Introduction to the SM (5)

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Yesterday

- Symmetries: $SU(3)$, $SU(2)$ and $U(1)$
- Lie groups (I added a self-study notes)
- Local symmetires that implies gauge fields
- SSB: Gives relationes between parameters

Today: The SM

HW: Building invariants game

- Symmetry is $SU(3) \times SU(2) \times U(1)$
  - $U(1)$: Add the numbers ($\bar{X}$ had charge $-q$)
  - $SU(2)$: $2 \times 2 \ni 1$ and recall that 1 is a singlet
  - $SU(3)$: we need $3 \times \bar{3} \ni 1$ and $3 \times 3 \times 3 \ni 1$
- Fields are

$$Q(3,2)_1 \qquad U(3,1)_4 \qquad D(3,1)_{-2} \qquad H(1,2)_3$$

- What 3rd and 4th order invariants can we built?

$$(HH^*)^2 \qquad H^3 \qquad UDD \qquad QUD \qquad HQU^*$$

- Find more invariants

$$HQ^*D \qquad QQD$$

Fermions

Fermions fields and power counting

- A fermion kinetic term is more complicated

$$\mathcal{L} \sim \bar{\psi}\partial_\mu\gamma^\mu\psi$$

- Since $\mathcal{L}$ has dimension 4, we see that ψ is dimension $3/2$
- So for fermions when we expand up to 4th order we can have at most two fermion fields
- Under Lorentz, the basic fields are left-handed and right-handed. A mass term must involve both $m\bar{\psi}_L\psi_R$

The SM

The SM

Input: Symmetries and fields

- Symmetry: 4d Poincare and

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

- Fields:
  - 3 copies of QUDLE fermions

$$Q_L(3,2)_{1/6} \quad U_R(3,1)_{2/3} \quad D_R(3,1)_{-1/3}$$
$$L_L(1,2)_{-1/2} \quad E_R(1,1)_{-1}$$

  - One scalar

$$\phi(1,2)_{+1/2}$$

Then Nature is described by

Output: the most general $\mathcal{L}$

$$\mathcal{L} = \mathcal{L}_{kin} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$$

- The model can have SSB

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \quad \Rightarrow \quad SU(2)_L \times U(1)_Y \ \rightarrow \ U(1)_{EM}$$

- This model has a $U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau$ accidental symmetry
- It has 18 parameters, and we measure them all by now
- We then made many tests and the SM basically passes almost all of them

More on the SM

$$\mathcal{L} = \mathcal{L}_{kin} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$$

- The Higgs part gives SSB. 2 parameters (v, m_H)
- Kinetic terms give rise to
  - The universal gauge interactions
  - Gauge bosons masses after SSB
  - 3 parameters, g, g' and g_s
- Yukawa terms: $H\bar{\psi}_L\psi_R$
  - This is where flavor is
  - Give fermion masses and mixing
  - 13 parameters (9 masses, 3 mixing angles and 1 CPV phase)

The gauge interactions

The gauge part

$$SU(3)_C \times SU(2)_L \times U(1)_Y \;\to\; SU(3)_C \times U(1)_{EM}$$

Three parts, each look so different...

- QED - photon interaction: Perturbation theory
- QCD - gluon interaction: Confinement and asymptotic freedom
- Electro-weak: SSB and massive gauge bosons

SSB in the SM

$$-\mathcal{L}_{Higgs} = \lambda\phi^4 - \mu^2\phi^2 = \lambda(\phi^2 - v^2)^2$$

- The minimum is at $|\phi| = v$
- ϕ has 4 DOFs. We can choose

$$\langle\phi_1\rangle = \langle\phi_2\rangle = \langle\phi_4\rangle = 0 \quad \langle\phi_3\rangle = v$$

- It leads to

$$SU(2)_L \times U(1)_Y \ \to \ U(1)_{EM}$$

- We call the remaining symmetry EM. The fact that the vev is for the neutral component is by definition

Implications of the vev

- It breaks the symmetry
- Out of the 4 gauge bosons DOFs, 3 are massive, we call them, $W^{\pm}$ and Z^0. The photon is massless
- The mass come from the kinetic term of the Higgs
- We can tell electrons from the neutrinos (How? They have different couplings to the photon)
- Also the fermions are massive now. Before SSB we could not write a mass term, and now we can

Gauge boson masses

From the kinetic term of the Higgs we get mass for the gauge bosons

$$D^\mu \phi = \left(\partial^\mu + \frac{i}{2} g W_a^\mu \sigma_a + \frac{i}{2} g' B^\mu \right) \phi$$

$$|D^\mu \phi|^2 \sim \frac{1}{8} \left| \begin{pmatrix} gW_3 + g'B & g(W_1 - iW_2) \\ g(W_1 + iW_2) & -gW_3 + g'B \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix} \right|^2$$

which gives for mass terms

$$\frac{1}{4} g^2 v^2 W^+ W^- + \frac{1}{8} v^2 (gW_3 - g'B)^2$$

Masses

Define the mass eigenstates

$$W^\pm = \frac{1}{\sqrt{2}}(W_1 \mp iW_2) \qquad \tan\theta_W \equiv \frac{g'}{g}$$

$$Z = \cos\theta_W W_3 - \sin\theta_W B \qquad A = \sin\theta_W W_3 + \cos\theta_W B$$

