

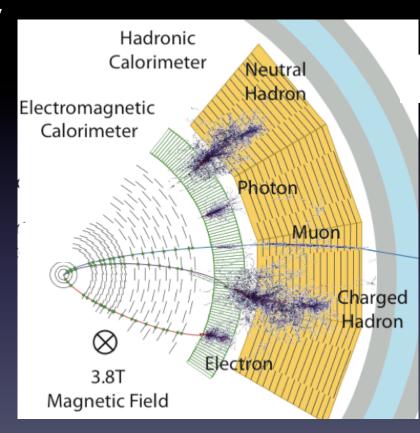
Detectors for Particle Physics

Calorimetry

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What is a calorimeter?

- In nuclear and particle physics calorimetry refers to the detection of particles through total absorption in a block of matter
 - The measurement process is destructive for almost all particle
 - The exception are muons (and neutrinos) → identify muons easily since they penetrate a substantial amount of matter
- In the absorption, almost all particle's energy is eventually converted to heat → calorimeter
- Calorimeters are essential to measure neutral particles



Electromagnetic shower

- Dominant processes at high energies (E > few MeV) :
- Photons: Pair production

$$\sigma_{pair} \approx \frac{7}{9} \left(4\alpha r_e^2 Z^2 \ln \frac{183}{Z^{1/3}} \right) = \frac{7}{9} \frac{A}{N_A X_0}$$

$$I(x) = I_0 e^{-\mu x}$$
 $\mu = \frac{7}{9} \frac{\rho}{X_0}$

 μ = attenuation coefficient X_0 = radiation length in [cm] or [g/cm²]

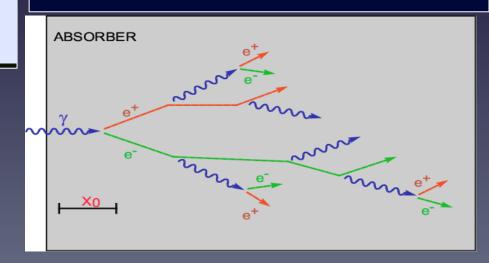
$$X_0 = \frac{A}{4\pi N_A Z^2 r_e^2 \ln \frac{183}{Z^{1/3}}}$$

Electrons: Bremsstrahlung

$$\frac{dE}{dx} = 4\alpha N_A \frac{Z^2}{A} r_e^2 E \ln \frac{183}{Z^{1/3}} = \frac{E}{X_0}$$

$$E = E_0 e^{-x/X_0}$$

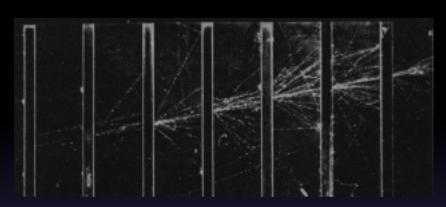
After traversing x=X₀ the electron has only 1/e=37% of its initial energy

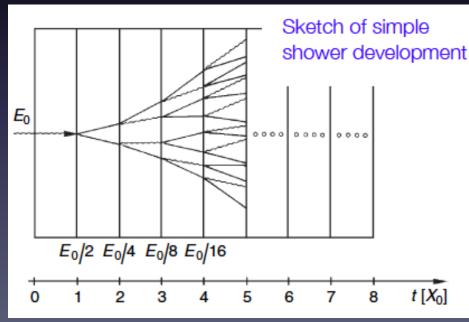


Analytic shower Model

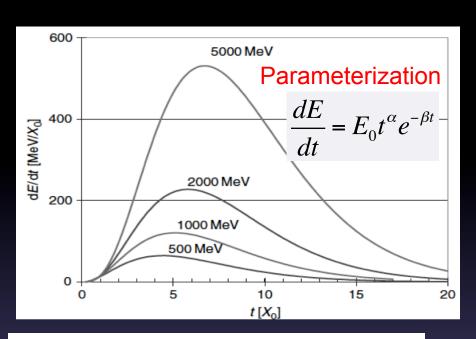
- Simplified model [Heitler]: shower development governed by X₀
 - e^{-} loses [1 1/e] = 63% of energy in 1 X_0 (Brems.)
 - the mean free path of a γ is 9/7 X_0 (pair prod.)
- Assume:
 - E > E_c: no energy loss by ionization/excitation
- Simple shower model:
 - $N(t)=2^t$ particles after $t=x/X_0$
 - each with energy $E(t)=E_0/2^t$
 - Stops if E (t) < E_c=E₀2^{tmax}
 - Location of shower maximum at

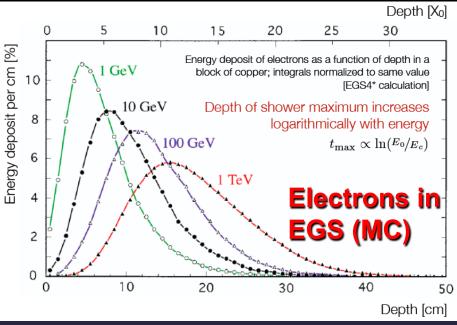
$$t_{\text{max}} = \frac{\ln(E/Ec)}{\ln 2} \propto \ln\left(\frac{E}{E_c}\right)$$

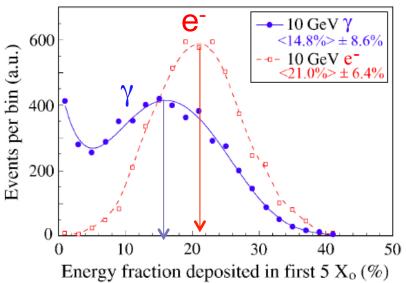




Longitudinal shower distribution







- Differences between electrons and photons generated showers
- Some photons penetrating (almost) the entire slab without interacting (peak at 0)

$$t_{\rm max} = \ln \left(\frac{E_0}{E_c}\right) + C_{e\gamma}$$
 C_{ey}=-0.5 for photons C_{ey}=-1 for electrons

Longitudinal shower containment

- EM calorimeter can be quite compact. Since t_{max}≈ ln(E) → calorimeter thickness must increase as ln(E)
- After shower max e⁺/e⁻ will stop in ≈ 1X₀
- To absorb 95% of photons after shower max ≈ 9X₀ of material are needed
- The energy leakage is mainly due to photons
- A useful expression to indicate 95% shower containment is:

$$L(95\%) = t_{max} + 0.08 Z + 9.6 [X_0]$$

$$E_{C} \approx 10 MeV$$
 $E_{0} = 1 GeV$ $\Rightarrow t_{\text{max}} = \ln 100 / \ln 2 \approx 6.6$ $N_{\text{max}} = 100$ $E_{0} = 100 GeV$ $\Rightarrow t_{\text{max}} = \ln 10,000 / \ln 2 \approx 9.9$ $N_{\text{max}} = 10,000$

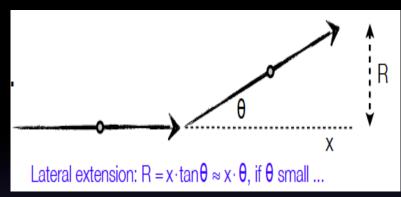
	Scint.	LAr	Fe	Pb	W
X ₀ (cm)	34	14	1.76	0.56	0.35

t_{max} for a 100 GeV is 17.5 cm Fe or 5.6 cm Pb

Lateral development of EM shower

- Opening angle:
 - bremsstrahlung and pair production

$$\left\langle \theta^2 \right\rangle \approx \left(\frac{m_e c^2}{E_e} \right)^2 = \frac{1}{\gamma^2}$$



multiple coulomb scattering [Molière theory]

$$\left| \left\langle \theta \right\rangle \right| = \frac{E_s}{E_e} \sqrt{\frac{x}{X_0}}$$
 where $E_s = \sqrt{\frac{4\pi}{\alpha}} \left(m_e c^2 \right) = 21.2 MeV$

- Main contribution from low energy electrons as <e>6> ~ 1/E_e, i.e. for electrons with E < E_c
 - Molière Radius

$$R_M = \frac{E_s}{E_c} X_0 \approx \frac{21.2 MeV}{E_c} X_0$$

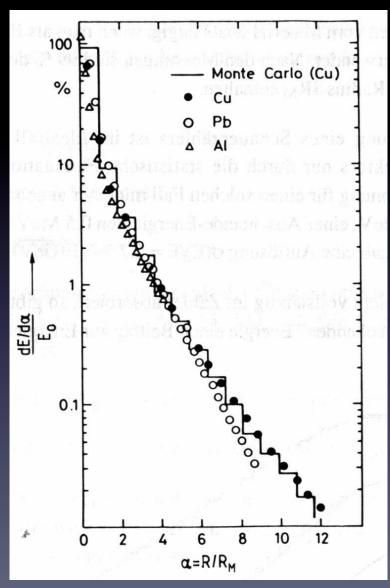
Assuming the approximate range of electrons to be X₀ yields <**0**>≈ 21.2 MeV/E_e→lateral extension: R =<**0**>X₀

