Theoretical Concepts
in Particle Physics

Lecture III: Towards beyond the Standard Model

Géraldine SERVANT
ICREA@IFAE-Barcelona
The beauty of the SM comes from the identification of a unique dynamical principle describing interactions that seem so different from each others.

The most general lagrangian given the particle content

\[
\mathcal{L} = - \frac{1}{4g'^2} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4g^2} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4g'^2_s} G^a_{\mu\nu} G^{a\mu\nu} \\
+ \bar{Q}_i i \slashed{D} Q_i + \bar{u}_i i \slashed{D} u_i + \bar{d}_i i \slashed{D} d_i + \bar{L}_i i \slashed{D} L_i + \bar{e}_i i \slashed{D} e_i \\
+ Y^{ij}_u \bar{Q}_i u_j \tilde{H} + Y^{ij}_d \bar{Q}_i d_j H + Y^{ij}_l \bar{L}_i e_j H + |D_\mu H|^2 \\
- \lambda (H^\dagger H)^2 + \lambda \nu^2 H^\dagger H + \frac{\theta}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma}
\]

What about baryon and lepton numbers? \(\rightarrow\) accidental symmetries!
The Standard Model: matter

**LEPTONS**

- electron
  - $\nu_e$
  - $e^-$
- muon
  - $\nu_\mu$
  - $\mu^-$
- tau
  - $\nu_\tau$
  - $\tau^-$

**QUARKS**

Each of the 6 quarks exists in three colors:

- up
- charm
- top
- down
- strange
- bottom

**composite states** (white objects)

- 0 baryons
  - proton: $p = (u, u, d)$
  - neutron: $n = (u, d, d)$
- 0 mesons

**the elementary blocks:**

- no composite states made of leptons

+ antiparticles
Symmetries

I- Continuous global space-time (Poincaré) symmetries
   all particles have (m, s)
   -> energy, momentum, angular momentum conserved

II- Global (continuous) internal symmetries
   -> B, L conserved
   (accidental symmetries)

III- Local or gauge internal symmetries
   \( SU(3)_c \times SU(2)_L \times U(1)_Y \)
   -> color, electric charge conserved

IV- Discrete symmetries
   -> CPT
When the positron was discovered, it raised a naive question: why can’t a proton decay into a positron and a photon \( p \rightarrow e^+ \gamma \) ?

This process would conserve momentum, energy, angular momentum, electric charge and even parity.

This can be understood if we impose conservation of baryon number.

Similarly, when the muon was discovered, it raised the question: why doesn’t a muon decay as \( \mu^- \rightarrow e^- \gamma \) ?

This led to propose another quantum number: lepton family number.
The following processes have not been seen. Explain which conservation law forbids each of them.

\[ n \rightarrow p \mu^- \bar{\nu}_\mu \]
\[ \mu^- \rightarrow e^- e^- e^+ \]
\[ n \rightarrow p \nu_e \bar{\nu}_e \]
\[ p \rightarrow e^+ \pi^0 \]
\[ \tau^- \rightarrow \mu \gamma \]
\[ K^0 \rightarrow \mu^+ e^- \]
\[ \mu^- \rightarrow \pi^- \nu_\mu \]
The following processes have not been seen. Explain which conservation law forbids each of them

\[ n \rightarrow p \mu^- \bar{\nu}_\mu \]  
energy

\[ \mu^- \rightarrow e^- e^- e^+ \]  
muon number or electron number

\[ n \rightarrow p \nu_e \bar{\nu}_e \]  
electric charge

\[ p \rightarrow e^+ \pi^0 \]  
baryon number or electron number

\[ \tau^- \rightarrow \mu \gamma \]  
tau number or muon number

\[ K^0 \rightarrow \mu^+ e^- \]  
muon number or electron number

\[ \mu^- \rightarrow \pi^- \nu_\mu \]  
energy
The hierarchy problem

As soon as we introduce a fundamental scalar field in the theory (the Higgs), this generates a puzzle: the so-called “hierarchy problem”.