- The masses are

$$M_W^2 = \frac{1}{4}g^2 v^2 \qquad M_Z^2 = \frac{1}{4}(g^2 + g'^2)v^2 \qquad M_A^2 = 0$$

- We have a rotation from W_3, B to the mass basis Z, A by an angle θ

Remarks

- $W^\pm$ are charged under EM. A and Z are not
- Using gauge boson interactions we can measure g and g'
- We get the following testable relation

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1 \qquad \tan\theta_W \equiv \frac{g'}{g}$$

- The above is a signal of SSB

Interactions

Charged current interactions

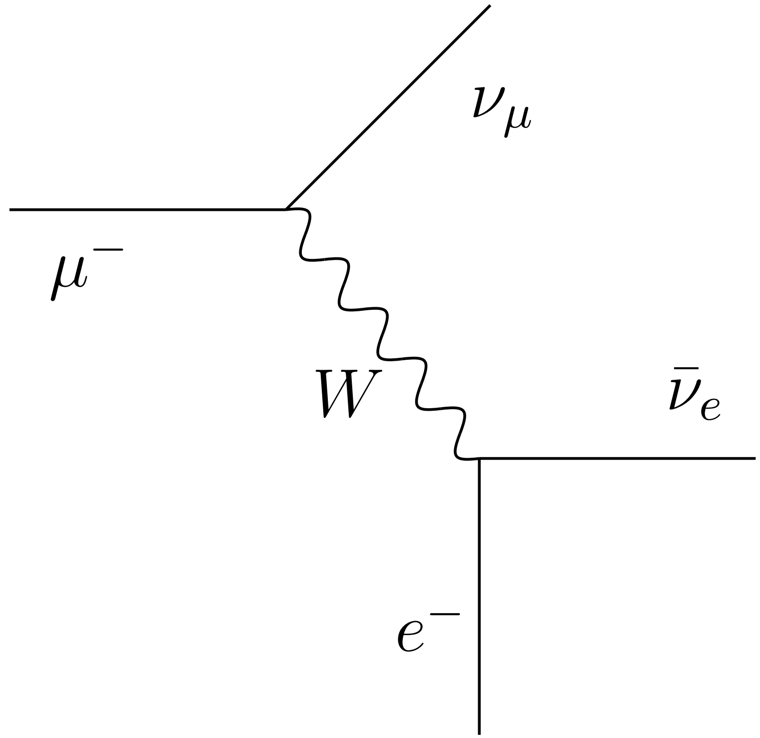
$$-\frac{g}{\sqrt{2}}\,\overline{\nu_{eL}}\,W^{\mu}\gamma_{\mu}e_L^{-}+h.c.$$

- Only left-handed fields take part in charged-current interactions. Therefore the W interaction violate parity
- Universality: the couplings of the W to $\tau\bar{\nu}_\tau$, to $\mu\bar{\nu}_\mu$ and to $e\bar{\nu}_e$ are equal
- At low energy we can “integrate out” the W

$$G_F=\frac{\sqrt{2}g^2}{8M_W^2}=\frac{1}{\sqrt{2}v^2}$$

  - The idea of NR terms!
  - Almost direct measurement of the vev, $V=246\ GeV$

Muon decay



$$\mathcal{A} \sim \frac{g^2}{p^2 - m_W^2} \sim \frac{g^2}{m_W^2} \sim G_F \qquad \Gamma(\mu) \sim G_F^2$$

Neutral currents

$$\mathcal{L}_{\rm int} = \frac{e}{\sin\theta\cos\theta}(T_3 - \sin^2\theta_W Q)\,\bar\psi\gamma^\mu\psi\,Z_\mu + e\,\bar\psi\gamma^\mu\psi\,A_\mu,$$

- $Q = T_3 + Y$
- Photon coupling is parity invariant
- Z couples to both LH and RH fermions but in a parity violating way
- Diagonal couplings. No flavor violation at tree level

Experimental tests

$$\mathcal{L}_{\rm int} = \frac{e}{\sin\theta\cos\theta}(T_3 - \sin^2\theta_W Q)\ \bar\psi\gamma^\mu\psi\, Z_\mu,$$

Of course, the model was built from experimental data

- High energy: Open your pdg and check W and Z decays to leptons. What do you expect to see?
- Z decays to lepton actually measures $\sin^2\theta_W \approx 0.23$ (HW: Calculate $\Gamma(Z\to\nu\bar\nu)/\Gamma(Z\to e^+e^-)$, get $\sin^2\theta_W$ from the data and check the $\rho=1$ prediction)
- Low energy data