Lateral development of EM shower

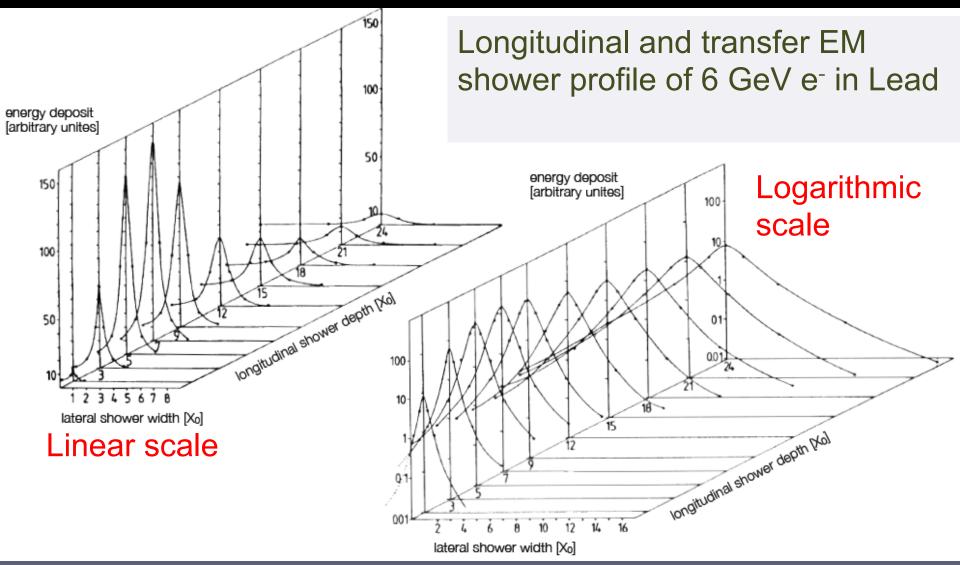
- Inner part is due to Coulomb's scattering of electron and positron
- Outer part is due to low energy γ produced in Compton's scattering, photo-electric effect etc.
 - Predominant part after shower max especially in high Z absorbers

$$\frac{dE}{dr} = \alpha e^{-r/RM} + \beta e^{-r/\lambda_{\min}}$$

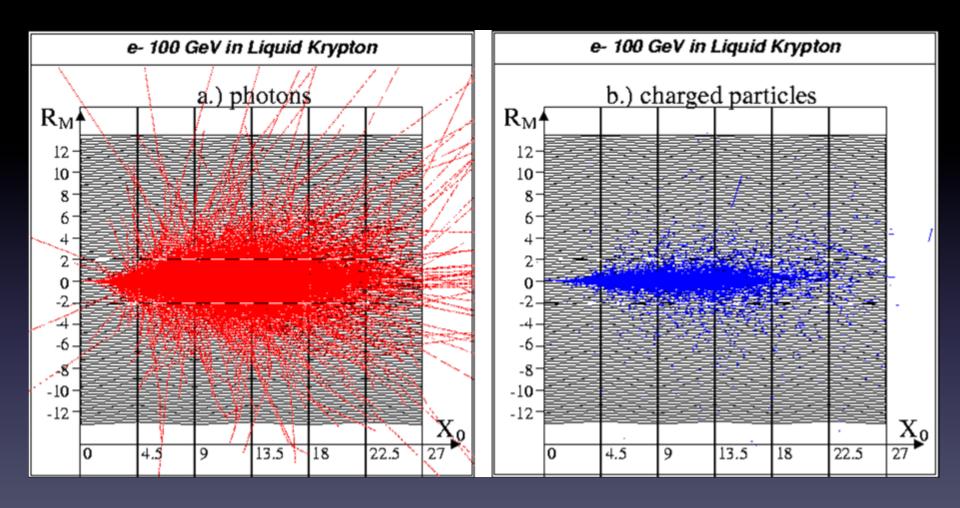
- The shower gets wider at larger depth
- An infinite cylinder of radius 2R_M contains 95% of the shower



3D EM Shower development



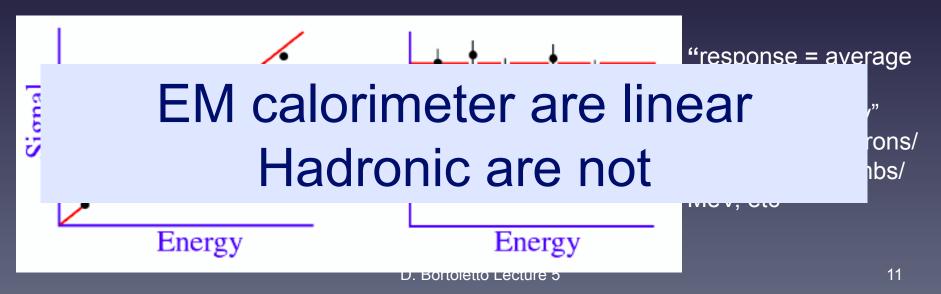
EM shower development in liquid krypton (Z=36, A=84)



GEANT simulation of a 100 GeV electron shower in the NA48 liquid Krypton calorimeter (D.Schinzel)

Energy Measurement

- How we determine the energy of a particle from the shower?
 - Detector response → Linearity
 - The average calorimeter signal vs. the energy of the particle
 - Homogenous and sampling calorimeters
 - Compensation (for hadronic showers)
 - Detector resolution → Fluctuations
 - Event to event variations of the signal
 - What limits the accuracy at different energies?



Sources of Non Linearity

- Instrumental effects
 - Saturation of gas detectors, scintillators, photo-detectors, Electronics
- Response varies with something that varies with energy
- Examples:
 - Deposited energy "counts" differently, depending on depth
 - And depth increases with energy
- Leakage (increases with energy)

Signal linearity for electromagnetic showers

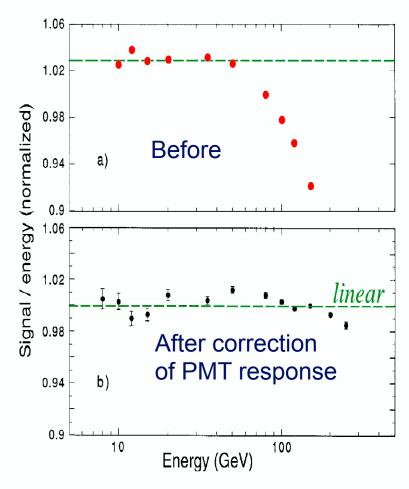


FIG. 3.1. The em calorimeter response as a function of energy, measured with the QFCAL calorimeter, before (a) and after (b) precautions were taken against PMT saturation effects. Data from [Akc 97].

EM Calorimeter configurations

Total absorption

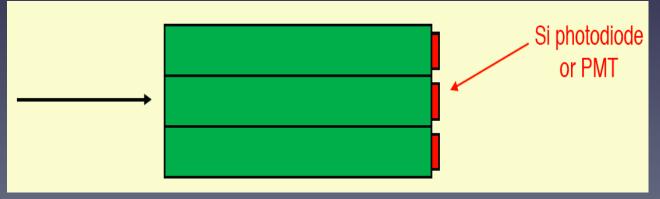
- Electrons and photons stop in calorimeter
- Scintillation proportional to energy of electron
- Usually non-organic scintillator (BGO, PbWO_{4,...}) or liquid Xe
- Advantage: Excellent energy resolution
 - see all charged particles in the shower (but for shower leakage) → best statistical precision
 - Uniform response → good linearity
- Disadvantages:
 - cost and limited segmentation

If W is the mean energy required to produce a signal (eg an electron-ion pair in a noble liquid or a 'visible' photon in a crystal)

$$\frac{\sigma_E}{E} = \frac{1}{\sqrt{n}} = \frac{1}{\sqrt{E/W}}$$

Examples:

- B factories: small photon energies
- CMS ECAL which was optimized for H→γγ



Homogenous calorimeters

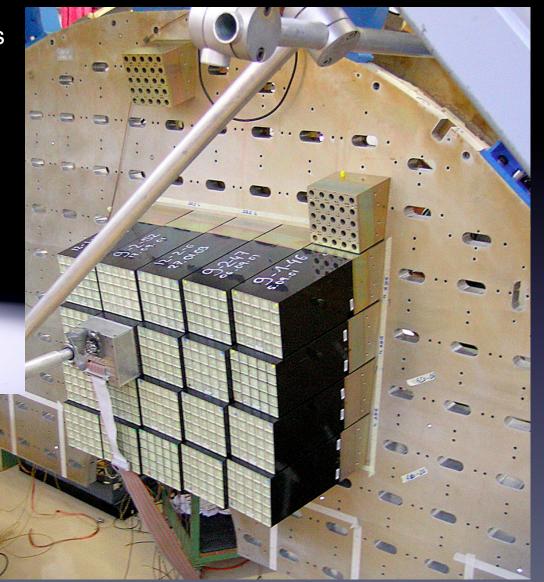
Barrel: 62K 2.2x2.2x23 cm³ crystals

Endcap: 15K 3x3x22 cm³ crystals

Development of PbWO₄ radiation

hard crystals

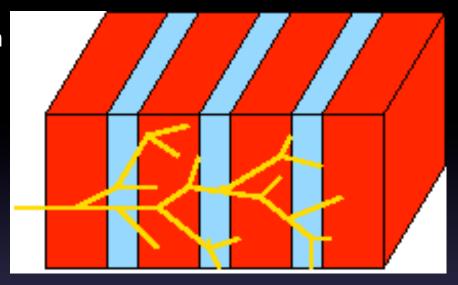
1% resolution at 30 GeV



EM Calorimeter configurations

Sampling Calorimeter

- One material to induce showering (high Z)
- Another to detect particles (typically by counting number of charged tracks)
- Many layers sandwiched together
- Resolution ∝ E^{-1/2}
- Advantages
 - Can segment in depth and have better spatial segmentation
- Disadvantages:
 - Only part of shower seen, less precise
- Examples
 - ATLAS ECAL
 - Most HCALs

