Beyond the Standard Model

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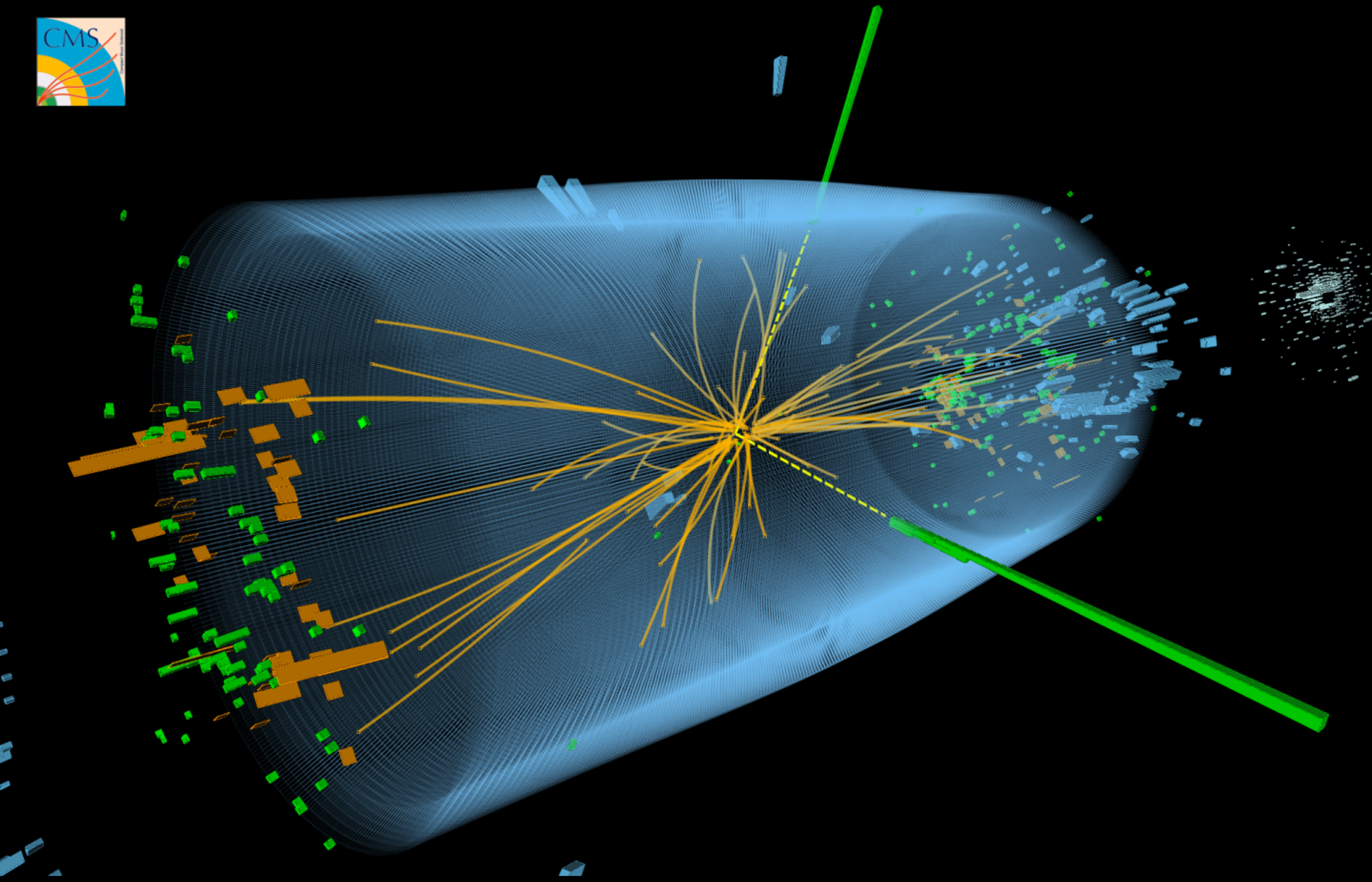
The LHC is a project aiming at exploring a new energy regime

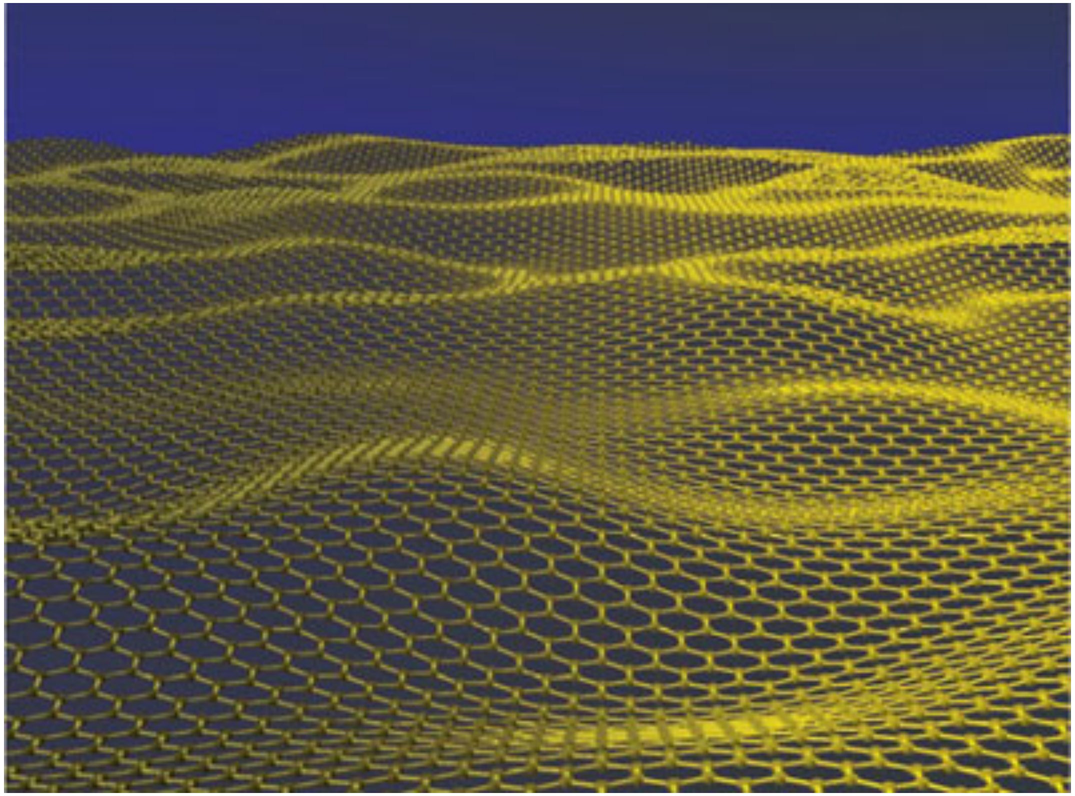
The goal is the exploration of small distances ($< 10^{-19}$ m) searching for new phenomena

- The engine that drives us to build accelerators is our understanding that the key to physical laws is hidden in the microcosm.
- The same laws help us to understand the large-scale structure of the universe and its early history.


CMS

At 10^{-10} seconds after the Big Bang:
Space crystallized into a new form

Nature filled space because she saved energy

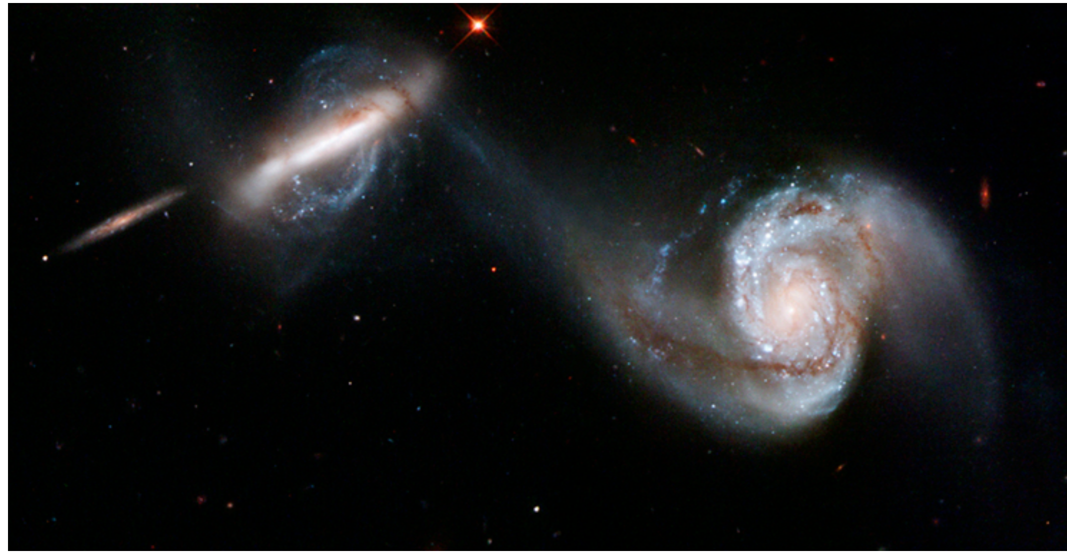


All particles are described by fields, but Higgs field is special

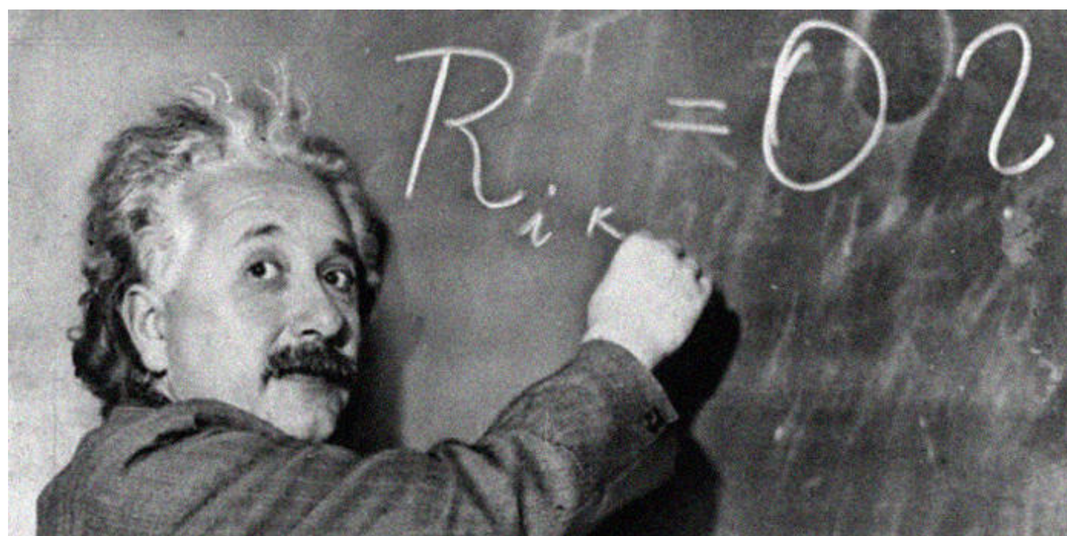
Higgs condensate: special arrangement of Higgs particles such that, in the "vacuum", the average Higgs field is constant in space-time. → spin zero

What caused the Bang?

