HST-2014 QUESTIONS
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Problem Beginning with $E^2 = p^2c^2 + m^2c^4$ and $p = \frac{E}{c^2}v$,

show that $E = \frac{mc^2}{\sqrt{1-v^2/c^2}}$ and $p = \frac{mv}{\sqrt{1-v^2/c^2}}$

Answer

- $E^2 = \left(\frac{Ev}{c^2}\right)^2 c^2 + m^2c^4$

$\Rightarrow E^2\left(1-\frac{v^2}{c^2}\right) = m^2c^4 \Rightarrow E = \frac{mc^2}{\sqrt{1-v^2/c^2}}$

- $p = \frac{E}{c^2}v \Rightarrow E^2 = \frac{p^2c^4}{v^2}$

Substituting this into $E^2 = p^2c^2 + m^2c^4$ yields

$$\frac{p^2c^4}{v^2} = p^2c^2 + m^2c^4 \Rightarrow p^2c^4 = p^2c^2v^2 + m^2c^4v^2$$

$$\Rightarrow p^2c^4\left(1-\frac{v^2}{c^2}\right) = m^2c^4v^2$$

$$\Rightarrow p = \frac{mv}{\sqrt{1-v^2/c^2}}$$

COMMENT Bearing in mind that (following OKUN's argument) a particle only has one mass, which does NOT depend on its velocity, we can accomodate the relations just derived by saying something like "relativistic particles respond to forces AS IF their mass depended on velocity!