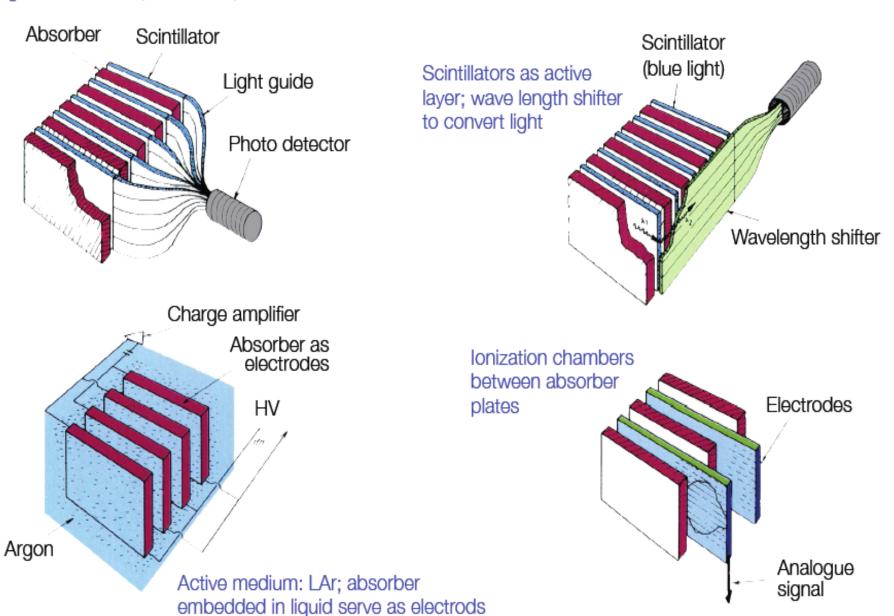


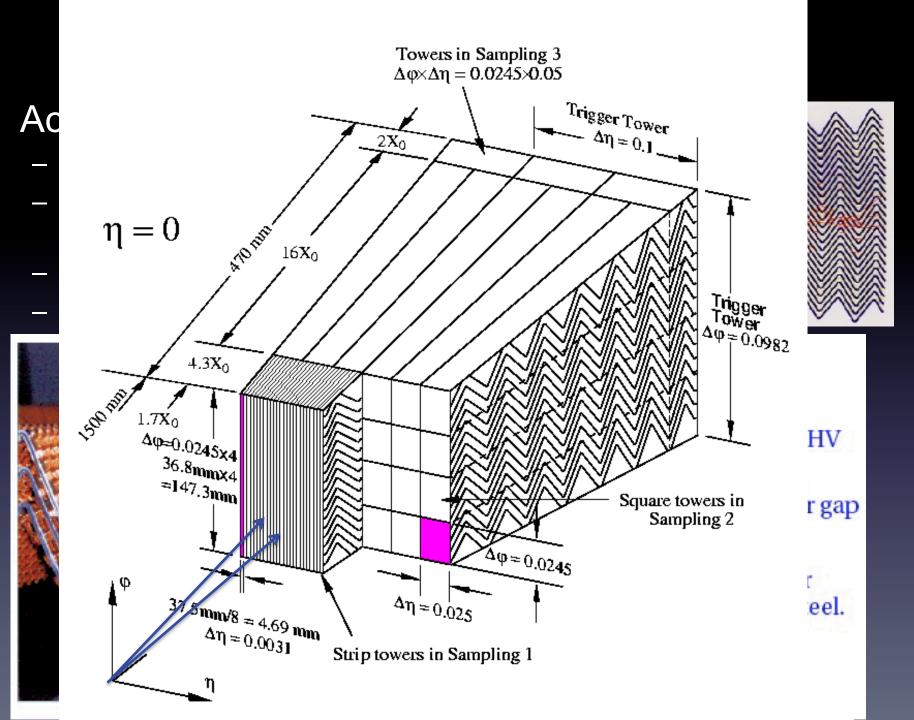
Sampling fraction

$$f_{sampling} = \frac{E_{visible}}{E_{deposited}}$$

Possible setups

Scintillators as active layer; signal readout via photo multipliers





Energy resolution

Ideally, if all shower particles counted:

$$E \propto N$$
 $\sigma_E \approx \sqrt{N} \approx \sqrt{E}$

In practice

$$\sigma_E = a\sqrt{E} \oplus bE \oplus c$$

$$\frac{\sigma_E}{E} = \frac{a}{\sqrt{E}} \oplus b \oplus \frac{c}{E}$$

- a: stochastic term
 - intrinsic statistical shower fluctuations
 - sampling fluctuations
 - signal quantum fluctuations (e.g. photo-electron statistics)

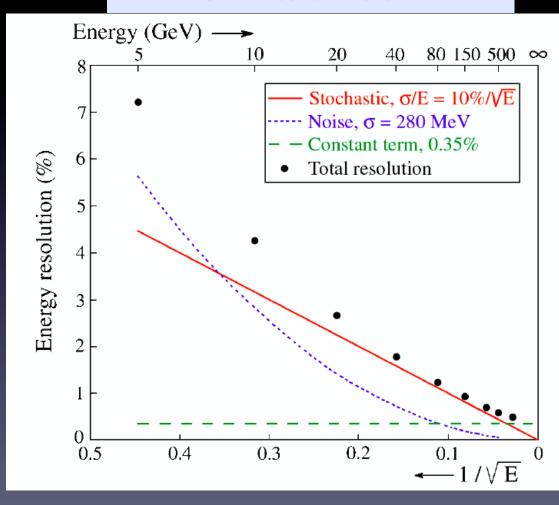
- b: constant term
 - inhomogeneities (hardware or calibration)
 - imperfections in calorimeter construction (dimensional variations, etc.)
 - non-linearity of readout electronics
 - fluctuations in longitudinal energy containment (leakage can also be ~ E-1/4)
 - fluctuations in energy lost in dead material before or within the calorimeter

- c: noise term
 - readout electronic noise
 - Radio-activity, pile-up fluctuations

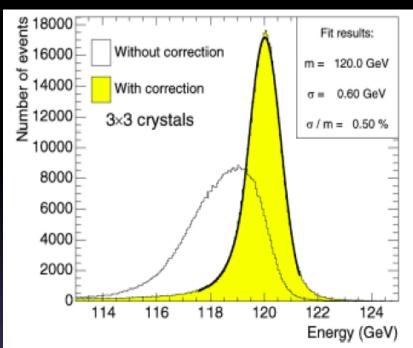
Effects on energy resolution

- Different effects have different energy dependence
 - Sampling fluctuations
 σ/Ε ~ E^{-1/2}
 - shower leakageσ/Ε ~ Ε-1/4
 - electronic noise $\sigma/E \sim E^{-1}$
 - structural nonuniformities:σ/E = constant
- $\sigma_{\text{tot}}^2 = \sigma_{1}^2 + \sigma_{2}^2 + \sigma_{3}^2 + \sigma_{4}^2 + \dots$

ATLAS EM calorimeter



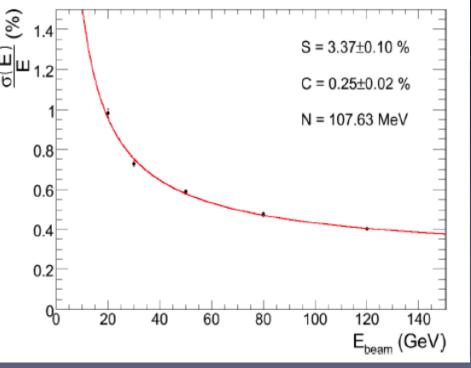
CMS ECAL resolution



Correction for radial loss

The sampling term is 3 times smaller than ATLAS; other terms are similar

$$\left(\frac{\sigma}{E}\right)^2 = \left(\frac{3.37\%}{\sqrt{E}}\right)^2 + \left(\frac{0.107}{E}\right)^2 + \left(0.25\%\right)^2$$
 stoch. noise const.



