!*eoretical Concepts in Pa*"*icle Physics*

Lecture II : From gauge theories to Standard Model observables

Géraldine SERVANT

ICREA@IFAE-Barcelona

2 main ingredients of Quantum Field Theory: Quantum Mechanics and Special Relativity

Invariance under Poincaré transformations (Lorentz transformations+translations)

-The different types of fields (scalar spin-0, vector spin-1 and spinor spin1/2) are different representations of Lorentz transformations

- symmetry -> conserved quantity

relativistic wave equation not enough, only able to describe a single particle

We need a formalism to describe processes in which number of particles change

◆ For fully consistent description, we need to reinterpret the field as a field operator which can destroy or create particles: : we need to quantize fields -> Quantum Field Theory

Classical Field theory

a system is described by $\ S=$

 Ω

classical mechanics & lagrangian formalism

action principle determines classical trajectory:

a system is described by
$$
S = \int dt \mathcal{L}(q, \dot{q})
$$
position momentum

z
Z

 $\delta S=0$ --> Euler-Lagrange equations

$$
\frac{\partial \mathcal{L}}{\partial q_i} - \frac{\partial}{\partial t} \frac{\partial \mathcal{L}}{\partial \dot{q}_i} = 0
$$

conjugate momenta
$$
p_i = \frac{\partial L}{\partial \dot{q}_i}
$$
 hamiltonian $H(p, q) = \sum_i p_i \dot{q}_i - \mathcal{L}$
extend lagrangian formalism
to dynamics of fields
$$
S = \int d^4x \mathcal{L}(\varphi, \partial_\mu \varphi) \qquad \qquad \partial_\mu = \frac{\partial}{\partial x^\mu}
$$

$$
\delta S = 0 \quad \rightarrow \quad \frac{\partial \mathcal{L}}{\partial \varphi_i} - \partial_\mu \frac{\partial \mathcal{L}}{\partial (\partial_\mu \varphi_i)} = 0 \qquad \qquad \partial_0 = \frac{\partial}{\partial x^0} = \frac{\partial}{\partial t}
$$

3 $H(x) = \sum$ *i* $\Pi_i(x)\partial_0\varphi_i(x) - \mathcal{L}$ $\Pi_i =$ @*L* $\partial(\partial_0\varphi_i)$ conjugate momenta $\Pi_i = \frac{1}{\sqrt{2}}$ hamiltonian

Classical Field theory and Noether theorem

Invariance of action under continuous global transformation

There is a conserved current/charge

$$
\partial_{\mu}j^{\mu} = 0 \qquad Q = \int d^{3}x j^{0}(x, t)
$$

example of transformation:

$$
\varphi \to \varphi e^{i\alpha} \quad \ (*)
$$

if small increment $\alpha \ll 1 \;\; \varphi \rightarrow \varphi + i \alpha \varphi$

$$
\delta \varphi = i \alpha \varphi
$$

\n
$$
\delta \varphi' = i \alpha \varphi'
$$

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$$
\delta \varphi' = i \alpha \varphi'
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\delta \varphi' = i \alpha \varphi'
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\delta \varphi' = i \alpha \varphi'
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\delta \varphi' = i \alpha \varphi'
$$

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$$
\frac{\partial \varphi}{\partial \varphi} \varphi + \frac{\partial \mathcal{L}}{\partial \varphi'} \varphi' \bigg) \bigg\{ \frac{\partial \varphi}{\partial x} (\varphi \frac{\partial \mathcal{L}}{\partial \varphi'}) = 0 \bigg\}
$$

\nEuler-Lagrange equations:
$$
\frac{\partial \varphi}{\partial x} (\frac{\partial \mathcal{L}}{\partial \varphi'}) - \frac{\partial \mathcal{L}}{\partial \varphi} = 0 \bigg\}
$$

\n
$$
\equiv J
$$

conserved current

Scalar Field theory

Lorentz invariant action of a complex scalar field

$$
S=\int d^4x(\partial_{\mu}\varphi^*\partial^{\mu}\varphi-m^2\varphi^*\varphi)
$$

Euler-Lagrange equation leads to Klein-Gordon equation

$$
(\Box + m^2)\varphi = 0
$$

with solution a superposition of plane waves:

$$
\varphi(x) = \int \frac{d^3p}{(2\pi^3)\sqrt{2E_p}} (a_p e^{-ipx} + b_p^* e^{ipx})
$$

existence of a global U(1)
$$
\varphi(x) \to e^{i\theta} \varphi(x)
$$

symmetry of the action

 $Q_{U(1)} = \int d^3x j_0$ $j_\mu = i\varphi^* \overleftrightarrow{\partial}_\mu \varphi$ z
Zanada
Zanada conserved U(1) charge $\; Q_{U(1)} = \; \int d^3x j_0$

From first to second quantization

Basic Principle of Quantum Mechanics:

To quantize a classical system with coordinates q^{i} and momenta p^{i} , we promote q^i and p^i to operators and we impose $\left[\mathsf{q}^\mathsf{i}$, $\mathsf{p}^\mathsf{j}\right]$ = δ^{ij}

same principle can be applied to scalar field theory

$$
\text{where } \mathsf{q}^\mathsf{i}(\mathsf{t}) \text{ are replaced by } \; \varphi\bigl(t,x\bigr) \\ \text{ and } \mathsf{p}^\mathsf{i}(\mathsf{t}) \text{ are replaced by } \; \Pi(t,x)
$$

 \mathscr{L} and \prod are promoted to operators and we impose $[\varphi(t,x),\Pi(t,y)]=i\delta^3(x-y)$

Expand the complex field in plane waves:

$$
\varphi(x) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} (a_p e^{-ipx} + b_p^{\dagger} e^{ipx})
$$

scalar field theory is a collection of harmonic oscillators

destruction operator $\left(\begin{array}{l} a_p | 0 > = 0 \end{array} \right)$ $\;$ defines the \mid 0>

where a_p and b_p^+ are promoted to operators

 $[a_p, a_q^{\dagger}] = (2\pi^3)\delta^{(3)}(p-q) = [b_p, b_q^{\dagger}]$

 $p_1 \ldots a_{p_n}^{\intercal} |0>$

a generic state is obtained by acting on $\;\;|p_1...p_n>\equiv a_p^{\dagger}$ the vacuum with the creation operators

Scalar field quantization continued

$$
\mathcal{H}=\Pi\partial_0\varphi-\mathcal{L}=\int\frac{d^3p}{(2\pi)^3}\frac{E_p}{2}(a_p^\dagger a_p+b_p^\dagger b_p)
$$

the quanta of a complex scalar field are given by two different particle species with same mass created by a^+ and b^+ respectively

The Klein Gordon action has a conserved U(1) charge due to invariance $\varphi(x) \to e^{i\theta} \varphi(x)$

$$
Q_{U(1)} = \int d^3x j^0 = \int \frac{d^3p}{(2\pi)^3} (a_p^{\dagger} a_p \bigodot b_p^{\dagger} b_p)
$$

2 different kinds of quanta: each particle has
its antiparticle which has the same mass but
opposite U(1) charge

Field quantization provides a proper interpretation of "E<0 solutions"

$$
\varphi(x) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} (a_p e^{-ipx} + b_p^{\dagger} e^{ipx})
$$

coefficient of the positive energy solution e^{-ipx} becomes after quantization the destruction operator of a particle while the coefficient of the e^{ipx} becomes the creation operator of its antiparticle

 $a_{p}^{+}|0\rangle$ and $b_{p}^{+}|0\rangle$ represent particles with opposite charges

Spinor fields Ψ

The electromagnetic field . *A^µ ^L* ⁼ ¹ 4 *Fµ*⌫*F ^µ*⌫ *^Fµ*⌫ ⁼ @*µA*⌫ @⌫*A^µ* Maxwell eq. Lorentz inv. lagrangian where @*µFµ*⌫ = 0 Maxwell lagrangian inv. under *A^µ* ! *A^µ* @*µ*✓ Lorentz invariant lagrangian Dirac equation *^L* ⁼ ¯ (*i*@ *^m*) (*i*@ *m*) = 0 @ = *^µ*@*^µ { a*(*x, t*)*, † ^b*(*y, t*)*}* ⁼ (3)(*^x ^y*)*ab* anticommutation relations fermions:

