Theoretical Concepts in Particle Physics

Lecture II : From gauge theories to Standard Model observables

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Summary of Lecture I

◆ 2 main ingredients of Quantum Field Theory: Quantum Mechanics and Special Relativity

◆ Invariance under Poincaré transformations (Lorentz transformations+translations)

-The different types of fields (scalar spin-0, vector spin-1 and spinor spin1/2) are different representations of Lorentz transformations

- symmetry -> conserved quantity

◆ relativistic wave equation not enough, only able to describe a single particle

We need a formalism to describe processes in which number of particles change

◆ For fully consistent description, we need to reinterpret the field as a field operator which can destroy or create particles: : we need to quantize fields -> Quantum Field Theory

Classical Field theory

classical mechanics & lagrangian formalism

a system is described by $S = \int dt \mathcal{L}(q, \dot{q})$

position (q), momentum ($\dot{q}$)

action principle determines classical trajectory:

$\delta S = 0$ --> Euler-Lagrange equations $\frac{\partial \mathcal{L}}{\partial q_i} - \frac{\partial}{\partial t}\frac{\partial \mathcal{L}}{\partial \dot{q}_i} = 0$

conjugate momenta $p_i = \frac{\partial \mathcal{L}}{\partial \dot{q}_i}$ hamiltonian $H(p, q) = \sum_i p_i \dot{q}_i - \mathcal{L}$

extend lagrangian formalism to dynamics of fields

$$S = \int d^4x \mathcal{L}(\varphi, \partial_\mu \varphi)$$

$$\partial_\mu = \frac{\partial}{\partial x^\mu}$$

$$\delta S = 0 \quad \text{-->} \quad \frac{\partial \mathcal{L}}{\partial \varphi_i} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi_i)} = 0$$

$$\partial_0 = \frac{\partial}{\partial x^0} = \frac{\partial}{\partial t}$$

conjugate momenta $\Pi_i = \frac{\partial \mathcal{L}}{\partial(\partial_0 \varphi_i)}$ hamiltonian $H(x) = \sum_i \Pi_i(x) \partial_0 \varphi_i(x) - \mathcal{L}$

Classical Field theory and Noether theorem

Invariance of action under continuous global transformation ---> There is a conserved current/charge

$$\partial_\mu j^\mu = 0 \qquad Q = \int d^3x j^0(x,t)$$

example of transformation: $\varphi \to \varphi e^{i\alpha}$ (*)

if small increment $\alpha \ll 1$ $\varphi \to \varphi + i\alpha\varphi$

$$\delta\varphi = i\alpha\varphi$$
$$\delta\varphi' = i\alpha\varphi'$$

invariance of $\mathcal{L}$ under (*): $\delta\mathcal{L} = 0 = i\alpha\left(\frac{\partial\mathcal{L}}{\partial\varphi}\varphi + \frac{\partial\mathcal{L}}{\partial\varphi'}\varphi'\right)$

Euler-Lagrange equations: $\frac{\partial}{\partial x}\left(\frac{\partial\mathcal{L}}{\partial\varphi'}\right) - \frac{\partial\mathcal{L}}{\partial\varphi} = 0$

$$\frac{\partial}{\partial x}\Big(\underbrace{\varphi\frac{\partial\mathcal{L}}{\partial\varphi'}}_{\equiv J}\Big) = 0$$

conserved current

Scalar Field theory

Lorentz invariant action of a complex scalar field

$$S = \int d^4x (\partial_\mu \varphi^* \partial^\mu \varphi - m^2 \varphi^* \varphi)$$

Euler-Lagrange equation leads to Klein-Gordon equation

$$(\Box + m^2)\varphi = 0$$

with solution a superposition of plane waves:

$$\varphi(x) = \int \frac{d^3p}{(2\pi^3)\sqrt{2E_p}}(a_p e^{-ipx} + b_p^* e^{ipx})$$

existence of a global U(1) symmetry of the action

$$\varphi(x) \to e^{i\theta}\varphi(x)$$

conserved U(1) charge

$$Q_{U(1)} = \int d^3x j_0 \qquad j_\mu = i\varphi^* \overleftrightarrow{\partial}_\mu \varphi$$

From first to second quantization

Basic Principle of Quantum Mechanics:

To quantize a classical system with coordinates q^i and momenta p^i, we promote q^i and p^i to operators and we impose $[q^i, p^j] = \delta^{ij}$

same principle can be applied to scalar field theory

where $q^i(t)$ are replaced by $\varphi(t, x)$

and $p^i(t)$ are replaced by $\Pi(t, x)$

φ and Π are promoted to operators and we impose $[\varphi(t,x), \Pi(t,y)] = i\delta^3(x-y)$

Expand the complex field in plane waves:

$$\varphi(x) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_p}}(a_p e^{-ipx} + b_p^\dagger e^{ipx})$$

where a_p and b_p^+ are promoted to operators

$$[a_p, a_q^\dagger] = (2\pi^3)\delta^{(3)}(p-q) = [b_p, b_q^\dagger]$$

scalar field theory is a collection of harmonic oscillators

destruction operator $a_p|0>= 0$ defines the vacuum state |0>

a generic state is obtained by acting on the vacuum with the creation operators $|p_1 \dots p_n> \equiv a_{p_1}^\dagger \dots a_{p_n}^\dagger |0>$

Scalar field quantization continued

$$\mathcal{H} = \Pi\partial_0\varphi - \mathcal{L} = \int \frac{d^3p}{(2\pi)^3}\frac{E_p}{2}(a_p^\dagger a_p + b_p^\dagger b_p)$$

the quanta of a complex scalar field are given by two different particle species with same mass created by a^+ and b^+ respectively

The Klein Gordon action has a conserved U(1) charge due to invariance $\varphi(x) \to e^{i\theta}\varphi(x)$

$$Q_{U(1)} = \int d^3x j^0 = \int \frac{d^3p}{(2\pi)^3}(a_p^\dagger a_p - b_p^\dagger b_p)$$

2 different kinds of quanta: each particle has its antiparticle which has the same mass but **opposite U(1) charge**

Field quantization provides a proper interpretation of "E<0 solutions"

$$\varphi(x) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_p}}(a_p e^{-ipx} + b_p^\dagger e^{ipx})$$

coefficient of the positive energy solution e^{-ipx} becomes after quantization the destruction operator of a particle while the coefficient of the e^{ipx} becomes the creation operator of its antiparticle

$a_p^+|0\rangle$ and $b_p^+|0\rangle$ represent particles with opposite charges

Similarly, we are led to quantize:

Spinor fields Ψ

Lorentz invariant lagrangian $\mathcal{L} = \bar{\Psi}(i\partial\!\!\!/ - m)\Psi \qquad \partial\!\!\!/ = \gamma^\mu \partial_\mu$

Dirac equation $(i\partial\!\!\!/ - m)\Psi = 0$

fermions: → anticommutation relations $\{\Psi_a(x,t), \Psi_b^\dagger(y,t)\} = \delta^{(3)}(x-y)\delta_{ab}$

The electromagnetic field A_μ.