Gravity is
always attractive

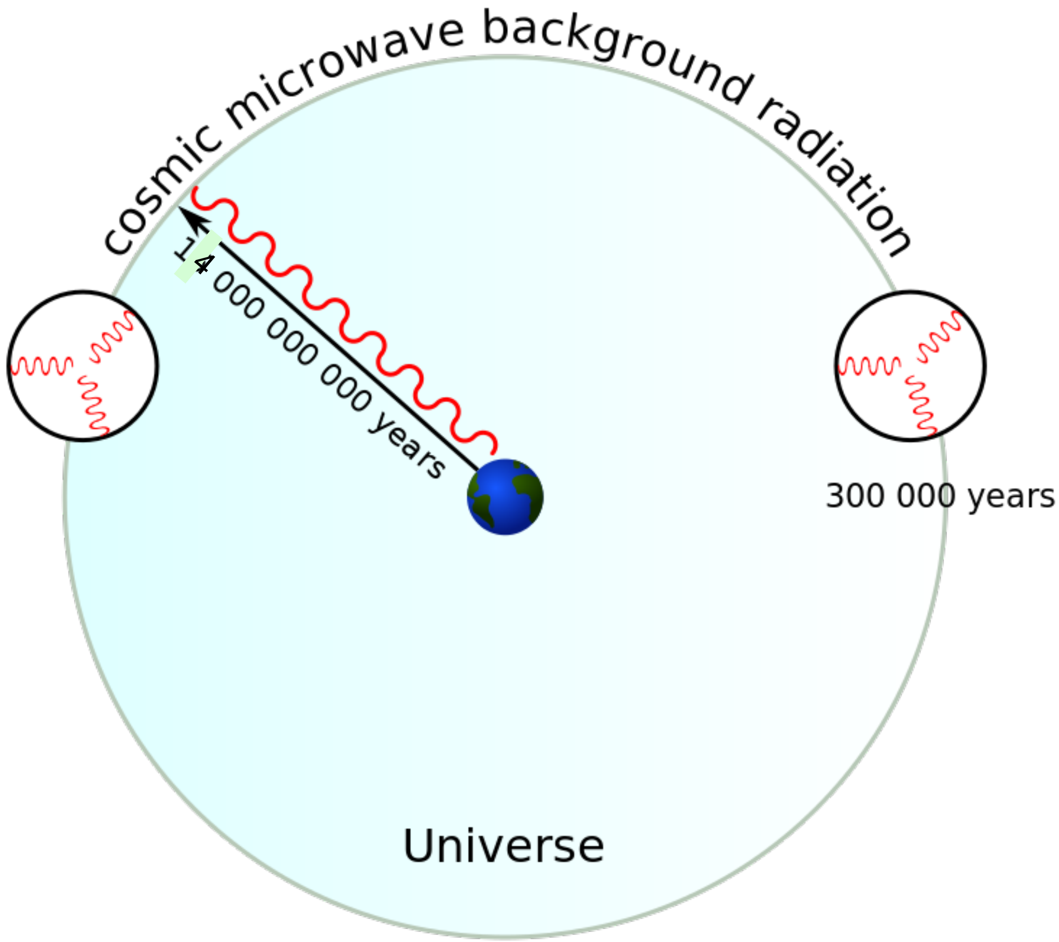


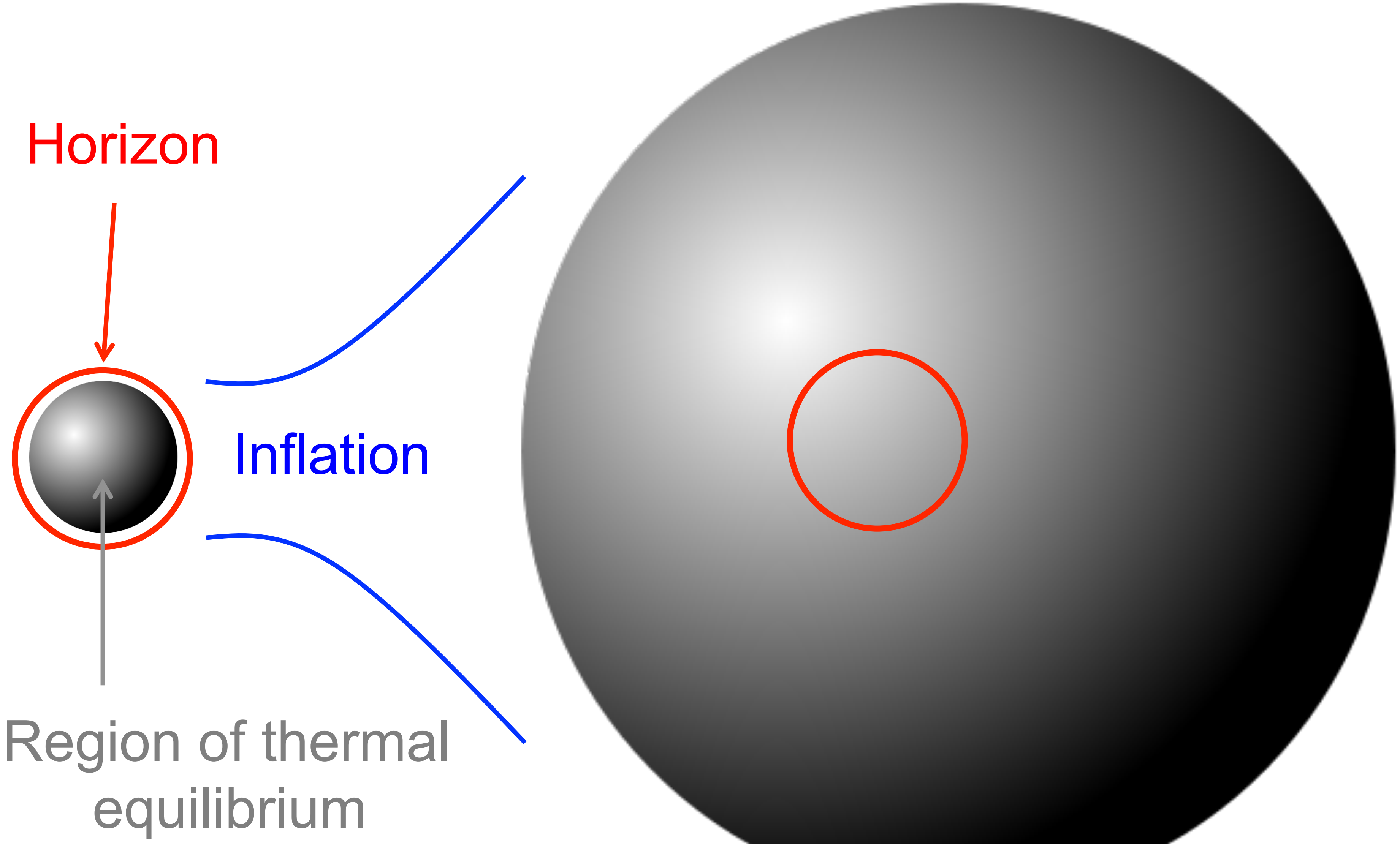
One exception in
General Relativity



Vacuum energy of a scalar field → inflation

Extraordinary space expansion sets the right initial conditions of the universe (uniform, flat, smooth, and expanding)