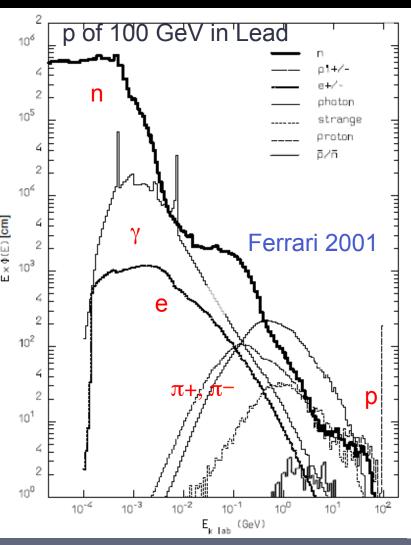
Homogeneous vs Sampling

E in GeV

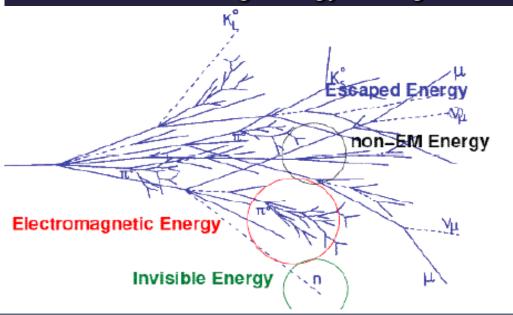
Technology (Experiment)	Depth	Energy resolution	Date
NaI(Tl) (Crystal Ball)	$20X_{0}$	$2.7\%/\mathrm{E}^{1/4}$	1983
$Bi_4Ge_3O_{12}$ (BGO) (L3)	$22X_0$	$2\%/\sqrt{E} \oplus 0.7\%$	1993
CsI (KTeV)	$27X_0$	$2\%/\sqrt{E} \oplus 0.45\%$	1996
CsI(Tl) (BaBar)	$16-18X_0$	$2.3\%/E^{1/4} \oplus 1.4\%$	1999
CsI(Tl) (BELLE)	$16X_0$	1.7% for $E_{\gamma} > 3.5~{\rm GeV}$	1998
PbWO ₄ (PWO) (CMS)	$25X_0$	$3\%/\sqrt{E} \oplus 0.5\% \oplus 0.2/E$	1997
Lead glass (OPAL)	$20.5X_0$	$5\%/\sqrt{E}$	1990
Liquid Kr (NA48)	$27X_{0}$	$3.2\%/\sqrt{E} \oplus\ 0.42\% \oplus 0.09/E$	1998
Scintillator/depleted U (ZEUS)	20-30X ₀	$18\%/\sqrt{E}$	1988
Scintillator/Pb (CDF)	$18X_0$	$13.5\%/\sqrt{E}$	1988
Scintillator fiber/Pb spaghetti (KLOE)	$15X_{0}$	$5.7\%/\sqrt{E} \oplus 0.6\%$	1995
Liquid Ar/Pb (NA31)	$27X_{0}$	$7.5\%/\sqrt{E} \oplus 0.5\% \oplus 0.1/E$	1988
Liquid Ar/Pb (SLD)	$21X_0$	$8\%/\sqrt{E}$	1993
Liquid Ar/Pb (H1)	$20 – 30X_0$	$12\%/\sqrt{E}\oplus 1\%$	1998
Liquid Ar/depl. U (DØ)	$20.5X_0$	$16\%/\sqrt{E} \oplus 0.3\% \oplus 0.3/E$	1993
Liquid Ar/Pb accordion (ATLAS)	$25X_0$	$10\%/\sqrt{E} \oplus 0.4\% \oplus 0.3/E$	1996

Hadron Showers

- Hadrons interact with detector material also through the strong interaction
- Hadron calorimeter measurement:



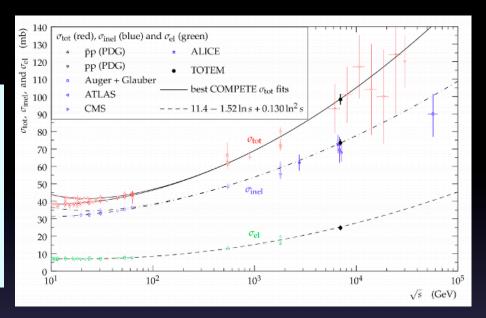
nentary to track measurement
ay to measure their energy
ondary particles are produced
reactions → hadronic cascades
ng particles (π,η) initiate EM shower
ed as nuclear binding energy or target recoil



Hadronic shower

Hadronic interaction Cross section

$$\sigma_{Tot} = \sigma_{el} + \sigma_{inel}$$
 $\sigma_{el} \approx 10mb$ $\sigma_{inel} \approx A^{2/3}$
 $\sigma_{Tot} = \sigma_{tot}(pp)A^{2/3}$
where: $\sigma_{tot}(pp)$ increases with \sqrt{s}



Hadronic interaction length

$$\lambda_{\text{int}} = \frac{1}{\sigma_{tot} \cdot n} = \frac{A\rho}{\sigma_{pp} A^{2/3} N_A} \approx \left(35g / cm^2\right) A^{1/3}$$

$$N(x) = N(0) e^{-x/\lambda_{\text{int}}}$$

λ_{int} characterizes both longitudinal and transverse shower profile

Rule of thumb argument: the geometric cross section goes as the square of the size of the nucleus, a_N^2 , and since the nuclear radius scales as $a_N \sim A^{1/3}$, the nuclear mean free path in gm/cm² units scales as $A^{1/3}$.

Hadronic vs EM showers

Hadronic vs. electromagnetic interaction length:

$$X_0 \sim rac{A}{Z^2} \ \lambda_{
m int} \sim A^{1/3} \
ightharpoons \ rac{\lambda_{
m int}}{X_0} \sim A^{4/3}$$

$$\lambda_{
m int}\gg X_0$$
[$\lambda_{
m int}/X_0>30$ possible; see below]

Typical

Longitudinal size: 6 ... 9 λ_{int}

... 9 ∧int [95% containment] [EM: 15-20 X₀]

Typical

Transverse size: one λ_{int}

[95% containment]

[EM: 2 R_M; compact]

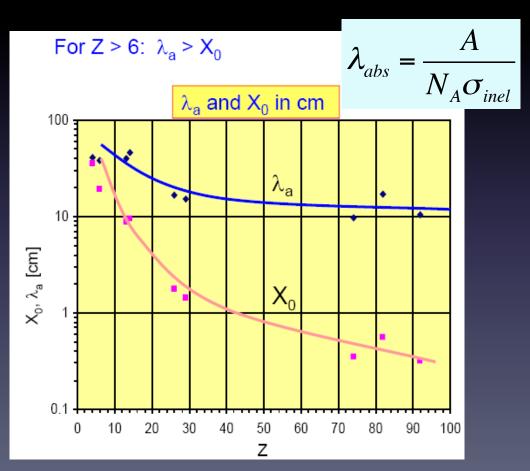
Hadronic calorimeter need more depth than electromagnetic calorimeter ...

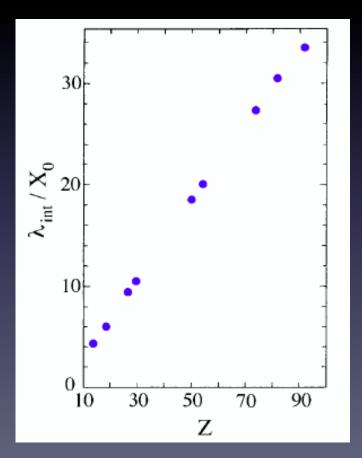
Some numerical values for materials typical used in hadron calorimeters

	λ _{int} [cm]	X ₀ [cm]
Szint.	79.4	42.2
LAr	83.7	14.0
Fe	16.8	1.76
Pb	17.1	0.56
U	10.5	0.32
ß	38.1	18.8

Material dependence

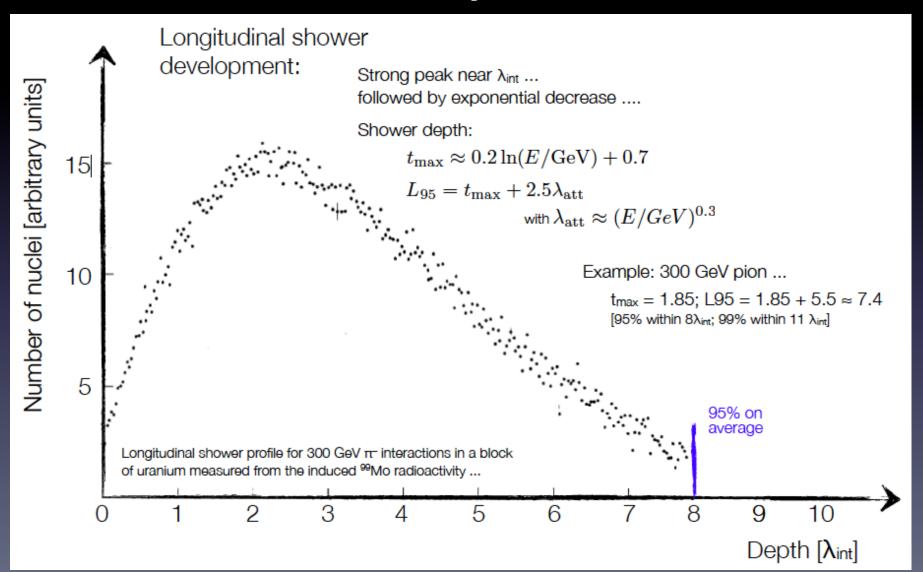
- λ_{int} : mean free path between nuclear collisions: λ_{int} (g cm⁻²) \propto A^{1/3}
- λ_{abs}: Hadronic absorption length for inelastic processes
- Hadron showers are much longer than EM ones. Length depends on Z