Lorentz inv. lagrangian $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Maxwell eq. $\partial_\mu F_{\mu\nu} = 0$

Maxwell lagrangian inv. under $A_\mu \to A_\mu - \partial_\mu \theta$

Summary of procedure for building a QFT

- Kinetic term of actions are derived from requirement of Poincaré invariance
- Promote field & its conjugate to operators and impose (anti) commutation relation
- Expanding field in plane waves, coefficients a_p, a_p^+ become operators
- The space of states describes multiparticle states

a_p destroys a particle with momentum p while a_p^+ creates it

e.g $|p_1 \ldots p_n >\equiv a^\dagger_{p_1} \ldots a^\dagger_{p_n} |0>$

→ crucial aspect of QFT: transition amplitudes between different states describe processes in which the number and type of particles changes

Gauge transformation and the Dirac action

Consider the transformation $\Psi \to e^{iq\theta}\Psi$ U(1) transformation

it is a symmetry of the free Dirac action if θ is constant $\mathcal{L} = \bar{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi$

no longer a symmetry if $\theta = \theta(x)$

However, the following action is invariant under

$$\begin{cases} \Psi \to e^{iq\theta}\Psi \\ A_\mu \to A_\mu - \partial_\mu \theta \end{cases}$$

$$\mathcal{L} = \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi$$

where $D_\mu \Psi = (\partial_\mu + iqA_\mu)\Psi$ **covariant derivative**

We have gauged a global U(1) symmetry, promoting it to a local symmetry

The result is a gauge theory and A_μ is the gauge field

conserved current: $j^\mu = \bar{\Psi}\gamma^\mu\Psi$

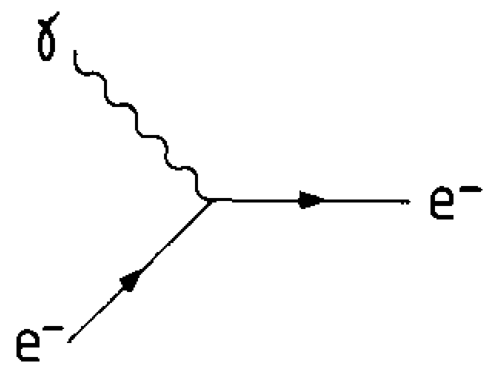
conserved charge: $Q = \int d^3x \bar{\Psi}\gamma^0\Psi = \int d^3x \Psi^\dagger \Psi$ → **electric charge**

Electrodynamics of a spinor field

$$\mathcal{L} = \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi$$ where $$D_\mu \Psi = (\partial_\mu + iqA_\mu)\Psi$$

$$\mathcal{L} = \bar{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi - qA_\mu \bar{\Psi}\gamma^\mu \Psi$$

Coupling of the gauge field A_μ to the current $j^\mu = \bar{\Psi}\gamma^\mu\Psi$



From Quantum Electrodynamics to the electroweak theory

These transformations are elements of U(1) group

$$\Psi \rightarrow e^{iq\theta}\Psi$$

In the electroweak theory , more complicated transformations, belonging to the SU(2) group are involved

$$\Psi \rightarrow \exp(ig\ \tau.\lambda)\Psi$$

where $\tau = (\tau_1, \tau_2, \tau_3)$ are three 2*2 matrices

Generalization to SU(N)

N^2-1 generators (N×N matrices)

$$\Psi(x) \rightarrow U(x)\Psi(x)$$
$$U(x) = e^{ig\theta^a(x)T^a}$$

$$A_\mu(x) \rightarrow UA_\mu U^\dagger - \frac{i}{g}(\partial_\mu U)U^\dagger$$

Gauge theories: Electromagnetism (EM) & Yang-Mills

EM U(1) $\phi \to e^{i\alpha}\phi$ but $\partial_\mu\phi \to e^{i\alpha}\,(\partial_\mu\phi) + \underbrace{i(\partial_\mu\alpha)}_{\neq 0 \text{ if local transformations}}\,\phi$

EM field and covariant derivative $\partial_\mu\phi + ieA_\mu\phi \to e^{i\alpha}(\partial_\mu\phi + ieA_\mu\phi)$

if $A_\mu \to A_\mu - \frac{1}{e}\partial_\mu\alpha$

the EM field keep track of the phase in different points of the space-time

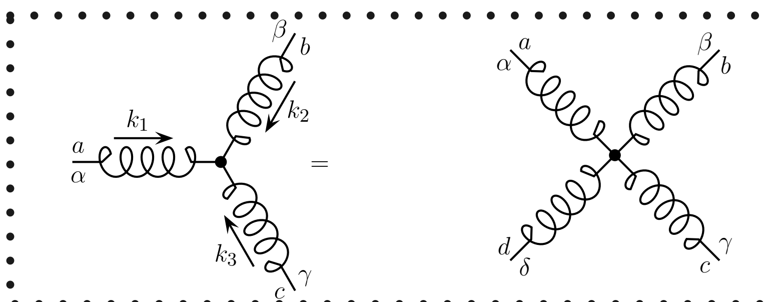
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Yang-Mills : non-abelian transformations $\phi \to U\phi$

$$\partial_\mu\phi + igA_\mu\phi \to U(\partial_\mu\phi + igA_\mu\phi)$$

if $A_\mu \to UA_\mu U^{-1} - \frac{i}{g}U\partial_\mu U^{-1}$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \underbrace{ig[A_\mu, A_\nu]}_{\text{non-abelian int.}}$$



The Standard Model: matter

the elementary blocks:

LEPTONS

electron: ν_e e^-

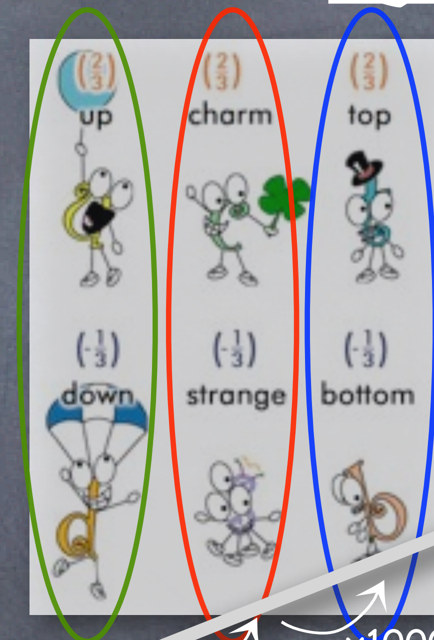
muon: ν_μ μ^-

tau: ν_τ τ^-

mass (x200, x20)