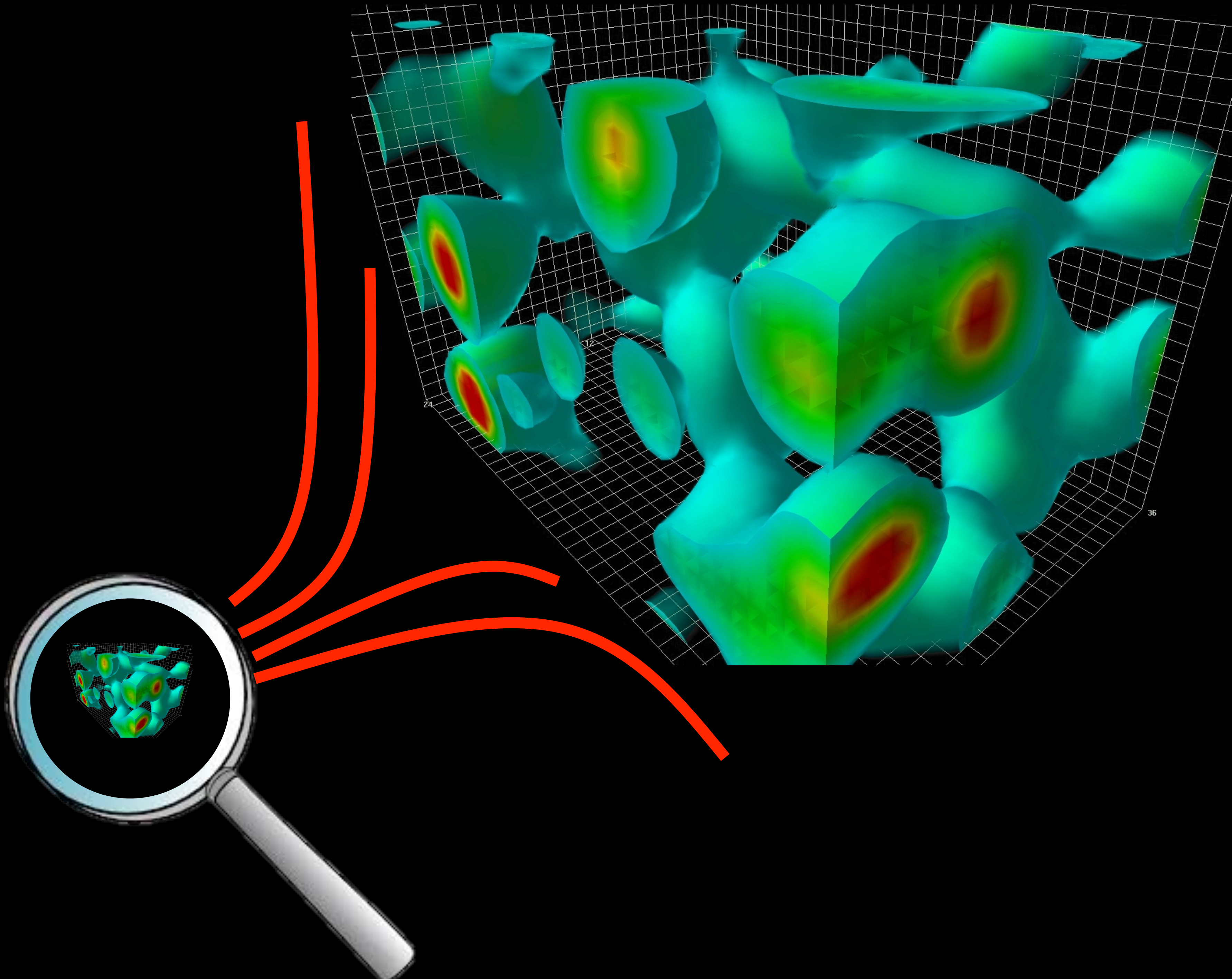

Horizon
Inflation
Region of thermal
equilibrium

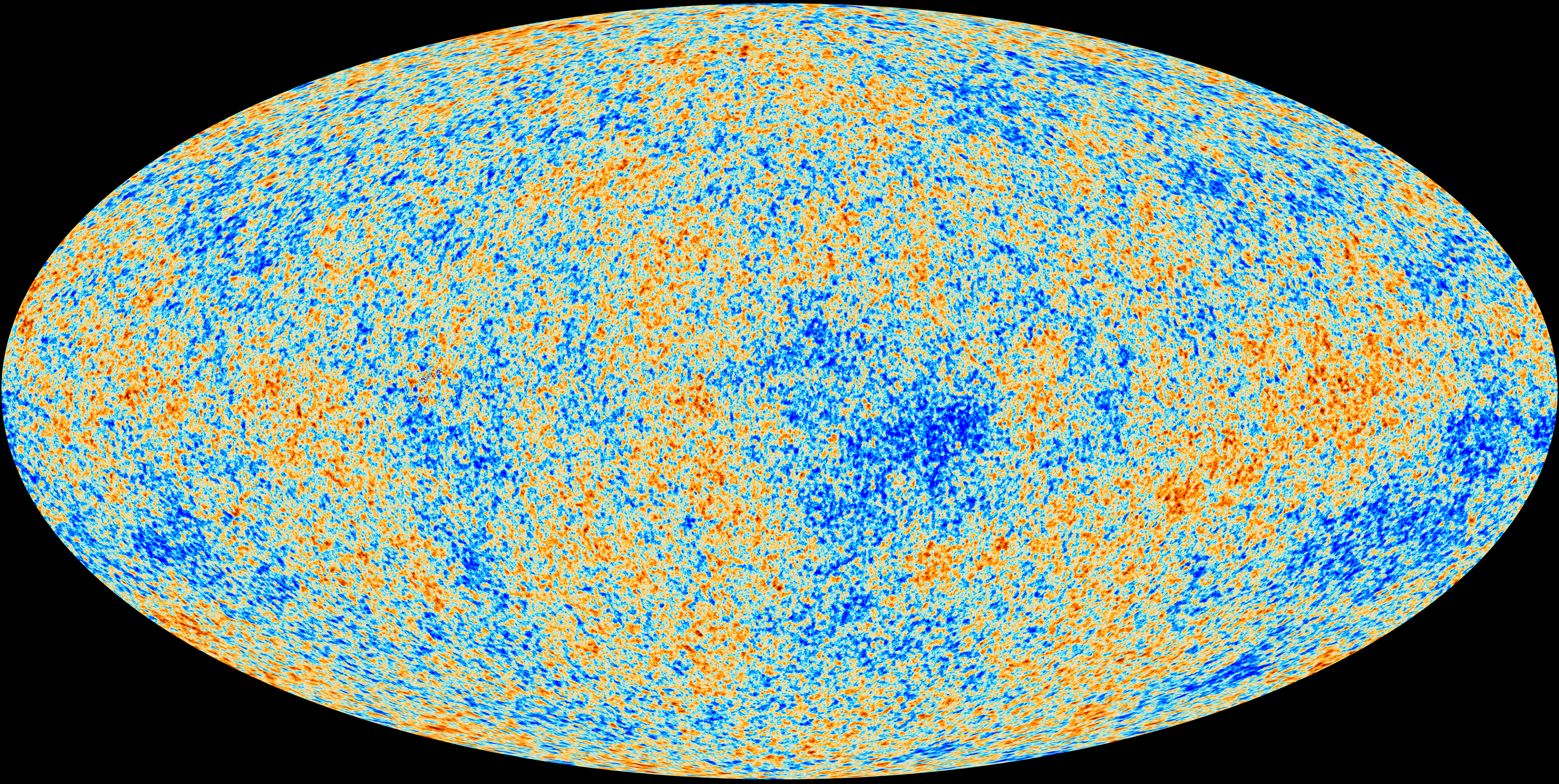
Inflation explains the initial conditions of the universe

