Theoretical Concepts in Particle Physics

Lecture III : Towards beyond the Standard Model

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The beauty of the SM comes from the identification of a unique dynamical principle describing interactions that seem so different from each others

gauge theory = spin-1

The most general lagrangian given the particle content

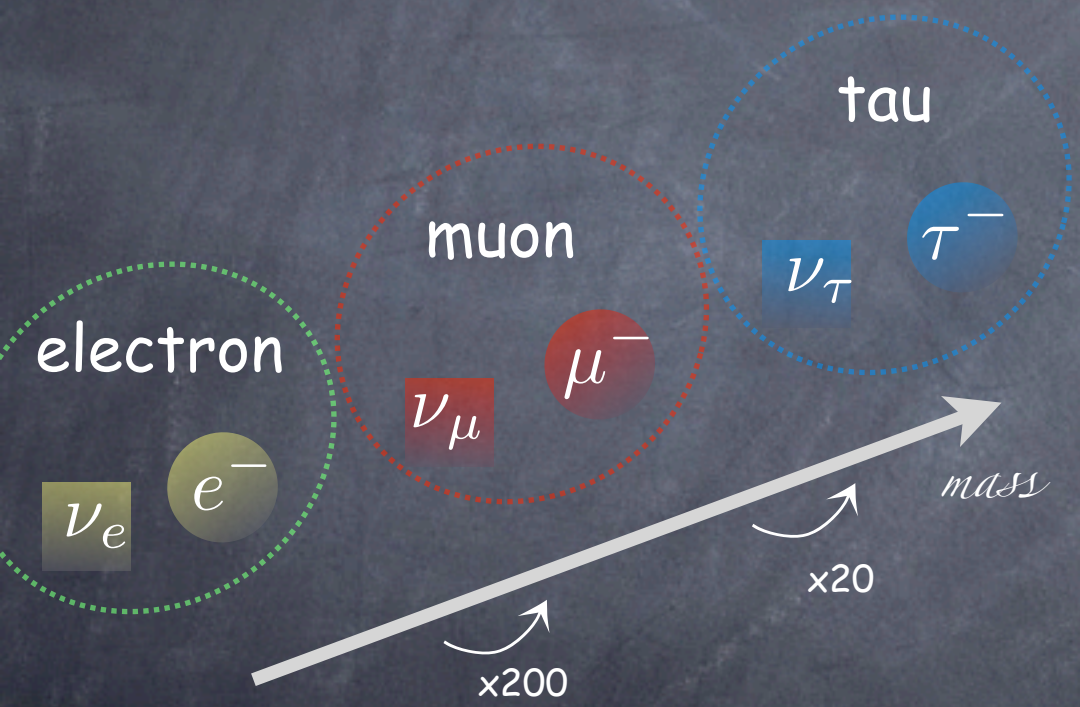
$$
\begin{aligned}
\mathcal{L} \quad = \quad & -\frac{1}{4g'^2}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4g^2}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4g_s^2}G^a_{\mu\nu}G^{a\mu\nu} \\
& +\bar{Q}_i i\not{D}Q_i + \bar{u}_i i\not{D}u_i + \bar{d}_i i\not{D}d_i + \bar{L}_i i\not{D}L_i + \bar{e}_i i\not{D}e_i \\
& +Y_u^{ij}\bar{Q}_i u_j \tilde{H} + Y_d^{ij}\bar{Q}_i d_j H + Y_l^{ij}\bar{L}_i e_j H + |D_\mu H|^2 \\
& -\lambda(H^\dagger H)^2 + \lambda v^2 H^\dagger H + \frac{\theta}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma}
\end{aligned}
$$

What about baryon and lepton numbers? -> accidental symmetries!

The Standard Model: matter

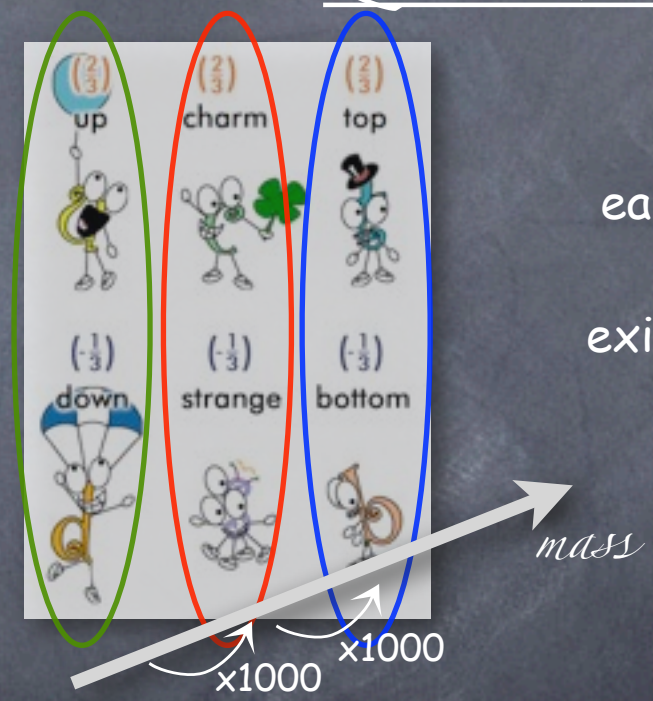
the elementary blocks:

LEPTONS



no composite states made of leptons

QUARKS



each of the 6 quarks exists in three colors

composite states (white objects

0 baryons

proton $p = (u, u, d)$

neutron $n = (u, d, d)$

0 mesons

+ antiparticles

Symmetries

I- Continuous global space-time (Poincaré) symmetries

all particles have (m, s)

-> energy, momentum, angular momentum conserved

II- Global (continuous) internal symmetries

-> B, L conserved

(accidental symmetries)

III- Local or gauge internal symmetries

-> color, electric charge conserved

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

IV- Discrete symmetries

-> CPT

Quantum numbers and Conservation laws

When the positron was discovered, it raised a naive question:
why can't a proton decay into a positron and a photon $p \to e^+ \gamma$?

This process would conserve momentum, energy, angular momentum, electric charge and even parity

This can be understood if we impose conservation of baryon number

Similarly, when the muon was discovered, it raised the question:
why doesn't a muon decay as $\mu^- \to e^- \gamma$?

This led to propose another quantum number: lepton family number

The following processes have not been seen.
Explain which conservation law forbids each of them

$$n \longrightarrow p\mu^-\bar{\nu}_\mu$$

$$\mu^- \rightarrow e^-e^-e^+$$

$$n \longrightarrow p\nu_e\bar{\nu}_e$$

$$p \longrightarrow e^+\pi^0$$

$$\tau^- \longrightarrow \mu\gamma$$

$$K^0 \rightarrow \mu^+e^-$$

$$\mu^- \rightarrow \pi^-\nu_\mu$$

The following processes have not been seen.
Explain which conservation law forbids each of them

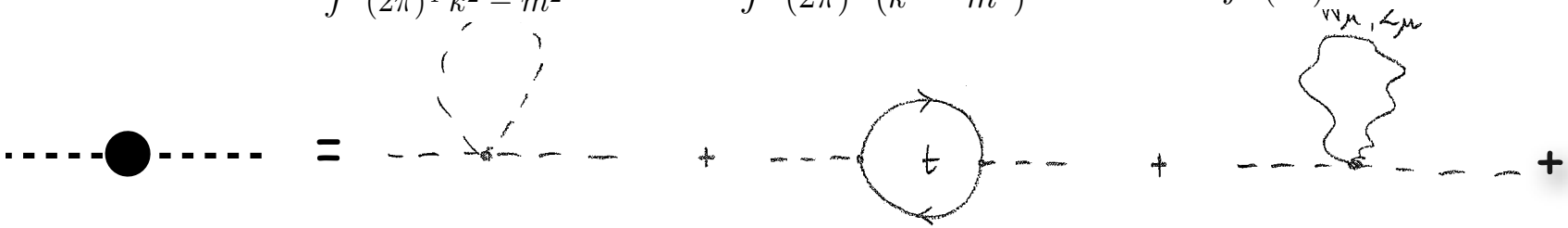
$n \longrightarrow p\mu^- \bar{\nu}_\mu$	energy
$\mu^- \rightarrow e^- e^- e^+$	muon number or electron number
$n \longrightarrow p\nu_e \bar{\nu}_e$	electric charge
$p \longrightarrow e^+ \pi^0$	baryon number or electron number
$\tau^- \longrightarrow \mu\gamma$	tau number or muon number
$K^0 \rightarrow \mu^+ e^-$	muon number or electron number
$\mu^- \rightarrow \pi^- \nu_\mu$	energy

The hierarchy problem

As soon as we introduce a fundamental scalar field in the theory (the Higgs), this generates a puzzle: the so-called "hierarchy problem".

= the fact that the Higgs self-energy receives radiative contributions that are quadratically divergent.

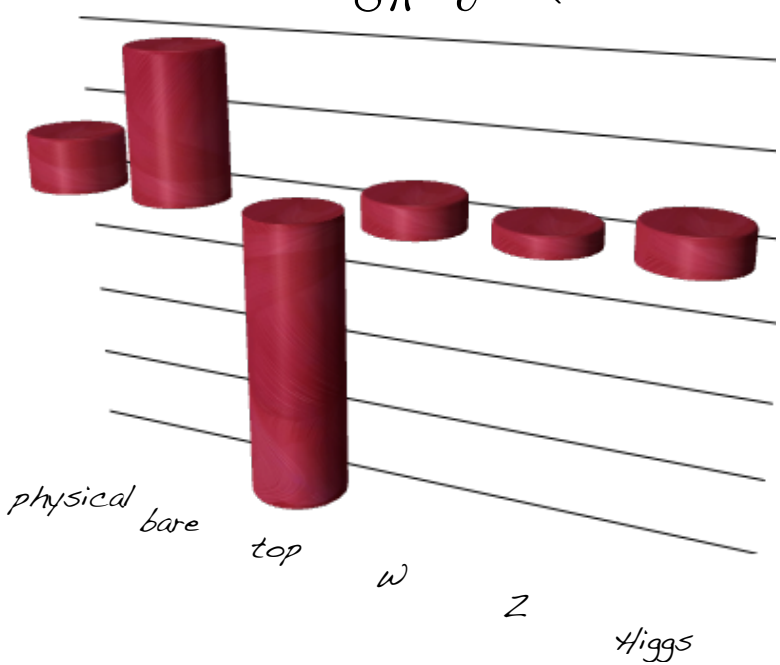
$$\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2-m^2} \propto \Lambda^2 \qquad \int \frac{d^4k}{(2\pi)^4}\frac{k^2}{(k^2-m^2)^2} \propto \Lambda^2 \qquad \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2-m^2} \propto \Lambda^2$$



Higher loops
Smaller Yukawa

$$\delta m_H^2 = \frac{3\Lambda^2}{8\pi^2 v^2}\left(2m_W^2 + m_Z^2 + m_H^2 - 4m_t^2\right) \sim -(0.23\ \Lambda)^2$$

strong sensitivity on high energy unknown physics



Λ , the maximum mass scale that the theory describes

To stabilise the Higgs mass at the EW scale against the Planck scale, we need to adjust the parameter of the Higgs potential at a level of 10^{-32}.

History repeats itself?

There are examples in physics where unexpected and precise parameter cancellations were actually the signal of the existence of new particles.