→ no composite states made of leptons

QUARKS



mass (x1000, x1000)

each of the 6 quarks exists in three colors

→ composite states (white objects

- baryons — proton $p = (u, u, d)$; neutron $n = (u, d, d)$
- mesons

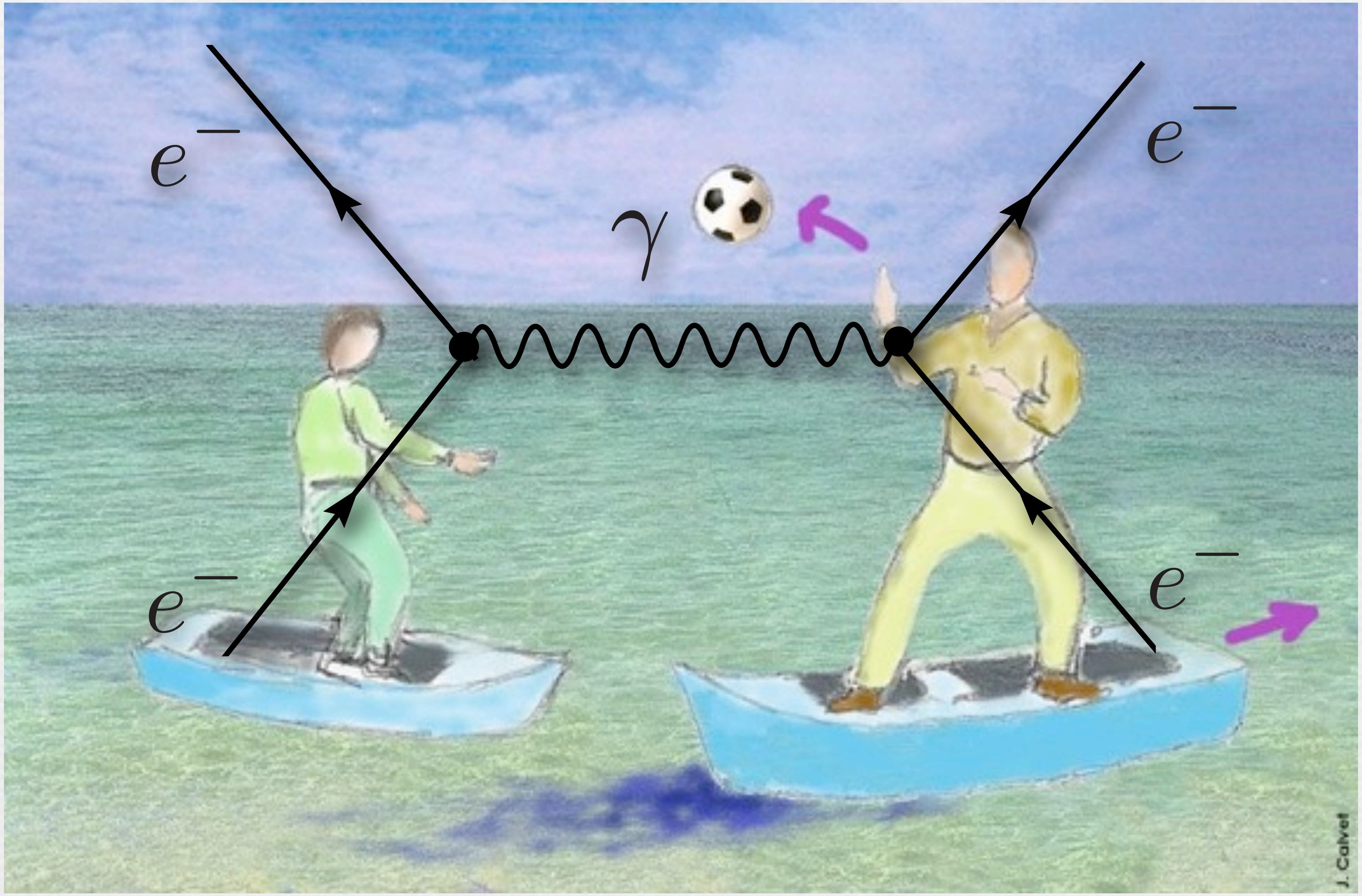
+ antiparticles

The Standard Model : interactions

- $U(1)_Y$ — *electromagnetic interactions*
  - Photon γ
  - light, atoms, molecules
  - 10^{-6}
- $SU(2)_L$ — *weak interactions*
  - bosons $W^{\pm}, Z^0$
  - β decay
  - $n \xrightarrow{W^{\pm}} p + e^- + \bar{\nu}_e$
  - 10^{-2}
- $SU(3)_C$ — *strong interactions*
  - gluons g^a
  - atomic nuclei
  - α decay
  - ${}^{238}_{92}U \to {}^{234}_{90}Th + {}^{4}_{2}He$
  - 1

strength

Interactions between particles



Elementary particles interact with each other by exchanging gauge bosons

The beauty of the SM comes from the identification of a unique dynamical principle describing interactions that seem so different from each others

gauge theory = spin-1

The most general lagrangian given the particle content

$$\mathcal{L} = -\frac{1}{4g'^2}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4g^2}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4g_s^2}G^a_{\mu\nu}G^{a\mu\nu}$$
$$+\bar{Q}_i i\not{D}Q_i + \bar{u}_i i\not{D}u_i + \bar{d}_i i\not{D}d_i + \bar{L}_i i\not{D}L_i + \bar{e}_i i\not{D}e_i$$
$$+Y_u^{ij}\bar{Q}_i u_j\tilde{H} + Y_d^{ij}\bar{Q}_i d_j H + Y_l^{ij}\bar{L}_i e_j H + |D_\mu H|^2$$
$$-\lambda(H^\dagger H)^2 + \lambda v^2 H^\dagger H + \frac{\theta}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma}$$

What about baryon and lepton numbers? -> accidental symmetries!