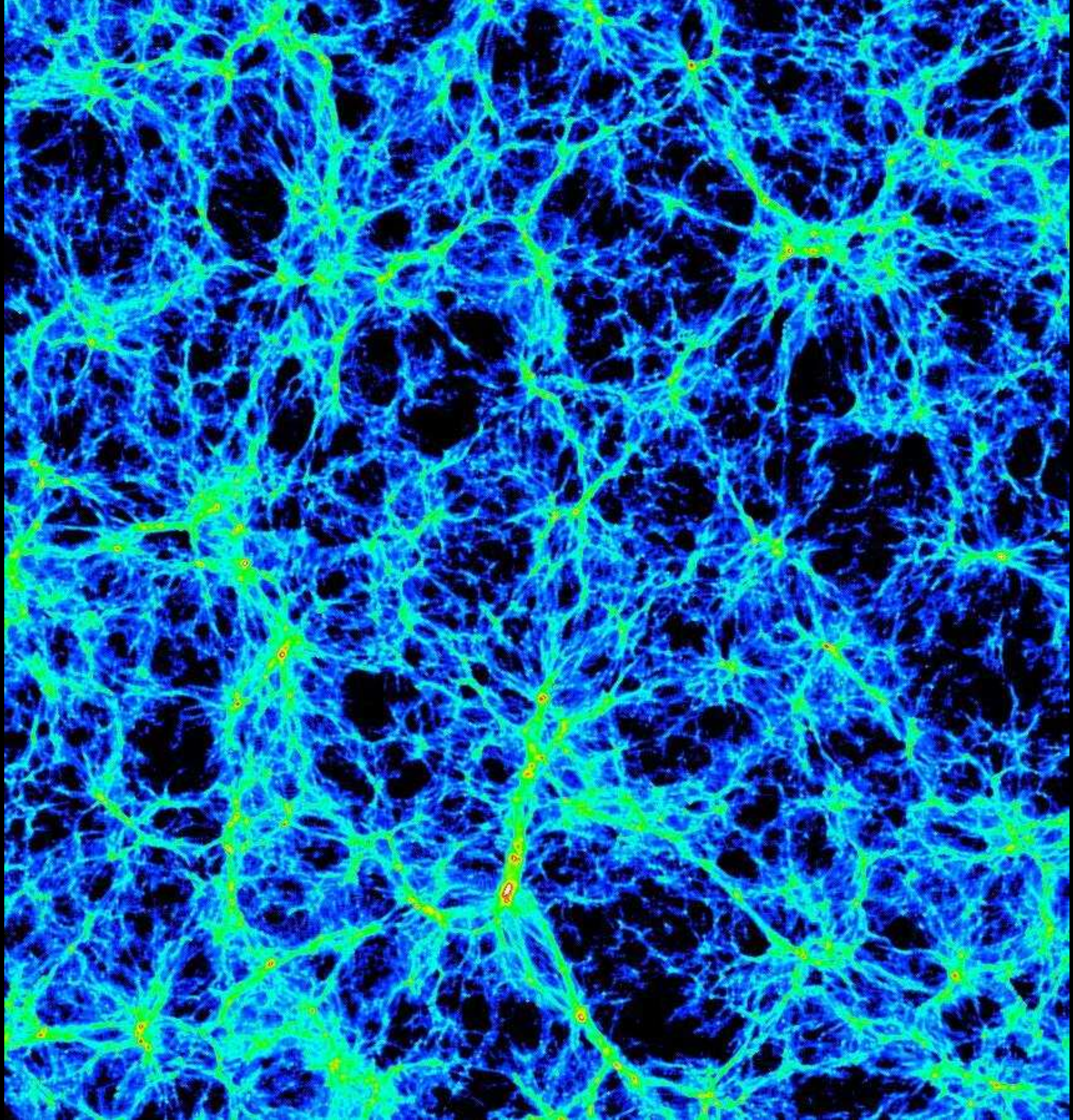
No bang, but

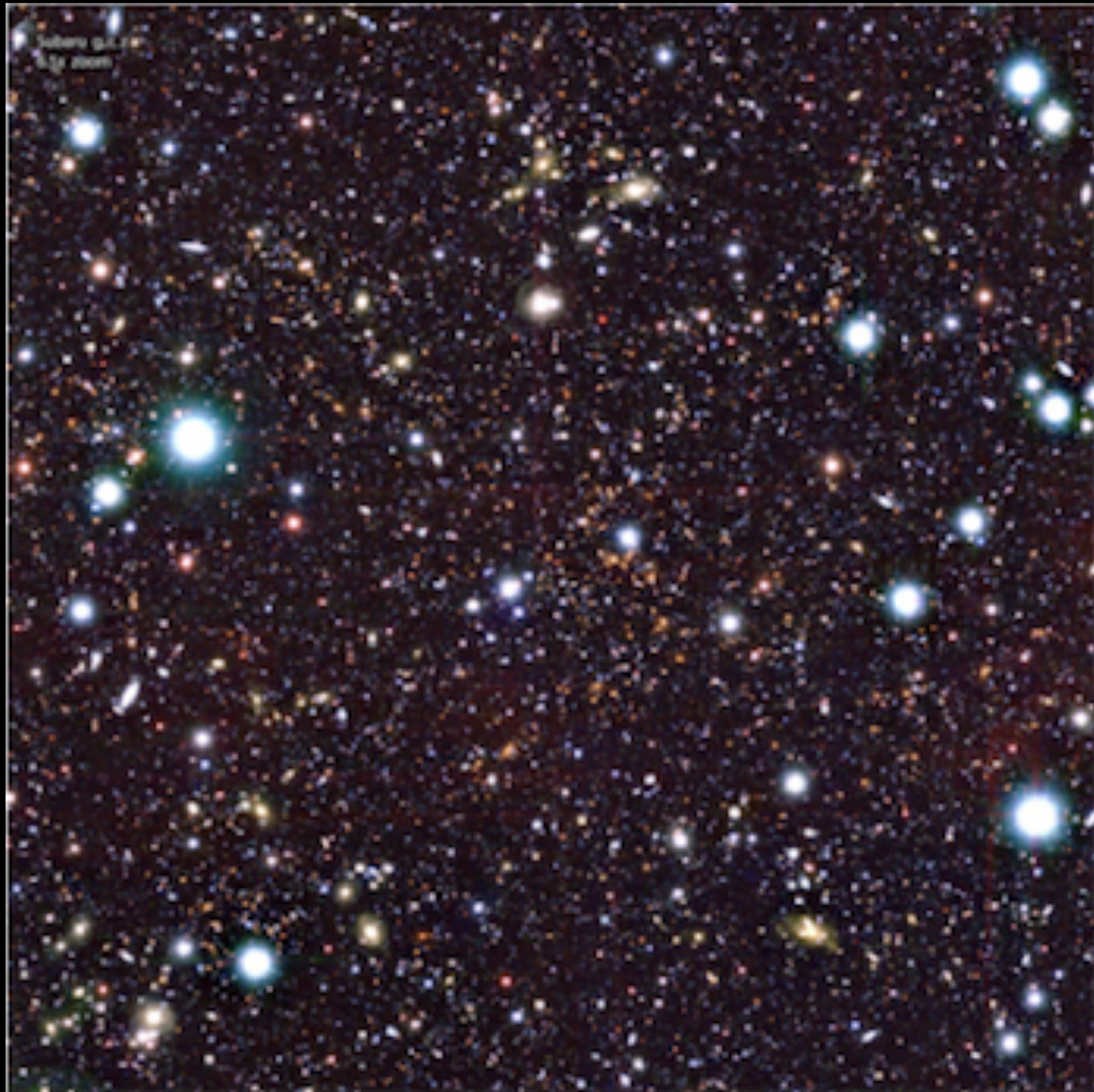
- Uniform and flat because of superluminal expansion
- Expanding because of initial kick from vacuum energy
- Low entropy
- Hot because, at the end of inflation, vacuum energy is released in the form of thermal energy

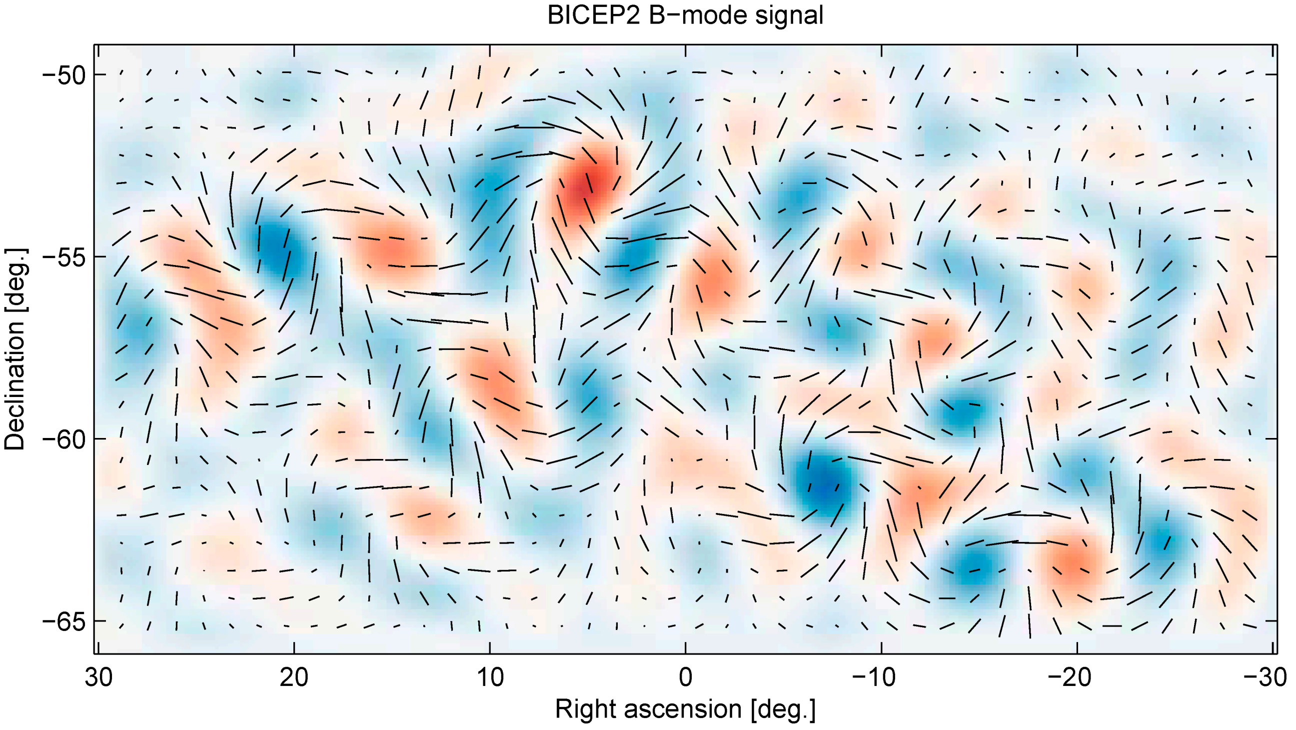
A new spin-0 field responsible for inflation?


12
24
36








BICEP2 B−mode signal
Declination [deg.]
−50
−55
−60
−65
Right ascension [deg.]
30
20
10
0
−10
−20
−30

Concept of **symmetry** central in modern physics

invariance of physics laws under transformation of dynamical variables

All physical phenomena in the microcosm can be understood in terms of a single symmetry principle

(simply connected) spherically symmetric object

Symmetry is the language of the fundamental laws of nature

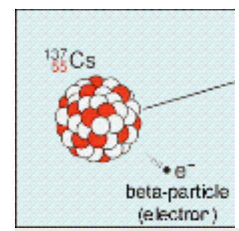
gauge symmetry

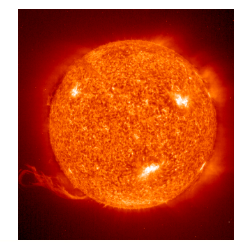
space-time

fields

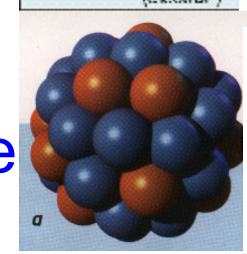
electro-magnetism

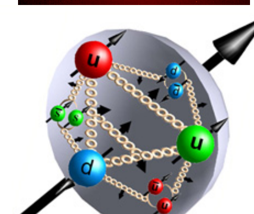
weak force





strong force



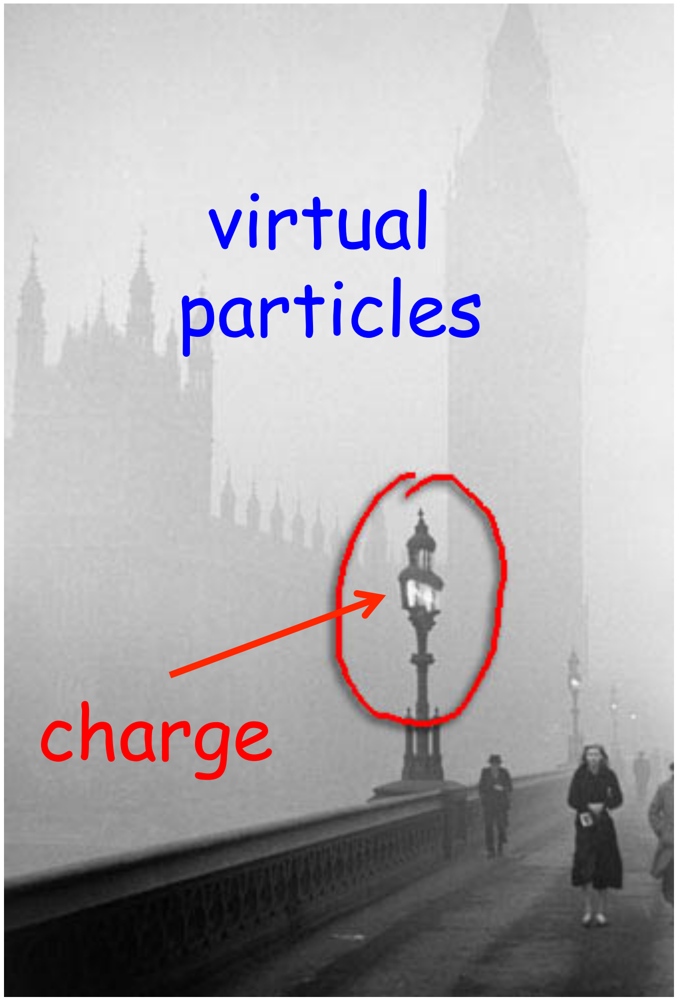


gravity

Can we go further?