= the fact that the Higgs self-energy receives radiative contributions that are quadratically divergent.

\[ \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m^2} \propto \Lambda^2 \]

\[ \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{(k^2 - m^2)^2} \propto \Lambda^2 \]

\[ \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m^2} \propto \Lambda^2 \]

\[
\delta m_H^2 = \frac{3\Lambda^2}{8\pi^2 v^2} \left( 2m_W^2 + m_Z^2 + m_H^2 - 4m_t^2 \right) \sim -(0.23 \ \Lambda)^2
\]

strong sensitivity on high energy unknown physics

\[ \Lambda \], the maximum mass scale that the theory describes

To stabilise the Higgs mass at the EW scale against the Planck scale, we need to adjust the parameter of the Higgs potential at a level of \(10^{-32}\).
There are examples in physics where unexpected and precise parameter cancellations were actually the signal of the existence of new particles.

For instance, the electron self energy has a power divergence that can be cured only by the introduction of the positron.

Similarly, the extreme sensitivity of the Higgs self energy with respect to physics at high momentum can be naturally reduced by introducing new symmetries and new degrees of freedom, such as supersymmetry, or extra spatial dimensions or additional global symmetries.
an electron makes an electric field which carries an energy

\[ \Delta E_{\text{Coulomb}} = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r_e} \]

electric charge
classical size of the electron

and interacts back to the electron and contributes to its mass
\[ \delta m c^2 = \Delta E_{\text{Coulomb}} \]

\[ \delta m \sim m_e \]
\[ r_e \sim \frac{e^2}{4\pi\varepsilon_0 m_e c^2} \sim 10^{-13} \text{ m} \]

The electron repels itself due to its charge, how to keep electric charge in a small pack? electron point like!

antimatter comes to rescue the 19th century “electron crisis”

\[ \Delta E = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r_e} \]

\[ \Delta E = -\frac{3\alpha}{4\pi} m_e c^2 \log \left( \frac{\hbar}{r_e m_e c} \right) \]

new states \( \approx \) softer UV behavior, small correction to the mass

Weisskopf '39
The hierarchy problem: What is keeping the Higgs boson light?

we need new degrees of freedom to cancel $\Lambda^2$ divergences and ensure the stability of the weak scale

a problem that arises for any elementary SCALAR particle

does not arise for fermions (protected by chiral symmetry) or gauge bosons (protected by the gauge symmetry)

What is cancelling the divergent diagrams?

$\Rightarrow \delta M_H^2 \propto \Lambda^2$

A light Higgs calls for New Physics at the TeV scale

the “hierarchy problem”: the main motivation for building the LHC
Addressing the hierarchy problem

\[ \left( \delta m_h^2 \right)_{1\text{-loop}} \sim -\frac{y_t^2}{8\pi^2} \Lambda_{\text{UV}}^2 \]

with a new symmetry

Supersymmetry

fermion

\[ \Psi \rightarrow e^{i\theta \gamma_5} \Psi \]

\( \Psi \) massless:

protected by
chiral symmetry

\[ \Psi^{\text{SUSY}} \leftrightarrow H \]
Supersymmetry can solve the big hierarchy and naturalness is preserved up to very high scales if superparticle masses are at the weak scale.

\[ \Delta(m_{h^0}^2) = h^0 - t - t + \tilde{t} + h^0 - \tilde{t} - \tilde{t} \]

\[ \delta m_H^2 \sim -\frac{3}{8\pi^2} \frac{h_t^2}{m_t^2} \log \frac{\Lambda^2}{m_t^2} \]
An elegant solution to the hierarchy problem: Supersymmetry

2 categories of particles:

Fermions
- matter particles
- fermions repel each other

Bosons
- force carriers
- bosons can pile up

[Enrico Fermi 1901-1954]
[Satyendra Bose 1894-1974]
String Theory

unification of the Standard Model forces with gravity

(observable universe)
$10^{-10}$ m

(Earth)
$10^{-17}$ m

(Hair)
$10^{-35}$ m

Atom
electrons + nucleus

Nucleus
quarks

(Super)String
String theories are (well) defined only in spacetime with 10 or 11 dimensions. These extra dimensions are assumed to be curled up.
Is the Standard Model a remnant of a Grand Unified Theory?