For instance, the electron self energy has a power divergence that can be cured only by the introduction of the positron.

Similarly, the extreme sensitivity of the Higgs self energy with respect to physics at high momentum can be naturally reduced by introducing new symmetries and new degrees of freedom, such as supersymmetry, or extra spacial dimensions or additional global symmetries.

Classical/Quantum Electromagnetism & Antimatter

an electron makes an electric field which carries an energy

$$\Delta E_{\text{Coulomb}} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r_e}$$

electric charge

classical size of the electron

and interacts back to the electron and contributes to its mass $\delta m c^2 = \Delta E_{\text{Coulomb}}$

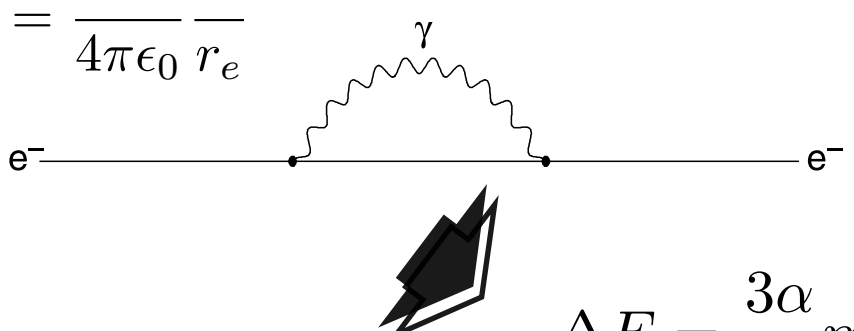
$$\delta m \sim m_e \quad \Rightarrow \quad r_e \sim \frac{e^2}{4\pi\epsilon_0 m_e c^2} \sim 10^{-13}\ \text{m}$$

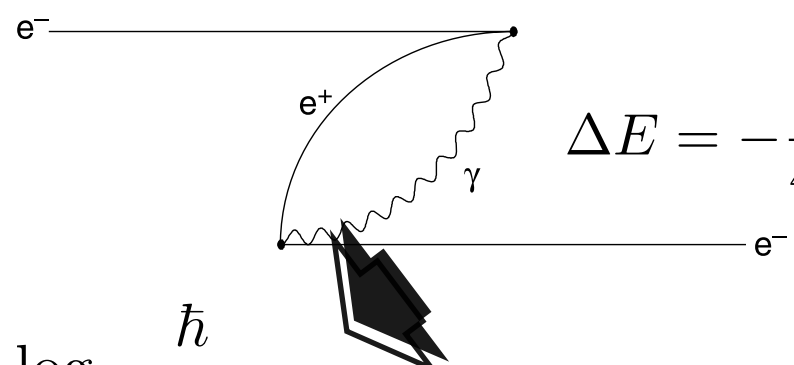
The electron repels itself due to its charge, how to keep electric charge in a small pack?

electron point like!

antimatter comes to rescue the 19th century "electron crisis"

$$\Delta E = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r_e}$$





$$\Delta E = -\frac{1}{4\pi\epsilon_0}\frac{e^2}{r_e}$$

$$\Delta E = \frac{3\alpha}{4\pi} m_e c^2 \log\frac{\hbar}{r_e m_e c}$$

Weisskopf '39

new states ≈ softer UV behavior, small correction to the mass

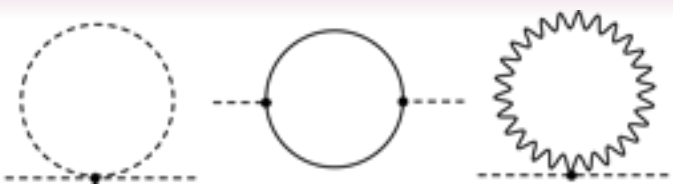
The hierarchy problem: What is keeping the Higgs boson light?

we need new degrees of freedom to cancel Λ^2 divergences and ensure the stability of the weak scale

a problem that arises for any elementary SCALAR particle

does not arise for fermions (protected by chiral symmetry) or gauge bosons (protected by the gauge symmetry)

What is cancelling the divergent diagrams?



$$\Rightarrow \delta M_H^2 \propto \Lambda^2$$

A light Higgs calls for New Physics at the TeV scale

the "hierarchy problem": the main motivation for building the LHC

Addressing the hierarchy problem

($\delta m_h^2|_{1-loop} \sim -\frac{y_t^2}{8\pi^2}\Lambda_{UV}^2$)

with a new symmetry

Supersymmetry

fermion

$$\Psi \longrightarrow e^{i\theta\gamma_5} \ \Psi$$

Ψ massless:

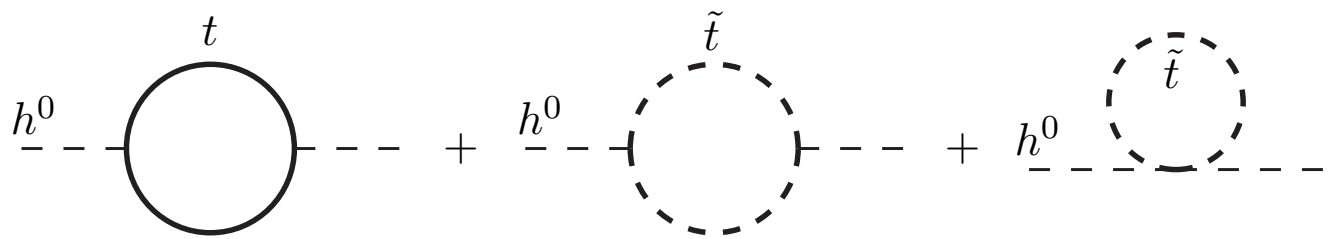
protected by
chiral symmetry

$\Psi \overset{\text{SUSY}}{\longleftrightarrow} H$

The Minimal Supersymmetric Standard Model (MSSM)

Supersymmetry can solve the big hierarchy and naturalness is preserved up to very high scales if superparticle masses are at the weak scale

$\Delta(m^2_{h^0}) =$



$$\delta m_H^2 \sim -\frac{3\,h_t^2}{8\pi^2} m_{\tilde{t}}^2 \log \frac{\Lambda^2}{m_{\tilde{t}}^2}$$

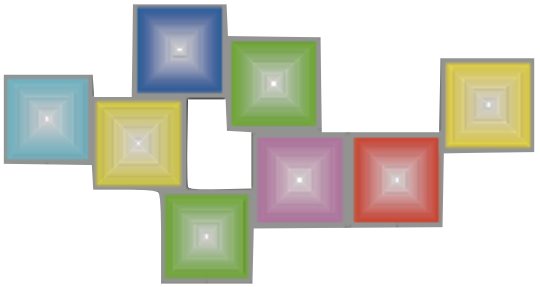
An elegant solution to the hierarchy pb: Supersymmetry

2 categories of particles:

Fermions

matter particles

fermions repel each other

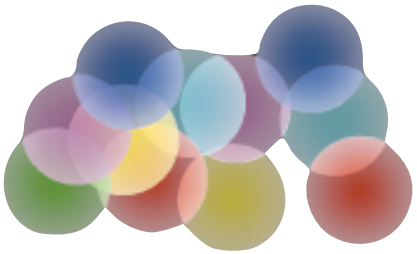


[Enrico Fermi 1901-1954]

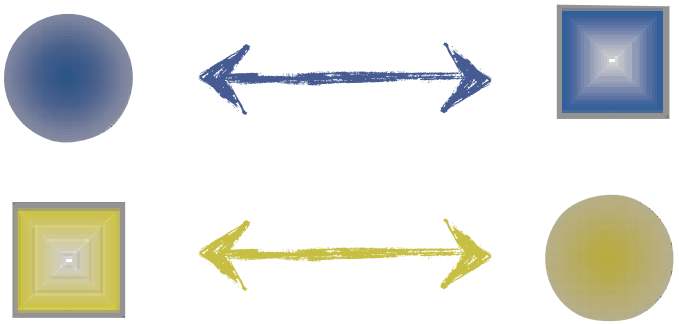
Bosons

force carriers

bosons can pile up



[Satyendra Bose 1894-1974]

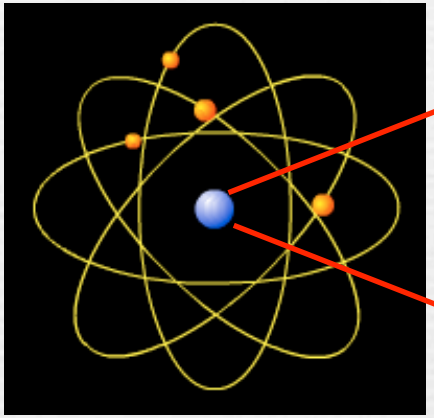


String Theory

unification of the Standard Model forces with gravity

(observable universe)

$10^{-10}\mathrm{m}$

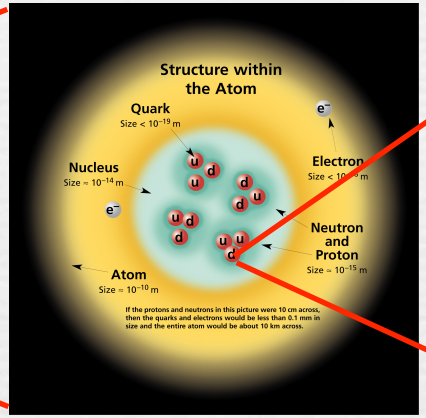


Atom

electrons + nucleus

(Earth)

$10^{-17}\mathrm{m}$

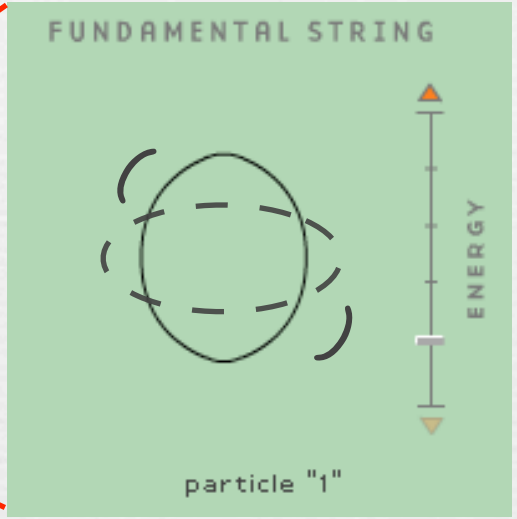


Nucleus

quarks

(Hair)

$10^{-35}\mathrm{m}$

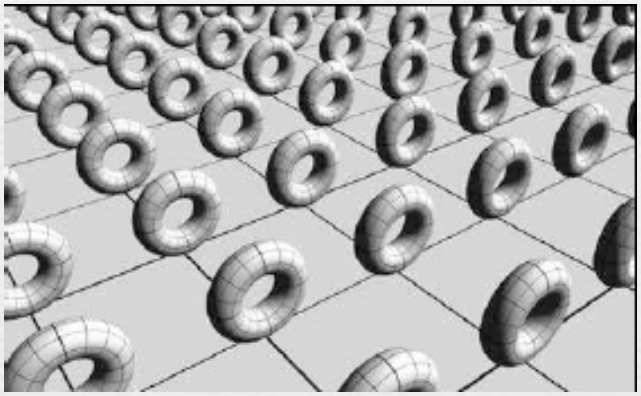


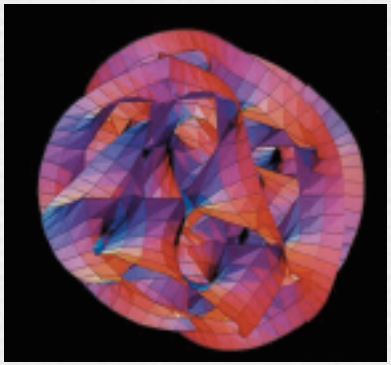
(Super)String

Extra Dimensions

String theories are (well) defined only in spacetime with 10 or 11 dimensions

These extra dimensions are assumed to be curled up





Is the Standard Model a remnant of a Grand Unified Theory ?