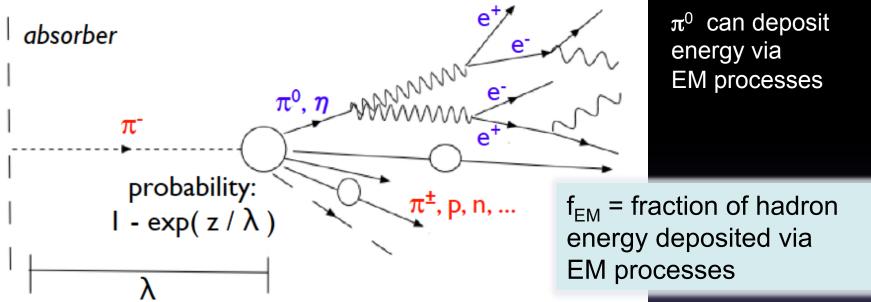


D. Bortoletto Lecture 5

Hadronic shower: Longitudinal development



Hadronic Shower



- Electromagnetic
 - ionization, excitation (e[±])
 - photo effect, scattering (γ)
- Hadronic
 - ionization (π[±], p)
 - invisible energy (binding, recoil)

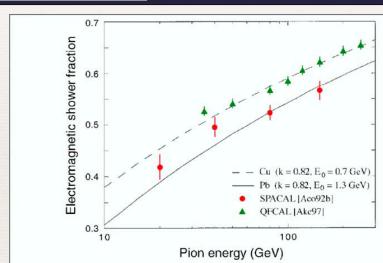


FIG. 2.22. Comparison between the experimental results on the em fraction of pion-induced showers in the (copper-based) QFCAL and (lead-based) SPACAL detectors. Data from [Akc 97] and [Aco 92b].

EM fraction in hadronic calorimeters

Charge conversion of $\pi^{+/-}$ produces electromagnetic component of hadronic shower (π^0)

- e = response to the EM shower component
- h = response to the non-EM component

$$\pi = f_{em} e + (1 - f_{em}) h$$

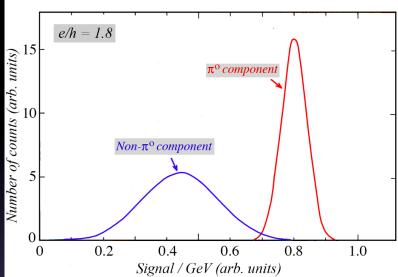
Comparing pion and electron showers:

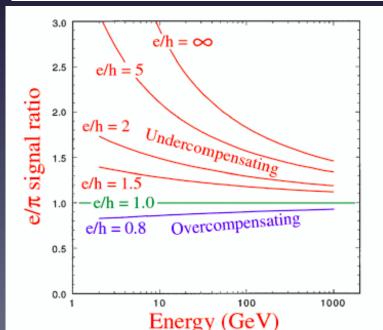
$$\frac{e}{\pi} = \frac{e}{f_{em}e + (1-f_{em})h} = \frac{e}{h} \frac{1}{1 + f_{em}(e/h-1)}$$

Calorimeters can be:

- Overcompensating e/h < 1
- Undercompensating e/h > 1
- Compensating e/h = 1

The origin of the non-compensation problems





Compensation

- Non-linearity determined by e/h value of the calorimeter
- Measurement of non-linearity is one of the methods to determine e/h
- Assuming linearity for EM showers, e(E1)=e(E2):

$$\frac{\pi(E_1)}{\pi(E_2)} = \frac{f_{em}(E_1) + [1 - f_{em}(E_1)] \cdot e/h}{f_{em}(E_2) + [1 - f_{em}(E_2)] \cdot e/h}$$

For e/h=1
$$\Rightarrow$$
 $\frac{\pi(E_1)}{\pi(E_2)} = 1$

 Response of calorimeters is usually higher for electromagnetic (e) than hadronic (h) energy deposits→ e/h>1

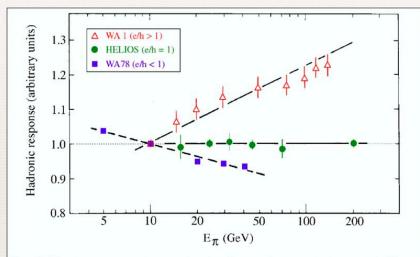
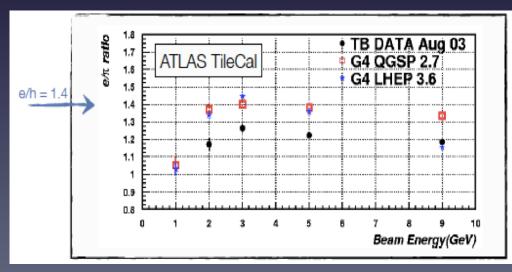
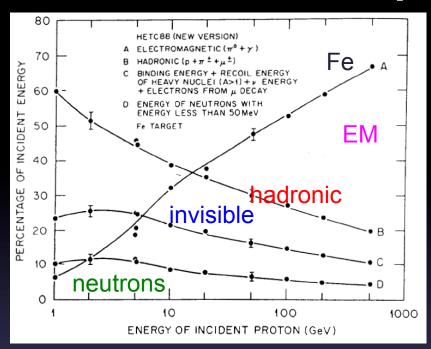


FIG. 3.14. The response to pions as a function of energy for three calorimeters with different e/h values: the WA1 calorimeter (e/h > 1, [Abr 81]), the HELIOS calorimeter ($e/h \approx 1$, [Ake 87]) and the WA78 calorimeter (e/h < 1, [Dev 86, Cat 87]). All data are normalized to the results for 10 GeV.



Compensation

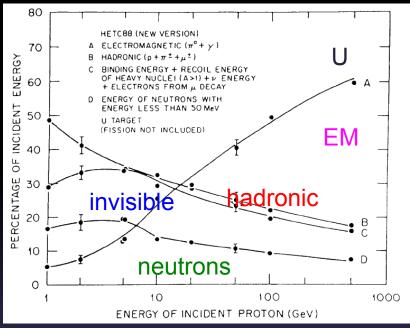


$$E_{p} = f_{em} e + (1 - f_{em})h$$

$$h = f_{rel} \cdot rel + f_{p} \cdot p + f_{n} \cdot n + f_{inv} \cdot inv$$

Compensation:

- Tuning the neutron response using hydrogenous active material (L3 Uranium/gas calorimeter)
- Compensation adjusting the sampling frequency



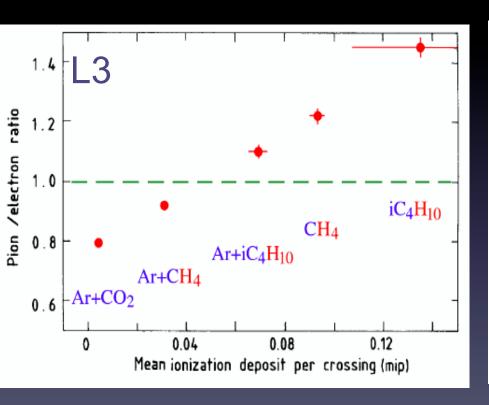
Energy deposition mechanisms

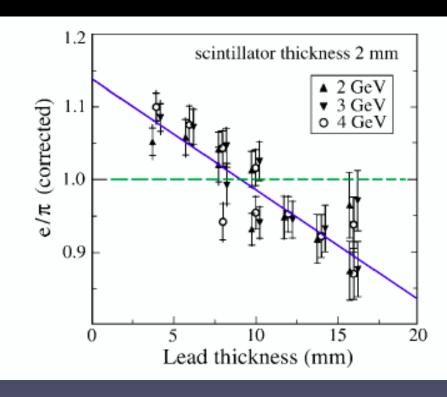
- f_{rel}= Ionization by charged pions (relativistic shower component)
- f_p=spallation protons
- f_n=neutrons evaporation
- finv=invisible energy by recoil nuclei

Compensation by tuning neutron response

Hydrogen in active material (gas mixture)

Pb/Scintillator



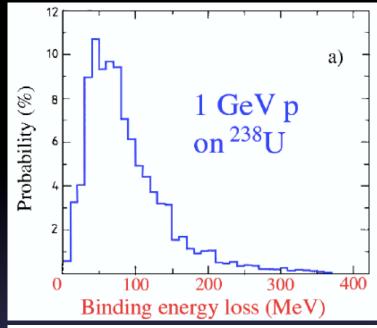


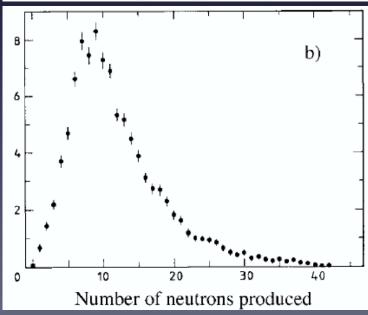
Elastic n-p scattering: efficient sampling of neutrons through the detection of recoiling proton

Sampling fraction can be tuned to achieve compensation

Energy resolution of hadronic showers

- Fluctuations in visible energy (ultimate limit of hadronic energy resolution)
 - fluctuations of nuclear binding energy loss in high-Z materials ~15%
- Fluctuations in the EM shower fraction, f_{em}
 - Dominating effect in most hadron calorimeters (e/h >1)
 - Fluctuations are asymmetric in pion showers
 - Differences between p, π induced showers (No leading π^0 in proton showers)
- Sampling fluctuations only minor contribution to hadronic resolution in noncompensating calorimeter