Abelian versus non-abelian gauge theories

The (Yang-Mills) action $\mathcal{L}_{YM} = \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi - \frac{1}{2}F_{\mu\nu}F^{\mu\nu}$ is invariant under

$$\Psi(x) \to U(x)\Psi(x)$$

Abelian U(1) symmetry

$$U(x) = e^{iq\theta(x)}$$

$$A_\mu(x) \to A_\mu + \frac{i}{e}(\partial_\mu U)U^\dagger$$

Non-abelian SU(N)

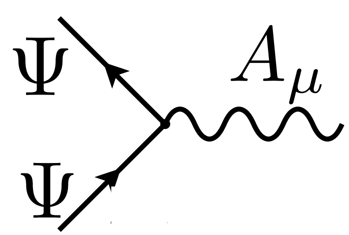
$$U(x) = e^{ig\theta^a(x)T^a}$$

T^a : N^2-1 generators (N×N matrices) acting on

$$A_\mu(x) = A_\mu^a T^a$$

$$A_\mu(x) \to U A_\mu U^\dagger - \frac{i}{g}(\partial_\mu U)U^\dagger$$

coupling constants (e, g)



infinitesimal transformation $U(x) = 1 + ig\theta^a(x)T^a + \mathcal{O}(\theta^2)$

$$A_\mu^a(x) \longrightarrow A_\mu^a + \partial_\mu\theta^a - gf^{abc}\theta^b A_\mu^c$$

$$D_\mu\Psi = (\partial_\mu + iqA_\mu)$$

$$D_\mu\Psi = (\partial_\mu - igA_\mu^a T^a)$$

The gauge symmetries of the Standard Model

Gauge Group $U(1)_Y$ (abelian)

$$\psi' = e^{+iY\alpha_Y}\psi,$$

$$B'_\mu = B_\mu - \frac{1}{g'}\partial_\mu \alpha_Y$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$D_\mu \psi_R = (\partial_\mu + i\, g'\, Y\, B_\mu)\, \psi_R$$

Gauge Group $SU(2)_L$ acts on the two components of a doublet Ψ_L=(u_L,d_L) or (ν_L,e_L)

$$\Psi_L \to e^{-iT^a\alpha^a}\psi_L \qquad U = e^{-iT^a\alpha^a} \qquad T^a = \sigma^a/2$$

Pauli matrices

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon^{abc}W^b_\mu W^c_\nu, \quad a = 1,\ldots,3$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = -i\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$D_\mu \psi_L = (\partial_\mu - i\, g\, W^a_\mu T^a)\, \psi_L$$

Gauge Group $SU(3)_c$ q=(q_1,q_2,q_3) (the three color degrees of freedom)

$$q \to e^{-iT^a\alpha^a} q \qquad U = e^{-iT^a\alpha^a} \qquad \left[T^a, T^b\right] = i f^{abc} T^c$$

(3×3) Gell-Man matrices

$$G^a_\mu T^a \to U G^a_\mu T^a U^{-1} - \frac{i}{g}\partial_\mu U U^{-1}$$

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g f^{abc} G^b_\mu G^c_\nu, \quad a = 1,\ldots,8$$

$$D_\mu q = (\partial_\mu - i\, g\, G^a_\mu T^a)\, q$$

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

The gauge symmetries of the Standard Model

Gauge Group $U(1)_Y$ (abelian)

$$\psi' = e^{+iY\alpha_Y}\psi,$$

$$B'_\mu = B_\mu - \frac{1}{g'}\partial_\mu \alpha_Y$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$D_\mu \psi_R = \left(\partial_\mu + i\, g'\, Y\, B_\mu\right)\psi_R$$

$$\mathcal{L}_{YM} = \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi - \frac{1}{2}F_{\mu\nu}F^{\mu\nu}$$

all Standard Model fermions carry U(1) charge

Gauge Group $SU(2)_L$

$$\Psi_L \rightarrow e^{-iT^a\alpha^a}\psi_L \qquad U = e^{-iT^a\alpha^a}$$

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon^{abc}W^b_\mu W^c_\nu, \quad a = 1,\ldots,3$$

$$D_\mu \psi_L = \left(\partial_\mu - i\, g\, W^a_\mu T^a\right)\psi_L$$

$\Psi_L=(u_L,d_L)$ or (ν_L, e_L)

only left-handed fermions charged under it -> chiral interactions

Gauge Group $SU(3)_c$

$$q \rightarrow e^{-iT^a\alpha^a}q \qquad U = e^{-iT^a\alpha^a}$$

$$G^a_\mu T^a \rightarrow U G^a_\mu T^a U^{-1} - \frac{i}{g}\partial_\mu U U^{-1}$$

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g f^{abc}G^b_\mu G^c_\nu, \quad a = 1,\ldots,8$$

$$D_\mu q = \left(\partial_\mu - i\, g\, G^a_\mu T^a\right) q$$

$q=(q_1,q_2,q_3)$

all quarks transform under it -> vector-like interactions

The lagrangian of the Standard Model

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

describe massless gauge bosons

$$\mathcal{L}_{\text{Fermion}} = \sum_{\text{quarks}} i\overline{q}\gamma^\mu D_\mu q + \sum_{\psi_L} i\overline{\psi_L}\gamma^\mu D_\mu \psi_L + \sum_{\psi_R} i\overline{\psi_R}\gamma^\mu D_\mu \psi_R$$

describe massless fermions and their interactions with gauge bosons

$$D_\mu \psi_R = \left[\partial_\mu + ig'YB_\mu\right]\psi_R$$

only left-handed fermions

all fermions carrying a $U(1)_Y$ charge i.e. all Standard Model fermions

$$\mathcal{L}_{\text{Higgs}} = (D_\mu\Phi)^\dagger D_\mu\Phi + \mu^2\Phi^\dagger\Phi - \lambda\left(\Phi^\dagger\Phi\right)^2$$

→ gives mass to EW gauge bosons $\frac{1}{2}M_Z^2 Z_\mu Z^\mu + M_W^2 W_\mu^+ W^{-\mu}$

$$D_\mu\Phi = \left[\partial_\mu - i\frac{g}{\sqrt{2}}\left(\tau^+W_\mu^+ + \tau^-W_\mu^-\right) - i\frac{g}{2}\tau_3 W_\mu^3 + i\frac{g'}{2}B_\mu\right]\Phi$$

: covariant derivative of the Higgs

H charged under $SU(2) \times U(1)_Y$

$$\mathcal{L}_{\text{Yukawa}} = -Y_l\,\overline{L}\,\Phi\,\ell_R - Y_d\,\overline{Q}\,\Phi\,d_R - Y_u\,\overline{Q}\,\widetilde{\Phi}\,u_R + \text{h.c.}$$

→ gives mass to fermions

responsible for electroweak symmetry breaking!

$$SU(3) \times SU(2)_L \times U(1)_Y \longrightarrow SU(3) \times U(1)_{em}$$

8 massless gluons

3 massive gauge bosons W^+ W^- Z_0

8 massless gluons

1 massless photon γ

remaining unbroken symmetry

The W and Z bosons interact with the Higgs medium, the γ doesn't.