Fermions

Lepton masses

- In a chiral theory fermions are massless
- In the SM they get mass from the interactions with the Higgs
- For leptons only the charged leptons get a mass. We need both LH and RH fields for a mass

$$Y_{ij}\left(\bar{L}_L\right)_i \phi\left(E_R\right)_j \to Y_{ij}\left(v+H\right)\bar{e}_L e_R + \ldots$$

- The mass is proportional to the Yukawa coupling and the vev $m_{ij} = Y_{ij} v$
- For leptons we can choose Y to be diagonal in flavor space and we get the known lepton masses

Quarks

$$Y^D_{ij}\left(\bar{Q}_L\right)_i \phi\left(D_R\right)_j + Y^U_{ij}\left(\bar{Q}_L\right)_i \tilde{\phi}\left(U_R\right)_j$$

- The Yukawa matrices, Y^F_{ij}, is a general complex matrix
- After the Higss acquires a vev, the Yukawa terms give masses to the fermions. Also, after the breaking we can talk about U_L and D_L, not about Q_L
- If Y is not diagonal, flavor is not conserved (soon we will go over the subtleties here)
- If Y carries a phase, CP is violated. C and P is violated to start with

The CKM matrix

It is all about moving between bases...

- We can diagonalize the Yukawa matrices

$$Y_{diag} = V_L Y V_R^\dagger, \qquad V_L, V_R \text{ are unitary}$$

- The mass basis is defined as the one with Y diagonal, and this is when

$$(d_L)_i \to (V_L)_{ij}(d_L)_j, \qquad (d_R)_i \to (V_R)_{ij}(d_R)_j$$

- The couplings to the photon is not modifies by this rotation

$$\mathcal{L}_\gamma \sim \bar{d}_i \delta_{ij} d_i \;\to\; \bar{d}_i V \delta_{ij} V^\dagger d \sim \bar{d}_i \delta_{ij} d_i$$

CKM, W couplings

- For the W the rotation to the mass basis is important

$$\mathcal{L}_W \sim \bar{u}_L^i \delta_{ij} d_L^i \;\to\; \bar{u}_i V_L^U \delta_{ij} V_L^{D\dagger} d \sim \bar{u}_i V_{CKM} d_i$$

  where

$$V_{CKM} = V_L^U V_L^{D\dagger}$$

- The point is that we cannot have Y_U, Y_D and the couplings to the W diagonal at the same basis
- In the mass basis the W interaction change flavor, that is flavor is not conserved

FCNCs

FCNC

FCNC=Flavor Changing Neutral Current

- Very important concept in flavor physics
- Important: Diagonal couplings vs universal couplings

FCNCs

In the SM there are no FCNCs at tree level. Very nice! In Nature FCNC are highly suppressed

- Historically, $K \to \mu\nu$ vs $K_L \to \mu\mu$
- The suppression was also seen in charm and B
- In the SM we have four neutral bosons, g, γ, Z, h. Their couplings are diagonal
- The reasons why they are diagonal, and what it takes to have FCNC, is not always trivial
- Of course we have FCNC at one loop (two charged current interactions give a neutral one)

Photon and gluon tree level FCNC

- For exact gauge interactions the couplings are always diagonal. It is part of the kinetic term

$$\partial_\mu \delta_{ij} \to (\partial_\mu + iqA_\mu)\delta_{ij}$$

- Symmetries are nice...
- In any extension of the SM the photon couplings are flavor diagonal

Higgs tree level FCNC

- The Higgs is a possible source of FCNC. With one Higgs doublet, the mass matrix is align with the Yukawa

$$\mathcal{L}_m \sim Y\, v\, \bar{d}_L d_R \qquad \mathcal{L}_{int} \sim Y H \bar{d}_L d_R$$

- With two doublets we have tree level FCNC

$$\mathcal{L}_m \sim \bar{d}_L (Y_1 v_1 + Y_2 v_2) d_R \qquad \mathcal{L}_{int} \sim H_1 \bar{d}_L Y_1 d_R$$

- There are “ways” to avoid it, by imposing extra symmetries

Z exchange FCNC

- For broken gauge symmetry there is no FCNC when: “All the fields with the same irreps in the unbroken symmetry also have the same irreps in the broken part”
- In the SM the Z coupling is diagonal since all $q = -1/3$ RH quarks are $(3,1)_{-1/3}$ under $SU(2) \times U(1)$
- What we have in the couplings is

$$\bar{d}_i \, (T_3)_{ij} \, d_j \;\to\; \bar{d} V \, (T_3)_{ij} \, V^\dagger d_j, \qquad V T_3 V^\dagger \propto I \text{ if } T_3 \propto I$$

- Adding quarks of different irreps generate tree level FCNC Z couplings
- It is the same for new neutral gauge bosons (usually denoted by Z')