◆ Kinetic term of actions are derived from requirement of Poincaré invariance

- ◆ Promote field & its conjugate to operators and impose (anti) commutation relation
- \blacklozenge Expanding field in plane waves, coefficients $a_{\rm p,}$ $a_{\rm p}^+$ become operators
- \blacklozenge The space of states describes multiparticle states

 a_p destroys a particle with momentum p while a_p creates it +

$$
\mathbf{e}.\mathbf{g}~|p_1\ldots p_n> \equiv a_{p_1}^\dagger \ldots a_{p_n}^\dagger|0>
$$

crucial aspect of QFT: transition amplitudes between different states describe processes in which the number and type of particles changes

Gauge transformation and the Dirac action

Consider the transformation it is a symmetry of the free Dirac action if θ is constant no longer a symmetry if $\;\;\theta=\theta(x)$ $\Psi \rightarrow e^{iq\theta}\Psi$ $\Psi \rightarrow e^{iq\theta}$ However, the following action is invariant under $A_{\mu} \rightarrow A_{\mu} - \partial_{\mu} \theta$ *{* $\mathcal{L} = \Psi(i\gamma^\mu \partial_\mu - m)\Psi$ where $[D_{\mu}\Psi = (\partial_{\mu} + iqA_{\mu})\Psi]$ covariant derivative $\mathcal{L} = \bar{\Psi}(i\gamma^{\mu}D_{\mu} - m)\Psi$ We have gauged a global U(1) symmetry, promoting it to a local symmetry **The result is a gauge theory and** A_μ is the gauge field conserved current: $j^\mu = \Psi \gamma^\mu \Psi$ conserved charge: z $d^3x\bar\Psi\gamma^0\Psi=$ z $d^3 x \Psi^\dagger \Psi \quad \rightarrow$ electric charge U(1) transformation

Electrodynamics of a spinor field

$$
\mathcal{L}=\bar{\Psi}(i\gamma^{\mu}D_{\mu}-m)\Psi\;\text{where}\;\;[\!D_{\mu}\Psi=(\partial_{\mu}+iqA_{\mu})\Psi]\!\;
$$

$$
\mathcal{L} = \bar{\Psi}(i\gamma^{\mu}\partial_{\mu} - m)\Psi - qA_{\mu}\bar{\Psi}\gamma^{\mu}\Psi
$$

Coupling of the gauge field \overline{A}_{μ} to the current $j^{\mu} = \bar{\Psi} \gamma^{\mu} \Psi$

11

From Quantum Electrodynamics to the electroweak theory

These transformations are elements of U(1) group

$$
\Psi \to e^{iq\theta}\Psi
$$

In the electroweak theory , more complicated transformations, belonging to the SU(2) group are involved

$$
\Psi \to \exp(ig~\tau.\lambda)\Psi
$$

where $\tau = (\tau_1, \tau_2, \tau_3)$ are three 2*2 matrices

Generalization to SU(N)

 N^2 -1 generators (N×N matrices)

$$
\Psi(x) \to U(x)\Psi(x)
$$

\n
$$
U(x) = e^{ig\theta^a(x)T^a}
$$

\n
$$
A_{\mu}(x) \to UA_{\mu}U^{\dagger} - \frac{i}{g}(\partial_{\mu}U)U^{\dagger}
$$

Gauge theories: Electromagnetism (EM) & Yang-Mills

EM U(1)
$$
\phi \rightarrow e^{i\alpha}\phi
$$
 but $\partial_{\mu}\phi \rightarrow e^{i\alpha}(\partial_{\mu}\phi) + i(\partial_{\mu}\alpha)\phi$
\n
$$
\phi \rightarrow e^{i\alpha}\phi
$$
 but $\partial_{\mu}\phi + ieA_{\mu}\phi \rightarrow e^{i\alpha}(\partial_{\mu}\phi + ieA_{\mu}\phi)$
\n
$$
\begin{array}{c}\n\text{in field and covariant derivative} \\ \n\text{the EM field keep track of the phase in} \\ \n\text{different points of the space-time} \\ \n\text{Hermite points of the space-time} \\ \n\text{Vang-Mills : non-abelian transformations} \\ \n\phi \rightarrow U\phi \\ \n\phi + igA_{\mu}\phi \rightarrow U(\partial_{\mu}\phi + igA_{\mu}\phi) \\ \n\vdots \\ \n\phi \rightarrow U\phi \\ \n\phi + igA_{\mu}\phi \rightarrow U(\partial_{\mu}\phi + igA_{\mu}\phi) \\ \n\vdots \\ \n\phi \rightarrow U\phi \\ \n\phi + igA_{\mu}\phi \rightarrow U(\partial_{\mu}\phi + igA_{\mu}\phi) \\ \n\vdots \\ \n\phi \rightarrow \psi A_{\mu} \\ \n\vdots \\ \n\phi \rightarrow \psi A_{\mu} \\ \n\phi
$$

• Quark-gluon vertex

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The Standard Model: matter

the elementary blocks:

The Standard Model : interactions

Interactions between particles

Elementary particles interact with each other by exchanging gauge bosons

The beauty of the SM comes from the identification of a unique dynamical principle describing interactions that seem so different from each others erince in the red
cribing interaction;
trom each others

 \vdots gauge theory = spin-1:

The most general lagrangian given the particle content

$$
\mathcal{L} = -\frac{1}{4g^{\prime 2}} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4g^2} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4g^2} G^a_{\mu\nu} G^{a\mu\nu}
$$

+ $\bar{Q}_i i \mathcal{P} Q_i + \bar{u}_i i \mathcal{P} u_i + \bar{d}_i i \mathcal{P} d_i + \bar{L}_i i \mathcal{P} L_i + \bar{e}_i i \mathcal{P} e_i$
+ $Y_u^{ij} \bar{Q}_i u_j \tilde{H} + Y_d^{ij} \bar{Q}_i d_j H + Y_l^{ij} \bar{L}_i e_j H + |D_\mu H|^2$
- $\lambda (H^{\dagger} H)^2 + \lambda v^2 H^{\dagger} H + \frac{\theta}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma}$

What about baryon and lepton numbers? -> accidental symmetries!

Abelian versus non-abelian gauge theories

The (Yang-Mills) action
$$
\mathcal{L}_{YM} = \bar{\Psi}(i\gamma^{\mu}D_{\mu} - m)\Psi - \frac{1}{2}F_{\mu\nu}F^{\mu\nu}
$$
 is invariant under
\n
$$
\{\Psi(x) \to U(x)\Psi(x)\}
$$

Abelian U(1) symmetry

 $A_\mu(x) \rightarrow A_\mu +$

Non-abelian SU(N)

$$
U(x) = e^{iq\theta(x)} \qquad \qquad U(x) = e^{ig\theta^a(x)T^a}
$$

 T^a : N²-1 generators (N×N matrices) acting on jeriel
I

$$
A_{\mu}(x) = A_{\mu}^{a} T^{a}
$$

$$
A_{\mu}(x) \rightarrow UA_{\mu}U^{\dagger} - \frac{i}{\Theta}(\partial_{\mu}U)U^{\dagger}
$$

Coupling constants

φ
φ Ψ Ψ A_μ

infinitesimal i rans pormation

$$
\mu \quad \text{infinitesimal} \quad \quad U(x) = 1 + i g \theta^a(x) T^a + \mathcal{O}(\theta^2)
$$

$$
A_{\mu}^{a}(x) \longrightarrow A_{\mu}^{a} + \partial_{\mu}\theta^{a} - gf^{abc}\theta^{b}A_{\mu}^{c}
$$

$$
D_{\mu}\Psi = (\partial_{\mu} + iqA_{\mu}) \qquad D_{\mu}\Psi = (\partial_{\mu} - igA_{\mu}^{a}T^{a})
$$

 \mathbf{w}

i

The gauge symmetries of the Standard Model [∂]^µ [−] igW^a $\frac{1}{2}$ $||$ The gay $\frac{1}{2}$ T_{max} if a T_{max} T_{max} if above T_{max} ∂µUU [−]¹ δG^a The gauge symmetries of τ is the second column is for infinitesimal transformations. We consider the second transformations on τ can verify the gauge symmetries of the ϵ the Klein-Gordon equation. Ine gauge symmetries of the Standard Model The gauge symmetries of the Standard Model

To do actual calculations it is very important to have all the Feynman rules with consistent

 $\mathcal{L}_{\mathcal{A}}$ transformation is group is

q = −iTaaaq = −iTaaaq

!