How come is the electric charge quantized? $Q_e = T_3 + Y$

- Eigen values of the generators of the abelian U(1) are continuous
  e.g. in the symmetry of translational invariance of time,
  there is no restriction in the (energy) eigen values.

- Eigen values of the generators of a simple non-abelian group are discrete

  e.g. in SO(3) rotations, the eigen values of the third component of angular momentum can take only integers or 1/2 integers values. In SU(5), since the electric charge is one of the generators, its eigen values are discrete and hence quantized.

simple unification group -> **Charge quantization** .

$SU(3)_c \times SU(2)_L \times U(1)_Y \subset SU(5)$

SM matter content fits nicely into SU(5)

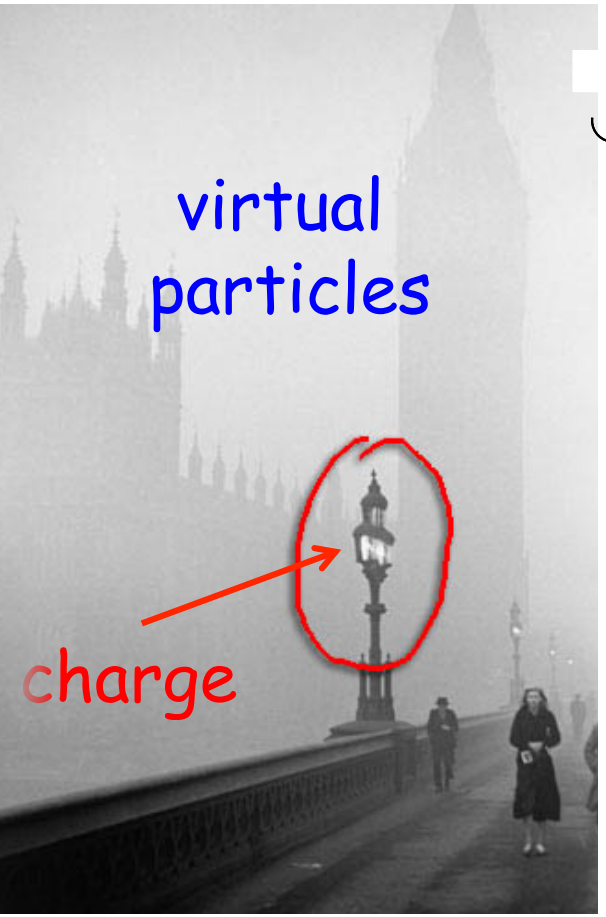
relation between color SU(3) and electric charge.

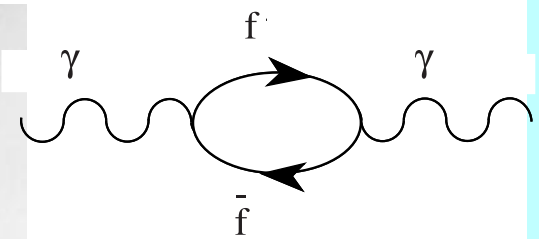
Quarks carry 1/3 of the lepton charge because they have 3 colors. The SU(5) theory provides a rationale basis for understanding particle charges and the weak hypercharge assignment in the SM

Evolution of coupling constants

Classical physics: the forces depend on distances

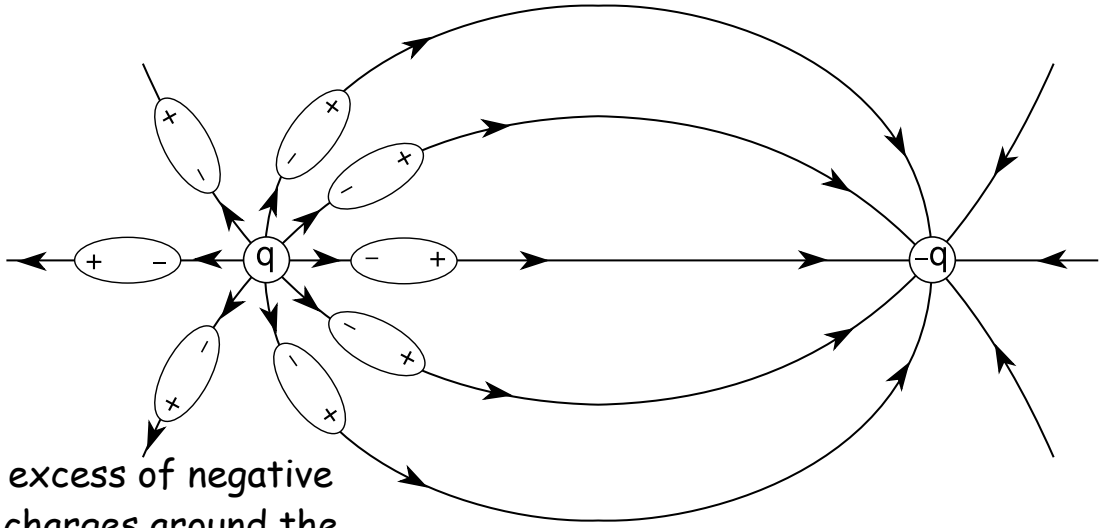
Quantum physics : the charges depend on distances





The electromagnetic coupling decreases at large distances.
Charge screening (vacuum polarization) due to virtual fermion-antifermion pairs

The vacuum behaves as a polarized dielectric medium



excess of negative charges around the positron: screening

An opposite effect for the strong coupling

because of the **non-abelian** nature of the underlying SU(3) gauge symmetry: the gauge boson self-interactions generate an **anti-screening** effect through gauge boson loops. This effect is larger than the one from fermion loops --> the strong coupling decreases at short distances

$$\frac{\partial \alpha_s}{\partial \log E} = \beta(\alpha_s) = \frac{\alpha_s^2}{\pi}\left(-\frac{11N_c}{6} + \frac{N_f}{3}\right)$$

$\alpha_s \nearrow$ when $d \nearrow$

$$\alpha_s = g_s^2/4\pi$$

property of 'asymptotic freedom'

quarks behave as free particles when the energy becomes very large

Evolution of gauge couplings

The evolution of gauge couplings is controlled by the renormalization group equations

$$\frac{d\alpha(\mu)}{d\log\mu} \equiv \beta(\alpha(\mu))$$

At one loop:

$$\beta(\alpha) \equiv \frac{d\alpha(\mu)}{d\log\mu} = \frac{-b}{2\pi}\,\alpha^2 + \mathcal{O}(\alpha^3)$$

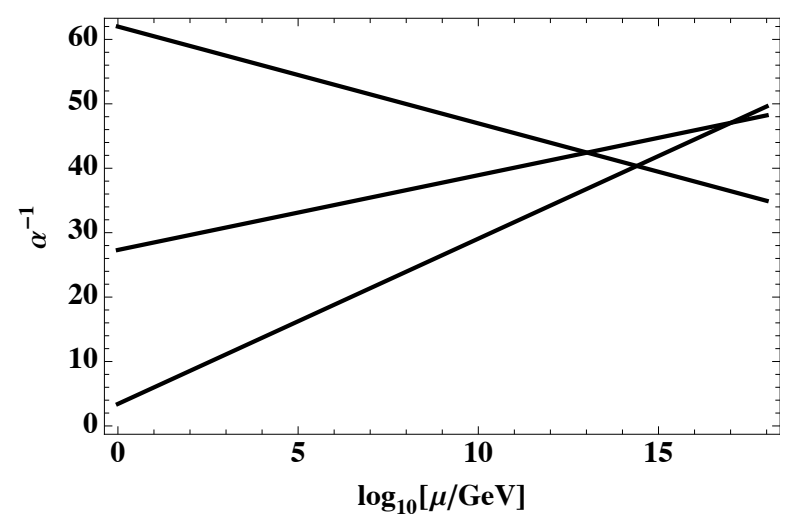
So couplings vary logarithmically as a function of the energy scale:

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(\mu_0)} + \frac{b_i}{2\pi}\log\frac{\mu}{\mu_0}$$

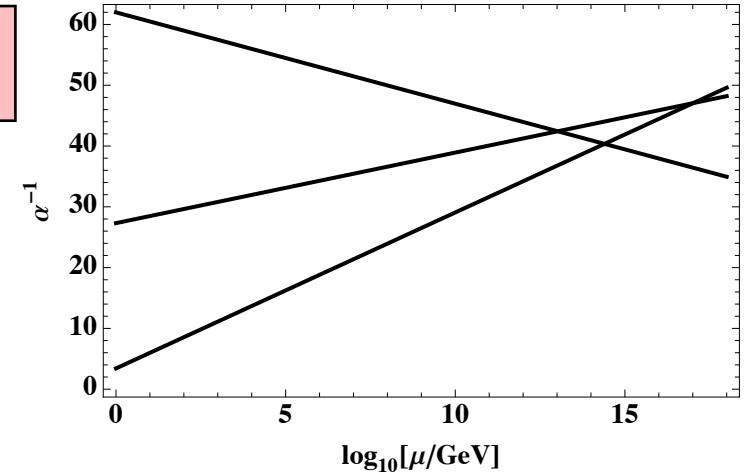
$$\alpha_i = g_i^2/4\pi \qquad i = SU(3), SU(2), U(1)$$

b_i : defined by the particle content

we observe different couplings but it looks like a low energy artefact



Unification?



