$$

- The interaction is proportional to q

Accidental symmetries

- We only impose local symmetries
- Yet, because we truncate the expansion, we can get symmetries as output
- They are global, and are called accidental
- Example: $U(1)$ with $X(q=1)$ and $Y(q=-4)$

$$V(XX^*, YY^*) \Rightarrow U(1)_X \times U(1)_y$$

  - X^4Y breaks this symmetry
- In the SM baryon and lepton numbers are accidental symmetries

SSB

Breaking a symmetry

SSB

- By choosing a ground state we break the symmetry
- We choose to expend around a point that does not respect the symmetry
- PT only works when we expand around a minimum

What is the different between a broken symmetry and no symmetry?

SSB

Symmetry is $x \to -x$ and we keep up to x^4

$$f(x) = a^2x^4 - 2b^2x^2 \qquad x_{\rm min} = \pm b/g$$

We choose to expand around $+b/g$ and use $u \to x - b/g$

$$f(x) = 4b^2u^2 + 4bau^3 + a^2u^4$$

- No $u \to -u$ symmetry
- The $x \to -x$ symmetry is hidden
- A general function has 3 parameters $c_2u^2 + c_3u^3 + c_4u^4$
- SSB implies a relation between them

$$c_3^2 = 4c_2c_4$$

SSB and generation of k

Symmetry $x \to -x$ and $y \to iy$ and we do not have y^*

$$V = ax^2 + bx^4 + cxy^2 + dy^4$$

- For $a, b, c, d > 0$, to quadratic order, y is a k–less spring
- For $a < 0$ we expand around a minimum, $x = x_0 + u$ with $x_0^2 = -a/2b > 0$ and $y = 0$
- We get a quadratic term for y

$$c\,xy^2 \to c\,x_0 y^2 + ...$$

- SSB of the symmetry generated a spring from a k–less spring

SSB in QFT

- When we expand the field around a minimum that is not invariant under a symmetry

$$\phi \to v + H$$

- It breaks the symmetries that ϕ is not a singlet under
- Masses to other fields via Yukawa interactions

$$\phi X^2 \to (v + H) X^2 = v X^2 + \ldots$$

- Gauge fields of the broken symmetries also get mass

$$|D_\mu \phi|^2 \ni A^2 \phi^2 \to v^2 A^2$$

Fermions

Fermions fields and power counting

- A fermion kinetic term is more complicated

$$\mathcal{L} \sim \bar{\psi}\partial_\mu \gamma^\mu \psi$$

- Since $\mathcal{L}$ has dimension 4, we see that ψ is dimension $3/2$
- So for fermions when we expand up to 4th order we can have at most two fermion fields
- Under Lorentz, the basic fields are left-handed and right-handed. A mass term must involve both $m\bar{\psi}_L\psi_R$