How come is the electric charge quantized? \[ Q_e = T_3 + Y \]

- Eigen values of the generators of the abelian U(1) are continuous e.g. in the symmetry of translational invariance of time, there is no restriction in the (energy) eigen values.

- Eigen values of the generators of a simple non-abelian group are discrete e.g. in SO(3) rotations, the eigen values of the third component of angular momentum can take only integers or 1/2 integers values. In SU(5), since the electric charge is one of the generators, its eigen values are discrete and hence quantized.

**Charge quantization**

**SM matter content fits nicely into SU(5)**

Simple unification group -> **Charge quantization**

\[ SU(3)_c \times SU(2)_L \times U(1)_Y \subset SU(5) \]

Relation between color SU(3) and electric charge.

Quarks carry 1/3 of the lepton charge because they have 3 colors. The SU(5) theory provides a rationale basis for understanding particle charges and the weak hypercharge assignment in the SM.
Classical physics: the forces depend on distances
Quantum physics: the charges depend on distances

The electromagnetic coupling decreases at large distances.
Charge screening (vacuum polarization) due to virtual fermion-antifermion pairs

The vacuum behaves as a polarized dielectric medium

Evolution of coupling constants

virtual particles
charge

excess of negative charges around the positron: screening
because of the non-abelian nature of the underlying SU(3) gauge symmetry: the gauge boson self-interactions generate an **anti-screening** effect through gauge boson loops. This effect is larger than the one from fermion loops --> the strong coupling decreases at short distances

\[
\frac{\partial \alpha_s}{\partial \log E} = \beta(\alpha_s) = \frac{\alpha_s^2}{\pi} \left( -\frac{11N_c}{6} + \frac{N_f}{3} \right)
\]

\(\alpha_s \uparrow \) when \(d \uparrow\)

\(\alpha_s = g_s^2/4\pi\) property of 'asymptotic freedom'

quarks behave as free particles when the energy becomes very large
The evolution of gauge couplings is controlled by the renormalization group equations

\[ \frac{d\alpha(\mu)}{d \log \mu} = \beta(\alpha(\mu)) \]

At one loop:

\[ \beta(\alpha) \equiv \frac{d\alpha(\mu)}{d \log \mu} = -\frac{b_i}{2\pi} \alpha^2 + \mathcal{O}(\alpha^3) \]

So couplings vary logarithmically as a function of the energy scale:

\[ \frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(\mu_0)} + \frac{b_i}{2\pi} \log \frac{\mu}{\mu_0} \]

\[ \alpha_i = \frac{g_i^2}{4\pi} \quad i = SU(3), SU(2), U(1) \]

\[ b_i \quad : \text{defined by the particle content} \]

we observe different couplings but it looks like a low energy artefact
we observe different couplings but it is a low energy artefact

SU(3)_c x SU(2)_L x U(1)_Y \subset SU(5)

SM matter content fits nicely into SU(5)

\[ Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} = (3, 2)_{1/6}, \quad u_R^c = (\bar{3}, 1)_{-2/3}, \quad d_R^c = (\bar{3}, 1)_{1/3}, \]

\[ L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = (1, 2)_{-1/2}, \quad e_R^c = (1, 1)_1 \]

\[ \bar{5} = (1, 2)_{-1/2} \sqrt{\frac{3}{5}} + (3, 1)_{1/3} \sqrt{\frac{3}{5}}, \quad \bar{5} = L + d_R^c \]

\[ 5 = (5 \times 5)_A = (\bar{3}, 1)_{-2/3} \sqrt{\frac{3}{5}} + (3, 2)_{1/6} \sqrt{\frac{3}{5}} + (1, 1)_{1/3} \sqrt{\frac{3}{5}}, \]

\[ 10 = u_R^c + Q_L + e_R^c \]

\[ \psi_{ab} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u^3 & -u^2 & -u_1 & -d_1 \\ -u^3 & 0 & u^1 & -u_2 & -d_2 \\ -u^2 & -u^1 & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -e^+ \\ d_1 & d_2 & d_3 & e^+ & 0 \end{pmatrix} \]