- **we observe different couplings but it is a low energy artefact**

- $SU(3)_c \times SU(2)_L \times U(1)_Y \subset SU(5)$

SM matter content fits nicely into SU(5)

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} = (3,2)_{1/6}, \quad u_R^c = (\bar{3},1)_{-2/3}, \quad d_R^c = (\bar{3},1)_{1/3},$$

$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = (1,2)_{-1/2}, \quad e_R^c = (1,1)_1$$

$$\bar{5} = (1,2)_{-\frac{1}{2}\sqrt{\frac{3}{5}}} + (\bar{3},1)_{\frac{1}{3}\sqrt{\frac{3}{5}}}$$

$$\bar{5} = L + d_R^c$$

$$\chi^a = \left(d^{c1}\ d^{c2}\ d^{c3}\ e^-\ -\nu_e\right)_L^T,$$

$$10 = (5\times 5)_A = (\bar{3},1)_{-\frac{2}{3}\sqrt{\frac{3}{5}}} + (3,2)_{\frac{1}{6}\sqrt{\frac{3}{5}}} + (1,1)_{\sqrt{\frac{3}{5}}}$$

$$10 = u_R^c + Q_L + e_R^c$$

$$\psi_{ab} = \frac{1}{\sqrt{2}} \left(\begin{array}{ccc|cc} 0 & u^{c3} & -u^{c2} & -u_1 & -d_1 \\ -u^{c3} & 0 & u^{c1} & -u_2 & -d_2 \\ u^{c2} & -u^{c1} & 0 & -u_3 & -d_3 \\ \hline u_1 & u_2 & u_3 & 0 & -e^+ \\ d_1 & d_2 & d_3 & e^+ & 0 \end{array}\right)_L.$$

SU(5) adjoint rep.

$$\left(\begin{array}{c|c} SU(2) & \\ \hline & SU(3) \end{array}\right)$$

$$T^{12} = \sqrt{\frac{3}{5}} \left(\begin{array}{cc|ccc} 1/2 & & & & \\ & 1/2 & & & \\ \hline & & -1/3 & & \\ & & & -1/3 & \\ & & & & -1/3 \end{array}\right) = \sqrt{\frac{3}{5}} Y$$

additional U(1) factor that commutes with SU(3)*SU(2)

$$\mathrm{Tr}(T^a T^b) = \frac{1}{2}\delta^{ab}$$

$$g_5 T^{12} = g' Y \qquad g_5 \sqrt{\frac{3}{5}} = g' \qquad g_5 = g = g_s$$

$\sin^2\theta_W = \frac{3}{8}$ @ M_{GUT}

Standard Model beta functions

$$b = \frac{11}{3}T_2(\text{spin-1}) - \frac{2}{3}T_2(\text{chiral spin-1/2}) - \frac{1}{3}T_2(\text{complex spin-0})$$

$$Tr(T^a(R)T^b(R)) = T_2(R)\delta^{ab} \qquad T_2(\text{fund}) = \frac{1}{2} \qquad T_2(\text{adj}) = N$$

universal contribution coming from complete SU(5) representations ($4N_F/3$ in SM in $4N_F/3$ *3/2 in susy)

from gauge bosons

So in the SM:

$$b_3 = \frac{11}{3} \times N_c - \frac{2}{3} \times N_f \left(\frac{1}{2} \times 2 + \frac{1}{2} \times 1 + \frac{1}{2} \times 1\right) = 7$$

$$b_2 = \frac{11}{3} \times 2 - \frac{2}{3} \times N_f \left(\frac{1}{2} \times 3 + \frac{1}{2} \times 1\right) - \frac{1}{3} \times \frac{1}{2} = \frac{19}{6}$$

$$b_Y = -\frac{2}{3} \times N_f \left((\frac{1}{6})^2 \times 2 \times N_c + (\frac{-2}{3})^2 \times N_c + (\frac{1}{3})^2 \times N_c + (\frac{-1}{2})^2 \times 2 + (1)^2\right) \times 3/5$$

$$-\frac{1}{3}(\frac{1}{2})^2 \times 2 = -\frac{41}{6} \qquad \longrightarrow \qquad b_1 = b_Y \times \frac{3}{5} = -\frac{41}{10}$$

from Higgs

$$\alpha_i^{-1}(M_Z) = \alpha_{GUT}^{-1} - \frac{b_i}{4\pi}\log\frac{M_{GUT}^2}{M_Z^2} + \Delta_i \qquad i = SU(3), SU(2), U(1)$$

$\alpha_3(M_Z), \alpha_2(M_Z), \alpha_1(M_Z)$: experimental inputs

b_3, b_2, b_1 : predicted by the matter content

3 equations and 2 unknowns (α_{GUT}, M_{GUT})

1 consistency relation for unification

Using $\alpha_1 = \frac{5}{3}\frac{1}{\cos^2\theta_W}\alpha_{em}$ and $\alpha_2 = \frac{\alpha_{em}}{\sin^2\theta_W}$

we obtain: $\epsilon_{ijk}(\alpha_i^{-1} - \Delta_i)(b_j - b_k) = 0$

If the Δ_i contributions are universal ($\Delta_1 = \Delta_2 = \Delta 3$) or negligible , this translates into

$$\sin^2\theta_W = \frac{3(b_3 - b_2) + 5(b_2 - b_1)\frac{\alpha_{em}(M_Z)}{\alpha_s(M_Z)}}{8b_3 - 3b_2 - 5b_1}$$

$\alpha_{em}(M_Z) \approx 1/128$

$\alpha_s(M_Z) \approx 0.1184 \pm 0.0007$

In the SM: $\sin^2\theta_W \approx 0.207$

Not so bad ...

to be compared with 0.2312+/-0.0002

Only the Higgs and the SM gauge bosons can affect the relative running

In the MSSM, extra contributions from the higgsinos and gauginos

SM

$$b = \frac{11}{3}T_2(\text{spin-1}) - \frac{2}{3}T_2(\text{chiral spin-1/2}) - \frac{1}{3}T_2(\text{complex spin-0})$$

$$Tr(T^a(R)T^b(R)) = T_2(R)\delta^{ab} \qquad T_2(\text{fund}) = \frac{1}{2} \qquad T_2(\text{adj}) = N$$

chiral superfield

complex spin-0
Weyl spin-1/2
in same representation R of gauge group

vector superfield

Weyl spin-1/2
real spin-1
in same representation V of gauge group

MSSM

$$b = \frac{11}{3}T_2(\texttt{vector}) - \frac{2}{3}T_2(\texttt{vector}) - \frac{2}{3}T_2(\texttt{chiral}) - \frac{1}{3}T_2(\texttt{chiral}) = 3\,T_2(\texttt{vector}) - T_2(\texttt{chiral})$$

gauginos scalars

$$b_{SU(3)} = \overset{g}{3\times 3} - \left(\overset{Q_L}{\frac{1}{2}\times 2\times 3} + \overset{U}{\frac{1}{2}\times 1\times 3} + \overset{D}{\frac{1}{2}\times 1\times 3}\right) = 3$$

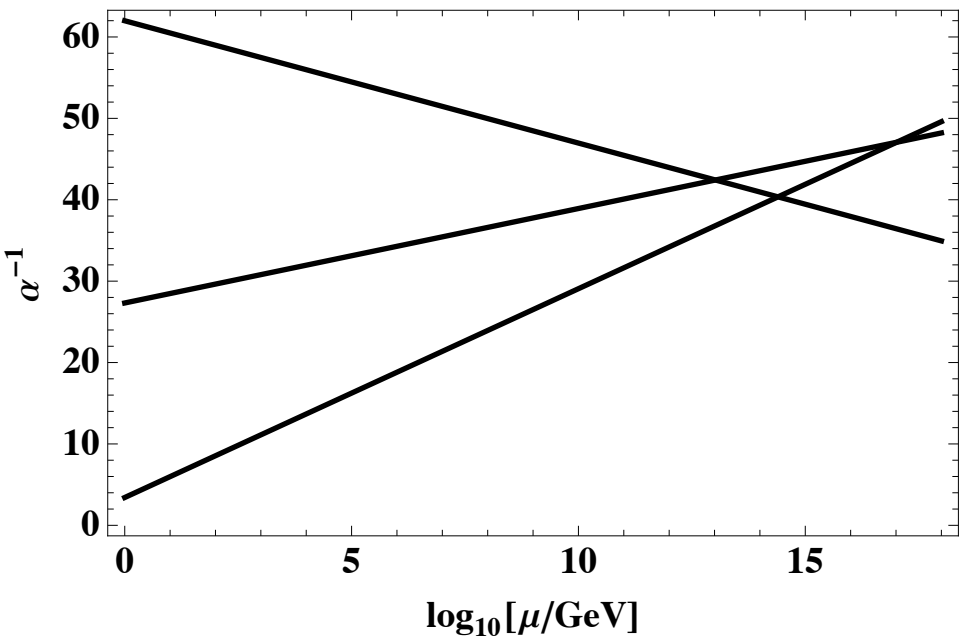
$$b_{SU(2)} = \overset{W^\pm, Z}{3\times 2} - \left(\overset{Q_L}{\frac{1}{2}\times 3\times 3} + \overset{L}{\frac{1}{2}\times 1\times 3}\right) - \overset{H_u}{\frac{1}{2}} - \overset{H_d}{\frac{1}{2}} = -1$$

$$b_Y = -\left(\overset{Q_L}{\left(\frac{1}{6}\right)^2 3\times 2\times 3} + \overset{U}{\left(-\frac{2}{3}\right)^2 3\times 3} + \overset{D}{\left(\frac{1}{3}\right)^2 3\times 3} + \overset{L}{\left(-\frac{1}{2}\right)^2 2\times 3} + \overset{E}{(1)^2\times 3}\right) - \overset{H_u}{\left(\frac{1}{2}\right)^2\times 2} - \overset{H_d}{\left(\frac{1}{2}\right)^2\times 2} = -11 \longrightarrow b_{T^{12}} = -\frac{33}{5}$$

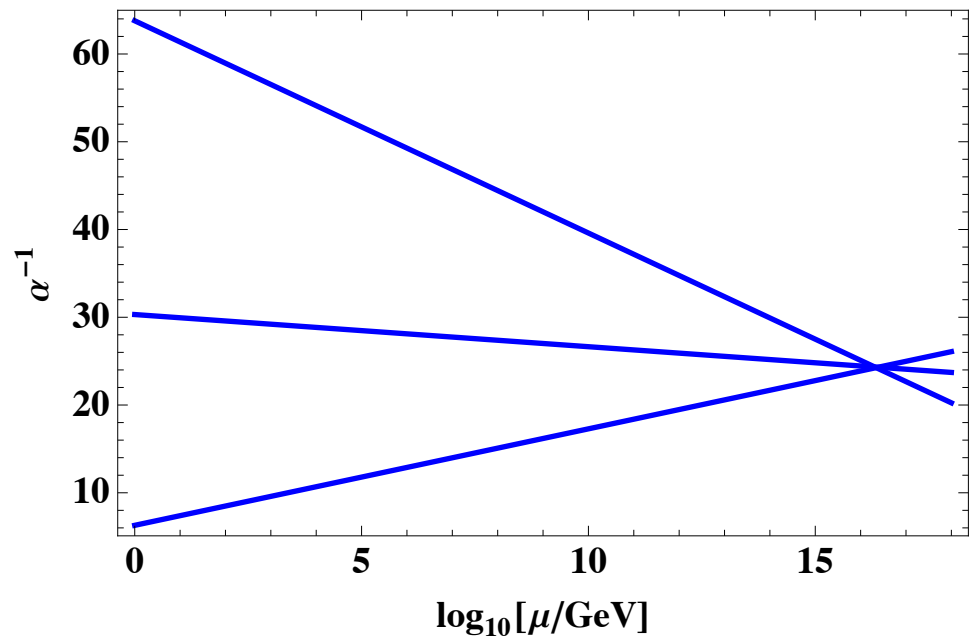
Comparison

1-loop evolution of gauge couplings

SM



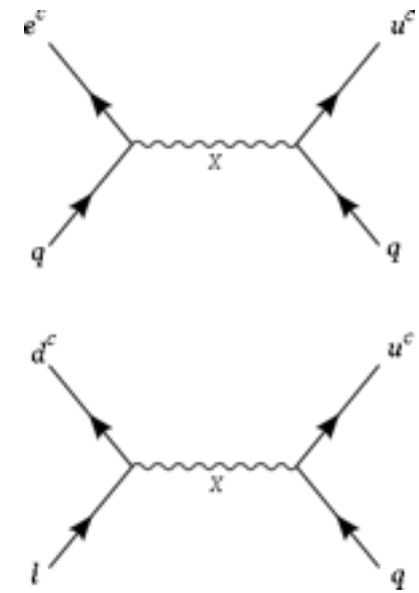
MSSM



Proton decay

Baryon number is violated via the exchange of GUT gauge bosons with GUT scale mass resulting in effective interactions suppressed by $1/M_{GUT}^2$