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

$SU(3)_c$ $\qquad$ $SU(2)_L$ $\qquad$ $U(1)_Y$

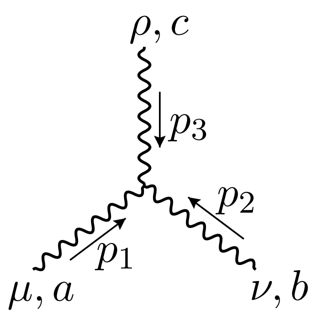
$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g f^{abc} G^b_\mu G^c_\nu \qquad W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon^{abc} W^b_\mu W^c_\nu, \qquad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

in mass eigen state basis

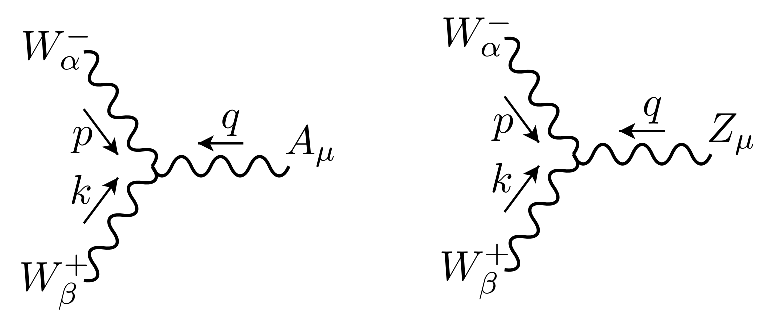
$$W^\pm_\mu = \frac{W^1_\mu \mp i W^2_\mu}{\sqrt{2}} \qquad Z_\mu = W^3_\mu \cos\theta_W + B_\mu \sin\theta_W$$

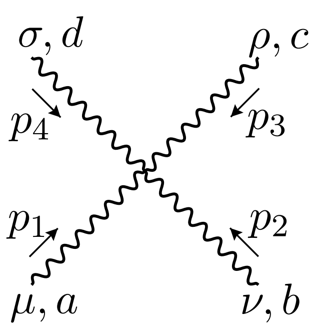
$$A_\mu = -W^3_\mu \sin\theta_W + B_\mu \cos\theta_W$$

$$\cos\theta_W = g/\sqrt{g^2 + g'^2} \qquad \sin\theta_W = g'/\sqrt{g^2 + g'^2}$$



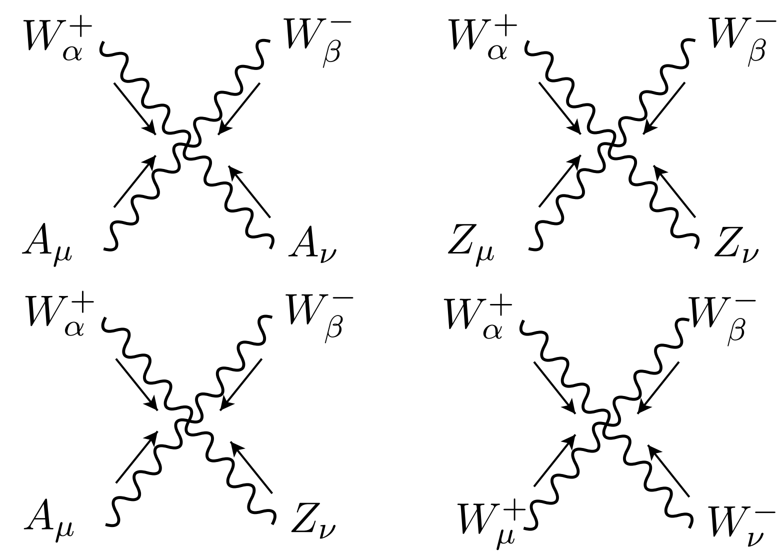
three gauge boson vertex





four gauge boson vertex

no such interactions for photon!



Charges of the Standard Model fields

Field	$SU(3)$	$SU(2)_L$	T^3	$\frac{Y}{2}$	$Q = T^3 + \frac{Y}{2}$
g^a_μ (gluons)	$\mathbf{8}$	$\mathbf{1}$	0	0	0
$(W^\pm_\mu, W^0_\mu)$	$\mathbf{1}$	$\mathbf{3}$	$(\pm 1, 0)$	0	$(\pm 1, 0)$
B^0_μ	$\mathbf{1}$	$\mathbf{1}$	0	0	0
$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\mathbf{3}$	$\mathbf{2}$	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$\frac{1}{6}$	$\begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \end{pmatrix}$
u_R	$\mathbf{3}$	$\mathbf{1}$	0	$\frac{2}{3}$	$\frac{2}{3}$
d_R	$\mathbf{3}$	$\mathbf{1}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$
$E_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$-\frac{1}{2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
e_R	$\mathbf{1}$	$\mathbf{1}$	0	-1	-1
$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$\frac{1}{2}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
$\Phi^c = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$-\frac{1}{2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$

The Standard Model of Particle Physics

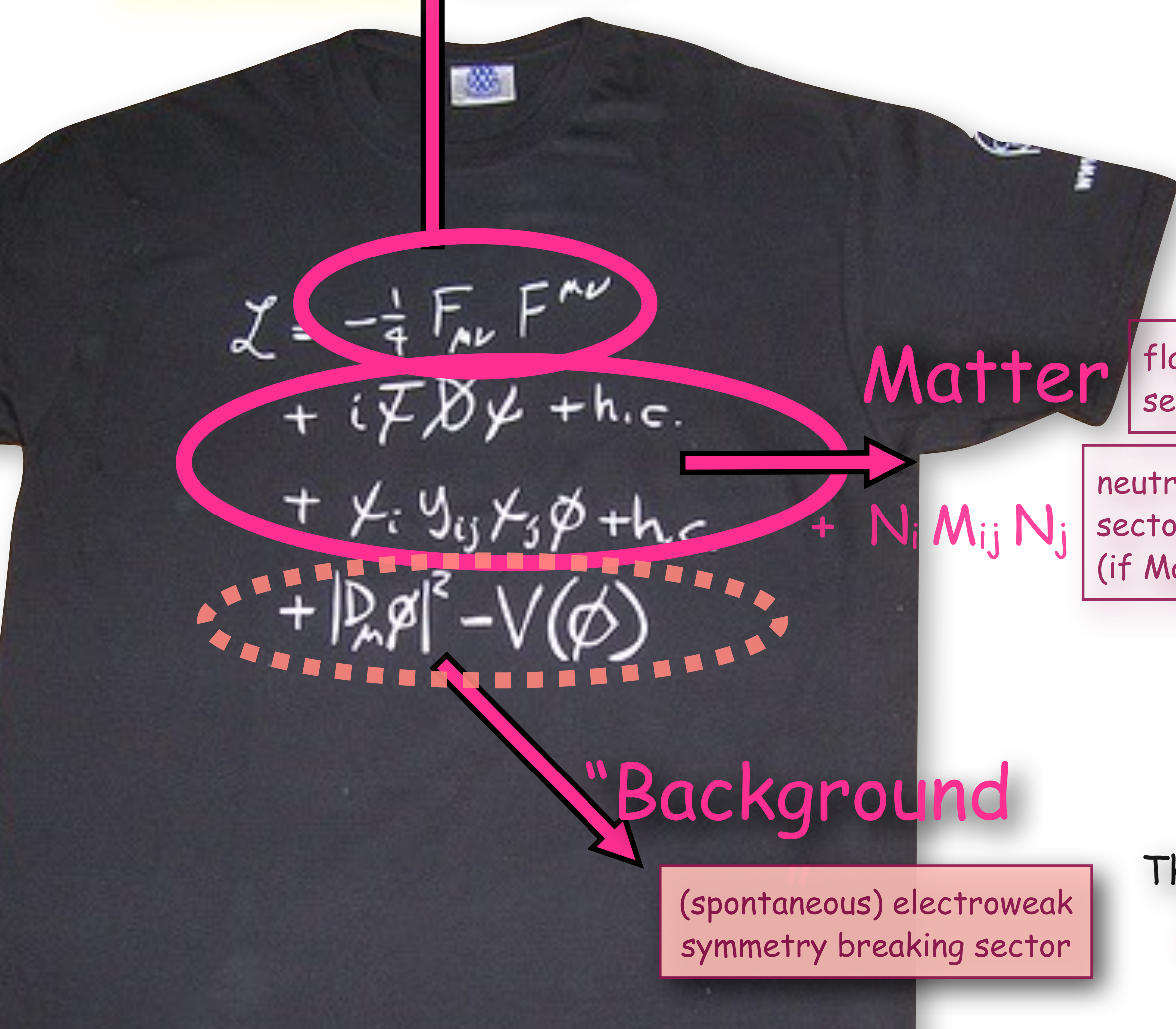
- one century to develop it
- tested with impressive precision
- accounts for all data in experimental particle physics