SU(5) adjoint rep.

\[ \left( \begin{array}{c} SU(2) \\ SU(3) \end{array} \right) \]

\[ T^{12} = \sqrt{\frac{3}{5}} \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & -1/3 \end{pmatrix} \begin{pmatrix} -1/3 \\ -1/3 \end{pmatrix} = \sqrt{\frac{3}{5}} Y \]

additional U(1) factor that commutes with SU(3)*SU(2)

\[ \text{Tr}(T^a T^b) = \frac{1}{2} \delta^{ab} \]

\[ g_5 T^{12} = g' Y \]

\[ g_5 \sqrt{\frac{3}{5}} = g' \quad g_5 = g = g_s \]

\[ \sin^2 \theta_W = \frac{3}{8} @ M_{\text{GUT}} \]
So in the SM:

\[
b = \frac{11}{3} T_2(\text{spin-1}) - \frac{2}{3} T_2(\text{chiral spin-1/2}) - \frac{1}{3} T_2(\text{complex spin-0}) \]

\[
Tr(T^a(R)T^b(R)) = T_2(R)\delta^{ab} \quad T_2(\text{fund}) = \frac{1}{2} \quad T_2(\text{adj}) = N
\]

universal contribution coming from complete SU(5) representations
(\(4N_F/3\) in SM in \(4N_F/3 \times 3/2\) in susy)

So in the SM:

\[
b_3 = \frac{11}{3} \times N_c - \frac{2}{3} \times N_f \left(\frac{1}{2} \times 2 + \frac{1}{2} \times 1 + \frac{1}{2} \times 1\right) = 7
\]

\[
b_2 = \frac{11}{3} \times 2 - \frac{2}{3} \times N_f \left(\frac{1}{2} \times 3 + \frac{1}{2} \times 1\right) - \frac{1}{3} \times \frac{1}{2} = \frac{19}{6}
\]

\[
b_Y = -\frac{2}{3} \times N_f \left(\frac{1}{6} \times \frac{1}{6} \times 2 \times N_c + \frac{-2}{3} \times \frac{2}{3} \times N_c + \frac{1}{3} \times 2 \times N_c + \frac{-1}{2} \times 2 + (1)^2\right)
\]

\[-\frac{1}{3} \times \left(\frac{1}{6} \times \frac{1}{6}\right) \times 2 = -\frac{41}{6} \quad \rightarrow \quad b_1 = b_Y \times \frac{3}{5} = -\frac{41}{10}
\]
\( \alpha_i^{-1}(M_Z) = \alpha_{GUT}^{-1} - \frac{b_i}{4\pi} \log \frac{M_{GUT}^2}{M_Z^2} + \Delta_i \)  

\( i = SU(3), SU(2), U(1) \)

\( \alpha_3(M_Z), \alpha_2(M_Z), \alpha_1(M_Z) \): experimental inputs

\( b_3, b_2, b_1 \): predicted by the matter content

3 equations and 2 unknowns \((\alpha_{GUT}, M_{GUT})\)

1 consistency relation for unification

Using

\[
\alpha_1 = \frac{5}{3} \frac{1}{\cos^2 \theta_W} \alpha_{em} \quad \text{and} \quad \alpha_2 = \frac{\alpha_{em}}{\sin^2 \theta_W}
\]

we obtain:

\[
\epsilon_{ijk}(\alpha_i^{-1} - \Delta_i)(b_j - b_k) = 0
\]

If the \( \Delta_i \) contributions are universal \((\Delta_1 = \Delta_2 = \Delta_3)\) or negligible,

this translates into

\[
\sin^2 \theta_W = \frac{3(b_3 - b_2) + 5(b_2 - b_1)\frac{\alpha_{em}(M_Z)}{\alpha_s(M_Z)}}{8b_3 - 3b_2 - 5b_1}
\]

\( \alpha_{em}(M_Z) \approx 1/128 \)

\( \alpha_s(M_Z) \approx 0.1184 \pm 0.0007 \)

In the SM: \( \sin^2 \theta_W \approx 0.207 \)

Not so bad ...