The dominant decay mode is $p \to e^+\pi_0$

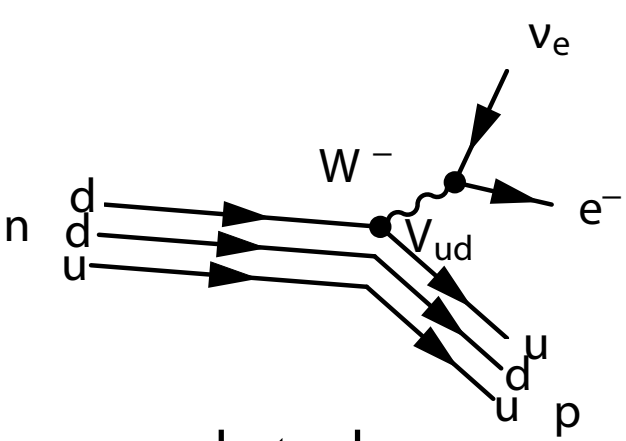


The proton lifetime is given by:

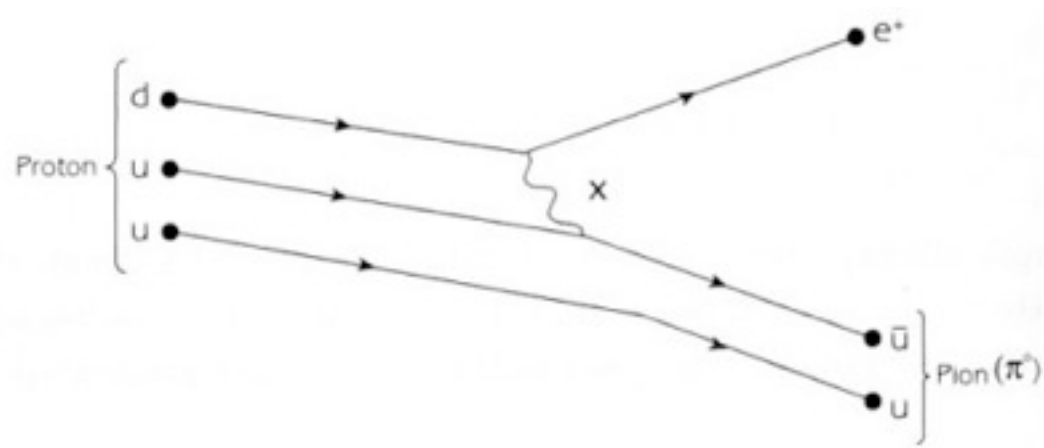
$$\tau(p \to \pi_0 e^+) \approx \left(\frac{M_{GUT}}{10^{16}}\right)^4 \left(\frac{1/35}{(\alpha_{GUT}}\right)^2 \times 4.4 \times 10^{34}\ \mathrm{yr}$$

Experimental constraints lead to: $\tau_p > 5.3 \times 10^{33}\ \mathrm{yr}$

i.e $$M_{GUT} > \left(\frac{\alpha_{GUT}}{1/35}\right)^{1/2} \times 6 \times 10^{15}\ \mathrm{GeV}$$



beta decay



The Dark Matter of the universe

Some invisible transparent matter (that does not interact with photons) which presence is deduced through its gravitational effects

15% baryonic matter (1% in stars, 14% in gas)

85% dark unknown matter

$$\Omega_{DM} = \frac{\text{energy density of the universe stored in dark matter}}{\text{total energy density of the universe}} \sim 25\ \%$$

$\Omega_{\text{dark energy}} \sim 70\ \%$

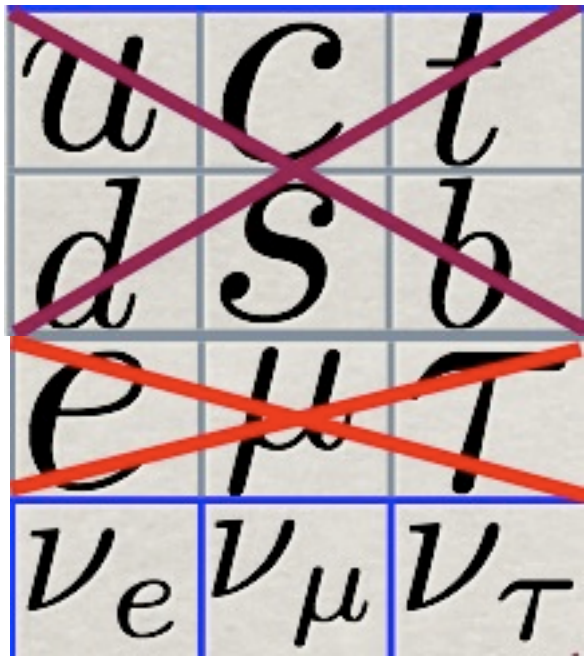
$\Omega_{SM} = \sim 5\ \%$

Dark matter can't be explained by the Standard Model

Matter

quarks

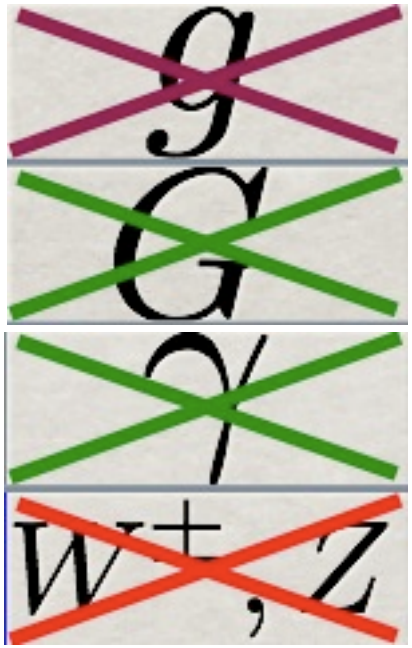
leptons



I II III

3 families of matter

Forces



force mediators

charged/unstable

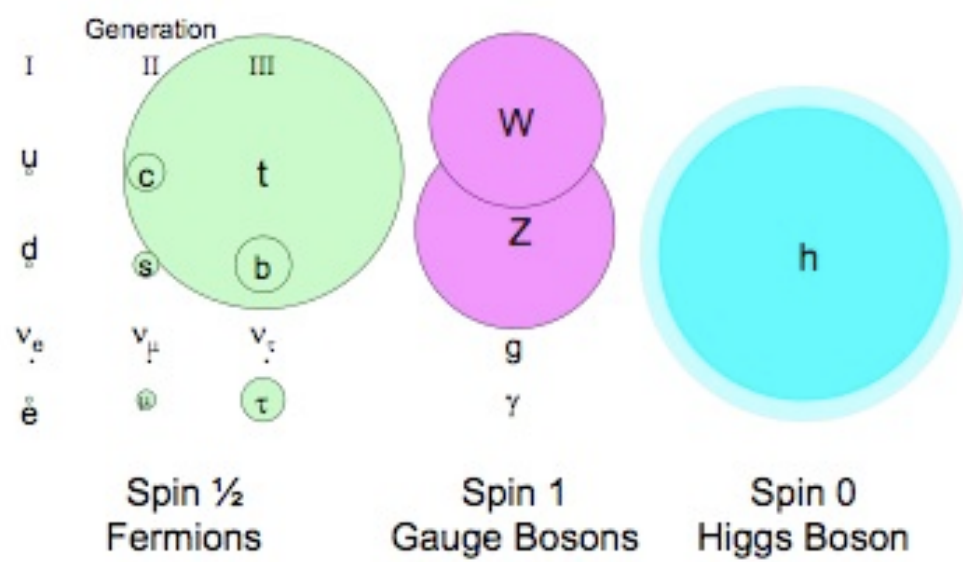
baryonic

massless

contribution to the energy budget of the universe

Particle	Ω	type
Baryons	4 - 5 %	cold
Neutrinos	< 2 %	hot
Dark matter	20 - 26 %	cold

radius of circle is proportional to the mass



Dark Matter candidates

Two main possibilities:

very light & only gravitationally coupled (or with equivalently suppressed couplings) -> stable on cosmological scales

Long-lived
(stable on cosmological scales)

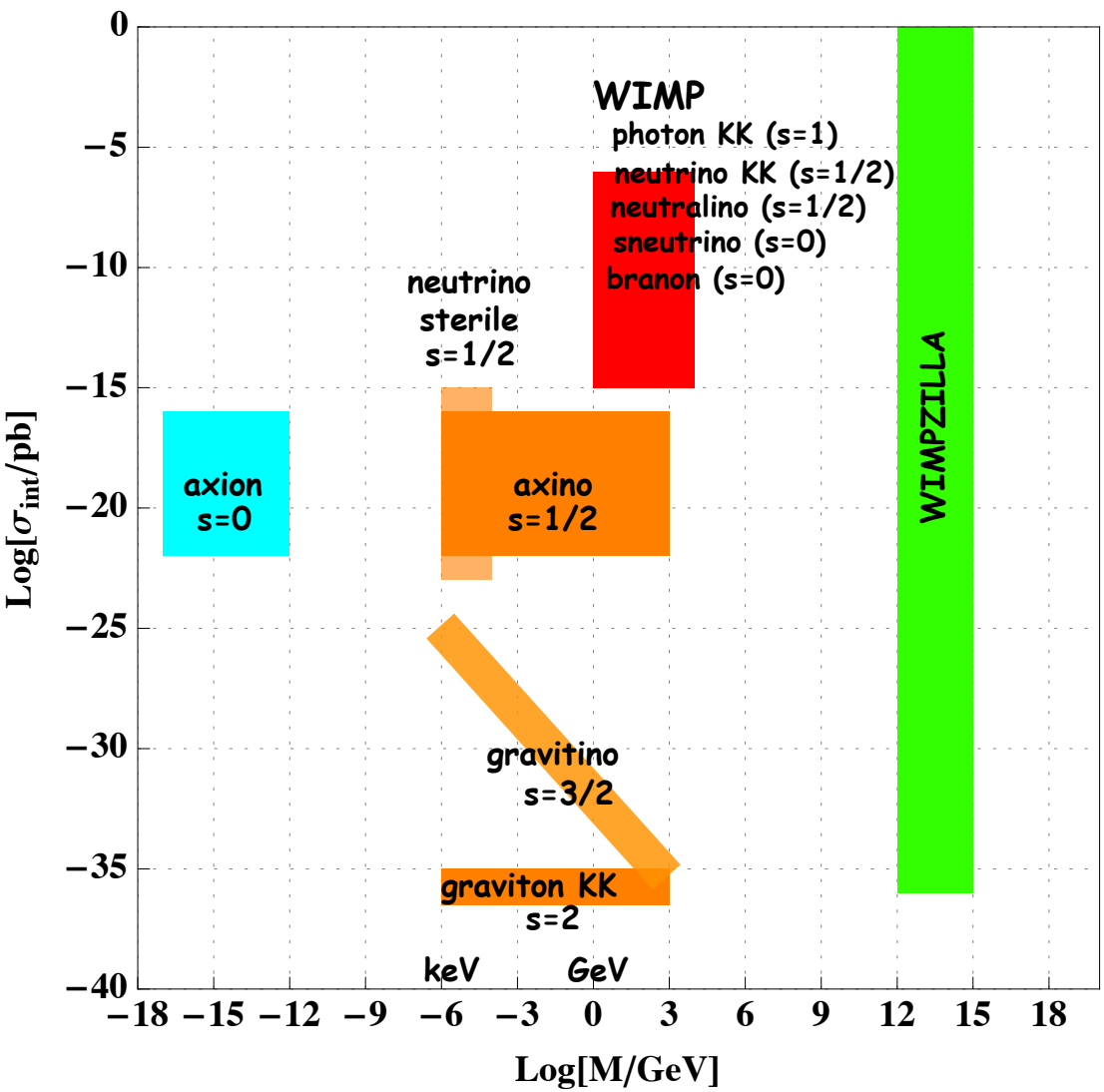
$$\tau_{DM} > \tau_{universe} \sim 10^{18}\ s$$