Grand unification: single force ➔ single coupling

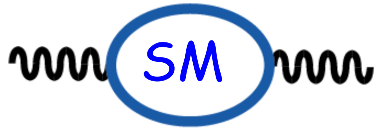
Classical physics: force depends on distance
Quantum physics: charge depends on distance



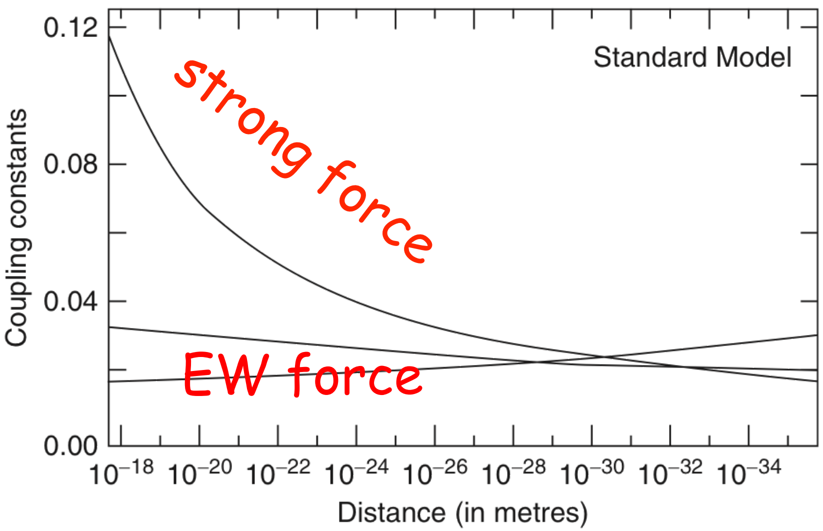
A strange phenomenon
QED: virtual particles screen the charge ➔ charge gets weaker as we move away

Even stranger
QCD: virtual particles antiscreen the charge ➔ charge gets stronger as we move away

$$\frac{dg_i^{-2}}{d\ln Q} = \frac{b_i}{4\pi}$$



b_3=−7, b_2=−19/6, b_1=41/6

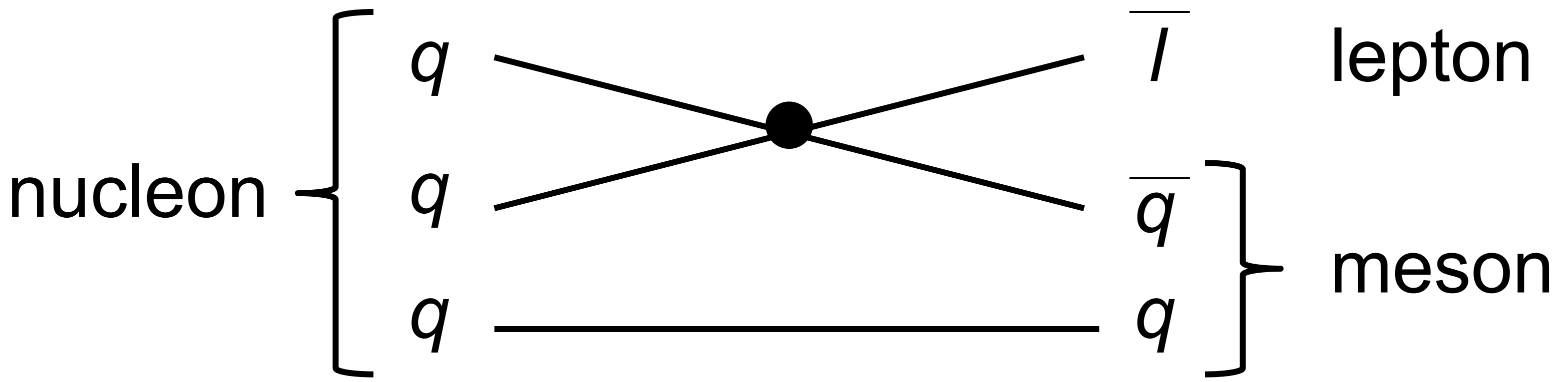


The screening (and antiscreening) depends on all species of existing particles

Extraordinary extrapolation to $M_X \sim 10^{14\text{-}16}$ GeV

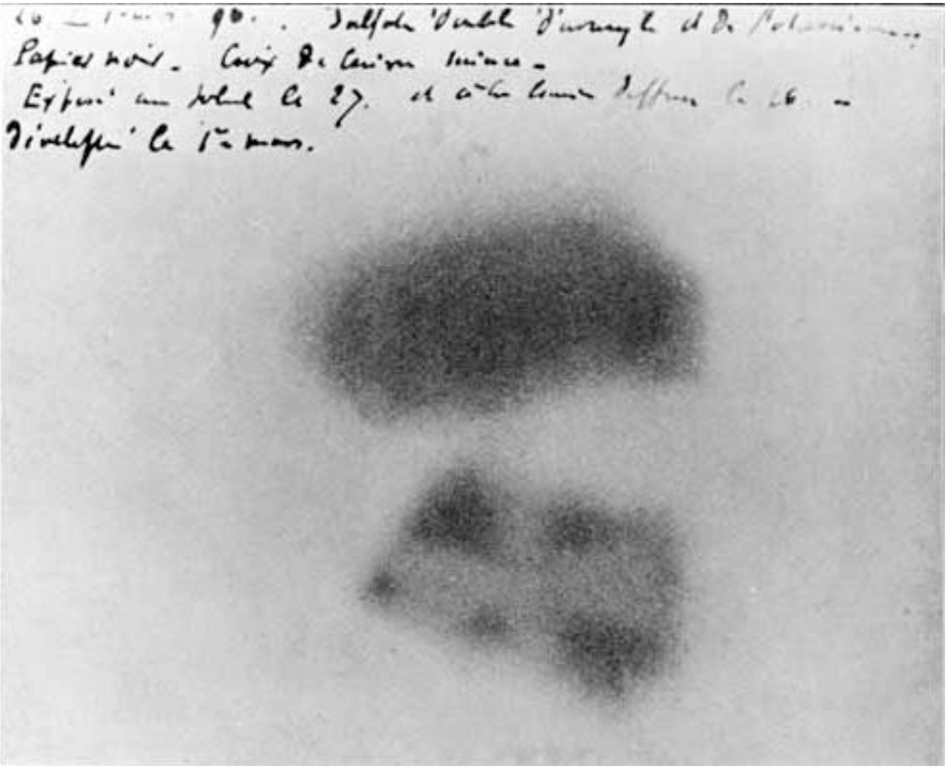
Above M_X theory with single coupling (SU_5, SO_{10})

Like quarks come in 3 colors, in GUT quarks & leptons are different components of the same particle



Matter is unstable?

Feb 26, 1896: Becquerel studies phosphorescence of a uranium salt



“Invisible phosphorescence radiation emitted with a persistence infinitely greater than the persistence of luminous radiation”

Discovery of many radioactive elements

(radium, thorium, polonium... + berzelium, carolinium...)

At a time when the nucleus was not known, the relation between radioactive and usual matter was unclear

In 1903, Rutherford and Soddy ask the question:

- Are all elements radioactive, but some have lifetimes too long to be measured?
- Radioactivity is due to small impurities of radium common in many materials?

Principle of simplicity: all matter is unstable

With the discovery of the nucleus and the understanding of its stability properties, it became clear that matter is stable

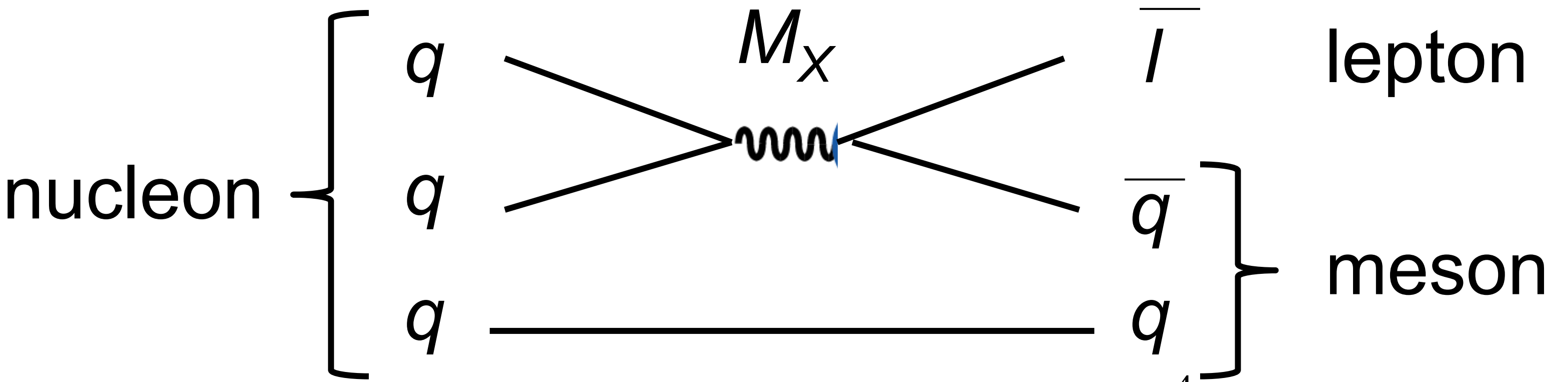
In 1929 Weyl formulated proton stability as conservation law

In 1948 Wigner talks about p-decay if conservation law is violated ($p \to e^+ \gamma$)

In 1954 Goldhaber says "we feel it in our bones that the proton lifetime is long": $\tau_p > 10^{18}$ yr

Experiments by Reines et al.: $\tau_p > 10^{21}$ yr; then 10^{26} yr

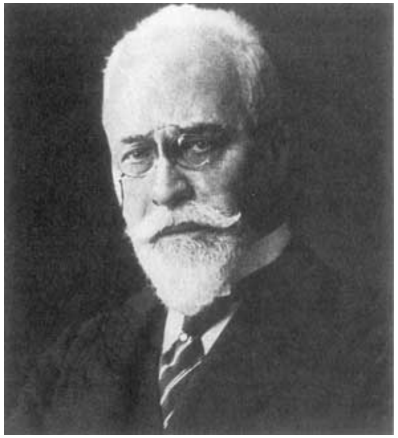
Shocking news from GUT: matter is unstable!