... to be compared with 0.2312 +/- 0.0002
Only the Higgs and the SM gauge bosons can affect the relative running

In the MSSM, extra contributions from the higgsinos and gauginos

\[
b = \frac{11}{3} T_2(\text{spin-1}) - \frac{2}{3} T_2(\text{chiral spin-1/2}) - \frac{1}{3} T_2(\text{complex spin-0})
\]

\[
Tr(T^a(R)T^b(R)) = T_2(R)\delta^{ab} \quad T_2(\text{fund}) = \frac{1}{2} \quad T_2(\text{adj}) = N
\]

CHIRAL SUPERFIELD

**complex spin-0**

**Weyl spin-1/2**

in same representation \( R \) of gauge group

VECTOR SUPERFIELD

**Weyl spin-1/2**

**real spin-1**

in same representation \( V \) of gauge group

\[
b = \frac{11}{3} T_2(\text{vector}) - \frac{2}{3} T_2(\text{vector}) - \frac{2}{3} T_2(\text{chiral}) - \frac{1}{3} T_2(\text{chiral}) = 3 T_2(\text{vector}) - T_2(\text{chiral})
\]

**gauginos**

**scalars**

\[
b_{SU(3)} = 3 \times 3 - \left( \frac{1}{2} \times 2 \times 3 + \frac{1}{2} \times 1 \times 3 + \frac{1}{2} \times 1 \times 3 \right) = \begin{pmatrix} 3 \\ 0 \end{pmatrix}
\]

\[
b_{SU(2)} = 3 \times 2 - \left( \frac{1}{2} \times 3 \times 3 + \frac{1}{2} \times 1 \times 3 \right) - \frac{1}{2} - \frac{1}{2} = -1
\]

\[
b_Y = - \left( \frac{1}{6} \right)^2 3 \times 2 \times 3 + \left( \frac{2}{3} \right)^2 3 \times 3 + \left( \frac{1}{3} \right)^2 3 \times 3 + \left( \frac{1}{2} \right)^2 2 \times 3 + (1)^2 \times 3 \right) - \left( \frac{1}{2} \right)^2 \times 2 - \left( \frac{1}{2} \right)^2 \times 2 = -11
\]
Comparison

1-loop evolution of gauge couplings

**SM**

**MSSM**

\[ \alpha^{-1} \text{ vs. } \log_{10}[\mu/\text{GeV}] \]

\[ \alpha^{-1} \text{ vs. } \log_{10}[\mu/\text{GeV}] \]
**Proton decay**

Baryon number is violated via the exchange of GUT gauge bosons with GUT scale mass resulting in effective interactions suppressed by $1/M_{GUT}^2$

The dominant decay mode is $p \rightarrow e^+ \pi_0$

The proton lifetime is given by:

$$\tau(p \rightarrow \pi_0 e^+) \approx \left(\frac{M_{GUT}}{10^{16}}\right)^4 \left(\frac{1/35}{\alpha_{GUT}}\right)^2 \times 4.4 \times 10^{34} \text{ yr}$$

Experimental constraints lead to: $\tau_p > 5.3 \times 10^{33} \text{ yr}$

i.e $M_{GUT} > \left(\frac{\alpha_{GUT}}{1/35}\right)^{1/2} \times 6 \times 10^{15} \text{ GeV}$

---

**Beta decay**

The proton lifetime is given by:

$$\tau(p \rightarrow \pi_0 e^+) \approx \left(\frac{M_{GUT}}{10^{16}}\right)^4 \left(\frac{1/35}{\alpha_{GUT}}\right)^2 \times 4.4 \times 10^{34} \text{ yr}$$