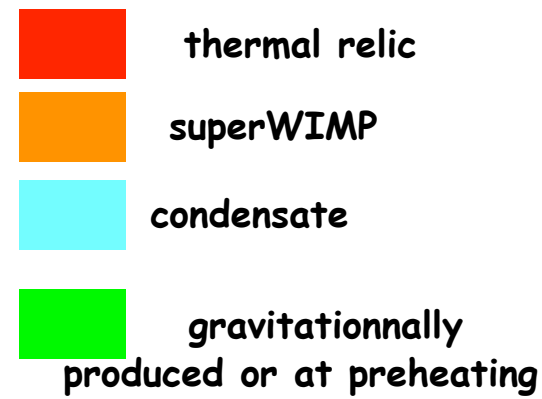
sizably interacting (but not strongly) with the SM -> symmetry needed to guarantee stability

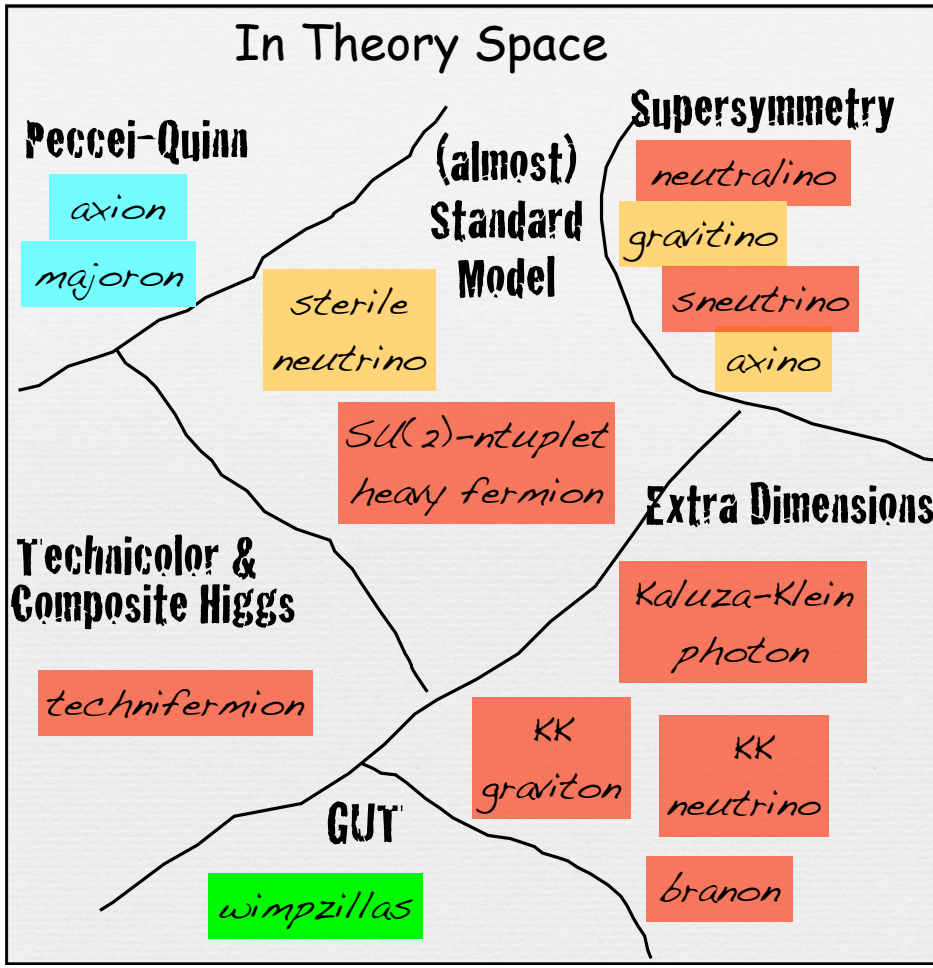
stable by a symmetry

-> WIMP

Dark Matter Candidates with Ω_{DM} ~1







What is the origin of the WIMP stability?

The lifetime of DM should be larger than the age of the universe $\tau_{\text{universe}} \sim 10^{18}$ s.

Actually even larger, $\tau_{DM} \geq 10^{26}$ s, not to overproduce e^+, p, γ fluxes

To get an idea, consider stable particles in the Standard Model:

- The photon is stable because it is the massless gauge boson of the exact electromagnetic $U(1)_{QED}$ gauge symmetry
- The electron is stable because it is the lightest particle charged under the $U(1)_{QED}$ gauge symmetry
- The lightest neutrino is stable because of Lorentz invariance since it is the lightest fermion
- The proton is stable because of the conservation of baryon number, which results accidentally from the SM gauge symmetries and the gauge charges assigned to the SM particles.

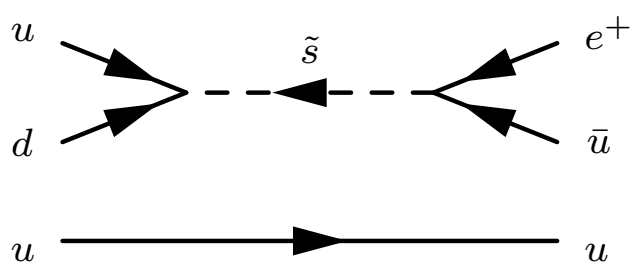
Can we use similar arguments for the dark matter particle?

Supersymmetric Dark Matter

stable by R-parity: $$R_p = (-1)^{3B+L+2s}$$

under which SM particles are even and superpartners are odd

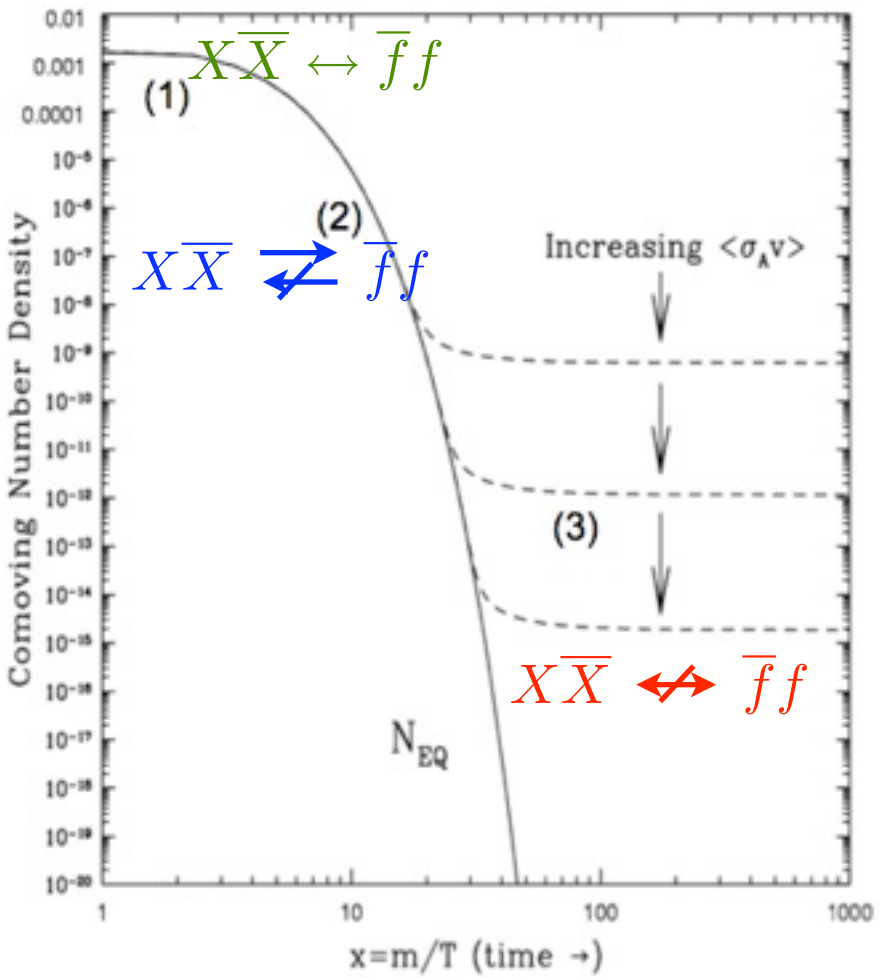
Primarily introduced to prevent fast proton decay in supersymmetry:



-> The Lightest Supersymmetric Particle (odd) is thus stable

The relic abundance of a stable particle follows from the generic thermal freeze-out mechanism in the expanding universe

$$\dot{n} + 3Hn = -\langle\sigma v\rangle(n^2 - n_{\rm T}^2)$$



expansion rate of universe

freese-out :