$$\text{GUT:}\quad \tau_p(p \to e^+\pi^0) = \left(\frac{M_X}{10^{15}\ \text{GeV}}\right)^4 10^{31-32}\ \text{yr}$$

$$\text{Exp:}\quad \tau_p(p \to e^+\pi^0) > 8.2 \times 10^{33}\ \text{yr}$$

SPACE DIMENSIONS AND UNIFICATION



Minkowski recognized special relativistic invariance of Maxwell's eqs $\Rightarrow$ connection between unification of forces and number of dimensions

$$\begin{cases} \vec{\nabla}\cdot\vec{E} = \rho & \vec{\nabla}\times\vec{E} = -\dfrac{\partial\vec{B}}{\partial t} \\ \vec{\nabla}\cdot\vec{B} = 0 & \vec{\nabla}\times\vec{B} = \dfrac{\partial\vec{E}}{\partial t} + \vec{J} \end{cases}$$

Electric & magnetic forces unified in 4D space time

$$\text{space - time} \quad t,\vec{x} \quad \rightarrow \quad x^{\mu} = (t,\vec{x})$$

$$\text{EM potentials} \quad \vec{E} = -\vec{\nabla}\phi - \frac{\partial\vec{A}}{\partial t}, \vec{B} = \vec{\nabla}\times\vec{A} \quad \rightarrow \quad A^{\mu} = (\phi,\vec{A})$$

$$\text{EM fields} \quad \vec{E},\vec{B} \quad \rightarrow \quad F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{pmatrix}$$

$$\text{current} \quad \rho,\vec{\mathrm{J}} \quad \rightarrow \quad \mathrm{J}_{\mu} = (\rho,\vec{\mathrm{J}})$$

$$\text{Maxwell's eqs} \quad \rightarrow \quad \partial_{\mu}F^{\mu\nu} = J^{\nu}$$



Next step:

UNIFICATION OF EM & GRAVITY

$\Rightarrow$ New dimensions?

1912: Gunnar Nordström proposes gravity theory with scalar field coupled to $T_{\mu}{}^{\mu}$

1914: he introduces a 5-dim A_{μ} to describe both EM & gravity

1919: mathematician Theodor Kaluza writes a 5-dim theory for EM & gravity. Sends it to Einstein who suggests publication 2 years later

1926: Oskar Klein rediscovers the theory, gives a geometrical interpretation and finds charge quantization

In the ‘80s the theory, known as Kaluza-Klein becomes popular with supergravity and strings

ELECTROMAGNETISM (QED)

Gauge principle: symmetry determines interactions

$$S_{Dirac} = \int d^4x\, \bar{\psi}\left(i\gamma^\mu \partial_\mu - m\right)\psi$$

Invariant under global $\psi \to e^{i\Lambda}\psi$

Under local transformations $\begin{cases} \psi \to e^{i\Lambda(x)}\psi \\ \partial_\mu \psi \to e^{i\Lambda(x)}\left(\partial_\mu \psi + i\psi\,\partial_\mu \Lambda\right) \end{cases}$

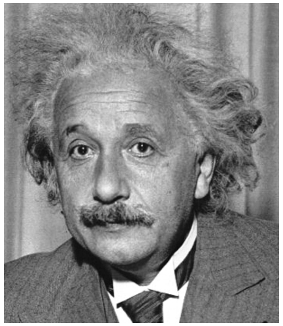
Action invariant under local gauge transformations

$$S_{QED} = \int d^4x \left[\bar{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right]$$

$$D_\mu = \partial_\mu - iA_\mu \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

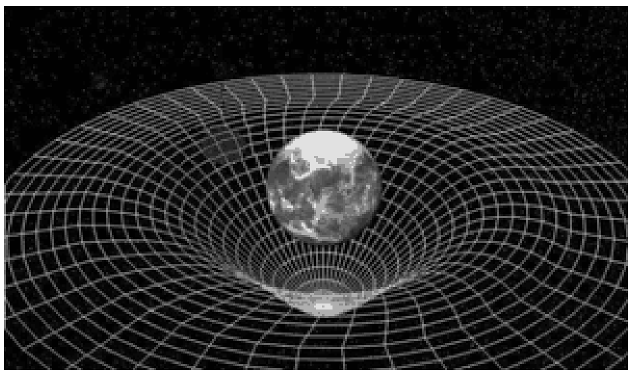
A_μ transforms as $A_\mu \to A_\mu + \partial_\mu \Lambda$

dynamical variable (photon)



GRAVITY

In General Relativity, metric $g_{\mu\nu}$ (4X4 symmetric tensor) dynamical variable describing space geometry (graviton)



$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

Dynamics described by Einstein action

$$S_G = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g}\, R(g)$$

- G_N Newton's constant
- R curvature (function of the metric)

Consider GR in 5-dim

$$\hat{S}_G = \frac{1}{16\pi \hat{G}_N} \int d^5x \sqrt{-\hat{g}}\, R(\hat{g})$$

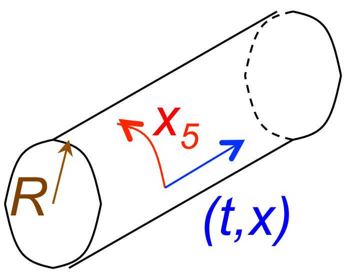
Choose

$$\hat{g}_{MN}(\hat{x}) = \begin{pmatrix} g_{\mu\nu} + \kappa^2 \phi A_\mu A_\nu & \kappa \phi A_\mu \\ \kappa \phi A_\nu & \phi \end{pmatrix} (\hat{x})$$

Dynamical fields

$$\hat{g}_{MN} \quad \Leftrightarrow \quad g_{\mu\nu}, A_\mu, \phi$$

Assume space is $M_4 \times S_1$



- First considered as a mathematical trick
- It may have physical meaning

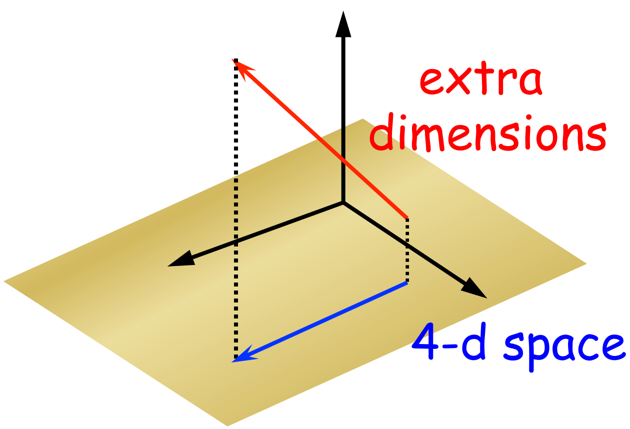
Extra dim is periodic or "compactified" $x_5 + 2\pi R = x_5$

All fields can be expanded in Fourier modes

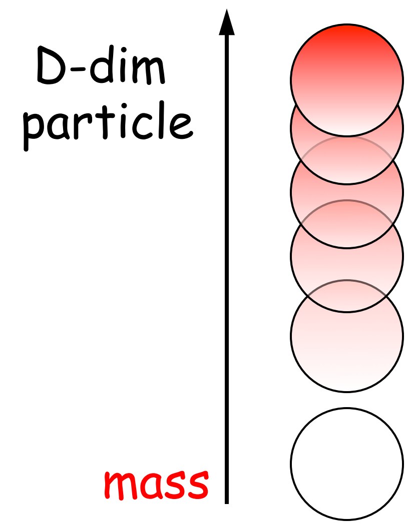
$$\varphi(\hat{x}) = \sum_{n=-\infty}^{+\infty} \frac{\varphi^{(n)}(x)}{\sqrt{2\pi R}} \exp\left(i\,\frac{n\,x_5}{R}\right)$$

5-dim field $\Leftrightarrow$ set of 4-dim fields: $\varphi^{(n)}(x)$ Kaluza-Klein modes

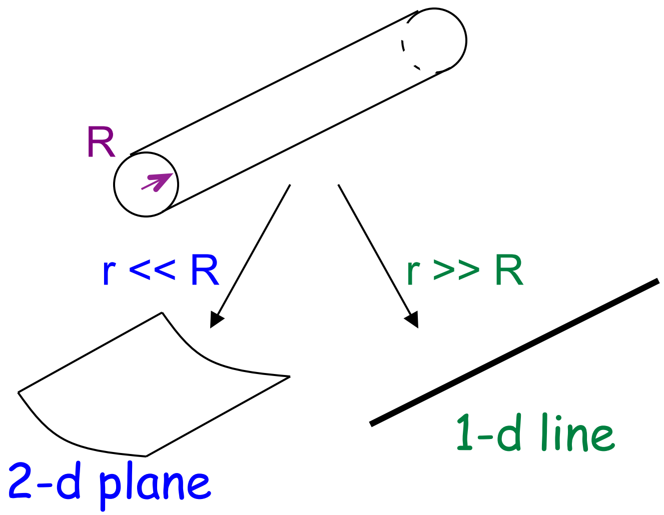
Each $\varphi^{(n)}$ has a fixed momentum $p_5 = n/R$ along 5th dim



$$E^2 = \vec{p}^{\,2} + \underbrace{p^2_{extra} + m^2}_{\text{KK mass}}$$



From KK mass spectrum we can measure the geometry of extra dimensions



Suppose typical energy << $1/R \Rightarrow$ only zero-modes can be excited

Expand S_G keeping only zero-modes and setting ϕ=1

$$\hat{S}_G(\hat{g}_{MN}) = S_G({g^{(0)}}_{\mu\nu}) + S_{EM}({A^{(0)}}_{\mu}) \begin{cases} S_G(g) = \dfrac{1}{16\pi G_N} \displaystyle\int d^4x \sqrt{-g}\, R(g) \\ S_{EM}(A) = -\dfrac{1}{4} \displaystyle\int d^4x\, F_{\mu\nu} F^{\mu\nu} \end{cases}$$

To obtain correct normalization:

$$S_G \;\to\; \frac{1}{G_N} = \frac{\int dx_5}{\hat{G}_N} = \frac{2\pi R}{\hat{G}_N}$$

$$S_{EM} \;\to\; \kappa = \sqrt{16\pi G_N}$$

Gravity & EM unified in higher-dim space: MIRACLE?