Experimental constraints lead to: $\tau_p > 5.3 \times 10^{33} \text{ yr}$

i.e $M_{GUT} > \left(\frac{\alpha_{GUT}}{1/35}\right)^{1/2} \times 6 \times 10^{15} \text{ GeV}$

---

**Proton decay**

The proton lifetime is given by:

$$\tau(p \rightarrow \pi_0 e^+) \approx \left(\frac{M_{GUT}}{10^{16}}\right)^4 \left(\frac{1/35}{\alpha_{GUT}}\right)^2 \times 4.4 \times 10^{34} \text{ yr}$$

Experimental constraints lead to: $\tau_p > 5.3 \times 10^{33} \text{ yr}$

i.e $M_{GUT} > \left(\frac{\alpha_{GUT}}{1/35}\right)^{1/2} \times 6 \times 10^{15} \text{ GeV}$
Some invisible transparent matter (that does not interact with photons) which presence is deduced through its gravitational effects

15% baryonic matter (1% in stars, 14% in gas)

85% dark unknown matter

$$\Omega_{DM} = \frac{\text{energy density of the universe stored in dark matter}}{\text{total energy density of the universe}} \sim 25\%$$

$$\Omega_{\text{dark energy}} \sim 70\%$$

$$\Omega_{SM} = \sim 5\%$$
Dark matter can't be explained by the Standard Model

<table>
<thead>
<tr>
<th>Particle</th>
<th>$\Omega$</th>
<th>type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baryons</td>
<td>4 - 5 %</td>
<td>cold</td>
</tr>
<tr>
<td>Neutrinos</td>
<td>&lt; 2 %</td>
<td>hot</td>
</tr>
<tr>
<td>Dark matter</td>
<td>20 - 26 %</td>
<td>cold</td>
</tr>
</tbody>
</table>
Dark Matter candidates

Two main possibilities:

very light & only gravitationally coupled (or with equivalently suppressed couplings) \(\rightarrow\) stable on cosmological scales

Long-lived (stable on cosmological scales)

\[ \tau_{DM} > \tau_{universe} \sim 10^{18} \text{ s} \]

sizably interacting (but not strongly) with the SM \(\rightarrow\) symmetry needed to guarantee stability

stable by a symmetry

\(\rightarrow\) WIMP
Dark Matter Candidates with $\Omega_{DM} \sim 1$

- Thermal relic
- SuperWIMP
- Condensate
- Gravitationally produced or at preheating

### In Theory Space
- Peccei-Quinn
- Supersymmetry
- Standard Model
- Extra Dimensions
- Technicolor & Composite Higgs
- GUT

- Axion
- Axino
- Gravitino
- Graviton KK
- Neutrino KK
- Neutralino
- Sneutrino
- Branon
- Sterile neutrino
- Majoron
- SU(2)-ntuplet
- Heavy fermion
- Technifermion
- Photon KK
- Graviton
- Neutrino
- Axino
- Axino
- GUT
- Wimpzillas
The lifetime of DM should be larger than the age of the universe $\tau_{\text{universe}} \sim 10^{18}$ s.

Actually even larger, $\tau_{\text{DM}} \geq 10^{26}$ s, not to overproduce $e^+, p, \nu$ fluxes.

To get an idea, consider stable particles in the Standard Model:

- The photon is stable because it is the massless gauge boson of the exact electromagnetic $\text{U}(1)_{\text{QED}}$ gauge symmetry.
- The electron is stable because it is the lightest particle charged under the $\text{U}(1)_{\text{QED}}$ gauge symmetry.
- The lightest neutrino is stable because of Lorentz invariance since it is the lightest fermion.
- The proton is stable because of the conservation of baryon number, which results accidentally from the SM gauge symmetries and the gauge charges assigned to the SM particles.