annihilation rate of particle

$$H \sim \frac{\sqrt{g}T^2}{M_{\rm P}} \sim \Gamma = n\sigma v$$

Thermal relic: $\Omega_{DM} \propto 1/\sigma_{anni}$

$\Rightarrow \sigma_{anni} \approx 1$ pb

$\sigma \sim \alpha^2/m^2$

$\Rightarrow m \sim 100$ GeV

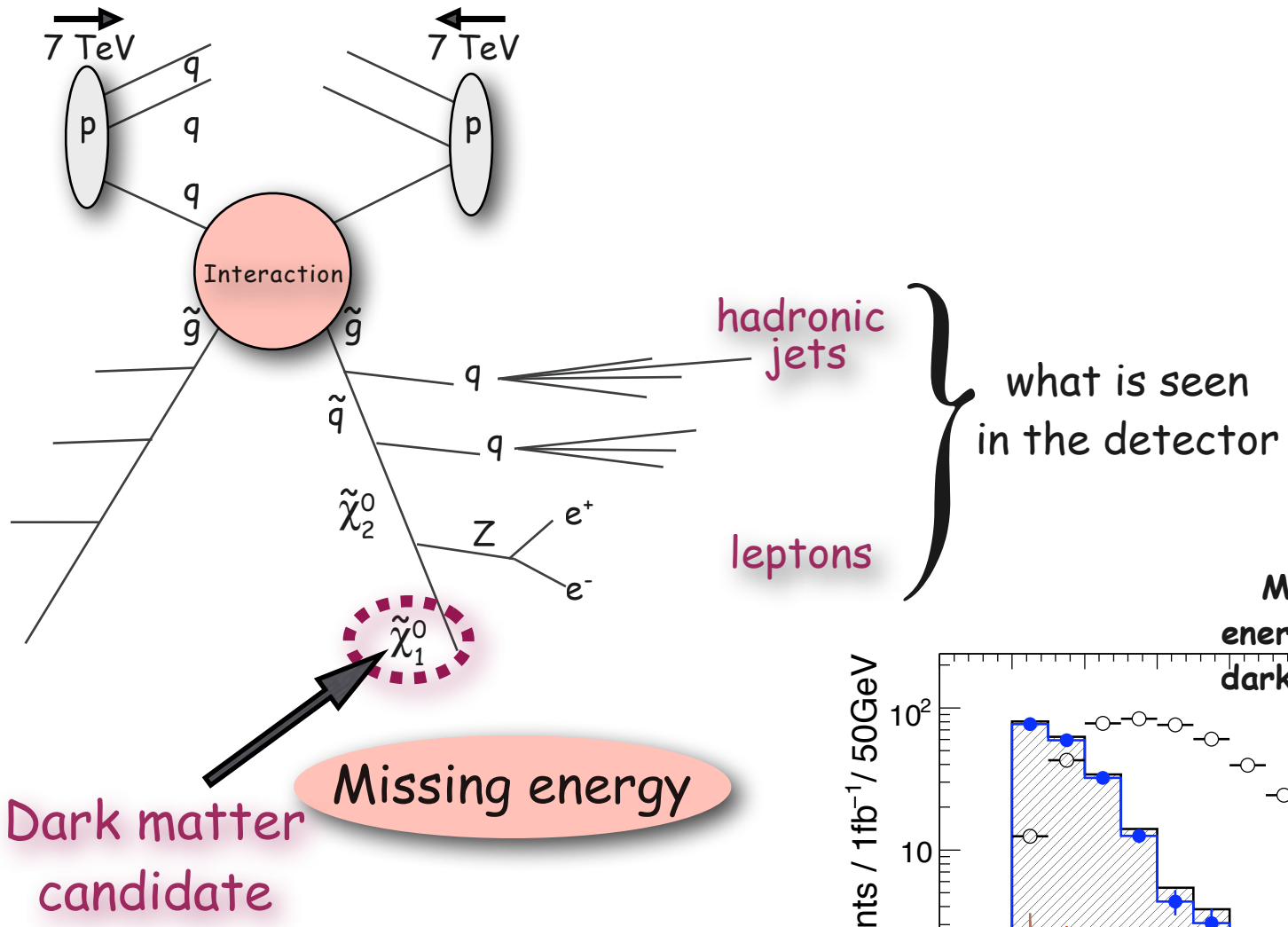
The "WIMP miracle"

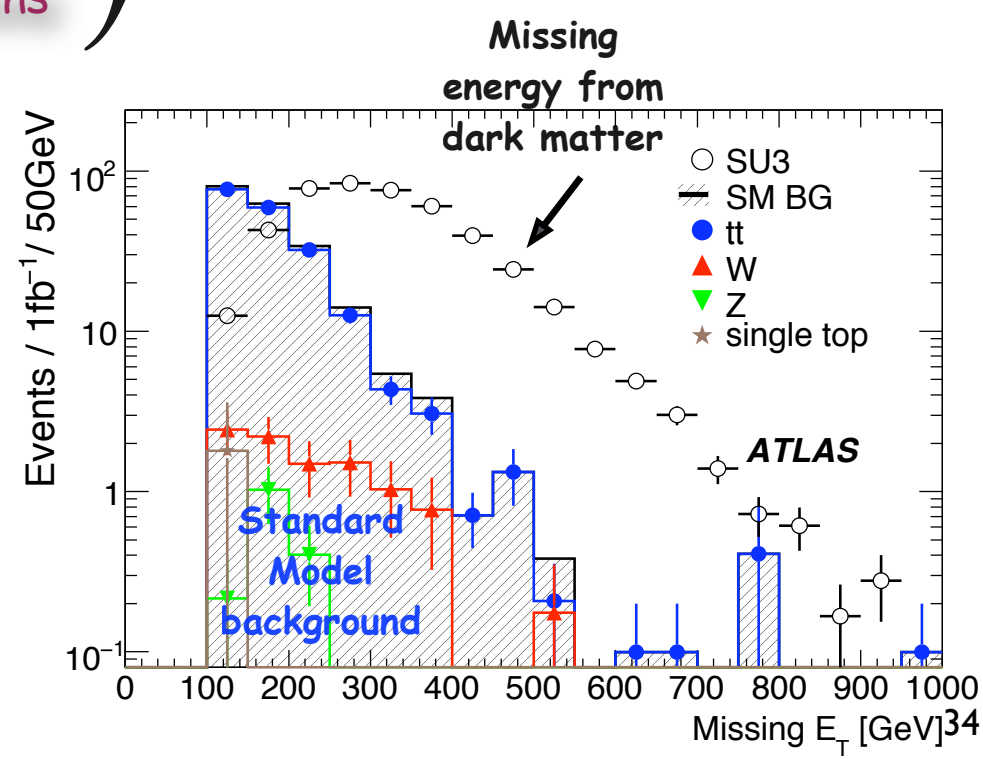
$$\Omega_{DM} \approx \frac{O(1)\ \text{pb}}{\sigma_{anni}}$$

→ a particle with a typical EW-scale cross section

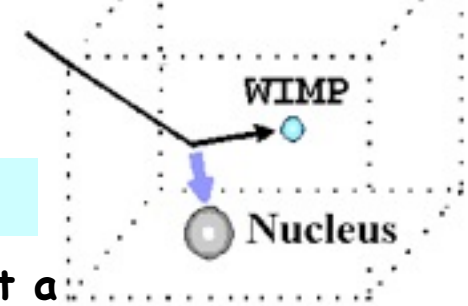
$\sigma_{anni} \approx 1$ pb leads to the correct dark matter abundance.

Producing Dark Matter at LHC = "Missing Energy" events



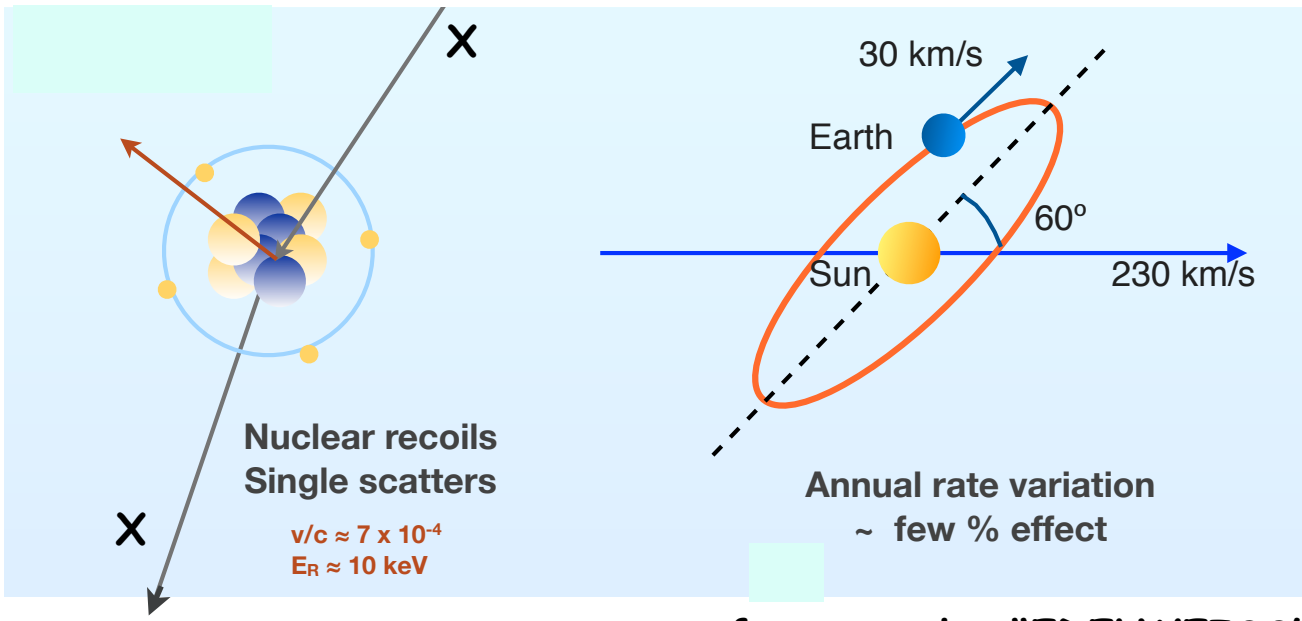


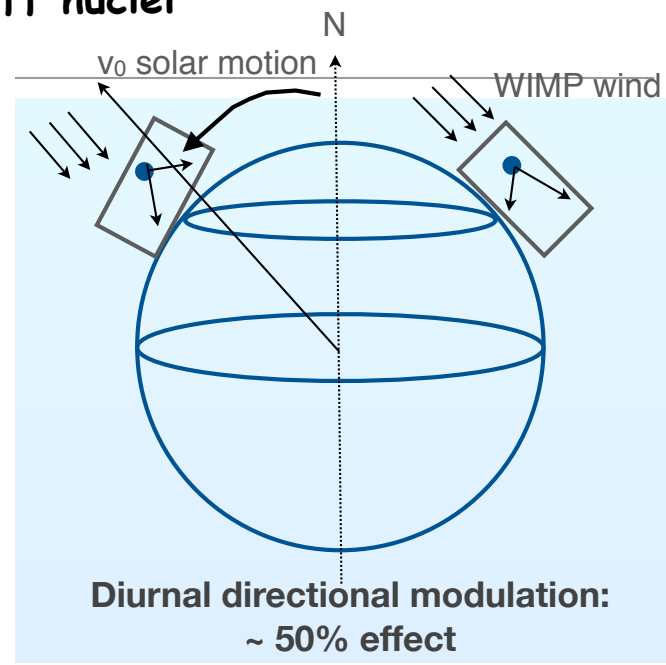
Direct Detection



WIMP flux on Earth: ~ 10^5 cm^{-2}s^{-1} (for a 100 GeV WIMP)