"

completely anti-symmetric tensor in 3 dimensions. The covariant derivative for any field

and the fields transform as

 \mathbb{R}^n transforming is, \mathbb{R}^n

 \mathcal{L}_{max} transforming non-trivially under this group is,

Gauge Group $U(1)_Y$ $\psi' = e^{+iY\alpha_Y}\psi,$ \mathcal{D} or \mathcal{D} or \mathcal{D} $\begin{bmatrix} 1 & \cdots & \cdots & 1 \\ 0 & \cdots & \cdots & 0 \\ 0 & \cdots & \cdots & 0 \\ 0 & \cdots & \cdots & 0 \end{bmatrix}$ μ is the μ is the different sign convention between sign convention between sign convention between μ Gauge $\operatorname{Group} \, SU(2)_L$ acts on the two components of a doublet $\,\,\Psi_L^{\vphantom{\dagger}}$ =(u_L,d_L) or ($\Psi_L \rightarrow e^{-iT^a\alpha^a}i\eta_L \qquad I\gamma - e^{-iT^a\alpha^a} \qquad T^a = \sigma^a/2$ \overline{u} is useful to write the covariant derivative in terms of the mass eigenstates \overline{u} Zµ. These are defined by the relations, \mathcal{L} = \mathcal{L} $D_\mu \psi_L = \left(\partial_\mu - i\,g\,W_\mu^a\right)$ $D_\mu \psi_L = (\partial_\mu - i \, q \, W_\mu^a T^a) \, \psi_L$ μ μ , ν ¹For this to be consistent one must also have, under hypercharge transformations, for a field of hyper- $U^a a^a$ $U = e^{-iT^a \alpha^a}$ $T^b = i$ f^{abc} T is interactions. It would have been possible to have been possible to have a minus signification \vert \rightarrow $G^a T^a$ $U G^a T^a T^{-1}$ \hat{i} $U T^{-1}$ \hat{j} $U T^{-1}$ \hat{j} $G_{\mu}^{a}T^{a} \rightarrow U G_{\mu}^{a}T^{a}U^{-1} - \frac{i}{c}\partial_{\mu}UU^{-1}$ $\frac{1}{\text{Gauge Group } U(1)_Y}$ (abelian) ϵ $B'_{\mu} = B_{\mu} - \frac{\partial}{g'}\partial_{\mu}\alpha_Y$ $B_{\mu\nu}=\partial_\mu B_\nu-\partial_\nu B_\mu$ D_μ EL^{\top} is to $\forall L$ $W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g \epsilon^{abc} W_\mu^b W_\nu^c, \quad a=1,\ldots,3$ Zµ. These are defined by the relations, ^µ = Z^µ cos θ^W − A^µ sin θ^W Gauge Group $SU($ $\frac{1}{2}$ expressed to $\frac{1}{2}$ e^{-iL} α q $U=e^{-iL}$ $\alpha \rightarrow e^{-i T^a \alpha^a}$ $I = e^{-i T^a \alpha^a}$ $\left[T^a \right]$ $G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g f^{abc} G^b_\mu G^c_\nu, \quad a=1,\ldots,8$ λ_4 = $D_{\mu}q = (\partial_{\mu} - i q G_{\mu}^a T^a) q$ $\begin{array}{ccc} \n\hline \n\end{array}$ Gauge Group U(1) $D_\mu \psi_R = \left(\partial_\mu + i\,g'\,Y\,B_\mu\right)\psi_R$ ψ_R $\mathbb{E}(\mathbf{T}^a \cdot \mathbf{a})$ γ μ . (Decree γ) is the set of α Zµ. These are defined by the relations, μ expansion μ ω_μ – \imath g vv $_\mu$ 1 $\left(\begin{array}{c} 1 \\ 1 \end{array} \right)$ $\, {\bf p} \,\, SU(3)_{\bm c} \,$ $q = (q_1, q_2, q_3)$ \overline{a} (\overline{b} \overline{c}) (the three color decrees of \overline{b}) $\mathcal{J}_{\mathcal{J}}$ μ is the hypercharge transformation in Eq. (D.11) maintaining the similar ¹For this to be consistent one must also have, under hypercharge transformations, for a field of hyper^µ ⁼ ^B^µ [−] ¹ $\mathcal{L}_{\mu} = \mathcal{L}_{\mu}$ is given possible to the given possible to have a minus sign possible to have a of the hypercharge transformation in $\mathcal{L}_{\mathcal{F}}$ maintaining the similar t $1+\mathrm{i} Y \alpha x$, under hypercharge transformations, for a field of hypercharge transformations, for a field of hyper- $B'_\mu = B_\mu - \frac{1}{g'} \partial_\mu \alpha_Y$ $D_{\mu\nu} - O_{\mu}D_{\nu} - O_{\nu}D_{\mu}$ $Gauge Group U(1)_Y$ (abelian) $\Psi_L \rightarrow e^{-i T^a \alpha^a} \psi_L \hspace{0.5cm} U = e^{-i T^a \alpha^a}$ $1, \ldots$ $\frac{1}{\sqrt{2\pi}}$ $\begin{array}{ll} \textbf{Gauge Group} \ U(1)_Y & \textbf{(abelian)} \ \end{array}$ ^µT^a [→] UG^a $\frac{1}{2}$ \mathbb{R}^l is for infinitesimal transformations on infinitesimal transformations on \mathbb{R}^l $D_{\mu} = D_{\mu} - \frac{1}{q'} \partial_{\mu} \alpha Y$ ν γ μ ο ο ο $\mathcal{L} = \mathcal{L} \times \mathcal{L} \times \mathcal{L}$ (D.7), 3 $W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g \epsilon^{abc} W^b_\mu W^c_\nu, \quad a = 1, \ldots, 3 \qquad \qquad \sigma_1 = \left(\begin{array}{cc} \circ & 1 \ 1 & 0 \end{array} \right), \sigma_2$ completely anti-symmetric tensor in 3 dimensions. The covariant derivative for any field α $D_\mu \psi_L = \left(\partial_\mu - i\,g\,W_\mu^a T^a \right) \psi_L$ Gauge Group $SU(3)_c$ In this case the group is abelian and we have $\lambda_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\lambda_2 = \begin{pmatrix} i & 0 \\ 0 & 0 \end{pmatrix}$ $G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + gf^{abc} G^b_\mu G^c_\nu, \quad a = 1, ..., 8$ $\lambda_4 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$ ω_{μ} is the field of the field. Notice the different sign convention between ω_{μ} \mathcal{L} to the T3 + Y . (Define the T3 + Y . (Define the T3 + Y . (Define the T3 + Y .) $\begin{split} \psi \; &\; - \, e \;\; \psi \,, \ B_\mu' &\; = B_\mu - \frac{1}{\epsilon} \partial_\mu \alpha_Y \,. \end{split}$ $e^{i\theta} = e^{\pm i \theta}$ $\psi,$ y $\overline{M}_{I} \rightarrow e^{-i T^a \alpha^a}$ _a $\overline{M}_{I} = e^{-i T^a \alpha^a}$ $\mu \nu$ and μ in μ in μ in μ in μ $-i T^a \alpha^a$ $\qquad \qquad \tau$ $-i T^a \alpha^a$ $\qquad \qquad$ $\qquad \qquad$ \overline{a} μ the group is abelian and we have the group is absolute the group is absolute that μ \mathbf{r} the covariant derivative given by \mathbf{r} $D_\mu q = \left(\partial_\mu - i\,g\,G_\mu^a T^a\right)q$ μ / 1 g can verify that the field is $iY\alpha_{Y,s}$. $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu.$ \overline{F} for the fundamental representation of SU(2)L we have Ta \overline{F} $\mathbf{r} \mathbf{r}^a$ and $\mathbf{r} \mathbf{r}^a$ in $\mathbf{r} \mathbf{r}^a$ dimensions. The covariant derivative for any field \mathbf{r} $W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g \epsilon^{abc} W^b_\mu W^c_\nu, \quad a = 1, \dots, 3 \qquad \qquad \sigma_1 = \left(\begin{array}{cc} 0 & 1 \ 1 & 0 \end{array} \right), \sigma_2$ ψ_L $a \rightarrow e^{-iT^a}$ $T^a \rightarrow U G^a_{\mu} T^a U^{-1} - \frac{i}{\partial \mu} U U^{-1}$ $(d$ belian) = $\frac{1}{2}$ Taga(Duq) (D.6) (D. $\begin{array}{cc} \varphi & -e & \varphi, \end{array}$ $\Psi_L \rightarrow e^{-\imath\,T^{\,\alpha}}\psi_L \hspace{0.5cm} U = e^{-\imath T^a \alpha^a} \hspace{0.5cm} T^a = \sigma^a/2 \hspace{1cm}$ Pauli mat completely anti-symmetric tensor in $\binom{3}{2}$ dimensions. The covariant derivative for any field $\binom{3}{2}$ $q \to e^{-\epsilon x} q$ $U = e$ $G^a_\mu T^a$) q $\left(\begin{array}{ccc} 1 & 0 & 0 \end{array} \right)$ $\left(\begin{array}{ccc} i & 0 \end{array} \right)$ $\lambda_7 = \begin{bmatrix} 0 & 0 & -i \end{bmatrix}$ $\lambda_8 =$ $\Psi_L \rightarrow e^{-i T^a \alpha^a} \psi_L$ $U = e^{-i T^a \alpha^a}$ $T^a = \sigma^a/2$ Pauli matrices of Eq. 5.3, known as Clifford algebra, then every solution $\mathcal{L}(\mathcal{L})$ of the Dirac Dir equation, will also be a solution of the Klein-Gordon equation. π ² α /2 **P**uli matrices and the theory $\frac{1}{2}$ $\sigma_1 =$ $\left(\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array}\right), \sigma_2 = -i$ $\left(\begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array}\right)$ $, \sigma_3 =$ $(1 \ 0)$ $0 -1$ $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}, \sigma_2 = -i \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \sigma_4 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ Gauge Group $SU(3)_c$ a=(q1,q2,q3) (the three color degrees of freedom) $(2, 2)$ $(1, 2)$ T^a , T^b = $i f^{abc} T^c$ (3.3) ben-manimalities matrices is, $\begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ For example, σ2 $\mathcal{I} = \mathcal{I}$ $\left(\begin{array}{cccc} 0 & 0 & 1 \end{array} \right)$ $\left(\begin{array}{cccc} 0 & 0 & -i \end{array} \right)$ $\begin{pmatrix} i & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & 1 & 0 \end{pmatrix}$ $\lambda_7 = \left(\begin{array}{ccc} 0 & 0 & 0 \ 0 & 0 & -i \end{array} \right)$ Gauge Group δU conventions. We give here those that are important for building the SM. We will separate $\tilde{H} = -i T^a \alpha^a$ and $\tilde{H} = -i T^a \alpha^a$ are $\begin{bmatrix} \overline{H}a & \overline{H}b \end{bmatrix}$ and the covariant defined the covariant derivatives. $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ y $\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\Delta \mu q = (\sigma \mu - \nu q) \sigma \mu$. $\frac{1}{1 + \sum_{i=1}^{n} \sum_{j=1}^{n} (1 - \sum_{j=1}^{n} \sum_{j=1}^{n}$ Gauge Group $SU(2)_L$ acts on the two components of a doublet U conventions. In this Appendix we will give the complete Feynman rules for the Standard $\Psi_I \rightarrow e$ conventions. We give here that are important for building the SM. We will separate the SM. We will separate th $q\rightarrow e^{-i T^a \alpha^a} q \qquad \ \ U = e^{-i T^a \alpha^a} \qquad \left[T^a,T^b \right] = i f^{abc} T^c \qquad \ \ \, \textbf{(3×3) }$ Gell-Man m $G^a_\mu T^a \to U G^a_\mu T^a U^{-1} - \frac{1}{c} \partial_\mu U U^{-1}$ $\lambda_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$ $D_\mu q = \left(\partial_\mu - i\,g\,G_\mu^a T^a\right)q$ To do actual calculations it is very important to have all the Feynman rules with consistent $Gauge Group U(1)_Y$ (abelian)
 $A^+ B^-$ Model in the general R^ξ gauge. \overline{D} a Gauge Group $SU(2)_L$ acts on the two components of a doublet Ψ_L =(u_L,d_L) or (ν _L,e_L) L $\frac{1}{\sqrt{2}}$ $\nabla \psi$ and Ta are the group. The group of α (quark) field α (quark) field α $q \hspace{2.5cm} (1 \hspace{.1cm} 0)$ $\lambda_7 =$ Gauge Group $SU(3)_c$ $G_{\mu}^{a}T^{a} \rightarrow U G_{\mu}^{a}T^{a}U^{-1} - \frac{i}{g}\partial_{\mu}U U^{-1}$ $\begin{array}{ccccc}\n\mu&&&&g\end{array}$, where $\begin{array}{ccccc}\n\mu&&&&g\end{array}$, where $\begin{array}{ccccc}\n\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2}$ $\lambda_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\lambda_2 = \begin{pmatrix} i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\lambda_3 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\mathcal{D}_{\alpha} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ $\alpha = \mu \mathcal{I}$ (since the field itself, $\alpha = \mu \mathcal{I}$) and field its like the field itself, $\alpha = \mu \mathcal{I}$ $\left(\begin{array}{cccc} 0 & i & 0 \end{array} \right)$ $\left(\begin{array}{cccc} 0 & 0 & -2 \end{array} \right)$ 374 APPENDIX D. FEYNMAN RULES FOR THE STANDARD MODEL $\partial_\mu U U^{-1}$ ^µ ⁼ [−]¹ $\lambda_1 = \left(\begin{array}{ccc} 0 & 1 & 0 \\ 1 & 0 & 0 \end{array} \right) \qquad \lambda_2 = \left(\begin{array}{ccc} 0 & 0 & 0 \\ i & 0 & 0 \end{array} \right)$ can verify that the covariant derivative transformation $\lambda_4 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\int q$ µG^c δ \overline{U} $\sigma^a_{\mu} + q \epsilon^{abc} W^b_{\mu} W^c_{\nu}, \quad a = 1, \dots, 3$ $D_\mu \psi_L = \left(\partial_\mu - i\,g\,W_\mu^a T^a\right)\psi_L$ Model in the general R^ξ gauge. $\mathcal{L} \mathcal{W},$ Θ \overline{B} \overline{A} \overline{B} conventions. We give here that are important for building that are important for building the SM. We will separate the SM. $t\left(\mathcal{O}_{\mu}+i\,g\right|\,Y\,\,B_{\mu}\right)\,\psi_{R}\,$ $\partial_\mu W_\nu^a - \partial_\nu V_\nu$ $\left[T^a,T^b\right]=if^{abc}T^c$ (3×3) Gell-Man matrices $\lambda_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\lambda_2 = \begin{pmatrix} i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\lambda_3 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\partial_{\mu}G_{\nu}^{a} - \partial_{\nu}G_{\mu}^{a} + gf^{abc}G_{\mu}^{b}G_{\nu}^{c}, \quad a = 1, \ldots, 8$ $\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} i & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$ In QCD the quarks are in the fundamental representation and T^a = λa/2 where λ^a are $\begin{array}{ccc} \n\text{F} & \text{I} & \$ $\left\lceil \begin{smallmatrix} \tau^a & \tau^b \end{smallmatrix} \right\rceil = {}_i\,f^{abc} \tau^c$ (3×3) Gell-Man matrices $\sqrt{ }$ $\overline{}$ 0 1 0 100 $0 \quad 0 \quad 0$ 1 $\lambda_2 =$ $\sqrt{ }$ $\overline{}$ $0 \t -i \t 0$ $i \qquad 0 \qquad 0$ $0 \qquad 0 \qquad 0$ 1 $\lambda_3 =$ $\sqrt{ }$ $\overline{ }$ 100 $0 \t -1 \t 0$ $0 \qquad 0 \qquad 0$ $\lambda_4 =$ $\sqrt{2}$ $\overline{}$ 0 0 1 $0\qquad 0\qquad 0$ 100 1 $\lambda_5 =$ $\sqrt{2}$ $\overline{ }$ $0 \t 0 \t -i$ 00 0 $i \quad 0 \quad 0$ 1 $\lambda_6 =$ $\sqrt{2}$ $\overline{ }$ $0\qquad 0\qquad 0$ 0 0 1 $0 \quad 1 \quad 0$ $\lambda_7 =$ $\sqrt{2}$ $\overline{ }$ 00 0 $0 \t 0 \t -i$ $0 \quad i \quad 0$ 1 $\lambda_8 = \frac{1}{\sqrt{25}}$ $\overline{\overline{3}}$ $\sqrt{2}$ $\overline{}$ 10 0 $0 \quad 1 \quad 0$ $0 \t 0 \t -2$ 1 $\overline{}$