Can we use similar arguments for the dark matter particle?
Supersymmetric Dark Matter

stable by R-parity:

\[ R_p = (-1)^{3B + L + 2s} \]

under which SM particles are even and superpartners are odd

Primarily introduced to prevent fast proton decay in supersymmetry:

\[ u \rightarrow \bar{s} \rightarrow e^+ \]

\[ d \rightarrow \bar{u} \]

\[ u \rightarrow u \]

\[ -> \text{The Lightest Supersymmetric Particle (odd) is thus stable} \]
The relic abundance of a stable particle follows from the generic thermal freeze-out mechanism in the expanding universe

\[ \dot{n} + 3Hn = -\langle \sigma v \rangle (n^2 - n_T^2) \]

Expansion rate of universe

Freeze-out:

\[ H \sim \frac{\sqrt{g} T^2}{M_P} \sim \Gamma = n \sigma v \]

 Thermal relic: \( \Omega_{DM} \propto \frac{1}{\sigma_{anni}} \)

\[ \Rightarrow \sigma_{anni} \approx 1 \text{ pb} \]

\[ \sigma \sim \frac{\alpha^2}{m^2} \]

\[ \Rightarrow m \sim 100 \text{ GeV} \]

The “WIMP miracle”

\[ \Omega_{DM} \approx \frac{O(1) \text{ pb}}{\sigma_{anni}} \]

→ a particle with a typical EW-scale cross section

\( \sigma_{anni} \approx 1 \text{ pb} \) leads to the correct dark matter abundance.
Producing Dark Matter at LHC = “Missing Energy” events

Dark matter candidate

Missing energy

hadronic jets

leptons

what is seen in the detector
even though WIMPs are weakly interacting, this flux is large enough so that a potentially measurable fraction will elastically scatter off nuclei.

**WIMP flux on Earth:** $\sim 10^5$ cm$^{-2}$s$^{-1}$ (for a 100 GeV WIMP)

for example, “EDELWEISS”:

- Nuclear recoils
- Single scatters
  - $v/c = 7 \times 10^{-4}$
  - $E_R = 10$ keV

**Annual rate variation**
- $\sim$ few % effect

**Diurnal directional modulation:**
- $\sim$ 50% effect
WIMP indirect detection

WIMP Dark Matter Particles
$E_{CM} \sim 100 \text{GeV}$

$\chi$ $\rightarrow$ WIMPs

$W^-/Z/q$

$W^+/Z/q$

Gamma-rays

$\gamma$

Neutrinos

$\pi^0$

$\pi^+$

$\pi^-$

$\nu_{\mu}$

$\nu_{\mu\nu_e}$

$\mu^+$

$\mu^-$

$e^+$

$e^-$

$\nu_{\mu\nu_e}$

+ a few $p/\bar{p}$, $d/\bar{d}$

Anti-matter
Indirect Detection

Search for neutrinos in the South Pole

IceCube

In the Mediterranean

Antarès

Search for antiprotons in space

AMS
Indirect Detection

Search for dark matter photons on Earth

and in space

Hess

Fermi
Matter antimatter asymmetry
Matter Anti-matter asymmetry:

characterized in terms of the baryon to photon ratio

\[ \eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} \equiv \eta_{10} \times 10^{-10} \]

\[ 5.1 < \eta_{10} < 6.5 \ (95\% \ CL) \]

The great annihilation

The Standard Model is unable to explain this asymmetry.
How much baryons would there be in a symmetric universe?

Nucleon and anti-nucleon densities are maintained by annihilation processes:

\[ n + \bar{n} \leftrightarrow \pi + \pi \leftrightarrow \gamma + \gamma + \ldots \]

Which become ineffective when:

\[ \Gamma \sim (m_N T)^{3/2} e^{-m_N / T / m_\pi^2} \sim H \sim \sqrt{g_* T^2 / m_{Pl}} \]

Leading to a freeze-out temperature:

\[ T_F \sim 20 \, \text{MeV} \]

\[ \frac{n_N}{s} \approx 7 \times 10^{-20} \]