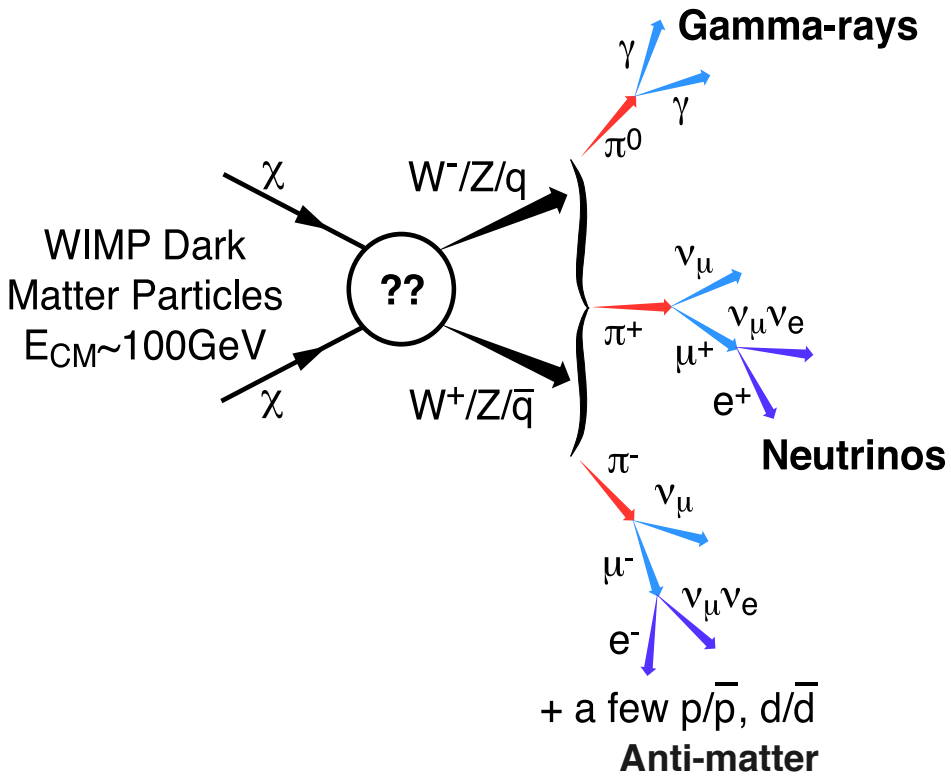
even though WIMPs are weakly interacting, this flux is large enough so that a potentially measurable fraction will elastically scatter off nuclei





for example, "EDELWEISS":

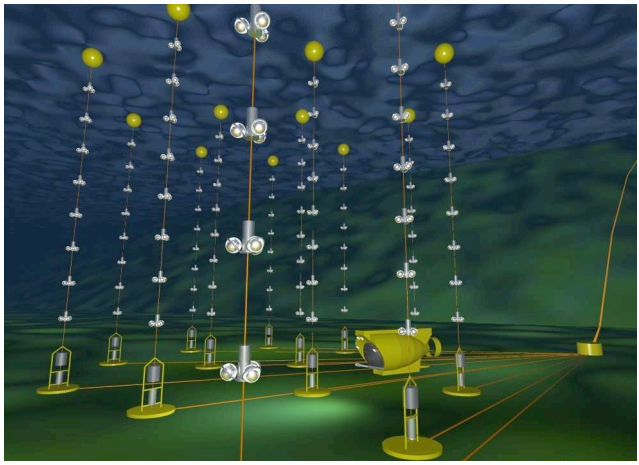
WIMP indirect detection



Indirect Detection

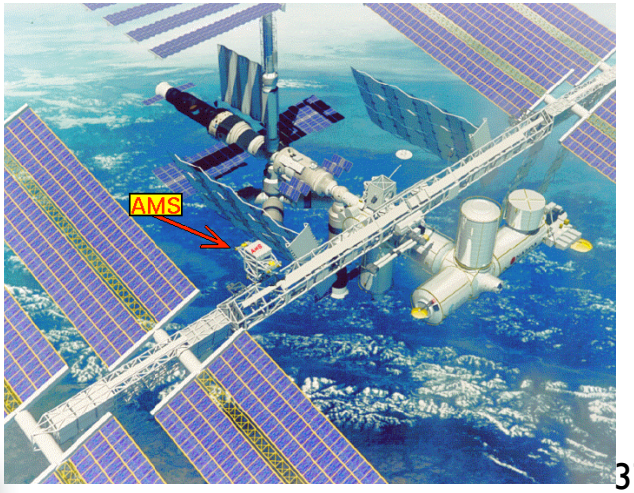
Search for neutrinos in the South Pole

In the Mediterranean



Antarès

Search for antiprotons in space



AMS

Indirect Detection

Search for dark matter photons on Earth

Hess

and in space

Fermi

Matter antimatter asymmetry

Matter Anti-matter asymmetry:

characterized in terms of the baryon to photon ratio

$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} \equiv \eta_{10} \times 10^{-10}$$

$$5.1 < \eta_{10} < 6.5 \ (95\% \ \mathrm{CL})$$

The great annihilation

10 000 000 001 Matter	10 000 000 000 Anti-matter

1
(us)

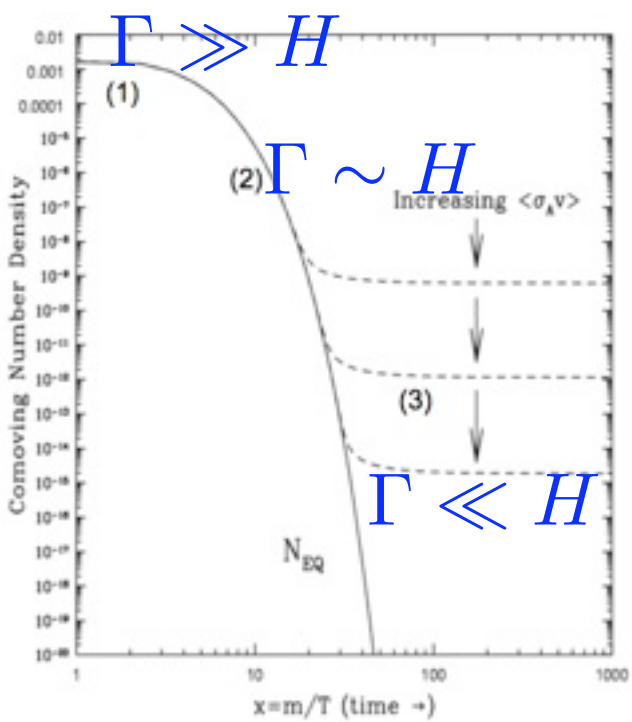
The Standard Model is unable to explain this asymmetry.

How much baryons would there be in a symmetric universe?

nucleon and anti-nucleon densities are maintained by annihilation processes

$$n + \overline{n} \longleftrightarrow \pi + \pi \longleftrightarrow \gamma + \gamma + \ldots$$

which become ineffective when



$$\Gamma \sim (m_N T)^{3/2} e^{-m_N/T}/m_\pi^2 \sim H \sim \sqrt{g_*} T^2/m_{Pl}$$

leading to a freeze-out temperature

$$T_F \sim 20 \text{ MeV}$$

$$\frac{n_N}{s} \approx 7 \times 10^{-20}$$

10^9 times smaller than observed,
and there are no antibaryons
-> need to invoke an initial asymmetry

Sakharov's conditions for baryogenesis (1967)

1) Baryon number violation

(we need a process which can turn antimatter into matter)

2) C (charge conjugation) and CP (charge conjugation ×Parity) violation

(we need to prefer matter over antimatter)

3) Loss of thermal equilibrium

(we need an irreversible process since in thermal equilibrium, the particle density depends only on the mass of the particle and on temperature --particles & antiparticles have the same mass , so no asymmetry can develop) $\Gamma(\Delta B > 0) > \Gamma(\Delta B < 0)$.

Need for either

-> Long-lived particles decay out of equilibrium

e.g leptogenesis, nicely connected to the explanation of neutrino masses

-> first-order phase transitions

The B+L anomaly

The charge B+L is not conserved by quantum fluctuations of gauge fields while the orthogonal combination B-L remains a good symmetry of electroweak interactions.

$$\partial_\mu j_B^\mu = \partial_\mu j_L^\mu = -N_f \left(\frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{a\mu\nu} - \frac{g'^2}{32\pi^2} f_{\mu\nu} \tilde{f}^{\mu\nu} \right)$$

The variation of the baryonic charge is given by $\Delta B = \int dt dx \partial_\mu j^\mu$

This integral is non-zero for certain gauge field configurations (instantons)

The topological charge of the instanton is defined by the Chern Simons number $N_{CS} = \int d^3x \; K^0$

where $\partial_\mu K^\mu = \frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{a,\mu\nu}$ $\quad K^\mu = \frac{g^2}{32\pi^2} \epsilon^{\mu\nu\alpha\beta} (F^a_{\nu\alpha} A^a_\beta - \frac{g}{3} \epsilon_{abc} A^a_\nu A^b_\alpha A^c_\beta)$

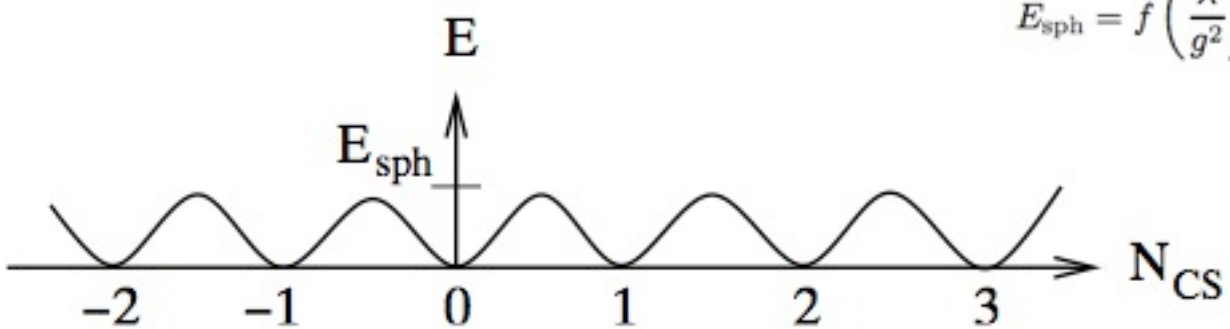
Baryon number violation in the Standard Model

due to chirality + topology of the SU(2) vacuum

$$N_{CS}(t_1) - N_{CS}(t_0) = \int_{t_0}^{t_1} dt \int d^3x \; \partial_\mu K^\mu = \nu$$

$$\partial_\mu K^\mu = \frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{a,\mu\nu}$$

$$E_{\rm sph} = f\left(\frac{\lambda}{g^2}\right)\frac{4\pi v}{g} \cong \frac{8\pi v}{g} = \frac{2M_W}{\alpha_W} f\left(\frac{\lambda}{g^2}\right)$$



Energy of gauge field configuration as a function of Chern Simons number

$$\Delta B = N_f \Delta N_{CS}$$

baryons are created by transitions between topologically distinct vacua of the $SU(2)_L$ gauge field

CP violation

Let M(i->j) be the amplitude for a transition from a state i to a state j, and let $\bar{i}$ be the state obtained by applying a CP transformation to i. Then the CPT theorem implies:

$$\mathcal{M}(i \to j) = \mathcal{M}(\bar{j} \to \bar{i})$$ (CPT invariance)

CP invariance (and hence, by CPT, T invariance) demands:

$$\boxed{\mathcal{M}(i \to j) = \mathcal{M}(\bar{i} \to \bar{j}) = \mathcal{M}(j \to i)}$$ (CP invariance)

The requirement of unitarity yields:

$$\sum_j |\mathcal{M}(i \to j)|^2 = \sum_j |\mathcal{M}(j \to i)|^2$$ (unitarity)

The sum over j includes states and antistates:

$$\sum_j |\mathcal{M}(i \to j)|^2 = \boxed{\sum_j |\mathcal{M}(j \to \bar{i})|^2 = \sum_j |\mathcal{M}(j \to i)|^2}$$ (CPT+unitarity)

In thermal equilibrium, interactions produce i and $\bar{i}$ in equal numbers. Thus no asymmetry may develop, even if CP is violated. And any preexisting asymmetry will be destroyed by interactions

How CP violation, Baryon number violation and out-of equilibrium dynamics are put together to explain the matter-antimatter asymmetry of the universe remains an open question ...