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The gauge symmetries of the Standard Model <u>no gaago op</u> ψ^L (D.8) τ_{ho} course dimensions. The covariant derivative for any field τ **I** − igWa <u>___</u> μ_μ (D.8), του καταστάστηκαν στην προσπάθηση στην προσπάθηση στην προσπάθηση στην προσπάθηση στην προσπάθηση στην πρ ne gauge symmetries of the $\frac{1}{2}$ auge symmetries of the Standard Model 374 APPENDIX D. FEYNMAN RULES FOR THE STANDARD MODEL STAN $\sqrt{\frac{1}{2}}$ The gauge D.2.1 Gauge Group SU(3)^c Here the important conventions are for the field strengths and the covariant derivatives.

. (D.13)

Gc

Gauge Group $U(1)_Y$ $\psi'=e^{+iY\alpha_Y}\psi,$ u_B ν u_B ν u_B ν u_B \overline{D} or \overline{D} or \overline{D} \overline{a} μ is the μ is the field. Notice the different sign convention between sign convention between μ \mathbf{G} auge \mathbf{G} roup $SU(2)_r$ $\mathcal{L} = \mathcal{L} \mathcal$ $\mathcal{L} \rightarrow \mathcal{C}$ ψ_L $\psi = e$ only $W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g \epsilon^{abc} W^b_\mu W^c_\nu, \quad a = 1, \dots, 3$ $D_\mu \psi_L = \left(\partial_\mu - i\, g\, W_\mu^a T^a \right) \psi_L$ ¹For this to be consistent one must also have, under hypercharge transformations, for a field of hyper- \mathbf{u} ge Group $\mathcal{D}\mathcal{C}(9)_c$ g: 2000 | 2011 | 2011 | 2011 | 2011 | 2011 | 2011 | 2011 | 2011 | 2011 | 2011 | 2011 | 2011 | 2011 | 2011 | 20
2011 | 2011 | 2011 | 2011 | 2011 | 2011 | 2011 | 2011 | 2011 | 2011 | 2011 | 2011 | 2011 | 2011 | 2011 | 2011 $\sigma \rightarrow e^{-i\Gamma(\alpha\alpha)}\sigma$ is the ghost interactions. It would have a minus significant when σ in Eq. (D.10), with a definition \mathcal{L} and \mathcal{L} with independent sign in the exponent of exponent \mathcal{L} α^a α^a α^a α^a α^a α^a α^a α^a α^b α^b α^b α^b $\lim_{L \to \infty} G$ roup $U(1)_V$ $(\text{ch}_2 \mid \text{in})$ $B'_{\mu} = B_{\mu} - \frac{\partial}{g'}\partial_{\mu}\alpha_Y$ $B_{\mu\nu} = \partial_{\mu}B_{\nu} - \partial_{\nu}B_{\mu}$ $D_{\mu}\psi_R = \left(\partial_{\mu} + i\,g'\,Y\,B_{\mu}\right)\psi_R$ \mathbf{F} σ is the field of the field of the different sign convention between $-iT^a \alpha^a$ $\Psi_L \rightarrow e^{-i T^a \alpha^a} \psi_L \quad \quad U = e^{-i T^a \alpha^a}$ $W^a = \partial W^a - \partial W^a + \alpha \epsilon^{abc} W^b W^c$ $a = 1$ 3 Γ is useful to Γ is useful to Γ $D_\mu \psi$ \overline{G} G roup $SU(3)_c$ $-iT^a\alpha^a_{\ \ \ \alpha}$ $II = e^{-iT^a\alpha}$ ^µ ⁼ ^B^µ [−] ¹ ^g! [∂]µα^Y . (D.11) $\sigma_{\mu\nu} - \sigma_{\mu}\sigma_{\nu}$ $\sigma_{\nu}\sigma_{\mu} + g_{J}$ $\sigma_{\mu}\sigma_{\nu}$, $\alpha = 1, \ldots, \sigma$ in Eq. (D.10), with a definition $\mathcal{L} = \mathcal{L} \mathcal{L}$ with a definition by the exponent the exponent the exponent $D_{\mu}q = (\partial_{\mu} - i\,g\,G^a_{\mu}T^a)\,q$ Gauge Group U(1) ψ_R Gauge Group $56(2)$ $W_{\mu\nu} = O_{\mu}W_{\nu} - O_{\nu}W_{\mu} + g\epsilon \quad W_{\mu}W_{\nu}, \quad a = 1, ...$ \overline{a} $C₂u$ go $C₂u$ go $SI₁(3)$ ^µ sin θ^W + B^µ cos θ^W ¹For this to be consistent one must also have, under hypercharge transformations, for a field of hyper- $\begin{array}{ccc} \text{A} & \text{B} & \text{C} & \text{A} \\ \text{C}^0 & \text{D}^0 & \text{D}^0 & \text{D}^1 & \text{D}^1 & \text{D} \\ \end{array}$ \mathcal{L} $G^{\alpha}_{\mu}T^{\alpha} \to U G^{\alpha}_{\mu}T^{\alpha}U^{-1} - \frac{\partial}{\partial \mu}U U^{-1}$ \mathcal{O} $G^a_{\mu\nu} = \partial_\mu G^a_{\nu} - \partial_\nu G^a_{\nu} + af^{abc} G^b_{\nu} G^c_{\nu}, \quad a = 1.$ ^µ ⁼ ^B^µ [−] ¹ $\mathcal{L}_{\mu} = \mathcal{L}_{\mu}$ is interactions. It would have been possible to have a minus significant possible to have a minus significant value of $\mathsf{U}(1)$ of the hypercharge transformation in Eq. (D.11) maintaining the similar the s $\overline{U(1)}$ (skeliau) $1-\frac{1}{2}iY_{OSE}$ $\begin{aligned} \varphi \, &= \, c & \varphi \,, \ \beta'_\mu = B_\mu - \frac{1}{g'} \partial_\mu \alpha_Y \, , \end{aligned}$ $D_{\mu\nu} - O_{\mu}D_{\nu} - O_{\nu}D_{\mu}$ (abelian) Gauge Group $U(1)_Y$ (abelian) ${\cal L}_{YM} = \bar\Psi (i\gamma^\mu D_\mu - m)\Psi - \frac{1}{\alpha} F_\mu$ Gauge Group $SU(2)_L$ $Gauge Group U(1)_V$ (abelian) $C = \sqrt{I}$ $\overline{H(1)}$ $\overline{G(x)}$ $y_1' = e^{+iY\alpha_{Y}}y_2$ ∂µUU [−]¹ δG^a g_r γ γ μ \sim \sim $W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g \epsilon^{abc} W^b_\mu W^c_\nu, \quad a = 1, \dots, 3$ completely anti-symmetric tensor in 3 dimensions. The covariant derivative for any field $T^a = \frac{1}{2}$ $\frac{q}{q}$ is absolute the group is absolute the case of $\frac{q}{q}$ μ = μ \overline{p} \overline{p} \overline{p} \overline{p} and the fields transform as $\overline{}$ $\frac{\partial}{\partial x}$ $\omega_{\mu\nu} - \omega_{\mu}\omega_{\nu}$ $\omega_{\nu}\omega_{\mu}$ $\Psi_L \rightarrow e^{-i T^a \alpha^a} \psi_L$ $U = e^{-i T^a \alpha^a}$ $\mu \nu$ and μ in μ in μ in μ $\frac{1}{2}$ \mathcal{L} $G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a + gf^{abc} G_\mu^b G_\nu^c, \quad a=1,\ldots,8$ $\frac{1}{2}$ ^µ ⁼ [−]¹ g \mathcal{L} $\psi'=e^{\mp iY\alpha_Y}\psi,$ $B'_\mu = B_\mu - \frac{1}{g'} \partial_\mu \alpha_Y$ $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu$ \overline{F} for the fundamental representation of \overline{F} α and α in α in α in α dimensions. The covariant derivative for any field α $W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g \epsilon^{abc} W^b_\mu W^c_\nu, \quad a = 1, \dots, 3$ ψ_L $\overline{\text{G}}$ $\alpha \rightarrow e^{-i}T^a\alpha^a$ and $\alpha \rightarrow T^r$ $\alpha^{-i}T^a\alpha^a$ and the field ${}_{\alpha}^{a}T^{a} \rightarrow U G_{\alpha}^{a}T^{a}U^{-1} - \frac{i}{2} \partial_{\mu}UU^{-1}$ μ $D_a - (\partial_a - i a C^a T^a) a$ $D_{\mu}q = (0\mu - \iota g)q_{\mu}$ $-\mu\nu$ $\partial_\mu\nu$ c $T_{\rm eff}$ is very important to have all the Feynman rules with consistent to have all the Feynman rules with consistent \sim M_{\odot} is the general R \mathbf{D} The Standard Model M conventions. We give here those that are important for building the SM. We will separate Gauge Group $SU(3)_c$ ^ν , a = 1,..., 8 (D.1) Gc y \mathcal{L} Gauge Group $U(1)_Y$ (abelian) $\mathcal{L}_{YM}=$ Gauge Group $SU(2)_L$ $c\tau^a$ and σ^a will give the complete Ψ $\Psi_L \rightarrow e$ $q\rightarrow e^{-iT^a\alpha^a}q$ $U=e^{-iT^a\alpha^a}$ all quarks t $G^a_\mu T^a \to U G^a_\mu T^a U^{-1} - \frac{1}{c} \partial_\mu U U^{-1}$ $D_\mu q = \left(\partial_\mu - i\,g\,G_\mu^a T^a\right)q$ ϵ in the conventions. In this Appendix we will give the complete Feynman rules for the Standard Feynman rules for the Standar Model in the general R^ξ gauge. C_{value} C_{round} $\frac{SII(0)}{S}$ \overline{L} $\frac{D}{\Box}$ \overline{a} , and \overline{a} (D) \overline{a} (D \overline{g} q $W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g \epsilon^{abc} W_\mu^b W_\nu^c, \quad a = 1, \dots, 3$ under it -> chiral interactions Gauge Group $SU(3)_c$ $G_{\mu}^{a}T^{a} \rightarrow U G_{\mu}^{a}T^{a}U^{-1} - \frac{i}{g}\partial_{\mu}U U^{-1}$ $G^a_{\mu\nu} = \partial_{\mu}G^a_{\mu} - \partial_{\nu}G^a_{\mu} + af^{abc}G^b_{\nu}G^c_{\nu}, \quad a = 1, \ldots, 8$ $D_{\alpha} = \left(\partial_{\alpha} - i\partial_{\alpha}C^{a}T^{a}\right)$ $-\mu$ μ verify that the field itself is the field itself, i 374 APPENDIX D. FEYNMAN RULES FOR THE STANDARD MODEL $\partial_\mu U U^{-1}$ μ verify that the covariant derivative transforms like the field itself, μ $\int q$ ν **Γ**
Γ ^γ Γ _δνα _σνα της κατά \overline{a} $\frac{1}{2}$ δ of $I^a \rightarrow e^{-i T^a \alpha^a}$ $D_\mu \psi_L = \left(\partial_\mu - i\, g\, W_\mu^a T^a\right) \psi_L$ Γ auge Γ roup $\mathcal{S}U(2)$