Gauge transformation has a geometrical meaning

$$d\hat{s}^2 = \hat{g}_{MN}(\hat{x})\, d\hat{x}^M\, d\hat{x}^N \qquad \hat{g}_{MN}(\hat{x}) = \begin{pmatrix} g_{\mu\nu} + \kappa^2 \phi A_\mu A_\nu & \kappa\,\phi\, A_\mu \\ \kappa\,\phi\, A_\nu & \phi \end{pmatrix}(\hat{x})$$

Keep only zero-modes:

$$d\hat{s}^2 = g^{(0)}{}_{\mu\nu}\, dx^\mu\, dx^\nu + \phi^{(0)}\left(dx^5 + \kappa\, A^{(0)}{}_\mu\, dx^\mu\right)^2$$

Invariant under local $\begin{cases} x^5 \to x^5 - \kappa\,\Lambda \\ A^{(0)}{}_\mu \to A^{(0)}{}_\mu + \partial_\mu \Lambda \end{cases}$ (where g and ϕ do not transform)

- Gauge transformation is balanced by a shift in 5th dimension
- EM Lagrangian uniquely determined by gauge invariance

CHARGE QUANTIZATION

Matter EM couplings fixed by 5-dim GR

Consider scalar field φ

$$S = \int d^5\hat{x}\sqrt{-\hat{g}}\,\hat{g}^{MN}\partial_M\varphi\,\partial_N\varphi$$

Expand in 4-D KK modes:

$$S = \underbrace{\int dx_5}_{2\pi R}\sum_n \int d^4x\sqrt{-g^{(0)}}\left[\left|\left(\partial^\mu - i\frac{n\kappa}{R}A^{(0)\mu}\right)\varphi^{(n)}\right|^2 - \frac{n^2}{R^2}\frac{\varphi^{(n)2}}{\phi}\right]$$

Each KK mode *n* has: mass *n/R* charge *nκ/R*

- charge quantization
- determination of fine-structure constant

$$\alpha = \frac{\kappa^2}{4\pi R^2} = \frac{4G_N}{R^2} \quad\Rightarrow\quad R = \sqrt{\frac{4G_N}{\alpha}} \approx 4\times 10^{-31}\,\mathrm{m} = \left(5\times 10^{17}\,\mathrm{GeV}\right)^{-1}$$

- new dynamics open up at Planckian distances

Not a theory of the real world

- ϕ=1 not consistent (ϕ dynamical field leads to inconsistencies: *e.g.* $F^{(0)}_{\mu\nu}F^{(0)\mu\nu}=0$ from eqs of motion)
- Charged states have masses of order M_{Pl}
- Gauge group must be non-abelian (more dimensions?)

Nevertheless

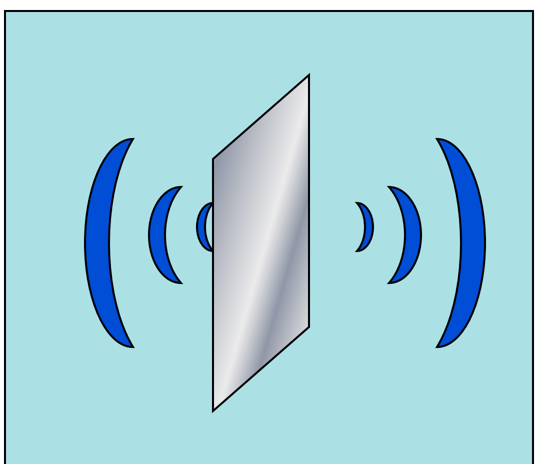
- Interesting attempt to unify gravity and gauge interactions
- Geometrical meaning of gauge interactions
- Useful in the context of modern superstring theory
- Relevant for the hierarchy problem?

Usual approach: fundamental theory at M_{Pl}, while Λ_W is a derived quantity

Alternative: Λ_W is fundamental scale, while M_{Pl} is a derived effect

New approach requires
- extra spatial dimensions
- confinement of matter on subspaces

Natural setting in string theory $\Rightarrow$ Localization of gauge theories on defects (D-branes: end points of open strings)



We are confined in a 4-dim world, which is embedded in a higher-dim space where gravity can propagate

COMPUTE NEWTON CONSTANT

Einstein action in D dimensions

$$S_E^D = \frac{1}{16\pi\,\hat{G}_N}\int d^D x\sqrt{-\hat{g}}\,R(\hat{g})$$

Assume space $R^4 \times S^{D-4}$: $g_{\mu\nu}$ doesn't depend on extra coordinates

Effective action for $g_{\mu\nu}$

$$S_E = \frac{V_{D-4}}{16\pi\,\hat{G}_N}\int d^4 x\sqrt{-g}\,R(g)$$

$$\Rightarrow \frac{1}{G_N} = \frac{V_{D-4}}{\hat{G}_N}$$

$$\left.\begin{aligned} \hat{G}_N &= \frac{1}{M_D^{D-2}} \\ V_{D-4} &= R^{D-4} \end{aligned}\right\} \Rightarrow \boxed{M_{Pl} = M_D\left(RM_D\right)^{\frac{D-4}{2}}}$$

Suppose fundamental mass scale $M_D \sim \mathrm{TeV}$

$$M_{Pl} = M_D \left(RM_D\right)^{\frac{D-4}{2}}$$ very large if R is large (in units of M_D^{-1})

Radius of compactified space

$$R = \begin{cases} \left(5\times10^{-4}\ \mathrm{eV}\right)^{-1} \approx 0.4\ \mathrm{mm} & D-4=2 \\ \left(20\ \mathrm{keV}\right)^{-1} \approx 10^{-5}\ \mu\mathrm{m} & D-4=4 \\ \left(7\ \mathrm{MeV}\right)^{-1} \approx 30\ \mathrm{fm} & D-4=6 \end{cases}$$

- Smallness of G_N/G_F related to largeness of RM_D
- Gravity is weak because it is diluted in a large space (small overlap with branes)
- Need dynamical explanation for $RM_D \gg 1$

Gravitational interactions modified at small distances

$$F_N(r) = G_N \frac{m_1 m_2}{r^2} \qquad \text{at } \; r > R$$

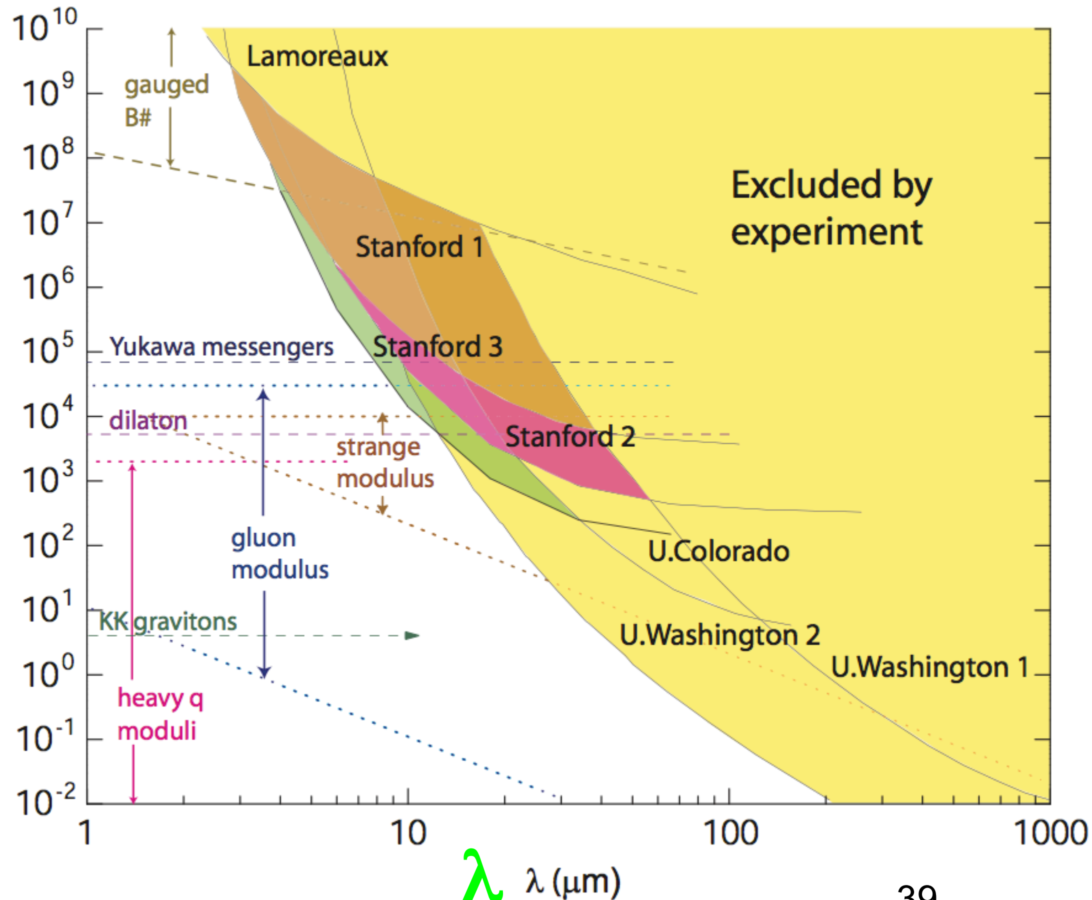
At $r < R$, space is $(3+\delta)$-dimensional $\qquad (\delta = D-4)$