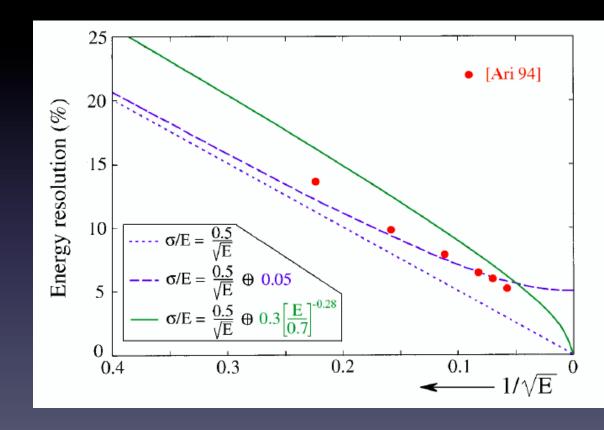
Energy resolution of hadron showers

■ Hadronic energy resolution of non-compensating calorimeters does not scale with 1/√E but as

$$\frac{\sigma_E}{E} = \frac{a}{\sqrt{E}} \oplus b \left(\frac{E}{E_0}\right)$$

But in practice we use

$$\frac{\sigma_E}{E} = \frac{a}{\sqrt{E}} \oplus b$$



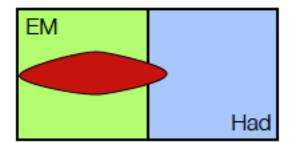
A realistic calorimetric system

Typical Calorimeter: two components ...

Schematic of a typical HEP calorimeter

Electromagnetic (EM) + Hadronic section (Had) ...

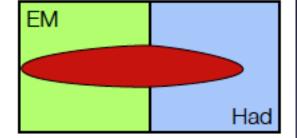
Different setups chosen for optimal energy resolution ... Electrons Photons



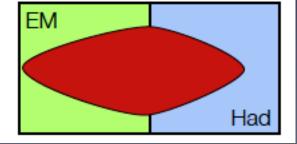
But:

Hadronic energy measured in both parts of calorimeter ...

Needs careful consideration of different response ... Taus Hadrons



Jets



LHC CALORIMETERS

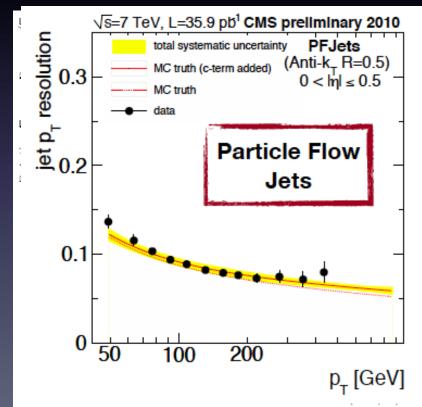


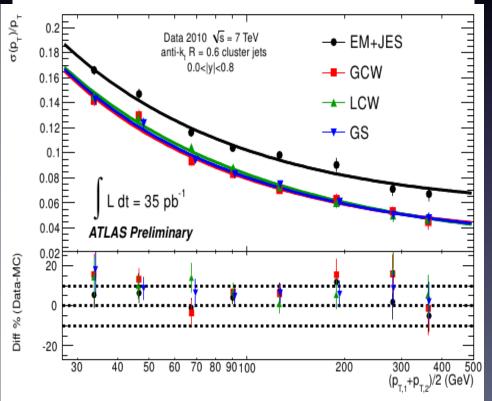
5 cm brass / 3.7 cm scint. Embedded fibres, HPD readout 14 mm iron / 3 mm scint. sci. fibres, read out by phototubes

Hadronic calorimeters resolution

- HCAL only $\sigma/E = (93.8 \pm 0.9)\%/\sqrt{E} \oplus (4.4 \pm 0.1)\%$
- ECAL+HCAL $\sigma/E = (82.6 \pm 0.6)\%/\sqrt{E} \oplus (4.5 \pm 0.1)\%$
- Improved resolution using full calorimetric system (ECAL+HCAL)

ATLAS LAr + Tile for pions:
$$\frac{\sigma(E)}{E} = \frac{42\%}{\sqrt{E}} \oplus 2\%$$



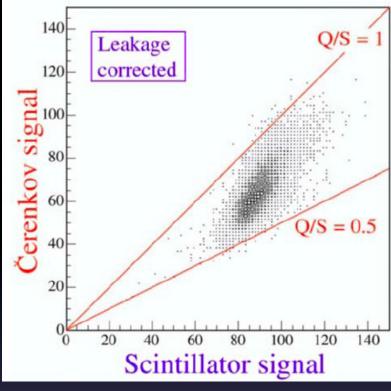


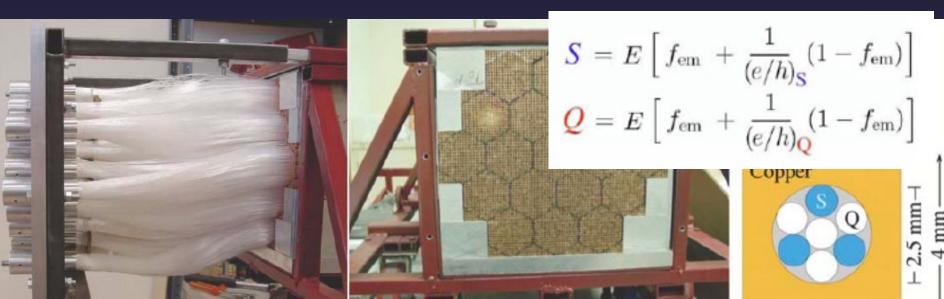
Future calorimeters

- Concentrate on improvement of jet energy resolution to match the requirement of the new physics expected in the next 30-50 years:
- Two approaches:
 - minimize the influence of the calorimeter and measure jets using the combination of all detectors → Particle Flow
 - measure the shower hadronic shower components in each event & weight directly access the source of fluctuations → Dual (Triple) Readout

DREAM

- Measure f_{EM} cell-by-cell by comparing Cherenkov and dE/dx signals
- Densely packed SPAgetti CALorimeter with interleaved Quartz (Cherenkov) and Scintillating Fibers
- Production of Cerenkov light only by emparticles (f_{EM})
- Aim at: σ_E/E ~ 15%/√E

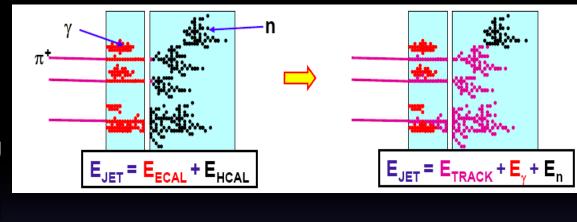




PF calorimetry (CALICE)

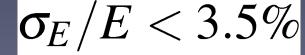
- Design detectors for Pflow
 - ECAL and HCAL: inside solenoids
 - Low mass tracker
 - High granularity for imaging calorimetry

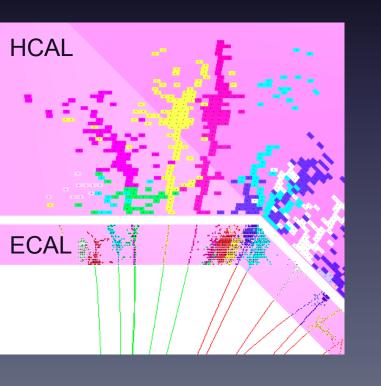
It also require sophisticated software



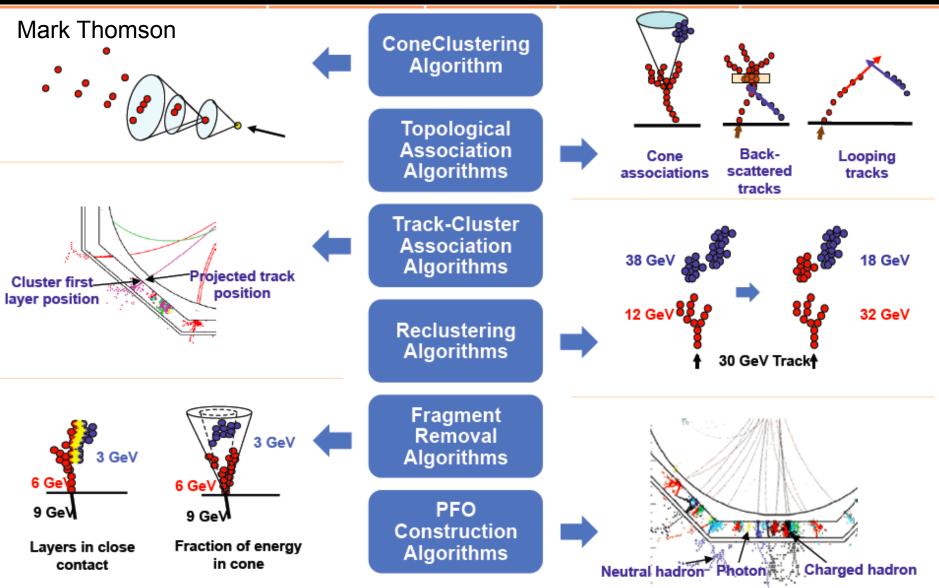
Two proto-collaborations for ILC (ILD and SLD)