completely anti-symmetric tensor in 3 dimensions. The covariant derivative for any field α

∂µα^a + f abcα^b

where $f(\omega)$ the fundamental representation of ω and ω and ω is the same Ta $=$ ω

∂µUU [−]¹ δG^a

and the fields transform as

Appendix D

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 \mathbb{R}^n transforming is,

 \mathcal{L}_eff transforming is, and this group is, and this group is, and this group is, and

$$
\begin{array}{ll}\text{Gauge Group}\ U(1)_Y&\text{(abelian)}\\ &\mathcal{L}_{YM}=\bar{\Psi}(i\gamma^{\mu}D_{\mu}-m)\Psi-\frac{1}{2}F_{\mu\nu}F^{\mu\nu}\\ &\psi'=e^{+iY\alpha_Y}v \end{array}
$$

. (D.13)

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Tall Standard Model fermions $\begin{array}{c} \text{C} \\ \text{C} \\ \text{D} \end{array}$ $\begin{array}{c} \text{C} \\ \text{D} \end{array}$ carry U(1) charge

 $\Psi_L = (u_L, d_L)$ or (ν_L, e_L)

 \overline{D} only left-handed fermions charged under it -> chiral interactions

q=(q1,q2,q3)

∂ $\frac{1}{2}$ all quarks transform under it -> vector-like interactions

The lagrangian of the Standard Model Standard Model e = g sin θ^W = g 376 APPENDIX D. FEYNMAN RULES FOR THE STANDARD MODEL 376 APPENDIX D. FEYNMAN RULES FOR THE STANDARD MODEL $F_{\rm eff}$ and gauge field $\frac{1}{2}$

D.2.4 The Gauge Field Lagrangian

The hypercharge of this doublet is 1/2 and the covariant derivative reads the covariant derivative reads of the covaria