Forces

gauge sector

$SU(3)_C \times SU(2)_L \times U(1)_Y$

EW unification



Matter

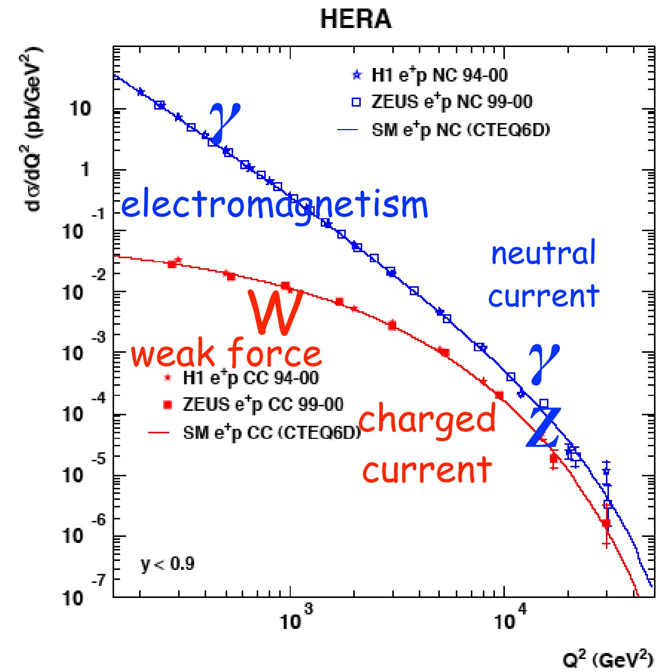
flavour sector

$+ \; N_i \, M_{ij} \, N_j$

neutrino mass sector (if Majorana)

"Background"

(spontaneous) electroweak symmetry breaking sector



This room is full of photons
but no W/Z
The symmetry between W, Z and γ
is broken at large distances

EM forces ≈ long ranges
Weak forces ≈ short range

$$m_\gamma < 6 \times 10^{-17} \text{ eV}$$

$$m_{W^\pm} = 80.425 \pm 0.038 \text{ GeV}$$

$$m_{Z^0} = 91.1876 \pm 0.0021 \text{ GeV}$$

The Higgs was the only remaining unobserved piece
and is a portal to new physics hidden sectors

Historically

Fermi Theory

(paper rejected by Nature: declared too speculative !)

$$n \xrightarrow{W^\pm} p + e^- + \bar{\nu}_e$$

$$\mathcal{L} = G_{\mathcal{F}} (\bar{n} p)(\bar{\nu}_e e)$$

exp: $G_F = 1.166 \times 10^{-5}$ GeV^{-2}

$$\mathcal{A} \propto G_{\mathcal{F}} E^2$$

- no continuous limit
- inconsistent above 300 GeV

Gauge theory

microscopic theory

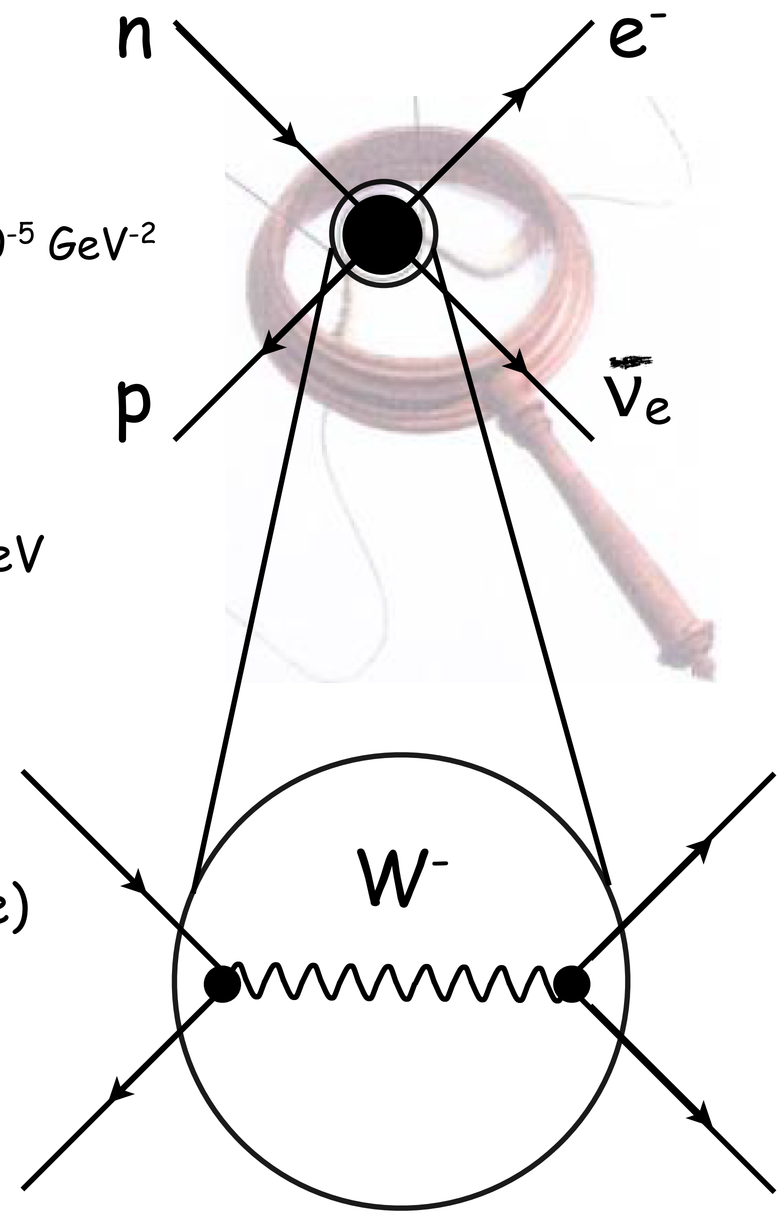
(exchange of a massive spin 1 particle)