- ECAL: Highly segmented SIW or Scintillator-W sampling calorimeters
 - Transverse segmentation: ~5 x 5 mm²
 - ~30 longitudinal sampling layers
- HCAL: Highly segmented sampling calorimeters
 Steel or W absorber+ active material (RPC, GEM)
 - Transverse segmentation: 1x1 cm² 3x3 cm²
 - ~50 Longitudinal sampling layers!
- Aiming at





Particle flow



Proposed CMS Si-based Endcap Calorimeter

- The CMS endcap calorimeters will be replaced for the high luminosity LHC running that aims to record an integrated luminosity of 3000 fb⁻¹.
- A dense and compact approach is proposed for both electromagnetic and hadronic calorimetry that uses a high lateral and longitudinal granularity.
- Recent advances in Si sensors in terms of cost per unit area and radiation tolerance, and advances in electronics and data transmission bring up the possibility of their use in such high granularity calorimetry.
- High granularity calorimeters are proposed for future ILC/CLIC detectors, for which they have been shown to provide very high resolving power for single particles in dense jet environments, with energies of several hundred GeV's.
- The challenges faced for high-luminosity LHC operation are mainly in the area of engineering (mechanical and thermal), data transmission and Level-1 trigger formation

CMS HGC

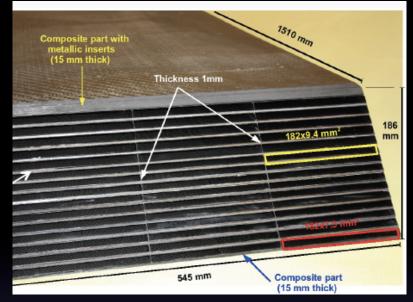
Electromagnetic Calorimeter Longitudinal sampling (25 X_0 , ~ 1 λ)

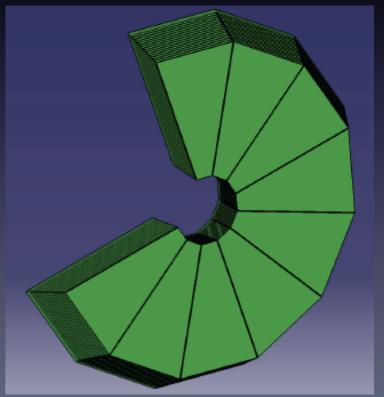
- 1 plane of silicon,
- 10 layers: $0.5 X_0$ thickness absorber followed by a plane of silicon,
- 10 layers: 0.8 X₀ thickness absorber followed by a plane of silicon,
- 10 layers: $1.2 X_0$ thickness absorber followed by a plane of silicon.

Lateral sampling
Average pad size ~ 1cm²

Front Hadronic Calorimeter Longitudinal sampling (3.5 λ) 12 samplings of ~0.3 λ Average cell size ~ 1-2 cm²

Total number of channels: 6-9 million



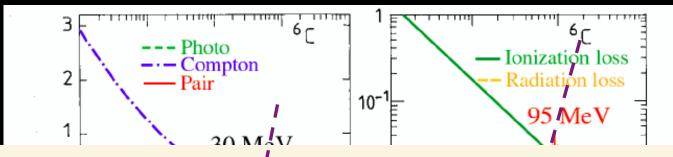


References

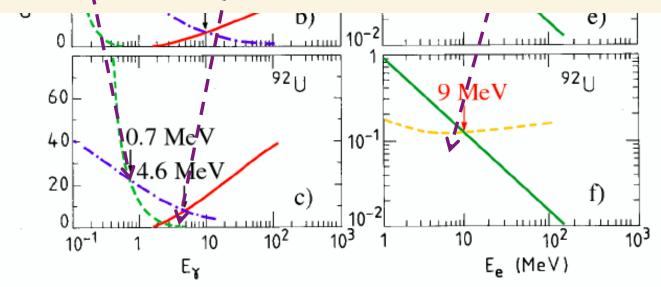
- Particle flow- M. Thompson
- Calorimetry for Particle Physics- C. Fabjan and F. Gianotti- CERN-EP/ 2003-075

BACKUP

Material dependence



Even though calorimeters are intended to measure GeV, TeV energy deposits, their performance is determined by what happens at the MeV - keV - eV level



Summary

Radiation length:

$$X_0 = \frac{180A}{Z^2} \frac{\mathrm{g}}{\mathrm{cm}^2}$$

Problem:

Calculate how much Pb, Fe or Cu is needed to stop a 10 GeV electron.

Pb: Z=82, A=207, $\rho=11.34$ g/cm³ Fe: Z=26, A=56, $\rho=7.87$ g/cm³ Cu: Z=29, A=63, $\rho=8.92$ g/cm³

Critical energy:

[Attention: Definition of Rossi used]

$$E_c = \frac{550 \text{ MeV}}{Z}$$

, acondon Boll later of Frood C

$$t_{\rm max} = \ln \frac{E}{E_c} - \left\{ \begin{array}{ll} 1.0 & {\rm e^-} \, {\rm induced \, shower} \\ 0.5 & {\rm y \, induced \, shower} \end{array} \right.$$

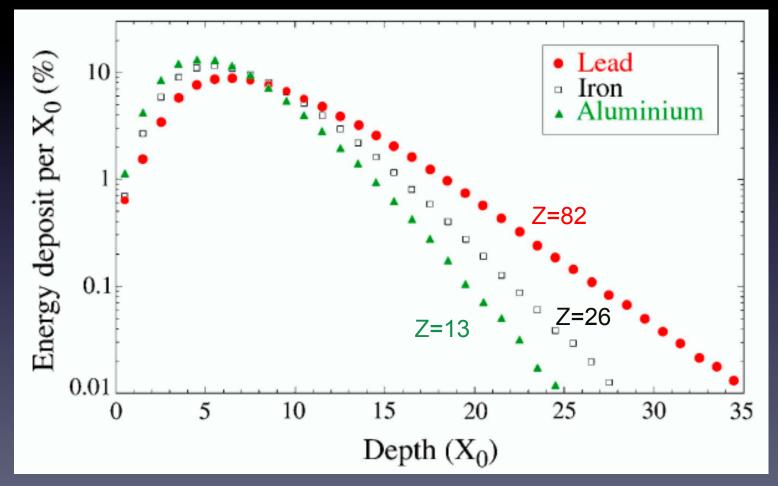
Shower maximum:

$$L(95\%) = t_{\text{max}} + 0.08Z + 9.6 [X_0]$$

$$R(90\%) = R_M$$
$$R(95\%) = 2R_M$$

Longitudinal development of EM

• Shower decay: after the shower decays slowly through ionization and Compton scattering→ proportional to X₀



Resolution in Homogenous calorimeters

- Homogeneous calorimeters: signal = sum of all E deposited by charged particles with E>E_{threshold}
- If W is the mean energy required to produce a 'signal quantum' (eg an electron-ion pair in a noble liquid or a 'visible' photon in a crystal) the mean number of 'quanta' produced is \(\ n \) = E / W
- The intrinsic energy resolution is given by the fluctuations on n.

$$\frac{\sigma_E}{E} = \frac{1}{\sqrt{n}} = \frac{1}{\sqrt{E/W}}$$

i.e. in a semiconductor crystals W \approx 3 eV (to produce e-hole pair) 1 MeV \mathbf{y} = 350000 electrons \rightarrow 1/ $\sqrt{}$ n = 0.17% stochastic term

 Fluctuations on n are reduced by correlation in the production of consecutive e-hole pairs: the Fano factor F

$$\frac{\sigma_E}{E} = \frac{1}{\sqrt{FE/W}}$$

The Fano factor depends on the material

Resolution in Sampling calorimeters

- Main contribution: sampling fluctuations, from variations in the number of charged particles crossing the active layers.
- Increases linearly with incident energy and with the finess of the sampling.
- Thus:

$n_{ch} \propto E / t$ where (is the thickness of each absorber layer)

 For statistically independent sampling the sampling contribution to the stochastic term is:

$$\frac{\sigma_{samp}}{E} = \frac{1}{\sqrt{n_{ch}}} \propto \sqrt{\frac{t}{E}}$$

- Thus the resolution improves as t is decreased.
- For EM calorimeters the 100 samplings required to approach the resolution of homogeneous devices is not feasible
- Typically

$$\frac{\sigma_{samp}}{E} = \frac{10\%}{\sqrt{E}}$$

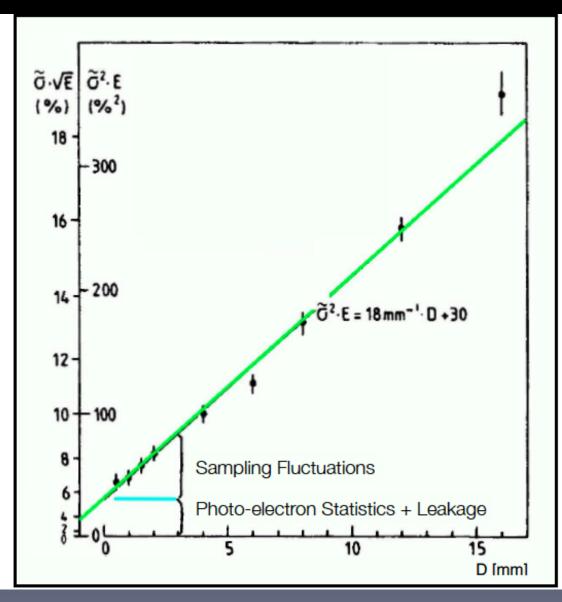
D. Bortoletto Lecture 5

Dependence on sampling

Measure energy resolution of a sampling calorimeter for different absorber thicknesses