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21 − in de la cos de la
∕2 de la cos de la c ($\hspace{0.1cm}\textcolor{black}{\mathscr{C}}$ describe massless gauge bosons describe massless fermions of $i \psi_R \gamma^\mu D_\mu \psi_R$ interactions with agu D_{μ} \mathbf{S} $-[\partial + i\partial']$ \int $V R \, \frac{1}{2}$ gg " v2W³ ^µB^µ + 1 ϵ annying a $11(1)$, change $\overline{\operatorname{carrv}}$ ing a U(1) $\overline{\operatorname{c}}$ ch \overline{a} only left-handed all fermions carrying a U(1)_y charge
fermions i.e. all Standard Model fermions an fermions can fing a $O(1)$, charge
i.e. all Standard Model fermions one, the $\mathcal{L}_{\mathcal{A}}$ with the relations given in Eq. (D.13), while the fourth gives the mass to th , TO L VV
JSONS 1 2 $M_Z^2 Z_\mu Z^\mu + M_W^2 W_\mu^+ W^{-\mu}$ **∣responsible for** $\tau_0 W^3 + i \frac{g'}{R} R^{\dagger}$ to consider deviating of the U Φ : covariant derivative of the Higgs $\qquad \qquad$ $\frac{1}{2}$ cutch $\frac{1}{2}$ **v** symmetry B cridiged ander $SU(2)$ xv (1) y \parallel besoluted bosons, we also got a problem. In fact the terms in the last line are quadratic in the fields in the fields in the fields of the fields of and complicate the definition of the propagators. We now see how one can use the needed —→ gives mass t and the usual definition, $4\frac{G}{\mu\nu}$ $4\frac{F}{\mu\nu}$ $4\frac{D\mu\nu}{\mu\nu}$ describe massiess gauge bosons $\overline{\mathbf{C}}$ $\mathcal{F}_{D\to b\tau}=\sum_{i\neq b}\frac{1}{2\sqrt{2}}\mathcal{F}_{D\to b\tau}^{\mu}$ describe mas $D_{\mu}\psi_R = \left[\partial_{\mu} + ig^{\prime}YB_{\mu}\right]\psi_R$ \mathcal{L} collect in Table D.1 the EW is L the L H changed under $SU(2)$ vi $U(1)$. Symmetry ss to fermior $Z_d\overline{Q}\Phi d_P - Y_u\overline{Q}\widetilde{\Phi} u_P + h.c.$ \longrightarrow gives mass to fermions ^µνWaµ^ν [−] ¹ $w \times U(1)_Y \longrightarrow SU(3) \times U(1)_{em}$ In the SM we use an Higgs doublet with the following assignments, assignments, assignments, assignments, assignments, assignments, and the following assignments, assignments, assignments, assignments, assignments, assignme μ $\frac{1}{2}$ $\frac{4}{1}$ $4^{-\mu\nu}$ acsortion $\mathcal{L}_{\text{Fermion}} = \sum_{\nu} i \overline{q} \gamma^{\mu} D_{\mu} q + \sum_{\nu} i \psi_{L} \gamma^{\mu} D_{\mu} \psi_{L} + \sum_{\nu} i \psi_{R} \gamma^{\mu} D_{\nu}$) τ ⁺W⁺ ^µ + τ [−]W[−] µ * − i $\overline{\mathbf{x}}$ $\overline{}$ $D_{\mu} \nu$ fermions of the SM we use an Extensive with the following assignments of the following assignments of the following \mathcal{L} $\mathcal{L}_{\text{Higgs}} = (D_{\mu} \Phi)^{\dagger} D_{\mu} \Phi + \mu^2 \Phi^{\dagger} \Phi - \lambda$ $\int \Phi^\dagger \Phi$ \setminus^2 e bos $\mathcal{I}(\mathcal{A})$ we find the following terms of the following $D_\mu \Phi = \left[\partial_\mu - i \frac{\partial}{\partial x}\right]$ 8 $\frac{\partial}{\partial 2}(\tau^+W^+_\mu + \tau^-W^-_\mu)$ $D_{\mu}\Phi = \left[\partial_{\mu} - i\frac{g}{\sqrt{2}}\left(\tau^+W^+_{\mu} + \tau^-W^-_{\mu}\right) - i\frac{g}{2}\tau_3W^3_{\mu} + i\frac{g}{2}B_{\mu}\right]\Phi$ $\mathcal{L}_{\text{Yukawa}} = -I_L L \Psi_L R - I_d Q \Psi u_R - I_u Q \Psi u_R + \text{h.c.}$ $\overline{O}\left(\frac{1}{2}\right)$ $\overline{O}\left(\frac{1}{2}\right)$, $\overline{O}\left(\frac{1}{2}\right)$, $\overline{O}\left(\frac{1}{2}\right)$, $\overline{O}\left(\frac{1}{2}\right)$, $\overline{O}\left(\frac{1}{2}\right)$ $SU(3)$ \times $\mathcal{L}=\mathcal{L}$ 8 massless ⊍ 3 mass *aluons* ^µ ∂µϕ−* (D.25) D.2.5 The Fermion Fields Lagrangian $\mathcal{L}_{\text{max}} = -\frac{1}{2}G^a G^{a\mu\nu} - \frac{1}{2}W^a W^{a\mu\nu} - \frac{1}{2}R_{\mu\nu}R^{\mu\nu}$ decembe mass ${\cal L}_{\rm gauge} = -\frac{1}{4} G^a_\mu$ μ_{D} $\sum \overline{\mu_{D}}$ where the covariant derivatives are obtained with the rules in Eqs. (D.3), (D.3), (D.3), (D.3), (D.14) and (D.1 $\left(1 + \frac{1}{2}\right)^2$ Φ^\dagger (D.21) In the SM we use an Higgs doublet with the following assignments, $\left[\partial_\mu -i\frac{g}{\sqrt{2}}\left(\tau^+W_\mu^++\tau^-W_\mu^-\right)-i\right.$ g 2 $\tau_3 W_\mu^3 + i$ g' 2 B_μ $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ H charge \geq Φ $\mathcal{L}_{\text{Yukawa}} = - Y_l \, \overline{L} \, \Phi \,\, \ell_R - Y_l$ $\overline{O(t)}$ $\overline{O(t)}$ $\overline{O(t)}$ $\overline{O(t)}$ \overline{O} (\overline{O}) \land \overline{O} (\overline{Q}) \overline{L} \land \overline{O} $\frac{2}{3}$ m issive gaug $M - Z₀$ o massiess is massive gauge bosons **8 massless** 1 massless photon γ 2 2 2 ^µ [∂]µϕ⁺ [−] ⁱ **remaining unbroken symmetry** The W and Z bosons interact with the Higgs medium, the γ doesn't. $G^a_{\mu\nu}G^{a\mu\nu}-\frac{1}{4}$ $W^a_{\mu\nu}W^{a\mu\nu}-\frac{1}{4}$ $B_{\mu\nu}B^{\mu\nu}$ describe mass where the field strengths are given in \Box $\overline{1}$ The Fermion Fields Lagrangian Fi $4 \mu \mu$ is the kinetic part and gauge interaction, leaving the Yukawa interaction for an $\mathcal{L}_{\text{Fermion}} = \sum$ quarks $i\overline{q}\gamma^{\mu}D_{\mu}q + \sum$ ψ_L $i\overline{\psi_L}\gamma^\mu D_\mu\psi_L + \sum$ ψ_R $i\overline{\psi_R}\gamma^\mu D_\mu\psi_R$ (interesting $m_{\mu\gamma}$ are obtained with the rules in Eqs. (D, 3), $\Phi^{\dagger}\Psi = \lambda \left(\Psi^{\dagger}\Psi\right)$ $\tau^{-W^{-}}$ i $^g\tau$ $\big)$ $D_\mu \Phi = \left[\partial_\mu - i \frac{g}{\epsilon}\left(\tau^+ W_\mu^+ + \tau^- W_\mu^-\right) - i \frac{g}{2} \tau_3 W_\mu^3 + i \frac{g'}{\epsilon} B_\mu\right]\Phi$: covariant deriva H charged under SU(2) ×U(1)_Y symmeling! $-Y_l \overline{L} \Phi \ell_R - Y_d \overline{Q} \Phi d_R Y_u \overline{O} \widetilde{\Phi} u$ r \overline{a} $\frac{b}{b}$ (b) $\frac{c}{c}$ (b) $\frac{c}{c}$ (b) $\frac{c}{c}$ (b) $\frac{c}{c}$ (b) $\frac{c}{c}$ (b) $\frac{c}{c}$ (c) $\frac{c}{c}$ (b) $\frac{c}{c}$ (c) $\frac{c}{c}$ ([∂]^µ [−] ⁱ ^g $SU(3)\times SU(2)_L\times U$ $(1)_Y \longrightarrow S$ $\overline{\mathcal{L}}$ \bar{t} $(\, \cdot \,$ describe massless gauge bosons describe massless fermions and their interactions with gauge bosons $\mathcal{L}_{\text{Higgs}} = (D_{\mu} \Phi)^{\dagger} D_{\mu} \Phi + \mu^2 \Phi^{\dagger} \Phi - \lambda \left(\Phi^{\dagger} \Phi \right)^2$ \longrightarrow gives mass to EW after spontaneous symmetry breaking gives masses to the elementary fermions. We have, ${\cal L}_{\rm Yukawa} = - \, Y_l \, \overline{L} \, \Phi \,\, \ell_R - Y_d \, \overline{Q} \, \Phi \,\, d_R - Y_u \, \overline{Q} \, \Phi \,\, u_R + {\rm h.c.} \qquad \longrightarrow \qquad {\bf gives \ mass \ to \ fermions}$ $SU(3)\times SU(2)_L\times U(1)_Y\longrightarrow SU(3)\times U(1)_{em}$ ndssiess
gluons 3 massive ∣u>
∤ **3 massive gauge bosons 8 massless electroweak W+ W- Z0 gluons**

Charges of the Standard Model fields

Historically

The (adhoc) Higgs Mechanism (a model without dynamics)

EW symmetry breaking is described by the condensation of a scalar field D.2.6 The Higgs Lagrangian

The Higgs selects a vacuum state by developing a non zero background value. When it does so, it gives mass to SM particles it couples to. $^{\prime}$ √2 t couples to. \sim ie \sim in \sim

the puzzle:

We do not know what makes the Higgs condensate $V(h) = \frac{1}{2}\mu^2h^2 + \frac{1}{4}\lambda h^4$ We ARRANGE the Higgs potential so that the Higgs condensates but this is just a parametrization that we are unable to explain dynamically. \overline{c} $\frac{1}{2}$ 2 1×4 \cdots $\mathbf{u} = \mathbf{u} \cdot \mathbf{v}$

We have quantized free fields

We have introduced interactions ◆
◆

(particle creation and annihilation can only take place in theory with interactions)