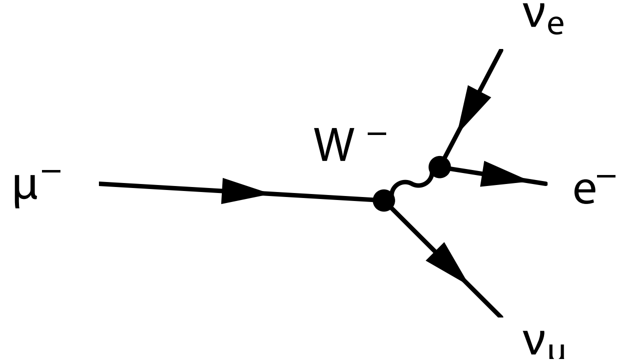
$$G_{\mathcal{F}} = \frac{\sqrt{2} g^2}{8 m_W^2}$$

exp: $m_W = 80.4$ GeV

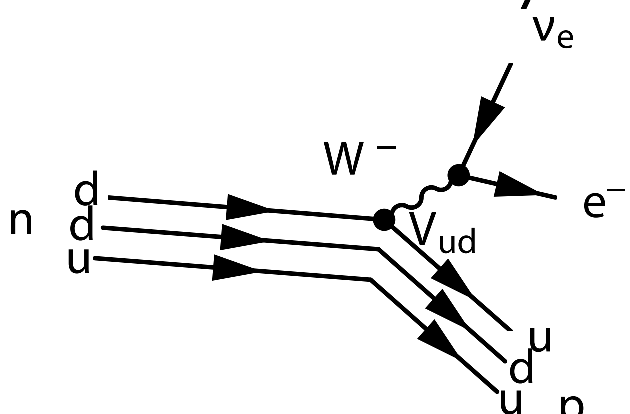
➡ $g \approx 0.6$, ie, same order as $e = 0.3$

unification EM & weak interactions





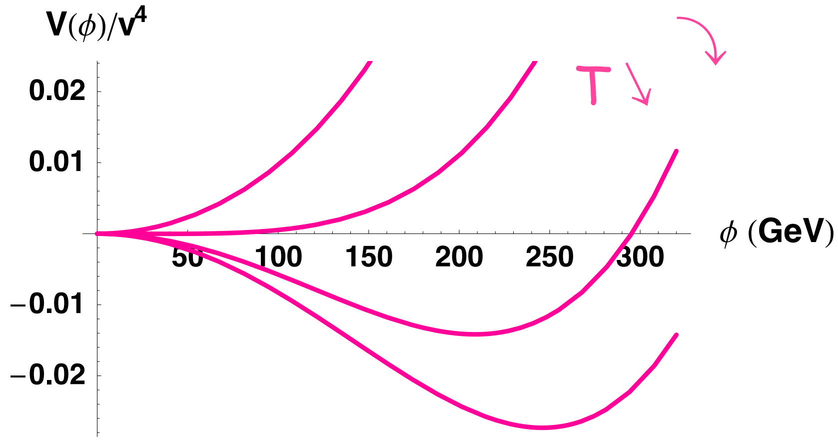
muon decay



beta decay

The (adhoc) Higgs Mechanism (a model without dynamics)

EW symmetry breaking is described by the condensation of a scalar field

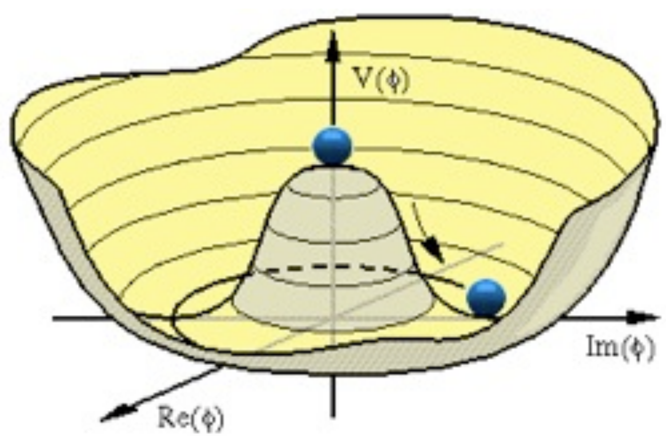


$$\Phi = \begin{bmatrix} \phi^+ \\ \frac{v + H + i\varphi_Z}{\sqrt{2}} \end{bmatrix}$$

Background value, Higgs medium

Higgs boson: excitation of the higgs medium

The Higgs selects a vacuum state by developing a non zero background value. When it does so, it gives mass to SM particles it couples to.



$$V(\Phi) = \frac{\mu^2}{2}\Phi^\dagger\Phi + \frac{\lambda}{4}\Phi^\dagger\Phi$$

Why is μ^2 negative ?

the puzzle:

We do not know what makes the Higgs condensate.
We ARRANGE the Higgs potential so that the Higgs condensates but this is just a parametrization that we are unable to explain dynamically.

- We have quantized free fields

- We have introduced interactions

(particle creation and annihilation can only take place in theory with interactions)

We now would like to compute probability of processes like for instance a two-body decay a->c+d or a two-body reaction a+b->c+d

“S-matrix approach”-> calculate probability of transition between two asymptotic states

The S-matrix

We consider a state |a>(t) which at an initial time t_i is labelled |a>.
Similarly we consider a state |b>(t) which at a final time t_f is labelled |b>

At t_f the state |a>(t) as evolved as $e^{-iH(t_f - t_i)}|a>$

where H is the hamiltonian of the theory

The amplitude for the process in which the initial state |a> evolves into the final state |b> is given by

$$\mathcal{M} = < b|e^{-iH(t_f - t_i)}|a >$$

the final state is a set of well-separated particles long after the interaction

evolution operator "S-matrix"

the initial state is either a one-particle (decay) or two well-separated particles (scattering), long before interaction happens

|a> and |b> are both described by free fields

The probability of the process is given by $|\mathcal{M}|^2$

and that can be linked to a transition rate per volume unit as measured by an experiment