Sampling contribution:

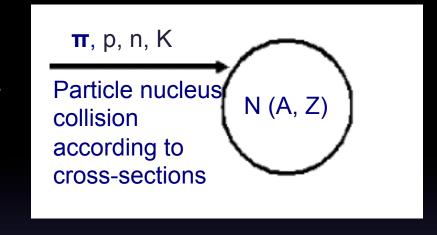
$$\frac{\sigma_E}{E} = 3.2\% \sqrt{\frac{E_c \,[\text{MeV}] \cdot t_{\text{abs}}}{F \cdot E \,[\text{GeV}]}}$$

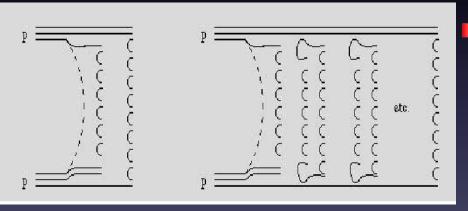


Hadronic interactions

1st stage: the hard collision

- pions travel 25-50% longer than protons (~2/3 smaller in size)
- a pion loses ~100-300 MeV by ionization (Z dependent)





Nucleon is split in quark di-quark Strings are formed String hadronisation (adding qqbar pair) fragmentation of damaged nucleus

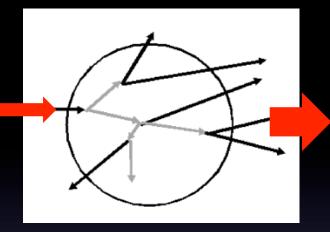
- Particle multiplication (string model)
 - average energy needed to produce a pion 0.7 (1.3) GeV in Cu (Pb)
 - Multiplicity scales with E and particle type
 - ~ 1/3 π°→ γγ produced in charge exchange processes: π⁺p → π°n and π⁻n → π°p
 - Leading particle effect: depends on incident hadron type e.g fewer π° from protons, barion number conservation

D. Bortoletto Lecture 5

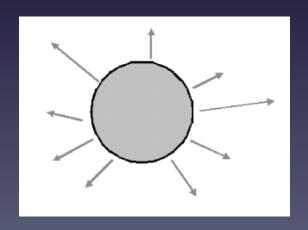
Hadronic interactions

2nd stage: spallation

- A fast hadron traversing the nucleus frees protons and neutrons in number proportional to their numerical presence in the nucleus.
- The nucleons involved in the cascade transfer energy to the nucleus which is left in an excited state
- Nuclear de-excitation
 - Evaporation of soft (~10 MeV) nucleons and α
 - fission for some materials
- The number of nucleons released depends on the binding E (7.9 MeV in Pb, 8.8 MeV in Fe)
- Mainly neutrons released by evaporation protons are trapped by the Coulomb barrier (12 MeV in Pb, only 5 MeV in Fe)



Dominating momentum component along incoming particle direction



isotropic process

Hadronic shower

Hadronic interaction:

Elastic:

 $p + \text{Nucleus} \rightarrow p + \text{Nucleus}$

Inelastic:

$$p + \text{Nucleus} \rightarrow$$

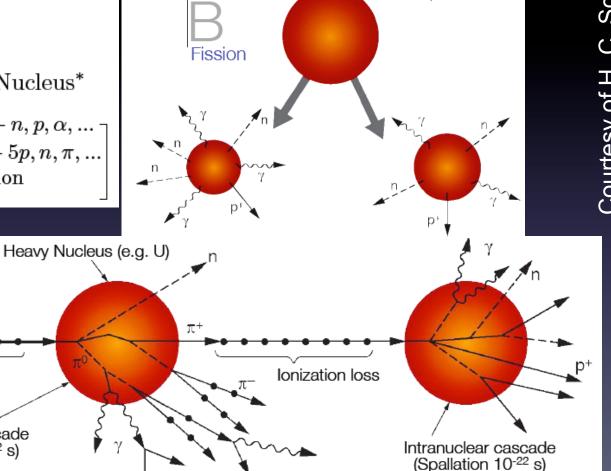
$$\pi^{+} + \pi^{-} + \pi^{0} + \ldots + \text{Nucleus}^{*}$$

- \rightarrow Nucleus B + 5p, n, π, \dots
- \rightarrow Nuclear fission

Ionization loss

Intranuclear cascade

Incoming hadron

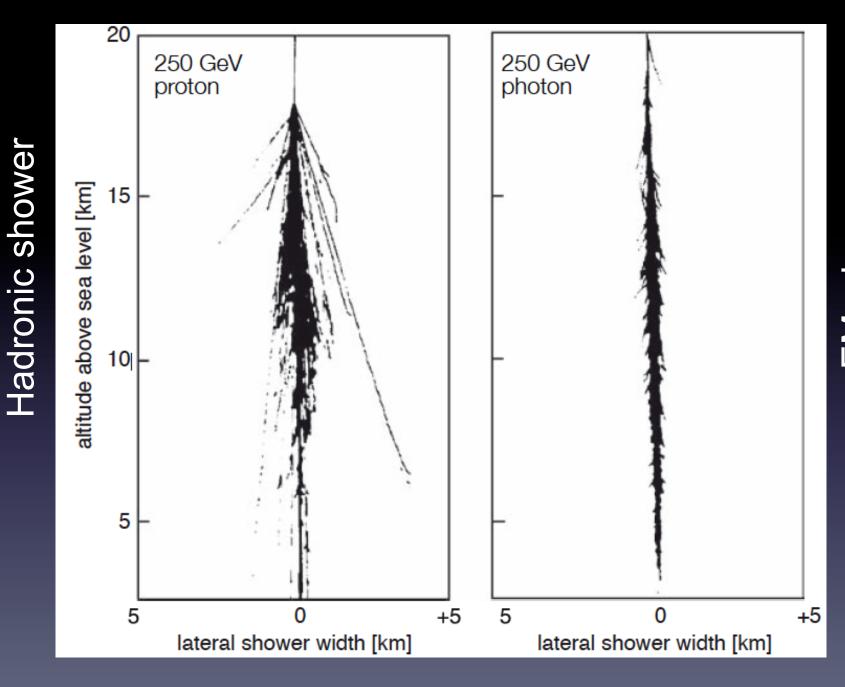


Internuclear cascade

Nuclear

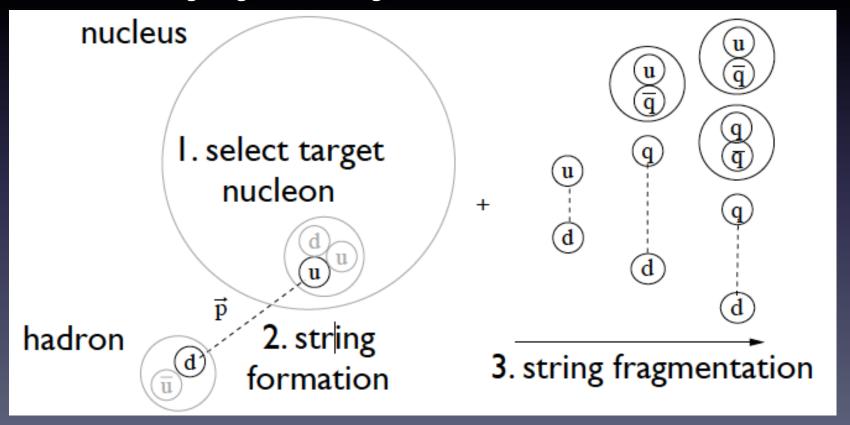
evaporation

(Spallation 10⁻²² s) Inter- and intranuclear cascade



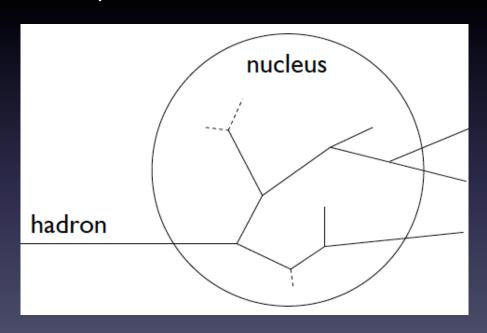
Simulation

- Interaction of hadrons with E > 10 GeV described by string models
 - projectile interacts with single nucleon (p,n)
 - a string is formed between quarks from interacting nucleons
 - the string fragmentation generates hadrons



Simulation

- Interaction of hadrons with 10 MeV < E < 10 GeV via intra-nuclear cascades
- For E < 10 MeV only relevant are fission, photon emission, evaporation, ...



Approximations

- λ_{deBroglie} ≤ d nucleon
- nucleus = Fermi gas (all nucleons included)
- Pauli exclusion: allow only secondaries above Fermi energy