We now would like to compute probability of processes like for instance a two-body decay a->c+d or a two-body reaction a+b->c+d

"S-matrix approach"-> calculate probability of transition between two asymptotic states

The S-matrix

We consider a state $|a\rangle$ (t) which at an initial time t_i is labelled $|a\rangle$. Similarly we consider a state $|b\rangle$ (t) which at a final time t_f is labelled $|b\rangle$

At t_f the state $|a\rangle(t)$ as evolved as $e^{-iH(t_f-t_i)}|a>$

where H is the hamiltonian of the theory

The amplitude for the process in which the initial state |a> evolves into the final state |b> is given by

and that can be linked to a transition rate per volume unit as measured by an experiment

Link to observables

◆ cross section: reaction rate per target particle per unit incident flux

[1/time] $\overline{[1/(\text{time length}^2)]}$ <mark>vab</mark>
art
_lath

 --> has units of a surface 1 measured in multiples of 1 barn= $10^{-24}\,$ ${\rm cm}^2$ g √**20 α**

−ieQ

typical relevant LHC cross sections \sim in pb

1 picobarn= 1 pb= 10^{-36} cm²

◆ Decay width (inverse of lifetime of a particle) =transition rate has dimension [1/time] **Example: decay width of EW gauge bosons** Link t
1 rate per
11
23 And units
1 picobar
1 picobar
1 f EW gaug
1 the couplint ra
as fθา ไ \cdot \cdot \cdot scales as the square of the coupling constant W g $\Gamma \propto |\mathcal{M}|^2$ coupling u d ν e

30

Z couplings to fermions

where $I_3 = \pm \frac{1}{2}$ is z-component of weak isospin and Q is electric charge 1 $\frac{1}{2}$ $\frac{1}{2}$ i g ortional to $I_2 = \sin^2 \theta_W Q$ 1 − γ⁵ to any fermion is proportional to $\,\, I_3-\beta$ $\overline{}$ φ
φ Z^2 μ \mathbf{c} Z_{μ} ψ_L ψ_L ψ_R \diagdown φ
φ \overline{a} .
II \mathbf{c} Z_{μ} ψ_{R} $I_3 - \sin^2 \theta_W Q$ $\psi_R \searrow \frac{\sin^2 \theta_W Q}{\sqrt{Q}}$ The coupling of Z to any fermion is proportional to $\,\,I_3 - \sin^2\theta_W Q$ $\sin^2 \theta_W = 0.231$

> $the₀$ $\overline{\mathsf{u}}$ أمره ^f [−] ^Q^f sin² ^θ^W , g^f

ψu,d

D.4.6 Fermion-Higgs and Fermion-Goldstone Interactions d_R $I_3 = 0$ $Q = -1/3$ m^f d_L *I*₃ = $-1/2$ *Q* = $-1/3$ u_L $I_3 = +1/2$ $Q = +2/3$ u_R *I*₃ = 0 *Q* = +2/3

is too f and similarly for c,s, and b (t is too heavy for the Z to decay into it)

for the lentons: gf ne leptons: T3 for the quarks: for the leptons:

ψu,d

 $-4/3$ *e*_L *I*₃ = $-1/2$ *Q* = -1 e_R *I*₃ = 0 *Q* = -1
 v_{ex} *I*₃ = +1/2 *Q* = 0 ν_{e_L} $I_3 = +1/2$

and similarly for $\nu, \tau, \nu_{\mu}, \nu_{\tau}$

Branching fractions for Z decay 1−22 $\overline{}$

 u_L $I_3 = +1/2$ $Q = +2/3$ *d*_{*L*} $I_3 = -1/2$ $Q = -1/3$ d_R $I_3 = 0$ $Q = -1/3$ u_R $I_3 = 0$ $Q = +2/3$ and similarly for c,s, and b (t is too heavy for the Z to decay into it) $\frac{1}{2}$ appendix $\frac{3}{2}$ by a decomposition of $\frac{1}{2}$ and $\$

for the quarks: for the leptons: e_R *I*₃ = 0 *Q* = -1 e_L *I*₃ = $-1/2$ *Q* = -1 ν_{e_L} $I_3 = +1/2$ $Q = 0$ 2 \overline{u} 55.54

ψ \mathbf{r}

ω a baransa da baransa da baransa

q q

γ

and similarly for $\nu,\tau,\nu_{\mu},\nu_{\tau}$

32

The decay rate is proportional to the square of the coupling constant $I_3 - \sin^2 \theta_W Q$ due to the additional gluon emission Also, for quarks, there is an additional factor $(1 + \frac{\alpha_s}{2})$ 2π) where $\alpha_s = g_s^2/4\pi = 0.118$ $\frac{1}{2\pi}$) where where $\alpha_s = g_s^2/4\pi = 0.118$ \overline{r}

$$
d_R \tI_3 = 0 \tQ = -1/3
$$

and similarly for c,s, and b (t is too
heavy for the Z to decay into it)
e decay rate is proportional to the square of the coupling constant $I_3 - \sin^2 \theta_W Q$
so, for quarks, there is an additional factor $(1 + \frac{\alpha_s}{2\pi})$ where $\alpha_s = g_s^2/4\pi = 0.118$
to the additional gluon emission

$$
B(Z \to e^+e^-) = B(Z \to e^+_L e^-_L) + B(Z \to e^+_R e^-_R)
$$

$$
B(Z \to e^+_L e^-_L) = \frac{\Gamma(Z \to e^+_L e^-_L)}{\Gamma(Z \to particle, antiparticle)}
$$

$$
B(Z \to e^+_L e^-_L) = \frac{\Gamma(Z \to e^+_L e^-_L)}{\sum_{all particles} \Gamma(Z \to particle, antiparticle)}
$$
gluon g_s
$$
B(Z \to \nu\bar{\nu}) = B(Z \to \nu_e \bar{\nu}_e) + B(Z \to \nu_\mu \bar{\nu}_\mu + B(Z \to \nu_\tau \bar{\nu}_\tau)
$$

$$
= 3B(Z \to \nu_e \bar{\nu}_e) = 20\%
$$

$$
Z \to e^+e^- = B(Z \to \mu^+ \mu^-) = B(Z \to \tau^+ \tau^-) = 3.33\%
$$

$$
B(Z \to \nu \bar{\nu}) = B(Z \to \nu_e \bar{\nu}_e) + B(Z \to \nu_\mu \bar{\nu}_\mu + B(Z \to \nu_\tau \bar{\nu}_\tau))
$$

= 3B(Z \to \nu_e \bar{\nu}_e) = 20\%

 $B(Z \to e^+e^-) = B(Z \to \mu^+\mu^-) = B(Z \to \tau^+\tau^-) = 3.33\%$ $B(Z \rightarrow all \; hadrons) = 3 \times [B(Z \rightarrow u\bar{u}) + B(Z \rightarrow d\bar{d}) + B(Z \rightarrow s\bar{s})$ $+ B(Z \rightarrow c\bar{c}) + B(Z \rightarrow b\bar{b})] = 69.9\%$

Branching fractions for W decay fermions in the standard model. However, the standard model model is to be producturity in definitions for the decay The quarks come in three colors. Of course, *d*⁰ and *s*⁰ contain all three genera**decompared to down are negligible compared to the Branching fractions for W decay and** *s***0 contain all three genera-**

tions of down quarks *d*, *s*, *b*. But their masses are negligible compared to the *W*

W <u>W 1, and hence we can ignore divided the second them. The method is the method them. The method is the method</u>

*b/m*²

$$
W^- \to e^- \bar{\nu}_e, \mu^- \bar{\nu}_\mu, \tau^- \bar{\nu}_\tau, d' \bar{u}, s' \bar{c}.
$$

$$
BR(W^{-} \to e^{-} \bar{\nu}_{e}) = BR(W^{-} \to \mu^{-} \bar{\nu}_{\mu}) = BR(W^{-} \to \tau^{-} \bar{\nu}_{\tau})
$$

= $\frac{1}{3 + 6(1 + \alpha_{s}/\pi)} = 0.108,$

$$
BR(W^{-} \to \text{hadrons}) = \frac{6(1 + \alpha_s/\pi)}{3 + 6(1 + \alpha_s/\pi)} = 0.675.
$$