Link to observables

- cross section: reaction rate per target particle per unit incident flux

$$\frac{[1/\text{time}]}{[1/(\text{time length}^2)]}$$

--> has units of a surface

measured in multiples of 1 barn= $10^{-24}\ \text{cm}^2$

typical relevant LHC cross sections ~ in pb

1 picobarn= 1 pb= 10^{-36}cm^2

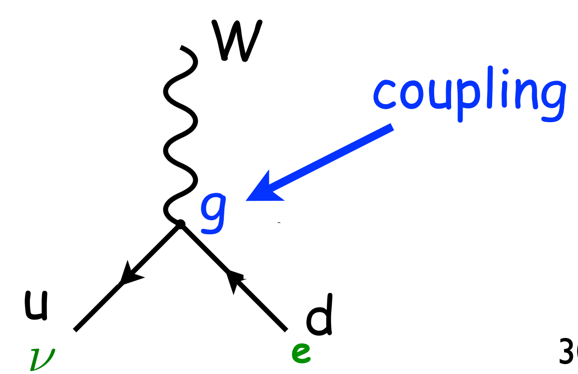
- Decay width (inverse of lifetime of a particle) =transition rate

has dimension [1/time]

Example: decay width of EW gauge bosons

$$\Gamma \propto |\mathcal{M}|^2$$

scales as the square of the coupling constant

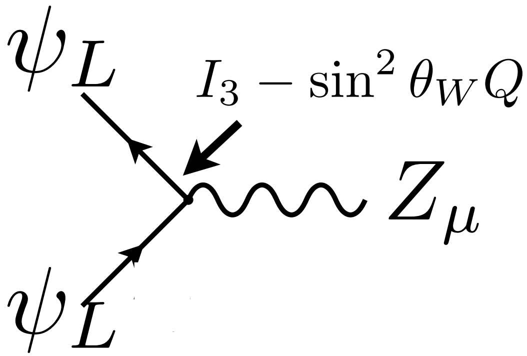


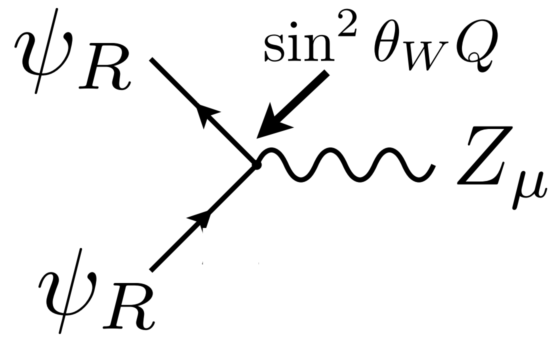
Z couplings to fermions

The coupling of Z to any fermion is proportional to $I_3 - \sin^2\theta_W Q$

where $I_3 = \pm\frac{1}{2}$ is z-component of weak isospin and Q is electric charge

$$\sin^2\theta_W = 0.231$$





for the quarks:

u_L	$I_3 = +1/2$	$Q = +2/3$
u_R	$I_3 = 0$	$Q = +2/3$
d_L	$I_3 = -1/2$	$Q = -1/3$
d_R	$I_3 = 0$	$Q = -1/3$

and similarly for c,s, and b (t is too heavy for the Z to decay into it)

for the leptons:

e_L	$I_3 = -1/2$	$Q = -1$
e_R	$I_3 = 0$	$Q = -1$
ν_{e_L}	$I_3 = +1/2$	$Q = 0$

and similarly for $\nu, \tau, \nu_\mu, \nu_\tau$

Branching fractions for Z decay

for the quarks:

u_L	$I_3 = +1/2$	$Q = +2/3$
u_R	$I_3 = 0$	$Q = +2/3$
d_L	$I_3 = -1/2$	$Q = -1/3$
d_R	$I_3 = 0$	$Q = -1/3$

and similarly for c,s, and b (t is too heavy for the Z to decay into it)

for the leptons:

e_L	$I_3 = -1/2$	$Q = -1$
e_R	$I_3 = 0$	$Q = -1$
ν_{e_L}	$I_3 = +1/2$	$Q = 0$

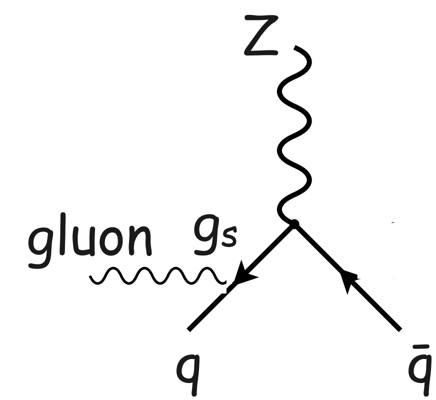
and similarly for $\nu, \tau, \nu_\mu, \nu_\tau$

The decay rate is proportional to the square of the coupling constant $I_3 - \sin^2\theta_W Q$

Also, for quarks, there is an additional factor $(1 + \frac{\alpha_s}{2\pi})$ where $\alpha_s = g_s^2/4\pi = 0.118$ due to the additional gluon emission

$$B(Z \to e^+e^-) = B(Z \to e^+_L e^-_L) + B(Z \to e^+_R e^-_R)$$

$$B(Z \to e^+_L e^-_L) = \frac{\Gamma(Z \to e^+_L e^-_L)}{\sum_{all\ particles} \Gamma(Z \to particle, antiparticle)}$$



$$B(Z \to \nu\bar{\nu}) = B(Z \to \nu_e\bar{\nu}_e) + B(Z \to \nu_\mu\bar{\nu}_\mu + B(Z \to \nu_\tau\bar{\nu}_\tau)$$
$$= 3B(Z \to \nu_e\bar{\nu}_e) = 20\%$$

$$B(Z \to e^+e^-) = B(Z \to \mu^+\mu^-) = B(Z \to \tau^+\tau^-) = 3.33\%$$
$$B(Z \to all\ hadrons) = 3 \times [B(Z \to u\bar{u}) + B(Z \to d\bar{d}) + B(Z \to s\bar{s})$$
$$+B(Z \to c\bar{c}) + B(Z \to b\bar{b})] = 69.9\%$$

Branching fractions for W decay

$$W^- \rightarrow e^-\bar{\nu}_e, \mu^-\bar{\nu}_\mu, \tau^-\bar{\nu}_\tau, d'\bar{u}, s'\bar{c}.$$

$$BR(W^- \rightarrow e^-\bar{\nu}_e) = BR(W^- \rightarrow \mu^-\bar{\nu}_\mu) = BR(W^- \rightarrow \tau^-\bar{\nu}_\tau)$$
$$= \frac{1}{3 + 6(1 + \alpha_s/\pi)} = 0.108,$$

$$BR(W^- \rightarrow \text{hadrons}) = \frac{6(1 + \alpha_s/\pi)}{3 + 6(1 + \alpha_s/\pi)} = 0.675.$$