$$F_N(r) = \hat{G}_N^{(4+\delta)} \frac{m_1 m_2}{r^{2+\delta}} =$$

$$= G_N R^{\delta} \frac{m_1 m_2}{r^{2+\delta}}$$

$$V(r) = -G_N \frac{m_1 m_2}{r} \left[1 + \alpha \exp(-r/\lambda) \right]$$

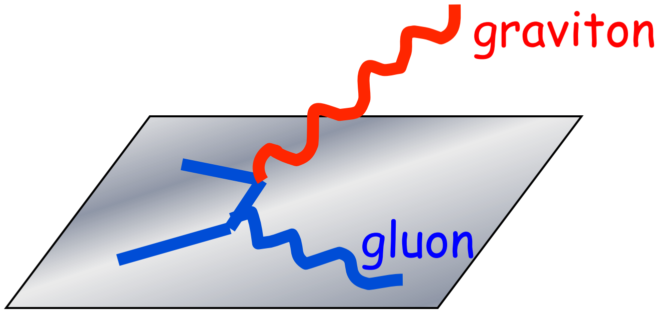
α

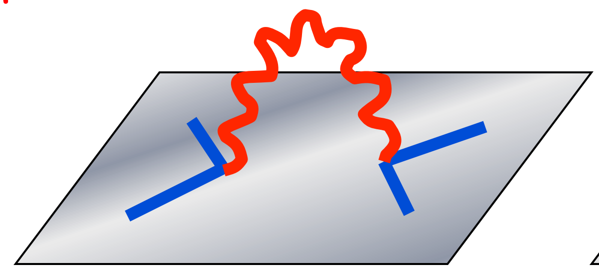


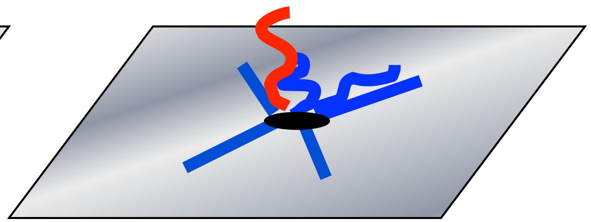
λ

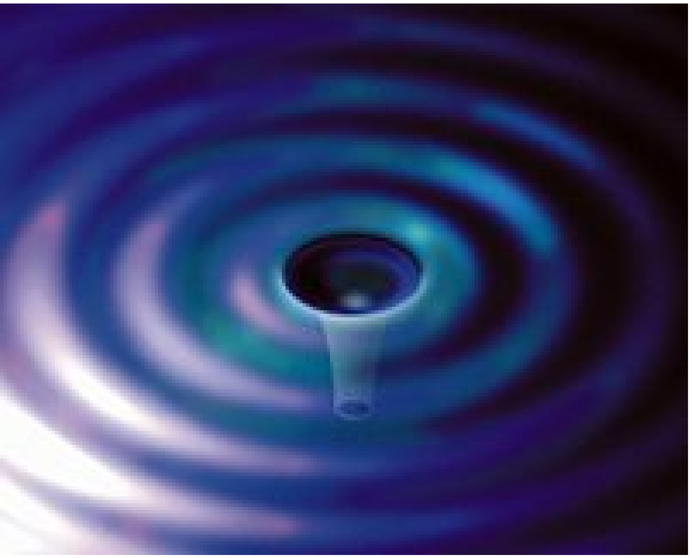
From SN emission and neutron-star heating: M_D>750 (35) TeV for δ=2(3)

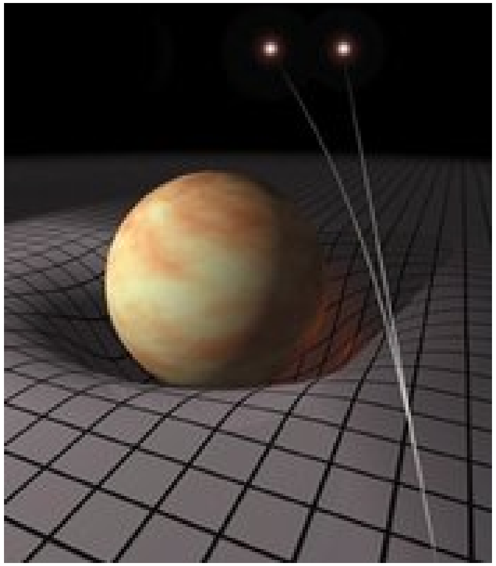
Probing gravity at the LHC?

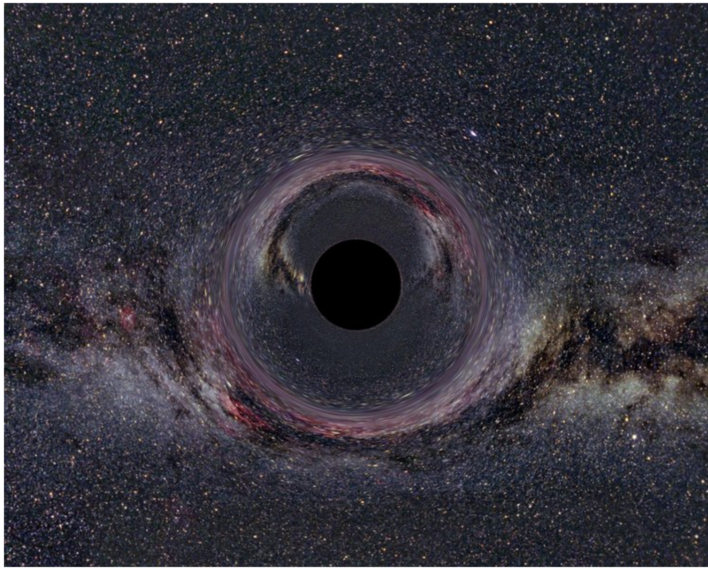












Gravitational wave
jet + $\not{E}_T$

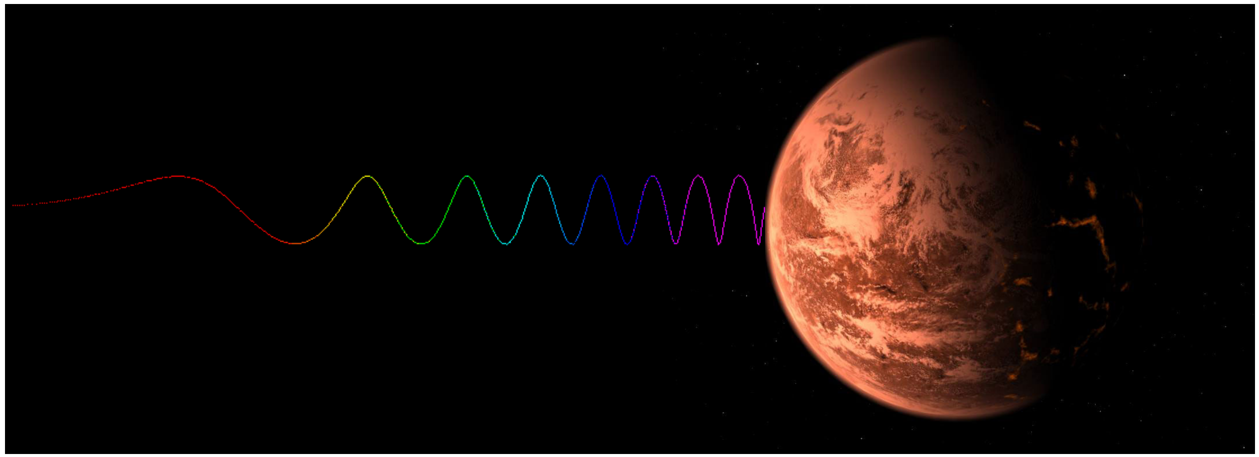
Gravitational deflection
dijet

Black hole
multiparticle event

Gravitational phenomena into collider arena

WARPED GRAVITY

A classical mechanism to make quanta softer



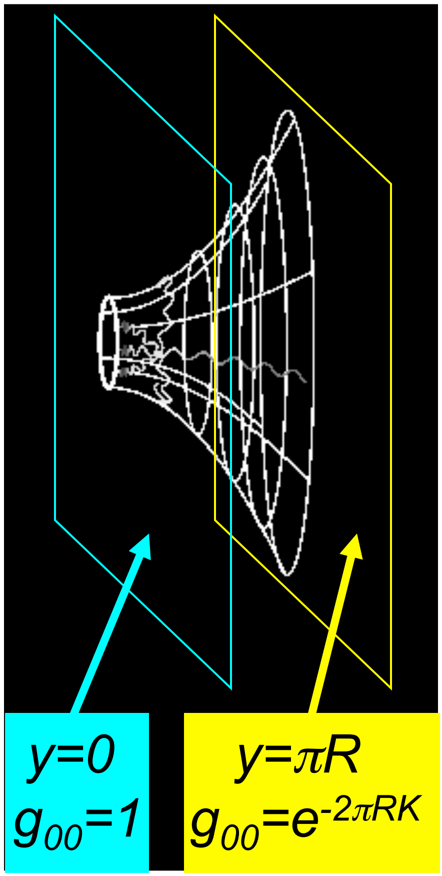
For time-indep. metrics with $g_{0\mu}=0 \Rightarrow E\,|g_{00}|^{1/2}$ conserved .

(proper time $d\tau^2 = g_{00}\, dt^2$)

Schwarzschild metric $g_{00} = 1 - \dfrac{2G_N M}{r} \quad \Rightarrow \quad \dfrac{E_{obs} - E_{em}}{E_{em}} = \sqrt{|g_{00}|} - 1 = -\dfrac{G_N M}{r_{em}}$

On non-trivial metrics, we see far-away objects as red-shifted

GRAVITATIONAL RED-SHIFT



$$ds^2 = e^{-2K|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$$

Masses on two branes related by

$$\frac{m_{\pi R}}{m_0} = e^{-\pi RK}$$

Same result can be obtained by integrating S_E over y

$$R \approx 10\,K^{-1} \quad \Rightarrow \quad \frac{m_{\pi R}}{m_0} \approx \frac{M_Z}{M_{GUT}}$$

PHYSICAL INTERPRETATION

- Gravitational field configuration is non-trivial
- Gravity concentrated at $y=0$, while our world confined at $y=\pi R$
- Small overlap $\Rightarrow$ weakness of gravity

WARPED GRAVITY AT COLLIDERS

- KK masses $m_n = Kx_n e^{-\pi RK}$ [x_n roots of $J_1(x)$] not equally spaced
- Characteristic mass $Ke^{-\pi RK} \sim$ TeV
- KK couplings

$$L = -T^{\mu\nu}\left(\frac{G^{(0)}_{\mu\nu}}{M_{Pl}} + \sum_{n=1}^{\infty}\frac{G^{(n)}_{\mu\nu}}{\Lambda_\pi}\right) \qquad \Lambda_\pi \equiv e^{-\pi RK} M_{Pl} \approx \mathrm{TeV}$$

- KK gravitons have large mass gap and are "strongly" coupled
- Clean signal at the LHC from $G \to l^+ l^-$