10^9 times smaller than observed, and there are no antibaryons,

-> need to invoke an initial asymmetry.
Sakharov’s conditions for baryogenesis (1967)

1) Baryon number violation
   (we need a process which can turn antimatter into matter)

2) C (charge conjugation) and CP (charge conjugation $\times$ Parity) violation
   (we need to prefer matter over antimatter)

3) Loss of thermal equilibrium
   (we need an irreversible process since in thermal equilibrium, the particle density depends only on the mass of the particle and on temperature -- particles & antiparticles have the same mass, so no asymmetry can develop) $\Gamma(\Delta B > 0) > \Gamma(\Delta B < 0)$.

Need for either
   -> Long-lived particles decay out of equilibrium
   e.g leptogenesis, nicely connected to the explanation of neutrino masses
   -> first-order phase transitions
The B+L anomaly

The charge B+L is not conserved by quantum fluctuations of gauge fields while the orthogonal combination B-L remains a good symmetry of electroweak interactions.

\[ \partial_\mu j_B^\mu = \partial_\mu j_L^\mu = -N_f \left( \frac{g^2}{32\pi^2} F_{\mu\nu} \tilde{F}^{a,\mu\nu} - \frac{g'^2}{32\pi^2} f_{\mu\nu} \tilde{f}^{\mu\nu} \right) \]

The variation of the baryonic charge is given by

\[ \Delta B = \int dt dx \partial_\mu j_\mu \]

This integral is non-zero for certain gauge field configurations (instantons)

The topological charge of the instanton is defined by the Chern Simons number

\[ N_{CS} = \int d^3 x \ K^0 \]

where

\[ \partial_\mu K^\mu = \frac{g^2}{32\pi^2} F_{\mu\nu} \tilde{F}^{a,\mu\nu} \]

\[ K^\mu = \frac{g^2}{32\pi^2} \epsilon^{\mu\nu\alpha\beta} (F_{\nu\alpha} A^a_\beta - \frac{g}{3} \epsilon_{abc} A^a_\nu A^b_\alpha A^c_\beta) \]
Baryon number violation in the Standard Model

due to chirality + topology of the SU(2) vacuum

\[ N_{CS}(t_1) - N_{CS}(t_0) = \int_{t_0}^{t_1} dt \int d^3x \, \partial_\mu K^\mu = \nu \]

\[ \partial_\mu K^\mu = \frac{g^2}{32\pi^2} F_\mu^\nu \tilde{F}^{\alpha,\mu\nu} \]

Energy of gauge field configuration as a function of Chern Simons number

\[ \Delta B = N_f \Delta N_{CS} \]

baryons are created by transitions between topologically distinct vacua of the SU(2)_L gauge field
Let $M(i\rightarrow j)$ be the amplitude for a transition from a state $i$ to a state $j$, and let $\overline{i}$ be the state obtained by applying a CP transformation to $i$. Then the CPT theorem implies:

$$M(i \rightarrow j) = M(\overline{j} \rightarrow \overline{i})$$

(CPT invariance)

CP invariance (and hence, by CPT, T invariance) demands:

$$M(i \rightarrow j) = M(\overline{i} \rightarrow \overline{j}) = M(j \rightarrow i)$$

(CP invariance)

The requirement of unitarity yields:

$$\sum_j |M(i \rightarrow j)|^2 = \sum_j |M(j \rightarrow i)|^2$$

(unitarity)

The sum over $j$ includes states and antistates:

$$\sum_j |M(i \rightarrow j)|^2 = \sum_j |M(j \rightarrow \overline{i})|^2 = \sum_j |M(j \rightarrow i)|^2$$

(CPT+unitarity)

In thermal equilibrium, interactions produce $i$ and $\overline{i}$ in equal numbers. Thus no asymmetry may develop, even if CP is violated. And any preexisting asymmetry will be destroyed by interactions.
How CP violation, Baryon number violation and out-of-equilibrium dynamics are put together to explain the matter-antimatter asymmetry of the universe remains an open question...